Fundamentals of Electric Circuits Notes

Last Updated: August 23, 2024

Contents

Notes based off of the textbook *Fundamentals of Electric Circuits* and the introductory circuits course taught by professor Stickel.

1 Basic Concepts

1.1 Charge and Current

- The only charges that occur in nature are integral amounts of $e = -1.602 \times 10^{-19}C$ which is the amount of charge in an electron.
- Charge can neither be created nor destroyed.
- Current is the rate of change of charge.
$$I = \frac{\mathrm{d}Q}{\mathrm{d}t} \tag{1}$$
- By convention, current flow is the movement of positive charges.
- Also by convention, upper case letters are usually used in DC circuits while lowercase letters are used in AC circuits.
- A direct current (DC) flows in only one direction and can be constant or time varying.
- An alternating current (AC) is a current that changes direction with respect to time.

1.2 Voltage

- Voltage or potential difference is the energy required to move a unit charge from a reference point to another reference point measured in volts (V).
$$V = \frac{\mathrm{d}W}{\mathrm{d}Q} \tag{2}$$
- $$V_{ab} = V_a - V_b \tag{3}$$

1.3 Power and Energy

1. Power is the time rate of expending or absorbing energy measured in watts (W).
$$P = \frac{\mathrm{d}W}{\mathrm{d}t} \tag{4}$$
$$= VI \tag{5}$$
2. Energy can be determined by the relationship of charge and voltage.
$$E = QV \tag{6}$$
3. By the passive sign convention, power is positive when current enters the positive terminal of an element (absorbing), and power is negative when current enters through the negative terminal (supplying).
4. By conservation of energy, the algebraic sum of power in a circuit at any instant in time must be 0.

1.4 Circuit Elements

- An ideal independent source is an active element that provide a specified voltage or current that is independent of other circuit elements.
- An ideal dependent source is an active element in which the source quantity is controlled by another voltage or current.
- Dependent sources are usually designated by diamond shaped symbols.
- There are four possible types of dependent sources:
  - A voltage-controlled voltage source.
  - A current-controlled voltage source.
  - A voltage-controlled current source.
  - A current-controlled current source.

(a) A Controlled Voltage Source. (b) A Controlled Current Source.

Figure 1: Symbols for dependent sources.

2 Basic Laws

2.1 Resistors

- The resistance of any material with a uniform cross-sectional area A depends on A, its length l, and the resistivity of the material ρ.

$$R = \rho \frac{l}{A} \tag{7}$$

- Ohm's Law states that the voltage is directly proportional to the current flowing the resister.

$$V = IR \tag{8}$$

- Resistance (R) is measured in ohms (Ω).
- A short circuit is when $R = 0$ and an open circuit is when $R \to \infty$.
- A variable resistor is known as a potentiometer (opposite is fixed resistor).
- Conductance G is the reciprocal of resistance measured in mhos (℧) or siemens (S).
- The total resistance of resistors in series can be calculated by the formula

$$R_{\text{eq}} = R_1 + R_2 + \cdots + R_n. \tag{9}$$

- The total resistance of resistors in parallel can be calculated by the formula

$$R_{\text{eq}} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}}. \tag{10}$$

2.2 Relationship between Loops, Nodes, and Branches

- A node is a point where two or more circuit elements are connected together.
- A branch is a path connecting two nodes.
- A loop is any closed path in a circuit.
- $B = L - N + 1$ where B is the number of branches, L is the number of loops, and N is the number of nodes.

2.3 Kirchhoff's Laws

- The algebraic sum of currents entering a node is 0.

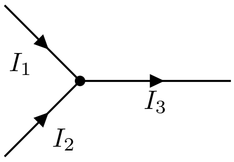


Figure 2: $I_1 + I_2 = I_3$.

$$\sum_{n=1}^{N} I_n = 0 \tag{11}$$

- The sum of all voltages in a loop is 0.

