Economic Growth
Supervision 1

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Question 1

Some of the relevant articles on the empirics of cross-country convergence are Mankiw, Romer, and Weil (1992) and Lucas (1990).

(a)

In the first article, Mankiw, Romer, and Weil test the Solow model with a cross-country regression based on the empirical specification

$$\ln\left(\frac{Y}{L}\right)) = a + \frac{\alpha}{1-\alpha}\ln(s) - \frac{\alpha}{1-\alpha}\ln(n+g+\delta) + \epsilon$$

where the estimated coefficients on $\ln(s)$ and $\ln(n+g+\delta)$ yield implied values of α.

Qualitatively, their results accord with the predictions of the Solow model. The coefficients on $\ln(s)$ and $\ln(n+g+\delta)$ have the predicted signs (positive and negative) and are statistically significant for 2 out of their 3 samples of countries. The hypothesis that the coefficients on $\ln(s)$ and $\ln(n+g+\delta)$ are equal and opposite in sign is not rejected in any sample. However, the values of α implied by the regressions are almost certainly too high: about 0.60 for all the samples except the last which only has OECD countries with a population greater than 1 million.

(b)

Lucas (1990) assumes the production technology is common between the US and India, meaning differences in output per worker is entirely due to differences in capital per worker. Under a Cobb-Douglas Solow-type model, he calculates the implied marginal product of capital in India as 58 times of that in the US, which raises the "Lucas paradox": why are the returns to capital not equalized?

The ratio is revised from 58 to 5 after his accounting for human capital, which is a substantial revision but "leaves the original paradox very much alive". The results suggest that either the standard Solow model explains little of the quantitative differences in income per worker we observe, or there are obstacles to capital flows that impede the equalisation of returns across countries.

Question 2

(a)

All firms have their production augmented by the productivity parameter $A^i(t) = Y(t)^\phi$. Firm i's productivity is increasing in aggregate output, and this could represent knowledge spillovers or agglomeration economies.

(b)

Aggregate output is

$$\begin{aligned}
Y(t) &= \sum_i Y^i(t) \\
&= \sum_i A^i(t) K^i(t)^\alpha L^i(t)^{1-\alpha} \\
&= \sum_i Y(t)^\phi K^i(t)^\alpha L^i(t)^{1-\alpha} \\
&= Y(t)^\phi \sum_i K^i(t)^\alpha L^i(t)^{1-\alpha} \\
Y(t)^{1-\phi} &= \sum_i K^i(t)^\alpha L^i(t)^{1-\alpha} \\
Y(t) &= \left[\sum_i K^i(t)^\alpha L^i(t)^{1-\alpha}\right]^{\frac{1}{1-\phi}}
\end{aligned}$$

In a closed economy, $I(t) = sY(t)$, so the aggregate capital stock evolves according to the following equation of motion:

$$\begin{aligned}
\dot{K}(t) &= I(t) - \delta K(t) \\
&= sY(t) - \delta K(t) \\
&= s\left[\sum_i K^i(t)^\alpha L^i(t)^{1-\alpha}\right]^{\frac{1}{1-\phi}} - \delta K(t)
\end{aligned}$$

On a balanced growth path, it must be that $\frac{\dot{K}(t)}{K(t)} = g_K$ where g_K is constant. In other words,

$$\frac{\dot{K}(t)}{K(t)} = \frac{s}{K(t)}\left[\sum_i K^i(t)^\alpha L^i(t)^{1-\alpha}\right]^{\frac{1}{1-\phi}} - \delta = g_K$$

which implies that

$$\sum_i \frac{K^i(t)^\alpha L^i(t)^{1-\alpha}}{K(t)^{1-\phi}} = \left(\frac{g_K + \delta}{s}\right)^{1-\phi}$$

and since firms are identical, with N firms, we have

$$\frac{K^i(t)^\alpha L^i(t)^{1-\alpha}}{K(t)^{1-\phi}} = \frac{1}{N}\left(\frac{g_K+\delta}{s}\right)^{1-\phi}$$

The right-hand side is a constant, so taking logarithms and differentiating yields

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\left[\alpha\log K^i(t) + (1-\alpha)\log L^i(t) - (1-\phi)\log K(t)\right] &= 0\\ \alpha g_{K^i} + (1-\alpha)g_{L^i} &= (1-\phi)g_K\end{aligned}$$

which yields a relationship between the factor share-weighted growth rates of firm capital and firm labour and the growth rate of aggregate capital when the economy is on the balanced growth path.

