Final Review Section

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Central Limit Theorem

- What does the Central Limit Theorem tell us?

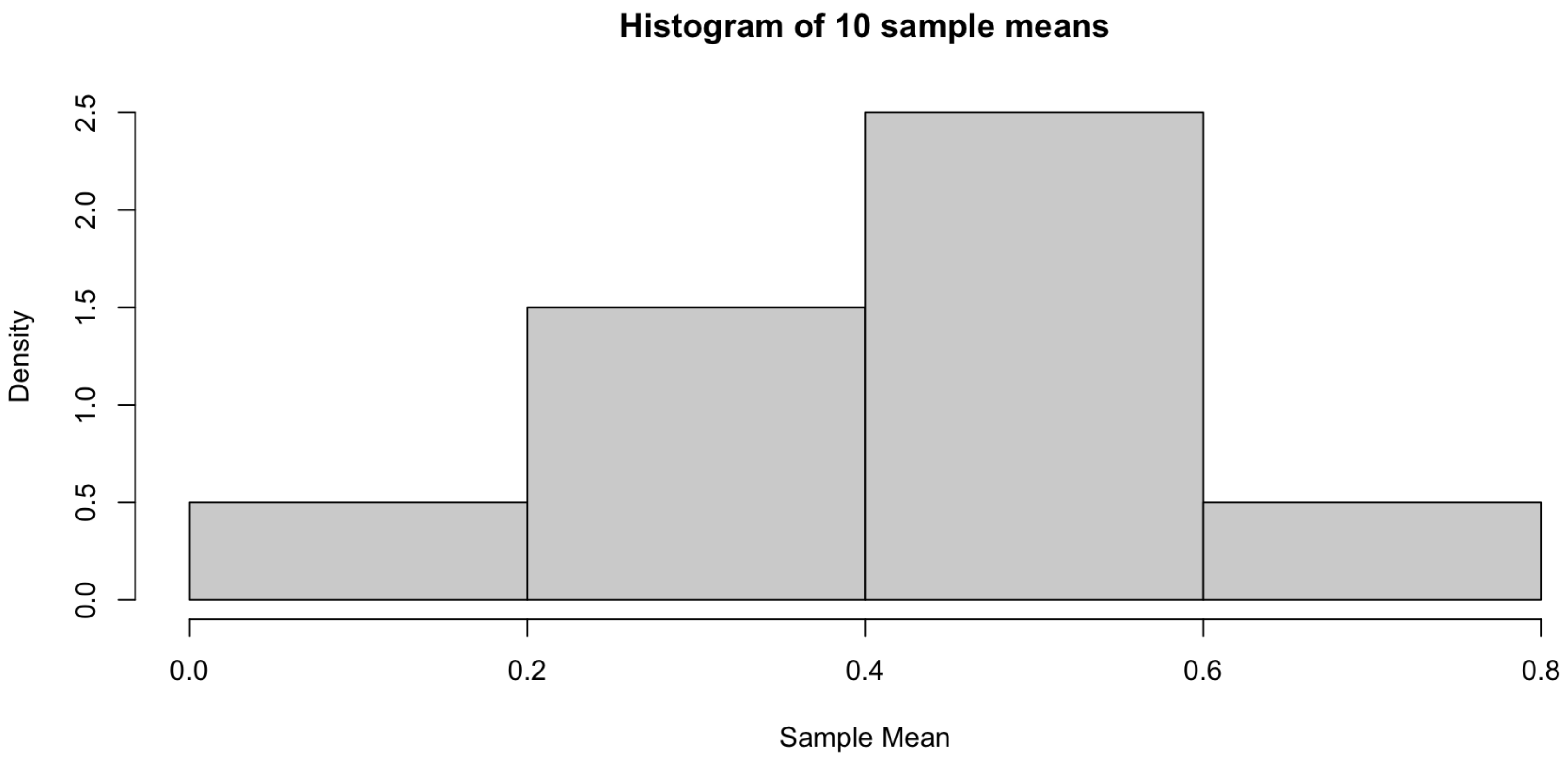
- Central Limit Theorem: the distribution of our sample means for n independent samples will approach a normal distribution as n goes to infinity with the mean equal to the population mean

