Problem Chosen	2020 MCM/ICM Summary Sheet	Team Control Number
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Title of you paper

From here, begin your summary

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Contents

1 Introduction

1.1 Problem Background

The registration fee for MCM/ICM is $100 per team. Please register only the teams that will take part in the contest. Registration fees are non-refundable. We accept payment via Credit Card, and payment must be made via our secure web site. Our secure site will process your credit card payment, so your credit card number is protected. Our system will not store your credit card number after it processes your payment. We regret that we are not able to accept other payment forms at this time.

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1.2 Problem Restatement

1.3 Our Work

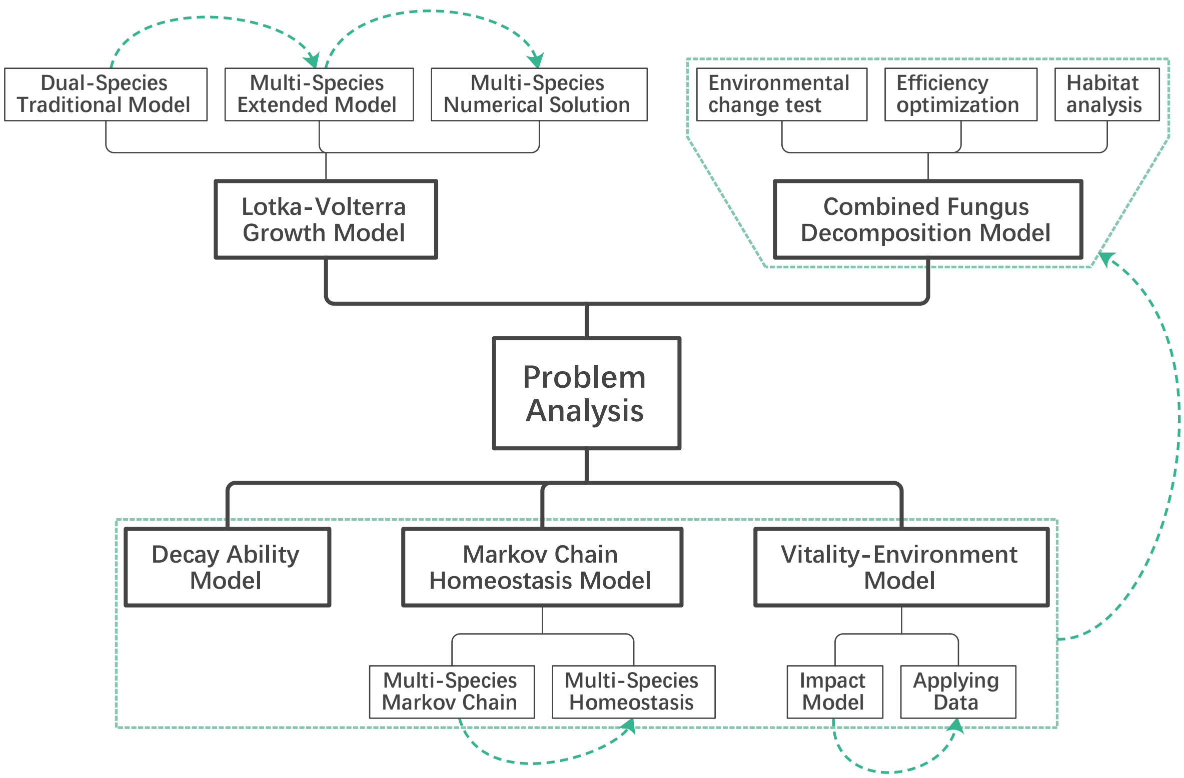
2 Basic Assumptions and Notations

Our model is based on several approximate but reasonable assumptions. For a certain ecological configuration in restricted region, such as the decomposition process of a log dominated by fungus, it is equivalent for considering the quantity of fungus individual, biomass or population density. Therefore, our modeling is based on the population density of each species of fungus.

Since our modeling is composed of several different parts, other necessary assumptions may be found in their own sections.

Assumption 1. The vital activity of fungus community, that is, the product of metabolization, has no effect on the environment humidity and temperature.

Figure 1: Work flow of our modelling



Justification In nature circumstances, the decomposition process happens in open air, the heat and moisture produced or absorbed by fungus can be promptly carried off or replenished by the external environment, hence the local environment dominates the conditions of the growth of fungus community.

Assumption 2. The decomposition is dominated by the fungus community, and no other species have effect on the system.