To find the long-run growth rate of output per worker, we first return to the aggregate output equation:

$$Y(t) = \left[\sum_i K^i(t)^\alpha L^i(t)^{1-\alpha}\right]^{\frac{1}{1-\phi}}$$

Noting again that firms are identical, and going through similar steps as before, we get

$$\begin{aligned}Y(t) &= \left[NK^i(t)^\alpha L^i(t)^{1-\alpha}\right]^{\frac{1}{1-\phi}}\\ \log Y(t) &= \frac{1}{1-\phi}\log N + \frac{\alpha}{1-\phi}\log K^i(t) + \frac{1-\alpha}{1-\phi}\log L^i(t)\\ g_Y &= \frac{1}{1-\phi}\left[\alpha g_{K^i} + (1-\alpha)g_{L^i}\right]\\ &= g_K\end{aligned}$$

which means that on the balanced growth path, aggregate output grows at the same rate as aggregate capital.

We are told that the aggregate labour force grows at rate n, so it is a simple matter to note that output per worker y grows at $g_y = g_K - n$. Furthermore, with identical firms, we can infer that $g_{K^i} = g_K$ and $g_{L^i} = n$. This means that

$$\alpha g_{K^i} + (1-\alpha)g_{L^i} = (1-\phi)g_K \implies g_K = \frac{1-\alpha}{1-\phi-\alpha}n$$

which means we have $g_y = \frac{\phi}{1-\phi-\alpha}n > 0$, as needed.

(c)

We found that on the balanced growth path, $\frac{K^i(t)^\alpha L^i(t)^{1-\alpha}}{K(t)^{1-\phi}}$ is equal to a constant. Again, with identical firms, we have $K^i(t) = \frac{K(t)}{N}$ and $L^i(t) = \frac{L(t)}{N}$. Therefore,

$$\frac{K^i(t)^\alpha L^i(t)^{1-\alpha}}{K(t)^{1-\phi}} = \frac{K(t)^\alpha L(t)^{1-\alpha}}{NK(t)^{1-\phi}}$$

Taking the reciprocal implies $\tilde{K}(t) = K(t)^{1-\phi-\alpha}L(t)^{-(1-\alpha)}$ is constant on the balanced growth path. We have

$$\begin{aligned}
\dot{\tilde{K}}(t) &= (1-\phi-\alpha)K(t)^{-\phi-\alpha}\dot{K}(t)L(t)^{-(1-\alpha)} - (1-\alpha)K(t)^{1-\phi-\alpha}L(t)^{-(1-\alpha)-1}\dot{L}(t) \\
&= (1-\phi-\alpha)K(t)^{-\phi-\alpha}\dot{K}(t)L(t)^{-(1-\alpha)} - (1-\alpha)K(t)^{1-\phi-\alpha}L(t)^{-(1-\alpha)}\frac{\dot{L}(t)}{L(t)} \\
&= (1-\phi-\alpha)K(t)^{-\phi-\alpha}\dot{K}(t)L(t)^{-(1-\alpha)} - (1-\alpha)\tilde{K}(t)n
\end{aligned}$$

Before proceeding further, we return to the law of motion for capital:

$$\begin{aligned}
\dot{K}(t) &= sY(t) - \delta K(t) \\
&= s\left[\sum_i K^i(t)^\alpha L^i(t)^{1-\alpha}\right]^{\frac{1}{1-\phi}} - \delta K(t) \\
&= s\left[N\left(\frac{K(t)}{N}\right)^\alpha\left(\frac{L(t)}{N}\right)^{1-\alpha}\right]^{\frac{1}{1-\phi}} - \delta K(t) \\
&= s\left[K(t)^\alpha L(t)^{1-\alpha}\right]^{\frac{1}{1-\phi}} - \delta K(t)
\end{aligned}$$

