Report of Assignment 1

Written by HUANG Guanchao, SID 11912309 and GONG Xinrui

The source code of this assignment can be retrieved at.

1.4

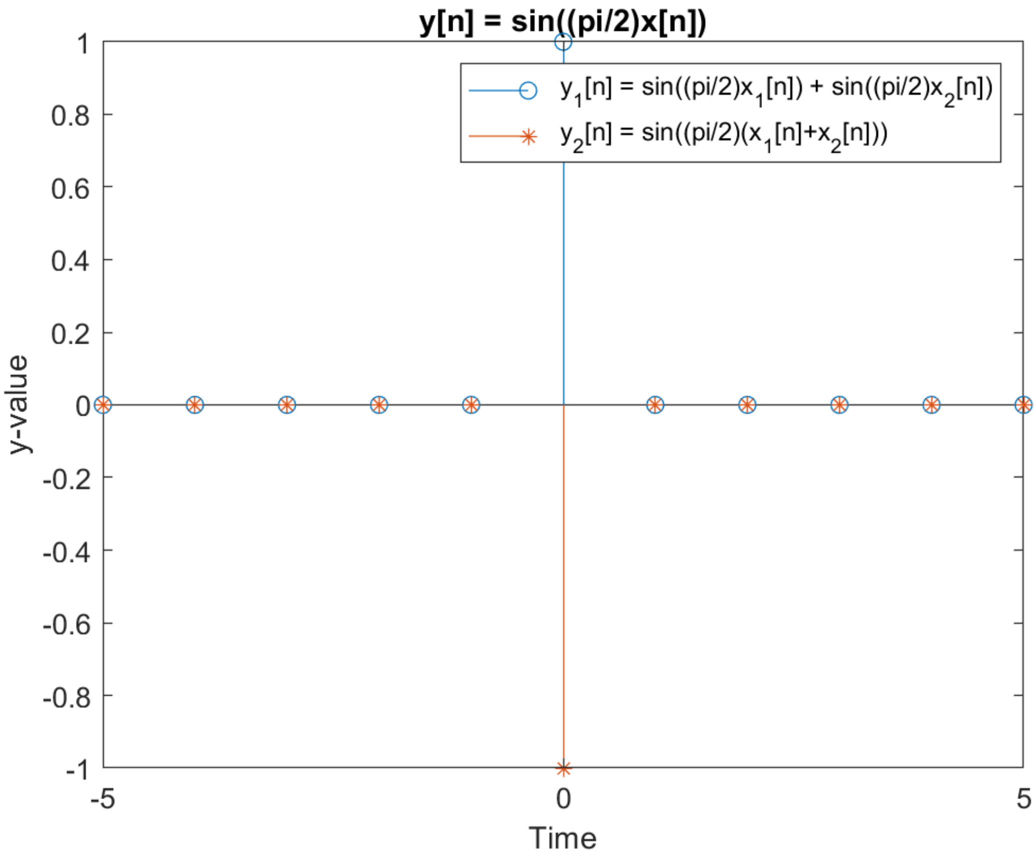
Basic Problem a

> The system $y[n] = \sin((\pi/2)x[n]$ is not linear. Use the signals $x_1[n] = \delta[n]$ and $x_2[n] = 2\delta[n]$ to demonstrate how the system violates linearity.

The MATLAB script is shown in the following code block.

```
% define scopes
l = 5;

% Basic Problem a
n = [-l:l];
x1 = [zeros(1, l) 1 zeros(1, l)];
x2 = 2 * x1;
x3 = x1 + x2;

y1 = L(x1);
y2 = L(x2);
y3 = y1 + y2
y4 = L(x3)

stem(n, y3, 'o')
hold on
stem(n, y4, '*')

title('y[n] = sin((pi/2)x[n])')
xlabel('Time');
ylabel('Output')
legend('y_1[n] = sin((pi/2)x_1[n]) + sin((pi/2)x_2[n])', ...
    'y_2[n] = sin((pi/2)(x_1[n]+x_2[n]))');

% define system
function y = L(x)
    y = sin(pi / 2 * x)
end
```

The plot is shown below.



Apparently, $y_1[n] = \sin\{(\pi/2)x_1[n]\} + \sin\{(\pi/2)x_2[n]\}$ is not identical with $y_2[n] = \sin\{(\pi/2)(x_1[n] + x_2[n])\}$, hence the system is not linear.