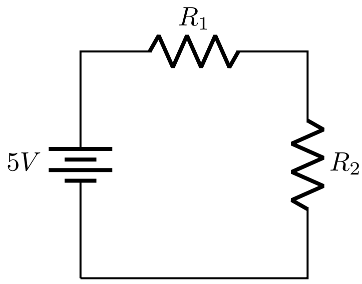
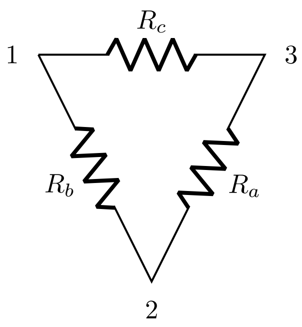


Figure 3: $5 - R_1 I - R_2 I = 0$

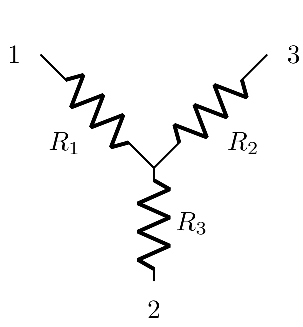
$$\sum_{m}^{M} V_m = 0 \tag{12}$$

2.4 Wye-Delta Transformations

For more complicated resistor circuits, it is often useful to transform a delta (Δ) circuit into a wye (Y) circuit or vice versa.



(a) Δ.



(b) Y.

Figure 4: Two forms of the same network.

The following transformations can be solved with these set of equations:

$$R_1 + R_2 = \frac{1}{\frac{1}{R_c} + \frac{1}{R_a+R_b}} \tag{13}$$

$$R_2 + R_3 = \frac{1}{\frac{1}{R_a} + \frac{1}{R_b+R_c}} \tag{14}$$

$$R_1 + R_3 = \frac{1}{\frac{1}{R_b} + \frac{1}{R_a+R_c}} \tag{15}$$

$$\tag{16}$$

2.4.1 Delta to Wye Conversion

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} \tag{17}$$

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c} \tag{18}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} \tag{19}$$

$$\tag{20}$$

2.4.2 Wye to Delta Conversion

$$R_a = \frac{R_1 R2 + R_2 R_3 + R_3 R_1}{R_1} \tag{21}$$

$$R_b = \frac{R_1 R2 + R_2 R_3 + R_3 R_1}{R_2} \tag{22}$$

$$R_c = \frac{R_1 R2 + R_2 R_3 + R_3 R_1}{R_3} \tag{23}$$

$$\tag{24}$$

2.5 Nodal Analysis

- In nodal analysis, KCL is applied at each node. Nodal analysis is best used when most of the circuit elements are current sources.
- However, there are cases where a dependent or independent voltage source connects two nodes. In this case, we must merge the two nodes to create a supernode.

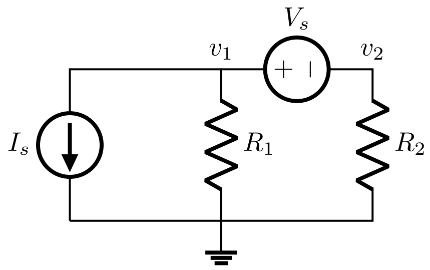


Figure 5: v_1 and v_2 can be merged to form a supernode.

- Analysis by inspection can be used if the circuit is made of independent current sources.

$$\begin{bmatrix} G_{11} & \cdots & G_{1N} \\ \vdots & \ddots & \vdots \\ G_{N1} & \cdots & G_{NN} \end{bmatrix} \begin{bmatrix} v_1 \\ \vdots \\ v_N \end{bmatrix} = \begin{bmatrix} i_1 \\ \vdots \\ i_N \end{bmatrix} \tag{25}$$

  Each of the diagonal terms is the sum of the conductances connected directly to the node, while the off-diagonal terms are the negatives of the conductances connected between the nodes

2.6 Mesh Analysis

- In mesh analysis, KVL is applied to each mesh. Mesh analysis is best used when most of the circuit elements are voltage sources.
- Similarly, there are cases where a dependent or independent current source connects two meshes. In this case, we must merge the two meshes to create a supermesh.

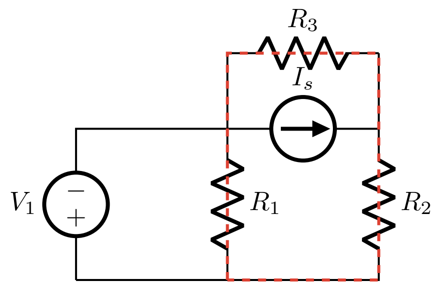


Figure 6: In this case, the two meshes beside the current source can be merged to form a supermesh shown by the red dotted line.

- Analysis by inspection can be used if the circuit is made of independent voltage sources.

$$\begin{bmatrix} R_{11} & \cdots & R_{1N} \\ \vdots & \ddots & \vdots \\ R_{N1} & \cdots & R_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} V_1 \\ \vdots \\ V_N \end{bmatrix} \tag{26}$$

  Each of the diagonal terms is the sum of the resistances in the related mesh, while each of the off-diagonal terms is the negative of the resistance common to both meshes.

