Supplementary Appendix

Risk Aversion in Share Auctions: Estimating Import Rents from TRQs in Switzerland.[*]

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Contents

[*]The main paper is available at https://dx.doi.org/10.2139/ssrn.3397027. Replicator files incl. data set are available at https://github.com/SamuelHaefner/RiskAversionInShareAuctions.

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A Proof of Proposition 1

The proof starts with an auxiliary lemma, which gives the interim utility for any opponent strategy profile μ_{-i} and will be useful later on. To this end, I define

$$W_i(q|b_i,\mu_{-i}) = \\ 1 - \int\limits_{\mathcal{V}^{n-1}} \int\limits_{\mathcal{B}^{n-1}} H_i^b(q)\mathrm{d}\mu_1(b_1|v_1)...\mathrm{d}\mu_{i-1}(b_{i-1}|v_{i-1})\mathrm{d}\mu_{i+1}(b_{i+1}|v_{i+1})...\mathrm{d}\mu_n(b_n|v_n)\mathrm{d}\eta_{-i}(v_{-i}),$$

which returns the (decreasing) probability that the allocated quantity q_i^c for bidder i with valuation v_i strictly exceeds $q \in [0,Q]$ when the submitted demand schedule is b_i and the opponent strategy profile is μ_{-i}. Writing $V_i(q) = \int_0^q v_i(q)\mathrm{d}q$ and $B_i(q) = \int_0^q \beta_{b_i}(q)\mathrm{d}q$ for the respective gross valuation and gross payment accruing to bidder i, we have:

Lemma A.1. *Given an opponent strategy profile $\mu_{-i} \in \mathcal{M}^{n-1}$, the interim utility $\Pi_i(b_i,v_i,\mu_{-i})$ for bidder $i \in \{1,...,n\}$ of type v_i when submitting a bid schedule $b_i \in \mathcal{B}$ is*

$$\Pi_i(b_i,v_i,\mu_{-i}) = \sum_{j=1}^{k} \int\limits_{q^{j-1}}^{q^j} \phi'\left(V_i(q) - B_i(q)\right)\left[v_i(q) - p_i^j\right] W_i(q|b_i,\mu_{-i})\mathrm{d}q. \tag{A.1}$$

Proof of Lemma A.1. Let

$$H_i^{(b_i,v_{-i})}(q) = \int\limits_{\mathcal{B}^{n-1}} H_i^{(b_i,b_{-i})}(q)\mathrm{d}\mu_1(b_1|v_1)...\mathrm{d}\mu_{i-1}(b_{i-1}|v_{i-1})\mathrm{d}\mu_{i+1}(b_{i+1}|v_{i+1})...\mathrm{d}\mu_n(b_n|v_n) \tag{A.2}$$

be the distribution of the quantity q_i^c that bidder i submitting b_i receives when his opponents play according to their strategies in μ_{-i} and the opponent type profile is v_{-i}. Combining (1) and (A.2), the interim utility $\Pi_i(b_i,v_i,\mu_{-i})$ of player i can be written as

$$\Pi_i(b_i,v_i,\mu_{-i}) = \int\limits_{\mathcal{V}^{n-1}} \int_0^Q \phi\left(V_i(q) - B_i(q)\right)\mathrm{d}H_i^{(b_i,v_{-i})}(q)\mathrm{d}\eta_{-i}(v_{-i}). \tag{A.3}$$

Because $\phi\left(V_i(q) - B_i(q)\right)$ is continuous and $H^{(b_i,v_{-i})}(q)$ is monotone, the inner integral of the right-hand side in (A.3) can be integrated by parts (cf. Apostol, 1974, Theorem 7.6), yielding

$$\Pi_i(b_i, v_i, \mu_{-i}) = \int\limits_{\mathcal{V}^{n-1}} \Bigg[-\int_0^Q \phi' \left(V_i(q) - B_i(q)\right) \left[v_i(q) - \beta_{b_i}(q)\right] H_i^{(b_i, v_{-i})}(q)\mathrm{d}q + \phi \left(V_i(q) - B_i(q)\right) H^{(b_i, v_{-i})}(q)\Big|_0^Q \Bigg] \mathrm{d}\eta_{-i}(v_{-i}). \quad \text{(A.4)}$$

We can rewrite (A.4) as

$$\Pi_i(b_i, v_i, \mu_{-i}) = -\int\limits_{\mathcal{V}^{n-1}} \int_0^Q \phi' \left(V_i(q) - B_i(q)\right) \left[v_i(q) - \beta_{b_i}(q)\right] \times H_i^{(b_i, v_{-i})}(q)\mathrm{d}q\mathrm{d}\eta_{-i}(v_{-i}) + \phi \left(V_i(Q) - B_i(Q)\right). \quad \text{(A.5)}$$

Because $f(v_{-i}, q) = \phi' \left(V_i(q) - B_i(q)\right) \left[v_i(q) - \beta_i(q)\right] H^{(b_i, v_{-i})}(q)$ is measurable and bounded on $\mathcal{V}^{n-1} \times [0, Q]$, the Fubini-Tonelli theorem (cf. Rudin, 1970, Theorem 8.8) can be applied to get that (A.5) is

$$\Pi_i(b_i, v_i, \mu_{-i}) = -\int_0^Q \phi' \left(V_i(q) - B_i(q)\right) \left[v_i(q) - \beta_{b_i}(q)\right] \times \int\limits_{\mathcal{V}^{n-1}} H^{(b_i, v_{-i})}(q)\mathrm{d}\eta_{-i}(v_{-i})\mathrm{d}q + \phi \left(V_i(Q) - B_i(Q)\right). \quad \text{(A.6)}$$

By definition we have

$$W_i(q|b_i, \mu_{-i}) = 1 - \int\limits_{\mathcal{V}^{n-1}} H^{(b_i, v_{-i})}(q)\mathrm{d}\eta_{-i}(v_{-i}),$$

which allows to rewrite (A.6) as

$$\Pi_i(b_i, v_i, \mu_{-i}) = -\sum_{j=1}^{k+1} \int_{q_i^{j-1}}^{q_i^j} \phi' \left(V_i(q) - B_i(q)\right) \left[v_i(q) - p_i^j\right] \left(1 - W_i(q|b_i, \mu_{-i})\right)\mathrm{d}q + \phi \left(V_i(Q) - B_i(Q)\right),$$

yielding (A.1), because $W_i(q|b_i, \mu_{-i}) = 0$ for all $q \in (q_i^k, Q]$ holds by assumption (a bidder never wins a quantity for which her price bid was zero). □

Now, for finite natural h, let $\mathcal{B}_{i,h}$ be a discrete action space for bidder i defined as

$$\mathcal{B}_{i,h} = \left\{ \{p_i^j, q_i^j\}_{j=1,\dots,k} \in [P_{i,h} \times Q_h]^k : p_i^j \geq p_i^{j+1}, q_i^j \leq q_i^{j+1}, q_i^{k+1} = Q, p_i^{k+1} = 0 \right\},$$

where

$$P_{i,h} = \left\{ 0, \frac{i}{nh^2}, \frac{1}{h}\left[\overline{p} + \frac{i}{nh}\right], \frac{1}{h}\left[2\overline{p} + \frac{i}{nh}\right], \dots, \frac{1}{h}\left[(h-1)\overline{p} + \frac{i}{nh}\right] \right\}$$
$$Q_h = \left\{ \frac{Q}{h}, 2\frac{Q}{h}, 3\frac{Q}{h}, \dots, Q \right\}.$$

For h large enough we have $\mathcal{B}_{i,h} \subset \mathcal{B}$. Moreover, $\lim_{h\to\infty} \mathcal{B}_{i,h} = \mathcal{B}$. The strategy of the proof is to first establish existence of an equilibrium μ_h^* in the restricted auction with bidder-specific action spaces $\mathcal{B}_{i,h}$ and, second, to use these equilibria to construct a sequence μ_h^* of equilibria whose limit μ^* is an equilibrium of the unrestricted auction with action space $\mathcal{B}$ for all bidders. For this approach it is crucial that ties cannot occur at positive prices by the construction of the action spaces $\mathcal{B}_{i,h}$ (cf. the proof to Lemma A.3 below).