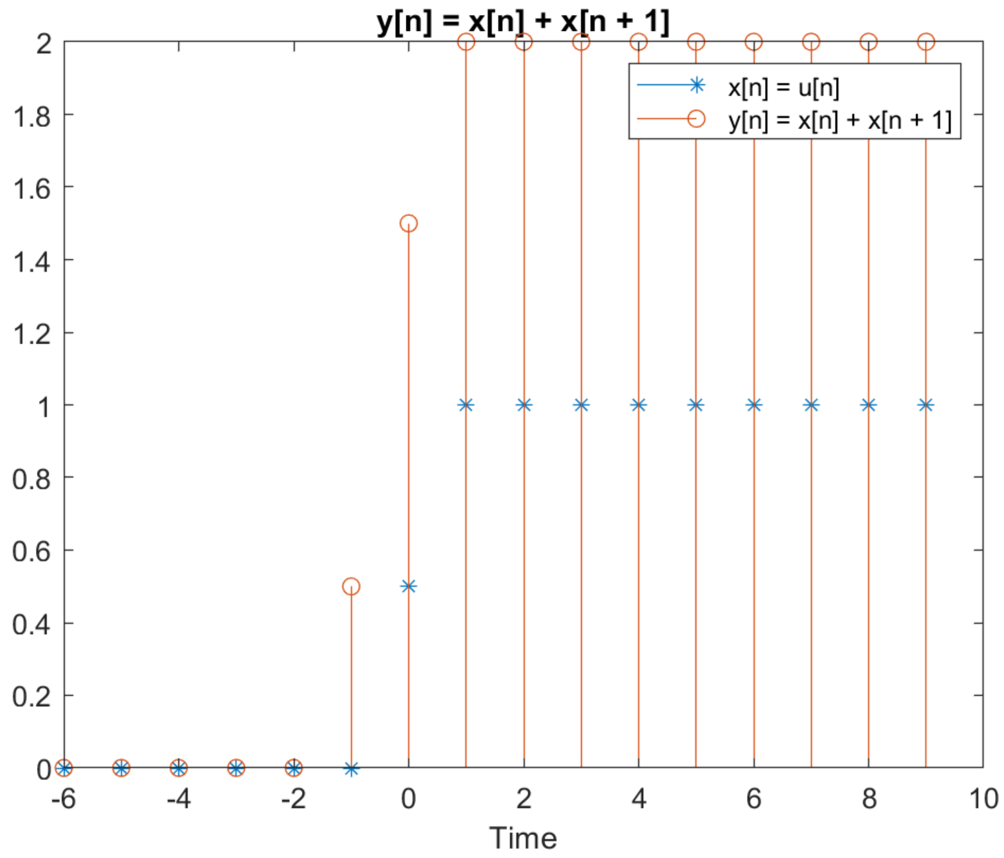
Basic Problem b

> The system $y[n] = x[n] + x[n+1]$ is not causal. Use the signal $x[n] = u[n]$ to demonstrate this. Define the MATLAB vectors x and y to represent the input on the interval $-5 \leq n \leq 9$, and the output on the interval $-6 \leq n \leq 9$, respectively.

The MATLAB script is shown in the following code block.

```
%Basic Problem b
n = [-6 : 9];
x = [zeros(1, 6) 1/2 ones(1, 9) ]
x0 = [zeros(1, 6) 1/2 ones(1, 9)];
x1 = [zeros(1, 5) 1/2 ones(1, 10)];
y = x0 + x1;

stem(n, x, '*');
stem(n, y);
```

The plot is shown below.



The excitation starts at time $n = 0$, however, before that at time $n = -1$, the output $y[n]$ is non-zero. Hence, the system is not causal.

Intermediate Problem c

The MATLAB script is shown in the following code block.

