Civ 102 Notes

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Contents

Notes based off of notes by Allan Kuan and Michael P. Collins and the Civ102 Course taught by professor Bentz.

The 3 Principles of Engineering

1. $F = ma$
2. You can't push on a rope.
3. A necessary condition for solving any given engineering problem is to know the answer before starting.

1 Moment

The moment is defined as the cross product between the radius vector and force.

$$\vec{M} = \vec{r} \times \vec{F} \tag{1}$$

A couple is defined as a special class of moments which occurs when two forces with the same magnitude F act in the opposite direction of each other while being separated by a perpendicular distance d.

2 Bridges

(a) The simplest bridge is a beam supported by vertical posts. This bridge may become unstable in the event of a flood.

(b) The truss bridge is stronger than the beam bridge; however, it requires high maintenance.

Figure 1: Simple bridges.

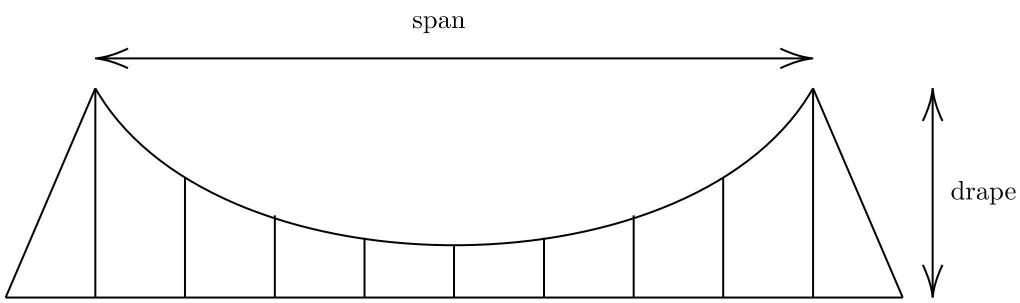


Figure 2: Another type of bridge is the suspension bridge. Two strong main cables run parallel to each other and are fastened to concrete pillars on either side of the bridge. Secondary vertical cables adds support to the horizontal bridge platform.

Catanery

The natural shape of a cable with no load is a catanery.

$$y = a\cosh\left(\frac{x}{a}\right) + b = \frac{a}{2}\left(e^{\frac{x}{a}} + e^{-\frac{x}{a}}\right) + b \tag{2}$$

The shape of the cable is a parabola when weights are equally spaced horizontally. Horizontal force is constant end to end.

3 Suspension Bridge Calculations

Uniform Distributed Load is equal to force per unit length MPa. The moment of a uniform distributed load is $M = wLd$ where L is length, and d is distance to the center of mass. For a triangular load, that would be $M = \frac{1}{2}wL \cdot \frac{1}{3}$.

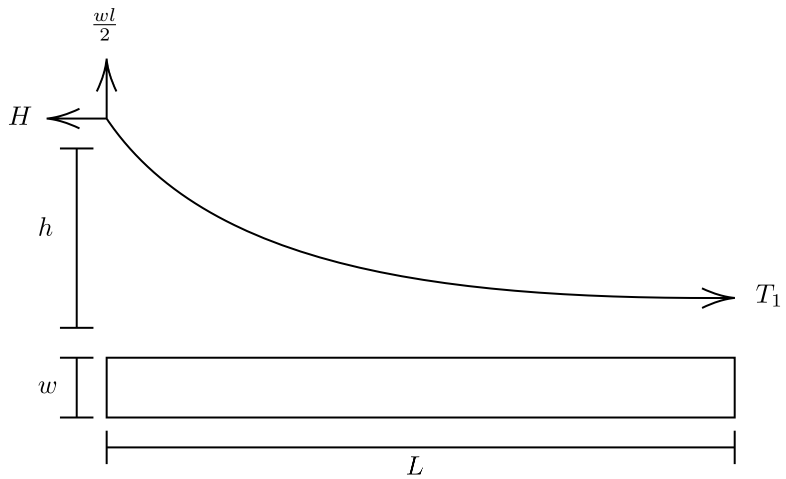


Figure 3: FBD of a suspension bridge.

$$\Sigma F_x = 0 = H - T_1 \tag{3}$$

$$\Sigma M = 0 = \frac{-wL}{2} \cdot \frac{L}{4} + H \cdot h \tag{4}$$

$$H = \frac{wL^2}{8h} \tag{5}$$

4 Stress, Strain and Young's Modulus

Stress is defined as:

$$\sigma = \frac{F}{A}. \tag{6}$$

Its units are in MPa which is the same as pressure. However, stress differs from pressure in that it is internal and that it has direction.

Strain is defined as:

$$\varepsilon = \frac{\Delta L}{L_0}. \tag{7}$$

Strain is dimensionless (in units of $\frac{\text{mm}}{\text{mm}}$) and represents deformation of a material.

Thomas Young used Hooke's Law to come up with a relation between stress and strain.

$$E = \frac{\sigma}{\varepsilon} \tag{8}$$

$$F = \frac{AE}{L_0}\Delta L \tag{9}$$

$$k = \frac{AE}{L_0} \tag{10}$$

5 Strain Energy

Strain energy density is defined as the area under the stress-strain curve.

$$U = \int \sigma d\varepsilon \tag{11}$$

Strain energy is defined as the area under the force displacement curve of a structure.

$$W = \int F d\Delta L \tag{12}$$

Under linearly elastic conditions, the force-displacement curve would be in the shape of a triangle.

$$W = \frac{1}{2} F \Delta L = \frac{1}{2} k (\Delta L)^2 \tag{13}$$

Strain energy can be related to strain energy density with the equation:

$$W = U \cdot V_0 \tag{14}$$

Where V_0 is the original volume before deformation.

