PC1201

MISC

Notation

• \vec{F}_{ab} : Force on a due to b

Common prefixes

Prefix	Abbrev.	Power of 10
mega	M	10^{6}
kilo	k	10^{3}
centi	c	10^{-2}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}

Units

Concept	\mathbf{Unit}	Alternative
Force	N	$ m kgms^{-2}$
Energy	J	Nm
Power	W	$\mathrm{J}\mathrm{s}^{-1}$
Charge	С	-
E-field	$ m NC^{-1}$	$ m Vm^{-1}$
Potential	V	$ m JC^{-1}$
Capacitance	F	$\mathrm{C}\mathrm{V}^{-1}$
Current	A	$\mathrm{C}\mathrm{s}^{-1}$
Resistance	Ω	VA^{-1}
B-field	Т	-
Flux	Wb	${ m Tm^2}$

Triangles

Given a triangle with side lengths a,b,c and angles A,B,C, with angle k opposite to side K,

Sine rule
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine rule $c^2 = a^2 + b^2 - 2ab\cos C$

Problem solving techniques

- Write down ALL info given, and unknowns
- Some relevant info might be expressed only in words (e.g. starts from rest)
- Change to SI units

KINEMATICS

Ticker tape Dots are drawn every fixed interval.

- Increasing gap size \Rightarrow acceleration

SUVAT

$$s = ut + \frac{1}{2}at^2 \qquad v = u + at$$

$$v^2 = u^2 + 2as \qquad s = \frac{1}{2}(u+v)t$$

Projectile Motion

Trajectory Parabola, only depending on the initial velocity \vec{u} .

$$y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$$

- Velocity is tangent to trajectory.
- $v_y = 0$ at top of trajectory.
- v_x is constant throughout trajectory.

Height
$$H = \frac{u^2 \sin^2 \theta}{2a}$$

Range
$$R = \frac{u^2 \sin(2\theta)}{q}$$

Circular motion

Centripetal acceleration keeps an object moving in a circle without changing its speed

$$a_c = \frac{v^2}{r}$$

- a_c is always \perp to v
- If speed is changing, there is a component of acceleration, a_t , not \perp to v

Relative motion

Notation $\vec{x}_{ ext{object|ref frame}}$

 $\bf Relation \,\,$ Applies also to velocity, and to 2D

$$\vec{x}_{A|B} = \vec{x}_{A|O} - \vec{x}_{B|O} = \vec{x}_{A|O} + \vec{x}_{O|B}$$

DYNAMICS

Forces

 $\underline{\mathbf{Gravity}} \quad W = mg$

Tension

- Same throughout the whole rope
- Direction is along the rope
- Acts away from body of interest
- Breaking tension is the max value of tension the rope can withstand before breaking

Normal A contact force that acts in the direction perpendicular to the surface

Friction

A contact force that acts opposite to the motion, resisting

- Motion (Kinetic friction, f_k)
- Tendency to move (Static friction, f_s)

Kinetic
$$f_k = \mu_k N$$

Static varies from 0 to max value $f_{s_{\text{max}}}$, depending on the force exerted. The maximum static friction obeys

$$f_{s_{\text{max}}} = \mu_s N$$

- In general, $f_k \leq f_{s_{\text{max}}}$
- An object cannot have static and kinetic friction at the same time

Elastic Known as a restoring force, acting in the opposite direction of $\Delta \vec{x}$.

$$\vec{F} = -k\Delta \vec{x}$$

Newton's laws

First An object will not change its motion unless a force acts on it

Second $\vec{F} = m\vec{a}$, and vector superposition

Third Every action has an equal and opposite reaction (that acts on a different object)

• Reaction pair for weight is not the normal force (both act on same object)

Two-body systems

- Can consider each body separately
- Can consider both bodies together (do not include any action-reaction pairs)

Circular motion

Centripetal force
$$F_c = ma_c = \frac{mv^2}{r}$$

Orbital motion

- If we launch a projectile horizontally at the right speed, the curve of the falling projectile matches the curve of the earth
- An object in such a <u>circular</u> orbit is in constant free fall, only being <u>acted</u> on by gravity. Hence,

$$mg = \frac{mv^2}{r}$$

$$v = \sqrt{rg}$$

• Natural orbits tend to be more elliptical

Apparent weight

Normal force Our apparent weight changes if $N \neq mq$.

- • If N < mg, we feel lighter (lift accelerating down)
- $\bullet \ \ \mbox{If} \ N>mg,$ we feel heavier (lift accelerating up)
- If N = 0, we are weightless (free fall)

We can experience this in elevators, vertical circular motion, orbitting satellites, etc.