3 Circuit Theorems

3.1 Linearity

- A linear circuit satisfies the following properties:

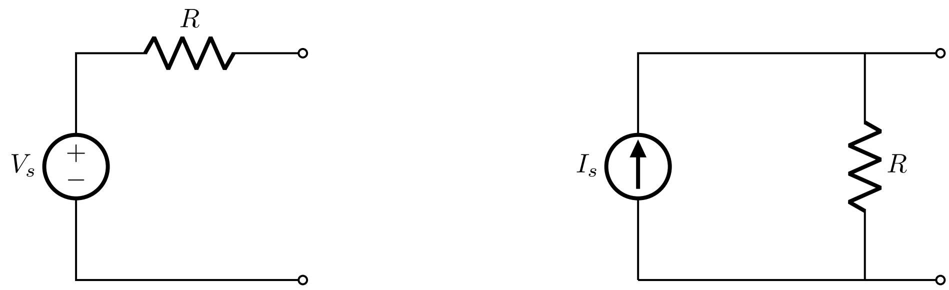
1. Homogeneity: If the input is multiplied by a constant, then the output is also multiplied by the same constant.
$$kIR = kV \tag{27}$$
2. Additivity: The response to a sum of inputs is the responses to each input applied separately.
$$V = (I_1 + I_2)R = I_1R + I_2R = V_1 + V_2 \tag{28}$$

3.2 Superposition

- The superposition theorem states that the voltage across an element of a linear circuit is the algebraic sum of the voltages generated by each source acting independently.

3.3 Source Transformation

- A source transformation is the process of replacing a voltage source in series with a resistor by a current source in parallel with the same resistor or vice versa.



(a) Voltage Source. (b) Current Source.

Figure 7: A voltage source can be transformed into a current source using Ohm's Law $V = IR$.

3.4 Thevenin and Norton's Theorems

- **Thevenin's Theorem** - A linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{th} in series with a resistor R_{th}.
- R_{th} can be calculated in the following ways:
  1. If the network has no dependent sources, R_{th} is the resistance between the terminals ab when we turn off all independent sources (open current sources and short terminals).
  2. If the network has dependent sources, R_{th} can be found by inserting a current source or voltage source between the terminals ab usually of 1A or 1V. Then after applying mesh or nodal analysis, we can use ohm's law to find R_{th}.
- V_{th} is simply the voltage difference between the two terminals ab.
- **Norton's Theorem** - A linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N. It is essentially a source transformation of Thevenin's Theorem.

3.5 Maximum Power Transfer

- The power transferred to a load can be calculated by the formula
$$P = I^2R_L = \left(\frac{V_{\mathrm{Th}}}{R_{\mathrm{Th}} + R_L}\right)^2 R_L \tag{29}$$

- The maximum power transfer occurs when $R_L = R_{\text{Th}}$.

$$P_{\max} = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}} \tag{30}$$

4 Operational Amplifiers

- An op amp is an electronic unit that behaves like a voltage-controlled voltage source.
- It is designed to perform mathematical operations of addition, subtraction, multiplication, and division.

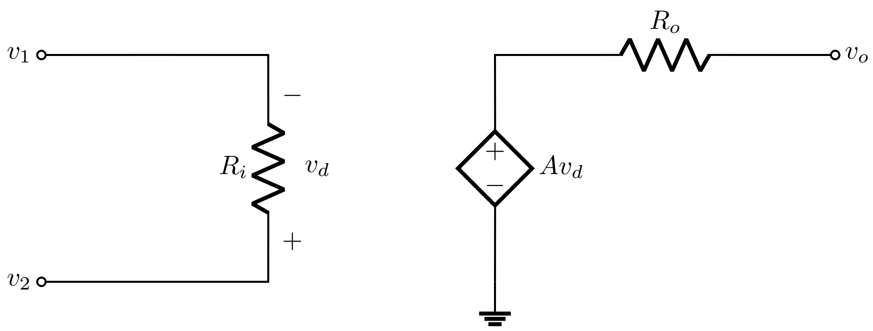


Figure 8: The equivalent circuit of an op amp.

$$v_o = Av_d = A(v_2 - v_1) \tag{31}$$

- A is called the open-loop voltage gain.
- $-V_{CC} \leq v_o \leq V_{CC}$ since if v_d is increased beyond the linear range, the op amp becomes saturated.
- An ideal op amp is an amplifier with infinite open-loop gain, infinite input resistance (no current or voltage drop through both input terminals), and zero output resistance.

