Topics in Economic Policy
Supervision 4

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Question 1

In the Alesina–Tabellini model, the government budget deficit bias doesn't arise because of some inherent preference for output above the natural rate. This is in contrast to the Barro-Gordon model, in which an inflationary bias arises due to the government/monetary authority's loss function which features a bliss point where output is above the natural level.

Instead, in the Alesina–Tabellini model, political parties simply have different preferences on the allocation of revenues to different public goods. Because of the uncertainty of re-election, incumbent parties strategically over-spend on their preferred public goods so as to constrain the opposing party in the case that they are not re-elected. Therefore the statement is false.

Question 2

The current UK fiscal policy objective is to return public finances to balance at the earliest possible date. One of the fiscal targets include the fiscal mandate, which is to bring the structural deficit down to below 2% of GDP by 2020–21. There is also an aim to have the public sector net debt fall as a percentage of GDP in 2020–21, and for welfare spending to lie below an inflation-adjusted welfare cap for 2022-2023.

One drawback of the fiscal mandate could be that it does not make any provisions for public investment, given that the optimal deficit ratio is countercyclical net of government investment. The fiscal mandate puts the burden of public investment on current taxpayers, whereas the next generation might benefit disproportionately from the investment.

Question 3

(a)

The government minimises the deadweight loss of distortionary taxes:

$$L = \frac{1}{2}\tau_1^2 Y_1 + \delta \frac{1}{2}\tau_2^2 Y_2$$

where τ_t is the income tax rate in period t, Y_t is national income in period t, and δ is the government's discount factor. The intertemporal budget constraint is given by

$$G_1 + \frac{1}{1+r}G_2 \leq \tau_t Y_1 + \frac{1}{1+r}\tau_2 Y_2$$

The Lagrangian for this optimisation problem is

$$\mathscr{L} = \frac{1}{2}\tau_1^2 Y_1 + \delta\frac{1}{2}\tau_2^2 Y_2 + \lambda\left(G_1 + \frac{1}{1+r}G_2 - \tau_t Y_1 - \frac{1}{1+r}\tau_2 Y_2\right)$$

and the first-order conditions are

$$\begin{aligned}\frac{\partial \mathscr{L}}{\partial \tau_1} &= \tau_1 Y_1 - \lambda Y_1 = 0\\ \frac{\partial \mathscr{L}}{\partial \tau_2} &= \delta\tau_2 Y_2 - \frac{\lambda}{1+r}Y_2 = 0\end{aligned}$$

with $\lambda \geq 0$ and the complementary slackness condition $\lambda\left(G_1 + \frac{1}{1+r}G_2 - \tau_t Y_1 - \frac{1}{1+r}\tau_2 Y_2\right) = 0$. The first equation implies $\tau_1 = \lambda$ and the second equation implies $\tau_2 = \frac{1}{\delta}\frac{\lambda}{1+r} = \lambda$ since $\delta = \frac{1}{1+r}$. This means $\tau_1 = \tau_2$; in other words, there is perfect tax smoothing. This is because the loss function is convex in the tax rates. Furthermore, raising τ_1 by 1 unit has the same effect on the budget balance as raising τ_2 by $1 + r$, but this difference is perfectly offset by δ in the loss function. It is therefore optimal to 'spread' the burden of taxation across both tax bases equally.

(b)

It is safe to assume that the budget constraint is active (otherwise $\lambda = 0$ which implies there are no taxes). Therefore,

$$G_1 + \frac{1}{1+r}G_2 = \tau_1 Y_1 + \frac{1}{1+r}\tau_2 Y_2$$

Using $\tau_1 = \tau_2 = \tau$, we have

$$\tau = \frac{G_1 + \frac{1}{1+r}G_2}{Y_1 + \frac{1}{1+r}Y_2}$$

which means the optimal income tax rate is the ratio of the present value of government liabilities to the present value of income. This makes sense: given that optimal tax rates are equal in both periods, we tax exactly as much as we need to fulfil government liabilities (the tax rate multiplied by the present value of income is equal to the present value of government liabilities).