Assumption 3. The inter-species relation among the populations in the community is merely competition.

Justification Other inter-species relations such as mutualism, parasitism and predation are rarely seen among fungus. Considering only competition enables us to utilize existed models.

Table 1: Notations and definitions

Symbol	Definition
n	Total number of fungus populations in the community
x	(Relative) Biomass of the fungi isolate
D	Decomposition rate
r	Hyphal extension rate
d	Moisture niche width
c	Competitive ranking
m	Moisture tolerance
w	Environmental moisture varying range

3 Decomposition Rate with Respect to Dominant Fungi Traits

In this problem, the **hyphal extension rate** and **moisture tolerance** are the two basic dominating traits we need to consider in modeling the decay ability of the fungi community.

3.1 Decomposition rate model regarding hyphal extension rate, moisture tolerance and competitive ranking

From Figure 3 and Figure 4, it is observed that the relation between the decomposition rate and hyphal extension rate, as well as between the log transformed decomposition rate and relative moisture tolerance are approximately linear.

$$\begin{cases} D = k_1 r + b_1 \\ \ln D = k_2 m + b_2 \end{cases} \tag{3.1}$$

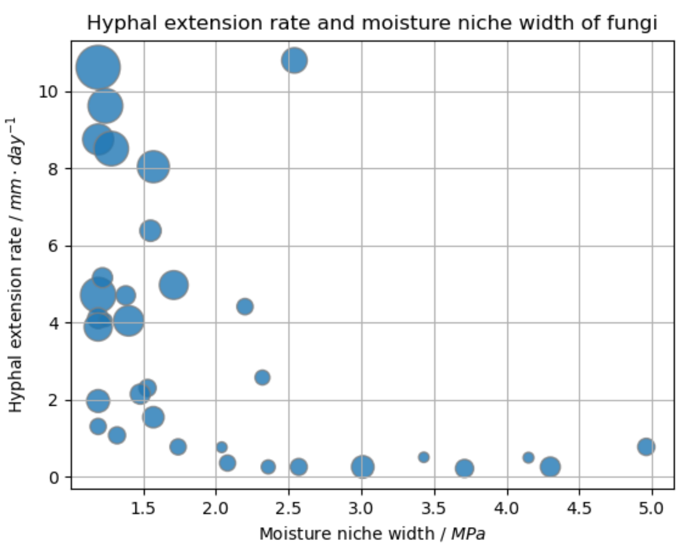


Figure 2: Hyphal extension rate and moisture niche width of the fungi, size of the bubble represents he decomposition rate of logs after 122 days of decay

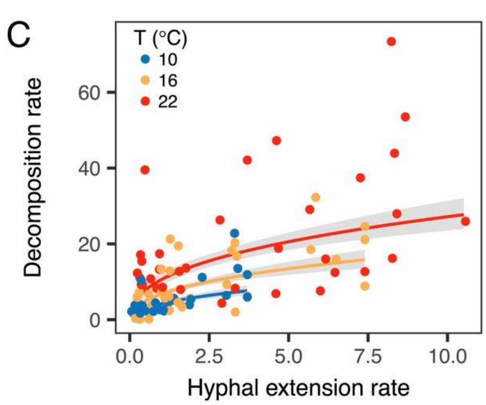


Figure 3: The relationship between the hyphal extension rate (mm/day) of various fungi and the resulting wood decomposition rate (% mass loss over 122 days) at various temperatures. This figure is adapted from [9].

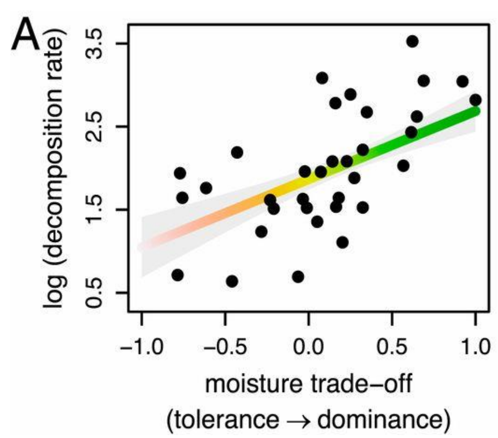


Figure 4: The relationship between the moisture tolerance (difference of each isolate's competitive ranking and their moisture niche width, both scaled to $[0, 1]$) of various fungi and the resulting wood decomposition rate (% mass loss over 122 days, log transformed). This figure is adapted from [9].

