hw4

October 8, 2024

1 BENG 37303 HW 4

[433]:
```
import numpy as np
import matplotlib.pyplot as plt
print("Transport HW4")

np.set_printoptions(precision=3, suppress=True, linewidth=200)
```

```
Transport HW4
```

2 Part D

2.1 Solve for all node temperatures including A, A^{-1}, C, and T.

Tiene la forma de matriz

$$AT = C$$

donde el T es el vector de las Temperatures y la C is el vector de columna de constantes en las equaciones. Luego, puedes invertir el matriz A a A^{-1} para que se encuentra las soluciones de la temperatura.

$$\begin{pmatrix}
-2 & 0.5 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.5 & -2 & 0.5 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.5 & -2 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.5 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & -4 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -4 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & -4 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -4 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & -4 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & -4 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & -4 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & -4 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -4 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & -4 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & -4 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & -4 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -4 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & -4 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & -4 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & -4
\end{pmatrix}
\cdot
\begin{pmatrix}
T_{1,2} \\ T_{1,3} \\ T_{1,4} \\ T_{1,5} \\ T_{2,2} \\ T_{3,2} \\ T_{4,2} \\ T_{5,2} \\ T_{2,3} \\ T_{3,3} \\ T_{4,3} \\ T_{5,3} \\ T_{2,4} \\ T_{3,4} \\ T_{4,4} \\ T_{5,4} \\ T_{2,5} \\ T_{3,5} \\ T_{4,5} \\ T_{5,5}
\end{pmatrix}
=
\begin{pmatrix}
-7.5 \\ 0 \\ 0 \\ -7.5 \\ -15 \\ -15 \\ -15 \\ -30 \\ 0 \\ 0 \\ 0 \\ -15 \\ 0 \\ 0 \\ 0 \\ -15 \\ -30 \\ -30 \\ -30 \\ -45
\end{pmatrix}$$

El vector de columna (temperatura) puede ser determinado por

$$T = A^{-1}C$$

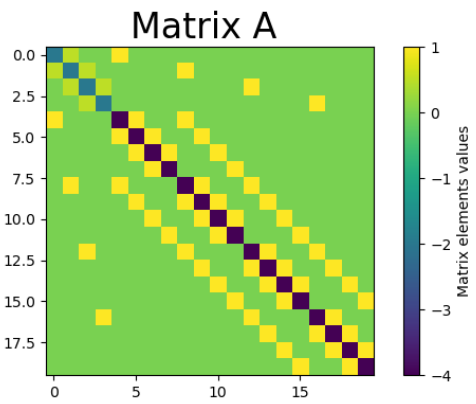
[434]:
```
A = np.array([
    [-2, 0.5, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],
    [0.5, -2, 0.5, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],
    [0, 0.5, -2, 0.5, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0],
    [0, 0, 0.5, -2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0],
    [1, 0, 0, 0, -4, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],
    [0, 0, 0, 0, 1, -4, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],
    [0, 0, 0, 0, 0, 1, -4, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0],
    [0, 0, 0, 0, 0, 0, 1, -4, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0],
    [0, 1, 0, 0, 1, 0, 0, 0, -4, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0],
    [0, 0, 0, 0, 0, 1, 0, 0, 1, -4, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0],
    [0, 0, 0, 0, 0, 0, 1, 0, 0, 1, -4, 1, 0, 0, 1, 0, 0, 0, 0, 0],
    [0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, -4, 0, 0, 0, 1, 0, 0, 0, 0],
    [0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, -4, 1, 0, 0, 1, 0, 0, 0],
    [0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, -4, 1, 0, 0, 1, 0, 0],
    [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, -4, 1, 0, 0, 1, 0],
    [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, -4, 0, 0, 0, 1],
    [0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, -4, 1, 0, 0],
    [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, -4, 1, 0],
    [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, -4, 1],
    [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, -4]
])

print("Orginal Matrix A: ")
print(A)

# Vector de soluciones
C = np.array([-7.5, 0, 0, -7.5, -15, -15, -15, -30, 0, 0, 0, -15, 0, 0, 0, -15,␣
 ↪-30, -30, -30, -45])

print("\nOriginal Solution Vector C: ")
print(C)

Ainv = np.linalg.inv(A)

print("\nInverted matrix A: ")
print(Ainv)

T = np.matmul(Ainv, C)

print("\nSolution for Right handed Temparatures Vector T: ")
print(T)

name_list = [
    "T_{1,2}", "T_{1,3}", "T_{1,4}", "T_{1,5}",
    "T_{2,2}", "T_{3,2}", "T_{4,2}", "T_{5,2}",
    "T_{2,3}", "T_{3,3}", "T_{4,3}", "T_{5,3}",
```

