Second Best Theory
Supervision 2

Samuel Lee

Question 1

(a)

The Ramsey taxation rule implies taxes should be set such that, for a marginal increase in taxes, the marginal deadweight cost per marginal unit of revenue collected is equalised for both taxes. Because the VAT is ad valorem, we don't have enough information to know the marginal unit of revenue collected for an increase in the tax rate. Therefore we cannot say whether it will be optimal to lower the VAT on highchairs and increase the VAT on car seats. To decide, we'd need to know for both goods: the amounts consumed, the current prices, and the own- and cross-price elasticities.

Even if we assume the cross-price elasticities of demand for car seats and demand for highchairs are independent (as they might be), we'd need the own-price elasticities for both goods to know if the taxes are suboptimal.

(b)

Looking only at tax collection gives us a partial picture of how progressive a fiscal system is. Outlays are just as important: we could apply a higher tax to food from Marks & Spencer relative to food from Tesco, and this may well be lauded by journalists as a "progressive" tax, but we'd be hard-pressed to call this a "pro-poor" policy if all the revenue is used to beautify golf courses. So an alternative to insisting on "progressive" taxes could be to organise spending in a "pro-poor" way. For instance, every once in a while in Singapore an argument re-surfaces which says that VATs (which we call GST) are regressive and should be lowered or at least not increased. But while the GST in Singapore is 7% across all goods, 84.2% of GST revenue was collected from the top 40% households by income in 2010. Also, there is a "GST voucher" scheme which makes flat transfers to citizens below some level of assessable income and housing type.

Another consideration is that in practice, non-linear taxation of income is feasible, which could obviate the need for differential VAT rates whether or not they are due to efficiency or equity concerns. We have the Atkinson-Stiglitz result which says that if income can be taxed non-linearly, and preferences are weakly separable between leisure and consumption, there is no gain to differential consumption taxes. Some of the assumptions will not hold in which case there are marginal deviations from this result, but assuming that non-linear income taxation is viable might make more sense as a default benchmark.

Question 2

(a)

There are two outputs x_1 and x_2 which have constant marginal costs of production c_1 and c_2. If there is a fixed cost, we have the cost function $C(x_1, x_2) = F + c_1x_1 + c_2x_2$. If the public enterprise charges prices p_1 and p_2 for the two goods, the break-even condition implies that we must have $p_1x_1 + p_2x_2 = F + c_1x_1 + c_2x_2$. Assuming the two goods are independent in demand, we have $x_1 = x_1(p_1)$ and $x_2 = x_2(p_2)$. And we can denote the indirect utility function of a representative consumer as $v(p_1, p_2)$. Given this, the regulator must solve

$$\max_{p_1,p_2} v(p_1, p_2) \text{ subject to } p_1x_1 + p_2x_2 \geq F + c_1x_1 + c_2x_2$$

The Lagrangian for this problem is

$$\mathscr{L} = v(p_1, p_2) + \lambda[(p_1 - c_1)x_1 + (p_2 - c_2)x_2 - F]$$

The first-order conditions are

$$\begin{aligned} \frac{\partial \mathscr{L}}{\partial p_1} &= \frac{\partial v}{\partial p_1} + \lambda \left[(p_1 - c_1)\frac{\partial x_1}{\partial p_1} + x_1\right] = 0 \\ \frac{\partial \mathscr{L}}{\partial p_2} &= \frac{\partial v}{\partial p_2} + \lambda \left[(p_2 - c_2)\frac{\partial x_2}{\partial p_2} + x_2\right] = 0 \end{aligned}$$

We know that $\dfrac{\partial v}{\partial p_i} = -\mu x_i$ where μ is the marginal utility of income. So we have

$$\begin{aligned} \lambda(p_1 - c_1)\frac{\partial x_1}{\partial p_1} &= -(\lambda - \mu)x_1 \implies \frac{p_1 - c_1}{p_1} = -\frac{\lambda - \mu}{\lambda}\frac{1}{\varepsilon_{11}} \\ \lambda(p_2 - c_2)\frac{\partial x_1}{\partial p_1} &= -(\lambda - \mu)x_1 \implies \frac{p_2 - c_2}{p_2} = -\frac{\lambda - \mu}{\lambda}\frac{1}{\varepsilon_{22}} \end{aligned}$$

which says that the markup for each good should be positive (since $\lambda - \mu$ represents the excess burden of taxation and should be positive) and inversely proportional to its own-price elasticity. Even though the markets for x_1 and x_2 are independent in demand, the positive and differentiated markups imply that product mix efficiency is not attained. This is because there is another interdependency created by the government budget constraint.