Central Limit Theorem Example

- For each statement below, plot a histogram of the sample means
  - Simulate and take the mean of 10 random samples from a binomial distribution with n = 10
  - Simulate and take the mean of 100 random samples from a binomial distribution with n = 10
  - Simulate and take the mean of 1000 random samples from a binomial distribution with n = 10
  - Simulate and take the mean of 10000 random samples from a binomial distribution with n = 10

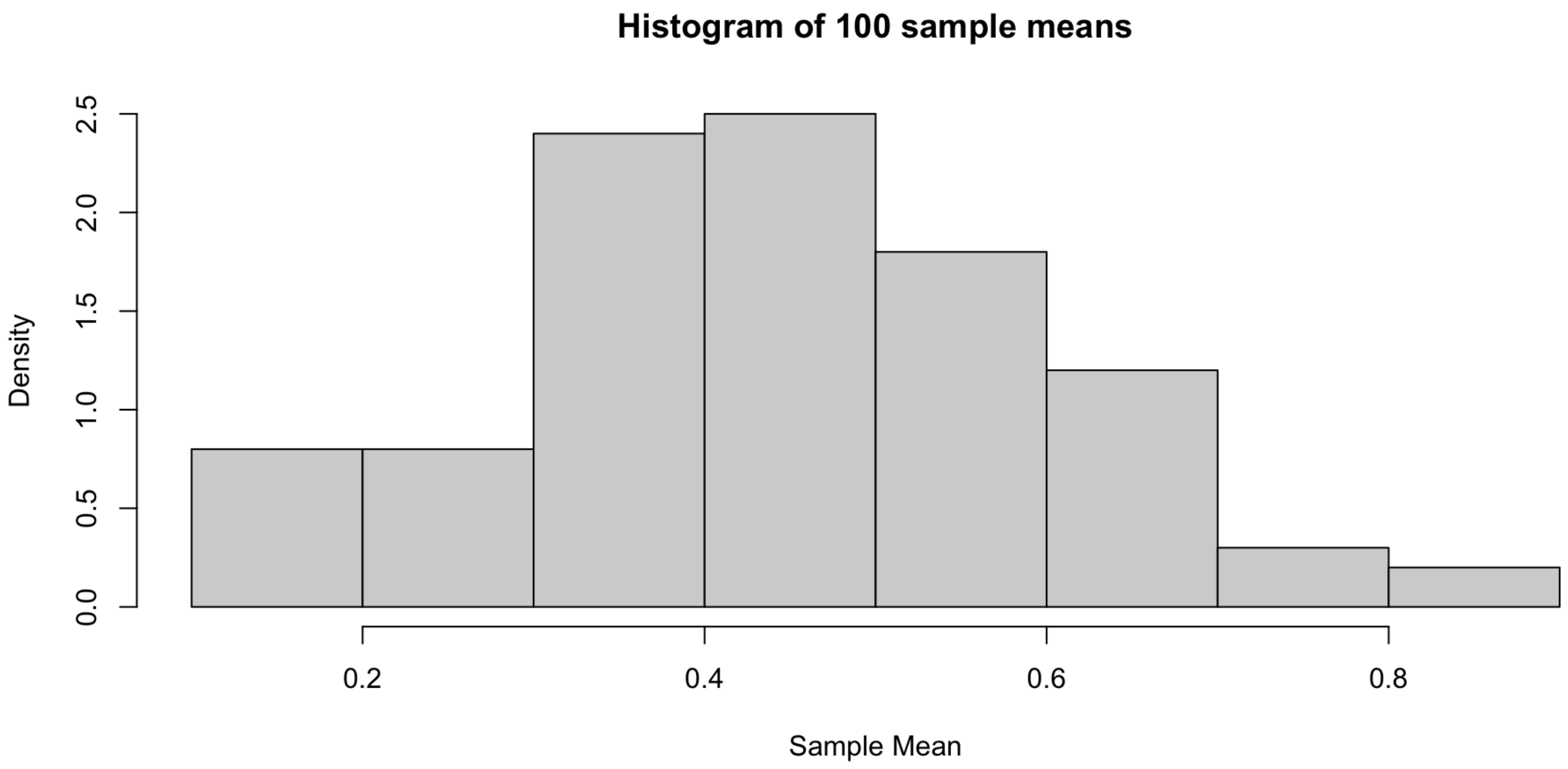
Central Limit Theorem

```r
set.seed(123)
hist(replicate(10, mean(rbinom(10, size = 1, prob = 0.5))),
     main = "Histogram of 10 sample means",
     xlab = "Sample Mean", freq = F)
```