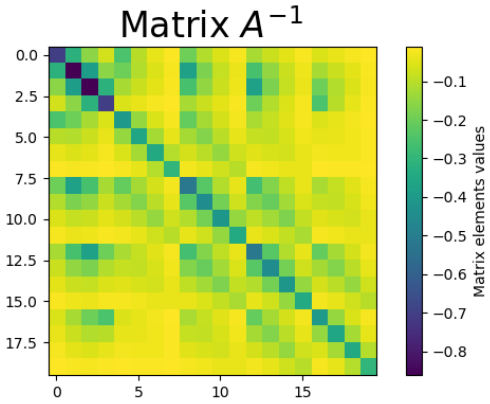
```
    "T_{2,4}", "T_{3,4}", "T_{4,4}", "T_{5,4}",
    "T_{2,5}", "T_{3,5}", "T_{4,5}", "T_{5,5}"
]

temperature_dict = dict(zip(name_list, T))
print("\nTemperatures: ")
for name, temperature in zip(name_list, T):
    print(f"{name}: {temperature} °C ")


fig2 = plt.figure(figsize=(12,4))
plt.subplot(121)
plt.imshow(A,interpolation='none')
clb=plt.colorbar()
clb.set_label('Matrix elements values')
plt.title('Matrix A ',fontsize=24)
plt.subplot(122)
plt.imshow(Ainv,interpolation='none')
clb=plt.colorbar()
clb.set_label('Matrix elements values')
plt.title(r'Matrix $A^{-1}$ ',fontsize=24)

fig2.tight_layout()
plt.show()
```

```
Orginal Matrix A:
[[-2.   0.5  0.   0.   1.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
0.   0.   0.   0. ]
 [ 0.5 -2.   0.5  0.   0.   0.   0.   0.   1.   0.   0.   0.   0.   0.   0.   0.
0.   0.   0.   0. ]
 [ 0.   0.5 -2.   0.5  0.   0.   0.   0.   0.   0.   0.   0.   1.   0.   0.   0.
0.   0.   0.   0. ]
 [ 0.   0.   0.5 -2.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.
1.   0.   0.   0. ]
 [ 1.   0.   0.   0.  -4.   1.   0.   0.   1.   0.   0.   0.   0.   0.   0.   0.
0.   0.   0.   0. ]
 [ 0.   0.   0.   0.   1.  -4.   1.   0.   0.   1.   0.   0.   0.   0.   0.   0.
0.   0.   0.   0. ]
 [ 0.   0.   0.   0.   0.   1.  -4.   1.   0.   0.   1.   0.   0.   0.   0.   0.
0.   0.   0.   0. ]
 [ 0.   0.   0.   0.   0.   0.   1.  -4.   0.   0.   0.   1.   0.   0.   0.   0.
0.   0.   0.   0. ]
 [ 0.   1.   0.   0.   1.   0.   0.   0.  -4.   1.   0.   0.   1.   0.   0.   0.
0.   0.   0.   0. ]
 [ 0.   0.   0.   0.   0.   1.   0.   0.   1.  -4.   1.   0.   0.   1.   0.   0.
```

```
0.   0.   0.   0. ]
 [ 0.   0.   0.   0.   0.   0.   1.   0.   0.   1.  -4.   1.   0.   0.   1.   0.
0.   0.   0.   0. ]
 [ 0.   0.   0.   0.   0.   0.   0.   1.   0.   0.   1.  -4.   0.   0.   0.   1.
0.   0.   0.   0. ]
 [ 0.   0.   1.   0.   0.   0.   0.   0.   1.   0.   0.   0.  -4.   1.   0.   0.
1.   0.   0.   0. ]
 [ 0.   0.   0.   0.   0.   0.   0.   0.   0.   1.   0.   0.   1.  -4.   1.   0.
0.   1.   0.   0. ]
 [ 0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   1.   0.   0.   1.  -4.   1.
