Unemployment and Labour Markets
Supervision 1

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Question 1

One form of the Shapiro-Stiglitz model is as such. An employed worker earns wage w, and in order to be productive, he must exert effort $\bar{e}$ which is a disutility to him. When not employed, he gets unemployment benefits b. If the worker chooses to shirk, there is a probability π that he gets caught and becomes unemployed. Therefore shirking gives the expected utility

$$\pi z + (1-\pi)w \tag{1}$$

where z is the value outside of his current employment. z is affected by the probability of finding another job (p), wages in other employment w^*, and unemployment benefits, through the following relationship:

$$z = pw^* + (1-p)b \tag{2}$$

Substituting (2) into (1), the expected utility from shirking is

$$\begin{aligned} &\pi[pw^* + (1-p)b] + (1-\pi)w \\ =&\pi pw^* + \pi(1-p)b + (1-\pi)w \end{aligned}$$

and the utility from exerting effort is

$$w - \bar{e}$$

Assuming that a firm wants to make it such that the worker chooses to exert effort, it must set wages such that

$$w - \bar{e} \geq \pi pw^* + \pi(1-p)b + (1-\pi)w$$

which is the no-shirking condition. In aggregate it is assumed that all firms are identical, and therefore $w^* = w$. The no-shirking condition then reduces to

$$\begin{aligned} w - \bar{e} &\geq \pi pw + \pi(1-p)b + (1-\pi)w \\ w - \pi pw - (1-\pi)w &\geq \bar{e} + \pi(1-p)b \\ [1 - \pi p - (1-\pi)]w &\geq \bar{e} + \pi(1-p)b \\ \pi(1-p)w &\geq \bar{e} + \pi(1-p)b \\ w &\geq b + \frac{\bar{e}}{\pi(1-p)} \end{aligned}$$

and now supposing the probability of finding a job p depends negatively on the unemployment rate u, in particular $p = 1 - u$, the no-shirking condition becomes

$$w \geq b + \frac{\bar{e}}{\pi u}$$

To determine labour demand, we let firms maximize their profits based on some production function $F(\bar{e}L)$ where $F'(\cdot) > 0$ and $F''(\cdot) < 0$. Therefore firms maximize