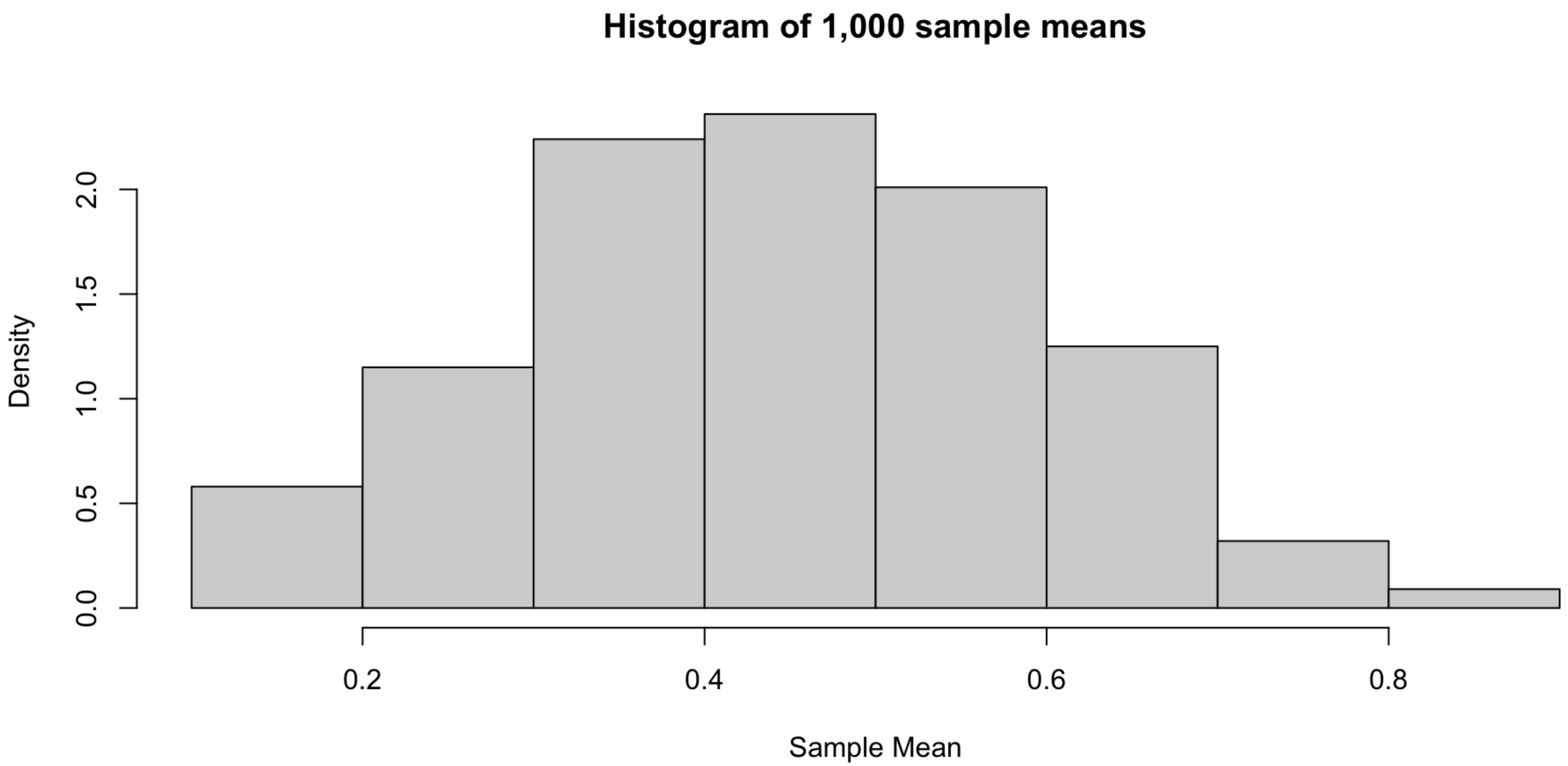
Central Limit Theorem

```r
set.seed(123)
hist(replicate(100, mean(rbinom(10, size = 1, prob = 0.5))),
     main = "Histogram of 100 sample means",
     xlab = "Sample Mean", freq = F)
```



