1. Q. (i) Can market prices be used as shadow prices when consumers have heterogeneous preferences over project outputs and other goods? (ii) What if preferences are uniform but individuals' levels of income differ?

   (i) With H consumers and social welfare measure $SWF(u_h)$ the net social marginal benefit from the project p (which in the absence of persistent spare capacity is assumed to draw resources from other activities o) can be expressed as $\sum_1^H \frac{\partial SWF}{\partial u_h}\frac{\partial u_h}{\partial x_p^h}\Delta x_p^h - \sum_1^H \frac{\partial SWF}{\partial u_h}\frac{\partial u_h}{\partial x_o^h}\Delta x_o^h$. [where the first term is the gross social marginal benefit from the project and the second term the corresponding gross social marginal loss from displacement of other activities].[1] Under individual consumer optimization the marginal utility terms can be replaced by the products of the consumer prices faced q_i and the individual marginal utilities of income μ_h so the net benefit (or shadow price) becomes $\sum_1^H \frac{\partial SWF}{\partial u_h}\mu_h q_p \Delta x_p^h - \sum_1^H \frac{\partial SWF}{\partial u_h}\mu_h q_o \Delta x_o^h$ or $q_p \sum_1^H \frac{\partial SWF}{\partial u_h}\mu_h \Delta x_p^h - q_o \sum_1^H \frac{\partial SWF}{\partial u_h}\mu_h \Delta x_o^h$ It follows that the answer to the question is thus in general negative since in effect the shadow price is specific to individuals. But if Δx_p^h and Δx_o^h coincide for each consumer, i.e., the resource allocation to the project equals the (physical) opportunity cost of other activities foregone, then the net benefit becomes $(q_p\Delta x_p - q_o\Delta x_o)\sum_1^H \frac{\partial SWF}{\partial u_h}\mu_h$ and the shadow price can be inferred from market prices

   (ii) With identical preferences net benefit is $\sum_1^H \frac{\partial SWF}{\partial u}\frac{\partial u}{\partial x_p^h}\Delta x_p^h - \sum_1^H \frac{\partial SWF}{\partial u}\frac{\partial u}{\partial x_o^h}\Delta x_o^h$, but with different incomes the individual marginal utilities of income differ; thus equality of preferences without equality of incomes does not justify use of market prices as shadow prices.

2. Q. see question paper. This requires reference to adjusted or rescaled welfare weights, the rescaling being intended to result in overall welfare-neutrality. The Green Book suggests that *marginal* funds are to be equally divided across members. In this case there are two issues: the utility functions specified in the question are ordinal rather than cardinal; and costs are in money terms while benefits are in utility terms.

   The welfare weights are originally $w_j = \frac{a_j}{x}$ with $a_o = 10,\ a_m = 5,\ a_y = 1.$

   If the new welfare weights are to divide the funds equally, $\sum \bar{w}_j \frac{100}{3} = 100$ so

[1] Δx_p^h is the increment in the project, $\frac{\partial u_h}{\partial x_p^h}$ the impact that increment has on individual marginal utility, and $\frac{\partial SWF}{\partial u_h}$ the contribution that change in individual marginal utility makes to social welfare, and likewise for the other term.

that the adjusted weights would be $\bar{w}_j = 3$. The rescaling factor is thus given by $f = \frac{3}{\sum w_j} = \frac{3}{\frac{10}{10}+\frac{5}{10}+\frac{1}{10}} = \frac{15}{8}$ and rescaled welfare weights would be
$\bar{w}_j = f w_j = \frac{15}{8}, \frac{15}{16}, \frac{15}{80}$
If the three policies are then evaluated with these weights:

A	Everything to the old	$\frac{15}{8}100 - 100 = 87.5$
B	Everything to the young	$\frac{15}{16}100 - 100 = -\frac{100}{16}$
C	Split between old and young	$\frac{15}{8}50 + \frac{15}{16}50 - 100 = 40.6$

and it looks as if option A is best, but A and C are also both better than the Green Book benchmark.
However 100 is very large in relation to initial consumption of 10 – it is hardly a *marginal* change – and so diminishing marginal utility needs to be taken into account. This being so, it not clear that the Green Book procedure makes sense in this case: for instance if the entire funds go to the old the utility gain appears to be $\Delta u^o = \frac{\partial u}{\partial x^o} = \frac{10}{10}100 = 100$ whereas instead, by specifying logarithmic utility function (and hence diminishing marginal utility) the overall change in utility turns out to be $10\ln(10+100) - 10\ln 10 \approx 24$.
A practical way around this difficulty is to identify rescaling weights that divide funds equally but make the non-marginal utility changes break-even. This sort of rescaling approach solves $f \sum \Delta u^i(\text{equal division}) - 100 = 0$
i.e., $f = \frac{100}{\left[10\ln\left(10+\frac{100}{3}\right) - 10\ln 10 + 5\ln\left(10+\frac{100}{3}\right) - 5\ln 10 + \ln\left(10+\frac{100}{3}\right)\right]} \approx 4.3$
Re-evaluating the policies with this modification to the weights:

A	All to old	$4.3 * 10[\ln(10+100) - \ln 10] - 100 \approx 2.15$
B	All to young	$4.3 * 5[\ln(10+100) - \ln 10] - 100 \approx -68.7$
C	Split old/young	

$$4.3 * (10[\ln(10+50) - \ln 10] + 5[\ln(10+50) - \ln 10]) - 100 \approx 14.5$$

So that now, taking into account diminishing marginal utility, option C looks best. For constant marginal utility, option A looks best, but the more accrues to the old, the more diminishing marginal utility causes the further marginal utility of further increments to diminish. In practice, for large programmes, sharing between the (two) groups that benefit significantly trumps giving it all to the group that benefits most, which, if you think about it, makes sense.

3. Q [see paper for detail]

a) A government agency imports r units of raw material for which there is an undistorted world price p^* and a national tariff τ. Without looking into the distribution of the tariff revenue, the associated social welfare can be written $W(p^*, r)$. The social valuation of the raw material (its shadow price) should be the marginal change in social welfare given a marginal change in material supply or $\partial W/\partial r$, and that must be the world price p^*. But this would be qualified if the world price were itself distorted, e.g., as a result of a non-internalized externality, for instance carbon-intensity. Note that this answer is fully congruent with the Diamond-Mirrlees principle.

b) Now a quadratic domestic social damage cost associated with r units of material is $d = r + \frac{r^2}{2}$. The social opportunity cost of an input is the direct cost of (or expenditure upon) the input, plus the difference between the increased social cost and the increased tariff revenue. So in this case $SOC = r(p^* + \tau) + \left(r + \frac{r^2}{2}\right) - r\tau = rp^* + r + \frac{r^2}{2}$ and the appropriate shadow price is then $\frac{\partial SOC}{\partial r} = p^* + r + 1$.

This shadow price increases in the volume of imports, because the damage function is also increasing in that volume.

As regards the numéraire, it is necessary for marginal damage and market price to be expressed in the same units. Accordingly, to consider whether a tariff-inclusive price $p^* + \tau$ might ever be appropriate as shadow price, consider that the marginal damage is $\frac{\partial d}{\partial r} = 1 + r$, so that the damage-internalizing tariff is given by $\frac{\partial \tau}{\partial r} = 1 + r$, and tariff-revenue is given by $\int_0^r (1 + y)dy = r + \frac{r^2}{2} = d(r)$ so that tariff revenue thus turns out to exactly recoup damage cost. So, when the tariff is set to be damage-internalizing the appropriate shadow price is indeed the tariff-inclusive world price.

In general the relative world price is not the appropriate shadow price under two circumstances: i) Either if the good or activity is non-tradable or the economy in question has monopoly or monopsony power in trade but does not apply an optimal tariff. ii) There are externalities to the good or activity that have not been internalized in the setting of the relative world price.