Using a precision interferometer in the Fabry-Perot and Michelson Morley configurations to ascertain a value of the Sodium Doublet.

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In this experiment we use a precision interferometer in both the Fabry-Perot and Michelson Morley configuration to ascertain the value of the sodium doublet to be $2.42 \pm 3 \times 10^{-6} nm$ and $2.39 \pm 8 \times 10^{-5} nm$ respectively. These values are roughly a quarter of our reference value[1]. In this report we aim to explain this difference and recommend improvements to our method that could reduce the difference between the calculated and reference values.

I. INTRODUCTION

When Sodium is excited it emits light of two very similar wavelengths, the difference between these wavelengths can be calculated using a precision interferometer. In this experiment we use a precision interferometer in the Fabry Perot and Michelson Morley configurations.

The Fabry Perot Configuration

The Fabry Perot configuration is depicted below. Light

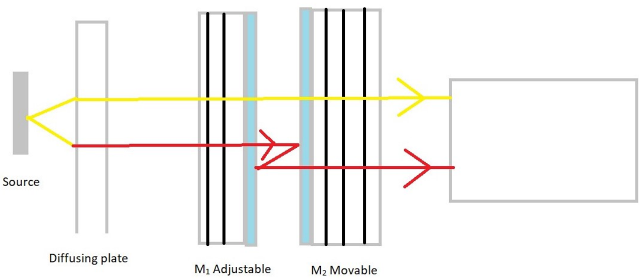


FIG. 1: Fabry-Perot Configuration

passes through a diffusing plate, a mirror, then a partially reflective mirror that reflects a fraction of the light back at the mirror (M_1), which is then reflected back towards the target. The reflected beam now is out of phase with the unreflected beam, this phase difference can be controlled by changing the separation of the mirrors.
The superposition of the two out of phase light waves results in an interference pattern on the target screen, As the separation of the mirrors changes so does the phase difference between the waves and so does interference pattern on the screen.

To quantify this suppose a position of maximum visibility occurs at a mirror displacement of s.

$$2s = m\lambda_1 \text{ and } 2s = n\lambda_2 \tag{1}$$

Where m and n are integers. The next position of maximum fringe visibility will occur at

$$2(s+t) = (m+p)\lambda_1 \text{ and } (n+p+1)\lambda_2 \tag{2}$$

Where P is the number of fringes of λ_1 between two successive positions of maximum visibility.
By expanding both sets of brackets and substituting in $s = m\lambda_1 = n\lambda_2$ we find that

$$p = \frac{2t}{\lambda_1} \text{ and so } t = \frac{\lambda_1\lambda_2}{2(\lambda_1 - \lambda_2)} \approx \frac{\lambda^2}{2(\lambda_1 - \lambda_2)} \tag{3}$$

The Michelson Morley Interferometer

The Michelson Morley Interferometer is most widely known about due to its role in the Michelson Morley experiment which disproved the notion of the luminiferous aether. In this experiment it is being used to create an interference pattern to measure the Sodium Doublet (the small difference between the two emitted wavelengths).
This is accomplished by sending light towards a partially silvered mirror, which reflects 50% of the light and the rest is transmitted through the mirror. Both beams are reflected off of two mirrors and recombined into a single beam. The phase difference of the two light beams resulting from their different path lengths can be controlled by varying the distance between the two adjustable mirrors and the partially silvered mirror.

Let the distance between the partially silvered mirror and the first adjustable mirror be d_1, and the distance between the partially silvered mirror and the second adjustable mirror be d_2.