Problem solving techniques

- Draw sketch of entire system
- Isolate SINGLE body to draw FBD, with forces acting ON the body
- Action-reaction pairs should NOT appear in same FBD (they act on different bodies)
- Choose positive axis that aligns with net \vec{a}
- Start with $\sum F = 0$ or $\sum F = ma$
- $\bullet\,$ Do NOT assume N=mg

WORK, ENERGY, POWER

\mathbf{Work}

- Is a transfer of energy, done on a system by an external agent
- Positive work done should increase energy of system
- F is in same dir. as $s \Rightarrow +$ ve work done

$$W = F_{||} s = F s \cos \theta$$

where F_{\parallel} is component of F parallel to s

• Work done is area under graph of F - x

Energy

$$\underline{\textbf{Kinetic}} \quad W = K_f - K_i = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

Potential

- Stored energy due to condition of object/force of interaction between objects
- Only allowed by conservative forces, where work done depends only on position, and is independent of the path
- When conservative forces do +ve work on the object, the potential energy of the object decreases

Gravitational $\Delta U_g = mgh$

Elastic $\Delta U_s = \frac{1}{2}k(\Delta x)^2$

Conservation of energy

In an isolated/closed system, total energy remains constant

$$\Delta E = 0 \Rightarrow \sum E_f = \sum E_i$$

In a non-isolated/open system,

$$\Delta E = W \Rightarrow \sum E_f - \sum E_i = W$$

Power

Rate of energy transfer

$$P = \frac{\Delta E}{t} = \frac{W}{t}$$

Kinematics (if acceleration is 0)

$$P = \frac{Fs}{t} = Fv$$

MOMENTUM

$$p = mi$$

Collisi	on	Elastic	Inelastic
Momen	tum	Conserved	Conserved
KE	- (Conserved	Not conserved

Elastic collisions

Use equations 1 and 2, or 1 and 3 to solve. To check a collision is elastic, use 2.

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$
$$v_2 - v_1 = -(u_2 - u_1)$$

Impulse

J is the area under the F-t graph

$$J = p_f - p_i = \Delta p$$
$$J = F_{\text{avg}} \Delta t$$

ELECTRIC CHARGES

Charges

 ${\bf Types} \ \ \, + {\rm ve\ charges:\ Protons,\ -ve\ charges:\ Electrons}$

Charge of an object

- Neutral if it has equal amounts of +ve and -ve charges
- +vely charged if it has an excess of protons
- -vely charged if it has an excess of protons
- Protons are tightly bounded to nucleus, so they cannot be added/removed from an atom

Conservation of charge One cannot create/destroy charges; one can only transfer charges from one body to another

Interaction Like charges repel, unlike charges attract

Insulators

Definition Materials that do not allow electrons to flow freely

Charging By friction, molecular bonds are broken, allowing a neutral molecule to split into positive and negative parts

Conductors

Definition Materials that allow electrons to flow

Polarization is a separation of charges within an object

Charging

Charging by contact

- Touch a conductor with an initially charged ob-
- Charges will flow to/from the conductor, leaving it with a net charge
- The charges will spread through the conductor
- The initially charged object will have less charge

Discharging by contact (grounding)

- The earth can be considered as a large conduc-
- Grounding a charged conductor is basically allowing it to make contact with earth
- If we ground a -vely charged object, electrons flow from object to earth
- If we ground a +vely charged object, electrons flow from earth to to object

Charging by induction

- Bring a charged object very near a conductor (without touching)
- Conductor will be polarized
- Ground the conductor
- · Remove grounding wire and charged object
- Conductor wil now have a net charge

ELECTRIC FORCE

Coulomb's law

Two charges exert a force on each other

Magnitude
$$|\vec{F}_{12}| = |\vec{F}_{21}| = \frac{k|q_1||q_2|}{r^2}$$

Direction

- Along line joining two points
- Repulsive for like charges, attractive for unlike charges

Limitation

- Only applies to point charges, but objects may be modeled as point charges if the r is much larger than their size
- Only applies to static charges, but this module focuses on static charges

ELECTRIC FIELD

Explains how q_1 can be aware of another charge

E-field for point charge

Magnitude We use a test charge $q_{\rm test}$ to help quantify the E-field strength:

$$\left| \vec{E} \right| = \frac{\left| \vec{F} \right|}{q_{\text{test}}} = \frac{k \left| q_0 \right|}{r^2}$$