4.1 Inverting Amplifier

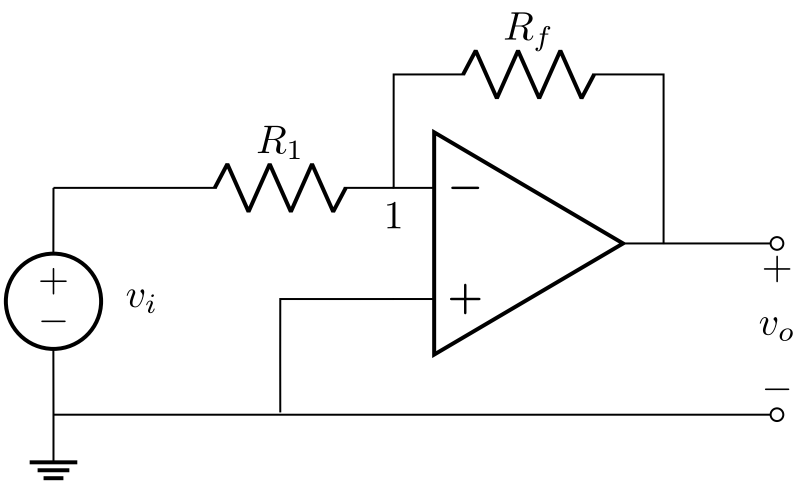


Figure 9: A diagram of the inverting amplifier

- Since the voltage at node 1 is 0 because there is 0 voltage difference between the two input terminals of the amplifier, we can perform nodal analysis to find the relationship between v_1 and v_i.

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$$\frac{v_1 - v_1}{R_1} + \frac{v_1 - v_o}{R_f} = 0 \tag{32}$$

$$v_o = -\frac{R_f}{R_1} v_i \tag{33}$$

4.2 Noninverting Amplifier

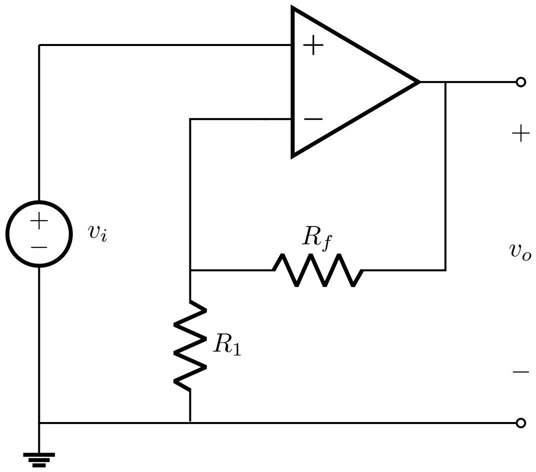

Figure 10: A diagram of the noninverting amplifier

- A noninverting amplifier is designed to provide a positive voltage gain.

$$v_o = \left(1 + \frac{R_f}{R_1}\right) v_i \tag{34}$$

4.3 Summing Amplifier

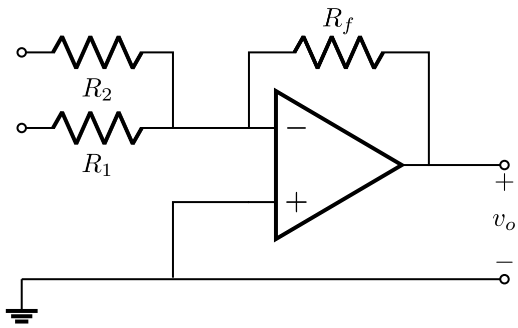

Figure 11: The summing amplifier is similar to the inverting amplifier. The sum of the output currents is the sum of the input currents, and output voltage can be calculated as