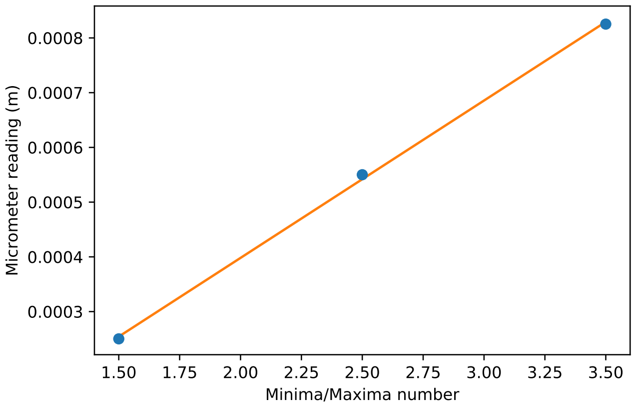
$$\text{path difference} = 2(d_1 - d_2) \text{ so } \frac{2(d_1 - d_2)}{\lambda} \tag{4}$$

Gives the path difference as a multiple of the wavelength. Thus t can be calculated in the same manner as that of the Fabry Perot configuration.

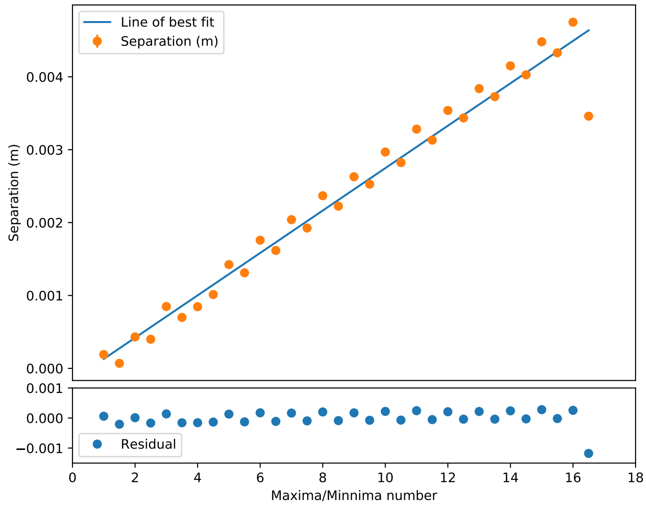
In both configurations when the bright fringes from λ_1 coincide with the bright fringes from λ_2 the contrast between the bright and dark fringes is at its maximum so the pattern is at maximum visibility. When the bright fringes from each wavelength lie between each other the view is almost uniformly yellow so the pattern is at minimum visibility.

II. METHODS

To calculate t (the average separation between maximum and minimum visibility) separation can be plotted against its corresponding maxima/minima number, the gradient will then give the value of t. Then using equation (3) $\lambda_1 - \lambda_2$ (The sodium doublet) can be calculated as λ has been measured as $589.3nm$. In both the Fabry Perot and



(a) Fabry Perot configuration (errors are too small to see)



(b) Michelson Morley configuration (errors are too small to see)

FIG. 2: Graphs of separation against maxima/minima number

Michelson Morley configurations this was done by varying the position of a mirror. The position of the mirror was adjusted using a screw gauge identical to that found on most micrometers.

The positions of the visibility maxima and minima were determined visually. The positions of the visibility maxima/minima were plotted against their respective maxima/minima numbers.

III. RESULTS

The gradient of graphs 2b and 2a give the value of t ($0.00029089 \pm 1 \times 10^{-8}m$ and $0.00028752 \pm 4 \times 10^{-10}m$ respectively). From equation (3) we find that

$$\lambda_1 - \lambda_2 = \frac{2\lambda^2}{t} = 2.39 \times 10^{-9} \pm 8 \times 10^{-14}m$$

For the Michelson Morley configuration, and $2.42 \times 10^{-9} \pm 3 \times 10^{-15}m$ For the Fabry Perot configuration.

