## Paper 10 Time Series Models 2019-2020

# Supervision Questions

**Supervision 2. More time series models**

(1) To investigate whether severe recessions affect the dynamic behaviour of output quarterly data for US GDP were obtained. All regressions are estimated over the period from 1987Q1 to 2012Q3 (102 observations).

First it is decided that an AR1 specification sufficiently describes the behaviour of the quarterly growth rate of output $y_t = ln(USGDP_t/USGDP_{t-1})$

$$\widehat{y}_t = \underset{(0.0008)}{0.0034} + \underset{(0.0890)}{0.4571}\, y_{t-1}$$
$$T = 102 \quad \bar{R}^2 = 0.2007 \quad F(1,100) = 26.354 \quad \hat{\sigma} = 0.0057$$
(standard errors in parentheses)

To investigate whether the AR coefficient differs when recent growth has been below average a dummy variable $DUM1_t$ is defined that is one when $(y_{t-1} + y_{t-2})/2$ is ABOVE the overall average growth rate of USGDP and zero otherwise. The following results are obtained.

$$\widehat{y}_t = \underset{(0.0008)}{0.0035} - \underset{(0.1574)}{0.0819}\, DUM1 * y_{t-1} + \underset{(0.1380)}{0.5118}\, y_{t-1}$$
$$T = 102 \quad \bar{R}^2 = 0.1948 \quad F(2,99) = 13.216 \quad \hat{\sigma} = 0.0057$$
(standard errors in parentheses)

A severe recession dummy is then defined that identifies when growth has been negative over the previous two quarters. That is, $DUM2_t$ is defined as one when $(y_{t-1}+y_{t-2})/2$ is ABOVE zero and zero when $(y_{t-1}+y_{t-2})/2$ is BELOW zero. The following results are obtained.

$$\widehat{y}_t = \underset{(0.0010)}{0.0050} - \underset{(0.2561)}{0.6032}\, DUM2 * y_{t-1} + \underset{(0.1926)}{0.8617}\, y_{t-1}$$
$$T = 102 \quad \bar{R}^2 = 0.2354 \quad F(2,99) = 16.550 \quad \hat{\sigma} = 0.0055$$
(standard errors in parentheses)

(i) To what extent do these results support the claim that shocks leading to severe recessions are more persistent than other shocks?

(ii) How would your interpretation change given the information that the recession dummy used was the result of a (lengthy) specification search?

(2) Suppose

$$y_t = \rho_1 y_{t-1} + \rho_2 y_{t-2} + \varepsilon_t$$

We can write this as

$$\left(1 - \rho_1 L - \rho_2 L^2\right) y_t = \varepsilon_t$$

Suppose we rewrite the lag polynomial in the form $(1 - \lambda_1 L)(1 - \lambda_2 L)$ where

$$1 - \rho_1 L - \rho_2 L^2 = (1 - \lambda_1 L)(1 - \lambda_2 L)$$

(i) Obtain the coefficients $\lambda_1$ and $\lambda_2$ in terms of $\rho_1$ and $\rho_2$.
(ii) Hence show that if $\rho_1 + \rho_2 = 1$ then $y_t$ is $I(1)$ (as long as $|\rho_2| < 1$).
(iii) How would you test $\rho_1 + \rho_2 = 1$. Explain carefully how you would calculate the test statistic. What critical values would be used?

(3) A popular way of thinking about some $I(1)$ processes is to decompose into a random walk or trend component, $\tau_t$, and a stationary AR(1) cycle component, $c_t$ (this is often called an unobserved components decomposition). So

$$\begin{aligned} Y_t &= c_t + \tau_t \\ c_t &= \lambda c_{t-1} + \varepsilon_t^c \\ \tau_t &= \tau_{t-1} + \varepsilon_t^\tau \end{aligned}$$

where $Y_t$ might for example be (the natural log of) the price level. Assume $\varepsilon_t^\tau \sim \left(0, \sigma_\tau^2\right)$, $\varepsilon_t^c \sim \left(0, \sigma_c^2\right)$ and assume $\sigma_{c\tau} = Cov\left(\varepsilon_t^c, \varepsilon_t^\tau\right) = 0$ ie the innovation to the random walk and the innovation to the cycle components are orthogonal. Typically we would expect $\lambda \geq 0$.
(i) Show this implies an ARMA(1,1) for $y_t = \triangle Y_t$.
(ii) Write the ARMA as

$$y_t = \lambda y_{t-1} + \eta_t - \theta \eta_{t-1}$$

and show that

$$-\frac{\theta}{1+\theta^2} = \frac{-\lambda - (1-\lambda) q}{1 + \lambda^2 + (1-\lambda^2) q}$$

where $q = \sigma_c^2 / \left(\sigma_\tau^2 + \sigma_c^2\right)$. What happens as $q \to 0$? As $q \to 1$? Interpret these limiting cases.
(iii) Suppose $\lambda = 0$. Show that this implies $\theta > 0$ in the $\eta_t - \theta \eta_{t-1}$ representation.
(iv) [Harder] This result generalises. For $\lambda$ non-zero show that $\theta > \lambda$. What does this imply for (innovations to) the trend in the Beveridge Nelson decomposition?
(v) Now suppose we add an intercept to the trend component ie we specify

$$\tau_t = g_t + \tau_{t-1} + \varepsilon_t^\tau$$

What are the implications for $Y_t$ of assuming
(a) $g_t = g$ for all $t$?
(b) $g_t = g_{t-1} + \varepsilon_t^g$ ie $g_t$ is itself random walk?

(4) To investigate the implications of the permanent income hypothesis an econometrician proposes to decompose income into trend and stationary components and then see whether innovations to consumption match the innovations to trend income. She first obtains the following results for log of consumption ($c_t$) and log of income ($y_t$) using US data from 1980Q1 to 2007Q1 (109 obs). Estimated coefficient standard errors are in parentheses.

$$\widehat{\Delta y_t} = \underset{(0.122)}{0.3036} - \underset{(0.015)}{0.0363} y_{t-1}$$

$$\widehat{\Delta c_t} = \underset{(0.108)}{0.3024} - \underset{(0.014)}{0.0379} c_{t-1}$$

(i) Test for stationarity of these processes (each equation also included four lags of the dependent variable and a time trend whose estimated coefficients are not reported).

She then estimates the following ARMA(1,1) process for the growth of income

$$\Delta y_t = \underset{(0.001)}{0.0141} + \underset{(0.100)}{0.7519} \Delta y_{t-1} + e_t - \underset{(0.140)}{0.5276} e_{t-1}$$

(ii) Explain briefly how given a time series of data you would choose a preferred ARMA model for the data.

She then approximates the stochastic innovation to trend income by

$$innovation_t = \frac{1 - 0.5276}{1 - 0.7519} e_t$$

where $e_t$ is the estimated residual in the ARMA.

(iii) Explain the reasoning behind this construction.

She then estimates the following equation

$$\widehat{\Delta c_t} = \underset{(0.0005)}{0.0158} + \underset{(0.042)}{0.2611} innovation_t$$

(iv) Test the hypotheses that the coefficient on income innovations

(a) differs from zero, and
(b) differs from unity

and interpret what each of these mean for the permanent income hypothesis.

Two explanations are often given for the fact that consumption does not seem to react fully to innovations in income: liquidity constraints or myopic behaviour. It is suggested that liquidity constrained consumers will have differing responses of consumption to positive and negative income innovations, whereas myopic consumers will show no such asymmetric response.

(v) Discuss how you would use this suggestion to extend the above procedure and formulate an econometric test between these rival explanations of the violation of the PIH.