0.   0.   1.   0. ]
 [ 0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   1.   0.   0.   1.  -4.
0.   0.   0.   1. ]
 [ 0.   0.   0.   1.   0.   0.   0.   0.   0.   0.   0.   0.   1.   0.   0.   0.
-4.   1.   0.   0. ]
 [ 0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   1.   0.   0.
1.  -4.   1.   0. ]
 [ 0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   1.   0.
0.   1.  -4.   1. ]
 [ 0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   0.   1.
0.   0.   1.  -4. ]]

Original Solution Vector C:
[ -7.5   0.    0.   -7.5 -15.  -15.  -15.  -30.    0.    0.    0.  -15.    0.
0.    0.  -15.  -30.  -30.  -30.  -45. ]

Inverted matrix A:
[[-0.704 -0.315 -0.158 -0.069 -0.251 -0.101 -0.043 -0.016 -0.2   -0.109 -0.055
-0.023 -0.124 -0.079 -0.044 -0.019 -0.058 -0.04  -0.024 -0.011]
 [-0.315 -0.862 -0.384 -0.158 -0.2   -0.109 -0.055 -0.023 -0.375 -0.18  -0.087
-0.036 -0.258 -0.149 -0.078 -0.033 -0.124 -0.079 -0.044 -0.019]
 [-0.158 -0.384 -0.862 -0.315 -0.124 -0.079 -0.044 -0.019 -0.258 -0.149 -0.078
-0.033 -0.375 -0.18  -0.087 -0.036 -0.2   -0.109 -0.055 -0.023]
 [-0.069 -0.158 -0.315 -0.704 -0.058 -0.04  -0.024 -0.011 -0.124 -0.079 -0.044
-0.019 -0.2   -0.109 -0.055 -0.023 -0.251 -0.101 -0.043 -0.016]
 [-0.251 -0.2   -0.124 -0.058 -0.403 -0.147 -0.059 -0.021 -0.212 -0.127 -0.066
-0.027 -0.119 -0.084 -0.049 -0.022 -0.054 -0.041 -0.026 -0.012]
 [-0.101 -0.109 -0.079 -0.04  -0.147 -0.36  -0.126 -0.042 -0.127 -0.169 -0.1
-0.043 -0.084 -0.089 -0.062 -0.03  -0.041 -0.04  -0.029 -0.015]
 [-0.043 -0.055 -0.044 -0.024 -0.059 -0.126 -0.344 -0.104 -0.066 -0.1   -0.146
-0.073 -0.049 -0.062 -0.069 -0.04  -0.026 -0.029 -0.029 -0.017]
 [-0.016 -0.023 -0.019 -0.011 -0.021 -0.042 -0.104 -0.302 -0.027 -0.043 -0.073
-0.103 -0.022 -0.03  -0.04  -0.039 -0.012 -0.015 -0.017 -0.014]
 [-0.2   -0.375 -0.258 -0.124 -0.212 -0.127 -0.066 -0.027 -0.521 -0.231 -0.108
-0.044 -0.266 -0.168 -0.091 -0.039 -0.119 -0.084 -0.049 -0.022]
 [-0.109 -0.18  -0.149 -0.079 -0.127 -0.169 -0.1   -0.043 -0.231 -0.449 -0.187
-0.072 -0.168 -0.209 -0.129 -0.058 -0.084 -0.089 -0.062 -0.03 ]
 [-0.055 -0.087 -0.078 -0.044 -0.066 -0.1   -0.146 -0.073 -0.108 -0.187 -0.413
