Coulombs Law Lab Report

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This report presents a method to calculate the permittivity of free space using Coulombs Law. The result is $\epsilon_0 = 1.15 \times 10^{-11} \pm 6 \times 10^{-13}\ Fm^{-1}$. This is within 30% of the reference value. This report aims to explain this discrepancy.

I. INTRODUCTION

The aim of this experiment is to compute a value for the permittivity of free space. To do this we will need to plot a graph and use the gradient.

It was discovered by Charles-Augustin de Coulomb that the force exerted by two charges on each other can be expressed by the equation:

$$F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2} \textbf{ Equation 1}$$

In this experiment we take advantage of the method of image charges. This simplifies the experimental process and calculations as we only need to charge one sphere and the resulting electric field between the metal plate and charged sphere will be as though there were two oppositely charged spheres with twice the separation of the sphere and metal plate.
From this we get:

$$F = \frac{Q^2}{4\pi\epsilon_0 (2s+d)^2}$$

Where $s =$ sphere plate separation and $d =$ the diameter of the sphere.

$$F^{\frac{1}{2}} = \frac{Q}{\sqrt{4\pi\epsilon_0}(2s+d)}$$

$$F^{-\frac{1}{2}} = \frac{2s+d}{Q}\sqrt{4\pi\epsilon_0} \textbf{ Equation 2}$$

This is in the form of $y = mx$ where $m = \sqrt{4\pi\epsilon_0}$, $y = F^{-\frac{1}{2}}$, and $x = \frac{2s+d}{Q}$

II. METHODS

Key Equipment

Metal sphere attached to an insulated rod so it can be handled whilst charged, and a grounded rod that will be used to remove any static charge from the electrometer.

Metal plate Used to mirror the electric field of the charged sphere .

High voltage power supply Connected to an exposed metal point that charges the sphere when they are in contact.

Electrometer Used to measure the charge of the conducting sphere.

Force meter Used to measure force exerted on the conducting sphere.

Vernier Caliper Used to measure the diameter of the sphere.

Process

First the force meter was calibrated using paperclips as weights and a scale to compare the measurements with in order to account for any systematic error. The difference in measurements between the force meter and the scale was negligible.

The diameter of the sphere was measured multiple times using a Vernier Caliper and the mean was calculated as well as its associated error.

Next the conducting sphere was charged using the power supply set to $25.4kv$, then the voltage on the sphere was measured. The sphere was then attached to the force meter and the sphere-plate separation was measured. The force was then measured at one second intervals for 60 seconds. This was repeated eight times with various separations.

Only the first four seconds of data was used to minimize the effect the decay of the charge off the sphere would have on the results. The average force in the first three seconds for each separation was calculated as well as all respective errors.
The charge on the sphere was also calculated using the measured voltage and capacitance of the sphere (The capacitance was assumed to be $10nF$ as that was the rated capacitance.).

Next $F^{-\frac{1}{2}}$ was plotted against $\frac{2s+d}{Q}$, the Y axis was set to zero as From Equation 2 the graph should be a straight line through the origin.

III. RESULTS

From figure 1 we see that

$$\sqrt{4\pi\epsilon_0} = 1.205 \times 10^{-5} \pm 3 \times 10^{-7} N^{-\frac{1}{2}} m^{-1} C$$

Therefore $\epsilon_0 = 1.15 \times 10^{-11} \pm 6 \times 10^{-13}\ Fm^{-1}$

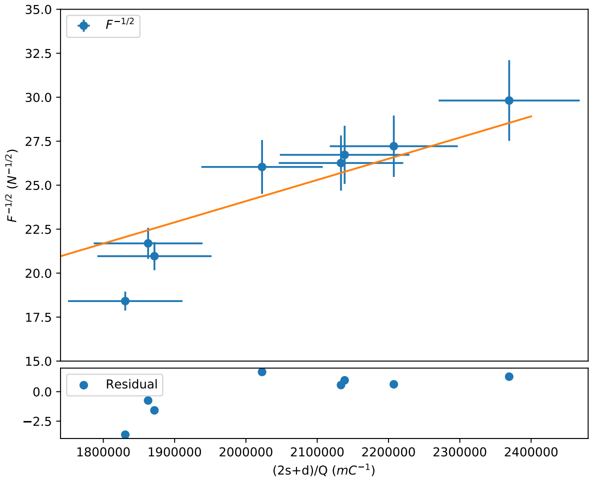

FIG. 1: Gradient = $1.205 \times 10^{-5} \pm 3 \times 10^{-7}$ $(N^{-\frac{1}{2}} m^{-1} C)$. The y intercept has been set to zero.

IV. DISCUSSION

This result for the value of ϵ_0 is 30% larger than the widely accepted value of $8.8541878128 \times 10^{-12} \pm 0.0000000013 \times 10^{-12}$ Fm^{-1} as measured by the National Institute of Standards and Technology. This discrepancy is too large to be explained by the uncertainty.

What could cause this discrepancy? This difference could be caused by the fact that the exponential decay of the charge from the conducting sphere was not taken into account and/or due to the fact that without forcing the Y intercept of the graph to zero the Y intercept was $-14.63 \pm 6 (N^{-\frac{1}{2}})$ (far from zero), this could indicate some systematic error or that our model is not taking account of some unknown variable.

V. CONCLUSIONS

Figure 1 shows that

$$F^{-\frac{1}{2}} \propto \frac{2s+d}{Q}$$

Therefore

$$F \propto \frac{Q^2}{(2s+d)^2}$$

So Coulombs Law is true.
In order to calculate a more accurate value of ϵ_0 the decay of the charge from the sphere must be taken into account.

Acknowledgments

References

[1] Ifan G. Hughes and Thomas P.A. Hase, Measurements and their Uncertainties, First edition, Oxford University Press, United States (2010),

Error Appendix

The uncertainty in d was $\pm 0.05mm$, the uncertainty in s was $\pm 0.5mm$, the uncertainty in Q was $\pm 1 \times 10^{-9} C$, and the uncertainty in F was calculated using the formula for

If $Z = A^n$ then $|\frac{\alpha_Z}{Z}| = |\frac{\alpha_A}{A}|$

All errors were calculated as described in Measurements and their uncertainties[1].

Error of the gradient of the graph in **FIG.1** was calculated using the method of least squares using the library matplotlib in python and Microsoft Excel. The error of the calculated value of ϵ_0 was calculated using the functional approach.

Scientific Summary for a General Audience

When you have two charges they exert a force on each other, this force is known as the coulomb force. The magnitude of this force is directly proportional to the magnitude of the charge, and inversely proportional to the separation of the charges squared. Putting this together we get this expression:

$$F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}$$

Where q_1 and q_2 are the charges of the two objects, r is the separation of the two charges, and ϵ_0 is the permittivity of free space.

The purpose of this experiment is to calculate a value for ϵ_0. We do this by plotting the graph that can be found in **FIG.1** and finding the gradient.
From **Equation 2** we can equate the gradient to $\sqrt{4\pi\epsilon_0}$, and thus rearrange to compute the value of ϵ_0.

Why is this important?

It is important to know an accurate value of ϵ_0 as this constant is required to calculate the energy stored by capacitors which are vital for all computers and advanced electronics. Without knowing this we would not be able to design much of the technology we have today.