University of Asia Pacific
Department of CSE, UAP

Course Title: Math II: Linear Algebra

Course Code: MTH 103

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Assignment-1

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1. a.

$$\begin{bmatrix} 1 & -1 & 2 & -1 \\ 2 & 1 & -2 & -2 \\ -1 & 2 & -4 & 1 \\ 3 & 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \\ 1 \\ -3 \end{bmatrix}$$

$[A|B]$

b. $$\left[\begin{array}{cccc:c} 1 & -1 & 2 & -1 & -1 \\ 2 & 1 & -2 & -2 & -2 \\ -1 & 2 & -4 & 1 & 1 \\ 3 & 0 & 0 & 3 & -3 \end{array}\right]$$

c. $$\left[\begin{array}{cccc:c} 1 & -1 & 2 & -1 & -1 \\ 2 & 1 & -2 & -2 & -2 \\ -1 & 2 & -4 & 1 & 1 \\ 3 & 0 & 0 & -3 & -3 \end{array}\right]$$

$$= \left[\begin{array}{cccc:c} 1 & -1 & 2 & -1 & -1 \\ 0 & 3 & -6 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \end{array}\right] \quad \begin{array}{l} r_2' = r_2 - 2r_1 \\ r_3' = r_1 + r_3 \\ r_4 = r_4 - 3r_1 \end{array}$$

$$= \left[\begin{array}{cccc:c} 1 & -1 & 2 & -1 & -1 \\ 0 & 3 & -6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \quad \begin{array}{l} r_3' = r_2 - 3r_3 \\ r_4 = r_2 - r_3 \end{array}$$

Infinitely many solutions.

d. $\begin{vmatrix} 1 & -1 & 2 & -1 \\ 2 & 1 & -2 & -2 \\ -1 & 2 & -4 & 1 \\ 3 & 0 & 0 & -3 \end{vmatrix}$

$= -3 \begin{vmatrix} -1 & 2 & -1 \\ 1 & -2 & -2 \\ 2 & -4 & -1 \end{vmatrix} - 3 \begin{vmatrix} 1 & -1 & 2 \\ 2 & 1 & -2 \\ -1 & 2 & -4 \end{vmatrix}$

$= -3 \{ -1(-2-8) - 2(1+4) - 1(-4+4) \}$

$- 3 \{ 1(-4+4) + 1(-8-2) + 2(4+1) \}$

$= -3(10-10) - 3(-10+10)$

$= 0$

Inversion is not possible.

e. Inverse doesn't exist

2. $2\sin\alpha - \cos\beta + 3\tan\gamma = 3$

$4\sin\alpha + 2\cos\beta - 2\tan\gamma = 2$

$6\sin\alpha - 3\cos\beta + \tan\gamma = 9$

$$\left[\begin{array}{ccc:c} 2 & -1 & 3 & 3 \\ 4 & 2 & -2 & 2 \\ 6 & -3 & 1 & 9 \end{array}\right]$$

$$\left[\begin{array}{ccc:c} 1 & -\frac{1}{2} & \frac{3}{2} & \frac{3}{2} \\ 4 & 2 & -2 & 2 \\ 6 & -3 & 1 & 9 \end{array}\right]$$

$$\left[\begin{array}{ccc:c} 1 & -\frac{1}{2} & \frac{3}{2} & \frac{3}{2} \\ 0 & -4 & 8 & 4 \\ 0 & 0 & 8 & 0 \end{array}\right]$$

$\tan\gamma = 0 \; ; \; \gamma = 0$

$-4\cos\beta = 4$

$\cos\beta = -1$

$\beta = \pi$

$\sin\alpha - \frac{1}{2}\cos\beta + \frac{3}{2}\tan\gamma = \frac{3}{2}$

$\sin\alpha + 0.5 = \frac{3}{2}$

$\sin\alpha = \frac{3}{2} - \frac{1}{2}$

$\sin\alpha = 1 \; ; \; \alpha = \frac{\pi}{2}$ $\quad \therefore (\alpha, \beta, \gamma) = \left(\frac{\pi}{2}, \pi, 0\right)$