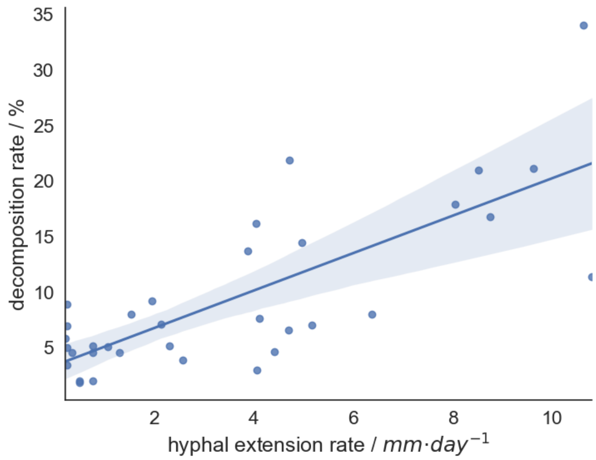


Figure 5: The relationship between the hyphal extension rate of various fungi and the resulting wood decomposition rate. The hyphal extension rate is the geometrical average of the value under 10, 16 and 22 Celsius in [9].

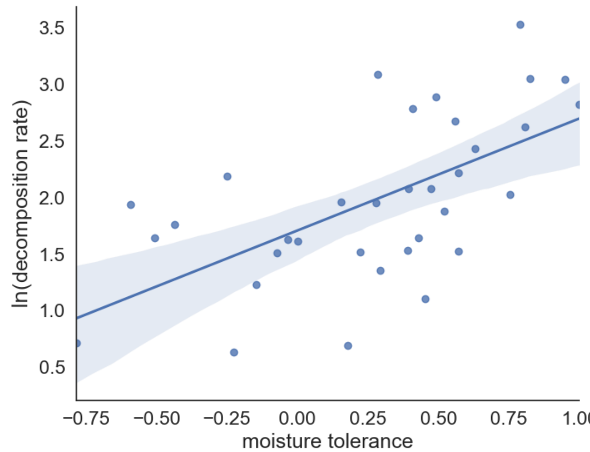


Figure 6: The relationship between the moisture tolerance. The tolerance-dominance trade-off is commonly found in the competition behavior of tested fungi. [11]

Table 2: The fitted parameters for the decay ability of an isolate over a short period

k_1	b_1	k_2	b_2
1.6850	3.4082	0.5831	0.4896

Note that, in 4, the x-axis variable is not exactly the moisture tolerance (moisture niche width), instead is the difference of each isolate's competitive ranking and their moisture niche width, in range $[-1, 1]$. Hence in the model, competitive ranking c is taken into consideration. According to [11], the moisture niche width is normalized as

$$\hat{d}_i = \frac{d_i - d_{\min}}{d_{\max}}. \tag{3.2}$$

3.2 Decomposition rate data fitting for single fungus isolate

Based on the traits data of various species of fungus provided in [11], and the resulting experimental decomposition rate in [9], the fitting result is shown in 2 and [].

3.3 Extending the model to multi-species system

For multi-species case, according to 7, the decomposition of logs increases with the hyphal extension rate of the community, and is approximately characterized by a linear model with respect to the community-weighted mean extension rate. Therefore, in order to describe the decay ability of a community consists of multiple species of fungus, the relative quantity relation

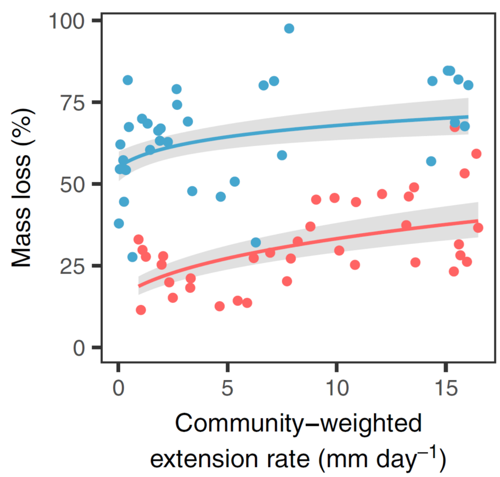


Figure 7: The decomposition of logs increases with the hyphal extension rate of the fungal community that colonized them. This figure is adapted from [9].