Let $\mathcal{M}_{i,h}$ be the space of distributional strategies on $\mathcal{B}_{i,h} \times \mathcal{V}$ for player i. The next Lemma, which is a direct application of Milgrom and Weber (1985), establishes existence of an equilibrium μ_h^* in the restricted auction for any $h \in \mathbb{N}_+$.

Lemma A.2. *There is an equilibrium $\mu_h^* \in \times_{i\in\{1,\dots,n\}}\mathcal{M}_{i,h}$ in the restricted auction for any h.*

Proof. From the Helly's selection theorem (Exercise 13 in Chapter 7 of Rudin, 1964), the space of decreasing functions from $[0, Q]$ to $[0, \overline{v}]$ is compact. From the Arzela-Ascoli Theorem (Theorem 7.25 in Rudin, 1964), the space of Lipschitz continuous fuctions from $[0, Q]$ to $[0, \overline{v}]$ is compact. Because the intersection of two compact stets is itself compact, we obtain that $\mathcal{V}$ is compact, and thus complete and separable. Because the action spaces $\mathcal{B}_{i,h}$ are finite, they are compact and condition (a) in Proposition 1 of Milgrom and Weber (1985) is satisfied. Together with the type space assumption (A1) the assumptions of Theorem 1 in Milgrom and Weber (1985) are thus satisfied, and we have existence of an equilibrium that we denote by μ_h^*. □

Having existence of an equilibrium $\mu_h^* \in \times_{i\in\{1,\dots,n\}}\mathcal{M}_{i,h}$ allows me to show existence of an equilibrium with a distinct structure that we will need in the following. For $q \in Q_h$, let

$$\theta_i(q; v_i) = \max\left\{p \in P_{i,h} : p \leq v_i(q - Q/h)\right\},$$

and let $\bar{\mathcal{M}}_{i,h} \subset \mathcal{M}_{i,h}$ be the set of strategies on

$$\{(b_i, v_i) \in \mathcal{B}_{i,h} \times \mathcal{V} \,:\, \beta_{b_i}(q) \leq \theta_i(q; v_i),\ \forall q \in Q_h\}\,.$$

That is, the support of the strategies in $\bar{\mathcal{M}}_{i,h}$ consists of bids such that, in the limit $h \to \infty$, the corresponding step functions lie weakly below the marginal valuation function. For further reference, let $\bar{\mathcal{M}} = \lim_{h\to\infty} \bar{\mathcal{M}}_{i,h} \subseteq \mathcal{M}$ be the strategy space in this limit, which is independent of the bidder's identity, because $\lim_{h\to\infty} \mathcal{B}_{i,h} = \mathcal{B}$ as observed above.

Lemma A.3. *An equilibrium μ_h^* satisfying $\mu_h^* \in \times_{i\in\{1,...,n\}} \bar{\mathcal{M}}_{i,h}$ exists for any h.*

Proof. Fix an equilibrium $\mu_h^* \in \times_{i\in\{1,...,n\}} \mathcal{M}_{i,h}$. I first argue that it is without loss of generality to assume that all bids b_i in the support of the equilibrium strategy of any bidder i with valuation $v_i \in V$ satisfy

$$q_i^j \leq q_i^{j+1} - Q/h \text{ whenever } p_i^j > 0$$

for all $j \in \{1, ..., k-1\}$. To see this, consider a bid b_i for which this is not the case, that is, suppose we have b_i with $q_i^j = q_i^{j+1}$ for at least one $j \in \{1, ..., k-1\}$. As the price-quantity-pairs (p_i^{j+1}, q_i^{j+1}) will be ignored by the auctioneer and there is zero probability to win a quantity q for which the price bid is zero, such a bid b_i yields the same payoff as a bid b_i' that is equal to b_i except that the price-quantity-pairs (p_i^{j+1}, q_i^{j+1}) are deleted and there is an equal number of price-quantity pairs $(0, Q)$ appended at the end. For example, if there is one such pair j, then we have

$$b_i' = \left\{(p_i^1, q_i^1), ..., (p_i^j, q_i^j), (p_i^{j+2}, q_i^{j+2}), ..., (p_i^k, q_i^k), (0, Q)\right\}.$$

Any bid b_i' that is thus altered does not change the utility of the other players, and hence for any equilibrium strategy $\mu_{i,h}^*$ having b_i in its support there is an alternative strategy $\mu_{i,h}'$ constructed from $\mu_{i,h}^*$ with all the mass on b_i appropriately shifted to b_i', so that $\mu_h^{*'} = (\mu_{i,h}', \mu_{-i}^*)$ is also an equilibrium.

So, suppose $b_i = \{(p_i^1, q_i^1), ..., (p_i^k, q_i^k)\} \in \mathcal{B}_{i,h}$ with $q_i^j \leq q_i^{j+1} - Q/h$ for $j \in \{1, ..., \ell_i - 1\}$ where $\ell_i \leq k$, and, if $\ell_i < k$, with $(p_i^j, q_i^j) = (0, Q)$ for $j \in \{\ell_i + 1, ..., k\}$ is in the support of the equilibrium strategy of bidder i with valuation v_i. I now show that $\beta_{b_i}(q) \leq \theta_i(q; v_i)$ holds for all $q \in Q_h$. By optimality, we get from interim utility (A.1) that it holds, for any

$j \in \{1, ..., \ell_i\}$ (taking $q_i^{k+1} = Q$ and $p_i^{k+1} = 0$),

$$\begin{aligned}\int_{q_i^{j-1}}^{q_i^j} \phi'\left(V_i(q) - B_i(q)\right)\left[v_i(q) - p_i^j\right] W_i^j(q|b_i, \mu_{-i})\mathrm{d}q \\ + \int_{q_i^j}^{q_i^{j+1}} \phi'\left(V_i(q) - B_i(q)\right)\left[v_i(q) - p_i^{j+1}\right] W_i^{j+1}(q|b_i, \mu_{-i})\mathrm{d}q \geq \\ \int_{q_i^{j-1}}^{q_i^j - Q/h} \phi'\left(V_i(q) - B_i(q)\right)\left[v_i(q) - p_i^j\right] W_i^j(q|b_i', \mu_{-i})\mathrm{d}q \\ + \int_{q_i^j - Q/h}^{q_i^{j+1}} \phi'\left(V_i(q) - B_i(q)\right)\left[v_i(q) - p_i^{j+1}\right] W_i^{j+1}(q|b_i', \mu_{-i})\mathrm{d}q, \quad \text{(A.7)}\end{aligned}$$

where b_i' is equal to b_i except for the j-th quantity point, q_i^j, which is replaced by $q_i^j - Q/h$.

Because ties at positive prices cannot happen and no quantities for which the price bid is zero are ever allocated, the probability to win a certain quantity only depends on the price bid for that quantity. That is, we have $W_i^j(q|b_i, \mu_{-i}) = W_i^j(q|b_i', \mu_{-i})$ on $[q_i^{j-1}, q_i^j - Q/h]$ and $W_i^{j+1}(q|b_i, \mu_{-i}) = W_i^{j+1}(q|b_i', \mu_{-i})$ on $[q_i^j, q_i^{j+1}]$. Consequently, it follows from (A.7) that

$$\begin{aligned}\int_{q_i^j - Q/h}^{q_i^j} \phi'\left(V_i(q) - B_i(q)\right)\left[v_i(q) - p_i^j\right] W_i^j(q|b_i, \mu_{-i})\mathrm{d}q \\ \geq \int_{q_i^j - Q/h}^{q_i^j} \phi'\left(V_i(q) - B_i(q)\right)\left[v_i(q) - p_i^{j+1}\right] W_i^{j+1}(q|b_i', \mu_{-i})\mathrm{d}q. \quad \text{(A.8)}\end{aligned}$$

First, consider step $j = \ell_i$. Because no quantities for which a prize of zero is bid are ever won, condition (A.8) becomes

$$\int_{q_i^{\ell_i} - Q/h}^{q_i^{\ell_i}} \phi'\left(V_i(q) - B_i(q)\right)\left[v_i(q) - p_i^{\ell_i}\right] W_i^{\ell_i}(q|b_i, \mu_{-i})\mathrm{d}q \geq 0,$$

which together with the assumptions that $\phi'(.) > 0$ and that the marginal value v_i is decreasing gives us that

$$p_i^{\ell_i} \leq v_i(q_i^{\ell_i} - Q/h) \quad \text{(A.9)}$$

must hold.

