Part II Paper 4 Game Theory Supervision 1

Michaelmas 2019

1. Show that the game below has a unique equilibrium (hint: start by showing that, in equilibrium, it cannot be that $pr(M) + pr(U) > 0$ and $pr(U) \geq pr(M)$).

	L	M	R
U	$1, -2$	$-2, 1$	$0, 0$
M	$-2, 1$	$1, -2$	$0, 0$
D	$0, 0$	$0, 0$	$1, 1$

2. A group of 19 firms choose simultaneously (i) whether to enter a market and (ii) how much to invest (a non-negative amount less than or equal to 1). The cost of entry for each firm is c. If i does not enter then its investment is zero. The payoff (gross of entry cost if any) that firm i obtains is equal to its investment multiplied by (1 minus the sum of all the investments).

(a) Formulate this as a strategic game.

(b) Assume that $c = 0$. Find a symmetric Nash equilibrium.

(c) Assume $c = 0.01$. Find a Nash equilibrium in which k firms stay out and the rest enter and behave symmetrically. What is k?

3. Two players play the following game. First, player 1 decides whether to exit ('*out*') or not ('*in*'). If he plays *out*, both players get payoff 2. If he plays *in*, then they play the simultaneous move game

	L	R
U	$3, 1$	$0, 0$
D	$0, 0$	$1, 3$

(a) Draw the extensive form and write the normal (strategic) form.

(b) Find all the pure-strategy Nash equilibria.

(c) Which strategies, for each player, survive iterated deletion of strictly dominated strategies?

(d) Which strategy profiles survive iterated deletion of weakly dominated strategies? (Think about the strategic reasoning which this solution captures).

4. Show that if a player has two weakly dominant strategies then, for every strategy choice by the opponents, the two strategies yield equal payoffs for him.

5.

	LL	L	M	R
U	$100, 2$	$-100, 1$	$0, 0$	$-100, -100$
D	$-100, -100$	$100, -49$	$1, 0$	$100, 2$

The game above is played with simultaneous moves and no preplay communication. Payoffs are in pence.

(a) If you were player 2 (the column player) what strategy would you play?

(b) Find all the Nash equilibria, pure and mixed. Is the strategy you chose in (a) part of a Nash equilibrium?

(c) For each strategy, establish whether it is ever a best-reply. Is the strategy you chose in part (a) rationalizable?

(d) If there were communication with player 1 before play, what would you play?

6. Consider the following game. Player 1 chooses either T or B; player 2 simultaneously chooses either L or R. The payoffs are either as described in Game 1 or as in Game 2, each being equally likely. Player 1 knows which of Game 1 and Game 2 is being played, but player 2 does not.

Game 1:

	L	R
T	$1, 1$	$0, 0$
B	$0, 0$	$0, 0$

Game 2:

	L	R
T	$0, 0$	$0, 0$
B	$0, 0$	$2, 2$

(a) Represent the above as a Bayesian game.

(b) Find all the pure strategy Bayes-Nash equilibria.

7. [Only to be discussed in the supervision if there's time. A solution will be posted on the course website later] Show that a strictly dominated strategy is never a best reply. Show that if iterated deletion of strictly dominated strategies yields a unique prediction in a game then this prediction also results from iterated deletion of strategies which are never a best reply (hint: use an induction argument).