# International Macroeconomics
# Supervision 2

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## Section A

### A.1

The flexible price monetary model is a long-run model, where prices are determined by money market equilibrium:

$$\frac{M}{P} = L(i, Y) \implies P = M/L(i, Y)$$

which means that price levels endogenously adjust to equate money supply with money demand. Similarly, nominal exchange rates are determined by purchasing power parity:

$$e\frac{P^*}{P} = \bar{\varepsilon} \implies e = \bar{\varepsilon}\frac{P}{P^*}$$

where nominal exchange rates endogenously adjust to satisfy purchasing power parity. Therefore,

$$e = \bar{\varepsilon}\frac{M/L(i, Y)}{M^*/L^*(i^*, Y^*)}$$

Knowing this, we can tell the statement is false. A decrease in the growth rate of domestic money supply lowers expected inflation, which means $i$ does fall, but this *increases* money demand, and domestic prices must fall, not rise, to satisfy money market equilibrium. And we know domestic interest rates must appreciate, not depreciate, to maintain purchasing power parity.

### A.2

The value of production from one unit of labour must be equal across sectors for internal labour market equilibrium:

$$P_T A_T = P_N A_N \implies P_N = P_T(A_T/A_N)$$

Then, the real exchange rate, defined as the relative price of Foreign vs. Home goods, is

$$\varepsilon = e\frac{{P_T^*}^{\gamma}{P_N^*}^{1-\gamma}}{P_T^{\gamma}P_N^{1-\gamma}} = \frac{eP_T^*(A_T^*/A_N^*)^{1-\gamma}}{P_T(A_T/A_N)^{1-\gamma}} = \left(\frac{A_T^*/A_N^*}{A_T/A_N}\right)^{1-\gamma}$$

where $P_T$ and $eP_T^*$ cancel out due to the law of one price for tradables. In logarithms, this becomes

$$\boldsymbol{\varepsilon} = (1-\gamma)[(\mathbf{A}_T^* - \mathbf{A}_T) - (\mathbf{A}_N^* - \mathbf{A}_N)]$$

The growth rate of the real exchange rate is approximately equal to

$$\frac{\mathrm{d}\boldsymbol{\varepsilon}}{\mathrm{d}t} = (1-\gamma)\left[\frac{\mathrm{d}}{\mathrm{d}t}(\mathbf{A}_T^* - \mathbf{A}_T) - \frac{\mathrm{d}}{\mathrm{d}t}(\mathbf{A}_N^* - \mathbf{A}_N)\right]$$

We can see that if $\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{A}_N^*$ and $\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{A}_N$ both increase by the same *level* ($x$ percentage points), then there is no effect on the growth rate of $\varepsilon$. If $\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{A}_N^*$ and $\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{A}_N$ increase by the same *proportion* ($x\%$ of their original levels), then the growth rate of $\varepsilon$ will decrease (increase) by $(1-\gamma)\frac{\mathrm{d}}{\mathrm{d}t}(\mathbf{A}_N^* - \mathbf{A}_N) \times \frac{x}{100}$ if the growth rate of $\mathbf{A}_N^*$ was greater (less) than the growth rate of $\mathbf{A}_N$ to begin with.

### A.3

The nominal exchange rate can only stay fixed following an unanticipated increase in the domestic money supply if domestic and foreign assets are perfect complements with rates of return determined in completely separate markets. Then the domestic exchange rate is entirely independent of the differential between foreign and domestic interest rates, although it will be indeterminate (within the DD-AA or IS-LM model) unless it is exogenously fixed at some level.

## Section B

### B.1

#### (a)

The expected rates of return on Home and Foreign assets are $\mathrm{E}[H] = \mathrm{E}[F] = \frac{5}{3}$. When $q > \frac{1}{3}$, $\mathrm{E}[H] > E[F]$, and vice versa.

#### (b)

We have $W = 1, H_1 = 3, H_2 = 1, F_1 = 1, F_2 = 2$, and $q = \frac{1}{3}$. Therefore, the Home investor maximizes the following with respect to $\alpha$:

$$U = -\frac{1}{3}e^{-2\alpha-1} - \frac{2}{3}e^{\alpha-2}$$

And the first-order condition implies

$$\left.\frac{\partial U}{\partial \alpha}\right|_{\alpha=\alpha^*} = \frac{2}{3}e^{-2\alpha^*-1} - \frac{2}{3}e^{\alpha-2} = 0 \implies -2\alpha^* - 1 = \alpha - 2 \implies \alpha^* = \frac{1}{3}$$

It is desirable for the Home investor to engage in international portfolio diversification since he is risk-averse, and would benefit from a portfolio with the same expected return but lower variance (deriving the variance of $C$ conditional on $\alpha$ and minimizing with respect to $\alpha$ yields the same answer).