But with inequality (A.9) at hand, I can now argue that $p_i^j \leq v_i(q_i^j - Q/h)$ must hold for every $j \in \{1, ..., \ell_i\}$: Suppose, to the contrary, that it does not hold for some $j < \ell_i$; i.e., we have $p_i^j > v_i(q_i^j - Q/h)$. Because v_i is decreasing, this implies that the left side of (A.8) is strictly negative, so that it must hold that $p_i^{j+1} > v_i(q_i^j)$, which is necessary for right side of (A.8) to be strictly negative, too. But because $q_i^j \leq q_i^{j+1} - Q/h$ holds and v_i is decreasing,

$p_i^{j+1} > v_i(q_i^j)$ in turn implies that $p_i^{j+1} > v_i(q_i^{j+1} - Q/h)$ holds, as well. Repeating this for every $j' > j$, ultimately yields $p_i^{\ell_i} > v_i(q_i^{\ell_i} - Q/h)$, which contradicts (A.9). As we can apply this argument to any $j < \ell_i$, we get that

$$p_i^j \leq v_i(q_i^j - Q/h), \quad \forall j \in \{1, ..., k\}$$

must hold for any v_i, thus giving us the claim. □

Next, consider a sequence of auctions with restricted action space $\times_{i\in\{1,...,n\}}\mathcal{B}_{i,h}$ having equilibria $\mu_h^* \in \times_{i\in\{1,...,n\}}\bar{\mathcal{M}}_{i,h}$ for $h \to \infty$. Because the space $\mathcal{M}$ of probability measures on $\mathcal{B} \times \mathcal{V}$ is compact in the weak*-topology (Milgrom and Weber, 1985), the sequence $\mu_h^* \in \times_{i\in\{1,...,n\}}\bar{\mathcal{M}}_{i,h} \subset \mathcal{M}^n$ has a converging subsequence. Pick such a subsequence and suppose it converges to some μ^*. By Remark 3.1 in Reny (1999) μ^* is an equilibrium in the unrestricted auction if the unrestricted auction with strategy space $\bar{\mathcal{M}}^n$ is better reply secure and if for every $\epsilon > 0$ there is $H > 0$ such that for all $h > H$ the profile μ_h^* in the respective subsequence is an ϵ-equilibrium of the unrestricted auction. In the following I write $U_i(\mu) = \int \Pi_i(b_i, v_i, \mu_{-i})\mathrm{d}\mu_i$ for player i's payoff function, where Π_i is player i's interim utility as defined in Lemma A.1.

Definition 1 (Better-Reply Security, cf. Reny 1999)**.** *Game $G = (\bar{\mathcal{M}}, U_i)_{i\in\{1,...,n\}}$ where each player $i = 1, ..., n$ has a strategy space $\bar{\mathcal{M}}$ and a payoff function $U_i(\mu)$, $\mu \in \bar{\mathcal{M}}^n$, is better-reply secure if whenever (μ^*, u^*) is in the closure of the graph of its vector payoff function U, $\{(\mu, u) : u = U(\mu)\}$, and μ^* is not a Nash equilibrium, then some player i can secure a payoff strictly above u_i^* at μ^*: There exists some $\tilde{\mu}_i$ such that $U_i(\tilde{\mu}_i, \mu_{-i}) > u_i^*$ for all μ_{-i} in some open neighborhood of μ_{-i}^*.*

Definition 2 (ϵ-equilibrium)**.** *A strategy profile $\mu \in \bar{\mathcal{M}}^n$ is an ϵ-equilibrium of game $G = (\bar{\mathcal{M}}, U_i)_{i\in N}$ if for all players $i \in N$ it holds $U_i(\hat{\mu}_i, \mu_{-i}) - U_i(\mu) \leq \epsilon$ for every $\hat{\mu}_i \in \bar{\mathcal{M}}$.*

Step I: Better-Reply Security. I start by showing better reply security, for which I adapt the argument given in Reny (1999) for the multi-unit auction case. For opponent profile $\mu_{-i} \in \bar{\mathcal{M}}^{n-1}$, let

$$B^\epsilon(\mu_{-i}) = \left\{ \mu_i \in \bar{\mathcal{M}} : \left| U_i(\mu_i, \mu_{-i}) - \sup_{\tilde{\mu}_i \in \bar{\mathcal{M}}} U_i(\tilde{\mu}_i, \mu_{-i}) \right| \leq \epsilon \right\}$$

be the set of strategies μ_i that yield utility within $\epsilon > 0$ of the supremum. The following observation is needed below.

Lemma A.4. *Fix $\tilde{\mu}_{-i} \in \bar{\mathcal{M}}^{n-1}$. Then, for every $\epsilon > 0$ sufficiently small and for any $\mu_i \in B^\epsilon(\tilde{\mu}_{-i})$, $U_i(\mu_i, \mu_{-i})$ is continuous in μ_{-i} at $(\mu_i, \tilde{\mu}_{-i})$.*

Proof. By contradiction. Take $\mu_i \in B^\epsilon(\tilde{\mu}_{-i})$ and suppose $U_i(\mu_i, \mu_{-i})$ is not continuous in μ_{-i} at $(\mu_i, \tilde{\mu}_{-i})$. If $U_i(\mu_i, \mu_{-i})$ is not continuous in μ_{-i} at $(\mu_i, \tilde{\mu}_{-i})$, then there must be a bidder $j \in \{1, ..., n\} \setminus i$ and a clearing price p^c such that bidder i and bidder j tie at p^c with positive probability. That is, there are $X, Y \subset \mathcal{V}$ with $\eta_i(X), \eta_j(Y) > 0$ such that both bidders have price points $p_i^{m_i} = p^c$ and $p_j^{m_j} = p^c$ (where the steps m_i and m_j might be distinct for the two bidders) in the support of their strategies $\mu_i(.|v_i)$ and $\mu_j(.|v_j)$ whenever they are of a type $v_i \in X$ and $v_j \in Y$, respectively.

The discontinuity together with the fact that $\sum_{i \in N} q_i^c = Q$ implies that at least one tying bidder is rationed with positive probability. Without loss suppose this to be bidder i. From Assumption (A4) we then obtain that the expected allocated quantity lies in $[q_i^{m_i-1}, \min\{q_i^{m_i}, Q - \lim_{p \searrow p^c} \sum_{j \neq i} \beta_{b_j}^{-1}(p)\})$. Moreover, (A4) implies that bidder i could secure a quantity of at least $\min\{q_i^{m_i}, Q - \lim_{p \searrow p^c} \sum_{j \neq i} \beta_{b_j}^{-1}(p)\}$ by marginally raising the price point $p_i^{m_i}$. Now, recall that it follows from (A2) that there are $X', X'' \subset X_1$ with $\eta_i(X'), \eta_i(X'') > 0$ where $\forall f \in X'$ and $\forall g \in X''$ it holds that $f(q) > g(q)$, $\forall q \in [0, 1]$. Because ϕ is strictly increasing, this gives us that there is indeed a set of bidder i types with strictly positive measure that strictly prefer to avoid the tie by marginally raising $p_i^{m_i}$. As the increase in utility is strict, we have, for any $\epsilon > 0$ sufficiently small, a contradiction to the assumption that $\mu_i \in B^\epsilon(\tilde{\mu}_{-i})$. $\square$

Next, consider some (μ^*, u^*) which is in the closure of the graph of the payoff function (i.e., there is a sequence $\mu_m \to \mu^*$ such that $u^* = \lim_{m \to \infty} U(\mu_m)$) without μ^* being an equilibrium. To show better-reply security, we need to establish that there is some bidder i that can secure a payoff strictly above u_i^* by deviating from μ_i^* even if the other bidders also slightly deviate. Two cases need to be considered: (i) $U(.)$ is continuous at μ^* and (ii) $U(.)$ is not continuous at μ^*.