(c)

Because taxes are optimally set to be equal in both periods, the effects of either change on tax rates are the same for both periods.

i. Following a temporary increase ΔG_1 in government purchases G_1, taxes in both periods will go up by $\frac{\Delta G_1}{(1+r)Y_1+Y_2}$. This is exactly the increase needed to raise the additional amount of revenue in present value terms. Because national income is exogenous, the increase in G_1 doesn't affect Y_2. With a constant G_2 and Y_2 but a higher τ_2, the primary budget deficit in period 2 must shrink (or the surplus must grow) relative to before. In period 1, the increase in government purchases is met with an increase in revenues of $\Delta\tau_1 Y_1 = \frac{\Delta G_1}{(1+r)Y_1+Y_2}Y_1$. Therefore the change in the primary budget deficit is

$$\Delta(G_1 - \tau_1 Y_1) = \Delta G_1 - \frac{\Delta G_1}{(1+r)Y_1 + Y_2}Y_1 = \Delta G_1\left(1 - \frac{Y_1}{(1+r)Y_1 + Y_2}\right) = \Delta G_1\left(\frac{rY_1 + Y_2}{(1+r)Y_1 + Y_2}\right)$$

which means the primary budget deficit in period 1 grows (or the surplus shrinks) relative to before.

ii. With an anticipated future increase in Y_2, the tax rate in both periods will decrease by $-\Delta\tau = \frac{G_1+\frac{1}{1+r}G_2}{Y_1+\frac{1}{1+r}(Y_2+\Delta Y_2)} - \frac{G_1+\frac{1}{1+r}G_2}{Y_1+\frac{1}{1+r}Y_2}$. With a larger tax base, this is exactly the decrease in tax rates which brings the intertemporal budget surplus to 0. In period 1, G_1 and Y_1 are the same but τ_1 would have decreased, which means the primary budget deficit will grow (or the surplus will shrink) relative to before. Without going through the algebra, we know from this that the budget deficit will shrink (or the surplus will grow) in period 2, since at the optimum the budget constraint is met with equality. This means

$$\underbrace{G_1 - \tau_1 Y_1}_{\text{Period 1 deficit}} + \frac{1}{1+r}\underbrace{(G_2 - \tau_2 Y_2)}_{\text{Period 2 deficit}} = 0$$

which means if the deficit in period 1 grows by Δd_1, the deficit in period 2 must shrink by $(1+r)\Delta d_1$.

Question 4

(a)

We define the following: B_t is the stock of national debt at the start of period t, G_t is government expenditure in period t, and T_t is government tax revenue in period t. The primary deficit D_t is equal to $G_t - T_t$. The government's budget constraint is then

$$B_{t+1} = B_t + \underbrace{rB_t + G_t - T_t}_{\text{Budget deficit}}$$

Then, letting $\Delta B_t = B_{t+1} - B_t$ be the net debt issue in period t, we have $\Delta B_t = G_t - T_t + rB_t$. We also let $\Delta Y_t = Y_{t+1} - Y_t$ be absolute income growth over period t. Therefore,

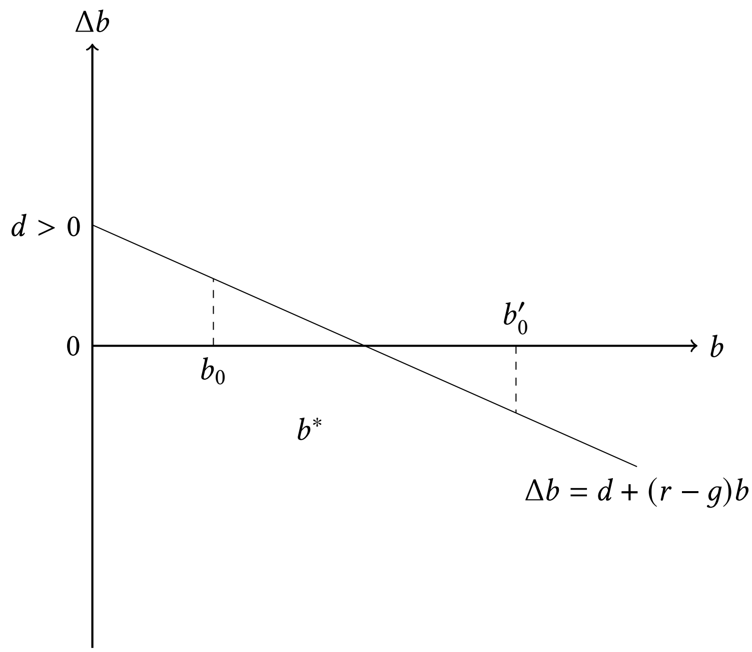
$$\underbrace{\frac{\Delta B_t}{B_t} - \frac{\Delta Y_t}{Y_t}}_{\approx\Delta\%b} = \frac{G_t - T_t + rB_t}{B_t} - g = \frac{G_t - T_t}{Y_t}\frac{Y_t}{B_t} + r - t = \frac{d}{b} + r - g$$