```

```
-0.144 -0.091 -0.129 -0.175 -0.09  -0.049 -0.062 -0.069 -0.04 ]
 [-0.023 -0.036 -0.033 -0.019 -0.027 -0.043 -0.073 -0.103 -0.044 -0.072 -0.144
-0.341 -0.039 -0.058 -0.09  -0.118 -0.022 -0.03  -0.04  -0.039]
 [-0.124 -0.258 -0.375 -0.2   -0.119 -0.084 -0.049 -0.022 -0.266 -0.168 -0.091
-0.039 -0.521 -0.231 -0.108 -0.044 -0.212 -0.127 -0.066 -0.027]
 [-0.079 -0.149 -0.18  -0.109 -0.084 -0.089 -0.062 -0.03  -0.168 -0.209 -0.129
-0.058 -0.231 -0.449 -0.187 -0.072 -0.127 -0.169 -0.1   -0.043]
 [-0.044 -0.078 -0.087 -0.055 -0.049 -0.062 -0.069 -0.04  -0.091 -0.129 -0.175
-0.09  -0.108 -0.187 -0.413 -0.144 -0.066 -0.1   -0.146 -0.073]
 [-0.019 -0.033 -0.036 -0.023 -0.022 -0.03  -0.04  -0.039 -0.039 -0.058 -0.09
-0.118 -0.044 -0.072 -0.144 -0.341 -0.027 -0.043 -0.073 -0.103]
 [-0.058 -0.124 -0.2   -0.251 -0.054 -0.041 -0.026 -0.012 -0.119 -0.084 -0.049
-0.022 -0.212 -0.127 -0.066 -0.027 -0.403 -0.147 -0.059 -0.021]
 [-0.04  -0.079 -0.109 -0.101 -0.041 -0.04  -0.029 -0.015 -0.084 -0.089 -0.062
-0.03  -0.127 -0.169 -0.1   -0.043 -0.147 -0.36  -0.126 -0.042]
 [-0.024 -0.044 -0.055 -0.043 -0.026 -0.029 -0.029 -0.017 -0.049 -0.062 -0.069
-0.04  -0.066 -0.1   -0.146 -0.073 -0.059 -0.126 -0.344 -0.104]
 [-0.011 -0.019 -0.023 -0.016 -0.012 -0.015 -0.017 -0.014 -0.022 -0.03  -0.04
-0.039 -0.027 -0.043 -0.073 -0.103 -0.021 -0.042 -0.104 -0.302]]

Solution for Right handed Temparatures Vector T:
[16.997 19.    20.782 21.168 16.994 16.867 16.51  15.869 19.11  18.966 18.302
16.967 21.48  21.586 20.767 18.695 24.444 25.129 24.485 22.045]

Temperatures:
T_{1,2}: 16.996687041101918 °C
T_{1,3}: 18.999698103939426 °C
T_{1,4}: 20.78207339798611 °C
T_{1,5}: 21.16762618097767 °C
T_{2,2}: 16.993525030234125 °C
T_{3,2}: 16.867397091499747 °C
T_{4,2}: 16.50970717011274 °C
T_{5,2}: 15.869053570948786 °C
T_{2,3}: 19.110015988334837 °C
T_{3,3}: 18.966356165652122 °C
T_{4,3}: 18.302378018002422 °C
T_{5,3}: 16.96650711368241 °C
T_{2,4}: 21.480484653513678 °C
T_{3,4}: 21.5856335647715 °C
T_{4,4}: 20.766941622562413 °C
T_{5,4}: 18.694596865778436 °C
T_{2,5}: 24.44421566296228 °C
T_{3,5}: 25.128751817357767 °C
T_{4,5}: 24.48515804169728 °C
T_{5,5}: 22.044938726868928 °C
```





