Contracts & Regulations, Education & Health
Supervision 5

Samuel Lee

Short Questions

Question 1

Fronstin (2007) states that in 2007, over 60% of the non-elderly US population was covered by employer-sponsored insurance, which represented 90.1% of all private coverage. Buchmueller and Monheit (2009) provide a historical context for this development: the 1942 Stabilisation Act imposed wage and price controls during World War II, but benefits such as health insurance were exempted. Many employers therefore competed along the axis of fringe benefits to attract scarce labour. [*I don't really know what I'm supposed to explain here?*]

Question 2

If getting a cold is a near-certainty, there is little point of insurance specifically for colds since there is not much benefit to be gained from diversification. If anything, what the individual would like to smooth is utility across time (healthy days versus days with a cold) rather than across states of the world. Furthermore, if we think the costs of a cold are relatively small, there is less utility to be gained from offloading the risk from risk-averse individuals to risk-neutral insurers. The biggest benefits to diversification come from low-probability, high-cost events, since expected utility losses are convex in costs while the expected monetary cost is close to 0 for low enough probabilities.

Question 3

Rate-of-return regulation imposes a legal cap on the rate of return from some asset base. Assuming this asset base is the stock of capital k, the rate of return is equal to $\frac{\pi}{k}$ where π is the profit of the firm. Assuming a cap of $s - r$ where r is the interest rate, the firm solves

$$\max_{k,l} pf(k,l) - rk - wl$$

subject to

$$\frac{\pi}{k} = \frac{pf(k,l) - rk - wl}{k} \leq s - r \implies pf(k,l) - wl \leq sk$$

where p is the price of the firm's product, w is the real wage, and l is labour. The Lagrangian for this problem is

$$\mathscr{L} = pf(k,l) - rk - wl - \lambda\left[pf(k,l) - wl - sk\right]$$

and we have the first-order conditions

$$\left.\begin{aligned} \frac{\partial \mathscr{L}}{\partial k} &= (1-\lambda)pf_k - r + \lambda s = 0 \\ \frac{\partial \mathscr{L}}{\partial l} &= (1-\lambda)pf_l - (1-\lambda)w = 0 \end{aligned}\right\} \implies \frac{f_k}{f_l} = \frac{r - \lambda s}{(1-\lambda)w} = \frac{r}{w} - \frac{\lambda(s-r)}{(1-\lambda)w} < \frac{r}{w}$$

which means that there is over-investment in capital (or under-investment in labour) since the marginal rate of capital is decreasing. Intuitively, the cap on the rate of return is defined with respect to the capital stock. Investing in labour raises profits but past a certain point the benefit is not internalised by the firm when there is a rate-of-return cap. Investing in capital broadens the asset base so that the firm can earn more gross profits, and this continues until the return to capital driven to the maximum rate of return. Beyond that point there are no benefits to capital investment.

Question 4

RPI-X regulation sets a price cap ex-ante for a given period of time. Any cost reduction the firm is able to achieve beyond that given by the price cap accrues to the firm as pure profits. Prices are allowed to adjust over time according to the following formula:

$$\%\Delta P_t = \%\Delta RPI_t - X_t$$

where P_t is the price at time t, RPI_t is the Retail Price Index at time t, X_t is some efficiency target set by the regulator, and $\%\Delta$ denotes percentage changes. If X_t is too high, that is, the regulator demands too much of a price decrease over a given period, reducing the cost of production below the price cap becomes very difficult. Also, if X_T is updated too frequently and the price cap is frequently set to be close to the true cost of production, then firms will anticipate low innovation rents and have little incentive to reduce costs.

Problems

Question 1

(a)

Profits are correlated with effort, so the firm owner could introduce an incentive scheme based on profits. An example might be a linear wage increasing in profits. Suppose we want to introduce the following linear wage scheme:

$$w = \alpha + \beta\pi$$

Given values of α and β, the worker will seek to maximise

$$\begin{aligned} U(w,e) &= \mathbb{E}[\alpha + \beta\pi \mid e] - \frac{1}{2}\rho \operatorname{Var}[\alpha + \beta\pi \mid e] - \frac{e^{1+\lambda}}{1+\lambda} \\ &= \alpha + \beta e - \frac{1}{2}\rho\beta^2\sigma_x^2 - \frac{e^{1+\lambda}}{1+\lambda} \end{aligned}$$

assuming $\mathbb{E}[x] = 0$ for simplicity. The solution to this is $e = \beta^{\frac{1}{\lambda}}$. The firm will set α such that the worker's expected utility given the optimal effort level is equal to the outside option (assumed to be 0):

$$\alpha + \beta^{\frac{1+\lambda}{\lambda}} - \frac{1}{2}\rho\beta^2\sigma_x^2 - \frac{\beta^{\frac{1+\lambda}{\lambda}}}{1+\lambda} = 0 \implies \alpha = \frac{1}{2}\rho\beta^2\sigma_x^2 - \frac{\lambda}{1+\lambda}\beta^{\frac{1+\lambda}{\lambda}}$$