IV. DISCUSSION

Our reference value[1] calculates the difference in wavelengths to be $0.59 \pm 0.01nm$. This is a quarter of our calculated value from the Michelson Morley and Fabry Perot configurations.
In the case of the Fabry-Perot configuration this could be due to the small sample size (three data points). The reason for this is the low contrast which is explained at the end of the introduction (section 1). In the Michelson Morley configuration this could be caused by the last data point that is far from the trend line. If we exclude this data point we find $\lambda_1 - \lambda_2 = 2.28 \times 10^{-9} \pm 2 \times 10^{-14}m$. So this outlier is not the cause of the discrepancy. Another potential cause is that our model is incomplete, the residual plot in figure 2b shows that there may be a sinusoidal term missing from our model with a period of one minima/maxima number as the data points oscillate around the trendline with that period.
The third possible cause of this discrepancy is that the characteristic wavelength of Sodium's emission lines was measured incorrectly. The fourth possible reason for this discrepancy could be due to systematic error as the y intercept of our graph was not zero (as predicted by equation (3)). As both the Michelson Morley and Fabry Perot configurations produced similar results systematic error is unlikely as it would have to have occurred in both configurations and had the same effect on the readings. As such the most likely cause for the difference between the calculated and accepted values of the sodium doublet is that our model is incomplete.

What improvements can be made?

A method to measure maximum contrast at a given mirror placement would allow accurate determination of the positions of minimum visibility in the Fabry Perot Configuration. This will reduce the uncertainty in the gradient of the line of best fit allowing a more precise and accurate measurement of t in the Fabry-Perot configuration. Repeating the experiment with different Sodium light sources could show that our bulb had some effect on the emitted light influencing our readings.

V. CONCLUSIONS

In the Fabry Perot Configuration the sodium doublet was calculated to be $\lambda_1 - \lambda_2 = 2.39 \times 10^{-9} \pm 1 \times 10^{-14}m$. In the Michelson Morley configuration the sodium doublet was calculated to be $2.42 \times 10^{-9} \pm 3 \times 10^{-15}m$ The reference value[1] for the sodium doublet is $0.59nm$, this is one order of magnitude different and a quarter of the value of the sodium doublet calculated in this experiment.

Acknowledgments

References

[1] D'Anna, Michele & Corridoni, Tommaso. (2017). Measuring the separation of the sodium D-doublet with a Michelson interferometer. European Journal of Physics. 39.10.1088/1361-6404/aa8e76

Error Appendix

All errors were calculated in line with Measurements and their uncertainties (authored by Hughes and Hase).

The gradients, intercepts, gradient errors, and intercept errors of the graphs 2a and 2b were calculated using the method of least squares. In the case of 2b this is not ideal as it is very sensitive to outliers (as squaring the residuals greatly amplifies the effect of outliers) so may have given a gradient that is too low due to the outlier that is the last data point.

The positions of the minimum and maximum visibility were repeated four times (in the Michelson Morley configuration), the mean was taken and the error was calculated using the following formula from measurements and their uncertainties.

$$\alpha_Z = Z\sqrt{\left(\frac{\alpha_A}{A}\right)^2 + \left(\frac{\alpha_B}{B}\right)^2 + \cdots + \left(\frac{\alpha_N}{N}\right)^2}$$

For the errors of N variables where α_i is the error of the i^{th} variable.

In the Michelson Morley configuration the screw gauge reading was a fifth of the distance moved by the second mirror (the gearing factor was 5). Because of this the errors were divided by the gearing factor as the data was also divided by the gearing factor.

The residuals in graph 2b were calculated by computing the difference between the value of the line of best fit at the minima/maxima value and the value of the data point.

Scientific Summary for a General Audience

All atoms contain a nucleus surrounded by a cloud of orbiting electrons. These electrons can only exist at specific energy levels. This means that electrons can only absorb specific (quantized) amounts of energy that will take them to the next permitted energy level.

If an electron is excited to a higher energy level it can return to a lower energy level. To do this it must lose an amount of energy equal to the difference in energy between these levels. This lost energy is emitted as a photon. This principle is what governs how most lightbulbs work. In this experiment we excite Sodium such that its electrons gain and lose energy emitting light in the process. Sodium is unique in the fact that its electrons when they lose energy can fall two different but very similar amounts emitting two very similar wavelengths in the process.

In this experiment we measure the difference between these wavelengths using a piece of equipment called a precision interferometer in two different configurations.