Intertemporal Macroeconomics
Supervision 2

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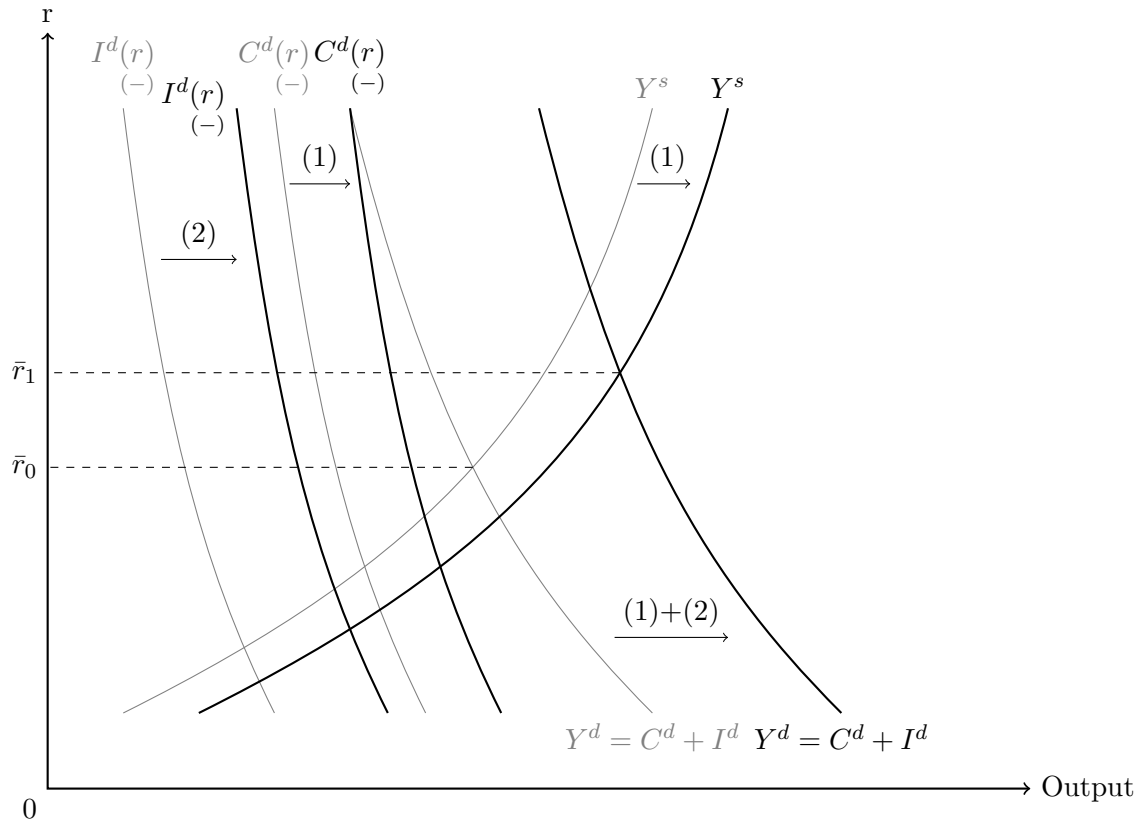
Question 1

Given the relationships between the interest rate and supply, consumption, and investment, the changes given can be analysed graphically for any functional form that adheres to these relationships. Supply depends positively on the real interest rate, while consumption and investment demands depend negatively. Additionally, the marginal propensity to consume out of a permanent change in income is 1, while a marginal propensity to consume out of a temporary change in income is less than 1. An anticipated increase in income leads to lower savings (or more borrowing if there are no savings) in the current period. These are results from the previous supervision.

(a)



A permanent productivity increase first shifts Y^s out by (1): at a given level of interest more can be profitably produced in period 1 (and also in all other periods). As mentioned before, this

leads to a shift in C^d also of magnitude (1) since the MPC for a permanent change in income is 1; the entire increase in income is consumed rather than saved for a given interest rate.

