The Impact of Smoothing Mortality Rates on Life Insurance

Samuel Hudec Jolana Gubalová Petra Medveďová Jana Špirková

Faculty of Natural Sciences
and
Faculty of Economics
Matej Bel University

November 2018

Council Directive 2009/138/EC - Solvency II

Article 77

Calculation of technical provisions

2. The best estimate shall correspond to the probability-weighted average of future cash-flows, taking account of the time value of money (expected present value of future cash-flows), using the relevant risk-free interest rate term structure.

Yield Curve of European Central Bank

- Svensson yield curve

Aggregation function

$\mathbb{I} = [a, b] \subset \overline{\mathbb{R}} = [-\infty, \infty]$,
$\mathbb{I}^n = \{\mathbf{x} = (x_1, \ldots, x_n) \mid x_i \in \mathbb{I}, i = 1, \ldots, n\}$

Definition

A function $F : \mathbb{I}^n \to \mathbb{I}$ is called an n-ary aggregation function if the following conditions hold:

(A1) F satisfies the boundary conditions $F(a, a, \ldots, a) = a$ and $F(b, b, \ldots, b) = b$,

(A2) F is standard monotone increasing.

Mixture function (Calvo, Mayor, Mesiar, 2002)

Definition

A mapping $M_g : \mathbb{I}^n \to \mathbb{I}$ given by

$$M_g(x_1, \ldots, x_n) = \frac{\sum_{i=1}^{n} g(x_i) \cdot x_i}{\sum_{i=1}^{n} g(x_i)},$$

where $g : \mathbb{I} \to]0, \infty[$ is a continuous weighting function, is called

a mixture function.

Linear Weighting Function

Proposition 1

Let $M_g : [0,1]^n \to [0,1]$ be a mixture function with the weighting function

$$g_c(x) = cx + 1 - c \ , \ c \in [0,1].$$

Then M_g is monotone increasing with respect to (2) and (3) for

$$c \in [0, 0.5].$$

Quadratic Weighting Function

Proposition 2

Let $M_g : [0,1]^n \to [0,1]$ be a mixture function with the weighting function $g_\gamma(x) = 1 + \gamma x^2$, $\gamma > 0$.

Then M_g is monotone increasing with respect to (2) and (3) for $\gamma \in [0,1]$, or $\gamma \in [0,3]$.

Exponential Weighting Function

Proposition 3

Let $M_g : [0,1]^n \to [0,1]$ be a mixture function with the weighting function $g_a(x) = a \cdot \left(\frac{1}{a}\right)^x$, $0 < a < 1$.

Then M_g is monotone increasing with respect to (2) and (3) for $a \in [\frac{1}{e}, 1[$.

Parameters of Using Weighting Functions

Calculation of basic probabilities

- L_x - number of living at age x
- D_x - number of deaths at age x

$$\mu_x = \frac{D_x}{L_x}$$

$$q_x = 1 - exp(-\mu_x)$$

Mortality Rate Smoothing by Mixture Function

Calculation of Moving Mixture Averages

$$\hat{q}_x = \frac{\sum_{j=0}^{3} g(q_{x\pm j}) \cdot q_{x\pm j}}{\sum_{j=0}^{3} g(q_{x\pm j})}$$

