PC1201 Mechanics

Miscellaneous

Math

Standards

- Use of SI units is encouraged (m, s, kg, A)
- Precision of answer should be the same as the **least** significant value provided in question

Angles An angle is conventionally defined by going counter-clockwise from the positive x-axis.

Vectors Consider a vector $\vec{v}$, making an angle θ with respect to the positive x-axis.

- Magnitude: $|\vec{v}|$ or $\sqrt{(\vec{v}_x)^2 + (\vec{v}_y)^2}$
- Direction θ or $\tan\theta = \dfrac{\vec{v}_y}{\vec{v}_x}$ (take care of sign of tangent appropriately)
- Usually 2 components in PC1201
- Addition: attach tails, sum up each component
- Subtraction: adding -1 times
- Scalar multiplication: scales vector

Length of vector components Consider a vector $\vec{v}$, making an angle θ with respect to the positive x-axis.

- Hypothenuse: $|\vec{v}|$
- Adjacent: $|\vec{v}|\cos\theta$
- Opposite: $|\vec{v}|\sin\theta$

Problem solving

Tilt axes Do not always need to choose "standard" cartesian axis with x direction parallel to ground.

Decompose into axes Sometimes, it is possible to decompose an event into perpendicular components and we should do so.

Kinematics

Ticker tape Dots are drawn every fixed interval.

- Larger gap $\Rightarrow$ faster speed
- Increasing gap size $\Rightarrow$ positive acceleration

Scalar vs vector

Scalar	Vector
Distance	Displacement
Speed	Velocity
-	Acceleration

Graphs We consider position-time (x-t), velocity-time (v-t), acceleration-time (a-t). Relations (due to calculus)

- Gradient of x-t is velocity
- Gradient of v-t is acceleration
- Area under v-t is displacement

SUVAT

$$s = ut + \frac{1}{2}at^2$$
$$v = u + at$$
$$v^2 = u^2 + 2as$$
$$s = \frac{1}{2}(u+v)t$$

Note: we generally deal with constant acceleration, so if there are sudden changes in acceleration, we deal with the motion separately.

Projectile motion

Trajectory Parabola, only depending on the initial velocity $\vec{u}$.

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

- Velocity is tangent to trajectory.
- $\vec{v}_y = 0$ at top of trajectory.
- $\vec{v}_x$ is constant throughout trajectory.

Height and range

$$H = \frac{u^2\sin^2\theta}{2g}$$
$$R = \frac{u^2\sin 2\theta}{g}$$

Hence, $\theta = 90\,\text{deg}$ gives max height, while $\theta = 45\,\text{deg}$ gives max range.

Circular motion

Properties

- Velocity is not constant, as direction is always changing
- Thus an acceleration exists:

$$a_c = \frac{|\vec{v}|^2}{r}$$

This is known as centripetal acceleration, and its direction is perpendicular to $\vec{v}$.

Non-uniform

1. Implies speed is changing
2. Implies net acceleration is not perpendicular to velocity
3. $|\vec{v}|$ is not constant

Relative motion

Velocity is technically a relative quantity. We use the following notation:

$$\vec{v}_{\text{object}|\text{reference frame}}$$

We can compare relative velocities:

$$\vec{v}_{A|B} = \vec{v}_{A|O} - \vec{v}_{B|O} = \vec{v}_{A|O} + \vec{v}_{O|B}$$

Problem solving

List information Before trying a question, list all information.

- Number of unknowns determines number of equations to use
- Draw diagrams for understanding
- Name unknowns for easy reference
- Not all information is numerical (Starts from rest $\Rightarrow \vec{u} = 0$)

Dynamics

Newton can be expressed in base units:

$$\text{N} = \text{kg m s}^{-2}$$

Forces

Gravity $W = mg$

Tension A contact force that is same throughout the rope. The tension at which the rope breaks is called breaking tension.

Normal A contact force that acts perpendicular to the surface.

Friction A contact force that acts opposite to the motion, resisting

- Motion (Kinetic, f_k)
- Tendency to move (Static, f_s)

The general equation is:

$$f = \mu N$$

where μ is the relevant friction coefficient.

