Karlsruhe Institute of Technology

# Understanding Small Separators in Road Networks


Master's Thesis of

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I declare that I have developed and written the enclosed thesis completely by myself. I have not used any other than the aids that I have mentioned. I have marked all parts of the thesis that I have included from referenced literature, either in their original wording or paraphrasing their contents. I have followed the by-laws to implement scientific integrity at KIT.

**Karlsruhe, July 1, 2025**

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(Samuel Born)

# Abstract

# Zusammenfassung

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## 3.1 Planarity

Road networks can be modeled as graphs that are nearly planar, meaning they can be embedded in the plane with only a small number of edge crossings. It is a well-known result in graph theory that planar graphs admit $\frac{2}{3}$-balanced separators of size $\mathcal{O}(\sqrt{n})$, where $n$ denotes the number of vertices.

A relevant inquiry is whether the near-planarity of road networks is a critical feature that influences their structural properties, or if the occasional non-planar elements are merely incidental and do not substantially affect the network's overall characteristics. This prompts the question of how the separator sizes of road networks are affected when they are transformed into strictly planar graphs, for instance, by altering edges to eliminate crossings.

To study road networks as planar graphs, we represent roads as linear segments between points. At each intersection of these segments, a new vertex is introduced, and the original edges are replaced accordingly. This process transforms the graph into a planar form by eliminating crossings.

There exist other algorihtms like bentley ottman or even linear time but it was easier to implement this one. not performance critical.

For efficient execution, we employ a spatial index that stores the bounding boxes of all edges. Under the assumptions of short edges this structure enables rapid identification of potential intersections by querying overlapping bounding boxes, followed by verification of actual crossings. Given that a single edge may intersect multiple times, we sort the intersection points along each edge and introduce new edges accordingly Pseudo-code for this planarization algorithm is provided in Algorithm 3.1.

**Algorithm 3.1:** Planarization algorithm

**Input:** Non-planar graph $G = (V, E, pos)$.
**Output:** Planarized version of $G$.

```
spatial_index ⟵ load(bounding_boxes(E))
crossings ⟵ {}
forall e in E do
    forall candidates c in spatial_index.query(e) do
        if c intersects e then
            crossings[e].append(c)
            crossings[c].append(e)
forall (e, crossed) in crossings do
    G.remove(e)
    vertices ⟵ get_intersection_vertices(e, crossed)
    sort vertices along e
    add_new_edges(e, vertices)
```

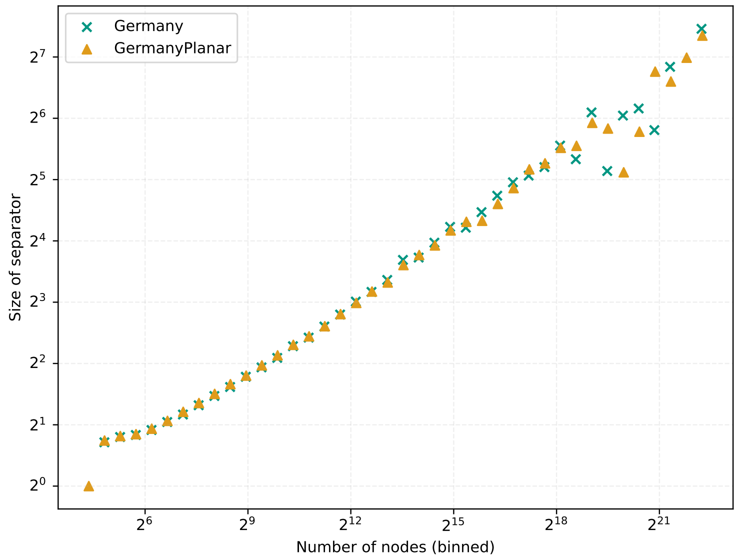



**Figure 3.1:** Comparison of separator sizes in the German road network: planar vs. non-planar.

We applied this planarization method to real-world road networks. The Karlsruhe network, with approximately 120,000 nodes and 150,000 edges, revealed around 2,500 intersections, while the Germany network, comprising about 5.8 million nodes and 7.2 million edges, exhibited approximately 100,000 intersections. These figures slightly exceed the $\mathcal{O}(\sqrt{n})$ intersection counts reported in prior studies but remain within a similar magnitude [**eppstein_linear-time_2010**]. These differences could be explained by the unoptimal linear assumption of edges and might be mitigated by using a more modeled road network like OpenStreetMap.

Analysis of separator sizes showed minimal variation post-planarization. We identified $\frac{2}{3}$-balanced separators of size approximately $\sqrt[3]{n}$, aligning with the values from non planar graphs. A comparison of the separator sizes in the planar and non-planar versions of the Germany network is depicted in Figure 3.1.