3. The given eq^n

$$ax^2 + ay^2 + bx + cy + d = 0$$

for $(-2, 7)$

$$4a + 49a - 2b + 7c + d = 0$$

$$\Rightarrow 53a - 2b + 7c + d = 0$$

for $(-4, 5)$

$$16a + 25a - 4b + 5c + d = 0$$

$$\Rightarrow 41a - 4b + 5c + d = 0$$

for $(4, -3)$

$$16a + 9a + 4b - 3c + d = 0$$

$$\Rightarrow 25a + 4b - 3c + d = 0$$

4. $$\begin{bmatrix} Y \\ I \\ Q \end{bmatrix} = \begin{bmatrix} .299 & .587 & .114 \\ .596 & -.275 & -.321 \\ .212 & -.523 & .311 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

b A X

$$\left[\begin{array}{ccc|ccc} .299 & .587 & .114 & 1 & 0 & 0 \\ .596 & -.275 & -.321 & 0 & 1 & 0 \\ .212 & -.523 & -.311 & 0 & 0 & 1 \end{array}\right]$$

$$|A| = 0.299(-0.082) - .587(-0.117) + .114(-0.263)$$
$$= 0.0153$$

$C_{11} = -0.0823$ $C_{12} = 0.1173$ $C_{13} = -0.253$

$C_{21} = 0.123$ $C_{22} = -0.117$ $C_{23} = 0.281$

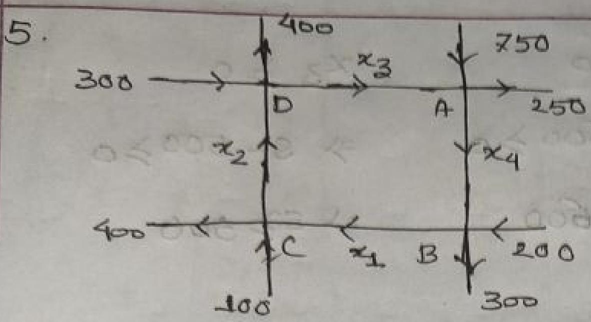
$C_{31} = -0.157$ $C_{32} = 0.164$ $C_{33} = -0.432$

$$\text{Adj}|A| = \begin{bmatrix} -0.0823 & 0.1173 & -0.253 \\ 0.123 & -0.117 & 0.281 \\ -0.157 & 0.164 & -0.432 \end{bmatrix}^T$$

$$= \begin{bmatrix} -0.0823 & 0.123 & -0.157 \\ 0.1173 & -0.117 & 0.164 \\ -0.253 & 0.164 & -0.432 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{Adj}$$

5.



Flowing in = Flowing Out

Node A ⇒ $x^3 + 750 = x_4 + 250$

Node B ⇒ $x_4 + 200 = x_1 + 300$

Node C ⇒ $x_1 + 100 = x_2 + 400$

Node D ⇒ $x_2 + 300 = x^3 + 400$

$x^3 - x^4 = -500$

⇒ $x_4 - x_1 = 100$

⇒ $x_1 - x_2 = 300$

⇒ $x^2 - x_3 = 100$

Let, $x_4 = S$

$x_3 - S = -500$

$x_3 = S - 500$

$x_1 - S = -100$

$x_1 = -100 + S$; $x_1 = S - 100$

$x_2 - S - 500 = 100$

$x_2 = S + 600$

$x_1 = S - 100$, $x_2 = S - 500$

$x_3 = S - 500$; $x_4 = S$

(c) $x_1 \geq 0$ $\quad$ $x_2 \geq 0$ $\quad$ $x_3 \geq 0$

$\Rightarrow \cancel{170 + s \geq 0}$ $\quad$ $\Rightarrow s - 600 \geq 0$ $\quad$ $\Rightarrow s - 500 \geq 0$

$\cancel{\Rightarrow s \geq 170}$ $\quad$ $\Rightarrow s \geq 600$ $\quad$ $\Rightarrow s \geq 500$

$\Rightarrow s - 100 \geq 0$

$\Rightarrow s \geq 100$

From A to B minimum flow is - 100 vehicles per hour.

6. $(v_1, v_2, v_3) = \{x(1, 1, 2) + y(1, 0, 1) + z(2, 1, 3)\}$

$$x + y + 2z = v_1$$
$$x + \quad + z = v_2$$
$$2x + y + 3z = v_3$$

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 3 \end{bmatrix}$$

$|A| = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 3 \end{vmatrix} = 0$ doesn't span linearly dependent

$v_1 = (2, -1, 3)$, $v_2 = (4, 1, 2)$, $v_3 = (8, -1, 8)$

$(v_1, v_2, v_3) = x(2, -1 + 3) + y(4, 1, 2) + z(8, -1, 8)$

$= (2x + 4y + 8z, -x + y - z, 3x + 2y + 8z)$

$$\begin{bmatrix} 2 & 4 & 8 \\ -1 & 1 & -1 \\ 3 & 2 & 8 \end{bmatrix}$$

$$= \begin{vmatrix} 2 & 4 & 8 \\ -1 & 1 & -1 \\ 3 & 2 & 8 \end{vmatrix}$$

$= 0$, dosen't span and linieanly dependent.

