Causality, Regression, and Fixed Effects

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Causality

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In randomized experiments, we observe:

$$Outcome_i(Treatment_i = 1) = y_i$$
$$Outcome_j(Treatment_j = 0) = y_j$$

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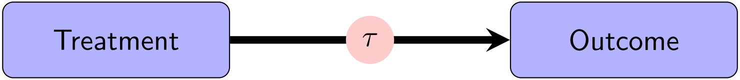
Randomization $\implies$ treatment is *only* difference between i and j

Causality

Estimate treatment effect by subtraction:

$$\begin{aligned}
Outcome_i(Treatment_i = 1) - Outcome_j(Treatment_j = 0) \\
= y_i - y_j \\
= \tau
\end{aligned}$$

Causality



Example: Estimating Treatment Effects in R

```r
experiment <- read.csv("fakeexperiment.csv")
mean(experiment$y[experiment$treatment ==
    1]) - mean(experiment$y[experiment$treatment ==
    0])
```

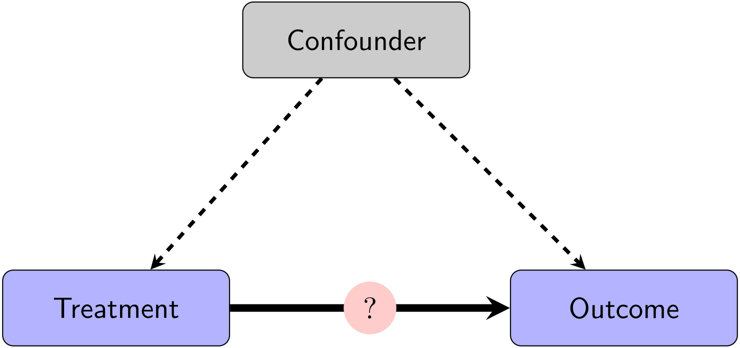
```
[1] 2.946722
```

```r
experiment <- read.csv("fakeexperiment.csv")
lm(y ~ treatment, data = experiment)
```

```
Call:
lm(formula = y ~ treatment, data = experiment)

Coefficients:
(Intercept)    treatment
      2.037        2.947
```

Causality in Observational Data



Causality in Observational Data

In observational data, we observe:

$$Outcome_i(Treatment_i = 1, Confounder_i) = y_i$$
$$Outcome_j(Treatment_j = 0, Confounder_j) = y_j$$

Treatment is not *only* difference between i and j

Estimating Causal Effects with Observational Data

True relationship: $y_i = 3 + 5 * treatment_i + 10 * confound_i + \varepsilon_i$

```
obs <- read.csv("fakeobservational.csv")
lm(y ~ treatment, data = obs)
```

```
Call:
lm(formula = y ~ treatment, data = obs)

Coefficients:
(Intercept)    treatment
     -58.18       179.05
```

What Can We Do?

- Think clearly about potential confounders
- Adjust/control for potential confounders identified

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- Think clearly about potential confounders
- Adjust/control for potential confounders identified

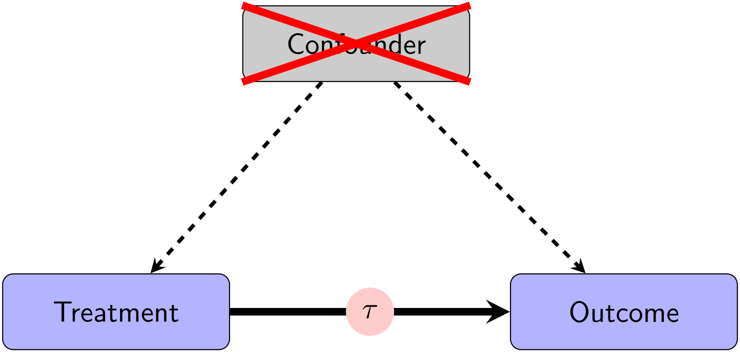
- Remember true equation is:
  $y_i = 3 + 5 * treatment_i + 10 * confound_i + \varepsilon_i$

```
lm(y ~ treatment + confound, data = obs)
```

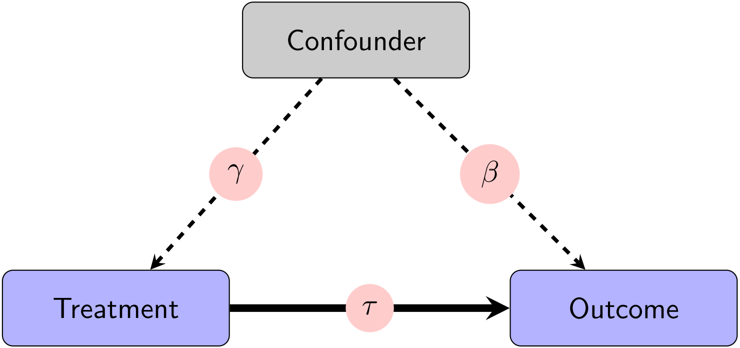
```
Call:
lm(formula = y ~ treatment + confound, data = obs)

Coefficients:
(Intercept)    treatment     confound
      3.254        5.042        9.985
```

What Can We Do?



