PC1201 E&M

Misc

Charges

Net charge An object is

- +vely charged if it has an excess of protons
- -vely charged if it has an excess of electrons
- Neutral otherwise
- has units C

Conservation of charge One cannot create/destroy charges; one can only transfer charges from one body to another

Interaction Like charges repel, unlike charges attract

L12 QC6

- Charges move because of a PD
- If two conductors are connected via a wire, charges redistribute equally only if the two conductors are identical.

Insulators

Definition Do not allow electrons to flow freely

Charging By friction molecular bonds are broken, allowing a neutral molecule to split into positive and negative parts

Conductors

Definition Allow electrons to flow freely

Polarization Charged object brought near to a conductor. Freely moving electrons will congregate accordingly to one side of the conductor

Charging by contact Touch a conductor with an initially charged object. Charges will spread through the conductor. The initially charged object gets discharged

Grounding Electrons will flow to the object that is "more positive"

Charging by induction Polarization with grounding on the opposite side

Electric force

- $\vec{F}_{ab}$ is the force on a due to b
- Is a vector, so can apply superposition

Coulomb's law

Magnitude For two charges q_1 and q_2 seperated by distance r:

$$\left|\vec{F}_{12}\right| = \left|\vec{F}_{21}\right| = \frac{k\,|q_1|\,|q_2|}{r^2}$$

Direction

- Along the line joining two charges
- Unlike charges attract; like charges repel

Superposition Electric force on a charge is the vector sum of the individual force due to all the other charges

Limitations

- Only applies to point charges, but objects may be modeled as point charges if the r is much larger than their size
- Only applies to static charges, but this module focuses on static charges

E-field

- Explains how a charge can be aware of the presence of another charge
- Force is exerted on a charged object, while field is exerted on a point in space
- has units $\mathrm{NC^{-1}}$ or $\mathrm{Vm^{-1}}$

E-field for a point charge

Magnitude We obtain magnitude by placing a test charge at the point we are interested in.

$$\left|\vec{E}\right| = \frac{\left|\vec{F}\right|}{q_{\text{test}}} = \frac{k\,|q_0|}{r^2}$$

where q_0 is the charge that created the e-field.

Direction Defined to be the direction of the force experienced by a positive q_{test}

	+ve q_{test}	-ve q_{test}
Field	Points away	Points towards
Force	Same dir. as field	Opposite dir. from field

Superposition E-field at a point is the vector sum of the individual fields due to all the charges

E-field lines

Exist at every point in space, just a representative sample

Magnitude Higher density of field lines $\Rightarrow$ stronger field

Direction

- E-field vector is tangent to e-field line
- Field lines do not cross
- Field lines start from +ve charges and end at -ve charges

E-field for sphere

Let R be the radius of the sphere. The following applies to:

- Thin insulating shell with uniform distribution
- Thin conducting shell (assume no charge nearby)
- Conducting sphere (assume no charge nearby)

Outside sphere Let Q be the total charge on the sphere. Let r be the distance to the centre of the sphere.

$$\left|\vec{E}\right| = \frac{k\,|Q|}{r^2}$$

Inside sphere Inside the sphere, there is no e-field due to symmetry

E-field for infinite sheet/plate

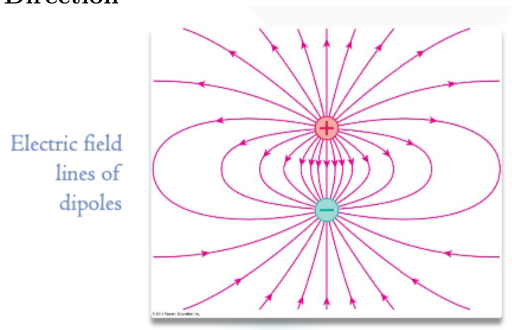
- We can approximate most large sheets/plates as infinite sheets/plates
- We use this setup for parallel plate capacitors

Magnitude Let Q be the total charge on the plate. Let A be the area of each plate

$$\left|\vec{E}\right| = \frac{|Q|}{2\varepsilon_0 A}$$

E-field for dipole

Direction



E-field in conductors

- We only consider conductors in electric equilibrium, so $\vec{E} = 0$
- Excess charges reside only on the surface, congregating at sharp points
- E-fields are perpendicular to surface

Misc

Dielectric breakdown When e-fields within insulators get too strong, it may fail to act as an insulator, turning into a conductor, allowing charges to flow