To find the optimal wage scheme, the firm owner solves

$$\max_{\alpha,\beta}\{\mathbb{E}[\pi - w]\} = \max_{\alpha,\beta}\{\mathbb{E}[x + e - \alpha - \beta x - \beta e]\}$$

subject to the employee choosing the optimal effort level $e = \beta^{\frac{1}{\lambda}}$. Therefore, the owner maximises

$$(1-\beta)\beta^{\frac{1}{\lambda}} - \frac{1}{2}\rho\beta^2\sigma_x^2 + \frac{\lambda}{1+\lambda}\beta^{\frac{1+\lambda}{\lambda}}$$

The first-order condition is

$$\frac{1}{\lambda}\beta^{\frac{1-\lambda}{\lambda}} - \frac{1}{\lambda}\beta^{\frac{1}{\lambda}} - \rho\beta\sigma_x^2 = 0 \implies \frac{1}{\lambda}\left(\beta^{\frac{1-\lambda}{\lambda}-1} - \beta^{\frac{1-\lambda}{\lambda}}\right) = \rho\sigma_x^2$$

There is no closed-form solution for this, but we can see that the more risk-averse the employee and the more volatile the economy is, the less the owner off-loads the risk onto the employee.

(b)

The economy-wide shock now follows an AR(1) process. Since effort is chosen each period, the employee deciding on their effort level for period $t+1$ will have to maximise expected utility conditional on their information set at the end of period t. Since the employee knows how much effort they put in this period, they can deduce the realised shock x_t given π_t and e_t, and their information set at time t is

$$\Omega_t = \{\mathbf{x}_t, \mathbf{e}_{t+1}, \boldsymbol{\pi}_t, \boldsymbol{\alpha}, \boldsymbol{\beta}\}$$

where $\mathbf{x}_t$, $\mathbf{e}_t$, and $\boldsymbol{\pi}_t$ are vectors containing values of x_t, e_t, π_t for $t = \{0, \ldots, t\}$, and $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are vectors containing all values of α_t and β_t. Then, we have

$$\begin{aligned} \mathbb{E}[x_{t+1}|\Omega_t] &= \mathbb{E}[\rho x_t + \eta_{t+1}|\Omega_t] = \rho x_t \\ \operatorname{Var}[x_{t+1}|\Omega_t] &= \sigma_\eta^2 \end{aligned}$$

While the employee has information on the expected value of x_t, the conditional variance is constant and independent of time. We can solve for the worker's optimal level of effort given α_t

and β_t. The worker maximises

$$\begin{aligned}
&\mathbb{E}[\alpha_{t+1} + \beta_{t+1}\pi_{t+1} \mid \Omega_t] - \frac{1}{2}\rho \operatorname{Var}[\alpha_{t+1} + \beta_{t+1}\pi_{t+1} \mid \Omega_t] - \frac{e_{t+1}^{1+\lambda}}{1+\lambda} \\
=&\alpha_{t+1} + \beta_{t+1}(\rho x_t + e_{t+1}) - \frac{1}{2}\rho\beta_{t+1}^2\sigma_\eta^2 - \frac{e_{t+1}^{1+\lambda}}{1+\lambda}
\end{aligned}$$

and the solution to this is the same as before:

$$e_{t+1} = \beta_{t+1}^{\frac{1}{\lambda}}$$

This time, α_{t+1} is equal to

$$\begin{aligned}
\alpha_{t+1} &= \frac{1}{2}\rho\beta_{t+1}^2\sigma_\eta^2 + \frac{\beta_{t+1}^{\frac{1+\lambda}{\lambda}}}{1+\lambda} - \beta_{t+1}(\rho x_t + \beta_{t+1}^{\frac{1}{\lambda}}) \\
&= \frac{1}{2}\rho\beta_{t+1}^2\sigma_\eta^2 - \frac{\lambda}{1+\lambda}\beta_{t+1}^{\frac{1+\lambda}{\lambda}} - \beta_{t+1}\rho x_t
\end{aligned}$$

and the owner must now maximise $\mathbb{E}[\pi_{t+1} - \alpha_{t+1} - \beta_{t+1}e_{t+1} \mid \Omega_t^O]$, where we have

