Macroeconometrics
Supervision 3

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Question 1

(a)

The t-statistic for the coefficient on c_{t-1} is as usual: the estimated coefficient minus the coefficient under the null hypothesis divided by the standard error. In this case, $t = \frac{0.798-1}{0.098} = -2.06122$. However, under the null hypothesis, the usual central limit theorem doesn't apply, such that the t-statistic is not t-distributed but follows a Dickey-Fuller distribution. According to the statistical table, the 5% critical value for a unit root test with

- a null hypothesis where c_t follows a random walk with drift
- an alternative hypothesis where c_t is stationary around a linear trend

is -3.41. Therefore, the null hypothesis is not rejected.

(b)

In equation (2), y_t has already been first-differenced, so the null hypothesis is that the coefficient on y_{t-1} is 0, not 1 as before. Therefore, the t-statistic is $\frac{-0.239}{0.099} = -2.4141414$. THe critical value is the same as before, and the null hypothesis is not rejected.

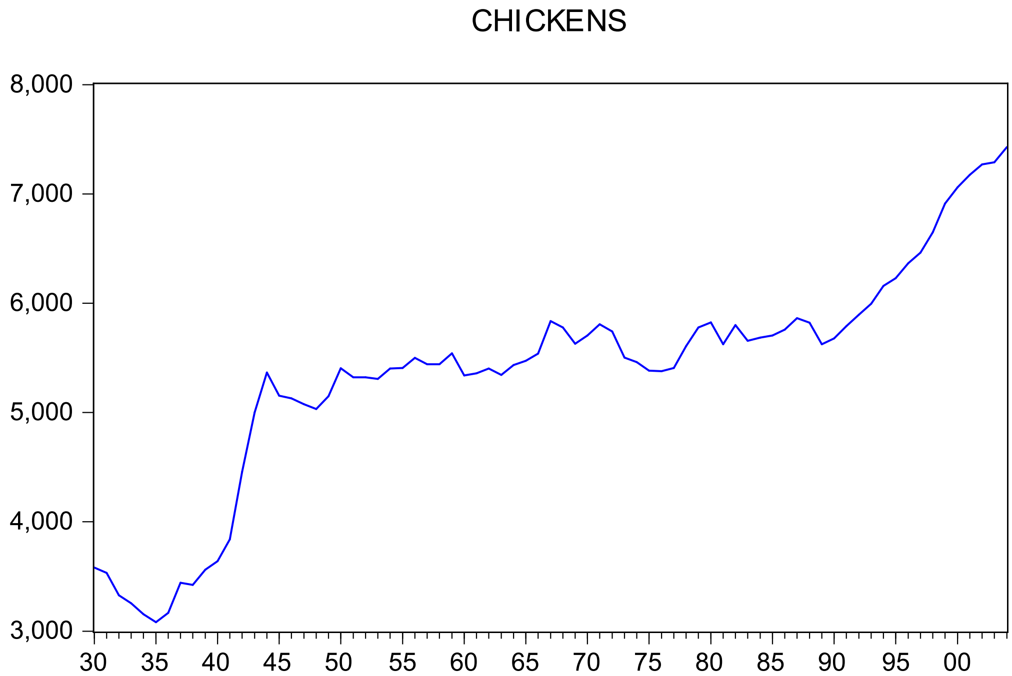
(c)