#### (c)

As mentioned, when $q > \frac{1}{3}$, the expected rate of return on Home assets exceeds that on Foreign assets. It should now be optimal to allocate more of the portfolio to Home assets, especially since the agent exhibits constant absolute risk aversion.

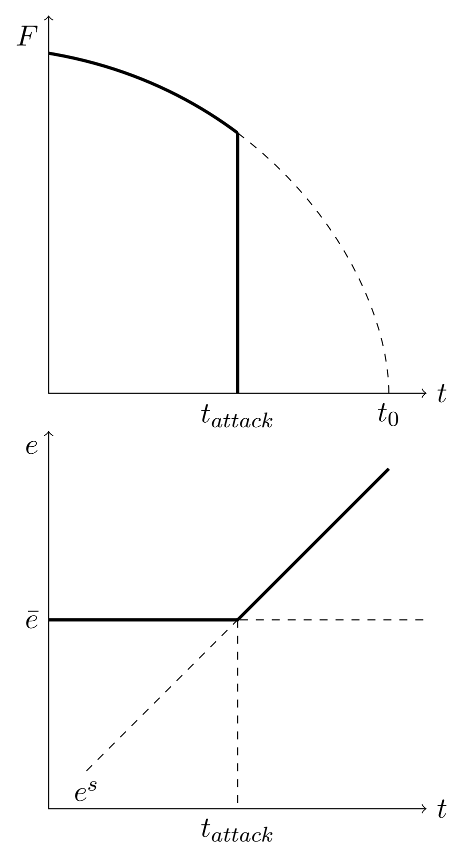
# Section C

## C.1

A speculative attack on foreign reserves can be rational even if it takes place long before reserves run out. If a central bank is financing government debt, it will build up its stock of domestic assets. However, to maintain a fixed exchange rate, the central bank must offset this buildup of domestic assets with a drawdown on foreign assets in order to keep constant the value of liabilities on the central bank balance sheet (keeping the money base and money supply constant). The central bank can do this until it runs out of foreign reserves, after which the exchange rate must be allowed to float if the central bank is to continue accumulating domestic assets.

If the central bank did not try to fix the exchange rate, and instead allowed domestic assets to build up without an offsetting drawdown on foreign assets, we know that this leads to a positive growth rate of the money supply, which requires a positive growth rate of domestic prices (or positive inflation) to equate real money supply with money demand, which requires a more depreciated exchange rate to maintain purchasing power parity (the same story as in A.1, which assumes that output is fixed throughout). So we can conceive of a 'shadow' exchange rate $e^s$ which is the exchange rate which would prevail under a floating currency, and $e^s$ must be depreciating over time as the central bank finances government debt.

From this stylistic view, we can work out why it would be rational for a speculative attack to occur long before foreign assets would be depleted from just the offsetting of domestic asset accumulation:




In the top graph, the dashed line represents the stock of foreign assets if they were to decline at the rate of accumulation of domestic assets. Left to their own, the foreign assets would only deplete at $t = t_0$. However, the bold line drawn shows a speculative attack occuring at $t_{attack}$, where the continuously depreciating shadow exchange rate $e^s$ crosses $\bar{e}$ which the central bank is trying to peg the exchange rate at. This is the only timing of the speculative attack which precludes any untapped arbitrage opportunities.

If the attack occurred at $t < t_{attack}$, the exchange rate would jump discontinuously from $\bar{e}$ to $e^s < \bar{e}$, leading to a sudden appreciation. If so, every informed investor will have an opportunity to *sell* foreign assets to the central bank before the attack; they can simply be bought back at a cheaper price after the attack occurs. Buying up the central bank's foreign reserves at $t < t_{attack}$ is uneconomical given the predictable capital loss that will occur thereafter.

If the attack occurred at $t > t_{attack}$, the exchange rate would jump discontinuously from $\bar{e}$ to $e^s > \bar{e}$, leading to a sudden depreciation. If so, every informed investor will have an incentive to buy foreign assets from the central bank at a lower price just before the attack occurred. Buying up the central bank's foreign reserves at $t > t_{attack}$ is uneconomical given the untapped arbitrage opportunity from buying just before $t_{attack}$.

Therefore, it is fully rational for the speculative attack to occur long before $t_0$, where foreign reserves deplete through the predictable drawdown that must occur when the central bank finances government debt while trying to maintain an exchange rate peg.