 $\left|\vec{E}\right| = \frac{\left|\vec{F}\right|}{q_{\rm test}} = \frac{k\left|q_0\right|}{r^2}$ A charge q_1 in an E-field due to q_0 experiences a

$$\left| \vec{F} \right| = |q_1| \left| \vec{E} \right| = \frac{k |q_0| |q_1|}{r^2}$$

Direction Defined to be the direction of the force experienced by a positive q_{test}

	$+$ ve q_{test}	-ve q_{test}
Field	Points away	Points towards
Force	Same dir.	Opposite dir.
rorce	as field	from field

E-field lines

Exist at every point in space, just a representative

Magnitude Higher density of field lines ⇒ stronger field

Direction

- E-field vector is tangent to E-field line
- Field lines do not cross
- Field lines start from +ve charges and end at -ve charges

E-field for sphere

Let R be the radius of the sphere. The following applies to:

- Thin insulating shell with uniform distribution
- Thin conducting shell (assume no charge nearby)
- Conducting sphere (assume no charge nearby)

Outside sphere Let Q be the total charge on the sphere. Let r be the distance to the centre of the sphere.

$$\left| \vec{E} \right| = \frac{k \left| Q \right|}{r^2}$$

Inside sphere Inside the sphere, there is no efield due to symmetry

E-field for infinite sheet/plate

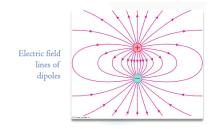
- We can approximate most large sheets/plates as infinite sheets/plates
- We use this setup for parallel plate capacitors

Magnitude Let Q be the total charge on the plate. Let A be the area of each plate.

$$\left| \vec{E} \right| = \frac{|Q|}{2\varepsilon_0 A}$$

E-field for dipole

Direction



E-field in conductors

- We only consider conductors in electric equilibrium (charges not moving), so E = 0
- Excess charges reside only on the surface, congregating at sharp points
- E-fields are perpendicular to surface

Misc

Dielectric breakdown When E-fields within insulators get too strong, it may fail to act as an insulator, turning into a conductor, allowing charges to flow

Screening in conductors When conductors are placed in an E-field, any void in the conductor will also have $\vec{E} = 0$ (Faraday's cage / electrostatic shielding)

ELECTRIC POTENTIAL ENERGY

Interaction EPE for a point charge

Magnitude A charge q_1 will have EPE if another charge q_2 is present

$$U_{q_1} = \frac{kq_1q_2}{r} = U_{q_2}$$

Alternative Based on the definition of electric potential later, we also have

$$\Delta U = q\Delta V$$

Zero point $U \to 0$ as $r \to \infty$, i.e. when there are no charges around

Sign Similar charges have positive U, while opposite charges have negative U.

Interaction EPE vs electric force

 $\Delta U \text{ vs } \vec{F}_{\text{ext}}$ Work done is given by

$$W = \left| \vec{F}_{\text{ext}} \right| d\cos\theta$$

where d is +ve in the direction of displacement, and θ is the angle between $\vec{F}_{\rm ext}$ and d. We can also define work done as

$$W = \Delta U = U_f - U_i$$

Hence,

- $\theta = 0 \Rightarrow \text{same dir.} \Rightarrow W = \vec{F}_{\text{ext}} d > 0 \Rightarrow \Delta U > 0$
- $\theta = 180 \Rightarrow \text{diff dir.} \Rightarrow W = \vec{F}_{\text{ext}} d < 0 \Rightarrow \Delta U <$

Here, ΔU is the work done on the charge to bring it from initial point i to final point f.

 ΔU vs Coulomb's force \vec{F} In order to move the charged object at constant velocity, we need $\vec{a}=0$, so we have to oppose Coulomb's force, so

$$\vec{F} = -\vec{F}_{
m ext}$$

This case has a negative sign, so

- \vec{F} and d same dir. $\Rightarrow \Delta U < 0$
- \vec{F} and d diff dir. $\Rightarrow \Delta U > 0$

Configuration EPE

Work required to bring the charges from infinity to build this configuration

$$U = \sum_{i < j} \frac{kq_i q_j}{r_{ij}}$$

ELECTRIC POTENTIAL

- Like E-field, electric potential explains how a charge can be aware of the presence of another charge
- $\bullet\;$ But E-field is about force per charge while electric potential is about work done/energy per

Electric potential for point charge

Magnitude Similar to E-field, we obtain magnitude by placing a test charge q_{test} at the point we are interested in.