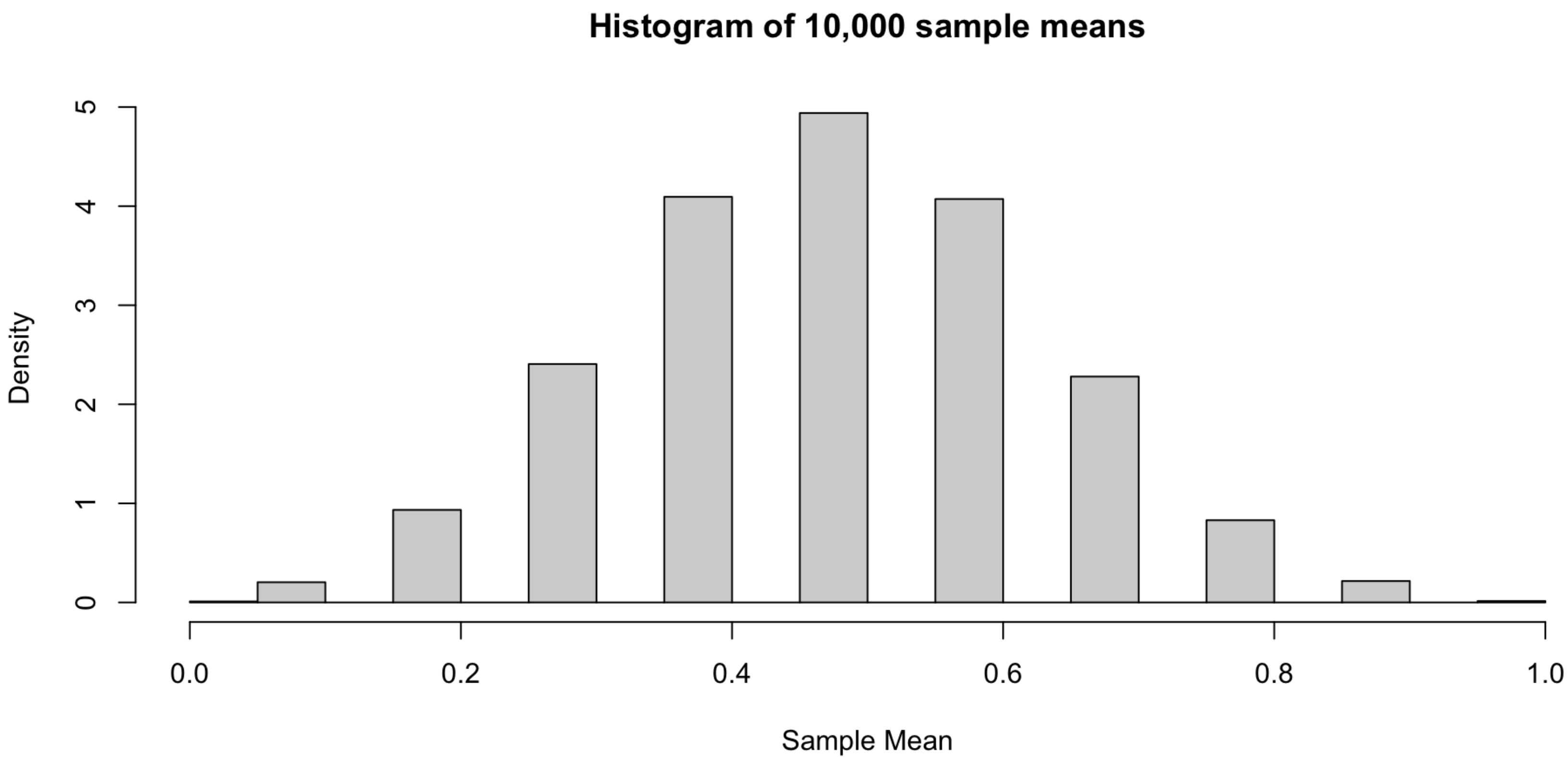
Central Limit Theorem

```r
set.seed(123)
hist(replicate(1000, mean(rbinom(10, size = 1, prob = 0.5))),
     main = "Histogram of 1,000 sample means",
     xlab = "Sample Mean", freq = F)
```



Central Limit Theorem

```r
set.seed(123)
hist(replicate(10000, mean(rbinom(10, size = 1, prob = 0.5))),
     main = "Histogram of 10,000 sample means",
     xlab = "Sample Mean", freq = F)
```



