FACULTY OF ECONOMICS

UNIVERSITY OF CAMBRIDGE

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Theory and Practice of Econometrics: II

Supervision Sheet 7

Question 1

The utility that individual i receives from choice j is given by

$$U_{ij} = \alpha_j + \mathbf{v}_{ij}\boldsymbol{\omega} + \varepsilon_{ij},$$

where $\mathbf{v}_{ij}$ is a $L \times 1$ vector, and $\boldsymbol{\omega}$, α_j denote unknown parameters. ε_{ij} is an error term. The choice set Ω_J contains J unordered alternatives.

a. Find a general expression for the probability that alternative j is chosen by decision-maker i, and thereby demonstrate that only the signs of the differences in utility are relevant.

b. In the case of two choices construct the log-likelihood function assuming that $\varepsilon_{ij} \sim N(0, \sigma^2)$.

c. Demonstrate that $\boldsymbol{\omega}$ and σ^2 are not separately identified.

d. How would you interpret α_j? In a two choice problem is it possible to identify α_1 and α_2? Explain your answer.

e. In some cases we may not observe choice attributes but only individual characteristics as in the following utility specification

$$U_{ij} = \alpha_j + \mathbf{x}_i\boldsymbol{\beta}_j + \varepsilon_{ij}$$

where ε_{ij} is an i.i.d error term. For a choice set comprised of 5 alternatives, which parameters are identified?

Question 2

The utility that decision maker i receives from alternative j is written as

$$\begin{aligned} U_{ij} &= V_{ij} + \varepsilon_{ij} && (1) \\ &= \mathbf{v}'_{ij}\boldsymbol{\omega} + \varepsilon_{ij} && (2) \end{aligned}$$

for $i = 1, ..., n$ and $j = 1, ..., J$. $\mathbf{v}_j$ is a $L \times 1$ vector of attributes and $\boldsymbol{\omega}$ is a $L \times 1$ vector of unknown parameters. ε_{ij} is an error term distributed type 1 extreme value. $\text{Var}(\varepsilon_{ij}) = \sigma^2 \times (\pi^2/6)$. Ω_J denotes the choice set.

i) Demonstrate that in (2) the ratio $\boldsymbol{\omega}^* = \boldsymbol{\omega}/\sigma$ is identifiable and not separate estimates of $\boldsymbol{\omega}$ and σ.

In view of this property, in what sense is estimation in willingness-to-pay space attractive?

ii)

a. Let $y_i = j$ if individual i chooses the j^{th} alternative. Write down an expression for $\Pr(y_i = j|\mathbf{v})$ and in doing so show that one can estimate the parameters $\boldsymbol{\omega}$ consistently on a subsample of observations.

What is the significance of this property?

b. Suggest a specification test based upon an efficient maximum likelihood estimator (MLE) using the full sample and an inefficient MLE based upon a subsample where choice is restricted to a subset of Ω_J.

iii) Instead of observing a single choice for each individual, the analyst now observe each individual over T choice occasions.

$$U_{ijt} = V_{ijt} + \varepsilon_{ijt} \quad (3)$$

$$= -\alpha_i p_{ijt} + \boldsymbol{\beta}_i' \mathbf{v}_{ijt} + e_{ijt} \quad (4)$$

where $t = 1, ..., T$ indexes choice occasions and $\mathbf{v}_{jt}$ is $L \times 1$. In (4) we differentiate between price, p_{ijt} and a vector ($\mathbf{v}_{ijt}$) of all other attributes.

a. Assuming a stated preference setting, provide a motivation why one might allow the variance of the errors to differ across individuals, such that we may write $\text{Var}(\varepsilon_{ij}) = \sigma_i^2 \times (\pi^2/6)$.

b. In what sense does the model in (4) circumvent the IIA property that characterises (2).

c. By writing utility in willingness-to-pay (wtp) space, demonstrate that variation in wtp is independent of scale, and is distinguished from the variation in the price coefficient, which incorporates scale.

Note: only attempt part (c) if covered in lectures.

iv) Table 1 provides estimates, presented in preference space, for residential choice of electricity supplier. In a stated choice experiment, each individual was presented with four alternatives, differentiated by price and a number of attributes, including contract length, and whether the supplier was their local utility company. Each individual was presented with 8-12 hypothetical choice situations.

All of the nonprice coefficients are specified to be normally distributed. The price coefficient is fixed across the population.

Using the estimates in Table 2, comment on the estimates of willingness-to-pay for the two non-price attributes.

Table 1: Energy Supplier Choice: Parameter Estimates

	Model 1
Price, kWH	−0.8547
	(0.0488)
Contract length, years	
mean	−0.1833
	(0.0289
std. deviation	1.5585
	(0.1264)
Known company	
mean	1.5247
	(0.1018)
std. deviation	0.9520
	(0.0998)
Log likelihood at convergence	−3646.51

Standard errors in parentheses.

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