Screening in conductors When conductors are placed in an e-field, as mentioned previously, $\vec{E} = 0$, so any void in the conductor will also have $\vec{E} = 0$ (Faraday's cage / electrostatic shielding)

Electric potential energy

Interaction EPE for a point charge

Magnitude A charge q_1 will have PE if another charge q_2 is present

$$U_{q_1} = \frac{kq_1q_2}{r} = U_{q_2}$$

Alternative Based on the definition of electric potential later, we also have

$$\Delta U = q\Delta V$$

Zero point $U \to 0$ as $r \to \infty$, i.e. when there are no charges around

Sign

- Similar charges have positive U, while opposite charges have negative U.
- Not a vector, so has no direction

Superposition EPE due to multiple charges is the scalar sum of individual terms

Interaction EPE vs electric force

ΔU vs $\vec{F}_{\text{ext}}$ Work done is given by

$$W = \vec{F}_{\text{ext}} d \cos\theta$$

where d is usually positive in the direction of displacement, and θ is the angle between $\vec{F}_{\text{ext}}$ and d. We can also define work done as

$$W = \Delta U = U_f - U_i$$

Hence,

- $\theta = 0 \Rightarrow$ same dir. $\Rightarrow W = \vec{F}_{\text{ext}} d > 0 \Rightarrow \Delta U > 0$
- $\theta = 180 \Rightarrow$ diff dir. $\Rightarrow W = \vec{F}_{\text{ext}} d < 0 \Rightarrow \Delta U < 0$

Here, ΔU is the work done on the charge to bring it from initial point i to final point f.

ΔU vs Coulomb's force $\vec{F}$ In order to move the charged object at constant velocity, we have to oppose Coulomb's force, so

$$\vec{F} = -\vec{F}_{\text{ext}}$$

This case has a negative sign, so

- $\vec{F}$ and d same dir. $\Rightarrow \Delta U < 0$
- $\vec{F}$ and d diff dir. $\Rightarrow \Delta U > 0$

Configuration EPE

Work required to bring the charges from infinity to build this configuration

$$U = \sum_{i<j} \frac{kq_iq_j}{r_{ij}}$$

Electric potential

- Like e-field, electric potential explains how a charge can be aware of the presence of another charge
- But e-field is about force per charge while electric potential is about work done/energy per charge
- has units V

Electric potential for a point charge

Magnitude Similar to e-field, we obtain magnitude by placing a test charge at the point we are interested in.

$$V = \frac{U}{q_{\text{test}}} = \frac{kq_0}{r}$$

where q_0 is the charge that created the electric potential.

Zero point We can choose the zero point of V.

Superposition Electric potential at a point is the scalar sum of potential due to each charge

Electric potential for a sphere/shell

Let R be the radius of the sphere. The following applies to:

- Thin insulating shell with uniform distribution
- Thin conducting shell (assume no charge nearby)
- Conducting sphere (assume no charge nearby)

Outside sphere Let Q be the total charge on the sphere. Let r be the distance to the centre of the sphere.

$$V = \frac{kQ}{r}$$

Inside sphere Inside the sphere, potential is constant.

$$V = \frac{kQ}{R}$$

Equipotential surfaces

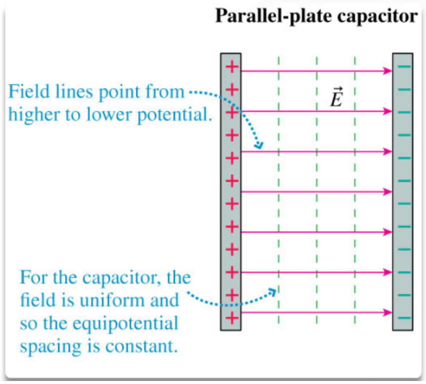
- Lines/surfaces of the same potential, so moving a charge along the line does not change EPE
- PD between each adjacent pair of equipotential lines is the same

Relationship to $\vec{E}$

- E-field points in direction of decreasing potential, so a PD causes an $\vec{E}$
- E-fields are perpendicular to equipotential lines
- If E-field is constant, then

$$\left|\vec{E}\right| = \frac{|\Delta V|}{d}$$

where d is the distance between the two points (parallel to $\vec{E}$), and ΔV is the PD between the two points.