The left-hand side is approximately equal to the percentage change in b when the percentage changes in B_t and Y_t are small, so we can make the following approximation:

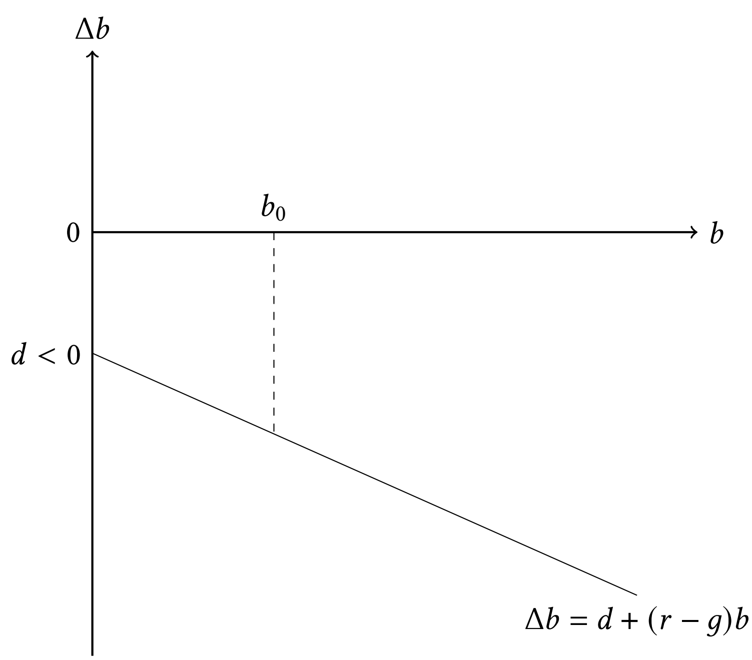
$$\Delta b = b\Delta\%b \approx d + (r-g)b$$

(b)

i. For $d > 0$, plotting the law of motion for national debt in $(b, \Delta b)$ space, we have



We can see that there are two possibiities: in one, the initial stock of debt is $b_0 < b^*$, in which case Δb is positive and b increases. This continues until $b_t = b^*$, in which case $\Delta b = 0$ and a steady state is reached. The other case is that the initial stock of debt is $b'_0 \geq b^*$, in which case the stock of debt declines until reaching the steady state. Therefore there is a globally stable steady state.
ii. If $d < 0$, we have