$$\Pi = \max_L \{F(\bar{e}L) - wL\}$$

with the first order condition

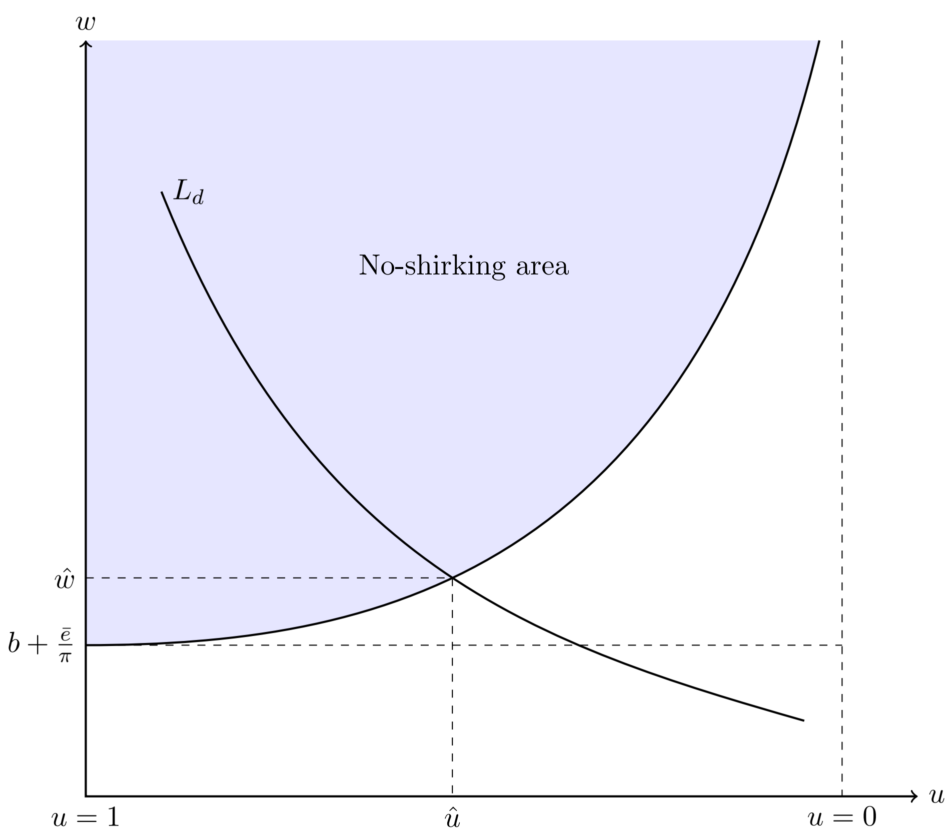
$$\bar{e}F'(\bar{e}L) = w$$

which is downward sloping in (w, L) space since $F''(\cdot) < 0$. Letting the labour demand that fulfills this condition be $L_d(w)$, the unemployment rate is determined by

$$u(w) = \frac{L_s - NL_d(w)}{L_s}$$

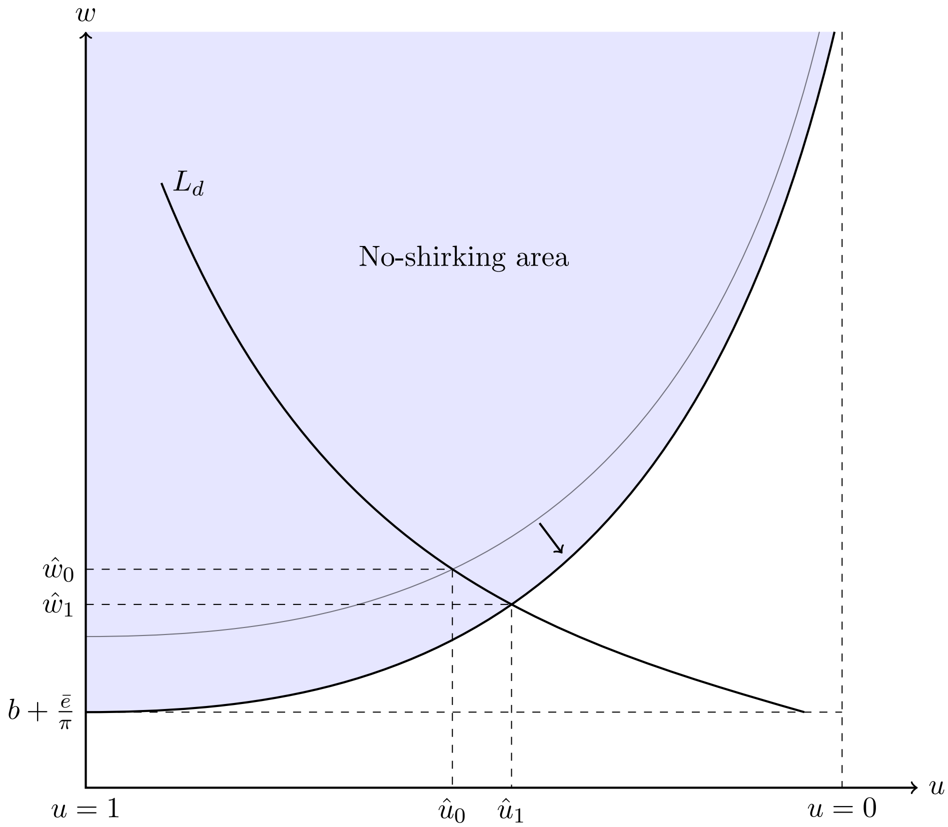
where L_s is the inelastic labour supply and N is the number of firms. Since $L_d(w)$ is decreasing in w, $u(w)$ is increasing in w.

