Unemployment and Labour Markets
Supervision 1

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Question 1

One form of the Shapiro-Stiglitz model is as such. An employed worker earns wage w, and in order to be productive, he must exert effort $\bar{e}$ which is a disutility to him. When not employed, he gets unemployment benefits b. If the worker chooses to shirk, there is a probability π that he gets caught and becomes unemployed. Therefore shirking gives the expected utility

$$\pi z + (1-\pi)w \tag{1}$$

where z is the value outside of his current employment. z is affected by the probability of finding another job (p), wages in other employment w^*, and unemployment benefits, through the following relationship:

$$z = pw^* + (1-p)b \tag{2}$$

Substituting (2) into (1), the expected utility from shirking is

$$\begin{aligned} &\pi[pw^* + (1-p)b] + (1-\pi)w \\ =&\pi pw^* + \pi(1-p)b + (1-\pi)w \end{aligned}$$

and the utility from exerting effort is

$$w - \bar{e}$$

Assuming that a firm wants to make it such that the worker chooses to exert effort, it must set wages such that

$$w - \bar{e} \geq \pi pw^* + \pi(1-p)b + (1-\pi)w$$

which is the no-shirking condition. In aggregate it is assumed that all firms are identical, and therefore $w^* = w$. The no-shirking condition then reduces to

$$\begin{aligned} w - \bar{e} &\geq \pi pw + \pi(1-p)b + (1-\pi)w \\ w - \pi pw - (1-\pi)w &\geq \bar{e} + \pi(1-p)b \\ [1 - \pi p - (1-\pi)]w &\geq \bar{e} + \pi(1-p)b \\ \pi(1-p)w &\geq \bar{e} + \pi(1-p)b \\ w &\geq b + \frac{\bar{e}}{\pi(1-p)} \end{aligned}$$

and now supposing the probability of finding a job p depends negatively on the unemployment rate u, in particular $p = 1 - u$, the no-shirking condition becomes

$$w \geq b + \frac{\bar{e}}{\pi u}$$

To determine labour demand, we let firms maximize their profits based on some production function $F(\bar{e}L)$ where $F'(\cdot) > 0$ and $F''(\cdot) < 0$. Therefore firms maximize

$$\Pi = \max_L \{F(\bar{e}L) - wL\}$$

with the first order condition

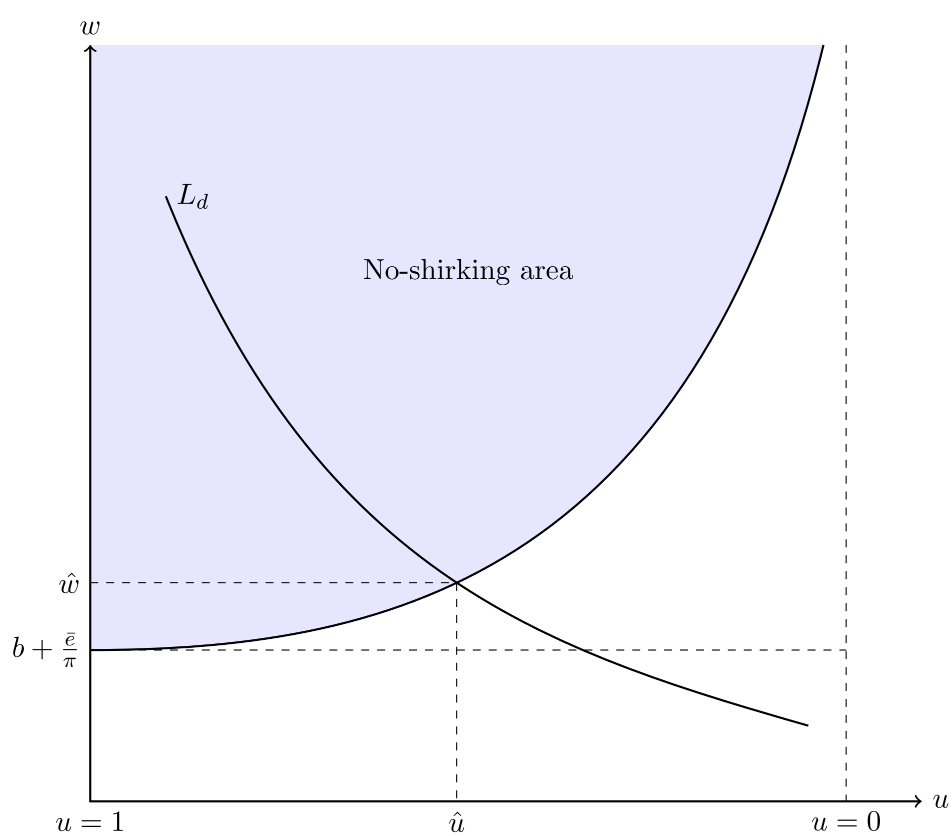
$$\bar{e}F'(\bar{e}L) = w$$

which is downward sloping in (w, L) space since $F''(\cdot) < 0$. Letting the labour demand that fulfills this condition be $L_d(w)$, the unemployment rate is determined by