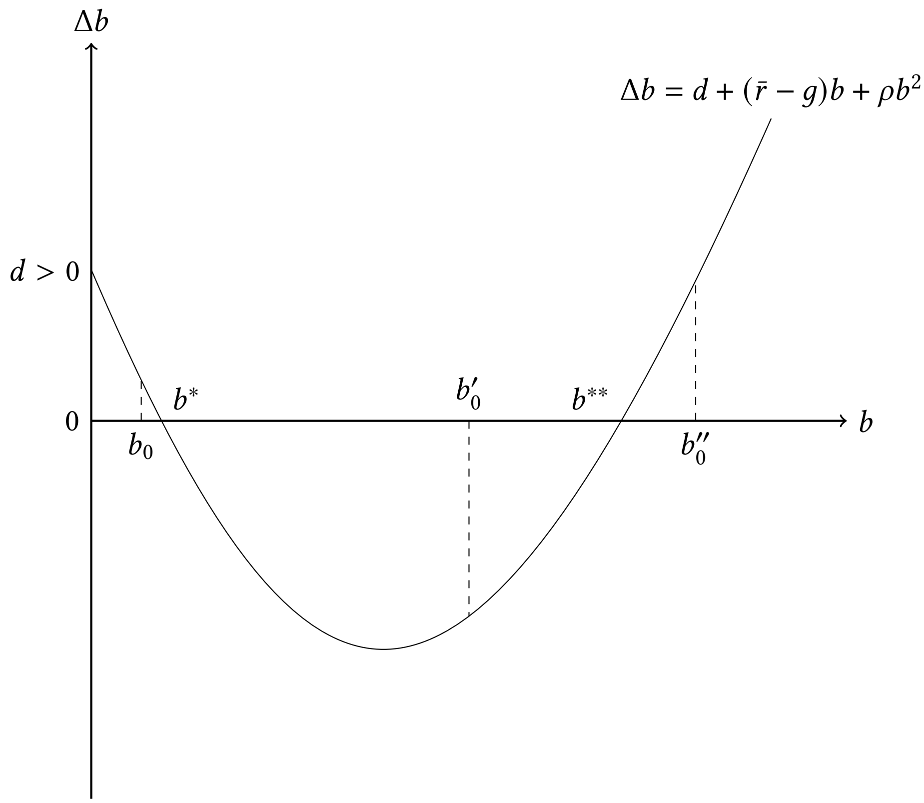
If the government persistently runs a primary surplus, it follows that the national debt will eventually go to zero since the growth of national income outpaces the growth of debt ($r < g$). This can be seen in the diagram; starting from any positive initial stock of debt b_0, the corresponding Δb is negative and b continues to dwindle until it reaches 0.

(c)

The law of motion for national debt is now

$$\Delta b = d + (r - g)b = d + (\bar{r} + \rho b - g)b = d + (\bar{r} - g)b + \rho b^2$$

which means it is now a parabola in the $(b, \Delta b)$ space: there is a point at which the risk premium attached to national debt wipes out the differential between $\bar{r}$ and g, after which additional debt causes existing debt to either shrink more slowly or grow more quickly. This is depicted below:



As before, if the initial stock of debt is b_0 or b_0', where $0 < b_0 < b^* \leq b_0' < b^{**}$, national debt will be on a locally stable trajectory towards b^*. If the initial stock of debt is exactly b^{**}, it will remain in an unstable steady state. If the initial stock of debt exceeds b^{**}, then the risk premium attached to national debt becomes exceedingly large and there is runaway growth of the national debt.

There can also be a case where d and/or ρ is sufficiently high, or $\bar{r}$ is sufficiently close to g, in which the turning point of the parabola above is above the b-axis; any positive amount of national debt is too much for investors to bear and the national debt explodes. The exact condition can be found by deriving the turning point:

$$
\begin{aligned}
d + (\bar{r} - g))b + \rho b^2 &= \rho \left[b^2 + \frac{\bar{r} - g}{\rho} b + \frac{d}{\rho} \right] \\
&= \rho \left[b^2 + \frac{\bar{r} - g}{\rho} b + \left(\frac{\bar{r} - g}{2\rho} \right)^2 - \left(\frac{\bar{r} - g}{2\rho} \right)^2 + \frac{d}{\rho} \right] \\
&= \rho \left[\left(b + \frac{\bar{r} - g}{2\rho} \right)^2 + \frac{4d\rho - (\bar{r} - g)^2}{4\rho^2} \right] \\
&= \rho \left(b + \frac{\bar{r} - g}{2\rho} \right)^2 + \underbrace{\frac{4d\rho - (\bar{r} - g)^2}{4\rho}}_{\text{Value of } \Delta b \text{ at turning point}}
\end{aligned}
$$

The above shows that any positive amount of initial national debt is unsustainable whenever $4d\rho - (\bar{r} - g)^2 > 0$.