$$V = \frac{U}{q_{\text{test}}} = \frac{kq_0}{r}$$

where q_0 is the charge that created the electric potential. Thus, a charge q_1 in a potential due to q_0 has energy

$$U = q_1 V = \frac{kq_0 q_1}{r}$$

Electric potential for sphere/shell

Let R be the radius of the sphere. The following applies to:

- Thin insulating shell with uniform distribution
- Thin conducting shell (assume no charge nearby)
- Conducting sphere (assume no charge nearby)

Outside sphere Let Q be the total charge on the sphere. Let r be the distance to the centre of the sphere.

$$V = \frac{kQ}{r}$$

Inside sphere Inside the sphere, potential is constant.

$$V = \frac{kQ}{R}$$

Equipotential surfaces

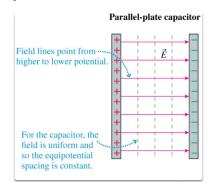
- Lines/surfaces of the same potential, so moving a charge along the line does not change EPE
- PD between each adjacent pair of equipotential lines is the same

Relationship to \vec{E}

- E-field points in direction of decreasing potential
 - PD establishes E-field
 - A charge in a PD experiences a force
- E-fields are perpendicular to equipotential lines
- If E-field is constant, then

$$\left| \vec{E} \right| = \frac{|\Delta V|}{d}$$

where d is the distance between the two points (parallel to \vec{E}), and ΔV is the PD between the two points.



Moving charge in a field

1. Charge q moving from A to B in a PD ΔV

$$\Delta U = q\Delta V$$

2. Charge q moving from A to B in a constant \vec{E} field

$$\Delta \vec{V} = \left| \vec{E} \right| d$$

L12 QC6

- Charges move because of a PD, so they stop moving once there is no PD
- If two conductors are connected via a wire, charges redistribute equally only if the two conductors are identical.

$\frac{\text{CAPACITANCE, CURRENT,}}{\text{RESISTANCE}}$

Capacitor

Capacitance A capacitor stores electric charges, and consists of two equally and oppositely charged parts that are some distance apart. The capacitance C is given as:

$$C = \frac{Q}{\Delta V} = \frac{Q}{Ed} = \kappa \frac{\varepsilon_0 A}{d}$$

- Q is the amount of charge the capacitor can hold, given PD between the plates ΔV
- κ is the dielectric constant, 1 in air, others will be given
- RHS is always true, even if the capacitor is dis-

Electric field Proved in assignment 2:

$$\left| \vec{E} \right| = \frac{Q}{\varepsilon_0 A}$$

Dielectric in a capacitor

- As mentioned above, it introduces κ into the equation
- (CA2 Q10) If there are multiple dielectrics, then treat them as capacitors, and handle series/parallel correctly

Energy Energy stored in a capacitor is equal to the work done to charge it

$$U = \frac{1}{2}Q\Delta V = \frac{Q^2}{2C} = \frac{1}{2}C(\Delta V)^2$$

Energy density The capacitor's energy is stored in the electric field between the plates.

$$U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}\frac{\varepsilon_0 A}{d}(Ed)^2 = \frac{1}{2}\varepsilon_0 E^2(Ad)$$

In order to be independent of the dimensions of the capacitor, define $u=\frac{U}{Ad}$ to be the energy density, so we have

$$u = \frac{C(\Delta V)^2}{2Ad} = \frac{1}{2}\varepsilon_0 E^2$$

Current

- Rate of flow of charge $I = \frac{Q}{\Delta t}$
- Defined to be the direction of flow of positive charges (although electrons are the ones flowing)
- ΔV exists $\Rightarrow \vec{E}$ exists \Rightarrow force exists \Rightarrow electrons move (in opposite direction of \vec{E})

EMF source, ε (Creates and) maintains a constant PD by moving +ve charges against the electric field, opposing Coulomb's force

Power supplied by EMF source The +ve charges gain potential energy $U=q\Delta V,$ so a battery supplies energy $U=q\varepsilon.$

$$P = \frac{U}{\Delta t} = \frac{q\Delta V}{\Delta t} = I\Delta V = I\varepsilon$$

Resistance

- • For ohmic materials, which have constant resistance, Ohm's law holds true: $R = \frac{\Delta V}{I}$
- Can assume the materials are ohmic unless otherwise stated

Computing resistance for a material

$$R = \frac{\rho l}{A}$$

where ρ is the resistivity (given), l is the length of the conductor, A is the cross sectional area.