Graphically, the equilibrium unemployment and wage is as such



(a)

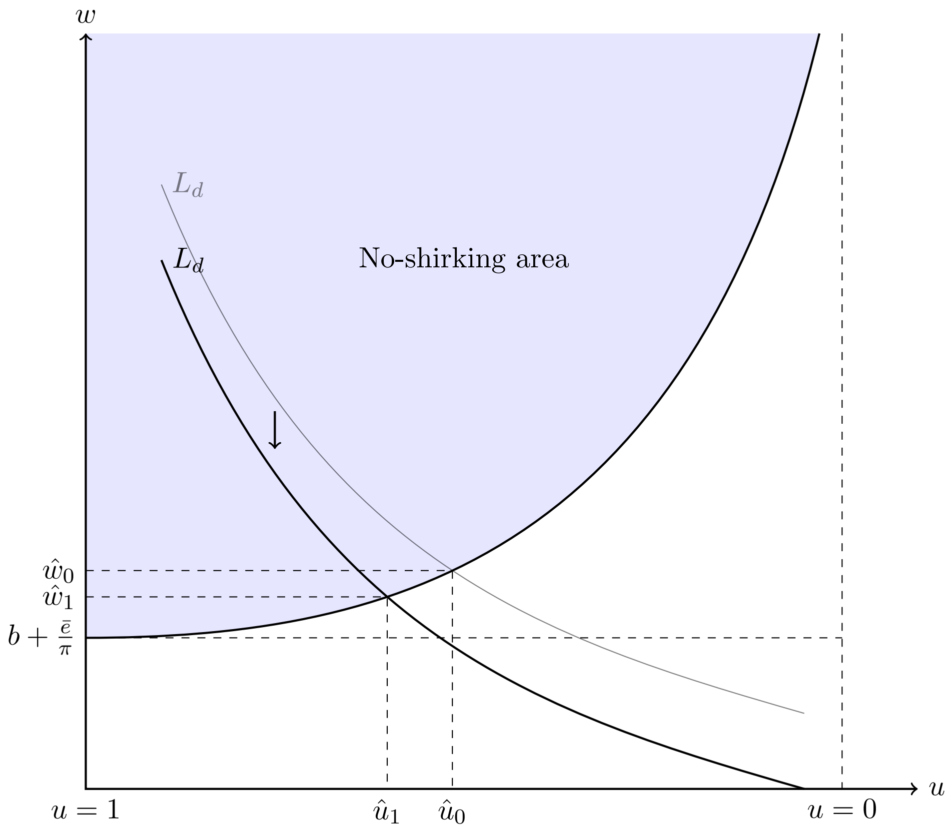
A decrease in unemployment benefits shifts the curve bounding the no-shirking area down by Δb, as such:



In equilibrium, wages fall from $\hat{w}_0$ to $\hat{w}_1$ while unemployment falls from $\hat{u}_0$ to $\hat{u}_1$. Essentially the lack of benefits makes shirking more dangerous, and wages no longer have to be as high to ensure there is no shirking. Firms are then willing to hire more workers given that they do not have to pay as much for a given level of effort.

(b)

A negative productivity shock reduces the marginal productivity of labour, that is, $F'(\bar{e}L)$ decreases for all values of L. Given that the firms seek to meet the condition $\bar{e}F'(\bar{e}L) = w$, it means that at every level of L they are now only willing to pay a lower wage w, and $L_d(w)$ shifts down.



The recession leads to a fall in wages from $\hat{w}_0$ to $\hat{w}_1$, and unemployment rises from $\hat{u}_0$ to $\hat{u}_1$, which is expected in a recession.

(c)

An increase in the probability that a shirker is detected has the exact same effects as (a) in all but magnitude. When π increases, the expected benefit of shirking decreases, and this shifts the curve bounding the no-shirking area down by $\frac{\bar{e}}{\pi+\Delta\pi} - \frac{\bar{e}}{\pi}$.

Question 2

(a)

Let the output price be normalized to 1. The firm seeks to maximize

$$\begin{aligned} \pi &= Y - wL \\ &= K^{\frac{1}{3}} L^{\frac{2}{3}} - wL \end{aligned}$$

and the first-order condition is

$$w = \frac{2}{3} \left(\frac{K}{L} \right)^{\frac{1}{3}}$$

Therefore the labour demand in this economy is

$$L_d = \frac{8K}{27w^3}$$

(b)

The real wage is where $L_d = L_s = 1000$ where $K = 1000$. Therefore

$$\begin{aligned} \frac{8000}{27w^3} &= 1000 \\ w^3 &= \frac{8}{27} \\ w &= \frac{2}{3} \end{aligned}$$

There is full employment by definition of the equilibrium, output is $Y = K^{\frac{1}{3}}L^{\frac{2}{3}} = 1000$, and the total amount earned by workers is $\frac{2000}{3}$ or $\frac{2}{3}$ of the total output since this is a Cobb-Douglas production function.

