Game Theory, Welfare, and Applications
Supervision 1

Samuel Lee

Question 1

A Nash equilibrium is an action profile in which all players are playing their mutual best responses. In that equilibrium, no player can get a better payoff by deviating from their current action, holding the actions of the other players constant.

One way to assign values of utility (or disutility) of the different outcomes which is consistent with the scenario is that the disutility from being charged with a petty crime is -2, the disutility from being charged with a grievous crime is -3, the disutility from being charged with a petty crime but under a reduced sentence is -1, and the disutility of being punished by the cartel is -4.

In the scenario given, the outcomes can be expressed in the form (u_1, u_2) where u_i is the utility of Prisoner i.

- If both prisoners keep quiet, they both get charged for a petty crime and the outcome is $(-2, -2)$.
- If Prisoner 1 finks while Prisoner 2 keeps quiet, Prisoner 1 goes free but gets punished by the cartel while Prisoner 2 gets charged for a grave crime, and the outcome is $(-4, -3)$.
- If both prisoners fink, they both get charged for a grave crime but get reduced sentences for cooperating. The outcome depends on whether the cartel can punish the prisoners while they are in prison.
  - If the prisoners do not get punished while they are in prison, the outcome will be $(-1, -1)$.
  - If the prisoners get punished while in prison, the outcome will be $(-5, -5)$.

Depending on which form the game takes, it can be represented by one of the payoff matrices below:

		Prisoner 2	
		Q	F
Prisoner 1	Q	$(-2,-2)$	$(-3,-4)$
	F	$(-4,-3)$	$(-1,-1)$

In the case where the cartel does not go after snitches in prison, the two Nash equilibria are (Q, Q) and (F, F). If Prisoner 1 deviates from (Q, Q), his utillty drops from -2 to -4. If he deviates from (F, F), his utility drops from -1 to -3. The game is symmetric so this applies to Prisoner 2 as well.

		Prisoner 2	
		Q	F
Prisoner 1	Q	$(-2,-2)$	$(-3,-4)$
	F	$(-4,-3)$	$(-5,-5)$

If the cartel does go after snitches in prison, the only Nash equilibrium is (Q, Q). (F, F) is no longer viable because Prisoner 1 can improve his utility from -5 to -3 by deviating from (F, F), and the same applies for Prisoner 2.

Question 2

(a)

The order of preferences are laid out quite clearly, so following the same conventions as Question 1:

		Passenger 2	
		Sit	Stand
Passenger 1	Sit	$(1,1)$	$(2,0)$
	Stand	$(0,2)$	$(0,0)$

Again, the game is symmetric, so what applies for Passenger 1 applies to Passenger 2. No matter what Passenger 2's choice is, Passenger 1 can get more comfortable by choosing to sit. If Passenger 2 was standing, Passenger 1 gets a payoff of 2, and if Passenger 2 was sitting, Passenger 1 gets a payoff of 1. If Passenger 1 was originally sitting, choosing to stand will always lead to a worse outcome for him no matter what Passenger 2's choice is. The only Nash equilibrium in this case is (Sit,Sit). This is different from the Prisoner's dilemma; whether one tries to map "Sit" onto "Quiet" or "Sit" onto "Fink", and vice-versa, the order of preferences between the different outcomes is different from that in the Prisoner's Dilemma. Another way to see it is that when one passenger is standing, he is indifferent towards the other passenger choosing to stand or sit. In the Prisoner's Dilemma, the prisoners are never indifferent towards the other prisoner's choices.

(b)

The scenario given is such that Passenger 1 would never dream of sitting while Passenger 2 is standing, but if forced to sit he would prefer if Passenger 2 could sit also, and this is preferable to having both of them stand up. Passenger 2 thinks exactly like Passenger 1.

		Passenger 2	
		Sit	Stand
Passenger 1	Sit	$(2, 2)$	$(0, 3)$
	Stand	$(3, 0)$	$(1, 1)$

In this case the game is exactly like the Prisoner's Dilemma, with "Sit" being the equivalent of "Quiet" and "Stand" the equivalent of "Fink". The unique Nash equilibria is (Stand,Stand), which is Pareto-inferior to (Sit,Sit), and the equilibrium level of comfort is lower than that in (a) for both passengers.