$$v_o = -\frac{R_f}{R_1} v_1 - \frac{R_f}{R_2} v_2 \tag{35}$$

4.4 Difference Amplifier

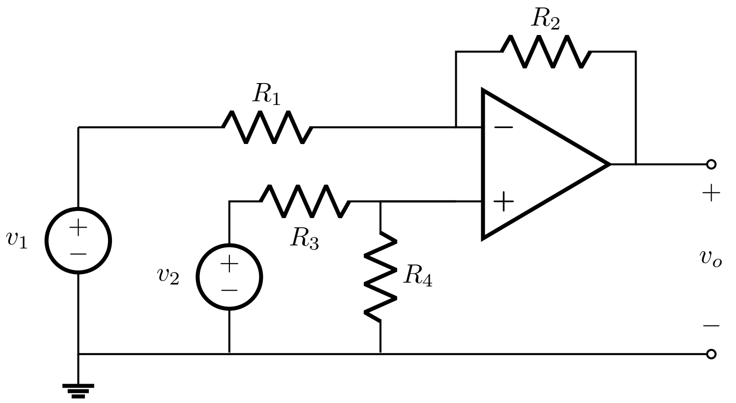

Figure 12: The output voltage for the difference amplifier can be calculated as

$$v_o = \frac{R_2(1 + R_1/R_2)}{R_1(1 + R_3/R_4)} v_2 - \frac{R_2}{R_1} v_1. \tag{36}$$

When $R_1 = R_3$ and $R_2 = R_4$, the op amp circuit becomes a subtractor.

$$v_o = v_2 - v_1 \tag{37}$$

5 Capacitors and Inductors

5.1 Capacitors

- A capacitor consists of two conductive plates separated by an insulator or dielectric material.
- Capacitance is the ratio of the charge on one plate to the voltage difference between the plates.

$$C = \frac{Q}{V} \tag{38}$$

- For a parallel plate capacitor, the capacitance can be calculated by the formula

$$C = \frac{\varepsilon A}{d} \tag{39}$$

  where ε is the permittivity of the dielectric material, A is the surface area of each plate, and d is the distance between the plates.
- The current, voltage, power, and energy stored with respect to time are given by the following equations:

$$i(t) = C\frac{\mathrm{d}v}{\mathrm{d}t} \tag{40}$$

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i(t)\, dt \tag{41}$$

$$p(t) = v(t)i(t) \tag{42}$$

$$w(t) = \frac{1}{2}Cv(t)^2 \tag{43}$$

- When voltage is not changing with time across a capacitor, the current is 0, so it acts as an open circuit.

- The voltage on a capacitor cannot change abruptly.
- The equivalent capacitance of parallel capacitors can be calculated as

$$C_{\text{eq}} = C_1 + C_2 + \cdots + C_n. \tag{44}$$

- The equivalent capacitance of series capacitors can be calculated as

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}. \tag{45}$$

5.2 Inductors

- An inductor consists of a coil of conducting wire.
- Inductance is the property whereby an inductor exhibits opposition to the change of current flowing through it, measured in Henrys (H).

$$v(t) = L\frac{\mathrm{d}i}{\mathrm{d}t} \tag{46}$$

- The inductance of a solenoid can be calculated by the formula

$$L = \frac{\mu N^2 A}{l} \tag{47}$$

  where μ is the permeability of the core, N is the number of turns, A is the cross-sectional area, and l is the length of the solenoid.
- The current and energy stored with respect to time are given by the following equations:

$$i(t) = \frac{1}{L}\int_{t_0}^{t} v(t)\,dt \tag{48}$$

$$w(t) = \frac{1}{2}Li(t)^2 \tag{49}$$

- When current is constant, an inductor acts like a short circuit.
- The current through an inductor cannot change instantaneously.
- The equivalent inductance of parallel inductors can be calculated as

$$\frac{1}{L_{\text{eq}}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_n}. \tag{50}$$

- The equivalent inductance of series inductors can be calculated as

$$L_{\text{eq}} = L_1 + L_2 + \cdots + L_n. \tag{51}$$

6 First-Order Circuits

6.1 Source-Free RC Circuits

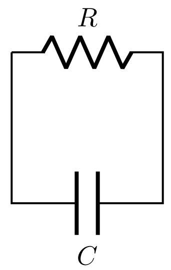

Figure 13: A simple source-free RC circuit.

- The relationship between voltage and time can be calculated by the formula

$$v(t) = V_0 e^{-\frac{t}{RC}} \tag{52}$$

- The time constant is $\tau = RC$.

Derivation

- Applying KCL, we get that $i_C + i_R = 0$ where i_C is the current going into the capacitor and i_R is the current going into the resistor.
- By definition, $i_c = C\frac{\mathrm{d}v}{\mathrm{d}t}$ and $i_R = \frac{v}{R}$.
- Thus,

$$C\frac{\mathrm{d}v}{\mathrm{d}t} + \frac{v}{R} = 0 \tag{53}$$

- Rearranging, we get

$$\frac{dv}{v} = -\frac{1}{RC}dt \tag{54}$$

- Integrating both sides, we get

$$\ln(v) = -\frac{t}{RC} + \ln(V_0) \tag{55}$$

- Solving for v, we get

$$v(t) = V_0 e^{-\frac{t}{RC}} \tag{56}$$

6.2 Source-Free RL Circuits

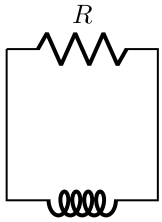

Figure 14: A simple source-free RL circuit.