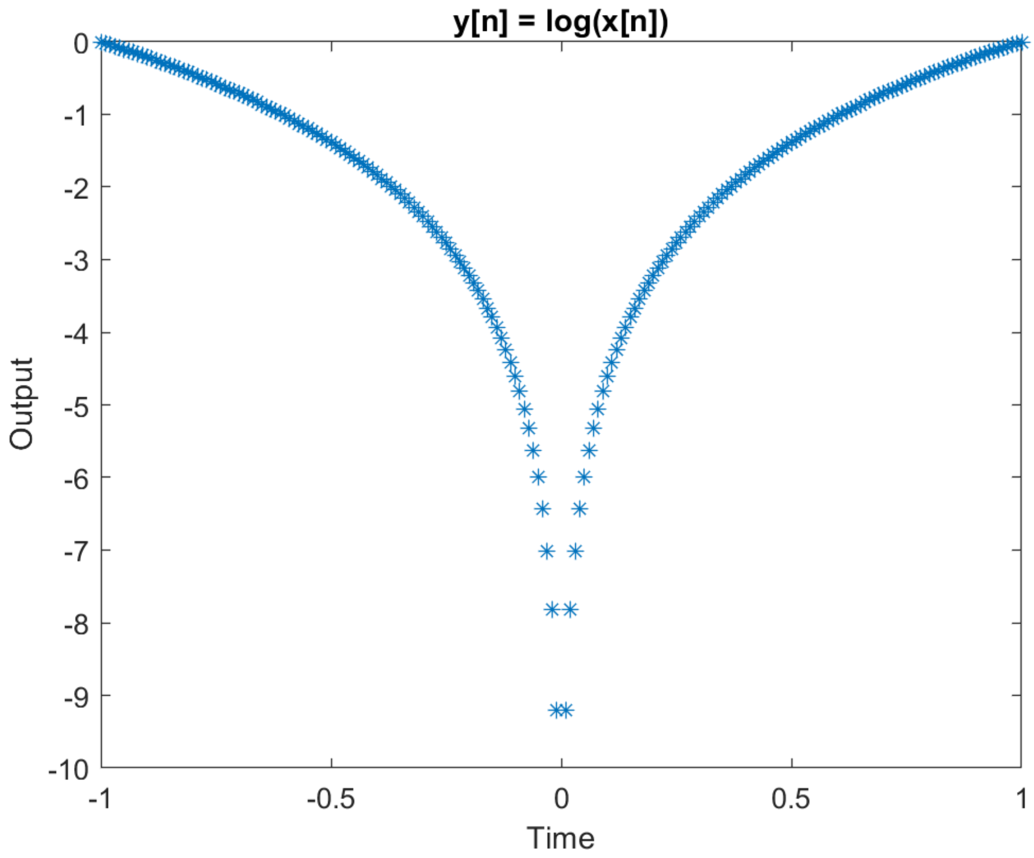
```
n = [-1 : 0.001 : 1];
x = n.^2;
y = log(x);
plot(n,y,'*')
```

The output in the terminal is

```
>> run("c:\Users\Guanc\Documents\GitHub\SigSys-lab\src\A1\A1_c.m")

...

Columns 97 through 112

   -6.4378    -7.0131    -7.8240    -9.2103       -Inf    -9.2103    -7.8240    -7.0131
  -6.4378    -5.9915    -5.6268    -5.3185    -5.0515    -4.8159    -4.6052    -4.4145

...

>>
```

In which there is `-Inf` output.

The plot is shown below.



The system is not stable, because when we give a bounded input the output approaches infinity at time $n = 0$, which contradicts the definition of stability.

Intermediate Problem d

Advanced Problems e, f, g

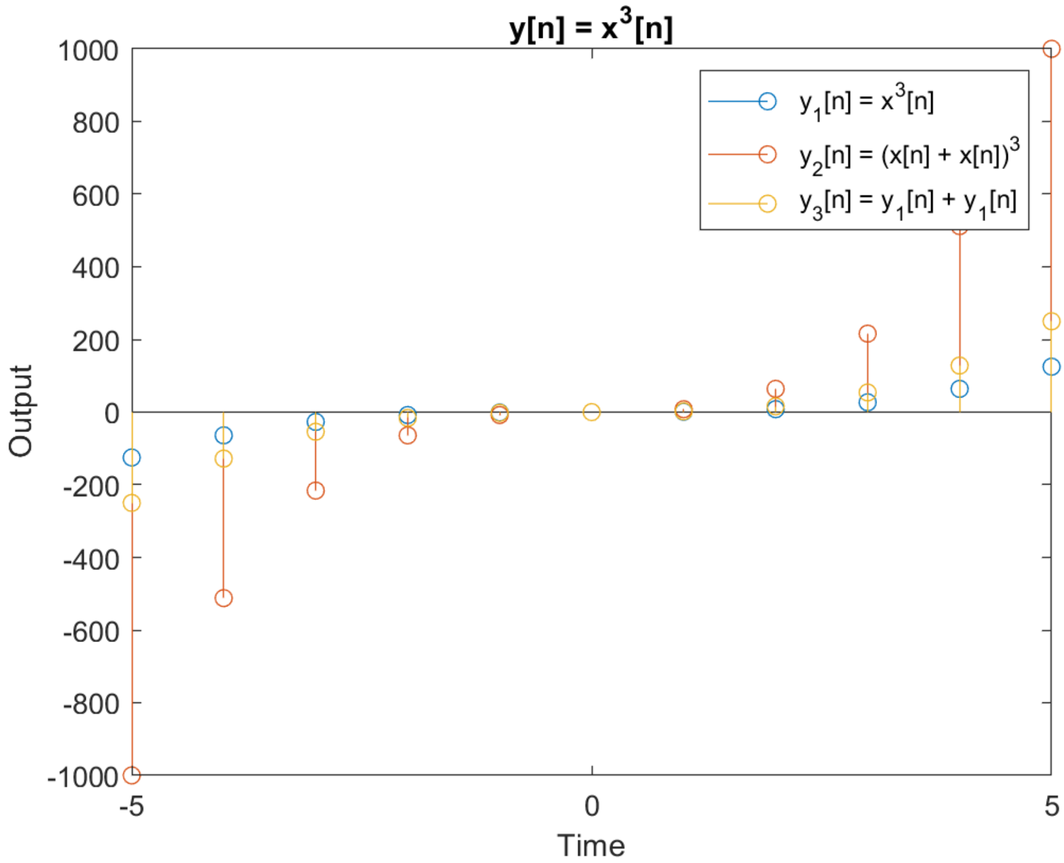
> For each of the following systems, state whether or not the system is linear, time-invariant, causal, stable, and invertible. For each property you claim the system does not possess, construct a counter-argument using MATLAB to demonstrate how the system violates the property in question.
>
> (e). $y[n] = x^3[n]$.

