Paper 10 Time Series Models 2019-2020

Supervision Questions

Supervision 3. Systems of Equations

Systems/IV Example Sheet

1. Consider the regression model

$$y_i = x_i^{'}\beta + u_i$$

where y_i is log wage and x_i is a vector of explanatory variables that includes constant, gender, race, union dummy, schooling, experience, and (experience)^2 . Labour economists believe that the conditional variance of log wage may itself depend on education, experience and some other explanatory variables.

a) What are the consequences of such a dependence for the OLS estimator?

b) How could one assess the accuracy of the OLS estimates (ie produce confidence intervals)?

c) If we propose the following model for the conditional heteroscedasticity

$$Var\left(u_i \mid x_i\right) = exp\left(x_i^{'}\gamma\right)$$

Discuss how we could use this model to produce a FGLS estimator of β.

2. Suppose you have a regression

$$y_i = \alpha + \beta x_i + u_i$$

where $E\left(u_i\right) = 0$ but x_i is an endogenous variable so that $E\left(x_i u_i\right) \neq 0$. Further, suppose that you consider a valid instrument w_i. In particular, you have $E\left(w_i u_i\right) = 0$ but $Cov\left(w_i, x_i\right) \neq 0$.

a) Your friend proposes to estimate β as follows. First, regress x_i on w_i without including a constant in the regression. Second, regress y_i on a constant and the fitted value from the first regression. Argue that such an estimator is inconsistent in general.

b) When you regress x_i on w_i and a constant using 100 observations, you get $R^2 = 0.05$. Is w_i a strong instrument? (Hint: think about how R^2 is linked to the F statistic)

3.

Consider a simple time series model where the explanatory variable has classical measurement error:

$$\begin{aligned} y_t &= \beta_0 + \beta_1 x_t^* + u_t \\ x_t &= x_t^* + e_t, \end{aligned}$$

where u_t has zero mean and is uncorrelated with x_t^* and e_t. We observe y_t and x_t only. Assume that e_t has zero mean and is uncorrelated with x_t^* and that x_t^* also has a zero mean (this last assumption is only to simplify the algebra).

a) Write $x_t^* = x_t - e_t$ and plug this into the first of the above equations. Show that the error term in the new equation, say, ν_t, is negatively correlated with x_t if $\beta_1 > 0$. What does this imply about the OLS estimator of β_1 from the regression of y_t on x_t?

b) In addition to the previous assumptions, assume that u_t and e_t are uncorrelated with all past values of x_t^* and e_t; in particular, with x_{t-1}^* and e_{t-1}. Show that $E(x_{t-1}\nu_t) = 0$, where ν_t is the error term in the model from part (a).

c) Are x_t and x_{t-1} likely to be correlated? Explain.

d) What do parts (b) and (c) suggest as a useful strategy for consistently estimating β_0 and β_1?

4. Consider the following two-equation model, linking inflation and the openness of a country.

$$\begin{aligned} Infl_i &= \beta_{10} + \alpha_1 Open_i + \beta_{11} \log(Inc_i) + \varepsilon_{1i} \\ Open_i &= \beta_{20} + \alpha_2 Infl_i + \beta_{21} \log(Inc_i) + \beta_{22} \log(land_i) + \varepsilon_{2i} \end{aligned}$$

where $Infl_i$ is inflation in country i, Inc_i is per capita income in country i, $land_i$ is the land area of country i, and $open_i$ is a measure of openness for contry i equal to the share of imports in the GDP. Assuming that Inc_i and $land_i$ are exogenous variables, are the above equations identified? Explain.

5.

Let *corn* denote per capita consumption of corn in bushels, at the county level, let *price* be the price per bushel of corn, let *income* denote per capita county income, and let $rainfall$ be inches of rainfall during the last corn-growing season. The following simultaneous equations model imposes the equilibrium condition that supply equals demand

$$\begin{aligned} corn &= \alpha_{01} + \alpha_1 price + \beta_1 income + u_1 \\ corn &= \alpha_{02} + \alpha_2 price + \beta_2 rainfall + \beta_3 rainfall^2 + u_2 \end{aligned}$$

Which is the supply equation, and which is the demand equation? Explain in detail how you would estimate the equations by 2SLS and by 3SLS.