- The relationship between current and time can be calculated by the formula

$$i(t) = I_0 e^{-\frac{t}{L/R}} \tag{57}$$

- The time constant is $\tau = L/R$.

Derivation

- Applying KVL, we get that $v_L + v_R = 0$ where v_L is the voltage across the inductor and v_R is the voltage across the resistor.
- By definition, $v_L = L\frac{\mathrm{d}i}{\mathrm{d}t}$ and $v_R = iR$.
- Thus,

$$L\frac{\mathrm{d}i}{\mathrm{d}t} + iR = 0 \tag{58}$$

- Rearranging, we get
$$\frac{di}{i} = -\frac{R}{L}dt \tag{59}$$
- Integrating both sides, we get
$$\ln(i) = -\frac{R}{L}t + \ln(I_0) \tag{60}$$
- Solving for i, we get
$$i(t) = I_0 e^{-\frac{t}{L/R}} \tag{61}$$

6.3 Singularity Functions

- **Singularity functions** are functions that either are discontinuous or have discontinuous derivatives.
- **Unit Step Function**
$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases} \tag{62}$$
- **Unit Impulse Function**
$$\delta(t) = \begin{cases} 0 & t \neq 0 \\ \infty & t = 0 \end{cases} \tag{63}$$
- **Unit Ramp Function**
$$r(t) = \begin{cases} 0 & t < 0 \\ t & t \geq 0 \end{cases} \tag{64}$$

6.4 Step Response of an RC Circuit

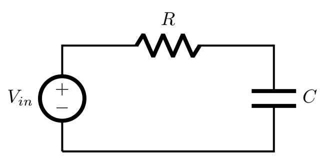

Figure 15: An RC circuit with a step response.

$$v(t) = \begin{cases} V_0 & t < 0 \\ V_s + (V_0 - V_s)e^{-t/\tau} & t \geq 0 \end{cases} \tag{65}$$

- The transient response is temporary while the stead-state response is permanent.
- The equation may be rewritten as
$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau} \tag{66}$$

6.5 Step Response of an RL Circuit

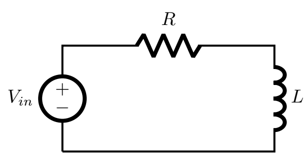

Figure 16: An RL circuit with a step response.

$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau} \tag{67}$$

7 Sinusoids and Phasors

- A sinusoid is a function that satisfies of the form

$$x(t) = A\cos(\omega t + \phi). \tag{68}$$

- The amplitude is A, the angular frequency is ω, the period is $2\pi/\omega$, and the phase is ϕ.
- A phasor is a complex number that represents the sinusoid of the form

$$X = A\underline{/\phi} = re^{j\phi}. \tag{69}$$

7.1 Phasor Relationships for Circuit Elements

- Suppose the phasor representation of current is given by $\mathbf{I} = I_m\underline{/\phi}$.
- For a resistor, the voltage is given by $\mathbf{V} = RI_m\underline{/\phi} = \mathbf{I}R$.
- For an inductor, the voltage is given by $\mathbf{V} = L\frac{\mathrm{d}i}{\mathrm{d}t} = \omega LI_m\underline{/\phi + 90^\circ} = j\omega\mathbf{I}L$.
- For a capacitor, the voltage is given by $\mathbf{V} = \frac{\mathbf{I}}{j\omega C}$.

7.2 Impedance and Admittance

- The impedance $\mathbf{Z}$ is the ratio of phasor voltage to phasor current measured in ohms Ω.
- The admittance $\mathbf{Y}$ is the ratio of phasor current to phasor voltage measured in siemens S.

Table 1: Impedance and Admittance of Circuit Elements.

Element	Impedance	Admittance
Resistor	R	$\frac{1}{R}$
Inductor	$j\omega L$	$\frac{1}{j\omega L}$
Capacitor	$\frac{1}{j\omega C}$	$j\omega C$

- Kirchhoff's voltage and current laws hold for phasors.

