Monetary Economics
Supervision 1

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Section A

A.1

The AS-AD model with rational expectations has the following set-up. The IS curve characterizes the goods market:

$$Y = \bar{Y} - \alpha(r - \bar{r}) + \eta$$

and monetary policy is conducted according to a reaction function:

$$r = \bar{r} + \mu_\pi(\pi - \pi^*) + \mu_Y(Y - \bar{Y}) + v$$

where η and v are stochastic shocks with mean 0. Substituting the monetary policy reaction function into the IS relationship gives rise to an AD relationship:

$$\begin{aligned} Y &= \bar{Y} - \alpha[\mu_\pi(\pi - \pi^*) + \mu_Y(Y - \bar{Y}) + v] + \eta \\ \alpha\mu_\pi(\pi - \pi^*) &= (1 + \alpha\mu_Y)(Y - \bar{Y}) - \alpha v + \eta \\ \pi &= \pi^* - \frac{1 + \alpha\mu_Y}{\alpha\mu_\pi}(Y - \bar{Y}) - \frac{v}{\mu_\pi} + \frac{\eta}{\alpha\mu_\pi} \end{aligned}$$

The aggregate supply relation is captured by the Phillips curve:

$$\pi = \pi^e + \theta(Y - \bar{Y}) + \varepsilon$$

where ε represents a stochastic shock with mean 0.

With rational expectations and perfect information, there is no forecast error, so $\pi^e = \pi$. This together with the AS relation is already enough to show that the output gap is just $-\frac{\varepsilon}{\theta}$. That is, the output gap is independent of any monetary policy actions that would affect AD, for example through the π^* (an intentional expansion) or v (an unintentional shock) terms in the AD equation. Under these restrictive assumptions, it is not true that an increase in aggregate demand decreases (or increases) output. Any changes in aggregate demand (stochastic or otherwise) are automatically built into the π^e term of the AS relation, offsetting the change in output and leaving the economy at a higher inflation rate. Even if we relax the assumption of perfect information, and introduce a time lag into expected inflation, this would lead to an increase in output rather than a decrease. In this sense the statement is not true.

In practice, a high inflation rate could distort price signals and lead to misallocations at the microeconomic level, and this may very well decrease output (the assumption of perfect information would of course have to be dropped). The effects of inflation on debt are also not captured in this model. If firms hold a substantial amount of debt denominated in foreign currencies as liabilities, the inflation and corresponding currency depreciation could lead to worsening balance sheet constraints, leaving firms unable to invest. If this outweighs the expansionary effects of currency depreciation on net exports, output will decrease as a result.

A.2

The policy ineffectiveness proposition follows from the derivation in A.1. It is true that in the model there are no effects of monetary policy on output. However, anticipated monetary policy does lead to an increase in inflation through inflation expectations. The statement is true if we only consider output and false if we consider inflation, although this is just in the context of the model; in practice monetary policy does have effects on both output and inflation.

Section B

B.1

The goods market equilibrium is described by the IS relation

$$y_t = -\alpha(r_t - \bar{r}) + \eta_t$$

where y_t denotes the output gap. Monetary policy is set according to the reaction function

$$r_t = \bar{r} + \mu_\pi(\pi_t - \pi^*) + \mu_y y_t + \upsilon_t$$

and aggregate supply is governed by the Phillips relation

$$\pi_t = \pi_t^e + \theta y_t + \varepsilon_t$$

(a)

The aggregate demand equation shows the relation between inflation and output (or the output gap), and is derived by substituting the monetary policy reaction function into the IS relation, as such:

$$\begin{aligned} y_t &= -\alpha[\mu_\pi(\pi_t - \pi^*) + \mu_y y_t + \upsilon_t] + \eta_t \\ \alpha\mu_\pi(\pi_t - \pi^*) &= -(1 + \alpha\mu_y)y_t - \alpha\upsilon_t + \eta_t \\ \pi_t &= \pi^* - \frac{1 + \alpha\mu_y}{\alpha\mu_\pi} y_t - \frac{\upsilon_t}{\mu_\pi} + \frac{\eta_t}{\alpha\mu_\pi} \end{aligned}$$

From this we can see that inflation is increasing in π^*, decreasing in υ_t, and increasing in η_t.

When inflation is above π_*, the central bank increases r_t according to the monetary policy reaction function. This reduces y_t according to the IS relation, which ultimately results in a downward sloping AD relation in (π_t, y_t) space. An increase in π_* means a higher level which the central bank finds desirable, and translates to a more loose monetary policy at every level of y_t compared to before.