Z-Scores

$$Z = \frac{\bar{X} - E[\bar{X}]}{\sqrt{Var(\bar{X})}}$$

Z-Scores

$$E[\bar{X}] = E[\frac{1}{n}\sum_{i=1}^{n} X_i] = \frac{1}{n}\sum_{i=1}^{n} E[X_i] = \frac{1}{n}\sum_{i=1}^{n} \mu = \frac{n}{n}\mu = \mu$$

$$Var(\bar{X}) = Var(\frac{1}{n}\sum_{i=1}^{n} X_i) = \frac{1}{n^2}\sum_{i=1}^{n} Var(X_i)$$

$$= \frac{1}{n^2} n * \sigma^2 = \frac{1}{n}\sigma^2$$

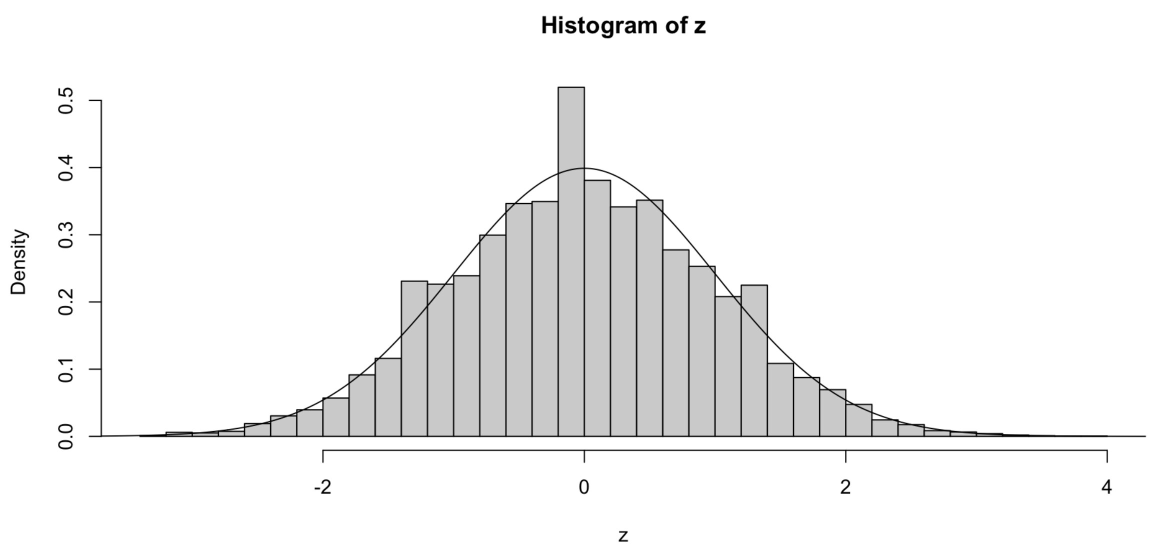
Central Limit Theorem

$$Z \overset{n\to\infty}{\to} N(0, 1)$$

As the number of samples goes to infinity, the distribution of Z is expected to be distributed approximately standard normal (i.e., N(0,1)).

Central Limit Theorem

```
p <- 0.5
n <- 1000
n.samp <- 10000
set.seed(123)
samp <- replicate(n.samp, mean(rbinom(n, 1, p)))
z <- (samp - 0.5)/sqrt(p*(1-p)/n)
stdnorm <- dnorm(seq(-4,40, 0.01))
hist(z, freq = F, breaks = 50)
lines(x = seq(-4,40, 0.01), y =stdnorm)
```



The Standard Normal Distribution

- Mean = 0, Standard Deviation = 1
- 68-95-99 approximation (***rough*** approximation)
  - 68% of density within about 1 standard deviation on either side of mean
  - 95% of density within about 2 standard deviations on either side of mean
  - 99% of density within about 3 standard deviations on either side of mean

Standard Errors

$$SE = \sqrt{Var(\bar{X})} = \sqrt{\frac{Var(X)}{n}} = \frac{\sigma}{\sqrt{n}}$$

- What's the Standard Error of our Sample Mean in the simulation above?

```
sqrt(p*(1-p)/n)
```

```
[1] 0.01581139
```

Confidence Intervals

What is the formula for a 95% Confidence Interval for a sample mean?

$$[\bar{X} - |z_{0.05/2}| * SE, \bar{X} + |z_{0.05/2}| * SE]$$

How do we calculate $|z_{0.05/2}|$?

```
abs(qnorm(0.025, mean = 0, sd = 1))
```

```
[1] 1.959964
```