ya nos hace falta reparar la matriz en total

3 Part E

3.1 Create a contour plot of the Temperature Distribution.

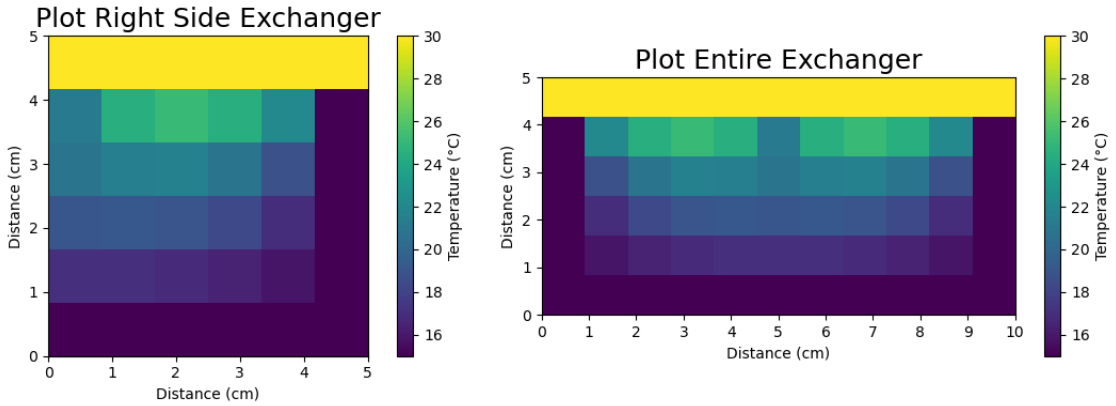
[435]:
```
T_right = np.zeros((6, 6))

# Set the bottom row to 15
T_right[5, :] = 15

# Set the right column to 15
T_right[:, 5] = 15

# Set the top row to 30
T_right[0, :] = 30

#print(T_right)

left_values = T[:4]
left_values = np.flip(left_values)

interior_values = T[4:]
row1 = interior_values[:4]
row2 = interior_values[4:8]
row3 = interior_values[8:12]
row4 = interior_values[12:16]

row1 = np.flip(row1)
row2 = np.flip(row2)
row3 = np.flip(row3)
row4 = np.flip(row4)
```