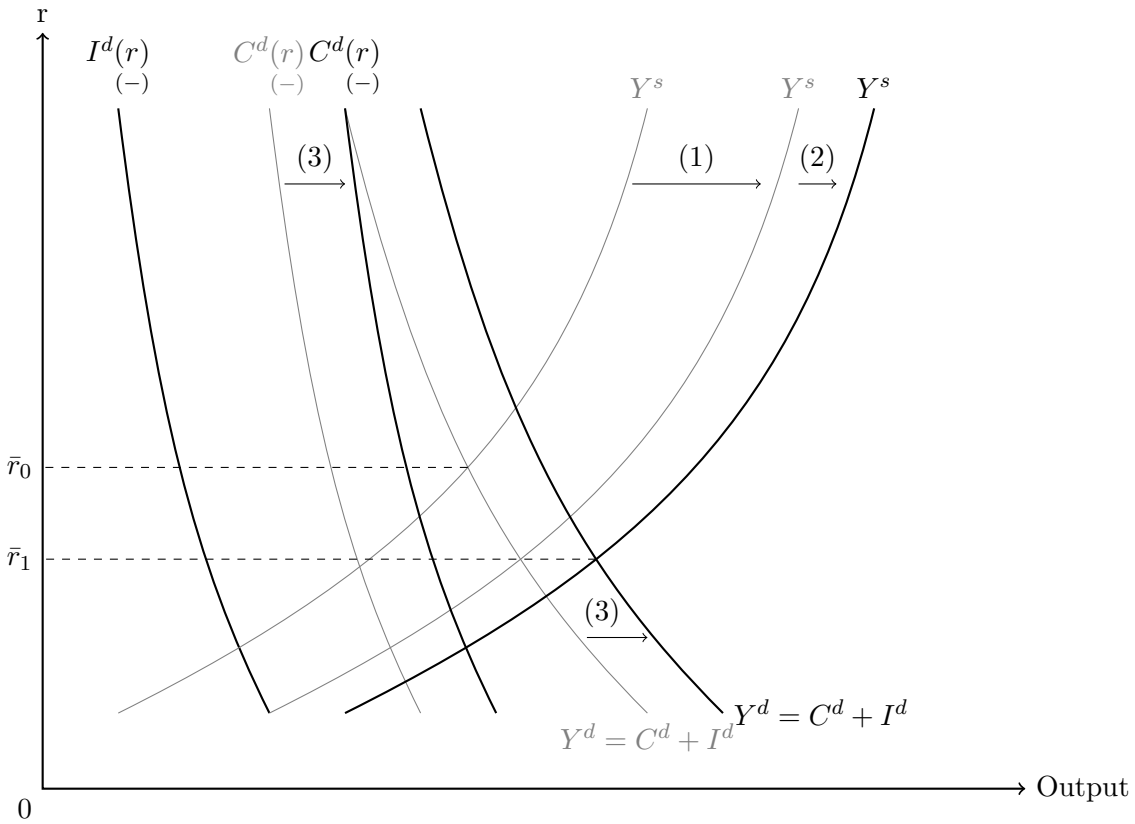
However, since the increase in A also increases $MPK_t = A_t \frac{\partial F(K_t, L_t)}{\partial K_t}$, this means it is profitable to invest in more capital until the returns on capital are brought back down to $r + \delta$. With a higher desired stock of capital (and no change to the current capital stock or depreciation rate), investment demand I^d shifts out by (2).

The total shift in demand is therefore (1)+(2). This leads to an increase in the equilibrium interest rate from $\bar{r}_0$ to $\bar{r}_1$, and an increase in output.

(b)

If investment was not considered, the shift in I^d would not take place and the shift in aggregate demand would only be (1); the same magnitude as the shift in aggregate supply. The equilibrium interest rate would therefore remain unchanged, while output would increase by a smaller amount than it does with investment. The higher interest rate due to increased investment demand also makes it such that consumption does not increase by the full amount of (1) in equilibrium.

(c)



The increase in productivity induces a rightward shift in Y^s of (1), as in (a). However, because the productivity shock is temporary, the increase in relative wages $\frac{w_0}{w_1}$ induces more labour effort in period 0, leading to a further shift of (2).

Consumption also increases due to the increase in income, but because the income shock is transitory, the additional consumption is smoothed out over future periods and thus the increase (3) is of a smaller magnitude than (1).