Define input signal to be $x[n] = n$, the crucial part of MATLAB script is shown below.

```
% define scope
l = 5;

% 1.4 Advanced Problem e
n = [-l:l];
x = n;

y1 = L(x);
y2 = L(x + x)
y3 = y1 + y1

stem(n, y1);
hold on
```

```
stem(n, y2);
hold on
stem(n, y3);

% define system
function y = L(x)
    y = x.^3;
end
```

According to the plot, $2x^3[n] \neq (2x[n])^3$, hence the system is nonlinear.



The system is time-invariant, causal, stable and invertible.

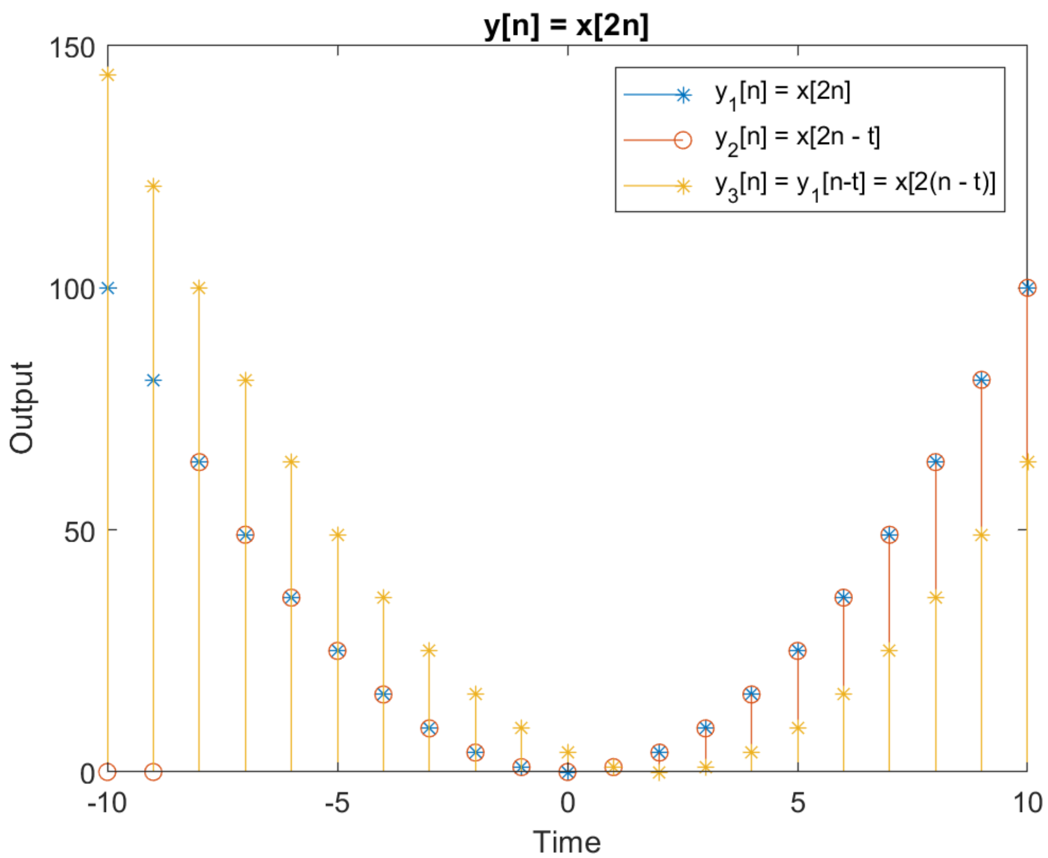
> (f). $y[n] = nx[n]$.