6 Stress-Strain Definitions

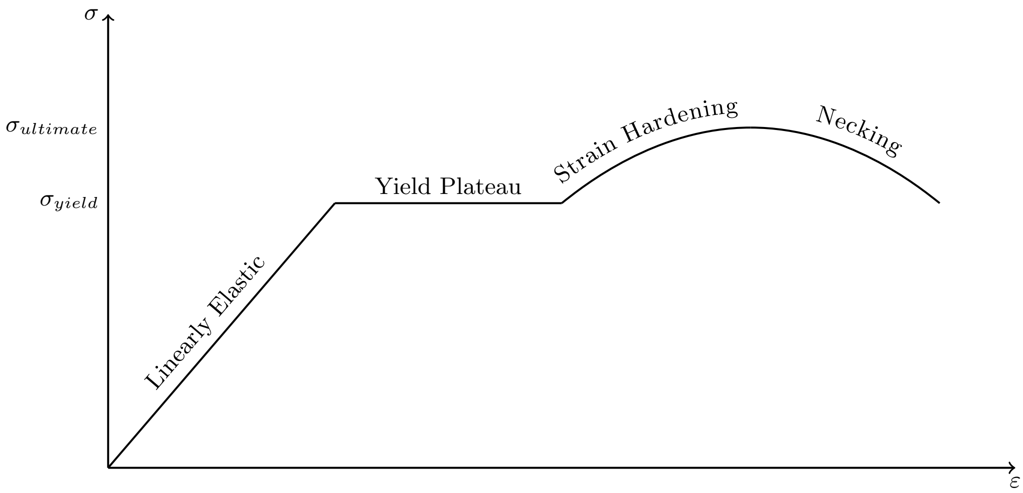


Figure 4: Stress-Strain Curve Graph

- **Yielding** occurs when atoms in the material are no longer vibrating back and forth between one another, but sliding across one another.
- **Strain hardening** is the phenomenon where a material gains strength and stiffness when strained beyond its yield point.
- **Necking** occurs where localization of tensile strains in a material causes the cross-sectional area to become noticeably smaller at one location causing it to resemble the shape of a neck. Usually precedes **rupture**.
- **Elastic deformations** refer to the recoverable deformations whereas **plastic deformations** refer to the permanent deformations.
- Materials which can sustain significant amounts of permanent deformations are **ductile** while those which cannot are **brittle**.

- Materials with a large Young's Modulus are generally known to be **stiff** whereas those who do not are generally referred to being **flexible**.
- **Resilience** is defined as the maximum amount of energy a structure can absorb before permanent deformation.
- **Toughness** is defined as the maximum amount of energy a structure can absorb before breaking.
- Coefficient of Thermal Expansion: $\varepsilon = \alpha \Delta T$.

7 Springs

$$F = -k\Delta x \tag{15}$$
$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = -\frac{k}{m}x \tag{16}$$
$$x = A\sin(\omega t + \phi) \tag{17}$$
$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = -A\omega^2 \sin(\omega t + \phi) \tag{18}$$

$\therefore$ the natural frequency of a spring can be defined as:

$$f = \frac{1}{2\pi}\omega = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{19}$$

8 Safety

Types of Loads

- Dead Loads (Loads which remain constant)
- Live Loads (Loads which vary)
- Earthquake Loads
- Wind Loads
- Thermal Loads

Applied loads must be less than that of applied resistance or else the structure will collapse. The Partial Safety Factor Concept scales down strength and scales up load to ensure that the applied resistance will never be less than the applied load. In the Allowable Stress Design:

$$\text{Factor of Safety} \leq \frac{\text{Resistance Average}}{\text{Applied Force Average}} \tag{20}$$
$$\sigma_{\text{allow}} = \frac{\sigma_{\text{fail}}}{FOS}. \tag{21}$$

9 Moment of Inertia

The moment of inertia of a point is defined as:

$$I_{m,i} = mL^2 \tag{22}$$
$$M = I\alpha = (mL^2)\alpha. \tag{23}$$

The moment of inertia of a body is defined as:

$$I_m = \int L^2 dm \tag{24}$$

When the body under consideration is a two-dimensional object having a uniform density, dm can be represented as ρdA where ρ is the density.

$$I_m = \rho \int L^2 dA \tag{25}$$

$$I = \int L^2 dA \tag{26}$$

The second moment of area, I, has units of m^4.

Example Calculation

Calculate the moment of area of a rectangular box about its centroid with width w and height h.

$$dm = wdL \tag{27}$$

$$I = \int_{h/2}^{-h/2} wL^2 dL \tag{28}$$

$$= \frac{1}{3}w\left(\left(\frac{h}{2}\right)^3 - \left(-\frac{h}{2}\right)^3\right) \tag{29}$$

$$= \frac{bh^3}{12} \tag{30}$$

10 Bending of Beams

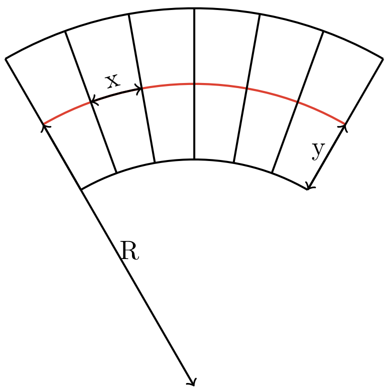


Figure 5: Plane sections remain plane.

Curvature is defined as:

$$\phi = \frac{\mathrm{d}\theta}{\mathrm{d}x} \tag{31}$$

$$R = \frac{1}{\phi} \tag{32}$$

And R is the radius of curvature. We can now calculate the deformed length and strain:

$$L' = \theta r \tag{33}$$
$$= (\phi L_0) \cdot \left(y + \frac{1}{\phi}\right) \tag{34}$$
$$= \phi L_0 y + L_0 \tag{35}$$
$$\varepsilon = \frac{\Delta L}{L} \tag{36}$$
$$= \phi y \tag{37}$$

Now, we will determine flexural stiffness which is the rotational version of the axial stiffness k.

$$\sigma = E\varepsilon = E\phi y \tag{38}$$
$$\Delta F = \sigma \Delta A \tag{39}$$
$$\Delta M = \Delta F \cdot y \tag{40}$$

Combining these three equations gives:

$$\Delta M = E\phi y^2 \Delta A \tag{41}$$
$$M = E\phi \int y^2 \Delta A \tag{42}$$
$$= EI\phi \tag{43}$$

$\therefore$ EI is the flexural stiffness of the structure. Moreover, axial force is similar to bending-moment, and displacement is similar to curvature in the same way as how axial stiffness is analogous to flexural stiffness.