(i) Consider first the case of $U(.)$ being continuous at μ^*. Then there is a bidder i, an $\epsilon > 0$ small enough, and some $\mu_i \in B^\epsilon(\mu_{-i}^*)$ such that $U_i(\mu_i, \mu_{-i}^*) > U(\mu^*) = u^*$. As $U_i(\mu_i, .)$ is continuous at μ_{-i}^* by Lemma A.4, we have better reply security.

(ii) Second, consider the case of $U(.)$ being discontinuous at μ^*: There must be at least two bidders i and j, a clearing price p^c, and a sequence $\mu_m \to \mu^*$ such that both bidders have a positive measure of types that have price points $p_i^{t_i} = p^c$ and $p_j^{t_j} = p^c$ (where the steps t_i and t_j might be distinct for the two bidders) in the support of their strategies in the limit μ^* but do not tie at p^c for any μ_m along the sequence whenever m is sufficiently high. Without loss, suppose $p_i^{t_i} < p_j^{t_j}$ for any μ_m with sufficiently high m.

To continue, observe that there is some sufficiently small $\epsilon > 0$ and some $\hat{\mu}_i \in B^\epsilon(\mu_{-i}^*)$ such that $U_i(\hat{\mu}_i, \mu_{-i}^*) > U_i(\mu^*)$, which, by the same argument as in the proof to Lemma

A.4, involves a positive mass of types marginally increasing the price point which was equal to the clearing price p^c. But then, $\hat{\mu}_i$ yields for these types a strictly higher utility than $\mu_{i,m}$ does against $\mu_{-i,m}$ for any sufficiently high m (because by marginally raising the respective price point discontinuously raises the winning probability for the respective quantities for all sufficiently high m); i.e. there is $\delta > 0$ and $M > 0$ such that

$$U_i(\hat{\mu}_i, \mu_{-i,m}) - U_i(\mu_m) > \delta, \forall m > M.$$

Because $U_i(\hat{\mu}_i, \mu_{-i})$ is continuous in μ_{-i} at μ^*_{-i} by Lemma A.4, it follows that

$$U_i(\hat{\mu}_i, \mu^*_{-i}) > \lim_{m \to \infty} U_i(\mu_m) = u^*_i,$$

giving us better-reply security in this case, too.

Step II: ϵ-Equilibria. I follow Reny (2011) and show that for every $\epsilon > 0$ there is h high enough such that for every feasible action b_i there is a feasible action $b_{i,h}$ such that the ex-post loss from choosing $b_{i,h}$ rather than b_i is smaller than ϵ, and that this holds uniformly in the strategies μ_{-i} of the other players.

Fix some finite natural h, some bidder i with type v_i and any b_i for which it holds that $\beta_{b_i}(q) \leq \theta_i(q; v_i)$, $\forall q \in Q_h$. If $b_i \in \mathcal{B}_{i,h}$ then we are done. So consider $b_i \notin \mathcal{B}_{i,h}$. Let

$$b_i = \left\{(p_i^1, q_i^1), ..., (p_i^k, q_i^k)\right\},$$

and define

$$b_{i,h} = \left\{(p_{i,h}^1, q_{i,h}^1), ..., (p_{i,h}^k, q_{i,h}^k)\right\},$$

with

$$\begin{aligned} p_{i,h}^j &= \min\left\{p \in P_{i,h} : p \geq p_i^j\right\} \\ q_{i,h}^j &= \max\left\{q \in Q_h : q \leq q_i^j \text{ and } p_{i,h}^j \leq v_i(q - Q/h)\right\}, \end{aligned}$$

for all $j \in \{1, ..., k\}$. Above definitions guarantee that $\beta_{b_{i,h}}(q) \leq \theta_i(q, v_i)$ holds for all $q \in Q_h$, and hence that $b_{i,h}$ is a feasible action, as well as that $q_{i,h}^j \to q_i^j$. The ex-post loss sources from switching from b_i to $b_{i,h}$ are threefold:

1. There might be shares q at which it holds that $\beta_{b_{i,h}}(q) > \beta_{b_i}(q)$ and that are won under b_i, that are also won under b_i^h yet at a higher price. The loss from such quantities is

bounded above by

$$\phi\left(V_i(q_i^k) - B_i(q_i^k)\right) - \phi\left(V_i(q_i^k) - B_i(q_i^k) - \sum_{j=1}^{k}(q_i^j - q_i^{j-1})\frac{\overline{p}}{h}\right). \tag{A.10}$$

2. There might be shares q at which it holds that $\beta_{b_{i,h}}(q) > v_i(q) \geq \beta_{b_i}(q)$ and that are not won under b_i, but that are won under b_i^h. The loss from such quantities is also bounded above by

$$\phi\left(V_i(q_i^k) - B_i(q_i^k)\right) - \phi\left(V_i(q_i^k) - B_i(q_i^k) - \sum_{j=1}^{k}(q_i^j - q_i^{j-1})\frac{\overline{p}}{h}\right). \tag{A.11}$$

3. There might be shares q at which it holds that $\beta_{b_{i,h}}(q) < \beta_{b_i}(q)$ and that are won under b_i, but that are not won under b_i^h. The loss from such quantities is bounded above by

$$\sum_{j=1}^{k} \int_{q_{i,h}^j}^{q_i^j} \phi'\left(V_i(q) - B_i(q)\right) v_i(q)\mathrm{d}q. \tag{A.12}$$

All three bounds (A.10)–(A.12) vanish as $h \to \infty$ independently, and hence uniformly, in the strategies μ_{-i} of the other players, because $\phi'(.)$ is bounded on $\mathbb{R}$. Consequently, we have that μ_h^* is a sequence of ϵ-equilibria of the unrestricted auction where $\epsilon \to 0$ when $h \to \infty$, as required.

We can conclude that an equilibrium exists. Moreover, absence of ties follows from the same argument as in the proof of Lemma A.4: If a tie were to happen with positive probability than there would always be a non-negligible set of types for at least one of the bidders that strictly prefer to avoid the tie.

B Inequality Violations By Auction Groups

This appendix provides more details on the shares of inequality violations in the three auction groups that I use for estimation.

For simplicity I assume that all bidders have the same risk-aversion parameter ρ. Let $\mathcal{T}_a \subset \{1, ..., T\}$ be the set of auctions in auction group $a = 1, 2, 3$. Analogous to the main analysis, I compute

$$\Theta_{Q,a}(\rho) \equiv \frac{1}{|\mathcal{T}_a|} \sum_{t \in \mathcal{T}_a} \left[\frac{1}{\sum_{i \in N_t} \ell_{i,t}} \left[\sum_{i \in N_t} \sum_{j=1}^{\ell_{i,t}-1} \left[\mathbf{1} \left\{ \min_{m \leq j} \{ \hat{\bar{v}}_i^m(\rho) \} < \max_{m \geq j+1} \{ \hat{\underline{v}}_i^m(\rho) \} \right\} \right] \right] \right].$$

For all "last" price-quantity pairs $j = \ell_{i,t}$ and whenever the inequality in (15) holds for a price-quantity pair $j < \ell_{i,t}$, I additionally check the inequalities in (16) for that j. To this end, I let the set of such price-quantity pairs j be $\hat{L}_{i,t}$ and define

$$\Theta_{P,a}(\rho) \equiv \frac{1}{|\mathcal{T}_a|} \sum_{t \in \mathcal{T}_a} \left[\frac{1}{\sum_{i \in N_t} \ell_{i,t}} \left[\sum_{i \in N_t} \sum_{j \in \hat{L}_{it}} \left[\mathbf{1} \{ (F_i^j(b_i, \hat{\underline{v}}_i(\rho), \rho) > 0) \text{ or } (F_i^j(b_i, \hat{\bar{v}}_i(\rho), \rho) < 0) \} \right] \right] \right].$$

From Figure 1, we see that the estimate $\Theta_{Q,a}(\rho) + \Theta_{P,a}(\rho)$ is U-shaped for all three groups. Ideally, the minimizers of this function are the same for all auction groups $a = 1, 2, 3$, which would indicate that risk preferences remain stable across auctions. Yet, the values of ρ that minimize the estimated $\Theta_{Q,a}(\rho) + \Theta_{P,a}(\rho)$ differ between groups $1 - 2$, for which the minimum is at $\rho = 0.0041$, and group 3, for which the minimum is at $\rho = 0.0111$.

