$$R = R_p^{1/2} r$$

Outer Solution

$$O(\alpha): u_1^{(0)}$$

$$O\left(\frac{\alpha}{r^2}\right) \to O\left(\frac{\alpha R_p}{R^2}\right)$$

$$O(\alpha Rp): u_1^{(1)} \longrightarrow O(\alpha R_p) \to O(\alpha R_p) \longrightarrow O(\alpha Rp): U_1^{(1)}$$

$$O(\alpha^2): u_2^{(0)}$$

$$O(\alpha^2 R_p r) \to O\left(\frac{\alpha R_p R}{R_c^{1/2}}\right)$$

Not used in calculation

$$O(\alpha^2 \text{Rp}): u_2^{(1)}$$

$$O\left(\frac{\alpha^2}{r^3}\right) \to O\left(\frac{\alpha^2 R_p^{3/2}}{R^3}\right)$$

$$O(\alpha^2 R_p^{3/2})$$
: $U_2^{(3/2)}$