Moving charge in a field

1. Charge q moving from A to B in a PD ΔV

$$\Delta U = q\Delta V$$

2. Charge q moving from A to B in a constant $\vec{E}$ field

$$\Delta V = \left|\vec{E}\right| d$$

Capacitance, current, resistance

Capacitor

Capacitance A capacitor stores electric charges, and consists of two equally and oppositely charged parts, that are some distance apart. The capacitance C is given as:

$$C = \frac{Q}{\Delta V} = \kappa \frac{\varepsilon_0 A}{d}$$

- Q is the amount of charge the capacitor can hold, given PD between the plates ΔV
- κ is the dielectric constant, 1 in air, others will be given
- RHS is always true, even if the capacitor is disconnected
- has units CV^{-1} or F

Electric field Proved in assignment 2:

$$\left|\vec{E}\right| = \frac{Q}{\varepsilon_0 A}$$

Energy Energy stored in a capacitor is equal to the work done to charge it

$$U = \frac{1}{2}Q\Delta V = \frac{Q^2}{2C} = \frac{1}{2}C(\Delta V)^2$$

Energy density The capacitor's energy is stored in the electric field between the plates.

$$U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}\frac{\varepsilon_0 A}{d}(Ed)^2 = \frac{1}{2}\varepsilon_0 E^2 (Ad)$$

In order to be independent of the dimensions of the capacitor, define $u = \dfrac{U}{Ad}$ to be the energy density, so we have

$$u = \frac{1}{2}\varepsilon_0 E^2$$

Current

- Rate of flow of charge

$$I = \frac{Q}{\Delta t}$$

- has units Cs^{-1} or A
- Direction of flow of positive charges
- Charges flow $\Rightarrow \Delta V$ exists $\Rightarrow$ electric field exists

EMF source, ε

- Maintains a constant PD by moving charges against the electric field, opposing Coulomb's force
- has units V

Power

$$P = \frac{U}{\Delta t} = \frac{q\Delta V}{\Delta t} = I\Delta V = I\varepsilon$$

Resistance

- For ohmic materials, which have constant resistance, Ohm's law holds true:

$$R = \frac{\Delta V}{I}$$

- Can assume the materials are ohmic unless otherwise stated
- has units Ω or VA^{-1}

Computing resistance for a material

$$R = \frac{\rho l}{A}$$

where ρ is the resistivity (given), l is the length of the conductor, A is the cross sectional area.

Power For ohmic resistors,

$$P = I^2 R = \frac{(\Delta V)^2}{R}$$

Power ratings in household appliances are based off a constant voltage (230V in Singapore)

DC Circuits

Battery Positive terminal is the longer line

Resistors

	Series	Parallel
PD	Sum	Same
Current	Same	Sum
Resistance	Sum	Reciprocal of (sum of reciprocals)

Capacitor

	Series	Parallel
PD	Sum	Same
Charge	Same	Sum
Capacitance	Reciprocal of (sum of reciprocals)	Sum

Kirchhoff

Parallel vs series

- Two elements are in parallel if any loop contains only those two elements
- Two elements are in series if every loop contains only both those elements

Junction rule charge in = charge out

Loop rule

- Loop rule: sum of PD along loop is 0
- Battery: Positive plate has higher potential than negative plate
- Resistor: Potential decreases in the direction of current (depends on the emf or the setup)

Magnetic field

- Is a vector, so can apply superposition
- has units T

Magnet

- Has "inherent magnetic moments" that are aligned.
- This is why dropping/heating will harm the magnet

Earth is a magnet

- The magnetic north pole is the geographic south pole of the earth
- A compass needle points north because the north pole of the compass needle is attracted to the south pole of the earth

Magnetic field lines

- Starts from north pole, ends at south pole
- Denser where field is stronger (near magnet)
- Never crosses

3D convention

- Cross means current into page, dot means current out of page
- Vertical plane (up/down, west/east): used in question paper
- Horizontal plane (north/south, west/east): surface of the earth
- Other plane (up/down, north/south): visualize earth's B-field aways away from the equator

Source of magnetic fields

Right hand grip rule (RHGR)

1. Point your right thumb in the direction of current
2. Wrap your fingers around the wire to indicate a circle
3. Your fingers curl in the direction of the B-field lines around the wire

Moving charge (in a wire)

- Magnitude is

$$\left|\vec{B}\right| = \frac{\mu_0 I}{2\pi r}$$

- Direction determined by RHGR

Current loop

- Magnitude at the centre is:

$$\left|\vec{B}\right| = \frac{\mu_0 N I}{2R}$$

- Direction determined by RHGR, or
- Fingers curl in the current direction, thumb points in direction of B (magnetic north)