Nevertheless, the estimates do not allow to conclude that the assumption of stable risk preferences does not hold. To see this, consider the value that minimizes $\Theta_{Q,a}(\rho) + \Theta_{P,a}(\rho)$ for groups 1 and 2, $\rho = 0.0041$. From Table 1, the 95% confidence interval around the estimated value of $\Theta_{Q,3}(\rho) + \Theta_{P,3}(\rho)$ at $\rho = 0.0041$ overlaps with the confidence interval around $\Theta_{Q,3}(\rho) + \Theta_{P,3}(\rho)$ at $\rho = 0.0111$. In other words, the share of inequality violations at the minimum of $\Theta_{Q,3}(.) + \Theta_{P,3}(.)$ does not substantially differ from that at $\rho = 0.0041$.

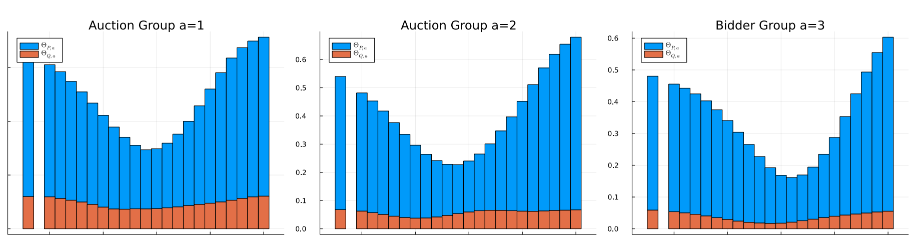

Figure 1: *The fractions of share of inequality (15)–(16) violations among the submitted price-quantity pairs for the different auction groups used for estimation.*

Table 1: *The table describes the estimates of $\Theta_a(\rho)$ for the three different auction groups that I use for estimation.*

	Auction Group 1				Auction Group 2				Auction Group 3			
ρ	$\Theta_{Q,1}$	$\Theta_{P,1}$	$\Theta_{Q,1}+\Theta_{P,1}$	s.e.	$\Theta_{Q,2}$	$\Theta_{P,1}$	$\Theta_{Q,2}+\Theta_{P,2}$	s.e.	$\Theta_{Q,3}$	$\Theta_{P,3}$	$\Theta_{Q,3}+\Theta_{P,3}$	s.e.
0.0	0.12	0.541	0.66	0.0266	0.0676	0.472	0.54	0.0223	0.0587	0.422	0.48	0.0305
$4.54e-5$	0.119	0.492	0.611	0.0243	0.0625	0.419	0.482	0.0202	0.0537	0.402	0.455	0.0299
$7.49e-5$	0.113	0.472	0.584	0.0229	0.057	0.396	0.453	0.0197	0.05	0.392	0.443	0.0291
0.000123	0.106	0.442	0.549	0.0219	0.0506	0.367	0.417	0.019	0.0456	0.379	0.425	0.0283
0.000203	0.0995	0.41	0.509	0.0211	0.0445	0.332	0.376	0.0182	0.0403	0.362	0.403	0.0266
0.000335	0.0907	0.377	0.468	0.019	0.0397	0.295	0.335	0.018	0.0349	0.34	0.375	0.0251
0.000553	0.0805	0.342	0.422	0.0179	0.0376	0.259	0.296	0.0179	0.0294	0.311	0.341	0.0227
0.000912	0.0736	0.305	0.379	0.0186	0.0381	0.226	0.264	0.0188	0.0241	0.28	0.304	0.0196
0.0015	0.0727	0.268	0.341	0.0183	0.0417	0.2	0.242	0.0202	0.0202	0.245	0.266	0.0167
0.00248	0.0744	0.236	0.311	0.0177	0.0471	0.181	0.228	0.0218	0.0181	0.209	0.227	0.0143
0.00409	0.0739	0.22	0.294	0.0185	0.0536	0.174	0.227	0.023	0.0169	0.176	0.192	0.0124
0.00674	0.0749	0.223	0.298	0.0208	0.0594	0.181	0.24	0.0233	0.0175	0.151	0.168	0.0142
0.0111	0.0781	0.241	0.319	0.0234	0.064	0.201	0.265	0.0228	0.0207	0.141	0.161	0.0154
0.0183	0.0813	0.271	0.352	0.0255	0.0655	0.236	0.301	0.0225	0.0253	0.144	0.169	0.017
0.0302	0.0862	0.314	0.4	0.0286	0.0651	0.282	0.347	0.0209	0.0301	0.164	0.194	0.0184
0.0498	0.0908	0.367	0.458	0.0296	0.064	0.333	0.397	0.02	0.0351	0.199	0.235	0.021
0.0821	0.095	0.425	0.52	0.0305	0.0624	0.39	0.452	0.0201	0.0395	0.248	0.287	0.0255
0.135	0.0998	0.481	0.581	0.0282	0.0618	0.449	0.511	0.0208	0.043	0.31	0.353	0.0321
0.223	0.106	0.529	0.636	0.0249	0.0631	0.507	0.57	0.0199	0.0463	0.379	0.425	0.0362
0.368	0.113	0.561	0.674	0.0197	0.0648	0.554	0.619	0.018	0.0494	0.444	0.493	0.0385
0.607	0.119	0.58	0.699	0.0162	0.0659	0.589	0.655	0.0162	0.0527	0.502	0.555	0.0382
1.0	0.121	0.592	0.713	0.0135	0.067	0.612	0.679	0.0144	0.0552	0.548	0.603	0.0361

C An alternative functional form for the cdf of $D_{g,t}(p)$

In this appendix, I analyze an alternative assumption about the distribution family for residual demand. Specifically, I assume that residual demand $D_{g,t}(p)$ follows a log-normal distribution on $(0, \infty)$ and compare it to the Gamma distribution used in the main analysis (cf. Assumption (A6) in the main text).

To this end, I compute the fraction of best-response violations under the assumption that all bidders have the same risk-aversion parameter ρ. Analogous to the main analysis, I compute

$$\Theta_P(\rho) \equiv \frac{1}{T}\sum_{t=1}^{T}\left[\frac{1}{\sum_{i\in N_t}\ell_{i,t}}\left[\sum_{i\in N_t}\sum_{j=1}^{\ell_{i,t}}\left[\mathbf{1}\left\{\min_{m\le j}\{\hat{\overline{v}}_i^m(\rho)\} < \max_{m\ge j+1}\{\hat{\underline{v}}_i^m(\rho)\}\right\}\right]\right]\right]$$

and, letting $\hat{L}_{it}$ be the set of price-quantity pairs j of bidder i in auction t for which $\min_{m\le j}\{\hat{\overline{v}}_i^m(\rho)\} \ge \max_{m\ge j+1}\{\hat{\underline{v}}_i^m(\rho)\}$ holds,

$$\Theta_Q(\rho) \equiv \frac{1}{T}\sum_{t=1}^{T}\left[\frac{1}{\sum_{i\in N_t}\ell_{i,t}}\left[\sum_{i\in N_t}\sum_{j\in\hat{L}_{it}}\left[\mathbf{1}\{(F_i^j(b_i,\hat{\underline{v}}_i(\rho),\rho) > 0) \text{ or } (F_i^j(b_i,\hat{\overline{v}}_i(\rho),\rho) < 0)\}\right]\right]\right].$$

Figure 2 compares the estimate $\Theta_P(\rho) + \Theta_Q(\rho)$ for the log-normal (left panel) and the gamma distribution (right panel). Under either assumptions, $\Theta_P(\rho) + \Theta_Q(\rho)$ is U-shaped and its the lowest value is at $\rho = 0.0067$. The numbers in Table 2 show that the fraction of best response violations are indeed comparable under the two assumptions. I conclude that the posited risk preference has a much higher impact on model fit than the specific assumption about the distribution of residual demand.