11 Statically Determinate Structures

Table 1: Types of Supports

Name	Permitted Degrees of Freedom	Restrained Degrees of Freedom	Support Forces
Roller	$\Delta x, \theta_{xy}$	$\Delta y = 0$	F_y
	$\Delta y, \theta_{xy}$	$\Delta x = 0$	F_x
Pin	θ_{xy}	$\Delta x = \Delta y = 0$	F_x, F_y
Fixed End	None	$\Delta x = \Delta y = \theta_{xy} = 0$	F_x, F_y, M_{xy}

A **hinge** freely rotates and is unable to resist moment.

- Structures whose reaction forces can be directly solved are called **statically determinate**.
- Structures who have fewer reaction forces than the number of equilibrium equations are called **mechanisms**.
- Structures which have more reaction forces that the number of equilibrium equations are **statically indeterminate**.

12 Truss Bridges

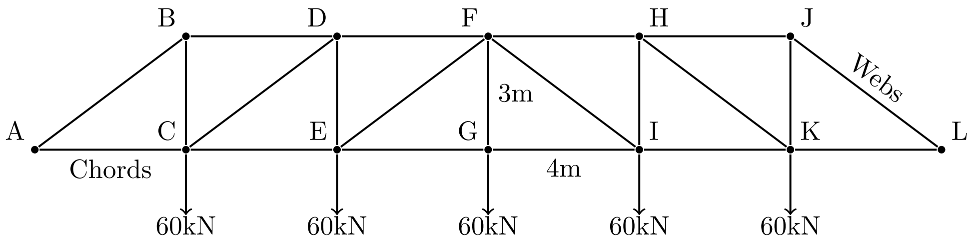


Figure 6: A Truss Bridge.

The loads applied to each of the joints are obtained by multiplying the area load by the tributary area of the deck which is the portion of the deck that the joint is responsible for supporting.

12.1 Method of Joints

We can start calculating forces from joint A. We know $A_y = 150$kN since the weight of the bridge is 300kN.

$$\Sigma F_x = 0 = AB_x + AC_x \tag{44}$$
$$\Sigma F_y = 0 = A_y + AB_y + AC_y \tag{45}$$

We find that $AB_y = 250$kN pointing towards A, and $AC_y = 200$kN pointing away from A. This method can be repeated for each joint onwards until all member forces have been found.

Helpful Tips

- Tension in members is usually represented as positive while compression is represented as negative.
- Tension points away from the joint and compression points towards the joint.

12.2 Method of Sections

We can divide the suspension bridge into 2 sections by slicing through DF, EF, and EG. We can solve for the unknown forces with the left section.

$$\Sigma F_x = 0 = DF + EF_x + EG \tag{46}$$
$$\Sigma F_y = 0 = A_y - 60 - 60 + EF_y \tag{47}$$
$$\Sigma M_E = 0 = 60 \cdot 4 - A_y \cdot 8 - DF \cdot 3 \tag{48}$$

We find that $DF = -320$kN, $EF = -50$kN, and $EG = 30$kN.

13 Euler Buckling

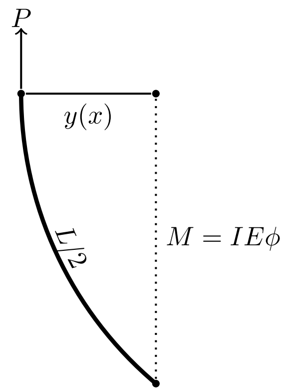


Figure 7: A compression force P causes the member to curve as it buckles.

$$Py = EI\phi \tag{49}$$

If we assume that for small angles, $\theta = \dfrac{\mathrm{d}y}{\mathrm{d}x}$,

$$Py = EI\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} \tag{50}$$

$$y = A\sin\left(\sqrt{\frac{P}{EI}}x\right) \tag{51}$$

$$\sqrt{\frac{P}{EI}}L = n\pi \tag{52}$$

$$P = \frac{n^2\pi^2 EI}{L^2} \tag{53}$$

Note that equation 52 was determined through boundary conditions: $y(0) = 0$ and $y(L) = 0$. The lateral buckling displacement for members with $\Delta_0 \neq 0$ because they are not straight is calculated by the equation

$$\Delta_{\text{lateral}} = \frac{\Delta_0}{1 - \frac{P}{P_{\text{euler}}}}. \tag{54}$$

14 Truss Bridge Design

Design of Members In Tension

$$A \geq FOS_{\text{yield}}\frac{F}{\sigma_y} \tag{55}$$

The factor of safety for members in tension is usually 2.0.

Design of Members In Compression

$$I \geq FOS\frac{FL^2}{\pi^2 E} \tag{56}$$

The factor of safety for members in compression is usually 3.0.

Euler Buckling Stress

$$\sigma_{\text{euler}} = \frac{P_{euler}}{A} = \frac{\pi^2 EI}{AL^2} = \frac{\pi^2 E}{\left(\frac{L}{r}\right)^2} \tag{57}$$

$r = \sqrt{\frac{I}{A}}$ is known as the radius of gyration, and $\frac{L}{r}$ is known as the slenderness ratio. Members with a large slenderness ratio tend to fail due to buckling, and those with a small slenderness ratio tend to fail by crushing.

15 Wind Loads

Wind pressure can be calculated by

$$w_{\text{wind}} = \frac{1}{2}\rho v^2 c_D, \tag{58}$$

where ρ is the density of the fluid, v is its velocity, and c_D is the drag coefficient. A low drag coefficient means that the shape of the object is quite aerodynamic. Usually, a drag coefficient of 1.5 is used for boxy objects like truss bridges to overestimate for wind pressure.

$$F_{\text{wind}} = A_{\text{tributary}} \cdot w_{\text{wind}} \tag{59}$$

Designing for Wind Loads

Lateral bracing is the term we use to refer to any pieces on a bridge that help keep the trusses from twisting.

- **Top Braces -** Account for buckling caused by gravitational loads and wind loads.
- **Bottom Braces -** Account for only wind loads.