(c)

Since prices are normalized to 1 the minimum wage is now $w = 1$. This is higher than the real wage in equilibrium which was $\frac{2}{3}$.

(d)

Now it is L_d which is endogenous and w which is exogenously given as 1. Hence

$$L_d = \left\lfloor \frac{8000}{27} \right\rfloor = 296$$

Employment is now 296, output is $Y = 1000^{\frac{1}{3}} \cdot 296^{\frac{2}{3}} \approx 444$, and the total amount earned by workers is just $w \cdot L_d = 296$.

(e)

At any one point in time the government's policy will have raised the standards of living of 296 of the working class while impoverishing the other 704.

(f)

No. Firstly, there are enough case studies reporting a negligible effect on employment with modest increases in the minimum wage, so there is at least some reason to be skeptical of the theory's applicability to all cases. Secondly, wages are taken to be independent of labour demand, which is inaccurate in firms with substantial market power. Standard analyses of monopsony power can show that there are levels of the minimum wage which would increase both employment and wages. Lastly, labour is taken to be inelastically supplied in this model and there is no intertemporal consumption/leisure tradeoff. In reality those earning lower wages (who are also likely to be liquidity constrained) are likely to have a higher marginal propensity to consume. It seems plausible to think of some model which could incorporate different classes of workers and owners of capital, and establish a link between the wages of low-wage workers and aggregate demand, such that a minimum wage would not result in much change in employment due to the boost to aggregate demand.

Question 3

The agent faces the following problem

$$\max_{C,\ell}\left\{\frac{C^\gamma-1}{\gamma}+\frac{\ell^\gamma-1}{\gamma}\right\}\text{ subject to } C = wL(1-\tau)+\pi-T$$
$$\ell+L=1$$

The two constraints can be combined into one, yielding $C = w(1-\ell)(1-\tau)+\pi-T$. Therefore, the Lagrangian for this problem is

$$\mathcal{L}=\frac{C^\gamma-1}{\gamma}+\frac{\ell^\gamma-1}{\gamma}-\lambda[C-w(1-\ell)(1-\tau)-\pi+T]$$

and the first-order conditions are

$$\frac{\partial\mathcal{L}}{\partial C}=C^{\gamma-1}-\lambda=0 \tag{1}$$
$$\frac{\partial\mathcal{L}}{\partial \ell}=\ell^{\gamma-1}-\lambda w(1-\tau)=0 \tag{2}$$
$$\frac{\partial\mathcal{L}}{\partial \lambda}=C-w(1-\ell)(1-\tau)-\pi+T=0 \tag{3}$$

Dividing (1) by (2),

$$\left(\frac{C}{\ell}\right)^{\gamma-1}=\frac{1}{w(1-\tau)}$$
$$C=\frac{\ell}{[w(1-\tau)]^{\frac{1}{\gamma-1}}} \tag{4}$$

Substituting (4) into (3),

$$\begin{aligned}
&\frac{\ell}{[w(1-\tau)]^{\frac{1}{\gamma-1}}}-w(1-\ell)(1-\tau)-\pi+T=0\\
&\frac{\ell}{[w(1-\tau)]^{\frac{1}{\gamma-1}}}+w(1-\tau)\ell-w(1-\tau)-\pi+T=0\\
&\left\{\frac{1}{[w(1-\tau)]^{\frac{1}{\gamma-1}}}+w(1-\tau)\right\}\ell=w(1-\tau)+\pi-T\\
&\frac{1+[w(1-\tau)]^{\frac{\gamma}{\gamma-1}}}{[w(1-\tau)]^{\frac{1}{\gamma-1}}}\ell=w(1-\tau)+\pi-T\\
&\ell=\frac{[w(1-\tau)]^{\frac{\gamma}{\gamma-1}}+(\pi-T)[w(1-\tau)]^{\frac{1}{\gamma-1}}}{1+[w(1-\tau)]^{\frac{\gamma}{\gamma-1}}}
\end{aligned}$$