What Can We Do?



$$outcome_i = \alpha + \tau * treatment_i + \beta * confounder_i + \varepsilon_i$$

$$confounder_i = \eta + \gamma * treatment_i$$

$$outcome_i = \nu + \lambda * treatment_i$$

$$\lambda = \tau + \gamma * \beta$$

Making Sense of Regression Results

- In experiments, treatment *causes* (leads to, generates, etc.) an effect of about τ

Making Sense of Regression Results

- In experiments, treatment *causes* (leads to, generates, etc.) an effect of about τ

- Not so in observational studies...
- Should be careful about interpretation of regression coefficients
- Generally, not "treatment" variable *causes* (leads to, generates, increases) the outcome

What Can We Say Instead?

- A one unit increase in the "treatment" variable is associated with a change in the outcome of about τ
- If the "treatment" variable were higher by Z, we would expect our outcome to be higher by about $Z * \tau$ on average
- The outcome variable is predicted to be about Z higher/lower for each increase/decrease of the "treatment" variable

A Note about R^2

$$R^2 = 1 - \frac{\sum_i^n (\hat{y}_i - y_i)^2}{\sum_i^n (y_i - \bar{y})^2}$$

- How much of variation in outcome "explained" by model
- Better description might be variation "captured" by model
- Not really helpful in evaluating causality
  - Experiments often have very low R^2
- Mostly helpful for prediction

A Note about R^2

```
fit <- lm(y ~ treatment, data = obs)
ssr <- sum((fit$fitted.values - obs$y)^2)
sst <- sum((obs$y - mean(obs$y))^2)
1 - ssr/sst
```

```
[1] 0.2289192
```

```
summary(fit)$r.squared
```

```
[1] 0.2289192
```

A Note about R^2

- R^2 will *always* increase if you add more variables
  - Even if those variables are unrelated to the outcome

```
summary(lm(y ~ treatment, data = experiment))$r.squared
```

```
[1] 0.08032865
```

```
summary(lm(y ~ treatment + unobserved, data = experiment))$
```

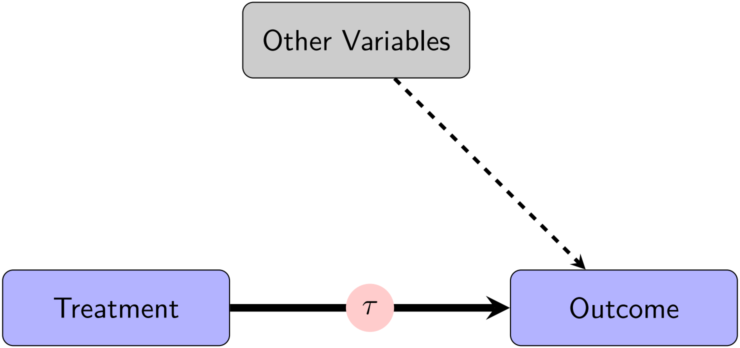
```
[1] 0.0804521
```

```
summary(lm(y ~ treatment + unobserved + X,
    data = experiment))$r.squared
```

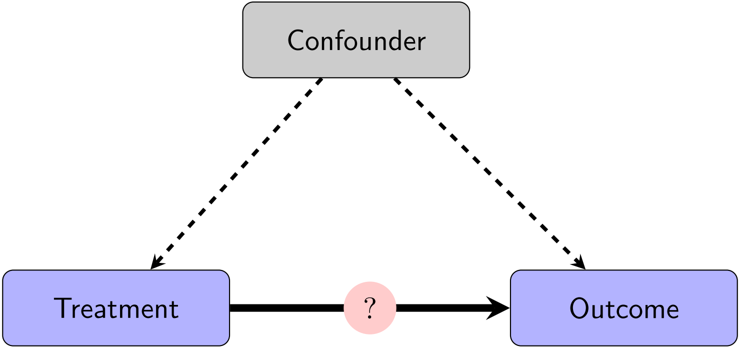
```
[1] 0.08063636
```

Causality vs. R^2

- Should we worry about causality in regression if we have data like this?
- What might we expect the R^2 of this regression to look like?