υ_t appears in the monetary policy reaction function. It captures imperfections in the conduct of monetary policy which are not biased towards monetary tightness or looseness in particular. A positive value of υ_t means that the central bank inadvertently sets a tighter monetary policy than it would have in the absence of uncertainty. This is the reason π_t is decreasing in υ_t.

η_t appears in the IS relation, and captures imperfections in the optimizing behaviour of firms and individuals that underlie the IS relationship. Again, these are not systematically biased in either direction. A positive value of η_t means that firms are producing more (or consumers are consuming more) than they would have given the prevailing interest rate in the absence of uncertainty. For illustrative purposes we have assumed that the central bank has already set interest rates. Thus η_t has the effect of increasing the level of output demanded at any given level of π_t.

If we follow the stories above it seems intuitive to interpret υ_t as a decrease along the π_t (vertical) axis, and η_t as an increase along the y_t (horizontal) axis, even though the choice of axes is of no great consequence when dealing with linear reliationships.

(b)

With perfect foresight, $\pi_t^e = \pi_t$. From the Phillips relation, this means

$$\begin{aligned}
\pi_t &= \pi_t + \theta y_t + \varepsilon_t \\
y_t &= -\frac{\varepsilon_t}{\theta}
\end{aligned}$$

and from the AD relation, this means

$$\pi_t = \pi^* + \frac{(1+\alpha\mu_y)\varepsilon}{\alpha\theta\mu_\pi} - \frac{\upsilon_t}{\mu_\pi} + \frac{\eta_t}{\alpha\mu_\pi}$$

which gives us the equilibrium levels of y_t and π_t. As discussed in Section A, the result for y_t is just one form of the policy ineffectiveness proposition. With perfect information and flexible prices, the values of π^*, υ_t, and η_t are all known and incorporated into inflation expectations immediately such that the output gap remains unaffected. Intuitively, the reason y_t is unaffected by any choice or chance variable other than ε is that when π_t is known to everyone, there is no channel to cause output to deviate from its 'natural' level other than the stochastic cost-push shocks.

The effects of the three variables on the equilibrium π_t are just as in (a).

(c)

Taking the expectation of the Phillips relation at time $t-1$ yields

$$\mathrm{E}_{t-1}[\pi_t] = \pi_t^e + \theta \mathrm{E}_{t-1}[y_t] + \mathrm{E}_{t-1}[\varepsilon_t]$$

and since $\pi_t^e = \mathrm{E}_{t-1}[\pi_t]$ and $\mathrm{E}_{t-1}[\varepsilon_t] = 0$, this means $\mathrm{E}_{t-1}[y_t] = 0$. Taking the expectation of the AD relation,

$$\begin{aligned}
\mathrm{E}_{t-1}[\pi_t] &= \pi^* - \frac{1+\alpha\mu_y}{\alpha\mu_\pi}\mathrm{E}_{t-1}[y_t] - \frac{\mathrm{E}_{t-1}[\upsilon_t]}{\mu_\pi} + \frac{\mathrm{E}_{t-1}[\eta_t]}{\alpha\mu_\pi} \\
&= \pi^*
\end{aligned}$$

This means that prior to the realization of any stochastic shocks or changes in policy targets, agents in the economy expect next period's inflation to be the central bank's inflation target and output to be the perceived natural rate of output.

(d)

Substituting $\mathrm{E}_{t-1}[\pi_t] = \pi_t^e = \pi^*$ into the Phillips relation gives a system of linear equations together together with the AD relation:

$$\begin{aligned}
\pi_t &= \pi^* + \theta y_t + \varepsilon_t \\
\pi_t &= \pi^* - \frac{1+\alpha\mu_y}{\alpha\mu_\pi} y_t - \frac{\upsilon_t}{\mu_\pi} + \frac{\eta_t}{\alpha\mu_\pi}
\end{aligned}$$

from this we can find y_t:

$$\begin{aligned}
\theta y_t + \varepsilon_t &= -\frac{1+\alpha\mu_y}{\alpha\mu_\pi} y_t - \frac{\upsilon_t}{\mu_\pi} + \frac{\eta_t}{\alpha\mu_\pi} \\
\left(\theta + \frac{1+\alpha\mu_y}{\alpha\mu_\pi}\right) y_t &= \frac{\eta_t}{\alpha\mu_\pi} - \frac{\upsilon_t}{\mu_\pi} - \varepsilon_t \\
\left(\frac{1+\alpha\mu_y+\alpha\theta\mu_\pi}{\alpha\mu_\pi}\right) y_t &= \frac{\eta_t}{\alpha\mu_\pi} - \frac{\upsilon_t}{\mu_\pi} - \varepsilon_t \\
y_t &= \frac{1}{1+\alpha\mu_y+\alpha\theta\mu_\pi}\eta_t - \frac{\alpha}{1+\alpha\mu_y+\alpha\theta\mu_\pi}\upsilon_t - \frac{\alpha\mu_\pi}{1+\alpha\mu_y+\alpha\theta\mu_\pi}\varepsilon_t
\end{aligned}$$

From the equation above, we can see that the equilibrium output gap does not depend on π^* as before. The output gap is increasing in η_t and decreasing in υ_t, which makes sense because a positive η_t represents more production in the goods market than would occur in the absence of uncertainty, while υ_t represents a tighter monetary policy than what would be pursued without uncertainty.

The parameters μ_π and μ_y appear in the denominators of the coefficients of η_t and υ_t. This means higher values of μ_π and μ_y lead to greater stabilization of the υ_t and η_t shocks. In the monetary policy reaction function, μ_y does not actually represent how averse the central bank is to output gaps relative to deviations in inflation; that factor is totally captured in the magnitude of μ_π. If the monetary policy reaction function is derived from a quadratic loss function, it can be shown that the only difference between μ_π and μ_y is driven by the slope of the IS curve; it has nothing to do with the relative weights given to inflation and output in the central bank's preferences.

Instead, μ_π and μ_y show how responsive the central bank is to inflation and output deviations. This is a purely descriptive statement that says interest rates change more in response to a given deviation in inflation or output. Since υ_t and η_t are demand-side shocks, the effect of μ_π and μ_y on interest rates in response to either of those shocks work in the same direction. That is, for a demand-side shock, output and inflation move away from the original state in the same direction. This is not true for a supply-side shock, and is shown by the fact that the effect of ε_t on output is amplified the larger μ_π is. When a cost-push shock increases inflation, a very strong interest rate response will worsen the output deviation. We should expect the reverse for inflation when π_t is derived later; a higher μ_y should amplify the effect of ε_t on π_t.

To find π_t, we can substitute the equilibrium y_t into the Phillips curve:

$$\begin{aligned}
\pi_t &= \pi^* + \theta\left(\frac{1}{1+\alpha\mu_y+\alpha\theta\mu_\pi}\eta_t - \frac{\alpha}{1+\alpha\mu_y+\alpha\theta\mu_\pi}\upsilon_t - \frac{\alpha\mu_\pi}{1+\alpha\mu_y+\alpha\theta\mu_\pi}\varepsilon_t\right) + \varepsilon_t \\
&= \pi^* + \frac{\theta}{1+\alpha\mu_y+\alpha\theta\mu_\pi}\eta_t - \frac{\alpha\theta}{1+\alpha\mu_y+\alpha\theta\mu_\pi}\upsilon_t - \frac{\alpha\theta\mu_\pi}{1+\alpha\mu_y+\alpha\theta\mu_\pi}\varepsilon_t + \varepsilon_t \\
&= \pi^* + \frac{\theta}{1+\alpha\mu_y+\alpha\theta\mu_\pi}\eta_t - \frac{\alpha\theta}{1+\alpha\mu_y+\alpha\theta\mu_\pi}\upsilon_t + \frac{1+\alpha\mu_y}{1+\alpha\mu_y+\alpha\theta\mu_\pi}\varepsilon_t
\end{aligned}$$

As the equation shows, the relationship between inflation and υ_t and η_t, and the stabilizing properties of μ_y and μ_π are qualitatively identical to that for output, and these are for the same reasons as before. The only difference is that inflation is increasing in the cost-push shock ε_t (as should be expected), and μ_π is now stabilizing while μ_y is amplifying with respect to ε_t as predicted earlier.

B.2

The central bank minimizes the loss function

$$L = \frac{1}{2}(\pi - \pi^*)^2 + \frac{1}{2}\lambda(y - y^*)^2$$

where λ denotes the relative aversion of the central bank to output deviations against deviations in inflation. The monetary policy instrument is the rate of money growth m, which affects inflation through the relationship

$$\pi = m + v$$

where v is a velocity shock. This relationship follows from the quantity theory of money, where $MV = PY$. Taking logarithms and differentiating with respect to time yields the above relationship if output is assumed to be constant. The aggregate supply relation is described by

$$y = \theta(\pi - \pi^e) + s$$

where s is an aggregate supply shock.