$$\Omega_t^O = \{\boldsymbol{\pi}_t, \boldsymbol{\alpha}, \boldsymbol{\beta}\}$$

Since the unconditional mean of x_t is 0, the owner maximises

$$\mathbb{E}\left[x_t + \beta_{t+1}^{\frac{1}{\lambda}} - \frac{1}{2}\rho\beta_{t+1}^2\sigma_\eta^2 + \frac{\lambda}{1+\lambda}\beta_{t+1}^{\frac{1+\lambda}{\lambda}} + \beta_{t+1}\rho x_t - \beta_{t+1}^{\frac{1+\lambda}{\lambda}}\right] = \beta_{t+1}^{\frac{1}{\lambda}} - \frac{1}{2}\rho\beta_{t+1}^2\sigma_\eta^2 - \frac{1}{1+\lambda}\beta_{t+1}^{\frac{1+\lambda}{\lambda}}$$

with the first-order condition

$$\frac{1}{\lambda}\beta_{t+1}^{\frac{1-\lambda}{\lambda}} - \rho\beta_{t+1}\sigma_\eta^2 - \frac{1}{\lambda}\beta_{t+1}^{\frac{1}{\lambda}} = 0$$

which is identical to that from part (a) except for the time subscripts and σ_η^2 in lieu of σ_x^2. Perhaps more importantly, the maximisation problem contains no time-varying variables other than the owner's choice variables, so it should follow that β_t does not vary with time under this framework. Intuitively, since the employee knows x_t, the conditional covariance between their effort and profits and the conditional variance of profits in the next period are constant over time, and the optimal wage structure involves the same profile of risk-sharing between the owner and the worker.

If $\mathbb{E}[x_t]$ cannot be treated as a known variable, and instead the owner acts like a Bayesian with prior beliefs on $\mathbb{E}[x_t]$, then there is the possibility of a ratchet effect. However, the data-generating process of x_t is explicitly stated, so that is perhaps less applicable here.

Question 2

(a)

Some assumptions, which are not mentioned in the question, are probably needed to make the problem interesting or tractable. We start with the fundamental ones: the banker is risk-averse, and x has finite variance. Next we take it that the banker maximises their certainty-equivalent payoff, and for this we use the following approximation:

$$\mathrm{CE}(e) = \mathbb{E}[w|e] - \frac{1}{2}\rho(e)Var[w|e] - c(e)$$

Finally, for simplicity we assume the banker has constant absolute risk aversion, such that $\rho(e) = \rho$. Although this would imply a specific functional form for the banker's utility function, we still confine our problem to maximising (the approximation of) the certainty-equivalent payoff, since we don't have enough information about the structure of the noise variable x.

With that, given the initial wage contract, the banker maximises

$$\alpha + \beta e - \frac{1}{2}\rho\beta^2\sigma_x^2 - c(e)$$

and the old optimal effort $e^*(\beta)$ satisfies

$$\beta = c'(e^*(\beta))$$

Since $c'(\cdot)$ is monotone, its inverse is also monotone and therefore $e^*(\beta)$ is increasing in β. This makes sense: the higher β is, the higher the marginal return to effort. Therefore, without assuming anything further about $c(\cdot)$, we know that the new wage contract with the coefficient on e being $\frac{\beta}{2}$ will lead to a lower optimal level of effort.

(b)

If by the phrase 'the participation constraint keeps unchanged $\bar{U} \leq U$' we mean that the new wage scheme keeps the banker on their participation constraint, then we have

$$\left.\begin{array}{l} \overbrace{\alpha + \beta e^*}^{\mathbb{E}[w^*|e^*]} - \frac{1}{2}\rho\beta^2\sigma_x^2 - c(e^*) = \bar{U} \\ \underbrace{2\alpha + \frac{\beta}{2}e^{**}}_{\mathbb{E}[w^{**}|e^{**}]} - \frac{1}{8}\rho\beta^2\sigma_x^2 - c(e^{**}) = \bar{U} \end{array}\right\} \implies \mathbb{E}[w^{**}|e^{**}] - \mathbb{E}[w^*|e^*] = \underbrace{-\frac{3}{8}\rho\beta^2\sigma_x^2}_{<0} + \underbrace{c(e^{**}) - c(e^*)}_{<0} < 0$$

where we used $e^{**} < e^*$ and $c'(\cdot) > 0$. This means the new expected wage is lower than the old expected wage.

(c)

Even though the coefficient on e is the same as in the initial contract in cases where the cap is not binding, the banker makes ex-ante decisions knowing that there is a non-zero chance his average (not marginal) return to effort will be lower than β due to hitting the cap. Therefore, when maximising ex-ante expected utility, the bonus cap has an effect analogous to that of a lower coefficient on e as in the second wage contract, and the optimal effort is lower than before. This brings the banker further from productive efficiency. However, as in the case where the coefficient on e is reduced, allocative efficiency is improved since the risk-neutral bank bears more of the risk from the noise variable x. (The bank captures the benefit, really, since it bears the 'upside risk' through the bonus cap. This is still allocatively more efficient since the banker's utility is diminishing in gains while the bank's profits are linear.)