> (g). $y[n] = x[2n]$.

Define input signal $x[n] = n^2$, the MATLAB script is shown in the code block below.

```
% define scope
l = 10;

% 1.4 advanced problem g
% time-invariance

n = [-l:l]
t = 2
y1 = L(n)
nt = n - t;
y2 = [zeros(1:t) y1(1, t + 1:2 * l + 1)]
```

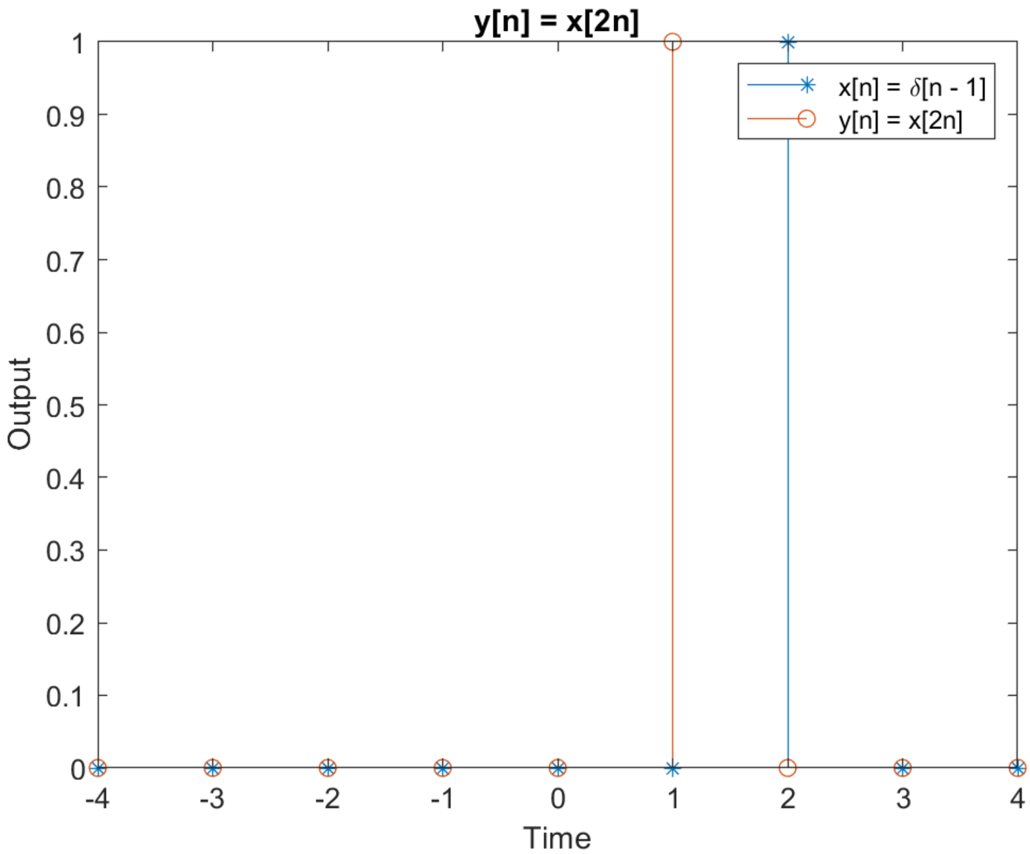
```
y3 = L(nt)


stem(n, y1, '*');
hold on
stem(n, y2);
hold on
stem(n, y3, '*');

...

%def function
function y = L(x)
    y = x.^2
end
```



According to the figure, $y_2[n] = x[2n - t]$ is not identical with $y_3[n] = y_1[n - t] = x[2(n - t)]$, hence the system is time-variant.