And with $\ell+L=1$, we get the nice expression for the optimal labour supply:

$$\begin{aligned}
L&=1-\frac{[w(1-\tau)]^{\frac{\gamma}{\gamma-1}}+(\pi-T)[w(1-\tau)]^{\frac{1}{\gamma-1}}}{1+[w(1-\tau)]^{\frac{\gamma}{\gamma-1}}}\\
&=\frac{1-(\pi-T)[w(1-\tau)]^{\frac{1}{\gamma-1}}}{1+[w(1-\tau)]^{\frac{\gamma}{\gamma-1}}}
\end{aligned}$$

(a)

With $\tau = 0$ and $\gamma = \frac{1}{2}$, the optimal labour supply reduces to

$$L = \frac{1 - (\pi - T)w^{-2}}{1 + w^{-1}}$$
$$= \frac{\frac{w^2 - (\pi - T)}{w^2}}{\frac{1+w}{w}}$$
$$= \frac{w^2 + T - \pi}{w(1+w)}$$

The optimal labour supply in this case is decreasing in π; the more non-labour income the worker has, the less he has to work. Consumption and leisure are both normal goods and the income effect from a higher π increases their amount at the expense of labour.

(b)

With $T = 0$ and $\pi = 0$, the optimal labour supply reduces to

$$L = \frac{1}{1 + [w(1-\tau)]^{\frac{\gamma}{\gamma-1}}}$$
$$= \frac{1}{1 + \frac{1}{[w(1-\tau)]^{\frac{\gamma}{1-\gamma}}}}$$
$$= \frac{[w(1-\tau)]^{\frac{\gamma}{1-\gamma}}}{1 + [w(1-\tau)]^{\frac{\gamma}{1-\gamma}}}$$

which is decreasing in τ since $\frac{\gamma}{1-\gamma}$ is positive with $\gamma < 1$, and the smaller $[w(1-\tau)]^{\frac{\gamma}{1-\gamma}}$ becomes, the more significant the 1 in the denominator becomes. This is true for all viable values of γ, so the qualitative result does not depend on the value of γ. A higher τ reduces the opportunity cost of leisure, but also reduces the value of the worker's time endowment. Whether or not leisure is a normal good, it is the former substitution effect which prevails for all γ.

Question 4

Diamond's critique is the so-called Diamond paradox: assuming that firms set wages and there are search frictions where job seekers can visit one firm at a time with some search cost, when a job seeker enters the wage bargaining phase with a firm, his search costs are sunk and do not affect the firm's behaviour. Because there are costs to declining the firm's offer to search for a better opportunity, the firm essentially has some market power over the job seeker at that moment. This means there is some $\epsilon > 0$ small enough that the firm can discount from the wage it offers while still making it weakly preferred for the job seeker to accept the offer. But running this through many iterations for all the firms in the economy should drive wages right down to the reservation wage of the representative job seeker, and there is no reason to offer a wage higher than the reservation wage. The paradox is that when wages being offered are only the reservation wage, there is no benefit to taking part in the job search in the first place.

This is a problem because workers do engage in search eveen though Diamond's setting applies to many labour markets: firms set wages and have some idea about reservation wages, job seekers incur search costs, and yet they engage in search. Some attempts to resolve this involve introducing imperfect information on the side of the firm, and the Shapiro-Stiglitz model does this by introducing imperfect information on worker effort. This resolves the paradox for some industries, but there are occupations where effort is more easily observed and yet search takes place, for example quality control inspectors.

Another attempt to resolve this is by introducing wage advertisements to the model. Poeschel (2010) shows that this resolves the paradox in theory only if firms will commit to the contracts they advertise, but there is an incentive for firms to renege once the offer has been accepted. The paper suggests that combining wage advertisements with efficiency wages resovles the paradox without relying on imperfect information or firm commitment. This solution relies on advertising and sign-up bonuses or base wages, which is said to be more widespread in real-world labour markets than the conditions of imperfect information and firm commitment.