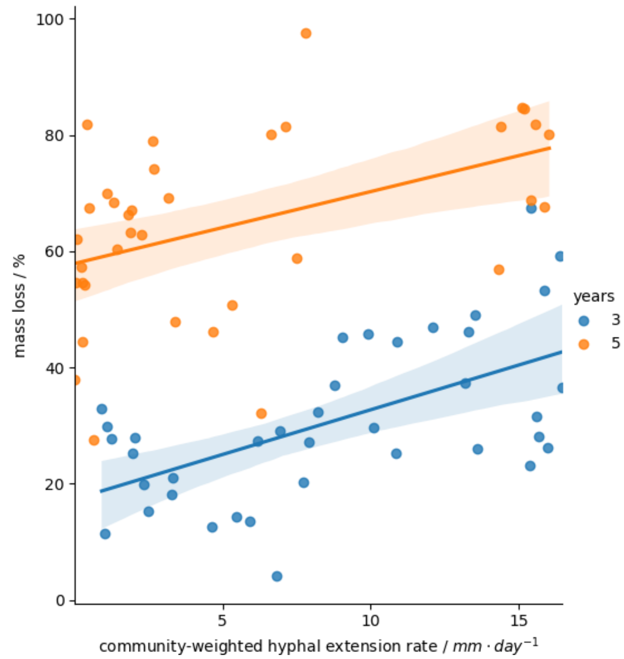


Figure 8: The mass loss rate increases with the community-weighted hyphal extension rate. The experiments are conducted on a variety of wood substrates for 3 or 5 years.

Table 3: The fitted parameters for the decay ability of a community over a long period

Years	k_3	b_3
3	1.539	17.32
5	1.237	57.87

between of each isolate in the community is necessary.

The decay ability of a fungi community can be described quantitatively as the following expression.

$$D_{\text{comm}} = k_3\bar{r} + b_3 = k_3 \frac{r_1 x_1 + \cdots + r_n x_n}{x_1 + \cdots + x_n} + b_3 = k_3 \frac{\sum_{i=1}^{n} r_i x_i}{\sum_{i=1}^{n} x_i} + b_3. \tag{3.3}$$

Research conducted in [9] presents the data obtained from experimental results on various wood substrate for 3 years or 5 years. With the data presented in [11], the coefficient is roughly determined as in 3, and the result is shown in figure 8.

For solving this model for practical prediction of the decay ability of fungi community, each x_i needs to be specified. This part is discussed in the following sections.

4 Competition Model of the Growth of Fungi Community

4.1 Traditional Lotka-Volterra model for dual-species system

In the early twentieth century, Alfred Lotka and Vito Volterra simultaneously derived a model that described how competition affects population growth from the perspective of differential equations, which is now known as the Lotka-Volterra, or LV model. [1]

First, for single population, if the resources are sufficient enough, the population size, or similarly, density, is expected to grow in the form of

$$\frac{\mathrm{d}x_1}{\mathrm{d}t} = r_1 x_1. \tag{4.1}$$

In which, the intrinsic rate of increase r is the per capita growth rate that could potentially be realized by a fungus population. However, unlimited growth is impossible in reality, environment itself has retardant effect on the population. Take the bounded carrying capacity into consideration, obtains

$$\frac{\mathrm{d}x_1}{\mathrm{d}t} = r_1 x_1 \left(1 - \frac{x_1}{K_1}\right). \tag{4.2}$$

The extra term places a limit on exponential growth, and the resulting equation represents the logistic growth.

In a dual-species system, the resistance against the growth of one population comes not only from the environment, but also another specie. It is natural that, when specie 2 has larger population density, specie 1 will be in face of larger competitive pressure, and similar for species 2. Such notion is also capable for intra-specie competition. Combining the inter-species and intr-specie competition into (4.2), obtains the differential equations set

$$\begin{cases} \dfrac{\mathrm{d}x_1}{\mathrm{d}t} = r_1 x_1 \left(1 - \dfrac{\alpha_{11}x_1 + \alpha_{12}x_2}{K_1}\right) \\ \dfrac{\mathrm{d}x_2}{\mathrm{d}t} = r_2 x_2 \left(1 - \dfrac{\alpha_{22}x_2 + \alpha_{21}x_1}{K_2}\right). \end{cases} \tag{4.3}$$

Which is the model for dual-species competition. Note that, neither Lotka nor Volterra explained this equation from the perspective of ecology, it is Tilman who made a convincing interpretation. His resource-based model included resources requirement, consumption and supply, which is concise and rigorous but not the topic of this literature. [15]