- Static friction is equal to the applied force until a certain $f_{s_{\max}} = \mu_s N$.
- Once the applied force exceeds this, the object starts moving, and kinetic friction is now present, instead of static friction, and now $f_k = \mu_k N$, where $\mu_k \leq \mu_s$.

Elastic Known as a restoring force, acting in the opposite direction of $\Delta\vec{x}$.

$$\vec{F} = -k\Delta\vec{x}$$

Circular motion

The net force that causes circular motion is known as centripetal force.

Orbital motion

- If we launch a projectile with a high enough speed, it can escape gravity.
- If we get the speed right, the object will be so fast that as it falls towards the earth, the earth starts to curve away.
- If it is too slow, the object will eventually return to Earth.
- If it is too fast, the object escapes Earth's gravity.
- An object in such a circular orbit is in constant free fall, only being acted on by gravity. Hence,

$$mg = \frac{mv^2}{r}$$
$$v = \sqrt{rg}$$

Apparent weight

Normal force Our apparent weight changes if $N \neq mg$.

- If $N < mg$, we feel lighter (low apparent weight)
- If $N > mg$, we feel heavier (high apparent weight)

We can experience this in elevators, vertical circular motion, orbitting satellites, etc.

Problem solving

- Draw sketch of entire system
- Isolate SINGLE body to draw FBD, with forces acting ON the body
- Action-reaction pairs should NOT appear in same FBD
- Choose axis that aligns with net acceleration
- If object is accelerating, net force is nonzero

Work, Energy, Power

Energy

- Examples: Kinetic, Potential (Gravitational, Elastic, Electric), Thermal, Chemical
- Unit: Joules (J)
- Is a scalar
- Energy transformation - changes within the system
- Energy transfer - exchanges with external bodies outside system

Work

- Work is transfer of energy - done on a system by external force
- Is a scalar, but positive work should increase the total energy of the system
- No movement = no work done
- The equation is given by

$$W = F_{||}s = Fs\cos\theta$$

  where $F_{||}$ is the component of the force parallel to displacement.
- Work is positive if the force is acting in the same direction as displacement
- Work done is area under graph of $F - x$

Types of energy

Kinetic $W = \frac{1}{2}mv^2$

Potential energy

- Stored energy
- Allowed by conservative forces (where work done is dependent on position, and independent of path)
- Examples of conservative forces: Gravitational, elastic, electric (and magnetic) forces
- Derive formula by using

$$|W| = |F\Delta s|$$

- When conservative forces do positive work, potential energy of object decreases

Gravitational potential $\Delta U_g = mg\Delta h$

Elastic potential $U_s = \frac{1}{2}k(\Delta x)^2$

Conservation of Energy

- In a closed system, total energy is constant.
- In a open system, we consider only mechanical energy, ignoring heat.

Power

- Rate of energy transfer

$$P = \frac{\Delta E}{t} = \frac{W}{t}$$

- For kinematics, if object is moving at constant speed,

$$P = \frac{Fs}{t} = Fv$$

- Unit: Watt (W)

Problem solving

- Focus is on before and after, ignoring what happens in between
- Be aware if system is open or close

Momentum

- Momentum is a vector quantity that quantifies motion:

$$\vec{p} = m\vec{v}$$

- Unit: kg m s^{-1}

Conservation of Momentum

- In a closed system, momentum of the **total system** is conserved
- But momentum of individual objects in the system may change
- Can decompose into perpendicular axes

Types of collision

Elastic KE is conserved, objects separate.

Consider two objects with mass m_i, initial velocity u_i and final velocity v_i.

$$m_1\vec{u_1} + m_2\vec{u_2} = m_1\vec{v_1} + m_2\vec{v_2}$$
$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

By algebra,

$$\vec{v_2} - \vec{v_1} = -(\vec{u_2} - \vec{u_1})$$

Thus, the relative velocity after collision is same in magnitude, and opposite in direction.

Completely inelastic KE is not conserved, objects stick

Inelastic KE is not conserved, objects separate (occurs when one body splits into two)

Impulse

Momentum is not conserved in open systems, and the change in momentum is called impulse.

$$\vec{J} = \vec{F}_{\text{avg}}\Delta t$$

It is also the area under the F-t graph.