(a)

The central bank adjusts m after π^e is set and s and v are observed. Thus these should appear in the central bank's minimization problem. Substituting the AS relation into the loss function,

$$L = \frac{1}{2}(\pi - \pi^*)^2 + \frac{1}{2}\lambda[\theta(\pi - \pi^e) + s - y^*]^2$$

The monetary authority adjusts m, so substituting $\pi = m + v$ into L,

$$L = \frac{1}{2}(m + v - \pi^*)^2 + \frac{1}{2}\lambda[\theta(m + v - \pi^e) + s - y^*]^2$$

The first-order condition for a minimum is then

$$\begin{aligned}
\frac{\partial L}{\partial m} &= m + v - \pi^* + \lambda\theta[\theta(m + v - \pi^e) + s - y^*] = 0 \\
m + v - \pi^* + \lambda\theta^2 m &+ \lambda\theta^2 v - \lambda\theta^2\pi^e + \lambda\theta s - \lambda\theta y^* = 0 \\
(1 + \lambda\theta^2)m &= \pi^* + \lambda\theta^2\pi^e + \lambda\theta(y^* - s) - (1 + \lambda\theta^2)v \\
m &= \frac{1}{1 + \lambda\theta^2}\pi^* + \frac{\lambda\theta^2}{1 + \lambda\theta^2}\pi^e + \frac{\lambda\theta}{1 + \lambda\theta^2}(y^* - s) - v
\end{aligned}$$

The optimal m is increasing in π^*, y^*, and π^e, and decreasing in s and v. A higher inflation target π^* means that the central bank is willing to let the money supply grow more for a given level of π^e, since the additional inflation that entails is more tolerable (or desirable). The same thing applies to y^*; a higher output target means the central bank is willing to move further up the AS curve, allowing greater inflation in the process.

The equation shows a lower rate of money growth given a positive s or v. That is, the central bank finds it optimal to put the brakes on monetary expansion when supply or money velocity is greater than usual. Monetary policy is set after v has already been realized. Since the central bank already knows v, it's a simple matter to offset v 1-for-1 to reach the most preferred point on the Phillips curve. Thus v appears in the expression for m without any coefficient.

As for s, in an AD-AS model, a positive supply shock would lead to higher output but lower inflation in equilibrium without any monetary intervention. There is no AD relation here, and furthermore the response of the central bank is always to reduce output in response to a positive s even though this would lower inflation as well. Even though there might be some ambiguity in theory as to whether it is more important to deal with the downward pressure on inflation or the upward pressure on output, the result derived shows that it is unambiguously preferable to offset the output shock.

This is a subtle result of the specific functional form of the loss function. Because the indifference curves implied by the loss function are concentric ellipses that expand outwards in a regular way, it just so happens that the optimal choice on the new Phillips curve following a supply shock always involves a change in money growth in the opposite direction:

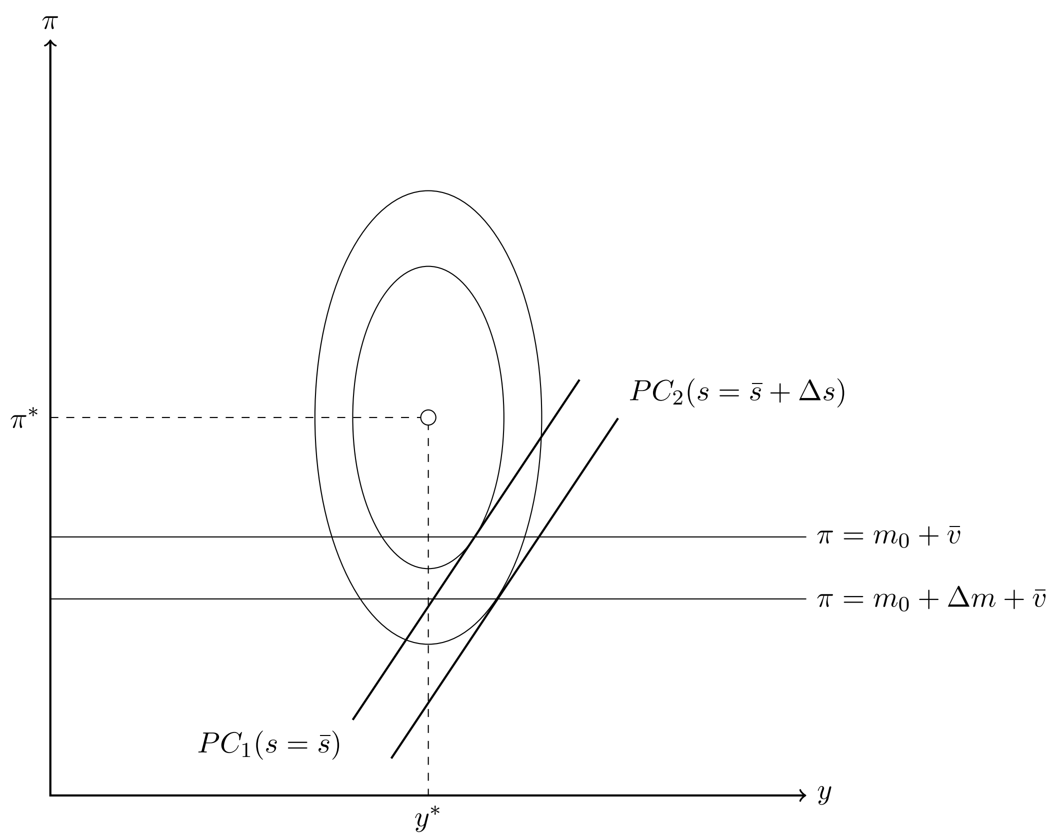

Figure 1: Optimal response to positive supply shock with quadratic loss function

In Figure 1, the loss function is a regular quadratic loss function. At first, the central bank sets $m = m_0$ to attain the most preferred state along PC_1. In response to a positive supply shock ($\Delta s > 0$), the Phillips curve shifts down to PC_2 and the new optimum is obtained by setting $\Delta m < 0$, that is, a slowdown in the rate of money growth. Hence the negative relation between the optimal m and s. It is possible to draw a set of indifference curves where this is not the case:

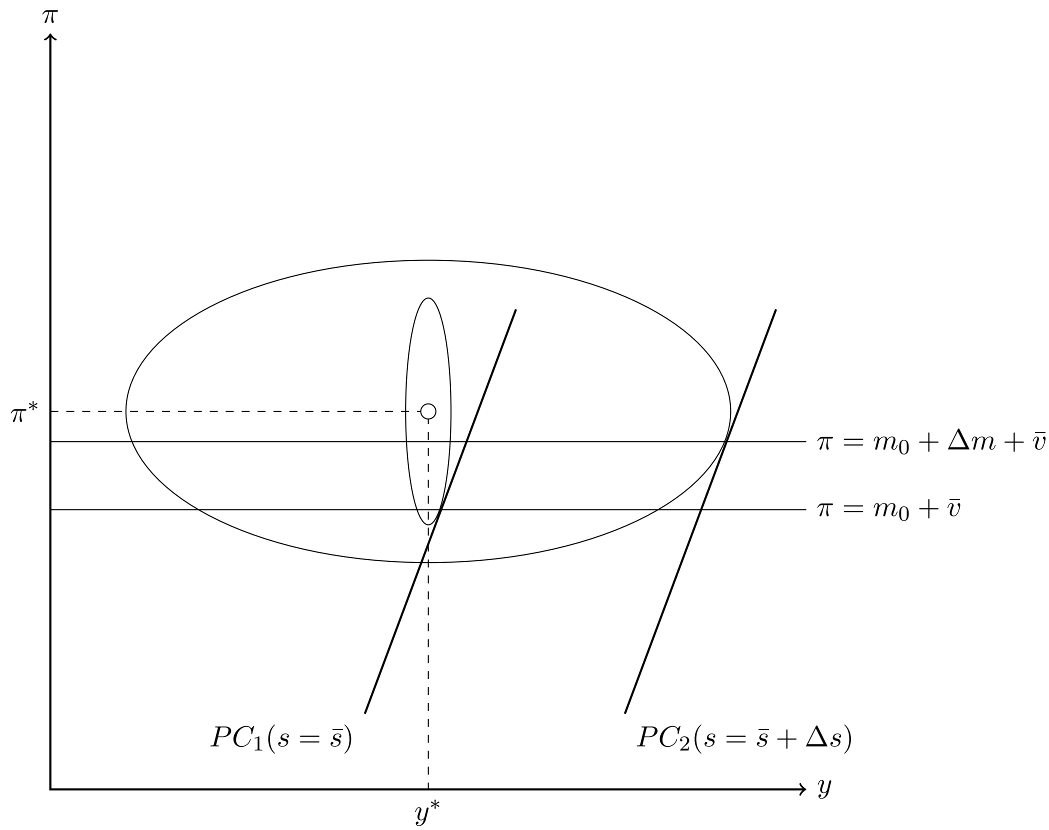

Figure 2: Optimal response to positive supply shock with non-conventional loss function