Confidence Interval Example

What is the 95% confidence interval for the mean of a sample of 1000 from the binomial distribution?

```
set.seed(123)
samp <- rbinom(1000, size = 1, prob = 0.5)
samp.mean <- mean(samp)
samp.mean
```

```
[1] 0.493
```

```
samp.mean - abs(qnorm(0.025))*sqrt(p*(1-p)/n)
```

```
[1] 0.4620102
```

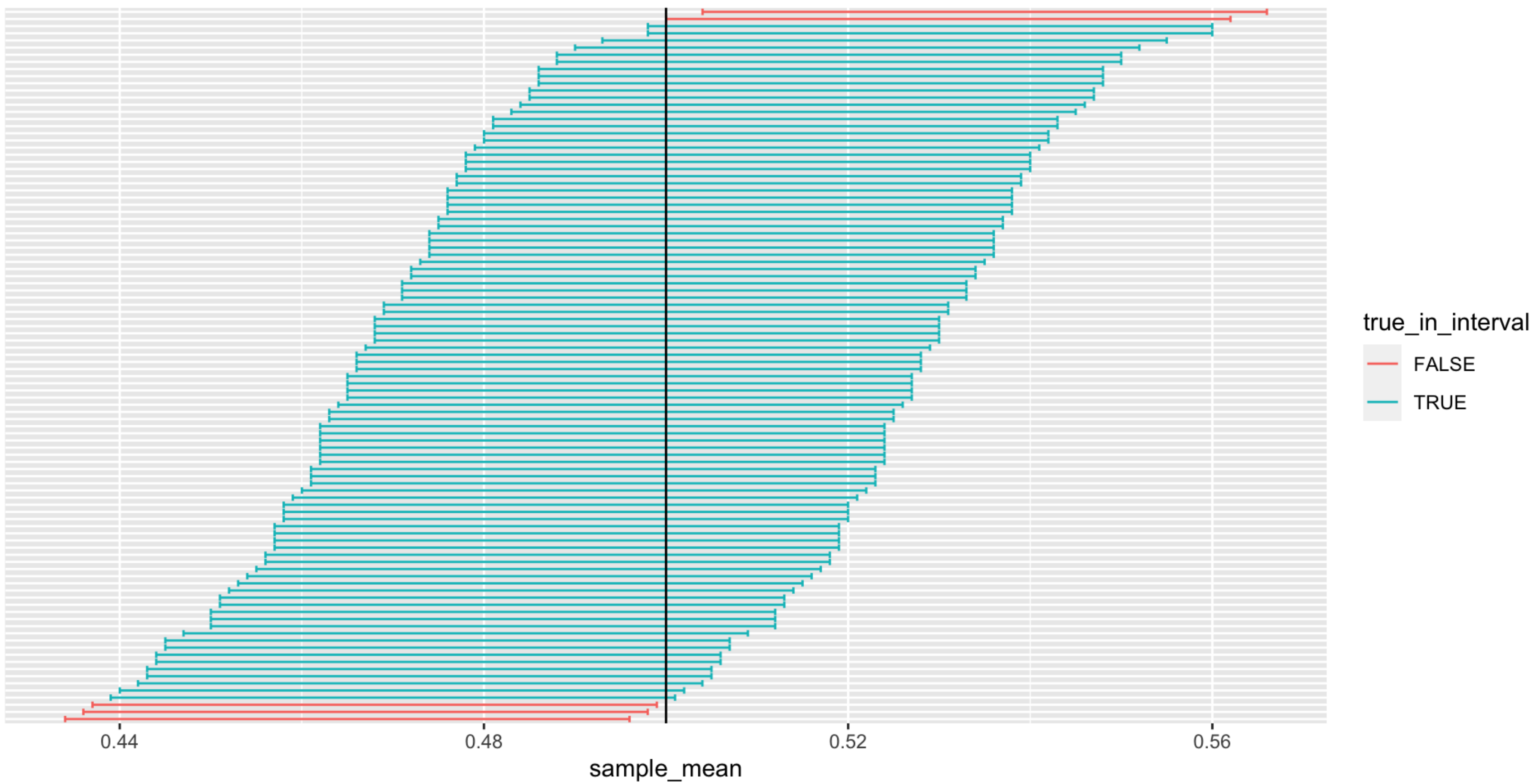
```
samp.mean + abs(qnorm(0.025))* sqrt(p*(1-p)/n)
```

```
[1] 0.5239898
```

95% Confidence Interval

- Does ***not*** mean we are 95% confident the true population mean is within the interval
- It means if we repeated the sampling process many times, about 95% of the confidence intervals would contain the true population mean

95% Confidence Interval



Hypothesis Testing

Two-Sided Hypothesis Test:

- **Null Hypothesis:** $H_0 \ : \mu = 0$
- **Alternative Hypothesis:** $H_1 \ : \mu \neq 0$
- How do we test the null hypothesis?