Define input signal $x[n] = \delta[n - 1]$, the MATLAB script is shown in the code block below.

```
l = 4;
n = [-l:l];
% define unit impulse input at n = 2
x1 = [zeros(1, l + 2) 1 zeros(1, l - 2)]

nt = 1:2:2 * l + 1
y4 = [zeros(1, l / 2) x1(nt) zeros(1, l / 2)]

stem(n, x1, '*')
hold on
stem(n, y4)
```



According to the figure, the impulse input is at time $n=2$, however, there is response at time $n=1$, hence the system is not causal.

The system is linear, stable and invertible.

1.5

> $$y[n] = ay[n-1] + x[n]. \tag{1.6}$$

Advanced Problem a

> Write a function `y = diffeqn(a, x, yn1)` which computes the output $y[n]$ of the causal system determined by Eq. (1.6). The input vector `x` contains $x[n]$ for $0 \leq n \leq N-1$ and `yn1` supplies the value of $y[-1]$. The output vector `y` contains y[n] for $0 \leq n \leq N-1$. The first line of your M-file should read
>
> ```
> function y = diffeqn(a, x, yn1)
> ```

> Hint: Note that $y[-1]$ is necessary for computing $y[0]$, which is the first step of the autoregression. Use a `for` loop in your M-file to compute $y[n]$ for successively larger values of n, starting with n = 0.

The function is shown in the code block below.

```
function y = diffeqn(a, x, yn1)
    N = length(x)
    y(1) = a * yn1 + x(1)

    for index = 2:N
        y(index) = a * y(index - 1) + x(index)
    end

end
```

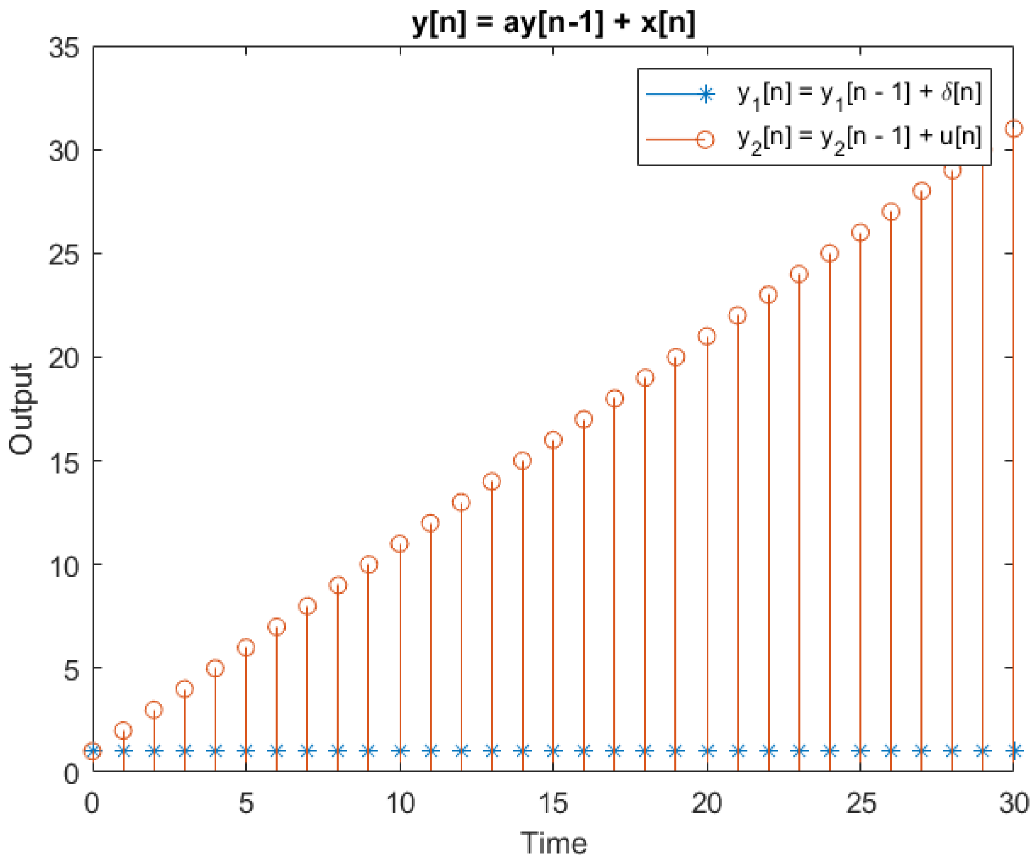
Advanced Problem b

> Assume that $a = 1$, $y[-1] = 0$, and that we are only interested in the output over the interval $0 \leq n \leq 30$. Use your function to compute the response due to $x_1[n] = u[n]$ and $x_2[n] = u[n]$, the unit impulse and unit step, respectively. Plot each response using `stem`.