In Figure 2, a similarly positive supply shock induces a faster monetary expansion instead ($\Delta m > 0$), and we end up with the opposite relation between m and s. So this relation is not that straightforward, and is not a natural conclusion without knowledge of the specific functional form of the loss function. Still, it is possible to mathematically show that for all indifference curves of a quadratic loss function, any two points with the same gradient always involves changes of π and y in opposite directions.

The optimal rate of money growth is increasing in π^e. This is for the same reasons as in the case of s, except that π_e affects the Phillips curve in the opposite direction.

(b)

Taking expectations of the aggregate supply relation,

$$\mathrm{E}[y] = \theta(\mathrm{E}[\pi] - \pi^e) + \mathrm{E}[s] = 0$$

and taking expectations of the optimal m,

$$\begin{aligned}
\mathrm{E}[m] &= \frac{1}{1+\lambda\theta^2}\mathrm{E}[\pi^*] + \frac{\lambda\theta}{1+\lambda\theta^2}(\mathrm{E}[y^*] + \theta\mathrm{E}[\pi^e] - \mathrm{E}[s]) - \mathrm{E}[v] \\
&= \frac{1}{1+\lambda\theta^2}\pi^* + \frac{\lambda\theta}{1+\lambda\theta^2}(y^* + \theta\pi^e)
\end{aligned}$$

Taking expectations of $\pi = m + v$ yields $\mathrm{E}[\pi] = \pi^e = \mathrm{E}[m]$, so

$$\begin{aligned}
\pi^e &= \frac{1}{1+\lambda\theta^2}\pi^* + \frac{\lambda\theta}{1+\lambda\theta^2}(y^* + \theta\pi^e) \\
\left(1 - \frac{\lambda\theta^2}{1+\lambda\theta^2}\right)\pi^e &= \frac{1}{1+\lambda\theta^2}\pi^* + \frac{\lambda\theta}{1+\lambda\theta^2}y^* \\
\frac{1}{1+\lambda\theta^2}\pi^e &= \frac{1}{1+\lambda\theta^2}\pi^* + \frac{\lambda\theta}{1+\lambda\theta^2}y^* \\
\pi^e &= \pi^* + \lambda\theta y^*
\end{aligned}$$

So we have $\mathrm{E}[y] = 0$ and $\mathrm{E}[\pi] = \pi^e = \pi^* + \lambda\theta y^*$. To compute $\mathrm{Var}[y]$ and $\mathrm{Var}[\pi]$, we first solve for π and y. We know $\pi^e = \pi^* + \lambda\theta y^*$, so we can solve for π:

$$\begin{aligned}
\pi = m + v &= \frac{1}{1+\lambda\theta^2}\pi^* + \frac{\lambda\theta^2}{1+\lambda\theta^2}\pi^e + \frac{\lambda\theta}{1+\lambda\theta^2}(y^* - s) \\
&= \frac{1}{1+\lambda\theta^2}\pi^* + \frac{\lambda\theta^2}{1+\lambda\theta^2}(\pi^* + \lambda\theta y^*) + \frac{\lambda\theta}{1+\lambda\theta^2}(y^* - s) \\
&= \pi^* + \lambda\theta y^* - \frac{\lambda\theta}{1+\lambda\theta^2}s
\end{aligned}$$

and we use this to solve for y:

$$\begin{aligned}
y = \theta(\pi - \pi^e) + s &= \theta\left(-\frac{\lambda\theta}{1+\lambda\theta^2}s\right) + s \\
&= \frac{1}{1+\lambda\theta^2}s
\end{aligned}$$