Hypothesis Testing

- How do we test the null hypothesis?

- If the null hypothesis is true, the Z-Score is $\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} = \frac{\bar{X}-0}{\sigma/\sqrt{n}}$

Using the Central Limit Theorem…

- Z-Scores should be distributed approximately standard normal
- Calculate the probability of observing a Z-Score at least as large in magnitude as observed *if the null hypothesis is true*
  - This is the “p-value”

Hypothesis Testing: Binomial

$$H_0 : p = 0.5$$

$$H_1 : p \neq 0.5$$

```
set.seed(123)
samp <- rbinom(1000, size = 1, prob = 0.5)
samp.mean <- mean(samp)
z.score <- (samp.mean - 0.5)/sqrt(0.5^2/1000)
z.score
```

```
[1] -0.4427189
```

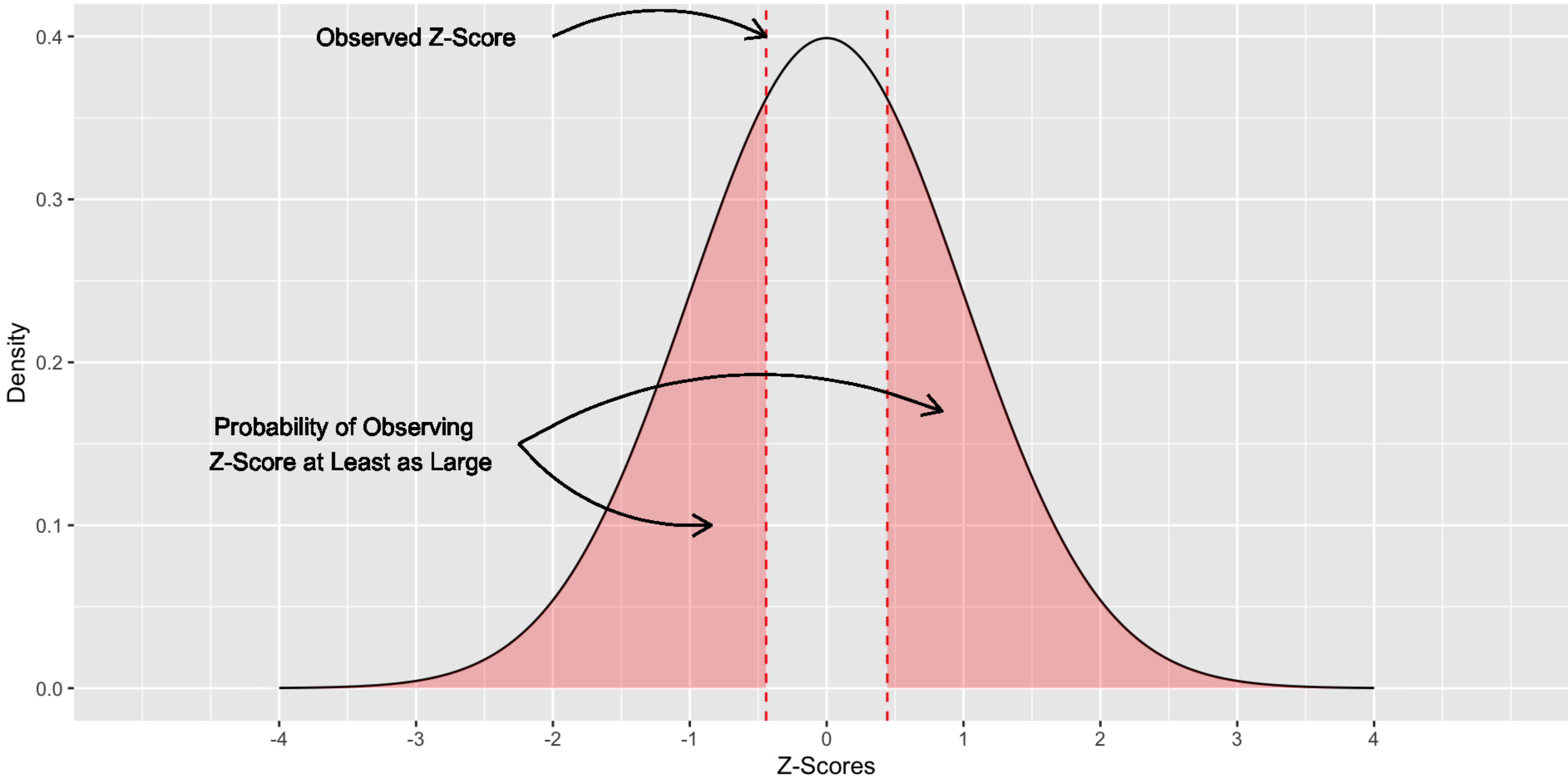
```
2*abs(pnorm(z.score))
```

```
[1] 0.6579691
```