7. a) $a_{ij} = i + j$

$$\begin{bmatrix} 1+1 & 1+2 & 1+3 & 1+4 \\ 2+1 & 2+2 & 2+3 & 2+4 \\ 3+1 & 3+2 & 3+3 & 3+4 \\ 4+1 & 4+2 & 4+3 & 4+4 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \\ 5 & 6 & 7 & 8 \end{bmatrix}$$

b) $a_{ij} = i^{j-1}$

$$\begin{bmatrix} 1^0 & 1^1 & 1^2 & 1^3 \\ 2^0 & 2^1 & 2^2 & 2^3 \\ 3^0 & 3^1 & 3^2 & 3^3 \\ 4^0 & 4^1 & 4^2 & 4^3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \\ 1 & 4 & 16 & 64 \end{bmatrix}$$

c) $a_{ij} = \begin{cases} 1 & \text{if } |i-j| > 1 \\ -1 & \text{if } |i-j| \le 1 \end{cases}$

$$\begin{bmatrix} -1 & -1 & 1 & 1 \\ -1 & -1 & -1 & 1 \\ 1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 \end{bmatrix}$$

8. (a) $AX = IX$ or $AX = X \Rightarrow (A - I)x = 0$

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 2 & 2 & -2 \\ 3 & 1 & 1 \end{bmatrix} \Rightarrow (A-I)x = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & -2 \\ 3 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{aligned} \Rightarrow x_1 + x_2 + 2x_3 &= 0 \\ 2x_1 + x_2 - 2x_3 &= 0 \end{aligned}\right\} \Rightarrow 3x_1 + 2x_2 = 0 \Rightarrow x_2 = 0$$

$$3x_1 + x_2 = 0 \qquad \Rightarrow x_1 = 0$$

$$\Rightarrow x_3 = 0$$

$$\Rightarrow x = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

(b) $AX = 4x \Rightarrow (A - 4I)X = 0$

$$(A - 4I)x = \begin{pmatrix} -2 & 1 & 2 \\ 2 & -2 & -2 \\ 3 & 1 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\left.\begin{aligned} \Rightarrow -2x_1 + x_2 + 2x_3 &= 0 \\ 2x_1 - 2x_2 - 2x_3 &= 0 \end{aligned}\right] \Rightarrow -x_2 = 0 \Rightarrow x_1 - x_3 = 0$$

$$3x_1 + x_2 - 3x_3 = 0 \qquad \Rightarrow x_1 = x_3 = t \text{ (say)}$$

$$\Rightarrow x = \begin{bmatrix} t \\ 0 \\ t \end{bmatrix}, \ t \in R$$

9. $x+2y-3z=4$

$3x-y+5z=2$

$4x+y+(a^2-14)z=a+2$

$$=\left[\begin{array}{ccc:c} 1 & 2 & -3 & 4 \\ 3 & -1 & 5 & 2 \\ 4 & 1 & a^2-14 & a+2 \end{array}\right] = \left[\begin{array}{ccc:c} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 0 & -7 & a^2-2 & a-14 \end{array}\right] \begin{array}{l} r_2' = r_2-3r_1 \\ r_3' = r_3-4r_1 \end{array}$$

$$=\left[\begin{array}{ccc:c} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 0 & 0 & a^2-16 & a-4 \end{array}\right] \quad r_3' = r_3-r_2$$

No solution if

$a^2-16=0$, $a-4 \neq 0$

$\Rightarrow a=4$ $\quad a \neq 4$

infinitely many solution if

$a^2-16=0$ $\quad a-4=0$

$\Rightarrow a=4$ $\quad \Rightarrow a=4$

only solution if

$a^2-16=a-4$

$\Rightarrow a^2-a-20=0$

$\Rightarrow a^2-5a+4a-20=0$

$\Rightarrow a(a-5)+4(a-5)=0$

$a=-4, 5$