4.2 Extended Lotka-Volterra model for multi-species

Based on LV model, we extended (4.3) to a multi-species system with n various populations.

$$
\begin{cases}
\dfrac{\mathrm{d}x_1}{\mathrm{d}t} = r_1 x_1 \left(1 - \dfrac{\sum_{j=1}^{n} \alpha_{1j} x_j}{K_1}\right) \\
\dots \\
\dfrac{\mathrm{d}x_i}{\mathrm{d}t} = r_i x_i \left(1 - \dfrac{\sum_{j=1}^{n} \alpha_{ij} x_j}{K_i}\right) \\
\dots \\
\dfrac{\mathrm{d}x_n}{\mathrm{d}t} = r_n x_n \left(1 - \dfrac{\sum_{j=1}^{n} \alpha_{nj} x_j}{K_n}\right)
\end{cases}
\tag{4.4}
$$

The equations set can be simplified with matrix differential equations as

$$
\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} = \boldsymbol{r} \cdot \boldsymbol{x} \left[\mathbf{1} - (\boldsymbol{A}\boldsymbol{x}) \cdot \hat{\boldsymbol{K}}\right]. \tag{4.5}
$$

In which the matrix $\hat{\boldsymbol{K}}$ is the element-wise reciprocal of $\boldsymbol{K}$, that is, $\hat{\boldsymbol{K}}_{ij}\boldsymbol{K}_{ij} = 1$.

The growth process of fungus community considering competition is characterized by this first-order linear ordinary differential equations group, with n unknown functions. Note that while the analytical can be difficult to be found, we can still figure out the development of the system using computer numerically.

4.3 Adapted Lotka-Volterra model for fungi community

As shown in (4.5), a significant drawback of LV model is that, in total $n^2 + n$ extra defined parameters, symbolizing the stress exerted from each specie onto each another and the carrying capacity of the environment for each specie are needed. It would be impractical for our model if such amount of factors without support from experimental result need to be considered.

Assumption 4. The stress from other species of fungi can be characterized by the ratio of the product of relative moisture tolerance and biomass of the two species, and normalized with an exponential function.

Justification The relative moisture tolerance is defined as the difference of each isolate's competitive ranking and their moisture niche width, indicating the significant tolerance-dominance trade-off property in the competition of fungi. While incorporating the effect of multiple species, the form of the differential equations are roughly consistent with LV model.

Considering features of various types of fungi with available data, and the compatibility to the composition rate model constructed, traditional LV model in dual-species case is redefined to adapt the fungi community circumstances as follows.

$$\begin{cases} \dfrac{\mathrm{d}x_1}{\mathrm{d}t} = r_1 x_1 \exp\left(1 - \dfrac{m_1 x_1}{m_1 x_1} \cdot \dfrac{m_2 x_2}{m_1 x_1}\right) \\ \dfrac{\mathrm{d}x_2}{\mathrm{d}t} = r_2 x_2 \exp\left(1 - \dfrac{m_1 x_1}{m_2 x_2} \cdot \dfrac{m_2 x_2}{m_2 x_2}\right) \end{cases} \tag{4.6}$$

When extended to multiple species, we have

$$\begin{cases} \dfrac{\mathrm{d}x_1}{\mathrm{d}t} = r_1 x_1 \exp\left(1 - \dfrac{m_1 x_1}{m_1 x_1} \cdots \dfrac{m_n x_n}{m_1 x_1}\right) \\ \cdots \\ \dfrac{\mathrm{d}x_i}{\mathrm{d}t} = r_i x_i \exp\left(1 - \dfrac{m_1 x_1}{m_i x_i} \cdots \dfrac{m_j x_j}{m_i x_i} \cdots \dfrac{m_n x_n}{m_i x_i}\right) \\ \cdots \\ \dfrac{\mathrm{d}x_n}{\mathrm{d}t} = r_n x_n \exp\left(1 - \dfrac{m_1 x_1}{m_n x_n} \cdots \dfrac{m_n x_n}{m_n x_n}\right). \end{cases} \tag{4.7}$$

Or simply expressed as

$$\frac{\mathrm{d}x_i}{\mathrm{d}t} = r_i x_i \exp\left(1 - \frac{\prod_{j=1}^{n} m_j}{m_i^n} \cdot \frac{\prod_{j=1}^{n} x_j}{x_i^n}\right). \tag{4.8}$$