Note: if z.score is positive, the p-value can be calculated:

```
2*(1- pnorm(z.score))
```

Hypothesis Testing: Binomial



Regression

Example from Assignment 5:

```r
progtax <- read.csv("progtax.csv")
```

```r
fit <- lm(toprate~wwi + gdppc+left_seatshare + factor(country) +
            factor(year), data = progtax)
round(summary(fit)$coefficients[c("wwi", "gdppc", "left_seatshare"),],3)
```

```
               Estimate Std. Error t value Pr(>|t|)
wwi              32.918      2.550  12.910    0.000
gdppc            -5.854      2.129  -2.750    0.007
left_seatshare    0.041      0.093   0.444    0.657
```

$$Z.\,Score_{wwi} = \frac{32.918-0}{2.55} = 12.9$$

```r
2*(1-pnorm(12.9))
```

```
[1] 0
```

Regression

```
fit <- lm(toprate~wwi + gdppc+left_seatshare + factor(country) +
            factor(year), data = progtax)
round(summary(fit)$coefficients[c("wwi", "gdppc", "left_seatshare"),],3)
```

```
               Estimate Std. Error t value Pr(>|t|)
wwi              32.918      2.550  12.910    0.000
gdppc            -5.854      2.129  -2.750    0.007
left_seatshare    0.041      0.093   0.444    0.657
```

95% Confidence Interval for $\hat{\beta}_1$:

```
32.918 - qnorm(0.975)*2.55
```

```
[1] 27.92009
```

```
32.918 + qnorm(0.975)*2.55
```

```
[1] 37.91591
```

```
confint(fit)[rownames(confint(fit))%in%
               c("wwi", "gdppc", "left_seatshare"),]
```

```
                     2.5 %     97.5 %
wwi             27.8880533 37.9478276
gdppc          -10.0532222 -1.6540168
left_seatshare  -0.1420066  0.2245598
```

Regression Tables: Stargazer

```
install.packages("stargazer")
library(stargazer)
fit1 <- lm(toprate~wwi, data = progtax)
fit2 <- lm(toprate~wwi + gdppc+left_seatshare+factor(country)+
                factor(year), data = progtax)
stargazer(fit1, fit2, type = "text",
          omit = "factor",
          add.lines = list(c("Country Fixed Effects", "No", "Yes"),
                           c("Year Fixed Effects", "No", "Yes")))
```

Stargazer

	Dependent variable:	
	toprate	
	(1)	(2)
wwi	34.616***	32.918***
	(2.024)	(2.550)
gdppc		-5.854***
		(2.129)
left_seatshare		0.041
		(0.093)
Constant	8.231***	29.573***
	(1.036)	(8.241)
Country Fixed Effects	No	Yes
Year Fixed Effects	No	Yes
Observations	248	228
R2	0.543	0.839
Adjusted R2	0.541	0.804
Residual Std. Error	14.019 (df = 246)	9.262 (df = 187)
F Statistic	292.421*** (df = 1; 246)	24.282*** (df = 40; 187)
Note:		*p<0.1; **p<0.05; ***p<0.01

Fixed Effects

Why might we include country fixed effects?

- Account for omitted country-specific confounders that do not vary over time

Why might we include year fixed effects?

- Account for omitted year-specific confounders that do not vary across countries

Questions