Substituting this back into $\dot{\tilde{K}}(t)$, we get

$$\begin{aligned}
\dot{\tilde{K}}(t) &= (1-\phi-\alpha)K(t)^{-\phi-\alpha}L(t)^{-(1-\alpha)}\left\{s\left[K(t)^\alpha L(t)^{1-\alpha}\right]^{\frac{1}{1-\phi}} - \delta K(t)\right\} - (1-\alpha)\tilde{K}(t)n \\
&= s(1-\phi-\alpha)\tilde{K}(t)K(t)^{-1}\left[K(t)^\alpha L(t)^{1-\alpha}\right]^{\frac{1}{1-\phi}} - \delta(1-\phi-\alpha)\tilde{K}(t) - (1-\alpha)\tilde{K}(t)n \\
&= s(1-\phi-\alpha)\tilde{K}(t)\left[K(t)^{-(1-\phi-\alpha)}L(t)^{1-\alpha}\right]^{\frac{1}{1-\phi}} - \delta(1-\phi-\alpha)\tilde{K}(t) - (1-\alpha)\tilde{K}(t)n \\
g_{\tilde{K}} &= s(1-\phi-\alpha)\tilde{K}(t)^{-\frac{1}{1-\phi}} - (1-\phi-\alpha)\delta - (1-\alpha)n
\end{aligned}$$

It is trivial to see that $\lim_{\tilde{K}\to 0} g_{\tilde{K}} = \infty$ and $\lim_{\tilde{K}\to\infty} g_{\tilde{K}} = -(1-\phi-\alpha)\delta - (1-\alpha)n < 0$. Therefore, by the Intermediate Value Theorem, there is some positive and finite $\tilde{K}$ such that $g_{\tilde{K}} = 0$. Furthermore, we have $\dfrac{\partial g_{\tilde{K}}}{\partial \tilde{K}} < 0$ for all $\tilde{K}$, which means $g_{\tilde{K}}$ globally converges to its balanced growth path-value (0), and the value of $\tilde{K}$ which delivers this is unique.

Therefore, the economy converges to its balanced growth path equilibrium.

(d)

Using what we have from before, we have

$$\begin{aligned}
\log y(t) &= \frac{\alpha}{1-\phi}\log K(t) + \frac{1-\alpha}{1-\phi}\log L(t) - \log L(t) \\
g_y(t) &= \frac{\alpha}{1-\phi}g_K(t) + \frac{1-\alpha}{1-\phi}n - n = \frac{\alpha}{1-\phi}g_K(t) - \frac{\alpha-\phi}{1-\phi}n
\end{aligned}$$

where growth rates are now a function of time and not a constant as assumed when on the balanced growth path.

Immediately after the decrease in n, the second term becomes less negative while the first term may become more or less positive. Whatever happens, the new long-run value of $g_y = \frac{\phi}{1-\phi-\alpha}n$ is lower than before. Therefore, the immediate impact of the decrease in n is that the growth rate of output per worker either jumps up or drops down (it may or may not overshoot the new equilibrium), before dampening over time and asymptotically reaching the new lower equilibrium value. On a log-linearised graph of $y(t)$, this will show up as a gradual dampening of the slope of $y(t)$ until it looks as though the curve is straight with a slope of $\frac{\phi}{1-\phi-\alpha}n'$.

