Real Business Cycles
Supervision 2

Samuel Lee

Question 1

(a)

The nominal profit for firm i is given by the following

$$P\Pi_i = P_i Q_i^D - WL_i = Y\left(\frac{P_i}{P}\right)^{-2}(P_i - W)$$

since the production function is $Q_i = L_i$. Real profits are then given by

$$\Pi_i = Y\left(\frac{P_i}{P}\right)^{-2}\left(\frac{P_i}{P} - \frac{W}{P}\right)$$

Maximising this over $\frac{P_i}{P}$, the first-order condition is

$$-2Y\left(\frac{P_i}{P}\right)^{-3}\left(\frac{P_i}{P} - \frac{W}{P}\right) + Y\left(\frac{P_i}{P}\right)^{-2} = 0 \implies \frac{P_i}{P} = 2\frac{W}{P}$$

The resulting pricing policy involves a real markup of 2 over unit costs.

(b)

In the goods market equilibrium, we must have $Y = \frac{M}{P}$. All firms behave identically, so production is the same for each firm and we have $Q_i = L_i = L = Y$ since the number of households is normalised to one. Substituting the labour supply relationship into the goods market equilibrium, we get

$$Y = L = \frac{W}{P} = \frac{M}{P}$$

and since firms optimally set a markup over wages, we have

$$\frac{P_i^*}{P} = 2\frac{M}{P} \implies P_i^* = 2M$$

(c)

Taking logs, we get

$$p^* = m + \log 2$$

since firms are identical and set the same price (which does not mean there is perfect competition, but rather that every firm sets prices with the same markup over costs). Since $Y = \frac{M}{P}$, we have

$$y^* = -\log 2$$

This is less than the socially optimal level; if firms set prices $\frac{P_i}{P}$ to the marginal cost $\frac{W}{P}$, then $p = m$ and $y = 0$.

(d)

With nominal rigidity, a share λ of firms set their prices to the optimal level p^* while the remaining share $1-\lambda$ keep them at a pre-determined price $\bar{p}$. For the λ share of firms, we must have $p_i = p^*$ since the firms behave identically, and we know their optimal pricing rule is to set a markup on prices (deflated by the price index) over unit costs, so we have

$$p_i - p = p^* - p = \log 2 + w - p \implies p^* = \log 2 + w = \log 2 + m$$

since $\frac{W}{P} = \frac{M}{P}$ in equilibrium. This is the same result as in (c), because the price index cancels out and the fact that some prices are sticky does not affect the optimal pricing. Therefore, we get

$$p = \lambda \log 2 + \lambda m + (1-\lambda)\bar{p}$$

We could have gotten this result above by simply substituting our previous result $p^* = m + \log 2$ from (c) into the price level $p = \lambda p^* + (1-\lambda)\bar{p}$, but the reasoning laid out makes it clear that the optimal price p^* remains the individually optimal price after having taken into account that some prices are sticky (again, because the price index cancels out in the optimal markup rule).

In equilibrium we have $y = m - p$, so

$$y = (1-\lambda)(m-\bar{p}) - \lambda \log 2$$

The equations above are log-linear, so the coefficients represent elasticities. So the elasticity of prices with respect to the money supply is λ, and the elasticity of output with respect to the money supply is $1-\lambda$. Intuitively, the more flexibility firms in aggregate have in setting prices (that is, as λ increases), the more influence the money supply has on the overall price level. Likewise, when firms do not have the flexibility to set prices, they have to make a quantity adjustment instead, so the response of output to the money supply is increasing in $1-\lambda$. The result in (c) is obviously a limiting case when $\lambda \to 1$.

Question 2

We want to know whether imperfect competition and price rigidity are substitutes or complements in generating the New Keynesian results. In the typical examples covered, there are at least two main results: the non-neutrality of money, and the sub-optimality of output levels. It is easiest to answer this question by going through a formal derivation like the one above, with the change that we set the parameters to arbitrary values. Firstly, we let the firm demand curve be equal to

$$Q_i^D = Y\left(\frac{P_i}{P}\right)^{-\eta}$$

In this formulation, η represents how close the economy is to perfect competition; when η takes a high value, deviations in firm prices above the price index lead to a huge drop in quantity demanded. Therefore, $\frac{1}{\eta}$ represents the degree of imperfect competition. Maximising real profits over $\frac{P_i}{P}$ then gives us the first order condition

$$-\eta Y\left(\frac{P_i}{P}\right)^{-\eta-1}\left(\frac{P_i}{P}-\frac{W}{P}\right)+Y\left(\frac{P_i}{P}\right)^{-\eta}=0 \implies \frac{P_i}{P}=\left(\frac{\eta}{\eta-1}\right)\frac{W}{P}$$

For simplicity we let the labour supply equation stay the same, which means we get

$$\frac{P_i^*}{P}=\left(\frac{\eta}{\eta-1}\right)\frac{M}{P} \implies P_i^*=\left(\frac{\eta}{\eta-1}\right)M$$

Assuming the same form of price rigidity, we go through the same steps as before to get

$$\begin{aligned}
p &= \lambda\log\left(\frac{\eta}{\eta-1}\right)+\lambda m+(1-\lambda)\bar{p}\\
y &= (1-\lambda)(m-\bar{p})-\lambda\log\left(\frac{\eta}{\eta-1}\right)=(1-\lambda)(m-\bar{p})-[1-(1-\lambda)]\log\left(\frac{1}{1-\frac{1}{\eta}}\right)
\end{aligned}$$

and we can see that imperfect competition has nothing to do with the result of money non-neutrality; the coefficients on m are independent of η. However, the sub-optimality of output is jointly determined by λ and η. To know whether imperfect competition and price rigidity are substitutes or complements in bringing output below the socially optimal level, we take the cross-derivative of y:

$$\frac{\partial^2 y}{\partial\left(\frac{1}{\eta}\right)\partial(1-\lambda)}=\frac{1}{1-\frac{1}{\eta}}>0$$

The above shows that imperfect competition ($\frac{1}{\eta}$) and price rigidity ($1-\lambda$) are substitutes in bringing output below the socially optimal level: the effect of increasing $\frac{1}{\eta}$ becomes less negative as $1-\lambda$ increases, and vice versa. Intuitively, the more imperfect competition there is, the smaller the opportunity cost of being unable to adjust prices (with perfect competition, not being able to adjust prices means losing the entire market share). When some firms are unable to adjust prices, their second-best option is to make the adjustment to quantities produced, but the size of the required adjustment is not that great when the costs of sticking to the status quo are small.