The MATLAB script is shown in the code block below.

```
a = 1;
yn1 = 0;
n = 0:1:30;
x1 = [1 zeros(1, 30)];
x2 = ones(1, 31);

y1 = diffeqn(a, x1, yn1)
y2 = diffeqn(a, x2, yn1)

stem(n, y1, '*')
hold on
stem(n, y2)
```

Using the function define in problem a to plot $y[n]$, the figure is shown below.



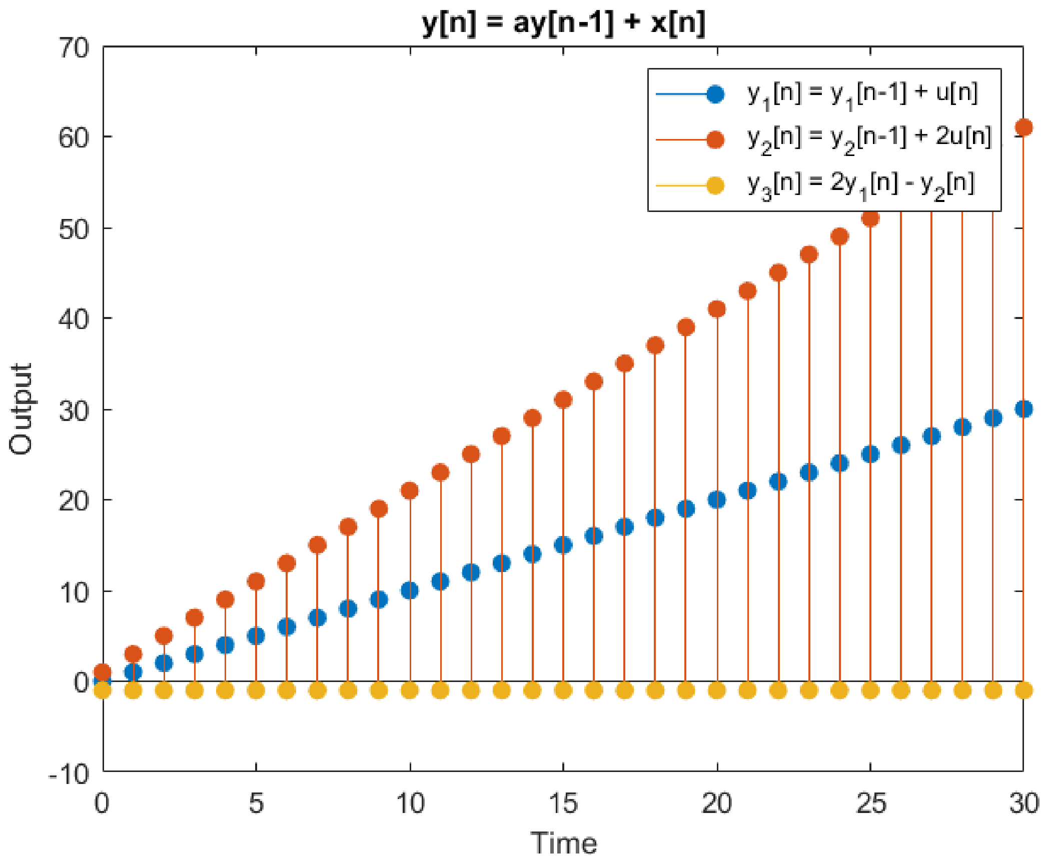
Advanced Problem c

> Assume again that $a = 1$, but that $y[-1] = -1$. Use your function to compute $y[n]$ over $0 \leq n \leq 30$ when the inputs are $x_1[n] = u[n]$ and $x_2[n] = 2u[n]$. Define the outputs produced by the two signals to be $y_1[n]$ and $y_2[n]$, respectively. Use `stem` to display both outputs. Use stem to plot $(2y_1[n] - y_2[n])$. Given that $\text{Eq. }(1.6)$ is a linear difference equation, why isn't this difference identically zero?

The MATLAB script is shown below.

```
a = 1;
yn1 = -1;
x1 = ones(1, 31);
x2 = 2 * x1;

y1 = diffeqn(a, x1, yn1)
y2 = diffeqn(a, x2, yn1)
y3 = y1 .* 2 - y2

stem(n, y1, '')
hold on
stem(n, y2, '')
hold on
stem(n, y3, '')
```

The plot is shown below.



