Part IIB Paper 4 Game Theory Supervision 5

This supervision is intended to be given in week 3 of the Lent Term.

1. A seller and buyer of an indivisible good have, respectively, production cost v_s and value v_b. v_s and v_b are independently uniformly distributed on $[0, 1]$ and are private information. If they trade at price p their payoffs are respectively $p - v_s$ and $v_b - p$ and are zero if they do not trade. They play a Double Auction and use the equilibrium in affine strategies discussed in the lectures. Show that some types of each player use weakly dominated strategies. Which types? Explain why these strategies are nevertheless compatible with equilibrium. Describe a Bayesian equilibrium strategy profile, with strategies continuous in types, in which no type of either player uses a dominated strategy and explain why it's an equilibrium.

2. In the model of Q1, consider a trading mechanism (not a Double Auction) with an ex-post efficient Bayes-Nash equilibrium. By redefining the type of the seller as $t \equiv 1 - v_s$, show that the expected equilibrium payoff of a seller with value (i.e., production cost) v_s is equal to $0.5(1 - v_s)^2$ plus the equilibrium expected payoff of the type with the highest cost. By using equivalence with a Groves mechanism, explain carefully why no such equilibrium can be *ex ante* budget balanced and interim incentive-compatible.

3. A seller (S) faces a single buyer (B), who has value v for the good. v takes the value 4 with probability 0.8 and 3 with probability 0.2, and is private information to B. Both agents are expected utility maximizers. There are two periods. In period 1 S asks a price p_1 and B either accepts or rejects it. If p_1 is rejected S names a second price p_2 in period 2, which B accepts or rejects. If trade takes place at price p in period t, their payoffs are respectively $\delta^{t-1}p$ and $\delta^{t-1}(v - p)$, where $\delta \in (0, 1)$. If B rejects both prices no trade takes place and both get payoff zero. In each period the price named by S must be either 3 or 3.5.

(a) Describe (fully and carefully) a pure strategy Weak Perfect Bayesian Equilibrium in which, on the equilibrium path, the second equilibrium price is less than the first. Explain why it's an equilibrium.

(b) Describe a pure strategy Weak Perfect Bayesian Equilibrium in which the second equilibrium price is equal to the first.

In each case, describe the set of discount factors for which the equilibrium exists.