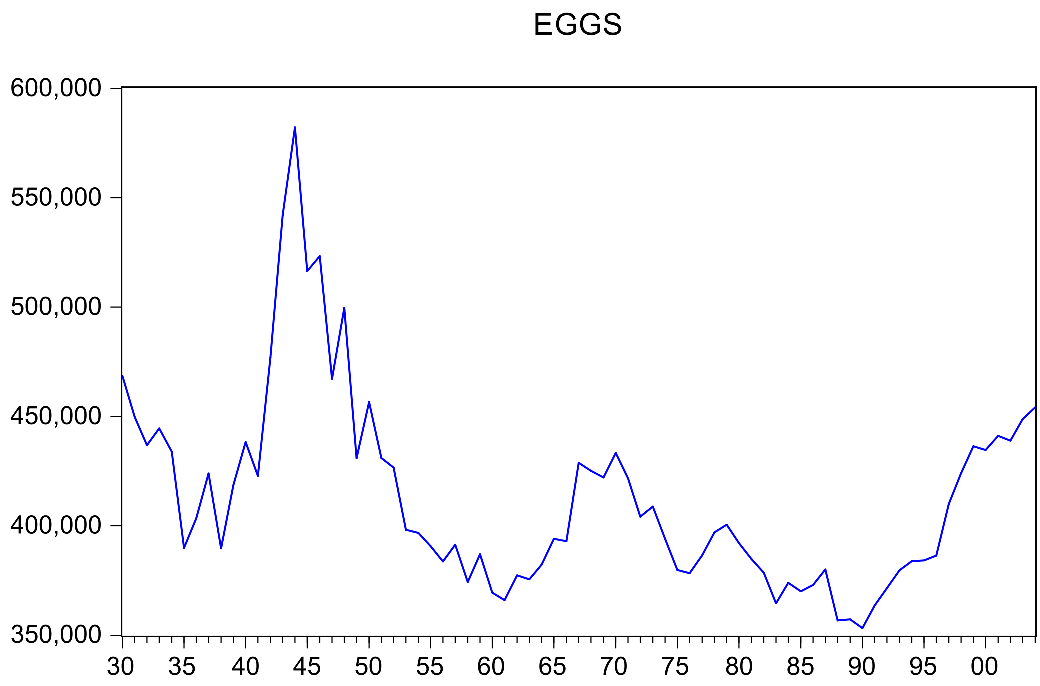
This time, we are doing a joint test where in the null hypothesis, both the intercept and coefficient on apc_{-1} are 0. Normally, we would do a likelihood-ratio or F-test as outlined in Dicker and Fuller (1981), but the log likelihood and SSR values are not tabulated. If we were to test the hypotheses that the intercept or coefficient are equal to 0 separately, we end up with the t-statistics $\frac{-0.0154}{0.0066} = -2.3333$ for the intercept and $\frac{-0.1452}{0.072} = -2.0166667$ for the coefficient. The hypothesis that the coefficient is 0 would not be rejected. Some sources state a critical value for the intercept *given* that the coefficient is 0, which is -2.54. The hypothesis that the intercept is 0 would not be rejected in this case. But regardless a test of the joint hypothesis is more appropriate.

Question 2

(a)

The plots of chickens and eggs are below:





The two series appear to exhibit some persistence, and therefore both variables don't look stationary, although the persistence is a lot more pronounced in the data for chickens than for eggs. It still seems somewhat plausible that the variable for eggs is not stationary.

(b)

The results of the test are below:

Null Hypothesis: CHICKENS has a unit root
Exogenous: Constant
Lag Length: 1 (Automatic - based on SIC, maxlag=11)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		−0.808983	0.8105
Test critical values:	1% level	−3.522887	
	5% level	−2.901779	
	10% level	−2.588280	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(CHICKENS)
Method: Least Squares
Sample (adjusted): 1932 2004
Included observations: 73 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
CHICKENS(-1)	−0.014145	0.017485	−0.808983	0.4213
D(CHICKENS(-1))	0.392446	0.110557	3.549712	0.0007
C	109.2147	94.71617	1.153073	0.2528

R-squared	0.154239	Mean dependent var	53.36986
Adjusted R-squared	0.130074	S.D. dependent var	154.7224
S.E. of regression	144.3093	Akaike info criterion	12.82202
Sum squared resid	1457762.	Schwarz criterion	12.91615
Log likelihood	−465.0038	Hannan-Quinn criter.	12.85953
F-statistic	6.382841	Durbin-Watson stat	1.983726
Prob(F-statistic)	0.002842		

and it can be seen that the null hypothesis of a unit root is not rejected up to the 10% level. The equation estimated includes a constant, and the test done is an augmented Dickey-Fuller test, although the augmentation is just done with a first-difference lag of one period based on the Schwarz Information Criterion. The interpretation of the t-statistic is just as in question 1.