5 Homeostasis of the Community with Markov Chain Model

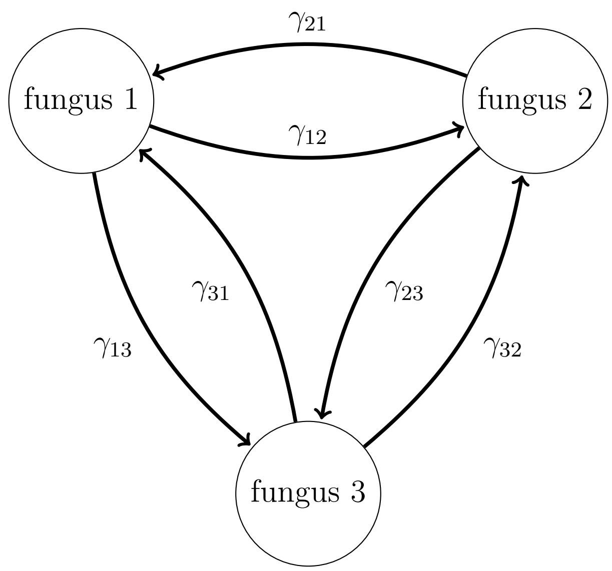
5.1 Feasibility of utilizing Markov chain model in fungus community

Since it is presumably impractical for finding the symbolic solution of the LV model, Markov chain model is introduced for predicting the stable state, that is, the relatively static composition of the fungus populations density in the community.

Assumption 5. Once the community component is stabilized, if possible, the population density of each fungi specie interchange in a rate proportional to its own population density, the coefficient is a constant.

Justification Though the Markov chain model is based on discrete time and state space, since the stable state is all we concerned in this attempt, we can still utilize such notion.

Figure 9: A Markov chain diagram containing 3 fungi species



5.2 Constructing Markov chain for fungus community

Consider a given time interval T, and the population density of each fungus species at time t is denoted as $\boldsymbol{x}(t) = [x_1(t), x_2(t), \cdots, x_n(t)]^T$. In the evolution process of the community, at each time interval, a specific percentage of fungi i is remained, and others can be considered as transformed into population density of other species of fungus.

For a simple dual-species system, such transition relation can be expressed as

$$\begin{cases} x_1(t+T) = \gamma_{11}x_1(t) + \gamma_{21}x_2(t) \\ x_2(t+T) = \gamma_{22}x_2(t) + \gamma_{12}x_1(t) \end{cases}. \tag{5.1}$$

More generally, in an n-species community, the system is characterized by

$$\begin{cases} x_1(t+T) = \gamma_{11}x_1(t) + \cdots + \gamma_{n1}x_n(t) \\ \cdots \\ x_i(t+T) = \sum_{j=1}^{n} \gamma_{ij}x_j \\ \cdots \\ x_n(t+T) = \gamma_{n1}x_1(t) + \cdots + \gamma_{nn}x_n(t). \end{cases} \tag{5.2}$$

Which can be further simplified with matrix notation.

$$\boldsymbol{x}(t+T) = \boldsymbol{x}(t)\boldsymbol{Y}. \tag{5.3}$$

In which, the matrix $\boldsymbol{Y}_{n\times n}$ is defined to be the transition matrix of the system, with element γ_{ij} denoting the transition rate from fungus specie i to j.

$$\boldsymbol{Y} = \begin{bmatrix} \gamma_{11} & \gamma_{12} & \cdots & \gamma_{1n} \\ \gamma_{21} & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ \gamma_{n1} & \cdots & \cdots & \gamma_{nn} \end{bmatrix} \tag{5.4}$$

Note that the transition rate in the biological configuration considerably differs from that in stochastic process. In a typical Markov chain model, each element in the transition matrix represents a certain probability of transition or decision towards another state. To implement this model in a continuous mathematical configuration, adjustments must be and evidently can be made to the definition of the population density x_i itself, ensuring that a unit of population density of different species of fungus takes up the same amount of resources in the system. With such adjustment, it is guaranteed that, for each element and row of the transition matrix, we have

$$\begin{cases} \boldsymbol{Y}_{ij} \in [0,\ 1],\ i,\ j \in \{1,\ 2,\ \cdots,\ n\} \\ \displaystyle\sum_{j=1}^{n} \boldsymbol{Y}_{ij} = 1,\ i \in \{1,\ 2,\ \cdots,\ n\}. \end{cases} \tag{5.5}$$

In addition, though during the growth process of the populations, and in macroscopic view the early stage of the decomposition, the transition matrix may not satisfy condition (5.5), and the sum of the rows could possibly even vary with time, it is shown that the final homeostasis is only related to the transition matrix after the community enters Markov process.