Causality vs. R^2



Causality vs. R^2

We can see this distinction using our datasets:

```
# Experimental Data
summary(lm(y ~ treatment, data = experiment))$r.squared
```

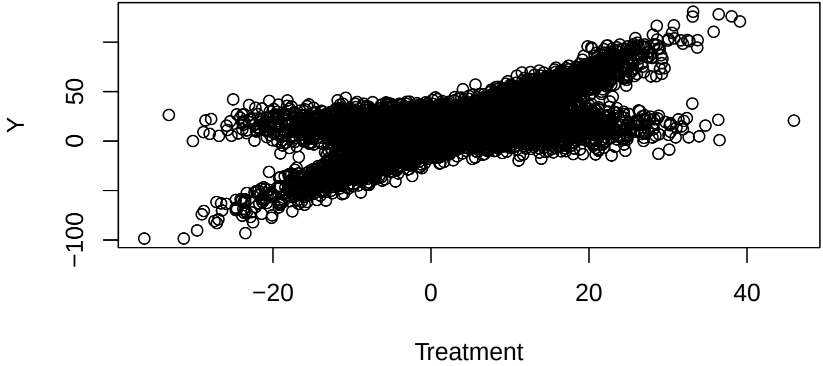
```
[1] 0.08032865
```

```
# Observational Data with Omitted
# Confounder
summary(lm(y ~ treatment, data = obs))$r.squared
```

```
[1] 0.2289192
```

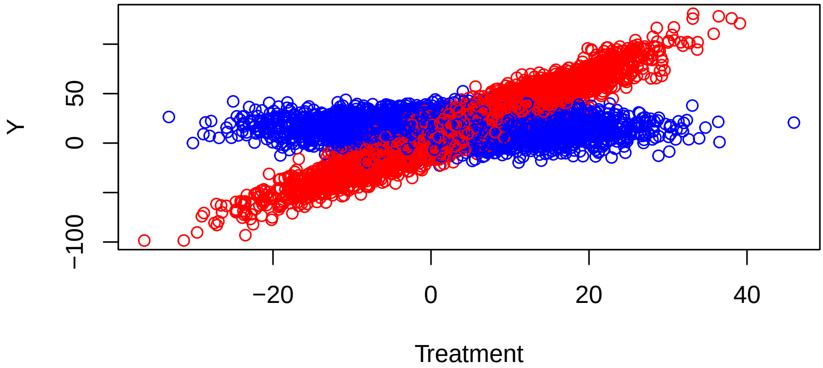
Interactions (Heterogeneity in Treatment Effects)

```
inter <- read.csv("fakeinteraction.csv")
plot(inter$treatment, inter$y, xlab = "Treatment",
     ylab = "Y")
```



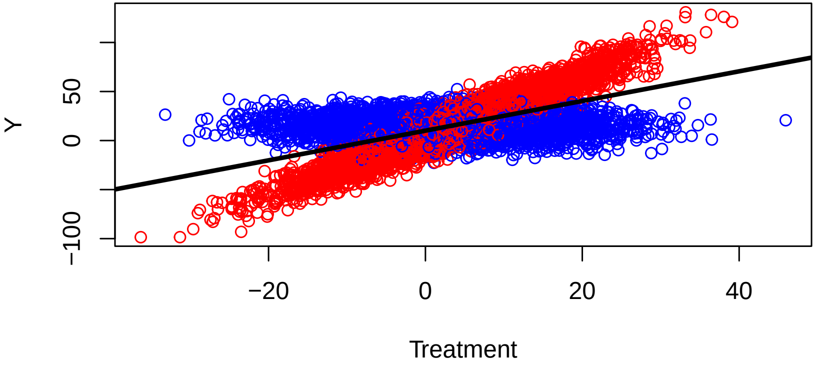
Interactions (Heterogeneity in Treatment Effects)

```
inter <- read.csv("fakeinteraction.csv")
plot(inter$treatment, inter$y, xlab = "Treatment",
    ylab = "Y", col = c("red", "blue")[factor(inter$democra
```



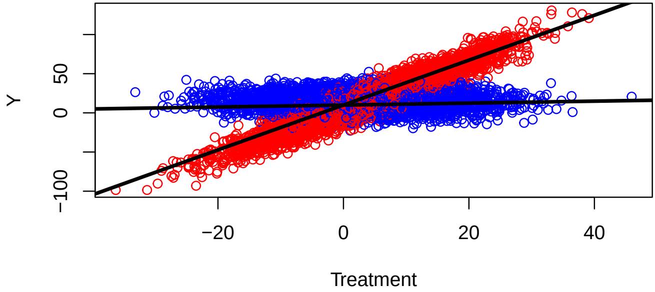
Estimating Interactions in R

```
fit <- lm(y ~ treatment, data = inter)
plot(inter$treatment, inter$y, xlab = "Treatment",
    ylab = "Y", col = c("red", "blue")[factor(inter$democra
abline(fit, lwd = 3)
```