Taking the variances, we get

$$\begin{aligned}
\mathrm{Var}[\pi] &= \mathrm{Var}\left[\pi^* + \lambda\theta y^* - \frac{\lambda\theta}{1+\lambda\theta^2}s\right] \\
&= \left(\frac{\lambda\theta}{1+\lambda\theta^2}\right)^2 \sigma_s^2 \\
\mathrm{Var}[y] &= \mathrm{Var}\left[\frac{1}{1+\lambda\theta^2}s\right] \\
&= \left(\frac{1}{1+\lambda\theta^2}\right)^2 \sigma_s^2
\end{aligned}$$

We have that $\mathrm{E}[y] = 0$ while $\mathrm{E}[\pi] = \pi^* + \lambda\theta y^*$. This means that the expectation of the output gap is totally independent of what output target the central bank chooses; agents price this into their inflation expectations such that y^* shows up in $\mathrm{E}[\pi] = \pi^e$ instead. As usual, the inflation target is built into inflation expectations. The results for $\mathrm{Var}[\pi]$ and $\mathrm{Var}[y]$ show that v does not contribute any volatility to either inflation or output at all; it is known when the central bank sets policy, so it can be perfectly offset. Whether inflation is more volatile than output depends on the magnitudes of λ and θ, but a comparison between the two is less insightful anyway since they are in different units.

(c)

The central bank now announces and commits to a rate of money growth m_C. At this point, s and v have not been observed, so a risk-neutral central bank has to minimize the expected value of the loss function:

$$\begin{aligned}
\mathrm{E}[L] &= \mathrm{E}\left[\frac{1}{2}(m_C + v - \pi^*)^2 + \frac{1}{2}\lambda[\theta(m_C + v - \pi^e) + s - y^*]^2\right] \\
&= \frac{1}{2}\mathrm{E}\left[(m_C + v - \pi^*)^2\right] + \frac{1}{2}\lambda\mathrm{E}\left[[\theta(m_C + v - \pi^e) + s - y^*]^2\right]
\end{aligned}$$

Since m_C is already announced when expectations are formed, π^e is simply equal to m_C. Therefore,

$$\mathrm{E}[L] = \frac{1}{2}\mathrm{E}\left[(m_C + v - \pi^*)^2\right] + \frac{1}{2}\lambda\mathrm{E}\left[[\theta v + s - y^*]^2\right]$$

Taking the derivative with respect to m_C and setting it to zero:

$$\mathrm{E}[m_C + v - \pi^*] = m_C - \pi^* = 0 \implies m_C = \pi^*$$

The shocks then take place after m_C has been set. As a result, the outcomes are

$$\begin{aligned}
\pi_C &= m_C + v = \pi^* + v \\
y_C &= \theta(\pi_C - \pi^e) + s = \theta v + s \\
\mathrm{E}[\pi_C] &= \mathrm{E}[\pi^* + v] = \pi^* \\
\mathrm{E}[y_C] &= \mathrm{E}[\theta v + s] = 0 \\
\mathrm{Var}[\pi_C] &= \mathrm{Var}[\pi^* + v] = \sigma_v^2 \\
\mathrm{Var}[y_C] &= \mathrm{Var}[\theta v + s] = \theta^2\sigma_v^2 + \sigma_s^2
\end{aligned}$$

With commitment, the inflation bias disappears. However, the effects on inflation volatility are ambiguous. σ_s^2 no longer appears in the expression for π_C, but in its place is σ_v^2; the central bank has given up its ability to offset v while the only effect of s is on output since it is realized after monetary policy has been set. Whether inflation volatility is improved depends on whether $\sigma_v^2 < \left(\frac{\lambda\theta}{1+\lambda\theta^2}\right)^2 \sigma_s^2$. This is reminiscent of a Part I question about whether fixing the money supply or interest rate is optimal. The mechanism in (b) is analogous to fixing interest rates (this behaviour is buried under the $\pi = m + v$ equation) while the mechanism in (c) is analogous to fixing the money supply (its growth rate, to be exact). In last year's context, if IS or output shocks are more prevalent (σ_s^2 is large), targeting the money supply is better for volatility, and if LM shocks are prevalent (σ_v^2 is large) then targeting the interest rate is better.

There is an unambiguous worsening of output volatility, since the central bank has lost any ability to stabilize output (σ_s^2 is fully felt) on top of losing the ability to offset any velocity shocks (σ_v^2 now appears in $\mathrm{Var}[y_C]$).

(d)

(i) Setting $\lambda_{CB} = 0$ means placing no weight at all on output deviations. This is likely to improve inflation volatility while worsening output volatility, since the central bank has perfect control over inflation in setting m after v has been realized and now does not have to care about the repercussions for output. In fact this probably means that π will always be set to π^*.