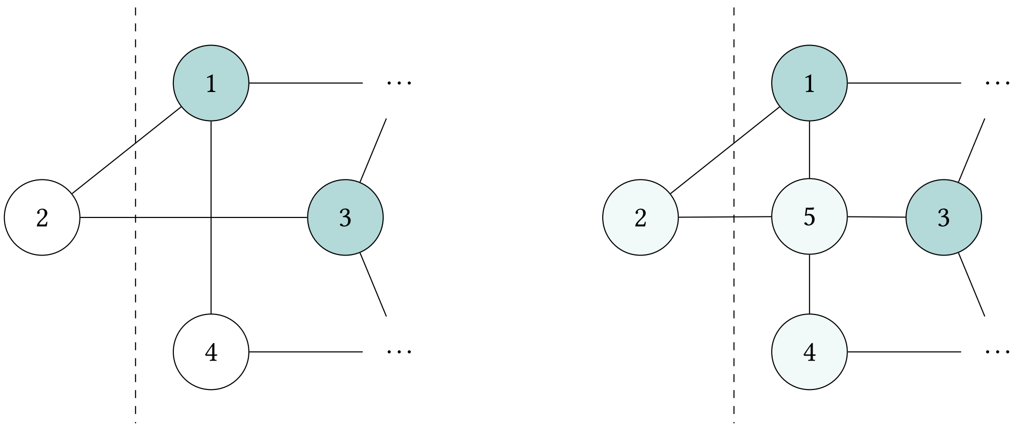
Our findings indicate that separators in non-planar road networks closely resemble those in their planarized versions. Frequently, non-planar separators are also separators in the planarized graph or can be adapted to planar ones with the addition of only a few nodes. This can be seen in Figure 3.4, which depicts a non-planar separator extended to be a separator in the planarized Karlsruhe network.

explain they can get larger and smaller

These findings highlight that the near-planar structure of road networks has minimal impact on separator size, suggesting that such networks can typically be analyzed as planar graphs.


**(a)** Separator in non-planar graph **(b)** Better separator in planarized graph

**Figure 3.2:** Example of a separator, where a better separator can be found in the planarized graph.




**(a)** Separator in non-planar graph

**(b)** If nodes 1 and 3 were forced to be part of the separator (because of other reasons), the separator of the planarized graph would be larger.

**Figure 3.3:** Example where the separator of the original graph is not a separator in the planarized graph.

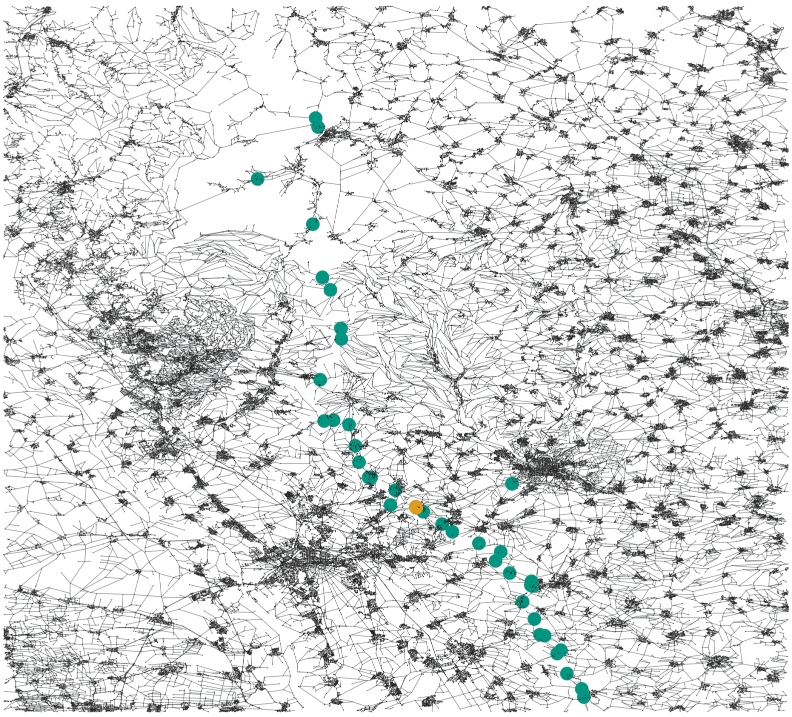


**Figure 3.4:** Example visualization of one possible top-level separator for Karlsruhe. Teal points represent the separator of the original graph, while orange points denote the additional nodes required to make it separator for the planarized version of Karlsruhe. Separators where computed with KaHIP.

# 4 Evaluation

# 5 Conclusion

## 5.1 Future Work