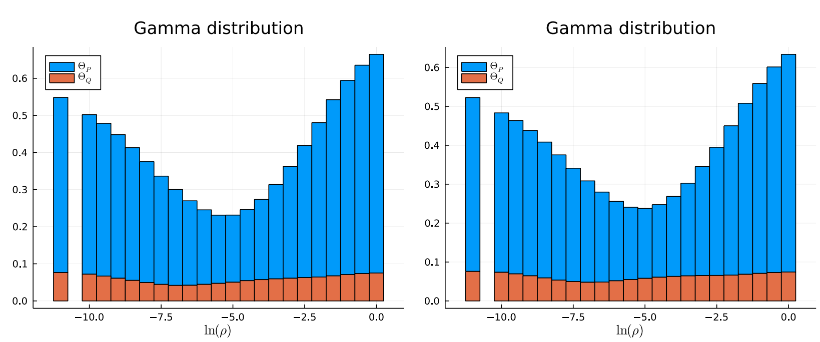

Figure 2: *The left graph shows the share of inequality (15)–(16) violations among the price-quantity pairs when W_i^* and w_i^* are estimated under the assumption of a log normal distribution for $D_{g,t}(p)$. The right graph shows the shares under the assumption of a gamma distribution.*

Table 2: *The tables show the values of $\Theta_P(\rho) + \Theta_Q(\rho)$ when W_i^* and w_i^* are estimated under the assumption of a log normal distribution for $D_{g,t}(p)$ and when assuming a gamma distribution (as in the main analysis). The reported standard errors are those of $\Theta_P(\rho) + \Theta_Q(\rho)$.*

Gamma distribution

ρ	Θ_Q	Θ_P	$\Theta_Q + \Theta_P$	s.e.
0.0	0.0762	0.472	0.548	0.0141
$4.54e-5$	0.0722	0.43	0.502	0.0131
$7.49e-5$	0.0671	0.412	0.479	0.0128
0.000123	0.0612	0.387	0.448	0.0125
0.000203	0.0552	0.358	0.413	0.0119
0.000335	0.0494	0.326	0.375	0.0114
0.000553	0.0445	0.292	0.336	0.0109
0.000912	0.0418	0.258	0.3	0.0108
0.0015	0.0422	0.228	0.27	0.0113
0.00248	0.0446	0.201	0.245	0.0115
0.00409	0.0473	0.184	0.231	0.012
0.00674	0.0506	0.181	0.231	0.0125
0.0111	0.0544	0.192	0.246	0.0125
0.0183	0.0572	0.216	0.273	0.0127
0.0302	0.0594	0.254	0.314	0.0125
0.0498	0.0613	0.301	0.363	0.0126
0.0821	0.0627	0.356	0.419	0.0134
0.135	0.0645	0.416	0.481	0.0146
0.223	0.0674	0.475	0.542	0.015
0.368	0.0707	0.524	0.594	0.0144
0.607	0.0734	0.562	0.635	0.0137
1.0	0.0753	0.589	0.665	0.0125

Log-normal distribution

ρ	Θ_Q	Θ_P	$\Theta_Q + \Theta_P$	s.e.
0.0	0.0758	0.447	0.523	0.0153
$4.54e-5$	0.0737	0.41	0.483	0.0145
$7.49e-5$	0.0695	0.394	0.464	0.0141
0.000123	0.0645	0.374	0.438	0.0138
0.000203	0.0591	0.349	0.408	0.0136
0.000335	0.0536	0.322	0.375	0.0133
0.000553	0.0497	0.291	0.341	0.013
0.000912	0.048	0.26	0.308	0.0128
0.0015	0.0486	0.231	0.28	0.0131
0.00248	0.0517	0.204	0.256	0.0131
0.00409	0.0547	0.186	0.241	0.0132
0.00674	0.058	0.18	0.238	0.0136
0.0111	0.061	0.186	0.247	0.0129
0.0183	0.0629	0.206	0.269	0.0131
0.0302	0.0643	0.238	0.303	0.013
0.0498	0.065	0.28	0.345	0.0132
0.0821	0.0654	0.33	0.395	0.0138
0.135	0.0664	0.384	0.45	0.0143
0.223	0.0684	0.439	0.508	0.0149
0.368	0.0707	0.488	0.559	0.0145
0.607	0.0727	0.528	0.601	0.0144
1.0	0.0743	0.559	0.634	0.0137

D All Estimates

The following table reports the estimated bounds for all auctions $t = 1, ..., 39$. The estimates are obtained using the standard bounds (Tight = no) both under risk neutrality ($\vec{\rho} = (0, 0, 0)$) and under risk aversion ($\vec{\rho} = \vec{\rho}^*$), as well as using the tighter bounds (Tight = yes) under risk aversion. Estimates are bagged from 200 bootstrap runs.