for $4 \leq x \leq 102$

by different types of appropriate weighting functions

for higher ages smoothing by

Gompert'z Makeham formula using the R enviroment

Model Selection

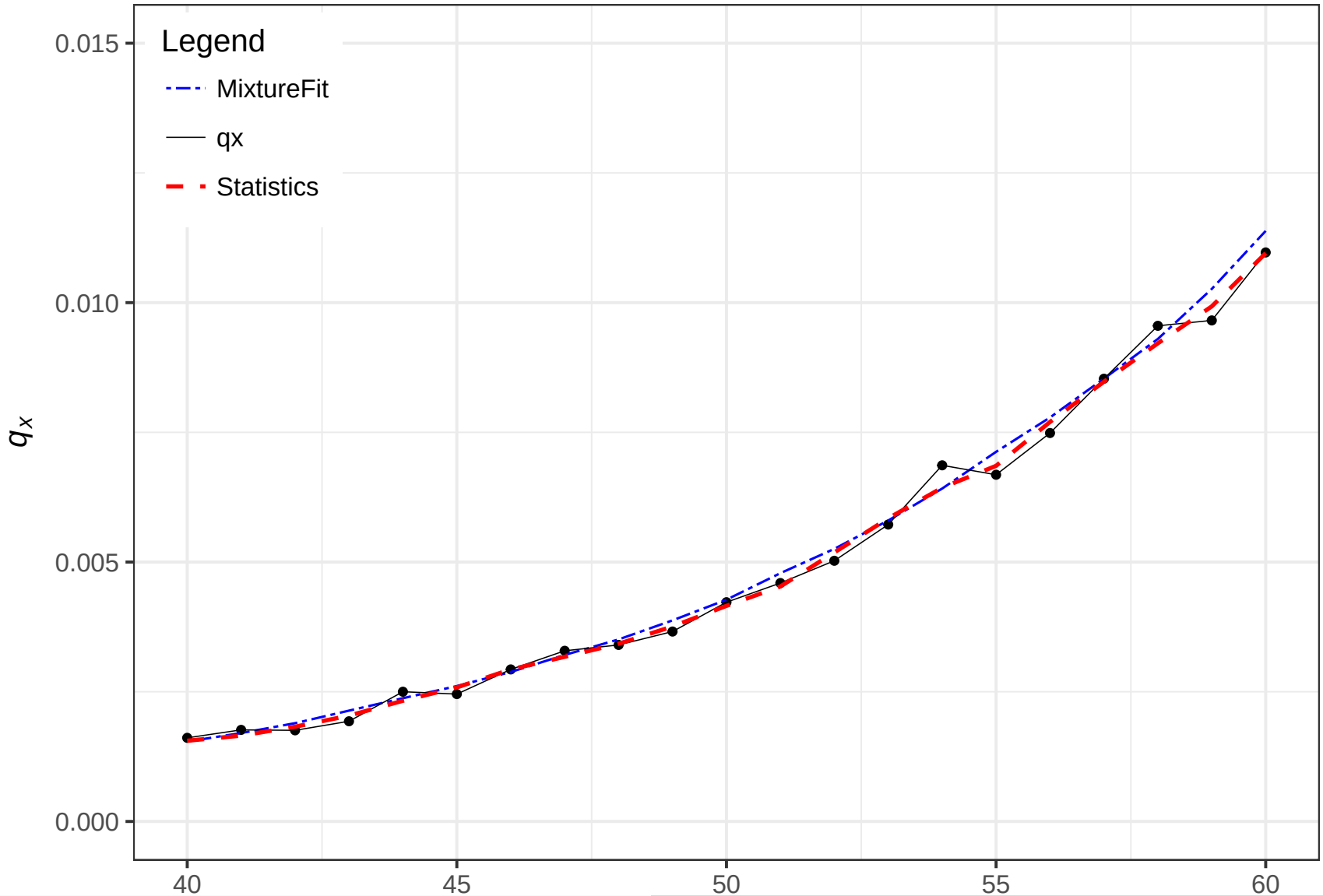
Mean Squared Error

$$MSE = \frac{\sum_{x=0}^{105} (q_x - \hat{q}_x)^2 \cdot (L_x + D_x)}{\sum_{x=0}^{105} (L_x + D_x)}$$