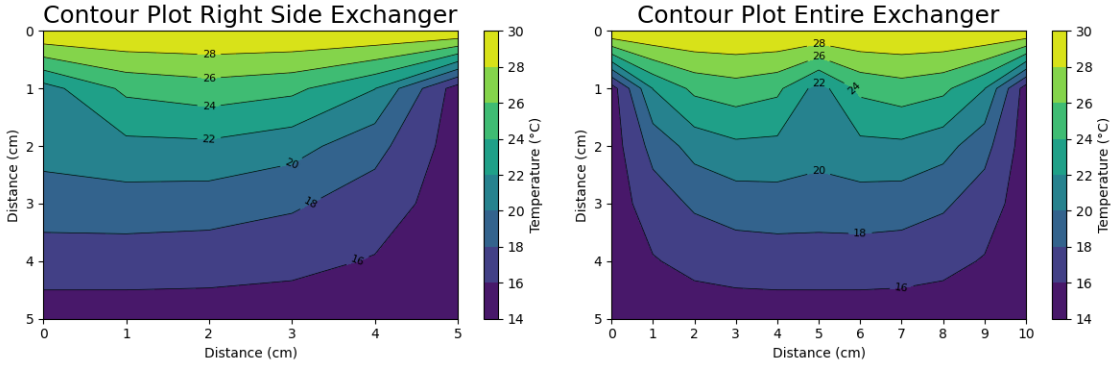
```
interior_values = np.concatenate((row1, row2, row3, row4))

#print("row 2" + str(row1))
interior_values = np.flipud(interior_values)

interior_values = interior_values.reshape((4,4))
#$print(interior_values)


# Fill in the interior values from T_vector
T_right[1:5, 0] = left_values

T_right[1:5, 1:5] = interior_values
#print(interior_values.reshape(4,4))

# Print the result
print("T matrix (right side 6x6): ")
print(T_right)


T_full= np.zeros((6,11))
#print(T_full)

T_full[:, 5:11] = T_right
T_full[:, 0:5] = np.fliplr(T_right[:, 1:6])
print("\n T matriz (full): ")
print(T_full)


joe = plt.figure(figsize=(12, 4))

# First subplot (Matrix A)
plt.subplot(121)
plt.imshow(T_right, interpolation='none', extent=[0, 5, 0, 5])  # Adjusting the␣
 ↪extent
clb = plt.colorbar()
clb.set_label('Temperature (°C)')
plt.title('Plot Right Side Exchanger', fontsize=18)
plt.xlabel('Distance (cm)')
plt.ylabel('Distance (cm)')
plt.xticks(np.arange(0, 6, 1))  # Set ticks for x-axis
plt.yticks(np.arange(0, 6, 1))  # Set ticks for y-axis

# Second subplot (Matrix A^(-1))
plt.subplot(122)
```

```
plt.imshow(T_full, interpolation='none', extent=[0, 10, 0, 5])  # Adjusting the␣
 ↪extent
clb = plt.colorbar()
clb.set_label('Temperature (°C)')
plt.title('Plot Entire Exchanger', fontsize=18)
plt.xlabel('Distance (cm)')
plt.ylabel('Distance (cm)')
plt.xticks(np.arange(0, 11, 1))  # Set ticks for x-axis
plt.yticks(np.arange(0, 6, 1))   # Set ticks for y-axis

# Adjust layout
joe.tight_layout()
plt.show()


fig = plt.figure(figsize=(12,4))
# Left subplot for the right-hand matrix
plt.subplot(121)
# Create a filled contour plot
contour_filled = plt.contourf(T_right, cmap='viridis', extent=[0,5,0,5])
# Add contour lines
contour_lines = plt.contour(T_right, colors='black', linewidths=0.5)
# Add labels to the contour lines
plt.clabel(contour_lines, inline=True, fontsize=8)
clb = plt.colorbar(contour_filled)
clb.set_label('Temperature (°C)')
plt.title('Contour Plot Right Side Exchanger', fontsize=18)
plt.xlabel('Distance (cm)')
plt.ylabel('Distance (cm)')
plt.gca().invert_yaxis()
plt.xticks(np.arange(0, 6, 1))  # Set ticks for x-axis
plt.yticks(np.arange(0, 6, 1))   # Set ticks for y-axis

# Right subplot for the full matrix
plt.subplot(122)
# Create a filled contour plot
contour_filled = plt.contourf(T_full, cmap='viridis', extent=[0,10,0,5])
# Add contour lines
contour_lines = plt.contour(T_full, colors='black', linewidths=0.5)
# Add labels to the contour lines
plt.clabel(contour_lines, inline=True, fontsize=8)
clb = plt.colorbar(contour_filled)
clb.set_label('Temperature (°C)')
plt.title('Contour Plot Entire Exchanger', fontsize=18)
plt.xlabel('Distance (cm)')
plt.ylabel('Distance (cm)')
plt.gca().invert_yaxis()
```

```
plt.xticks(np.arange(0, 11, 1))  # Set ticks for x-axis
plt.yticks(np.arange(0, 6, 1))   # Set ticks for y-axis

fig.tight_layout()
plt.show()
```

```
T matrix (right side 6x6):
[[30.    30.    30.    30.    30.    30.   ]
 [21.168 24.444 25.129 24.485 22.045 15.   ]
 [20.782 21.48  21.586 20.767 18.695 15.   ]
 [19.    19.11  18.966 18.302 16.967 15.   ]
 [16.997 16.994 16.867 16.51  15.869 15.   ]
 [15.    15.    15.    15.    15.    15.   ]]

 T matriz (full):
[[30.    30.    30.    30.    30.    30.    30.    30.    30.    30.    30.   ]
 [15.    22.045 24.485 25.129 24.444 21.168 24.444 25.129 24.485 22.045 15.   ]
 [15.    18.695 20.767 21.586 21.48  20.782 21.48  21.586 20.767 18.695 15.   ]
 [15.    16.967 18.302 18.966 19.11  19.    19.11  18.966 18.302 16.967 15.   ]
 [15.    15.869 16.51  16.867 16.994 16.997 16.994 16.867 16.51  15.869 15.   ]
 [15.    15.    15.    15.    15.    15.    15.    15.    15.    15.    15.   ]]
```