To check this, we derive the optimal m for $\lambda = 0$:

$$\begin{aligned}
L &= \frac{1}{2}(\pi - \pi^*)^2 = \frac{1}{2}(m + v - \pi^*)^2 \\
\frac{\partial L}{\partial m} &= m + v - \pi^*
\end{aligned}$$

which implies the optimal rate of money growth is $m = \pi^* - v$. Inflation is equal to $\pi = m + v = \pi^*$ and output is equal to

$$y = \theta(\pi - \pi^e) + s = s$$

since inflation expectations with rational expectations should equal to π^*. The results are as expected; inflation always reaches the target and supply shocks on output are fully felt. As such, $\mathrm{Var}[\pi] = 0$ and $\mathrm{Var}[y] = \sigma_s^2$. The inflation bias (if $y^* > 0$) is no longer present as it was in (b) but output is more volatile than before. So setting $\lambda_{CB} = 0$ improves inflation deviations and volatility if $y^* \neq 0$ but worsens output volatility. The expected output gap is 0 in both cases.
(ii) Appointing a new central banker with $y^*_{CB} = 0$ would remove the inflation bias. It should also allow for the central bank to stabilize the supply shocks and use the information from the realized v to offset velocity shocks totally. We can just substitute $y^* = 0$ into the expressions for π and y from (b) to get π_{CB} and y_{CB}:

$$\begin{aligned}
\pi_{CB} &= \pi^* - \frac{\lambda\theta}{1+\lambda\theta^2}s \\
y_{CB} &= \frac{1}{1+\lambda\theta^2}s \\
\mathrm{E}[\pi_{CB}] &= \pi^* \\
\mathrm{E}[y_{CB}] &= 0 \\
\mathrm{Var}[\pi_{CB}] &= \left(\frac{\lambda\theta}{1+\lambda\theta^2}\right)^2 \sigma_s^2 \\
\mathrm{Var}[y_{CB}] &= \left(\frac{1}{1+\lambda\theta^2}\right)^2 \sigma_s^2
\end{aligned}$$

which is as predicted. This is a Pareto improvement over (b). To restate the outcomes for (c), (ii), and (ii),

	Positive y^* (b)	Commitment to m_C (c)	Ignore y (i)	Set $y^*_{CB} = 0$ (ii)
π	$\pi^* + \lambda\theta y^* - \frac{\lambda\theta}{1+\lambda\theta^2}s$	$\pi^* + v$	π^*	$\pi^* - \frac{\lambda\theta}{1+\lambda\theta^2}s$
y	$\frac{1}{1+\lambda\theta^2}s$	$\theta v + s$	s	$\frac{1}{1+\lambda\theta^2}s$
$\mathrm{E}[\pi]$	$\pi^* + \lambda\theta y^*$	π^*	π^*	π^*
$\mathrm{E}[y]$	0			
$\mathrm{Var}[\pi]$	$\left(\frac{\lambda\theta}{1+\lambda\theta^2}\right)^2 \sigma_s^2$	σ_v^2	0	$\left(\frac{\lambda\theta}{1+\lambda\theta^2}\right)^2 \sigma_s^2$
$\mathrm{Var}[y]$	$\left(\frac{1}{1+\lambda\theta^2}\right)^2 \sigma_s^2$	$\theta^2\sigma_v^2 + \sigma_s^2$	σ_s^2	$\left(\frac{1}{1+\lambda\theta^2}\right)^2 \sigma_s^2$

All three alternatives remove the inflation bias for a positive y^*. The policy of setting $\lambda_{CB} = 0$ enjoys zero inflation volatility, and whether a commitment to m_C is better than setting $y^*_{CB} = 0$ in this regard depends on the magnitudes of λ, θ, σ_s^2, and σ_v^2. However, when it comes to reducing output volatility, the order is clear: (ii) is better than (i), which is better than (c). So someone who only cares about the expected level of inflation will be indifferent between the three alternatives. Someone who only cares about inflation volatility will prefer (i) to either (ii) or (c), but it is ambiguous whether (ii) is preferred to (c). Someone who only cares about output volatility will prefer (ii) to (i) and (i) to (c). Maybe if there were some satisfactory way to aggregate these individual preferences, we can know what is the most preferred alternative. Empirically it appears that (ii) is the one chosen the most. It happens to be the only alternative that Pareto dominates (b).