(b)

The distortion associated with a monopoly is that goods sold by a monopoly are under-produced. Therefore, relative to the case where the publicly-produced goods are independent of the monopoly-produced good x_3 in demand, a substitute for x_3 should be priced higher since there is the opportunity to induce greater production of x_3. The same reasoning tells us that a complement to x_3 should be priced relatively lower.

To show this, we can work through similar steps as before. We redefine the Lagrangian for the monopolist's new maximisation problem:

$$\mathscr{L} = v\left[p_1, p_2, p_3(x_3^M(p_1,p_2), p_1, p_2), I\right] + \lambda[(p_1-c_1)x_1 + (p_2-c_2)x_2 - F]$$

where I is the consumer's budget which includes monopoly profits $\pi_3(p_1,p_2)$. The first-order conditions are

$$\frac{\partial \mathscr{L}}{\partial p_1} = \frac{\partial v}{\partial p_1} + \frac{\partial v}{\partial p_3}\left(\frac{\partial p_3}{\partial x_3}\frac{\partial x_3^M}{\partial p_1} + \frac{\partial p_3}{\partial p_1}\right) + \frac{\partial v}{\partial I}\frac{\partial \pi_3}{\partial p_1} + \lambda\left[(p_1-c_1)\frac{\partial x_1}{\partial p_1} + x_1\right] = 0$$

$$\frac{\partial \mathscr{L}}{\partial p_2} = \frac{\partial v}{\partial p_2} + \frac{\partial v}{\partial p_3}\left(\frac{\partial p_3}{\partial x_3}\frac{\partial x_3^M}{\partial p_2} + \frac{\partial p_3}{\partial p_2}\right) + \frac{\partial v}{\partial I}\frac{\partial \pi_3}{\partial p_2} + \lambda\left[(p_2-c_2)\frac{\partial x_2}{\partial p_2} + x_2\right] = 0$$

Again, we have $\dfrac{\partial v}{\partial p_i} = -\mu x_i$. Also, μ is the marginal utility of income, which is also $\dfrac{\partial v}{\partial I}$. So we have

$$\frac{\partial \mathscr{L}}{\partial p_1} = -\mu x_1 - \mu x_3^M\left(\frac{\partial p_3}{\partial x_3}\frac{\partial x_3^M}{\partial p_1} + \frac{\partial p_3}{\partial p_1}\right) + \mu\frac{\partial \pi_3}{\partial p_1} + \lambda\left[(p_1-c_1)\frac{\partial x_1}{\partial p_1} + x_1\right] = 0$$

$$\frac{\partial \mathscr{L}}{\partial p_2} = -\mu x_2 - \mu x_3^M\left(\frac{\partial p_3}{\partial x_3}\frac{\partial x_3^M}{\partial p_2} + \frac{\partial p_3}{\partial p_2}\right) + \mu\frac{\partial \pi_3}{\partial p_2} + \lambda\left[(p_2-c_2)\frac{\partial x_2}{\partial p_2} + x_2\right] = 0$$

We should simplify this further by considering that the monopolist maximises profits as given by

$$p_3(x_3, p_1, p_2)x_3 - c_3x_3$$

The envelope theorem then tells us that $\dfrac{\partial \pi_3}{\partial p_i} = \dfrac{\partial p_3}{\partial p_i}x_3^M$, and the first-order condition tells us that $-\dfrac{\partial p_3}{\partial x_3}x_3^M = p_3 - c_3$. Substituting the envelope theorem result gives us

$$\begin{aligned}
\frac{\partial \mathscr{L}}{\partial p_1} &= -\mu x_1 - \mu x_3^M\left(\frac{\partial p_3}{\partial x_3}\frac{\partial x_3^M}{\partial p_1} + \frac{\partial p_3}{\partial p_1}\right) + \mu\frac{\partial p_3}{\partial p_1}x_3^M + \lambda\left[(p_1-c_1)\frac{\partial x_1}{\partial p_1} + x_1\right] \\
&= -\mu x_1 - \mu x_3^M\frac{\partial p_3}{\partial x_3}\frac{\partial x_3^M}{\partial p_1} + \lambda\left[(p_1-c_1)\frac{\partial x_1}{\partial p_1} + x_1\right] = 0 \\
\frac{\partial \mathscr{L}}{\partial p_2} &= -\mu x_2 - \mu x_3^M\frac{\partial p_3}{\partial x_3}\frac{\partial x_3^M}{\partial p_2} + \lambda\left[(p_2-c_2)\frac{\partial x_2}{\partial p_2} + x_2\right] = 0
\end{aligned}$$

and substituting the first-order condition gives us

$$\begin{aligned}
\frac{\partial \mathscr{L}}{\partial p_1} &= -\mu x_1 + \mu\frac{\partial x_3^M}{\partial p_1}(p_3-c_3) + \lambda\left[(p_1-c_1)\frac{\partial x_1}{\partial p_1} + x_1\right] \\
&= (\lambda-\mu)x_1 + \mu\frac{\partial x_3^M}{\partial p_1}(p_3-c_3) + \lambda(p_1-c_1)\frac{\partial x_1}{\partial p_1} = 0 \\
\frac{\partial \mathscr{L}}{\partial p_2} &= (\lambda-\mu)x_2 + \mu\frac{\partial x_3^M}{\partial p_2}(p_3-c_3) + \lambda(p_2-c_2)\frac{\partial x_2}{\partial p_2} = 0
\end{aligned}$$