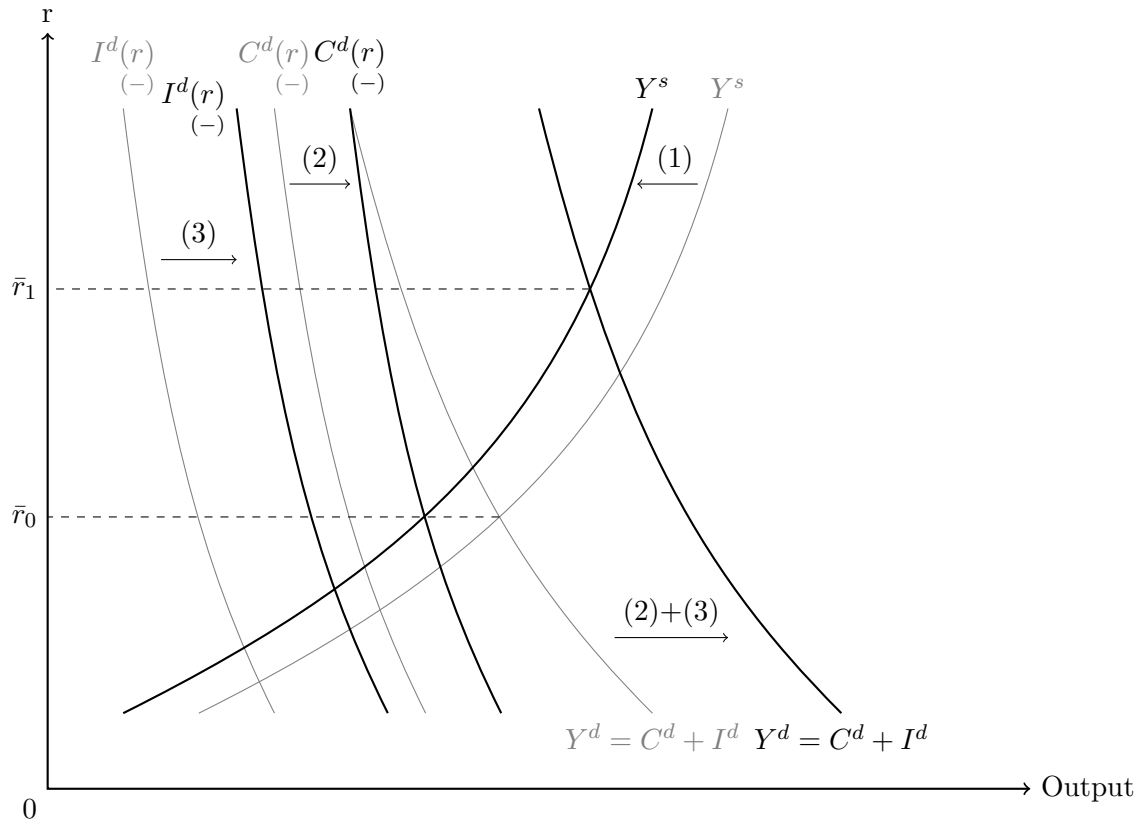
The desired stock of capital does not change since there would be a lag in capital formation, by which time the temporary productivity shock would have worn off. Therefore there is no change to I^d.

The equilibrium result is that interest rates decrease from $\bar{r}_0$ to $\bar{r}_1$, and output increases.

(d)

Investment does not play a special role here, so the analysis is identical to that when investment is not considered.

(e)



With an anticipated increase in productivity, there is no "direct" effect on Y^s in the current period. However, workers anticipate a higher wage ratio $\frac{w_1}{w_0}$, and allocate more labour to period 1. This leads to the so-called "indirect effect" which shifts Y^s to the left by (1).

While workers experience no increase in current income, their expected value of present value income increases with an anticipated productivity shock. This induces them to save less (or borrow more) in period 0 in order to consume more. Still, the increase in consumption is spread over current and future periods as in (c), and there is a shift of C^d by (2).

With an anticipated productivity shock, the increased expected return to capital can be realized in the next period unlike in (c). Therefore the desired stock of capital increases for period 1 and I^d shifts by (3).

The result is that there is an unambiguous increase in the equilibrium interest rate, with Y^s shifting left and Y^d shifting right. However, the equilibrium output can be more or less than the initial output. If the decrease in work effort is not compensated by the increase in current consumer

and investment demand, the equilibrium output will fall, and vice versa. This result is sensitive to the functional forms of C^d, I^d, and Y^s. In any case, although the expected increase in productivity is positively related to the contraction in current labour supply, it is also positively related to the increase in current consumption demand and the desired capital stock for period 1, so these are counteracting forces.

(f)

The difference is that of magnitude; without investment, there is a smaller expansionary demand response to the expected productivity shock.

Question 2

(a)

The person has the following optimization problem:

$$\max_{c,\ell,L}\{\ln c + \ln \ell\} \text{ subject to } c = Lw$$
$$1 = \ell + L$$

Substituting the budget constraints into the utility function, this is equivalent to

$$\max_{L}\{\ln(Lw) + \ln(1-L)\}$$

And the first-order condition is

$$\frac{\partial U}{\partial L} = \frac{1}{L} - \frac{1}{1-L} = 0$$
$$\frac{1-2L}{L(1-L)} = 0$$

Where the optimal choices are $L = \frac{1}{2}, c = \frac{w}{2}, \ell = \frac{1}{2}$. The optimal allocation of labour is invariant to w, and w only affects how much consumption the person can afford with the labour he provides. In this case the income and substitution effects on labour supply in response to changes in the wage rate are equal and opposite. This is because the utility function (which is a monotonic transformation of the Cobb-Douglas utility function) leads to a constant optimal share of the endowment allocated to both goods, and so changes in the wage rate will only affect the size of the endowment (on which c depends) without affecting the share of the endowment allocated to labour or leisure.