5.3 Incorporate hyphal extension rate and moisture tolerance

To model the system of fungi community, the transition matrix is further specified as follows in order to incorporate traits we concerned.

$$\gamma_{ij} = \mathrm{Softmax}\left(1 - \frac{r_i}{r_j}\right) = \frac{e^{1-r_i/r_j}}{\sum_{k=1}^{n} e^{1-r_i/r_k}} = \frac{e^{-r_i/r_j}}{\sum_{k=1}^{n} e^{-r_i/r_k}}. \tag{5.6}$$

In which the Softmax is for normalizing the data. The hyphal extension rate is a significant trait characterizing the **combative ability** of fungus, the ratio represents at what rate a fungus will be replaced another fungus isolate. It can be easily verified that, definition (5.6) is normalized for keeping consistent with prerequisite (5.5).

The final homeostasis can be expressed simply as

$$\boldsymbol{\pi} = \boldsymbol{x}(0) \lim_{n\to\infty} \boldsymbol{Y}^n. \tag{5.7}$$

Table 4: The four species chosen in verifying the model

Specie	Competitive Ranking	Hyphal Extension Rate
Phlebia acerina MR4280 B9G	1	8.75
Phlebiopsis flavidoalba FP150451 A8G	0.9864	10.8
Armillaria gallica HHB12551 C6C	0	0.49
Armillaria tabescens TJV93 261 A1E	0	1.07

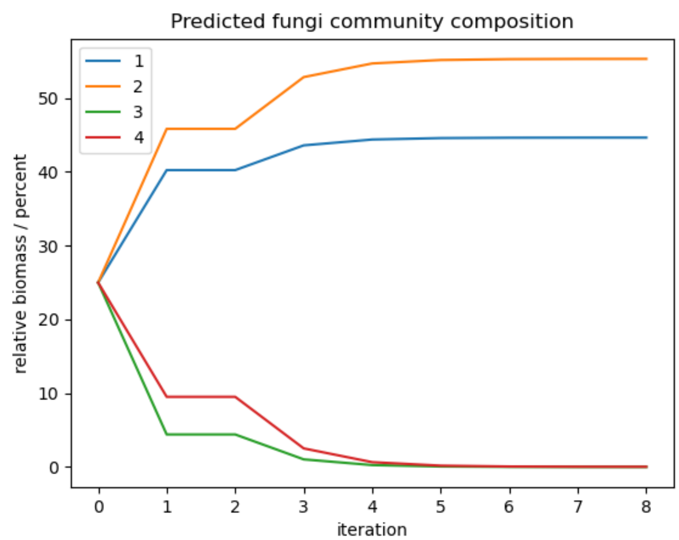


Figure 10: The community composition of a quad-species system, after several iteration, the relative biomass reaches homeostasis. 1 - Phlebia acerina, 2 - Phlebiopsis flavidoalba, 3 - Armillaria gallica, 4 - Armillaria tabescens

5.4 The homeostasis of the system

In a Markov process, the final homeostasis is only related to the transition matrix as shown in (5.7), but cannot be explicitly expressed.We postulate a quad-species fungi community, among them two in the top and two in the bottom of the competitive ranking, to visualize and verify this model.

The traits value of the four fungi is shown in table 4, and the transition matrix is

$$\begin{bmatrix} 0.45253 & 0.54712 & 0.00000 & 0.00035 \\ 0.44167 & 0.55827 & 0.00000 & 0.00006 \\ 0.32586 & 0.32935 & 0.12678 & 0.21801 \\ 0.38964 & 0.39879 & 0.04959 & 0.16198 \end{bmatrix}$$

As the iteration proceeds, the system reached the homeostasis gradually, and dominant species substitute weaker species completely in the end.

6 Environmental Effect on the Decay Ability of the Community

The purpose of this section, is to describe the environment affect the decomposition ability of a fungi community.

Since in this literature, moisture tolerance is the only trait of fungi concerned related to environment, we focus on how environment interact with this property.

6.1 Environmental impact on the hyphal extension rate

Moisture tolerance is defined as the difference of each isolate's competitive ranking and their moisture niche width. Competitive ranking can be obtained through pair wise competition experiment, and won't be affect by environment conditions since it is a relative ranking relation between species. Moisture niche width is the immediate property representing how tolerant a specie can be against varying environmental moisture condition, which is defined as the difference between the maximum and minimum moisture levels in which half of a fungal community can maintain its fastest growth rate.