8 AC Power Analysis

- Instantaneous power is the power at any instant of time.

$$p(t) = v(t)i(t) \tag{70}$$

- Let $v(t) = V_m\cos(\omega t + \phi_v)$ and $i(t) = I_m\cos(\omega t + \phi_i)$.
- Then, the instantaneous power can be expressed as

$$p(t) = \frac{1}{2}V_mI_m\cos(\phi_v - \phi_i) + \frac{1}{2}V_mI_m\cos(2\omega t + \phi_v + \phi_i) \tag{71}$$

- Note that the equation above has two components: the average power and the oscillating power.

- The average power is the average of the instantaneous power over one period.

$$P = \frac{1}{T}\int_0^T p(t)\,dt \tag{72}$$
$$= \frac{1}{2}V_m I_m \cos(\phi_v - \phi_i) \tag{73}$$
$$= \frac{1}{2}\mathrm{Re}[\mathbf{V}\mathbf{I}^*] \tag{74}$$

- Note that the asterisk denotes the complex conjugate.

8.1 Maximum Power Transfer

- If $\mathbf{Z}_{Th} = R_{Th} + jX_{Th}$ and $\mathbf{Z}_L = R_L + jX_L$, the power transferred to the load can be represented by the equation

$$P = \frac{|\mathbf{V}_{Th}|^2 R_L/2}{(R_{Th}+R_L)^2 + (X_{Th}+X_L)^2}. \tag{75}$$

- Maximum power is transferred when $R_L = R_{Th}$ and $X_L = -X_{Th}$.

$$P_{\max} = \frac{|\mathbf{V}_{Th}|^2}{8R_{Th}}. \tag{76}$$

Derivation

$$\mathbf{I} = \frac{\mathbf{V}_{Th}}{\mathbf{Z}_{Th}+\mathbf{Z}_L} \tag{77}$$
$$= \frac{\mathbf{V}_{Th}}{(R_{Th}+jX_{Th}) + (R_L + jX_L)} \tag{78}$$
$$P = \frac{1}{2}|\mathbf{I}|^2 R_L \tag{79}$$
$$= \frac{|\mathbf{V}_{Th}|^2 R_L/2}{(R_{Th}+R_L)^2 + (X_{Th}+X_L)^2} \tag{80}$$

Taking the derivatives,

$$\frac{\partial P}{\partial X_L} = \frac{|\mathbf{V}_{Th}|^2 R_L (X_{Th}+X_L)}{2[(R_{Th}+R_L)^2 + (X_{Th}+X_L)^2]^2} \tag{81}$$
$$\frac{\partial P}{\partial R_L} = \frac{|\mathbf{V}_{Th}|^2[(R_{Th}+R_L)^2 + (X_{Th}+X_L)^2 - 2R_L(R_{Th}+R_L)]}{2[(R_{Th}+R_L)^2 + (X_{Th}+X_L)^2]^2} \tag{82}$$

Setting both the partial derivatives to 0, we get the maximum power transfer conditions.

8.2 Effective or RMS Value

- The effective value of a periodic current is the value of a DC current that produces the same average power in a resistor.

$$P = \frac{1}{T}\int_0^T i^2(t) R\,dt \tag{83}$$
$$P = \frac{1}{2}I_{\text{eff}}^2 R \tag{84}$$
$$\tag{85}$$

- Equating the two, the effective value of the current is given by the formula

$$I_{\text{eff}} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\, dt}. \tag{86}$$

- Similarly,

$$V_{\text{eff}} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\, dt}. \tag{87}$$

- The effective value is also known as the root-mean-square (RMS) value.
- For sinusoidal signals, the effective value of voltage and current can be simplified to

$$V_{\text{eff}} = \frac{V_m}{\sqrt{2}} \quad \text{and} \quad I_{\text{eff}} = \frac{I_m}{\sqrt{2}}. \tag{88}$$

8.3 Power Factor

- Recall that average power is

$$P = \frac{1}{2} V_{\text{m}} I_{\text{m}} \cos(\phi_v - \phi_i) = V_{\text{rms}} I_{\text{rms}} \cos(\phi_v - \phi_i). \tag{89}$$

- The apparent power is defined as

$$S = V_{\text{rms}} I_{\text{rms}}. \tag{90}$$

- The power factor is defined as the ratio of real power to apparent power.

$$\text{pf} = \frac{P}{S} = \cos(\phi_v - \phi_i). \tag{91}$$

- It can also be calculated by taking the cosine of the angle of the impedance.
- If $\phi_v - \phi_i > 0$, current lags voltage which means that the power factor is lagging, implying that the circuit is inductive.
- If $\phi_v - \phi_i < 0$, current leads voltage which means that the power factor is leading, implying that the circuit is capacitive.