Power For ohmic resistors,

$$P = I\Delta V = I^2 R = \frac{(\Delta V)^2}{R}$$

- Power ratings in household appliances are based off a constant voltage (230V in Singapore), so they indirectly give us the resistance of the appliance
- $\bullet~$ Lower power rating \Rightarrow higher resistance
- Kilowatt hour (kWh) is the energy that a 1kW power source uses up in 1 hour

DC CIRCUITS

Battery Positive terminal is the longer line

Resistors

	Series	Parallel
PD	Sum	Same
Current	Same	Sum
		Reciprocal of
Resistance	Sum	(sum of
		reciprocals)

Capacitor

	Series	Parallel
PD	Sum	Same
Charge	Same	Sum
Capacitance	Reciprocal of (sum of reciprocals)	Sum

Kirchhoff

Parallel vs series

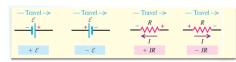
- Two elements are in parallel if any loop contains only those two elements
- Two elements are in series if every loop contains only both those elements

Junction rule
$$\sum I_{\mathrm{in}} = \sum I_{\mathrm{out}}$$

Loop rule sum of PD along loop is 0

How V changes across circuit elements

- Battery: Positive plate has higher potential than negative plate
- Resistor: Potential decreases in the direction of current (depends on the emf or the setup)
- If travel direction is towards lower potential, then ΔV is negative, vice versa



${\bf Strategy}$

- Label everything and write down all info
- Fix direction of currents
- Apply loop rule n-1 times, by going in a loop and comparing with current direction
- Apply junction rule using current direction

MAGNETIC FIELD

Magnet Has "inherent magnetic moments" that are aligned, so dropping/heating will harm the magnet

Earth is a magnet

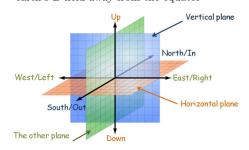
- The magnetic north pole is (near) the geographic south pole of the earth
- A compass needle points N because the N pole of the compass needle is attracted to the magnetic S pole of the earth, which is (near) the geographic N pole of the earth

Magnetic field lines

- Starts from north pole, ends at south pole
- Denser where field is stronger (near magnet)
- Never crosses

3D convention

- Cross means current into page, dot means current out of page
- Vertical plane (up/down, west/east): used in question paper
- Horizontal plane (north/south, west/east): surface of the earth
- Other plane (up/down, north/south): visualize earth's B-field away from the equator



Dip angle

- Magnetic field at the equator is approximately horizontal
- Magnetic field at the poles is nearly vertical
- Measured with respect to the surface of the earth

Sources of magnetic fields

Right hand grip rule (RHGR)

- Point your right thumb in the direction of current
- 2. Wrap your fingers around the wire to indicate a circle
- 3. Your fingers curl in the direction of the B-field lines around the wire

Moving charge (in a wire)

- Stationary charge produces E-field, while moving charge produces B-field
- Magnitude is

$$\left| \vec{B} \right| = \frac{\mu_0 I}{2\pi r}$$

• Direction determined by RHGR

Current loop

• Magnitude at the centre is:

$$\left| \vec{B} \right| = \frac{\mu_0 NI}{2R}$$

where N is the number of loops

- Direction determined by RHGR, or
- Fingers curl in the current direction (along loop), thumb points in direction of B (magnetic

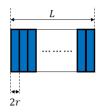
Solenoid

• Magnitude inside solenoid is almost uniform

$$\left| \vec{B} \right| = \frac{\mu_0 NI}{L} = \mu_0 nI = \frac{\mu_0 I}{2r}$$

where N is the number of turns, n = N/L is the number of turns per unit length, r is the radius of the cross-section of the wire

- Outside a solenoid is very small
- Magnitude outside solenoid is very small
- Direction: use RHGR like in current loop
- From Tut 6, N = L/(2r). This r is the radius of the cross-section of the wire, NOT the radius of the solenoid.



MAGNETIC FORCE

Right hand cross rule (RHCR)

- 1. Point 4 fingers in the direction of v
- 2. Rotate hand so that palm faces direction of B
- 3. Direction of thumb is direction of the force F
- 4. If charge is negative, flip the direction of the force

Force on moving charge

A magnetic force is exerted on a moving charge in a magnetic field

Magnitude $|\vec{F}| = |q| v_{\perp} B = |q| v B \sin \alpha$

Direction use RHCR

Trajectory

• If initial v is perpendicular to B, since F is always perpendicular to v, we have circular mo-

$$F = qvB = \frac{mv^2}{r}$$
 or $r = \frac{mv}{qB}$

Otherwise, there is circular motion in the horizontal plane, but also vertical motion in the vertical axis, due to the non-zero component of v parallel to the B-field. This is called helical

Force on current-carrying wire

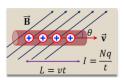
Magnitude A wire has Nq charge flowing in it, so we have

$$F = NqvB\sin\alpha$$

If we multiply by t/t, we have

$$F = \frac{Nq}{t}(vt)B\sin\alpha = IlB\sin\alpha$$

where vt = l is the length of the wire, and $I = \frac{Nq}{t}$ is the current.