(b)

The person now has the following optimization problem:

$$\max_{c,\ell,L}\{\ln c + \ln \ell\} \text{ subject to } c = L(1-\tau)w + R$$
$$1 = \ell + L$$

Again, substituting the budget constraints into the utility function, this is equivalent to

$$\max_{L}\{\ln(L(1-\tau)w + R) + \ln(1-L)\}$$

And the first order condition is

$$\begin{aligned}
\frac{\partial U}{\partial L} = \frac{(1-\tau)w}{L(1-\tau)w+R} - \frac{1}{1-L} &= 0 \\
\frac{(1-L)(1-\tau)w - L(1-\tau)w - R}{[L(1-\tau)w+R](1-L)} &= 0 \\
(1-2L)(1-\tau)w - R &= 0 \\
1-2L &= \frac{R}{w(1-\tau)} \\
L &= \frac{w(1-\tau)-R}{2w(1-\tau)}
\end{aligned}$$

From which it follows that the optimal choices are $L = \frac{w(1-\tau)-R}{2w(1-\tau)}$, $c = \frac{w(1-\tau)+R}{2}$, and $\ell = \frac{w(1-\tau)+R}{2w(1-\tau)}$. Now the optimal allocation of labour and leisure do depend on w. This is because the lump-sum rebate R can only be allocated to consumption, and thus the optimal ratio of consumption and leisure can no longer be achieved by simply allocating a fixed amount to labour. This is confirmed by setting $R = 0$, in which case the optimal choices are the same as before (except for c which depends on the purchasing power of one unit of labour and is $w(1-\tau)$ instead of w).

Assuming that the government rebates all revenue from the income tax, there is an additional constraint to be considered, which is $R = \tau wL$. **This can only be substituted in after solving for the optimal choice**; if this were substituted into the budget constraint at the start, the problem would be identical to (a). The reason is that even though we are working with a representative agent, when any single person makes his consumption decisions, he has to take the decisions of all others as exogenously given. Substituting this into the optimal choices, we get for L,

$$\begin{aligned}
L &= \frac{w(1-\tau)-\tau wL}{2w(1-\tau)} \\
2(1-\tau)L &= 1-\tau-\tau L \\
(2-\tau)L &= 1-\tau \\
L &= \frac{1-\tau}{2-\tau}
\end{aligned}$$

for c,

$$c = L(1-\tau)w + \tau wL = wL = \frac{w(1-\tau)}{2-\tau}$$

and for ℓ,

$$\ell = 1 - L = 1 - \frac{1-\tau}{2-\tau} = \frac{1}{2-\tau}$$

This is similar to (a) in that the optimal labour/leisure allocation is still unaffected by w, but is now affected by τ: L is decreasing in τ while ℓ is increasing in τ over $[0,1]$. The only effect τ has on c is through L (since $c = wL^*(\tau)$); a higher τ disincentivizes work which leads to a lower level of consumption that can be afforded.

(c)

The maximized utility function is

$$\begin{aligned} U\big|_{C^*,\ell^*} &= \ln \frac{w(1-\tau)}{2-\tau} + \ln \frac{1}{2-\tau} \\ &= \ln w + \ln(1-\tau) - 2\ln(2-\tau) \end{aligned}$$

and the level of τ that maximizes this satisfies

$$\begin{aligned} \frac{2}{2-\tau} - \frac{1}{1-\tau} &= 0 \\ 2(1-\tau) - (2-\tau) &= 0 \\ \tau &= 0 \end{aligned}$$

Intuitively, since the government is going to give back every dollar of tax it collects, the person's budget set with any positive tax is a subset of the budget set without any tax at all. From a revealed-preference argument this means that he cannot be better off after a tax; at most he will not be worse off.

Question 3

If "neoclassical business cycle models" refers to the sort of intertemporal model used in this supervision, i.e. 'real business cycle' models, then there are ways to set the parameters and shocks such that the model can fit some stylized facts on aggregate data. The result in 1(b) is that interest rates are countercyclical with respect to a temporary productivity shock. However, for US data, interest rates are weakly procyclical (Doppelhoefer 2006). The model can be reconciled with the facts by combining temporary and permanent productivity changes.

However, a more fundamental problem with such models is that no provision is made for involuntary unemployment; variations in employment are purely due to intertemporal allocation decisions. As Krugman asks, "was the Great Depression really the Great Vacation?" It is also difficult to suggest that any specific technological shock was responsible for the magnitude of the 2008 recession. Lastly, such models leave no room for monetary policy in alleviating a recession (due to the above). This probably contradicts the consensus views on periods such as the Great Depression where it is said that monetary policy (or lack thereof) prolonged the Depression in the US.