8.4 Complex Power

- Throughout the years, engineers have tried to express power relations as simply as possible. The following formulas summarize the power relations in AC circuits.

$$\begin{aligned}
\text{Complex Power} &= \mathbf{S} = P + jQ = \mathbf{V}_{\text{rms}} {\mathbf{I}_{\text{rms}}}^* = |\mathbf{V}_{\text{rms}}||\mathbf{I}_{\text{rms}}| \underline{/\phi_v - \phi_i} && (92)\\
\text{Apparent Power} &= S = |\mathbf{S}| = |\mathbf{V}_{\text{rms}}||\mathbf{I}_{\text{rms}}| && (93)\\
\text{Real Power} &= P = \text{Re}(\mathbf{S}) = S\cos(\phi_v - \phi_i) && (94)\\
\text{Reactive Power} &= Q = \text{Im}(\mathbf{S}) = S\sin(\phi_v - \phi_i) && (95)\\
\text{Power Factor} &= \text{pf} = \cos(\phi_v - \phi_i) && (96)
\end{aligned}$$

- The conservation of power states that the sum of the complex, real, and reactive powers of the sources equal the respective sums of the complex, real, and reactive powers of the individual loads.

8.5 Power Factor Correction

- Power factor correction is the process of improving the power factor without altering voltage or current to the original load.
- If the load is inductive, a capacitor can be added in parallel to the load to correct the power factor. The capacitance is given by the formula

$$C = \frac{Q_C}{V^2\omega} = \frac{P(\tan(\theta_1) - \tan(\theta_2))}{V^2\omega}, \tag{97}$$

  where Q_C is the reduction in reactive power, θ_1 is the original phase angle, and θ_2 is the corrected phase angle.
- Similarly, if the load is capacitive, an inductor can be across the load to correct the power factor.

$$L = \frac{V_{rms}^2}{Q_L\omega} \tag{98}$$

9 Magnetically Coupled Circuits

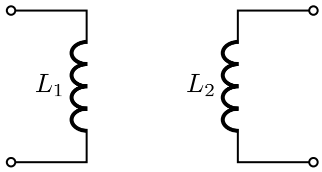

Figure 17: When two loops with or without contacts between them affect each other through magnetic fields, they are said to be magnetically coupled.

- According to Faraday's Law, the voltage induced in a coil is proportional to the rate of change of the magnetic flux through the coil, and the number of turns of the coils (N).

$$\begin{aligned} v &= N\frac{\mathrm{d}\phi}{\mathrm{d}t} && (99)\\ &= N\frac{\mathrm{d}\phi}{\mathrm{d}i}\frac{\mathrm{d}i}{\mathrm{d}t} && (100)\\ &= L\frac{\mathrm{d}i}{\mathrm{d}t} && (101)\\ L &= N\frac{\mathrm{d}\phi}{\mathrm{d}i} && (102) \end{aligned}$$

- This inductance is called the self-inductance of the coil.
- If current is supplied to the first inductor in the diagram above, then

$$\begin{aligned} v_1 &= L_1\frac{\mathrm{d}i_1}{\mathrm{d}t} && (103)\\ v_2 &= N_2\frac{\mathrm{d}\phi_{12}}{\mathrm{d}i_1}\frac{\mathrm{d}i_1}{\mathrm{d}t} = M\frac{\mathrm{d}i_1}{\mathrm{d}t} && (104) \end{aligned}$$

  where ϕ_{12} is the flux induced in coil 2, and M is the mutual inductance.

9.1 Ideal Transformer

- An ideal transformer is a unity-coupled, lossless transformer in which the primary and secondary coils have infinite self-inductances.

$$\frac{\mathbf{V}_2}{\mathbf{V}_1} = \frac{N_2}{N_1} = \frac{\mathbf{I}_1}{\mathbf{I}_2} = n \tag{105}$$

- n is the transformation ratio.
- A step-up transformer increases voltage while a step-down transformer decreases voltage.
- The complex power delivered to the primary coil is equal to the complex power delivered to the secondary coil.
- The input impedance for a load attached to a transformer as seen by the source is given by the formula

$$\mathbf{Z}_{\text{in}} = \frac{\mathbf{Z}_L}{n^2}. \tag{106}$$