4 Part F: Comparison

Part F: Compare your numerical solution to the exact solution (equation 4.19) for 4 selected calculated location within the dough (nodes 2,2; 3,3; 5,5; and 2,5)

$$\theta_{x,y} = \frac{T - T_1}{T_2 - T_1}$$

$$\theta_{x,y} = \frac{T - 15}{30 - 15}$$

where

$$\theta_{x,y} = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1} + 1}{n} \sin \frac{n\pi x}{L} \frac{\sinh(n\pi y/L)}{\sinh(n\pi W/L)}$$

where W is the width and L is the length. After θ is determined, then it can be subbed back into the transformation function to find the true value of T.

[436]:
```
#PART F
t1 = 15
t2 = 30

def theta_to_temperature(theta):
    #T = Theta * t2-t1  + T1
    temperature = ((t2-t1) * theta) + t1
    return temperature

def truetemperature(x, y, L, W, N = 350):
    theta_sum = 0
    for n in range (1, N+1):
        term1= (-1)**(n+1) + 1
        term2= np.sin(n * np.pi * x / L)
        sinh_term = (np.sinh(n * np.pi * y /L )) / (np.sinh(n * np.pi * W / L))

        if np.sinh(n * np.pi * W/L) == 0:
            continue
        theta_sum += (term1/n) * term2 * sinh_term

    theta_val = theta_sum * (2/np.pi)
    temptrue = theta_to_temperature(theta_val)

    return temptrue

#num = truetemperature(5, 5, 10, 5)
#print(num)
```

```
# trying part F
infsum22 = truetemperature(6, 1, 10,5)
finite22 = temperature_dict["T_{2,2}"]
print(f"Infinte Sum Solution for Node (2,2): {infsum22}")
print(f"Finite Difference Solution for Node (2,2): {finite22}")
print()

infsum33 = truetemperature(7, 2, 10, 5)
finite33 = temperature_dict["T_{3,3}"]
print(f"Infinte Sum Solution for Node (3,3): {infsum33}")
print(f"Finite Difference Solution for Node (3,3): {finite33}")
print()

infsum55 = truetemperature(9, 4, 10, 5)
finite55 = temperature_dict["T_{5,5}"]
print(f"Infinte Sum Solution for Node (5,5): {infsum55}")
print(f"Finite Difference Solution for Node (5,5): {finite55}")
print()

infsum25 = truetemperature(6, 4, 10, 5)
finite25 = temperature_dict["T_{2,5}"]
print(f"Infinte Sum Solution for Node (2,5): {infsum25}")
print(f"Finite Difference Solution for Node (2,5): {finite25}")
print()

print("These numbers are roughly in the same ballpark.")
```

```
Infinte Sum Solution for Node (2,2): 17.447648097867173
Finite Difference Solution for Node (2,2): 16.993525030234125

Infinte Sum Solution for Node (3,3): 19.582606176069465
Finite Difference Solution for Node (3,3): 18.966356165652122

Infinte Sum Solution for Node (5,5): 22.171785767274407
Finite Difference Solution for Node (5,5): 22.044938726868928

Infinte Sum Solution for Node (2,5): 26.383109973779252
Finite Difference Solution for Node (2,5): 24.44421566296228

These numbers are roughly in the same ballpark.
```

