Economics of Uncertainty and Information
Supervision 2

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Question 1

For a given utility function $u(x)$, the coefficient of absolute risk aversion $r(x)$ is defined as $-\frac{u''(x)}{u'(x)}$, and the coefficient of relative risk aversion $\rho(x)$ is defined as $-\frac{u''(x)x}{u'(x)}$.

(a)

With $u(x) = -\frac{1}{\alpha}e^{-\alpha x}$, we have $u'(x) = e^{-\alpha x}$ and $u''(x) = -\alpha e^{-\alpha x}$. Therefore $r(x) = \alpha$, and this utility function is characterized by constant absolute risk aversion.

(b)

i.

With $u(x) = \frac{1}{1-\beta}x^{1-\beta}$, we have $u'(x) = x^{-\beta}$ and $u''(x) = -\beta x^{-\beta-1}$. Therefore $\rho(x) = \beta$ and the utility function is characterized by constant relative risk aversion.

ii.

With $u(x) = \ln(x)$, we have $u'(x) = \frac{1}{x}$ and $u''(x) = -\frac{1}{x^2}$. Therefore $\rho(x) = 1$ and the utility function is characterized by constant relative risk aversion.

Question 2

It is apparent that the expected values of L_1 and L_2 are the same, whereas $\text{Var}[L_1] = 0 < \text{Var}[L_2]$. Thus L_2 is a mean-preserving spread of L_1, and any risk-averse agent will prefer L_1 to L_2; L_2 is riskier than L_1. $u(x) = e^x$ is a convex function, and thus an individual with utility $u(x)$ is risk-loving. Thus they will prefer L_2. To check that this is true, we can calculate the expected utility under both lotteries:

$$\text{E}[U(L_1)] = \frac{1}{3}(e^{10} + e^{20} + e^{30}) \approx 3.56 \times 10^{12}$$
$$\text{E}[U(L_2)] = \frac{5}{12}(e^{10} + e^{30}) + \frac{1}{6}e^{20} \approx 4.45 \times 10^{12}$$

Question 3

(a)

The utility function exhibits constant relative risk aversion with $\rho(x) = \gamma$, just as in (b)i. The student is risk averse, but her absolute risk aversion decreases as her wealth increases while her relative risk aversion stays constant. That is, her aversion to a given absolute loss decreases as her wealth increases, while her aversion to a loss which is a given proportion of her wealth stays constant whatever her wealth is.

(b)

For the student with utility function $u(c)$, her expected utility of making an investment of $c \in [0, 100]$ is

$$\begin{aligned} \mathrm{E}[U(c)] &= \frac{1}{2}u(100 - c + 2.5c) + \frac{1}{2}u(100 - c) \\ &= \frac{1}{2}u(100 + 1.5c) + \frac{1}{2}u(100 - c) \\ &= \frac{(100 + 1.5c)^{1-\gamma} + (100 - c)^{1-\gamma}}{2(1-\gamma)} \end{aligned}$$

Assuming the student chooses to make the investment, the value of c that maximizes her expected utility satisfies

$$\begin{aligned} \frac{1}{2}[1.5(100 + 1.5c)^{-\gamma} - (100 - c)^{-\gamma}] &= 0 \\ 1.5(100 + 1.5c)^{-\gamma} &= (100 - c)^{-\gamma} \\ \ln(1.5) - \gamma \ln(100 + 1.5c) &= -\gamma \ln(100 - c) \\ \gamma &= -\frac{\ln(1.5)}{\ln(100 - c) - \ln(100 + 1.5c)} \end{aligned}$$

and thus we can use the above formula to compute $\rho(x) = \gamma$ given her choice of c, although we cannot derive γ if she chooses to invest everything or not to invest at all (we can just assume γ goes to infinity as c goes to 0, and that γ is at most 0 when $c = 100$).

(c)

My $\rho(x) = \gamma$ will be calculated in the exact same way as before.

Question 4

(a)

(b)

The linear utility function implies risk-neutrality, so the decision that yields the highest expected value will be chosen. We omit the calculations, but Option A will yield the highest expected payoff for the first 4 rows while Option B will yield the highest expected payoff for the last 6 rows.

(c)

This time the utility function is concave and therefore implies risk aversion. To make things convenient, we assume the probabilities p and $1 - p$ take on any value from 0 to 1, and thus the expected utility from Option A is

$$p\sqrt{2} + (1-p)\sqrt{1.6}$$

while the expected utility from Option B is

$$p\sqrt{3.85} + (1-p)\sqrt{0.1}$$

Therefore, Option A will be chosen whenever

$$\begin{aligned} p\sqrt{2} + (1-p)\sqrt{1.6} &> p\sqrt{3.85} + (1-p)\sqrt{0.1} \\ (\sqrt{2} + \sqrt{0.1} - \sqrt{3.85} - \sqrt{1.6})p &> \sqrt{0.1} - \sqrt{1.6} \\ p &\lesssim 0.634 \end{aligned}$$

So this time Option A will be chosen for the first 6 rows while Option B will be chosen for the last 4 rows.

(d)

We do the same thing as before and assume the probabilities are continuous rather than discrete. The expected utility from Option A is

$$p\frac{2^{1-\gamma}}{1-\gamma} + (1-p)\frac{1.6^{1-\gamma}}{1-\gamma}$$

and the expected utility from Option B is

$$p\frac{3.85^{1-\gamma}}{1-\gamma} + (1-p)\frac{0.1^{1-\gamma}}{1-\gamma}$$

and so Option A will be chosen whenever

$$\begin{aligned} p\frac{2^{1-\gamma}}{1-\gamma} + (1-p)\frac{1.6^{1-\gamma}}{1-\gamma} &> p\frac{3.85^{1-\gamma}}{1-\gamma} + (1-p)\frac{0.1^{1-\gamma}}{1-\gamma} \\ \frac{1}{1-\gamma}(2^{1-\gamma} + 0.1^{1-\gamma} - 1.6^{1-\gamma} - 3.85^{1-\gamma})p &> \frac{1}{1-\gamma}(0.1^{1-\gamma} - 1.6^{1-\gamma}) \\ p &> \frac{0.1^{1-\gamma} - 1.6^{1-\gamma}}{2^{1-\gamma} + 0.1^{1-\gamma} - 1.6^{1-\gamma} - 3.85^{1-\gamma}} \end{aligned}$$