	AvP_l^{pre}			AvP_u^{pre}			AvP_l^{post}			AvP_u^{pre}			AvP_l^{ratio}			AvP_u^{ratio}		
$\vec{\rho}$	$\vec{0}$	$\vec{\rho}^*$	$\vec{\rho}^*$	0	$\vec{\rho}^*$	$\vec{\rho}^*$	$\vec{0}$	$\vec{\rho}^*$	$\vec{\rho}^*$	0	$\vec{\rho}^*$	$\vec{\rho}^*$	$\vec{0}$	$\vec{\rho}^*$	$\vec{\rho}^*$	$\vec{0}$	$\vec{\rho}^*$	$\vec{\rho}^*$
Tight	no	no	yes	no	no	yes	no	no	yes	no	no	yes	no	no	yes	no	no	yes
Auction																		
1	8.062	7.865	7.954	14.86	11.98	9.14	0.249	0.05164	0.1413	7.049	4.166	1.327	0.02643	0.006164	0.02189	0.4632	0.4171	0.1948
2	8.923	7.283	7.5	15.53	14.28	11.81	1.734	0.09349	0.3102	8.341	7.089	4.621	0.1295	0.01481	0.04725	0.5201	0.4735	0.2967
3	6.429	6.334	6.603	12.65	11.39	8.554	0.1306	0.03564	0.3044	6.355	5.089	2.255	0.02141	0.007634	0.02784	0.53	0.4961	0.2406
4	10.02	9.92	10.07	17.33	15.06	12.0	0.1519	0.05198	0.2043	7.467	5.191	2.134	0.01887	0.003664	0.02642	0.3978	0.3802	0.1849
5	15.3	10.84	11.82	20.26	19.18	17.31	5.511	1.055	2.035	10.47	9.399	7.53	0.3081	0.04688	0.1043	0.528	0.4878	0.3857
6	8.326	7.371	7.556	14.15	12.94	10.27	1.053	0.09839	0.2834	6.873	5.666	2.994	0.1124	0.01285	0.03467	0.4634	0.4142	0.2456
7	9.894	9.687	9.875	15.51	14.07	11.0	0.2652	0.05822	0.246	5.885	4.437	1.369	0.02613	0.00656	0.03945	0.4256	0.376	0.1721
8	15.52	12.35	13.22	20.41	20.0	18.41	3.923	0.7523	1.625	8.816	8.405	6.814	0.2556	0.05444	0.1122	0.4409	0.4292	0.3517
9	14.73	9.73	10.08	18.86	16.46	13.87	6.328	1.325	1.674	10.46	8.05	5.469	0.3529	0.08213	0.1324	0.5572	0.5047	0.3851
10	7.433	7.32	7.444	13.02	12.68	9.607	0.1492	0.03622	0.1599	5.734	5.401	2.324	0.02192	0.006211	0.02076	0.4624	0.4431	0.2045
11	5.072	4.928	5.852	14.3	11.18	8.611	0.1841	0.0403	0.9642	9.416	6.291	3.723	0.0366	0.01216	0.08319	0.6465	0.5713	0.3645
12	7.497	7.381	7.469	13.16	13.05	8.934	0.1384	0.02224	0.1099	5.803	5.687	1.575	0.0137	0.003632	0.01837	0.4673	0.4574	0.2113
13	6.188	6.11	6.253	11.43	10.57	7.364	0.1077	0.02998	0.1731	5.346	4.486	1.284	0.02242	0.00611	0.0181	0.5047	0.4808	0.2143
14	6.97	6.603	6.7	13.91	11.83	8.085	0.4087	0.04203	0.1389	7.352	5.273	1.524	0.02993	0.005866	0.01697	0.5554	0.5268	0.2512
15	9.067	8.655	8.746	16.44	14.58	10.48	0.4736	0.06122	0.1523	7.85	5.991	1.891	0.04294	0.007615	0.02424	0.5023	0.4526	0.2225
16	14.37	10.21	10.59	20.13	18.06	16.08	5.001	0.8435	1.222	10.77	8.7	6.711	0.2702	0.04632	0.08404	0.5421	0.5039	0.3657
17	10.18	9.344	9.489	18.63	14.04	11.57	0.9477	0.112	0.2569	9.395	4.803	2.334	0.05908	0.008032	0.03041	0.4992	0.4171	0.2158
18	10.45	8.526	8.719	17.35	14.98	12.31	2.123	0.1951	0.3882	9.017	6.651	3.984	0.1675	0.01663	0.0464	0.5124	0.4472	0.2853
19	7.52	6.777	6.889	14.48	14.02	9.39	0.8158	0.07265	0.1847	7.772	7.313	2.686	0.09242	0.01302	0.03644	0.5393	0.5172	0.2842
20	7.146	6.602	6.673	13.58	12.02	8.264	0.5976	0.05305	0.1244	7.036	5.469	1.715	0.06406	0.008968	0.02484	0.5538	0.5115	0.2596
21	7.359	7.045	7.149	14.52	12.73	8.691	0.361	0.04753	0.1506	7.522	5.733	1.693	0.03299	0.006014	0.01818	0.5705	0.5158	0.2558
22	8.78	8.461	8.534	14.35	12.59	9.792	0.3981	0.07938	0.1517	5.963	4.205	1.41	0.04036	0.005565	0.02182	0.5044	0.4489	0.2106
23	9.636	8.601	8.74	15.74	13.6	10.56	1.268	0.2339	0.3728	7.367	5.229	2.195	0.07917	0.01628	0.03721	0.5033	0.438	0.2338
24	14.1	11.73	12.07	19.9	18.74	15.91	2.711	0.3451	0.6884	8.509	7.353	4.519	0.157	0.02057	0.06031	0.4309	0.4065	0.2806
25	17.34	11.81	12.18	20.46	19.71	17.48	8.24	2.702	3.072	11.36	10.6	8.376	0.4365	0.1123	0.167	0.5635	0.5345	0.4437
26	18.28	16.43	16.65	20.52	20.35	20.09	4.347	2.495	2.72	6.586	6.413	6.154	0.1949	0.08618	0.114	0.3228	0.3207	0.3049
27	16.65	16.09	16.22	20.53	20.53	20.07	1.867	1.308	1.434	5.744	5.744	5.283	0.1359	0.0962	0.1157	0.2838	0.2838	0.2676
28	7.156	6.84	6.94	12.59	11.47	8.343	0.3662	0.04965	0.1501	5.796	4.679	1.554	0.0392	0.00701	0.01671	0.4847	0.4689	0.2217
29	5.213	5.089	5.734	17.63	11.09	9.128	0.177	0.05306	0.6979	12.59	6.049	4.092	0.04471	0.01368	0.06131	0.6541	0.4761	0.2914
30	6.283	6.053	6.433	13.38	12.05	7.941	0.2629	0.033	0.413	7.36	6.035	1.922	0.03236	0.006572	0.02413	0.5373	0.489	0.2337
31	9.956	9.854	9.875	20.53	20.53	14.4	0.1153	0.01277	0.03401	10.69	10.69	4.555	0.03395	0.004217	0.006088	0.5183	0.5183	0.2857
32	17.78	15.77	16.62	20.52	20.5	20.15	6.25	4.236	5.087	8.988	8.968	8.616	0.3215	0.1944	0.2973	0.451	0.4496	0.4305
33	14.74	14.15	14.19	20.53	20.53	20.17	1.264	0.6704	0.7173	7.053	7.053	6.691	0.07954	0.05124	0.0592	0.3542	0.3542	0.3395
34	8.997	8.498	8.593	14.16	12.64	9.783	0.5712	0.07248	0.1679	5.74	4.216	1.358	0.05626	0.007943	0.03634	0.4506	0.4069	0.2062
35	9.244	7.382	7.503	15.27	13.99	11.45	1.988	0.1266	0.2473	8.013	6.739	4.196	0.1723	0.02037	0.04784	0.5396	0.4791	0.3047
36	13.65	13.54	13.58	20.53	20.53	17.85	0.7469	0.6367	0.6788	7.625	7.625	4.948	0.08859	0.07047	0.08421	0.3719	0.3719	0.3399
37	13.42	10.55	10.65	18.93	17.25	15.36	3.483	0.6112	0.7156	8.989	7.316	5.424	0.2387	0.04107	0.06326	0.4759	0.4179	0.2995
38	16.92	13.33	13.82	20.36	19.1	16.75	5.91	2.317	2.813	9.351	8.093	5.734	0.3174	0.1094	0.1826	0.4647	0.4414	0.3686
39	13.89	13.87	13.9	20.53	20.53	20.21	0.2205	0.1923	0.2272	6.857	6.857	6.536	0.01431	0.01262	0.0193	0.3344	0.3344	0.3102

The following table reports the standard errors of the estimated bounds for all auctions $t = 1, ..., 39$. These are bootstrap standard errors obtained from 200 bootstrap estimates.