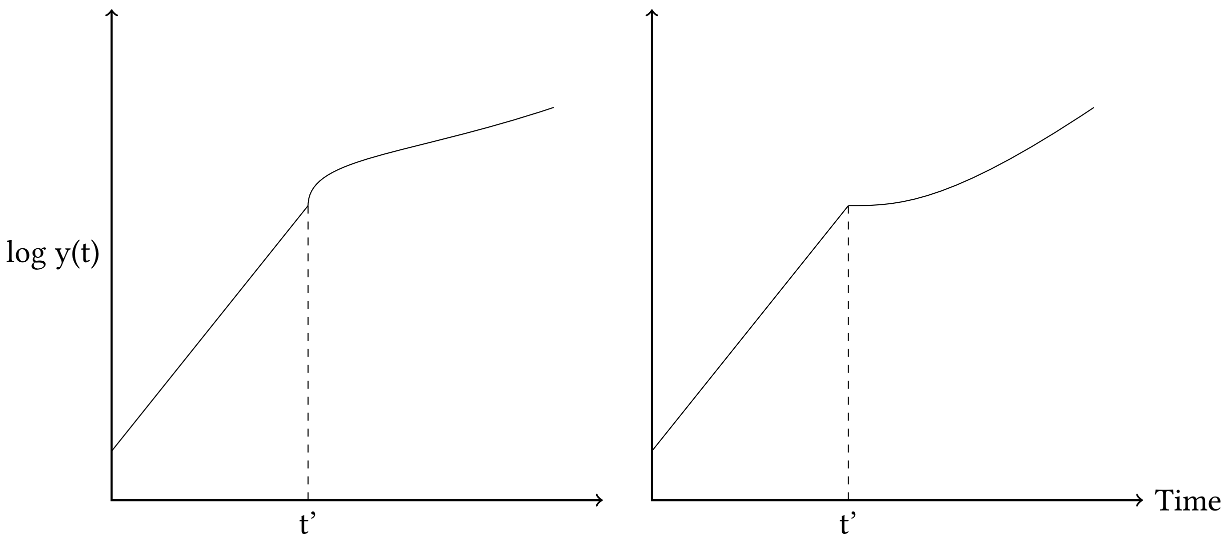


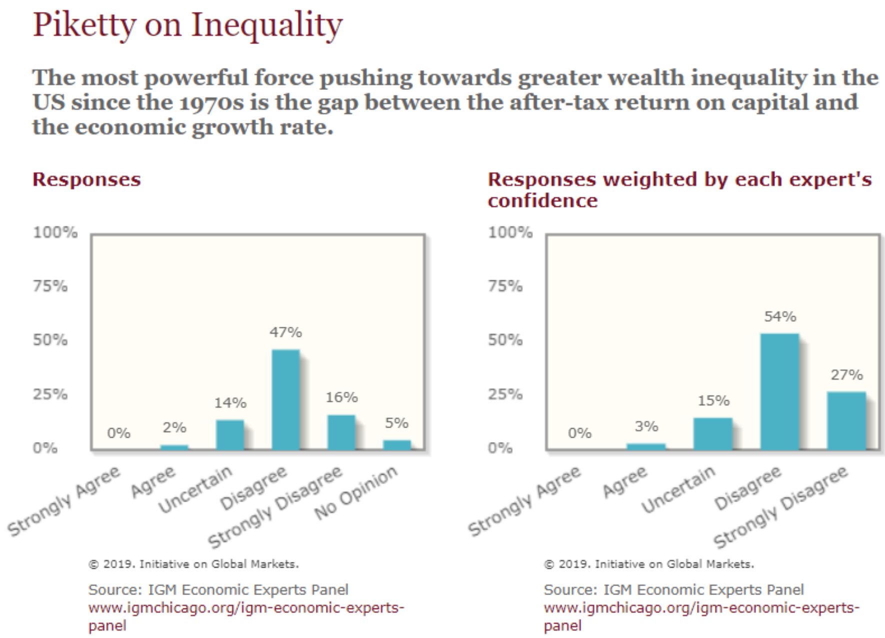
Figure 1: Evolution of per-capita output: two possibilities

Question 3

For what it's worth, the IGM Economic Experts Panel were asked to express an opinion on the statement:

> "The most powerful force pushing towards greater wealth inequality in the US since the 1970s is the gap between the after-tax return on capital and the economic growth rate."

Out of all the economists asked, 2% agreed, 19% were uncertain or had no opinion, and the rest either disagreed or strongly disagreed (nobody strongly agreed with the statement).



Many disagreed on empirical grounds. William Nordhaus quips,

> "Is this an inside joke? BEA estimates show little change in rate of return"

Some who disagreed remained open to the idea that $r - g$ may become an important driver in the future. Angus Deaton was one:

> "Maybe in the future, but right now it is high incomes that is increasing wealth inequality"

And Emmanuel Saez was a bit more sure:

> "Income and savings inequality increases are now fueling US wealth inequality. Down the road $r - g$ will be central as predicted by Piketty"

In a more academic setting, Krusell and Smith (2014) point out that the Solow-type theory driving Piketty's predictions differed from the "textbook" Solow model in that net savings, rather than gross savings, were the exogenous constant in his model. They note that when technological growth slows to 0, in order to maintain net savings at a constant rate (and therefore maintain an increase in the capital stock), gross savings eventually have to rise to 100% of gross income. They then suggest that standard theories of intertemporal utility maximisation are more in accord with the "textbook" Solow model than Piketty's formulation, and adduce some historical evidence that they say are inconsistent with a constant net savings rate.

Acemoglu and Robinson (2014) made similarly critical points on Piketty's central thesis (Acemoglu was also one of the respondents in the IGM panel). They note that $r > g$ is consistent with constant or even declining inequality; one needs additional assumptions on how much the owners of capital re-invest their returns (in a later clarification Piketty agreed as much). It would appear that Piketty's book drew a fair bit of scholarly criticism, not so much because the economists involved believed that inequality wasn't rising or that it was unimportant (one surely can't accuse the likes of Deaton or Saez of caring too little). They disagreed more with his characterisation of the mechanisms involved, and the prescriptions and predictions they imply.