16 Calculating the Deflection of a Loaded Structure

We know that the internal work done by members must be equal to the external work done by the applied loads. If only real forces are applied onto the system shown in figure 8,

$$\frac{1}{2}F_x\Delta B_x + \frac{1}{2}F_y\Delta B_y = \frac{1}{2}P_{AB}\Delta l_{AB} + \frac{1}{2}P_{BC}\Delta l_{BC}. \tag{60}$$

However, there are two unknown variables ΔB_x and ΔB_y which cannot be solved. That is why we must introduce a virtual force usually of 1 kN which acts in the direction along an axis.

$$\frac{1}{2}F^*\Delta B_y^* = \frac{1}{2}P_{AB}^*\Delta l_{AB}^* + \frac{1}{2}P_{BC}^*\Delta l_{BC}^* \tag{61}$$

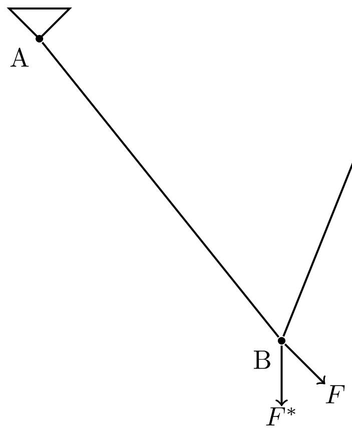
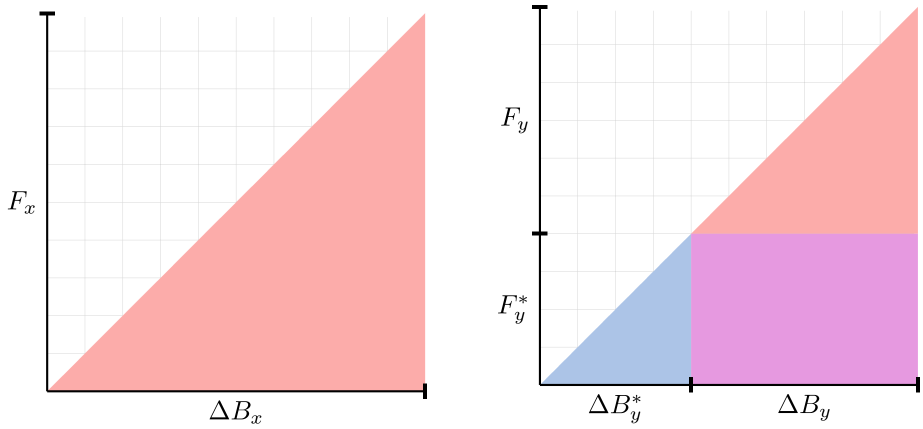


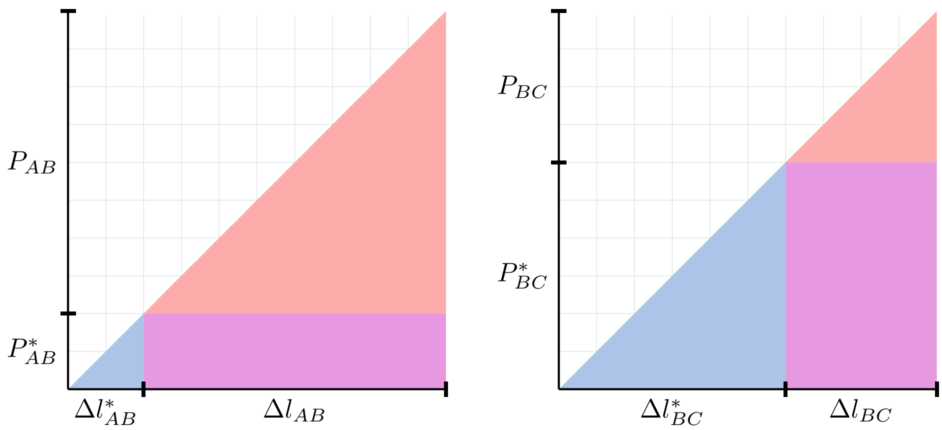
Figure 8: A simple system with real (F) and virtual (F^*) forces.

We can plot the sum of the real and virtual forces in work versus displacement graphs. The red area represents work done by real forces, and the blue area represents work done by virtual forces.



(a) Horizontal (b) Vertical

Figure 9: Work done by external loads.



(a) Member AB (b) Member BC

Figure 10: Work done by internal member forces.

The sum of the virtual and real work done by external loads must equal the sum of the virtual and real work done by internal member forces. Therefore, the purple area in the top two graphs must equal the purple area in the bottom two graphs.

$$F^* \Delta B_y = P_{AB}^* \Delta l_{AB} + P_{BC}^* \Delta l_{BC} \tag{62}$$

This equation can be generalized as

$$F^* \Delta = \sum_{i=1}^{n} P_i^* \Delta l_i. \tag{63}$$

17 Vibrations

The differential equation representing forced, damped oscillation is

$$mx''(t) + 2\beta\sqrt{mk}x'(t) + kx(t) = F_0 \sin \omega t, \tag{64}$$

where k is the axial stiffness of the spring, β is the damping ratio, and $F_0 \sin \omega t$ is the harmonic or sinusoidal load. The steady state solution to this equation is

$$x(t) = DAF \cdot \frac{F_0}{k} \sin(\omega t + \phi) + \Delta_0. \tag{65}$$

The Dynamic Amplification Factor, is calculated as

$$DAF = \frac{1}{\sqrt{\left(1-\left(\frac{f}{f_n}\right)^2\right)^2 + \left(\frac{2\beta f}{f_n}\right)^2}}. \tag{66}$$

$$F_{\text{eq}} = F_{\text{stationary}} + F_0 \cdot DAF \tag{67}$$

The ratio $\frac{w_{\text{eq}}}{w_{\text{stationary}}}$ can be multiplied by member displacements and internal forces to approximate the maximum member displacement and internal force.

Natural Frequency

The natural frequency (Hz) of a bridge can be approximated with the formulas:

$$\frac{15.76}{\sqrt{\Delta_0}} \tag{68}$$

for bridges with a point load and

$$\frac{17.76}{\sqrt{\Delta_0}} \tag{69}$$

for bridges with distributed loads.