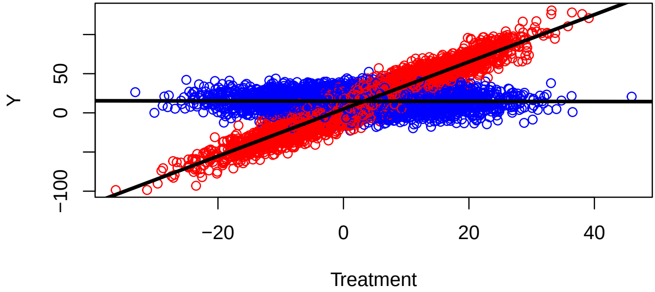
Estimating Interactions in R

```
fit <- lm(y ~ treatment + treatment:democrat,
    data = inter)
plot(inter$treatment, inter$y, xlab = "Treatment",
    ylab = "Y", col = c("red", "blue")[factor(inter$democra
abline(a = coef(fit)[1], b = coef(fit)[2],
    lwd = 3)
abline(a = coef(fit)[1], b = coef(fit)[2] +
    coef(fit)[3], lwd = 3)
```



Estimating Interactions in R

```
fit <- lm(y ~ treatment + democrat + treatment:democrat,
    data = inter)
plot(inter$treatment, inter$y, xlab = "Treatment",
    ylab = "Y", col = c("red", "blue")[factor(inter$democra
abline(a = coef(fit)[1], b = coef(fit)[2],
    lwd = 3)
abline(a = coef(fit)[1] + coef(fit)[3], b = coef(fit)[2] +
    coef(fit)[4], lwd = 3)
```



Interactions

$$y_i = \alpha + \beta_1 * treatment_i + \beta_2 * x_i + \beta_3 * treatment_i * x_i + \varepsilon_i$$

term	estimate
(Intercept)	4.903261
treatment	3.011608
democrat	10.035690
treatment:democrat	-3.023368

$y_i = 4.9 + 3.012 * treatment_i + 10.037 * democrat_i - 3.023 * democrat_i * treatment_i$

Interactions

```
lm(y ~ treatment, data = inter, subset = democrat ==
    1)
```

term	estimate
(Intercept)	14.9389500
treatment	-0.0117603

```
lm(y ~ treatment, data = inter, subset = democrat ==
    0)
```

term	estimate
(Intercept)	4.903261
treatment	3.011608

Interactions

Full Regression:

$y_i = 4.9 + 3.012 * treatment_i + 10.037 * democrat_i - 3.023 * democrat_i * treatment_i$

Democrats:

$$y_i = 14.94 - 0.012 * treatment_i$$

Equivalent to:

$$\begin{aligned} y_i &= 4.9 + 10.037 * 1 + 3.012 * treatment - 3.023 * 1 * treatment_i \\ &= 14.94 - 0.012 * treatment_i \end{aligned}$$

Republicans:

$$y_i = 4.9 + 3.012 * treatment_i$$

Equivalent to:

$$\begin{aligned} y_i &= 4.9 + 10.037 * 0 + 3.012 * treatment_i - 3.023 * 0 * treatment_i \\ &= 4.9 + 3.012 * treatment_i \end{aligned}$$

Fixed Effects

ind	year	ideology
Individual-1	2020	5
Individual-1	2022	4
Individual-2	2020	6
Individual-2	2022	5
Individual-3	2020	9
Individual-3	2022	8

Fixed Effects

```r
lm(ideology ~ factor(ind) + factor(year),
    data = example)
```

(Intercept)	Individual-2	Individual-3	2022	ideology
1	0	0	0	5
1	0	0	1	4
1	1	0	0	6
1	1	0	1	5
1	0	1	0	9
1	0	1	1	8

Fixed Effects

$$y_i = \alpha + \beta_1 * Individual2 + \beta_2 * Individual3 + \beta_3 * year_{2022} + \varepsilon_i$$

Fixed Effects

```r
lm(ideology ~ factor(ind) + factor(year),
    data = example)
```

term	estimate
(Intercept)	5
indIndividual-2	1
indIndividual-3	4
factor(year)2022	-1

Fixed Effects

$$y_i = 5 + 1 * Individual2 + 4 * Individual3 - 1 * year_{2022} + \varepsilon_i$$

ind	year	ideology
Individual-1	2020	5
Individual-1	2022	4
Individual-2	2020	6
Individual-2	2022	5
Individual-3	2020	9
Individual-3	2022	8

Fixed Effects

- What do fixed effects help us accomplish?

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  - Control for unobserved confounders

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- Confounders of what type?

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- What do fixed effects help us accomplish?
  - Control for unobserved confounders
- Confounders of what type?
  - Confounders that do not change within the fixed-effect unit
    - e.g., time-invariant confounders
    - e.g., person-invariant (constant) confounders