According to (3.1), the hyphal extension rate and moisture niche width are implicitly related, and can be quantified as

$$k_2(c-d)+b_2=\ln k_1 r \Rightarrow r=\frac{1}{k_1}\exp[k_2(c-d)+b_2]. \tag{6.1}$$

Assumption 6. The hyphal extension rate of fungi community is affected by the moisture range of the environment. When the humidity in the environment fluctuates in a remarkable range, the growth of fungi is depressed.

Justification In this model, the community can be treated as a whole, since moisture tolerance only has to do with the niche width, not the optimal moisture condition. The optimal moisture condition and the moisture niche can be varied a lot in a fungi community for different species, but the global trend with respect to the change of environment can be predicted in such assumption.

Define the moisture range in a certain environment as w, then the hyphal extension rate expression can be expressed as

$$\hat{r}=\frac{1}{k_1}\exp[k_2(c-d)+b_2]\cdot\frac{d}{w+d}. \tag{6.2}$$

The meaning of the extra factor in the equation is, if the environmental moisture width is exactly zero, then the hyphal extension rate of the community is considered as being right in the optimal moisture width of the system as a whole. Otherwise, if the environmental moisture

moisture width is equal to the feature moisture niche width of the community, then the hyphal extension rate is halved, which is consistent with the definition of niche width.

In addition, if the initial hyphal extension rate in a certain condition is available and denoted as r_0, we can predict the same trait in other environment as

$$\hat{r} = \frac{d}{w+d} r_0. \tag{6.3}$$

6.2 Applying the data in various climates

After collecting and processing climate property data, we can obtain the moisture width of different climate, then apply these data to the model.

Table 5: The moisture width w in different environments

climate	arid	semi-arid	temperate	arboreal	tropical rain forests
moisture width					

7 Combined Model of the Decay Ability Regarding Environment

In this section, the decay ability model, the growth model describing the composition of the fungi community, and the environmental factors are combined to determine the behaviors of the fungi community in breakdown process.

7.1 Combining the models

In (3.3), replace the hyphal extension rate with (6.3), obtains

$$\hat{D}_{\text{comm}} = k_3 \frac{\sum_{i=1}^{n} \hat{r}_i}{x} {}_i\sum_{i=1}^{n} x_i = k_3 \frac{\sum\limits_{i=1}^{n} \dfrac{d_i r_i x_i}{w+d_i}}{\sum\limits_{i=1}^{n} x_i}. \tag{7.1}$$

In (4.8), conduct the same substitution, obtains

$$\frac{\mathrm{d}x_i}{\mathrm{d}t} = \frac{d_i r_i}{w+d_i} x_i \exp\left(1 - \frac{\prod_{j=1}^{n} m_j}{m_i^n} \cdot \frac{\prod_{j=1}^{n} x_j}{x_i^n}\right). \tag{7.2}$$

Also, the transition matrix redefined in (5.6) is again transformed into

$$\gamma_{ij} = \frac{\hat{r}_i}{\sum_{k=1}^{n} \hat{r}_k} = \frac{\dfrac{d_i r_{ik}}{w + d_i}}{\displaystyle\sum_{k=1}^{n} \frac{d_i r_{ik}}{w + d_i}}. \tag{7.3}$$

Equation (7.1) determines the ability of a fungi community to decompose ground litter and woody fibers. In which, r_i, d_i are the traits of the fungus, w is determined by the environmental condition, k_3 is a coefficient and is determined in data fitting as shown in 3. x_i is the only unknown variable representing the biomass of different species of fungus, and can be inferred numerically through (7.2), in which m_i is the trait owned by fungus.

7.2 Solution of the model at different climatic conditions

Since a explicit analytical solution cannot be attained from (7.2), yet from model defined in section 5, we choose all 34 fungi species with available traits data from [], and evaluate their behaviors under different climatic conditions. For the purpose of brevity, [] model is chosen.

8 Analysis

8.1 Sensitivity Analysis

8.2 Strengths and Weaknesses

9 Conclusions

9.1 Conclusion of the problem

9.2 Applications of our models

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Appendices

A Source Code

From here, begin your first Appendix... your can include some program script, such as matlab, c/cpp, python.

A.1 First

```
import numpy as np

print('Hello world')
```

```
import numpy as np
import this

print('Hello world')
```