	AvP_l^{pre} (se)			AvP_u^{pre} (se)			AvP_l^{post} (se)			AvP_u^{pre} (se)			AvP_l^{ratio} (se)			AvP_u^{ratio} (se)		
$\vec{\rho}$	$\vec{0}$	$\vec{\rho}^*$	$\vec{\rho}^*$	0	$\vec{\rho}^*$	$\vec{\rho}^*$	$\vec{0}$	$\vec{\rho}^*$	$\vec{\rho}^*$	0	$\vec{\rho}^*$	$\vec{\rho}^*$	$\vec{0}$	$\vec{\rho}^*$	$\vec{\rho}^*$	$\vec{0}$	$\vec{\rho}^*$	$\vec{\rho}^*$
Tight	no	no	yes	no	no	yes	no	no	yes	no	no	yes	no	no	yes	no	no	yes
Auc.																		
1	0.05963	0.01559	0.03446	0.7331	0.05272	0.08192	0.05963	0.01559	0.03446	0.7331	0.05272	0.08192	0.005408	0.001487	0.00424	0.01405	0.002529	0.00753
2	0.9759	0.09121	0.1557	0.6734	1.199	1.116	0.9759	0.09121	0.1557	0.6734	1.199	1.116	0.0335	0.005442	0.008712	0.01753	0.01262	0.01894
3	0.03343	0.005291	0.1622	0.7165	0.1679	0.2138	0.03343	0.005291	0.1622	0.7165	0.1679	0.2138	0.004987	0.001176	0.004838	0.01441	0.003731	0.008997
4	0.1375	0.08587	0.119	1.664	1.357	1.487	0.1375	0.08587	0.119	1.664	1.357	1.487	0.005326	0.001125	0.0141	0.0146	0.007501	0.0116
5	2.052	1.024	1.136	0.2484	0.7869	0.8863	2.052	1.024	1.136	0.2484	0.7869	0.8863	0.0932	0.0387	0.05051	0.003634	0.01962	0.03315
6	0.3482	0.07454	0.1449	0.5714	0.5542	0.5895	0.3482	0.07454	0.1449	0.5714	0.5542	0.5895	0.02796	0.004149	0.006559	0.015	0.01629	0.02196
7	0.06018	0.004436	0.09371	0.8701	0.04275	0.08896	0.06018	0.004436	0.09371	0.8701	0.04275	0.08896	0.005967	0.001459	0.008243	0.02622	0.002376	0.006998
8	1.466	0.7889	0.9742	0.1578	0.3147	0.7421	1.466	0.7889	0.9742	0.1578	0.3147	0.7421	0.06767	0.04775	0.05136	0.002774	0.008585	0.03321
9	1.078	0.8933	0.8822	0.6542	0.7169	0.8844	1.078	0.8933	0.8822	0.6542	0.7169	0.8844	0.04222	0.04088	0.04235	0.01313	0.01925	0.03261
10	0.04561	0.008117	0.04711	0.1299	0.3373	0.4768	0.04561	0.008117	0.04711	0.1299	0.3373	0.4768	0.005863	0.001037	0.003896	0.007159	0.00253	0.006331
11	0.07792	0.02578	0.7504	1.281	0.6078	0.7602	0.07792	0.02578	0.7504	1.281	0.6078	0.7602	0.006916	0.00386	0.01451	0.02008	0.009029	0.01989
12	0.04095	0.004613	0.03033	0.1431	0.04805	0.1101	0.04095	0.004613	0.03033	0.1431	0.04805	0.1101	0.00412	0.0007904	0.003838	0.005914	0.004014	0.008006
13	0.01807	0.002419	0.1336	0.6816	0.1443	0.2048	0.01807	0.002419	0.1336	0.6816	0.1443	0.2048	0.004145	0.0005787	0.003959	0.01383	0.003834	0.008007
14	0.08761	0.00354	0.03362	0.6329	0.0789	0.1229	0.08761	0.00354	0.03362	0.6329	0.0789	0.1229	0.005504	0.0009061	0.001966	0.01031	0.005311	0.01056
15	0.137	0.003981	0.04092	0.8169	0.3911	0.3243	0.137	0.003981	0.04092	0.8169	0.3911	0.3243	0.01105	0.0009605	0.003403	0.01238	0.009275	0.012
16	2.239	0.7352	0.8047	0.3373	1.097	1.391	2.239	0.7352	0.8047	0.3373	1.097	1.391	0.08641	0.02959	0.03702	0.005538	0.01606	0.03374
17	0.3734	0.04881	0.07591	1.057	0.816	0.8661	0.3734	0.04881	0.07591	1.057	0.816	0.8661	0.01845	0.001748	0.004596	0.01999	0.01387	0.02165
18	0.9614	0.1743	0.2096	0.7639	0.6388	0.6476	0.9614	0.1743	0.2096	0.7639	0.6388	0.6476	0.0428	0.006756	0.009165	0.01651	0.01709	0.02645
19	0.2368	0.04576	0.0494	0.4878	0.188	0.1826	0.2368	0.04576	0.0494	0.4878	0.188	0.1826	0.01943	0.003689	0.004915	0.01281	0.006208	0.01288
20	0.1716	0.02227	0.02942	1.023	0.263	0.2431	0.1716	0.02227	0.02942	1.023	0.263	0.2431	0.01109	0.002131	0.003207	0.01125	0.00647	0.01101
21	0.07307	0.003051	0.02425	0.5043	0.2533	0.2395	0.07307	0.003051	0.02425	0.5043	0.2533	0.2395	0.00597	0.0004045	0.00169	0.008328	0.003754	0.006498
22	0.2452	0.05366	0.1914	0.7754	0.5075	0.6816	0.2452	0.05366	0.1914	0.7754	0.5075	0.6816	0.01638	0.002514	0.004908	0.01844	0.007671	0.0145
23	0.4842	0.1766	0.4547	0.7571	0.8822	1.094	0.4842	0.1766	0.4547	0.7571	0.8822	1.094	0.02102	0.006922	0.008912	0.01993	0.01298	0.02181
24	1.137	0.373	0.5333	0.5084	0.6961	0.9112	1.137	0.373	0.5333	0.5084	0.6961	0.9112	0.05167	0.0171	0.03137	0.01079	0.01921	0.04197
25	1.434	1.786	1.785	0.1336	0.6796	0.899	1.434	1.786	1.785	0.1336	0.6796	0.899	0.05544	0.07216	0.07396	0.002373	0.01502	0.02786
26	1.237	1.292	1.219	0.03662	0.2365	0.3838	1.237	1.292	1.219	0.03662	0.2365	0.3838	0.07454	0.05337	0.05039	0.0006282	0.003016	0.007336
27	0.6606	0.747	0.7502	$3.562e-15$	$4.681e-5$	0.0223	0.6606	0.747	0.7502	$1.781e-15$	$4.681e-5$	0.0223	0.03359	0.03876	0.04047	$5.565e-17$	$4.101e-5$	0.0005831
28	0.08719	0.009087	0.3149	1.409	0.7118	0.7736	0.08719	0.009087	0.3149	1.409	0.7118	0.7736	0.007371	0.0009051	0.001982	0.009746	0.004094	0.007986
29	0.08862	0.01141	0.7468	1.644	1.205	1.204	0.08862	0.01141	0.7468	1.644	1.205	1.204	0.01328	0.001921	0.008364	0.03384	0.01625	0.02674
30	0.04732	0.003367	0.4231	0.8076	0.1637	0.3618	0.04732	0.003367	0.4231	0.8076	0.1637	0.3618	0.005172	0.0005732	0.006774	0.01527	0.003747	0.007452
31	0.05949	0.008655	0.02569	$3.562e-15$	$3.562e-15$	0.05083	0.05949	0.008655	0.02569	$3.562e-15$	$3.562e-15$	0.05083	0.01495	0.002789	0.003781	$2.226e-16$	$2.226e-16$	0.005001
32	1.221	1.956	1.685	0.03242	0.06155	0.2267	1.221	1.956	1.685	0.03242	0.06155	0.2267	0.04734	0.07516	0.05336	0.001142	0.002561	0.009974
33	0.5691	0.449	0.4753	$3.562e-15$	$3.562e-15$	$3.562e-15$	0.5691	0.449	0.4753	0.0	0.0	$8.904e-16$	0.03088	0.02805	0.03264	$5.565e-17$	$5.565e-17$	$5.565e-17$
34	0.1847	0.02706	0.0421	0.7643	0.0836	0.09069	0.1847	0.02706	0.0421	0.7643	0.0836	0.09069	0.01577	0.002159	0.006224	0.01241	0.006748	0.01101
35	0.7686	0.09936	0.1208	0.7358	1.346	1.518	0.7686	0.09936	0.1208	0.7358	1.346	1.518	0.04485	0.007743	0.008539	0.01696	0.02093	0.03224
36	0.2682	0.2866	0.2985	$3.562e-15$	$3.562e-15$	$3.562e-15$	0.2682	0.2866	0.2985	$1.781e-15$	$1.781e-15$	0.0	0.0374	0.03523	0.04335	$1.113e-16$	$1.113e-16$	$5.565e-17$
37	1.119	0.4444	0.4411	0.7626	0.8035	0.9922	1.119	0.4444	0.4411	0.7626	0.8035	0.9922	0.04713	0.02021	0.01989	0.01655	0.01872	0.03151
38	1.438	1.405	1.443	0.3243	0.6475	1.059	1.438	1.405	1.443	0.3243	0.6475	1.059	0.07208	0.06888	0.07776	0.004784	0.01242	0.03825
39	0.1423	0.1246	0.1416	$7.123e-15$	$7.123e-15$	0.0	0.1423	0.1246	0.1416	$8.904e-16$	$8.904e-16$	0.0	0.008915	0.008164	0.01333	$2.783e-16$	$2.783e-16$	$5.565e-17$

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