18 Shear Force and Bending Moment Diagrams

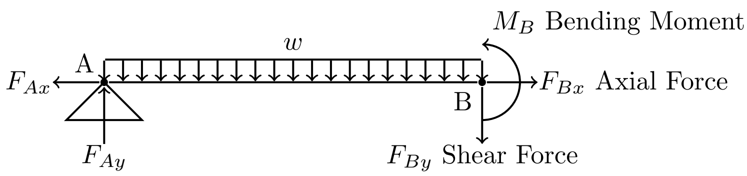


Figure 11: The member is cut at point B, and the internal forces are shown at B.

18.1 Calculating Bending Moment

$$M = F_{Ay}L - \frac{1}{2}wL^2 \tag{70}$$

18.2 Calculating Shear

$$F_{By} = F_{Ay} - wL \tag{71}$$

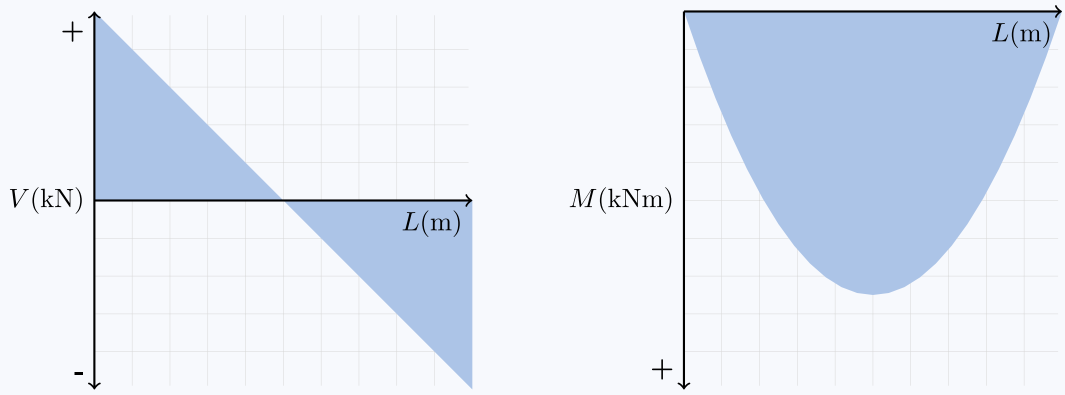
18.3 Shear Force Relationships

We find that the relationship between bending moment (M), vertical loads (W), and shear force (V) is

$$W = \frac{\mathrm{d}V}{\mathrm{d}x} \tag{72}$$

$$V = \frac{\mathrm{d}M}{\mathrm{d}x} \tag{73}$$

Tips for Drawing Bending Moment and Shear Force Diagrams



(a) Shear Force Diagram (b) Bending Moment Diagram

Figure 12: Example SFD and BMD for Figure 11.

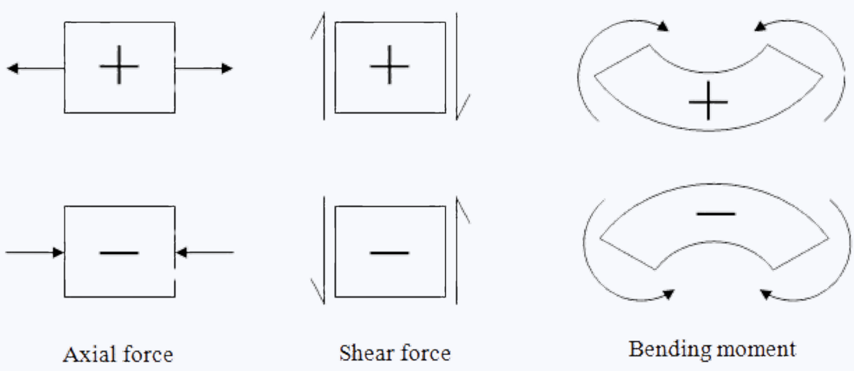


Figure 13: Sign Conventions Diagram (Image Source). Positive shear results in positive bending moment. An upward shear force from the left is positive.

19 Flexural Stress

From Equation 38, we know that,

$$dN = \sigma dA \tag{74}$$
$$dN = E\phi y dA \tag{75}$$
$$N = \int E\phi y dA = 0 \tag{76}$$

The centroidal axial force carried by the member is 0, so the first moment of area taken about the **centroidal axis** must equal 0.

$$\int y dA = 0 \tag{77}$$

Combining equation 38 and 43 produces another important equation which is the relationship between moment and flexural stress.

$$\sigma = \frac{My}{I} = \frac{M}{S} \tag{78}$$

The highest stress is when y is the greatest. $S = I/y_{\max}$ is known as the section modulus.

20 Second Moment of Area

Before we get to calculating I, we need to know where the centroidal axis is.

20.1 Finding the Centroidal Axis

Since the second moment of area must equal 0,

$$\sum_{i=1}^{n}(\overline{y} - y_i)A_i = 0. \tag{79}$$

$\overline{y}$ is the position of the centroidal axis relative to base of the cross-section, y_i is vertical distance from the base of the cross-section, and A_i is the area of the component.

$$\therefore \overline{y} = \frac{\sum_{i=1}^{n} y_i A_i}{\sum_{i=1}^{n} A_i} \tag{80}$$

20.2 Parallel Axis Theorem

$$\begin{align}
I &= \int (y+L)^2 dA \tag{81}\\
&= \int y^2 + 2Ly + L^2 dA \tag{82}\\
&= \int y^2 dA + \int 2Ly dA + \int L^2 dA \tag{83}\\
&= \int y^2 dA + 0 + d^2 dA \tag{84}\\
&= I_o + Ad^2 \tag{85}
\end{align}$$

For sections where all subparts have the same centroidal axis, we can add or subtract moments of area.

21 Deflection of Beams

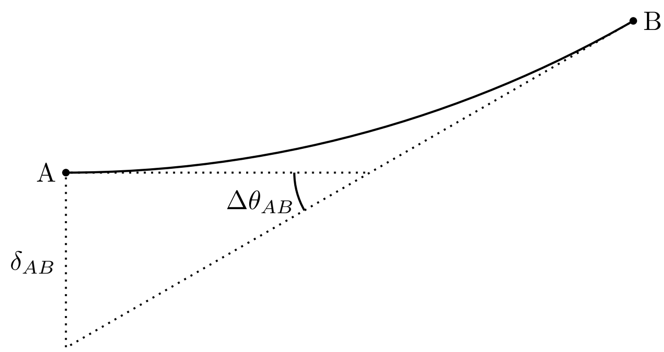


Figure 14: A deflected beam.