5 Part G: Error Analysis

[437]:
```
## loops through the entire matrix and finds maintains the highest amount of␣
↪error (squared)
```

```
## find a sum of the squared errors, take the root, then divide by the number of nodes (36) take the square root

print("Temperature Finite Difference Method:")
print(T_right)

Trightinf = np.zeros((6, 6))

# Set the bottom row to 15
Trightinf[0, :] = 15

# Set the right column to 15
Trightinf[:, 5] = 15

# Set the top row to 30
Trightinf[5, :] = 30
#Trightinf = np.fliplr(Trightinf)
#print(truetemperature(6, 4, 10, 5))
#print(Trightinf)

num_x = 5
num_y = 5


for i in range(num_x):
    for j in range(num_y):

        if j == 0: continue
        x = i + 5
        y = j

        posx = x+1-5
        posy = y+1

        #print(f"Calculating temp for x= {x} and y= {y}, position {posx}, {posy}")
        truetemp = truetemperature(x, y, 10, 5)
        #print(truetemp)

        #print(f"inputting true temp {truetemp} at matrix pos {posx-1}, {posy}")
        Trightinf[posy-1, posx-1] = truetemp

Trightinf = np.flipud(Trightinf)
print("\nTemperatures Fourier Fine Series:")
print(Trightinf)
maxError = 0
sumError = 0
```

```
sumPercentError = 0
maxPercentError = 0

for i in range(6):
    for j in range(6):

        inf = Trightinf[i,j]
        finite = T_right[i,j]

        error = abs(inf - finite)
        percentError = error/finite * 100

        sumPercentError += percentError
        sumError += error
        if error > maxError: maxError = error
        if percentError > maxPercentError: maxPercentError = percentError

avg_error = sumError / 36
ave_percenterror = sumPercentError / 36

print()
print(f"Average Error: {avg_error}")
print(f"Average Percent Error: {ave_percenterror}")
print(f"Maximum Error: {maxError}")
print(f"Maximum Percent Error: {maxPercentError}")
```

```
Temperature Finite Difference Method:
[[30.    30.    30.    30.    30.    30.   ]
 [21.168 24.444 25.129 24.485 22.045 15.   ]
 [20.782 21.48  21.586 20.767 18.695 15.   ]
 [19.    19.11  18.966 18.302 16.967 15.   ]
 [16.997 16.994 16.867 16.51  15.869 15.   ]
 [15.    15.    15.    15.    15.    15.   ]]

Temperatures Fourier Fine Series:
[[30.    30.    30.    30.    30.    30.   ]
 [26.498 26.383 25.964 24.922 22.172 15.   ]
 [23.206 23.038 22.455 21.19  18.764 15.   ]
 [20.227 20.078 19.583 18.617 17.055 15.   ]
 [17.532 17.448 17.176 16.676 15.927 15.   ]
 [15.    15.    15.    15.    15.    15.   ]]

Average Error: 0.5207871530159269
Average Percent Error: 2.505129356511474
Maximum Error: 5.330705877148141
```

```
Maximum Percent Error: 25.183295621209478
```

6 Part G Continued

6.1 Is this error acceptable?

This error is acceptable, the maximum error however is significant on the half node wall at the middle. This is a result of our methodology of using half-nodes and quarter-nodes as a part of our methodology. Splitting the system in half has caused a significant amount of error however the values outside of the border are very acceptable.

6.2 What could you do do reduce this error?

Either to add more nodes, or to solve the system as a whole without doing half or quarter nodes just as a evenly spaced grid with all four boundary conditions known.