(c)

The results of the test are below:

Null Hypothesis: EGGS has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 2 (Automatic - based on SIC, maxlag=11)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		−2.280852	0.4386
Test critical values:	1% level	−4.090602	
	5% level	−3.473447	
	10% level	−3.163967	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(EGGS)
Method: Least Squares
Sample (adjusted): 1933 2004
Included observations: 72 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
EGGS(-1)	−0.158430	0.069461	−2.280852	0.0257
D(EGGS(-1))	−0.032094	0.123687	−0.259474	0.7961
D(EGGS(-2))	0.274760	0.119862	2.292309	0.0250
C	68720.03	31659.50	2.170597	0.0335
@TREND("1930")	−83.88454	140.3534	−0.597666	0.5521

R-squared	0.137948	Mean dependent var	240.9722
Adjusted R-squared	0.086483	S.D. dependent var	22420.36
S.E. of regression	21428.95	Akaike info criterion	22.84979
Sum squared resid	$3.08E+10$	Schwarz criterion	23.00789
Log likelihood	−817.5924	Hannan-Quinn criter.	22.91273
F-statistic	2.680391	Durbin-Watson stat	2.004872
Prob(F-statistic)	0.038922		

and again, the null hypothesis of a unit root is not rejected up to the 10% level. This time, the equation estimated includes an intercept and trend, and the augmentation is done with the first-difference at lag 1 and 2, based on the SIC. An augmented Dickey-Fuller test reduces the sample size but may allow the estimated equation to capture higher-order autoregressive dynamics. With the lagged first differences included, under the null hypothesis, the first difference follows an autoregressive model.

(d)

The results of the Granger causality tests are below:

Pairwise Granger Causality Tests
Sample: 1930 2004
Lags: 4

Null Hypothesis:	Lag	F-Statistic	Prob.
D(EGGS) does not Granger Cause D(CHICKENS)	1	0.12067	0.7294
D(CHICKENS) does not Granger Cause D(EGGS)		12.3794	0.0008
D(EGGS) does not Granger Cause D(CHICKENS)	2	0.24720	0.7817
D(CHICKENS) does not Granger Cause D(EGGS)		4.42425	0.0157
D(EGGS) does not Granger Cause D(CHICKENS)	3	0.14607	0.9318
D(CHICKENS) does not Granger Cause D(EGGS)		3.36433	0.0239
D(EGGS) does not Granger Cause D(CHICKENS)	4	0.19951	0.9377
D(CHICKENS) does not Granger Cause D(EGGS)		4.16053	0.0048

Based on these tests it seems we should conclude that the lagged first differences of chickens provides useful information in forecasting the first difference of eggs beyond the lagged first differences of eggs alone, and not vice versa.

(e)

The equation estimated is ADL(2,2) with a constant, and the results of the Breusch-Godfrey test up to 2 lags are below:

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	0.039079	Prob. F(2,64)	0.9617
Obs*R-squared	0.087821	Prob. Chi-Square(2)	0.9570

Test Equation:
Dependent Variable: RESID
Method: ARDL
Date: 04/29/19 Time: 08:48
Sample: 1933 2004
Included observations: 72
Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
D(EGGS(-1))	−0.089342	1.629459	−0.054829	0.9564
D(EGGS(-2))	−0.106056	0.809592	−0.130999	0.8962
D(CHICKENS)	−0.008106	14.14346	−0.000573	0.9995
D(CHICKENS(-1))	7.757575	145.6082	0.053277	0.9577
D(CHICKENS(-2))	9.221193	67.77656	0.136053	0.8922
C	−903.3089	11242.04	−0.080351	0.9362
RESID(-1)	0.089017	1.634031	0.054477	0.9567
RESID(-2)	0.084068	0.302945	0.277503	0.7823

R-squared	0.001220	Mean dependent var	$-1.67E-12$
Adjusted R-squared	−0.108022	S.D. dependent var	15940.29
S.E. of regression	16779.16	Akaike info criterion	22.39810
Sum squared resid	$1.80E+10$	Schwarz criterion	22.65107
Log likelihood	−798.3317	Hannan-Quinn criter.	22.49881
F-statistic	0.011166	Durbin-Watson stat	1.999874
Prob(F-statistic)	0.999999		

Our LM statistic (0.087821) is small and the p-value is 0.957. This means that the null hypothesis of serially uncorrelated errors up to 2 lags is not rejected. However, doing the test up to 3 lags yields a p-value of 0.0668, and likewise for 4 lags yields a p-value of 0.0499. This has implications for the Granger causality test since the test is not valid when errors are serially correlated.