Moment of area theorems 1 and 2 can be used in conjunction to solve for deflection at a certain point along a member.

21.1 Moment of Area Theorem 1

The moment of area theorem 1 is used to calculate the change in slope between two points on a member, $\Delta\theta_{AB}$.

Recall that

$$\phi = \frac{\mathrm{d}\theta}{\mathrm{d}x} = \frac{M}{EI}. \tag{86}$$

ϕ can be plotted against length to produce a curvature diagram. The curvature diagram can also be found by dividing the bending moment diagram by EI. Thus, solving Equation 86 gives us the moment of area theorem 1.

$$\Delta\theta_{AB} = \int_A^B \phi(x)dx. \tag{87}$$

21.2 Moment of Area Theorem 2

The moment of area theorem 2 is used to calculate the tangential deviation of a member at a specified point. The notation δ_{AB} represents the distance between point A and the tangent drawn from point B. Since $d\theta$ is small, $\tan\theta \approx \theta$.

$$d\delta = xd\theta \tag{88}$$

From the curvature equation, we know that $d\theta = \phi dx$.

$$d\delta = x\phi dx \tag{89}$$

Therefore,

$$\delta_{AB} = \int_A^B x\phi dx. \tag{90}$$

Another way to picture this expression is that it represents first moment of area under the curvature diagram taken about point B.

$$y = \overline{x}\int_A^B \phi dx. \tag{91}$$

Note that $\overline{x}$ is the distance between the centroid of the curvature diagram and point B.

Example

Find the deflection of the beam below at point C.

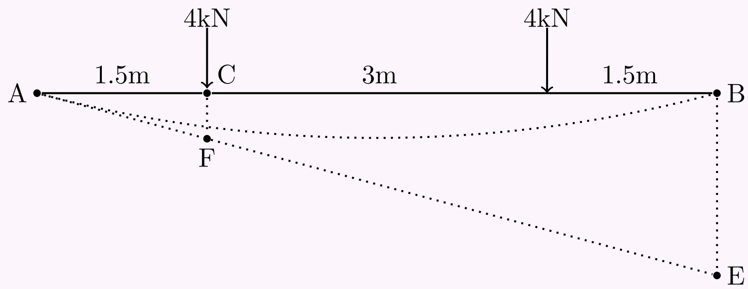


Figure 15: A 6 m long member with two 4 kN loads.

After drawing the bending moment diagram, we find that the maximum bending moment is 6 kNm. The bending moment diagram is in the shape of a trapezoid with its centroid at 3 m. Thus, we are able to calculate δ_{AB} using the moment of area theorem 2.

$$\delta_{BA} = \frac{6 \cdot \frac{3+6}{2}}{EI} \cdot 3 \tag{92}$$

$$= \frac{81}{EI} \tag{93}$$

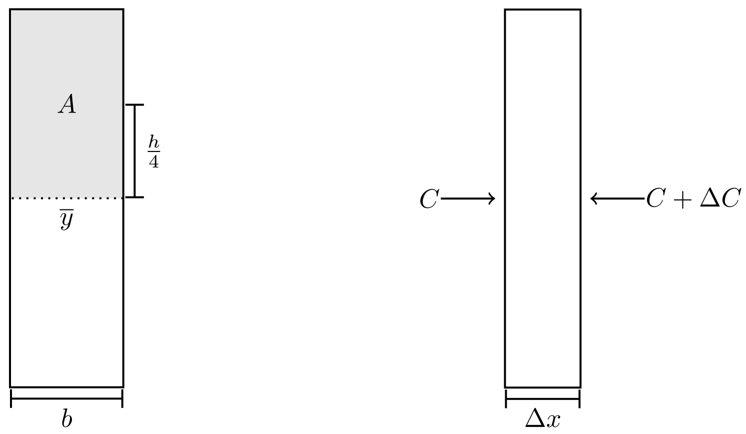
The bending moment diagram from A to C is in the shape of a triangle.

$$\delta_{CA} = \frac{\frac{1.5 \cdot 6}{2}}{EI} \cdot \frac{2}{3} \cdot 1.5 \tag{94}$$
$$= \frac{4.5}{EI} \tag{95}$$

Now we can use some simple geometry to determine the deflection at point C.

$$\frac{AB}{AC} = \frac{BE}{CF} \tag{96}$$
$$4 = \frac{\delta_{BA}}{\Delta_C + \delta_{CA}} \tag{97}$$
$$\Delta_C = \frac{15.75}{EI} \tag{98}$$

22 Shear Stress in a Beam



(a) Cross-sectional view of a beam. (b) Side view of a beam.

Figure 16: The shear stress of a beam. The first moment of area for the top half of the beam is $Q = \frac{Ah}{4}$.

Consider a slice of a beam. Because bending moment is higher on one side, this difference in moment will apply a force to the other side, ΔC.

$$\Delta C = \int_{y_{\text{bottom}}}^{y_{\text{top}}} \frac{\Delta M y}{I} dy \tag{99}$$
$$= \frac{\Delta M Q}{I} \tag{100}$$

Q is the first moment of area about the centroidal axis. The first moment of area is calculated by multiplying the cross-sectional area by the distance to the centroidal axis. The shear force which must resist this force is equal to the stress multiplied by the area at the bottom.

$$\Delta C = \tau b \Delta x \tag{101}$$

Combining these two equations produces

$$\tau = \lim_{\Delta x \to 0} \frac{\Delta M Q}{\Delta x I b} = \frac{VQ}{Ib}. \tag{102}$$

Wood Beams

Wood is **anisotropic** meaning that it has different mechanical properties in different directions. Steel is **isotropic**. To account for the variability in strength, the 5^{th} percentile strength is typically used in the design along with a factor of safety of 1.5.