We now have nicer expressions for the optimal prices:

$$
\begin{aligned}
p_1 - c_1 &= \frac{1}{\lambda \dfrac{\partial x_1}{\partial p_1}} \left[-(\lambda - \mu) x_1 - \mu \frac{\partial x_3^M}{\partial p_1} (p_3 - c_3) \right] \\
&= -\frac{\lambda - \mu}{\lambda} \frac{1}{\dfrac{\partial x_1}{\partial p_1} \dfrac{1}{x_1}} - \frac{\mu}{\lambda} \frac{1}{\dfrac{\partial x_1}{\partial p_1}} \frac{\partial x_3^M}{\partial p_1} (p_3 - c_3) \\
\frac{p_1 - c_1}{p_1} &= -\frac{\lambda - \mu}{\lambda} \frac{1}{\varepsilon_{11}} - \frac{\mu}{\lambda} \frac{1}{\dfrac{\partial x_1}{\partial p_1} p_1} \frac{\partial x_3^M}{\partial p_1} (p_3 - c_3) \\
&= -\frac{\lambda - \mu}{\lambda} \frac{1}{\varepsilon_{11}} - \underbrace{\frac{\mu}{\lambda} \frac{x_1}{\varepsilon_{11}} \frac{\partial x_3^M}{\partial p_1} (p_3 - c_3)}_{<0 \text{ since } \frac{\partial x_3^M}{\partial p_1} > 0} \\
\frac{p_2 - c_2}{p_2} &= -\frac{\lambda - \mu}{\lambda} \frac{1}{\varepsilon_{22}} - \underbrace{\frac{\mu}{\lambda} \frac{x_2}{\varepsilon_{22}} \frac{\partial x_3^M}{\partial p_2} (p_3 - c_3)}_{>0 \text{ since } \frac{\partial x_3^M}{\partial p_2} < 0}
\end{aligned}
$$

The first term in the optimal markup is the same as before. The sign of the second term depends on whether the good is a substitute or complement to x_3, and confirms our earlier intuition that substitutes are priced relatively higher and complements are priced relatively lower.

Question 3

(a)

There are seven conditions underlying the Diamond-Mirrlees result:

(i) All markets are competitive

(ii) The production technology exhibits constant returns to scale

(iii) The government has full flexibility in choosing the tax rates on all commodities and factors of production

(iv) The government has a revenue requirement R

(v) The government cannot levy lump-sum taxes

(vi) The social welfare function is individualistic

(vii) There is nonsatiation in at least one good

It is not explicitly mentioned that the production technology exhibits constant returns to scale, but the 100% profits tax ensures that consumers' incomes do not depend on producer prices. However, Diamond and Mirrlees (1971) state that

> Pure profits (or losses) associated with the violation of these assumptions imply that private production decisions directly influence social welfare by affecting household incomes. In such a case, it would presumably be desirable to add a profits tax to the set of policy instruments. Nevertheless, aggregate production efficiency would no longer be desirable in general; although it may be possible to get close to the optimum with efficient production if pure profits are small.

(b)

Notwithstanding the caveat about production technologies, a consumption tax should be favoured as it does not take the economy away from production efficiency.

(c)

It could be optimal to levy a tax on good y since good y is more price inelastic than good x; when faced with a revenue requirement the optimal tax on y should be higher than that on x since there is less of a substitution effect in y. Furthermore, x and y are substitutes, so a price increase in y will generate an added benefit through increased tariff revenues from x.

Therefore, because x and y are substitutes, a tariff on y generates increased tariff revenues from x due to a demand effect. Furthermore, there is also a supply effect: if x and y are the only two sectors in the economy, a tariff on y increases the domestic production of y and raises the wage rate in the economy. In the meantime the domestic production of x is reduced as labour is released, equating wages with marginal cost. There is both an increase in consumer demand and a decrease in domestic supply, so imports rise and tariff revenues increase even further. Therefore a tariff generates an added benefit relative to a consumption tax.

When there are many sectors in the economy and the aggregate demand for labour is infinitely elastic, a tariff on y doesn't affect the wage rate and no longer generates a benefit beyond that of a consumption tax. Therefore it is best to stay close to productive efficiency and impose a consumption tax to raise the remaining revenue.