INSPEX equations

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1 Introduction

This document lays out the mathematical formulae of the functions available in INSPEX for fitting.

2 Equations

In INSPEX, we currently provide the following power law fitting functions:

- Single power law
- Double power law
- Triple power law
- Quadruple power law
- Quintuple power law

The single power law function is defined as such:

$$F = AE^{\delta} \tag{1}$$

where A is the amplitude, and δ is the spectral index.

The double power law function is defined over two domains; above and below break energy E_b:

$$F = \begin{cases} A_1 E^{\delta_1} & E \leq E_b \\ A_2 E^{\delta_2} & E \geq E_b \end{cases} \tag{2}$$

where A_1 and A_2 are the amplitudes for each component, and δ_1 and δ_2 are the spectral indices of the components. The amplitudes are fixed so that the function is continuous at $E = E_b$.

The triple power law function is defined over three domains; below break energy E_{b_1}, above break energy E_{b_1} and below break energy E_{b_2}, and above break energy E_{b_2}:

$$F = \begin{cases} A_1 E^{\delta_1} & E \leq E_{b_1} \\ A_2 E^{\delta_2} & E_{b_1} \leq E \leq E_{b_2} \\ A_3 E^{\delta_3} & E \geq E_{b_2} \end{cases} \tag{3}$$

where A_1, A_2, and A_3 are the amplitudes for each component, and δ_1, δ_2, and δ_3 are the spectral indices of the components. The amplitudes are fixed so that the function is continuous at $E = E_{b_1}$ and $E = E_{b_2}$.

The quadruple power law function is defined over four domains; below break energy E_{b_1}, above break energy E_{b_1} and below break energy E_{b_2}, above break energy E_{b_2} and below break energy E_{b_3}, and above break energy E_{b_3}:

$$F = \begin{cases} A_1 E^{\delta_1} & E \leq E_{b_1} \\ A_2 E^{\delta_2} & E_{b_1} \leq E \leq E_{b_2} \\ A_3 E^{\delta_3} & E_{b_2} \leq E \leq E_{b_3} \\ A_4 E^{\delta_4} & E \geq E_{b_3} \end{cases} \tag{4}$$

where A_1, A_2, A_3, and A_4 are the amplitudes for each component, and δ_1, δ_2, δ_3, and δ_4 are the spectral indices of the components. The amplitudes are fixed so that the function is continuous at $E = E_{b_1}$, $E = E_{b_2}$, and $E = E_{b_3}$.

The quintuple power law function is defined over five domains; below break energy E_{b_1}, above break energy E_{b_1} and below break energy E_{b_2}, above break energy E_{b_2} and below break energy E_{b_3}, above break energy E_{b_3} and below break energy E_{b_4}, and above break energy E_{b_4}:

$$F = \begin{cases} A_1 E^{\delta_1} & E \leq E_{b_1} \\ A_2 E^{\delta_2} & E_{b_1} \leq E \leq E_{b_2} \\ A_3 E^{\delta_3} & E_{b_2} \leq E \leq E_{b_3} \\ A_4 E^{\delta_4} & E_{b_3} \leq E \leq E_{b_4} \\ A_5 E^{\delta_5} & E \geq E_{b_4} \end{cases} \tag{5}$$

where A_1, A_2, A_3, A_4, and A_5 are the amplitudes for each component, and δ_1, δ_2, δ_3, δ_4, and δ_5 are the spectral indices of the components. The amplitudes are fixed so that the function is continuous at $E = E_{b_1}$, $E = E_{b_2}$, $E = E_{b_3}$, and $E = E_{b_4}$.

In INSPEX, we currently provide the following thermal Maxwellian fitting functions:

- Single isothermal Maxwellian function
- Double isothermal Maxwellian function

These include a single isothermal Maxwellian function:

$$F = AEe^{\frac{-E}{k_B T}} \tag{6}$$

where A is an amplitude, T is the temperature of the originating plasma and k_B is the Boltzmann constant.

A pair of isothermal functions summed together is also included as an additional function for considering the contributions of hot, flaring plasma and the cooler corona/active region to the spectrum. It is formulated simply as

$$F = A_1 E e^{\frac{-E}{k_B T_1}} + A_2 E e^{\frac{-E}{k_B T_2}} \tag{7}$$

where A_1 and A_2 are amplitudes, T_1 and T_2 are the temperatures of the originating plasmas and k_B is the Boltzmann constant.

The double power law and single thermal component were combined to create a single function. The parameters were defined in such a way that the equation should be continuous between the two components, giving only a single amplitude for the whole combined function which then generates the other amplitudes according to their gradients under the condition that the components are equal at the point where they meet. This thus gives the form:

$$F = A_{therm} E e^{\frac{-E}{k_B T}} + \begin{cases} A_1 E^{\delta_1} & E \leq E_b \\ A_2 E^{\delta_2} & E \geq E_b \end{cases} \tag{8}$$

where A_{therm}, A_1 and A_2 are amplitudes, T is the temperature of the originating plasma, δ_1 and δ_2 are the spectral indices of the power law components, E_b is the spectral break energy, and k_B is the Boltzmann constant.

We include a form of the Kappa function:

$$F = A \frac{v_{th}^2}{m_i} \frac{n_i}{2\pi(\kappa w^2)^{3/2}} \frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)\Gamma(3/2)} (1 + \frac{v_{th}^2}{\kappa w^2})^{-(\kappa+1)}$$

(9)

where w is given by

$$w = \sqrt{\frac{(2\kappa-3)k_B T_k}{\kappa m_i}} \tag{10}$$

and v_{th} is the thermal velocity given by

$$v_{th} = \sqrt{\frac{2E}{m_i}} \tag{11}$$

A is an amplitude term, m_i is the mass of the species in question (fixed to electron mass by default), n_i is the number density of the plasma, $\Gamma(x)$ is the Gamma function, and κ is the spectral index which must take values above 3/2 for the function to be defined.

In addition to the physical functions, there is a Gaussian normal distribution. This has utility for fitting any regular perturbations present in the spectral shape, particularly when the true shape is unknown and an approximate shape is needed. Thus, it has no exact physical meaning, though that does not mean that physical information cannot be extracted if the user has context to the origin of the perturbation. It has the form:

$$F = A e^{\frac{-(E-E_0)^2}{2\sigma^2}} \tag{12}$$

where A is an amplitude parameter, E_0 is the energy at which the Gaussian is centred, and σ is the standard deviation.