23 Buckling of Thin Plates

Rearranging the terms from Equation 53, produces the following expression:

$$\sigma = \frac{\pi^2 E}{12}\left(\frac{t}{b}\right)^2 \tag{103}$$

However, if the edges of the plate are restrained from moving in the out-of-plane direction, and the width of the plate b is larger than its unrestrained length L, then the required stress to buckle the plate is:

$$\sigma = \frac{k\pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2. \tag{104}$$

μ is known as Poisson's ratio which is a measure of how much a material deforms in the directions orthogonal to the applied load ($\mu = -\frac{\varepsilon_y}{\varepsilon_x}$). k is a constant which depends on the applied loading conditions and boundary conditions. Equation 104 is the solution to a fourth order partial differential equation formulated by the American-Russian-Ukrainian engineer Stephen Timoshenko.

Boundary Condition	k
Both edges are restrained.	4
One edge is restrained	0.425
Triangular distributed load with both edges restrained.	6

The above equation can also be modified for shear stress.

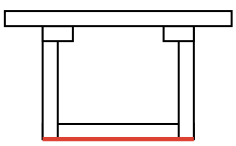
$$\tau = \frac{5\pi^2 E}{12(1-\mu^2)}\left(\left(\frac{t}{h}\right)^2 + \left(\frac{t}{a}\right)^2\right) \tag{105}$$

a is the spacing between diaphragms.

24 Design of a Thin-Walled Box Girder

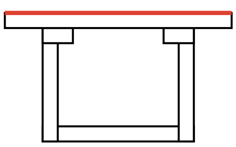
1. Tension Failure of Walls:

$$\sigma = \sigma^+_{\text{ult}} \tag{106}$$



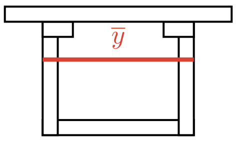
2. Compressive failure of walls:

$$\sigma = \sigma^-_{\text{ult}} \tag{107}$$



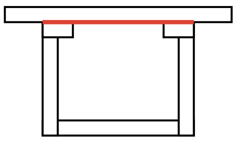
3. Shear failure of walls:

$$\tau = \tau_{\text{ult}} \tag{108}$$



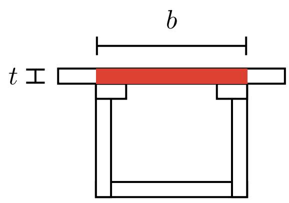
4. Shear failure of glue:

$$\tau = \tau_{\text{glue}} \tag{109}$$



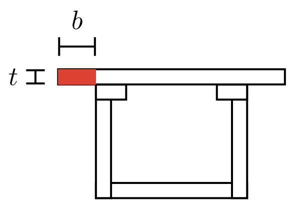
5. Buckling of the compressive flange between the webs:

$$\sigma = \frac{4\pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2 \tag{110}$$



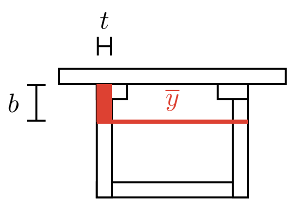
6. Buckling of the tips of the compressive flange:

$$\sigma = \frac{0.425\pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2 \tag{111}$$



7. Buckling of the webs:

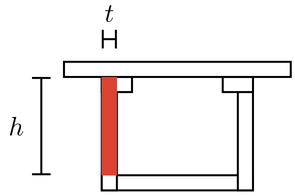
$$\sigma = \frac{6\pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2 \tag{112}$$



8. Shear buckling of the webs:

$$\sigma = \frac{5\pi^2 E}{12(1-\mu^2)}\left(\left(\frac{t}{h}\right)^2 + \left(\frac{t}{a}\right)^2\right)^2 \tag{113}$$

a is the distance between diaphragms.



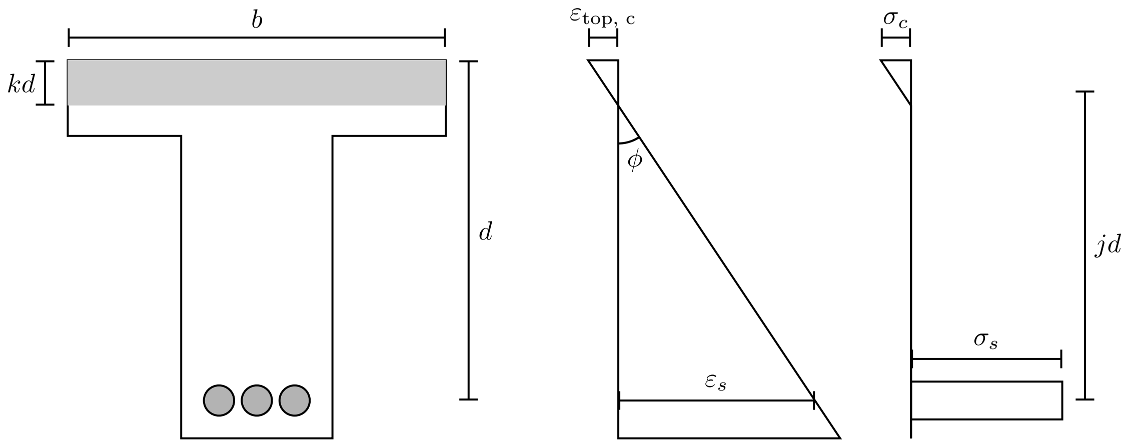
24.1 Girder Bridge Building Tips

If by any chance, you are planning to build a girder made of matboard and contact cement, here are some tips for you. The number one cause of failure for most matboard bridges is the contact cement. Make sure to apply enough contact cement to **both surfaces** of the matboards which are going to be glued, and wait for **15 minutes** before sticking them on. Leave it for **3 days**, and make sure to **apply enough pressure** with clamps, so the pieces don't separate.

25 Concrete

Concrete is made by mixing cement, water, and air together. Concrete has high compressive strength and low tensile strength. The tensile strength of the concrete structure can be improved on by adding steel reinforcements. Reinforcing steel also controls the crack width of the concrete by distributing the tensile force over multiple smaller cracks rather than fewer large ones.

26 Calculating the Flexural Strength of Reinforced Concrete



(a) The cross-section of a reinforced concrete beam. (b) Strain Diagram. (c) Stress Diagram.

Figure 17: Definitions.