Question 3

$$\begin{aligned}
y_{t+2} &= 0.5y_{t+1} + 0.1y_t + x_{t+2} + 0.3x_{t+1} + \varepsilon_{t+2} \\
&= 0.5(0.5y_t + 0.1y_{t-1} + x_{t+1} + 0.3x_t + \varepsilon_{t+1}) + 0.1y_t + x_{t+2} + 0.3x_{t+1} + \varepsilon_{t+2} \\
&= 0.35y_t + 0.05y_{t-1} + x_{t+2} + 0.8x_{t+1} + 0.15x_t + \varepsilon_{t+2} + 0.5\varepsilon_{t+1} \\
&= 0.35(0.5y_{t-1} + 0.1y_{t-2} + x_t + 0.3x_{t-1} + \varepsilon_t) \\
&+ 0.05y_{t-1} + x_{t+2} + 0.8x_{t+1} + 0.15x_t + \varepsilon_{t+2} + 0.5\varepsilon_{t+1} \\
&= 0.225y_{t-1} + 0.035y_{t-2} + x_{t+2} + 0.8x_{t+1} + 0.5x_t + 0.105x_{t-1} + \varepsilon_{t+2} + 0.5\varepsilon_{t+1} + 0.35\varepsilon_t
\end{aligned}$$

Therefore, a unit change in x_t has a causal effect on y_{t+2} of 0.5.

Question 4

(a)

The results are reported below:

	Dependent variable:
	d(WinTime)
L(d(WinTime))	-0.547^{***}
	(0.100)
Constant	-6.297
	(9.287)
Observations	66
R^2	0.320
Adjusted R^2	0.309
Residual Std. Error	75.426 (df = 64)
F Statistic	30.069^{***} (df = 1; 64)
Note:	*p<0.1; **p<0.05; ***p<0.01

(b)

If $WinTime_t = \gamma + \delta \times Year_t + v_t$, then

$$\begin{aligned}
DWT_t &= WinTime_t - WinTime_{t-1} = \delta(Year_t - Year_{t-1}) + v_t - v_{t-1} \\
&= \delta + v_t - v_{t-1} \\
\frac{3}{2}\delta - \frac{1}{2} \times DWT_{t-1} + u_t &= \frac{3}{2}\delta - \frac{1}{2}(\delta + v_{t-1} - v_{t-2}) + u_t \\
&= \delta - \frac{1}{2}v_{t-1} + \frac{1}{2}v_{t-2} + u_t \\
DWT_t &= \frac{3}{2}\delta - \frac{1}{2} \times DWT_{t-1} + u_t \\
\implies \delta + v_t - v_{t-1} &= \delta - \frac{1}{2}v_{t-1} + \frac{1}{2}v_{t-2} + u_t \\
\implies u_t &= v_t - \frac{1}{2}v_{t-1} - \frac{1}{2}v_{t-2}
\end{aligned}$$

We have that

$$\begin{aligned}
\mathrm{Cov}[u_t, DWT_{t-1}] &= \mathrm{Cov}\left[v_t - \frac{1}{2}v_{t-1} - \frac{1}{2}v_{t-2}, \delta + v_{t-1} - v_{t-2}\right] \\
&= -\frac{1}{2}\sigma^2 + \frac{1}{2}\sigma^2 = 0 \\
\mathrm{E}[u_t] &= \mathrm{E}\left[v_t - \frac{1}{2}v_{t-1} - \frac{1}{2}v_{t-2}\right] = 0
\end{aligned}$$

Therefore, u_t and DWT_{t-1} are not correlated and $\mathrm{E}[u_t] = 0$. In this case the probability limit of $\hat{\beta}$ will be $\frac{1}{2}$. This is because the regressors are contemporaneously exogenous, which guarantees consistency of the OLS estimator even if it is not sufficient to guarantee unbiasedness.

(c)

The default test will not be valid because u_t are serially correlated and the standard errors computed will not be appropriate. We can use heteroskedasticity and autocorrelation consistent standard errors to get around this, and carry out the usual inference techniques. However, if possible, it is preferable to explicitly model the serial correlation in errors if the true data-generating process is known. Even though the HAC standard errors are consistent, the tests performed will have less power than if the serial correlation is explicitly modeled and inference is done using the usual standard errors.