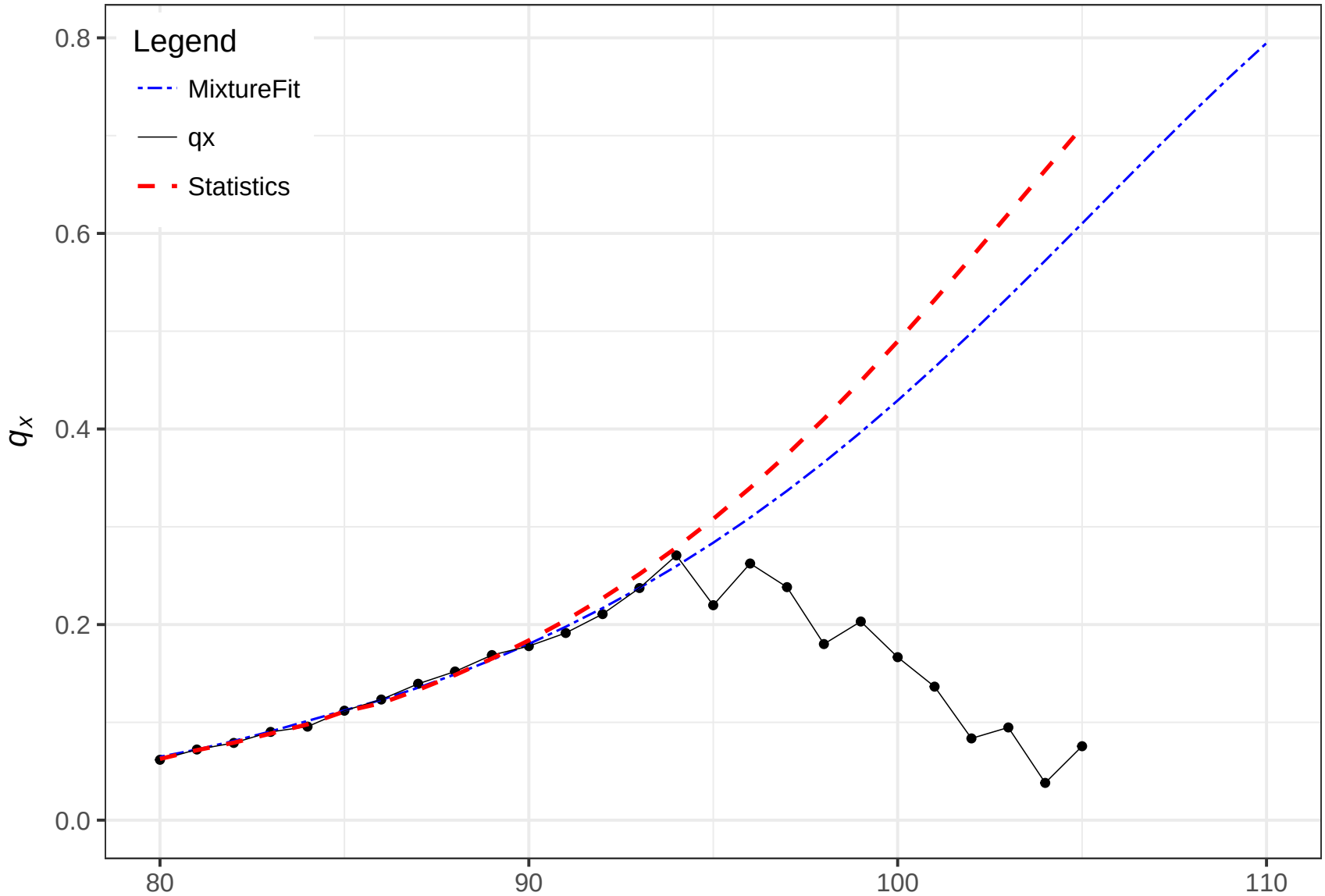
In Numbers

x	Statistical office	$g_c(x)$	$g_\gamma(x)$	$g_a(x)$
50	0.004155526	0.004133731	0.004133501	0.004133769
51	0.004529290	0.004600240	0.004599832	0.004600308
52	0.005183851	0.005185272	0.005184728	0.005185363
53	0.005842840	0.005721828	0.005721280	0.005721920
54	0.006432884	0.006431818	0.006431061	0.006431946
...	...	...	...	...
101	0.531883675	0.430605644	0.424598158	0.423442887
101	0.531883675	0.430605644	0.424598158	0.423442887
102	0.575499875	0.463498127	0.456572207	0.455390780
103	0.619920292	0.497657038	0.489791113	0.488590583
104	0.664552046	0.532897488	0.524089915	0.522878444
105	0.708716152	0.568989097	0.559262200	0.558049008

Probability of Death Using Mixture Aggregation



Probability of Death Using Mixture Aggregation



Premiums on insured sum 10,000 €, 47 old aged person, 20 years

	Statistical office €	$g_c(x)$ €	$g_\gamma(x)$ €	$g_a(x)$ €
Single premium Term Life Insurance	1,629.76	1,605.75	1,605.60	1,605.77
Single premium Pure Endowment Insurance	6,665.63	6,686.37	6,686.49	6,686.35
Monthly premium Term Life Insurance	11.70	11.58	11.58	11.58
Monthly Premium Pure Endowment	36.19	36.28	36.28	36.28

Thank you for your attention