- A_s - The cross-sectional area of steel.
- kd - The distance between maximum compression and the neutral axis.
- jd - The length of the flexural lever arm which is the distance between the compressive and tensile forces.
- F_c, F_s - The net compressive force in the concrete and the net tensile force in the steel.
- E_c, E_s - The Young's Modulus for concrete and steel.

Before we determine whether the concrete beam can carry the flexural stresses, we must first calculate the values of k and j, as shown in Figure 17.

$$n = \frac{E_s}{E_c} \tag{114}$$

$$\rho = \frac{A_s}{bd} \tag{115}$$

$$k = \sqrt{(n\rho)^2 + 2n\rho} - n\rho \tag{116}$$

$$j = 1 - \frac{1}{3}k \tag{117}$$

Derivation

Calculating Strain Previously, we found that the relationship between strain and curvature is

$$\varepsilon = \phi y. \tag{118}$$

Thus,

$$\begin{aligned} \varepsilon_{\text{top,c}} &= \phi kd && (119) \\ \varepsilon_s &= \phi d(1-k) && (120) \\ & && (121) \end{aligned}$$

Calculating Net Forces
F_c can be calculated by finding the area of the top triangle in Figure 17c.

$$F_c = \frac{1}{2} bkdE_c\varepsilon_{\text{top},c} \tag{122}$$

Similarly,

$$F_s = E_s \varepsilon_s A_s \tag{123}$$

Calculating k and j
Because the axial force is 0, the net tensile force in the steel must equal to the net compressive forces in the concrete.

$$F_c = F_s \tag{124}$$

$$\frac{1}{2}\phi b(kd)^2 E_c = \phi E_s A_s d(1-k) \tag{125}$$

$$\frac{1}{2}k^2 + k\frac{E_s A_s}{E_c bd} - \frac{E_s A_s}{E_c bd} = 0 \tag{126}$$

We can now substitute in the variables n and ρ as defined above to simplify the equation.

$$\frac{1}{2}k^2 + kn\rho - n\rho = 0 \tag{127}$$

$$k = \sqrt{(n\rho)^2 + 2n\rho} - n\rho \tag{128}$$

Because the distributed stress on the concrete is triangular, we know that

$$jd = d - \frac{1}{3}kd \tag{129}$$

An important property of the flexural lever arm is that the compression and tension forces at both ends of the arm form a couple.

$$M = F_c jd = F_s jd \tag{130}$$

Rearranging the equation, we get

$$\sigma_s = \frac{M}{A_s jd} \tag{131}$$

To calculate the stress on the concrete, we must substitute the steel stress back into the steel strain equation (Equation 120) to determine curvature.

$$\phi = \frac{M}{A_s E_s jd^2(1-k)} \tag{132}$$

Then, we must substitute the curvature into the concrete strain equation (Equation 119) to determine the maximum stress in the concrete.

$$\sigma_c = \frac{kM}{(1-k)nA_s jd} \tag{133}$$

Lastly, safety factors must be accounted for to ensure that the concrete beam can safely resist bending moments.

$$\sigma_s \leq 0.6\sigma_y \tag{134}$$

$$\sigma_c \leq 0.5\sigma_y \tag{135}$$

27 Calculating the Shear Strength of Reinforced Concrete

Reinforced concrete has two basic mechanisms for carrying shear stresses.

1. **Aggregate Interlock -** The rough, cracked surfaces of the concrete embedded with aggregate helps carry tension across the crack.

2. **Shear Reinforcement** - Perpendicular steel reinforcements also known as **stirrups** can be used to carry tension across the crack.

There are two primary ways at which reinforced concrete can fail. The first method of failure is when the shear stress exceeds the shear capacity of the beam. The second method of failure is when the top of the concrete beam fails by crushing due to the diagonal compression from shear.

Additional Definitions

- V_c - The shear strength of the concrete (Force).
- V_s - The shear strength of the steel (Force).
- τ_c - The maximum shear stress of the concrete.
- σ_c - The compressive strength of concrete.
- σ_s - The compressive strength of steel.
- b_w - The web width.
- A_v - The cross-sectional area of the stirrups.
- s - The spacing between stirrups.

The total shear resistance can be calculated by

$$V_r = V_c + V_s, \tag{136}$$

We must ensure that the shear resistance, V_r, is greater the applied shear force, V_a, but less than the shear force where concrete is crushed, V_{crush}. There is no point of adding extra stirrups to reinforce the concrete, if it fails by crushing first. Safety factors of 2 and 5/3 can be used for concrete and steel respectively to ensure that the member is safe.

$$V_a <= 0.5V_c + 0.6V_s <= 0.5V_{\text{crush}} \tag{137}$$

$$V_{\text{crush}} = 0.25f_c'b_2jd \tag{138}$$

27.1 Calculating V_c and V_s

By Jourawski's equation,

$$V = \tau b_w jd \tag{139}$$

The shear stress of concrete was found to be influenced by the size effect in that larger members tend to have larger cracks which decreases shear strength. Through experimentation, a good approximation of shear stress was found to be modeled by the piecewise function:

$$V_c = \begin{cases} \dfrac{230\sqrt{\sigma_c}}{1000 + 0.9d} b_w jd & \dfrac{A_v\sigma_s}{b_w s} < 0.06\sqrt{\sigma_c} \text{ or no stirrups} \\ 0.18\sqrt{\sigma_c} b_w jd & \dfrac{A_v\sigma_s}{b_w s} \geq 0.06\sqrt{\sigma_c} \end{cases} \tag{140}$$

Steel shear strength can be approximated as

$$V_s = \frac{A_v \sigma_s jd}{s} \cot 35^\circ. \tag{141}$$

28 Prestressed Concrete Members

Prestressed concrete members help reduce cracking from tensile forces in concrete. The equation calculating the stress of a prestressed member is

$$\sigma_{\text{top}} = -\frac{P}{A} + \frac{Pey_{\text{top}}}{I} - \frac{My_{\text{top}}}{I} \tag{142}$$

$$\sigma_{\text{bottom}} = -\frac{P}{A} - \frac{Pey_{\text{bottom}}}{I} + \frac{My_{\text{bottom}}}{I}. \tag{143}$$

P is the tensile force in the tendon, and e is the distance between the tendon and centroidal axis (positive when below).