The expression of difference can be transferred into

$$\begin{aligned}2y_1[n]-y_2[n]=&2y_1[n-1]+2u[n]-y_2[n-1]-2u[n]\\=&2y_1[n-1]-y_2[n-1]\\=&\cdots\\=&2y_1[-1]-y_2[-1]\\=&-1\end{aligned}$$

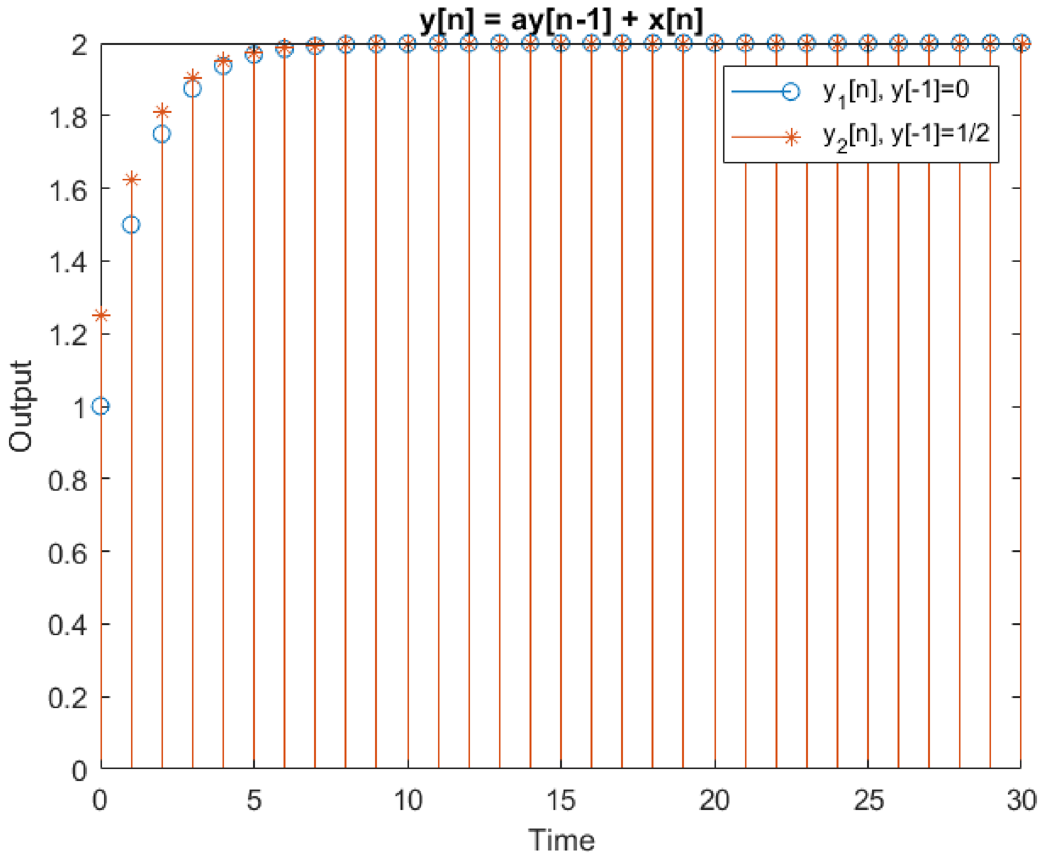
Hence, the signal $y_3[n]=2y_1[n]-y_2[n]$ is a non-zero constant.

Advanced Problem d

> The causal systems described by $\mathrm{Eq.}\ (1.6)$ are BIBO (bounded-input bounded-output) stable whenever $|a|<1$. A property of these stable systems is that the effect of the initial condition becomes insignificant for sufficiently large n. Assume $a=1/2$ and that `x` contains $x[n]=u[n]$ for $0\le n\le 30$. Assuming both $y[-1]=0$ and $y[-1]=1/2$, compute the two output signals $y[n]$ for $0\le n\le 30$. Use `stem` to display both responses. How do they differ?

The MATLAB script is shown in the code block below.

```
a = 1/2;
yn1_1 = 0;
yn1_2 = 1/2;

x = ones(1, 31);

y1 = diffeqn(a, x, yn1_1)
y2 = diffeqn(a, x, yn1_2)

stem(n, y1)
hold on
stem(n, y2, '*')
```



According to the plot, the magnitude of signal $y_1[n]$ is less than $y_2[n]$ at the beginning, and the two signals both approaches the same value.