Direction use RHCR, replacing v with I (they are in the same direction actually)

Force between (parallel) current-carrying wires

Magnitude Each wire exerts a force on the other, and it is symmetric:

$$F = F_{12} = F_{21} = I_2 l B = I_2 l \left(\frac{\mu_0 I_1}{2\pi r}\right)$$

where r is the distance between the wires. Alternatively,

$$F = \frac{\mu_0 I_1 I_2}{2\pi r}$$

Direction If their currents are flowing in the same direction, then the wires will attract. Otherwise they repel.

Note The B field generated by a wire does not affect itself.

Cross fields

If we want to filter charges, we can pass them through a cross field that has both electric and magnetic forces.

Direction

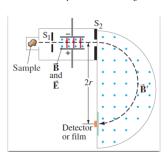
- Arrange in such a way that the forces are equal and opposite (for a particular type of charge, e.g. positive), then for the negative charge it will be equal but in the same direction
- Thus positive charges will move straight

Magnitude We have qvB = qE, i.e. $v = \frac{E}{R}$

Mass spectrometer

A mass spectrometer is a device used to measure the mass of an atom. It uses crossed fields for a velocity selector, and the circular trajectory of a charged particle in a magnetic field, to measure the mass of an atom.

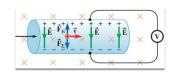
$$qvB = \frac{mv^2}{r} \Rightarrow m = \frac{qBr}{v}$$



Electromagnetic flowmeter

An electromagnetic flowmeter can measure speed of blood flow. In the presence of a B-field, positive and negative charges will separate, creating an Efield. When $F_E = F_B$, then ions travel through undeflected, and we can calculate

$$v = \frac{E}{B}$$



INDUCTION

Motional EMF

• Similar to electromagnetic flowmeter

- When a conductor moves perpendicular to a B-field (via F_{ext}), then F_B is exerted on the charges. But F_B acts in opposite directions for positive and negative charges, so there is a charge separation, leading to an EMF.
- The building of the EMF creates a PD, resulting in a F_E that will resist F_B . So charges continue accumulating until $F_B = F_E$.

$$qvB = qE$$

$$vB = \frac{\Delta V}{d}$$

$$\Delta V = Bdv$$

and we rename $\varepsilon = \Delta V$ and l = d to get the more well-known

$$\varepsilon = Bl$$

Induced current If the moving conductor is connected to a circuit, then a current is induced.

$$I=\frac{\varepsilon}{R}=\frac{Blv}{R}$$

Induced force With the induced current, and since the conductor is moving in a B-field, we have a magnetic force

$$F_B = IlB = \frac{B^2 l^2 v}{R}$$

 $F_B = IlB = \frac{B^2 l^2 v}{R}$ and direction can be obtained via RHCR (with current). Note that v is in the direction of F_{ext} .

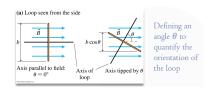
Magnetic flux

- Amount of B-field that passes through a loop
- Magnitude is

$$\phi = NBA\cos\theta$$

where N is the number of loops, A is the area of the loop, and θ is the angle between the axis of the loop to the B-field

Axis of the loop is the line passing through the centre of the loop, so $A\cos\theta$ is like the effective area of the loop



Faraday's law

An EMF is induced in a conducting loop if the magnetic flux through the loop changes

$$\varepsilon = \left| \frac{\Delta \phi}{\Delta t} \right|$$

and any of B, A, θ may cause a change in flux.

Lenz's law

Direction of induced current is such that the induced B-field opposes the change in flux.

 $\Delta \phi \to \varepsilon_{\mathrm{induced}} \to I_{\mathrm{induced}} \to B_{\mathrm{induced}} \to \phi_{\mathrm{induced}}$

How to apply

- 1. Determine direction of B-field
- 2. Determine if flux is increasing/decreasing
- 3. Lenz's law states that induced B-field will oppose this change
- 4. RHGR determines the direction of I that induces this B-field