Solenoid

- Magnitude inside solenoid is almost uniform

$$\left|\vec{B}\right| = \frac{\mu_0 N I}{L} = \mu_0 n I$$

where N is the number of turns, and $n = N/L$ is the number of turns per unit length
- From Tut 6, $N = L/(2r)$. This r is the radius of the cross-section of the wire, NOT the radius of the solenoid.
- Magnitude outside solenoid is very small
- Direction: use RHGR like in current loop

Magnetic force

Right hand cross rule (RHCR)

1. Point 4 fingers in the direction of v
2. Rotate hand so that palm faces direction of B
3. Direction of thumb is direction of the force F
4. If charge is negative, flip the direction of the force

Force on moving charge

Magnitude

$$\left|\vec{F}\right| = |q|\, v_\perp B = |q|\, vB \sin\alpha$$

Direction use RHCR

Trajectory

- If initial v is perpendicular to B, since F is always perpendicular to v, we have circular motion

$$F = qvB = \frac{mv^2}{r}$$

- Otherwise, there is circular motion in the horizontal plane, but also vertical motion in the vertical axis. This is called helical motion.

Force on current-carrying wire

Magnitude A wire has Nq charge flowing in it, so we have

$$F = NqvB \sin\alpha$$

If we multiply by t/t, we have

$$F = \frac{Nq}{t}(vt)B \sin\alpha$$

where vt is the length of the wire, and $I = \frac{Nq}{t}$ is the current.

Direction use RHCR, replacing v with I (they are in the same direction actually)

Force between (parallel) current-carrying wires

Magnitude Each wire exerts a force on the other, and it is symmetric:

$$F = F_{12} = F_{21} = I_2 l B = I_2 l \left(\frac{\mu_0 I_1}{2\pi r}\right)$$

where r is the distance between the wires. Alternatively,

$$F = \frac{\mu_0 I_1 I_2 l}{2\pi r}$$

Direction If their currents are flowing in the same direction, then the wires will attract. Otherwise they repel.

Cross fields

If we want to filter charges, we can pass them through a cross field that has both electric and magnetic forces.

Direction

- Arrange in such a way that the forces are equal and opposite (for a particular type of charge, e.g. positive), then for the negative charge it will be equal but in the same direction
- Thus positive charges will move straight

Magnitude We must have $qvB = qE$, i.e.

$$v = \frac{E}{B}$$

Induction

Motional EMF

How it works

- When a conductor moves perpendicular to a B-field (via F_{ext}), then F_B is exerted on the charges. But F_B acts in opposite directions for positive and negative charges, so there is a charge separation, leading to an EMF.
- The building of the EMF creates a PD, resulting in a F_E that will resist F_B. So charges continue accumulating until $F_B = F_E$.

$$\begin{aligned} F_B &= F_E \\ qvB &= qE \\ vB &= \frac{\Delta V}{d} \\ \Delta V &= Bdv \end{aligned}$$

and we rename $\varepsilon = \Delta V$ and $l = d$ to get the more well-known

$$\varepsilon = Blv$$

Induced current If the moving conductor is connected to a circuit, then a current is induced.

$$I = \frac{\varepsilon}{R} = \frac{Blv}{R}$$

Induced force With the induced current, and since the conductor is moving in a B-field, we have a magnetic force

$$F = IlB = \frac{B^2 l^2 v}{R}$$

and direction can be obtained via RHCR (with current). Note that v is in the direction of F_{ext}.

Magnetic flux

- Amount of B-field that passes through a loop
- Magnitude is

$$\phi = NBA\cos\theta$$

  where N is the number of loops, A is the area of the loop, and θ is the angle between the axis of the loop to the B-field
- Axis of the loop is the line passing through the centre of the loop, so $A\cos\theta$ is like the effective area of the loop
- has units Tm^2 or Wb

Faraday's law

An EMF is induced in a conducting loop if the magnetic flux through the loop changes

$$\varepsilon = \left|\frac{\Delta\phi}{\Delta t}\right|$$

and any of B, A, θ may cause a change in flux.

Lenz's law

Direction of induced current is such that the induced B-field opposes the change in flux.

$$\Delta\phi \to \varepsilon_{\text{induced}} \to I_{\text{induced}} \to B_{\text{induced}} \to \phi_{\text{induced}}$$

How to apply

1. Determine direction of B-field
2. Determine if flux is increasing/decreasing
3. Lenz's law states that induced B-field will oppose this change
4. RHGR determines the direction of I that induces this B-field