$$u(w) = \frac{L_s - NL_d(w)}{L_s}$$

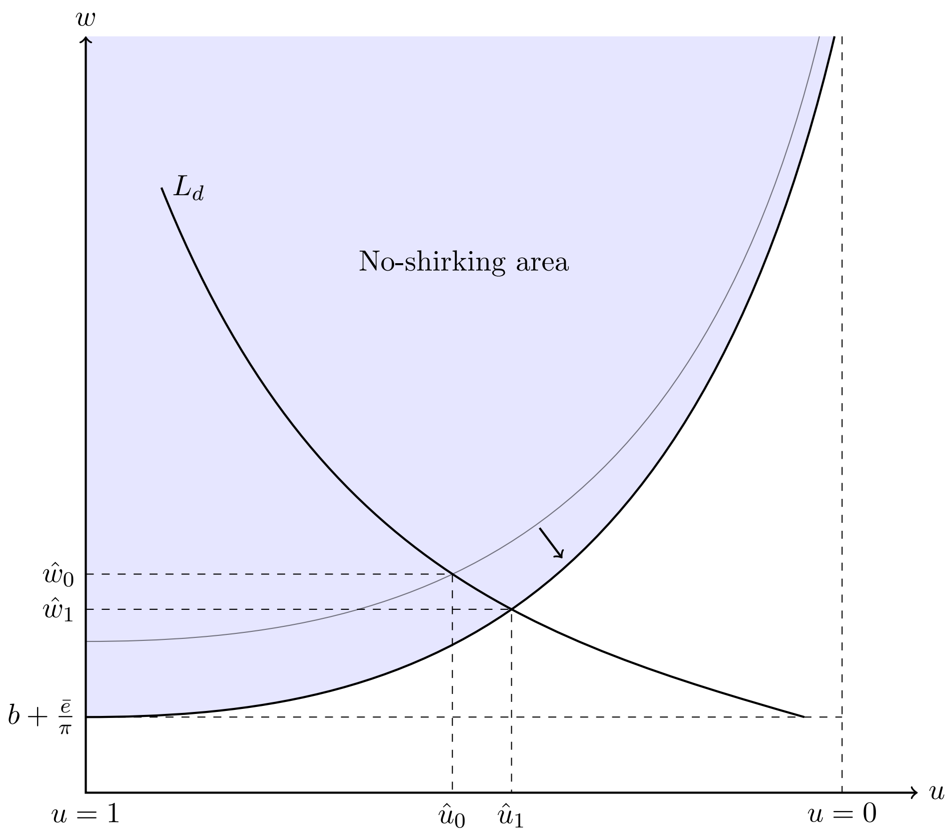
where L_s is the inelastic labour supply and N is the number of firms. Since $L_d(w)$ is decreasing in w, $u(w)$ is increasing in w.

Graphically, the equilibrium unemployment and wage is as such



(a)

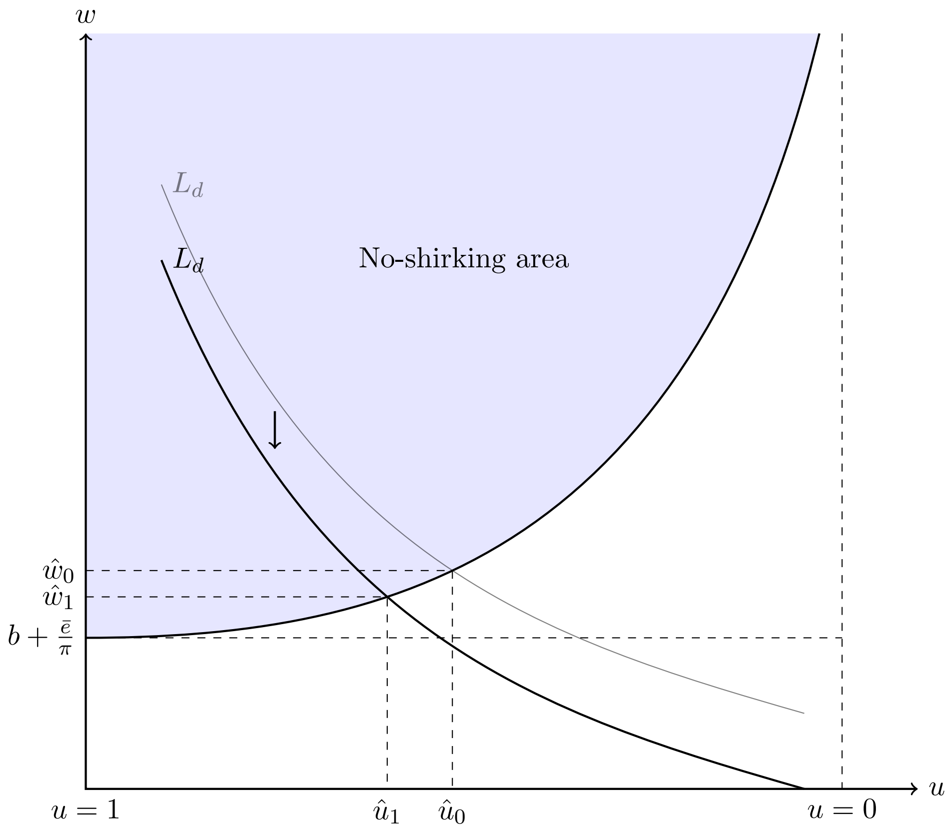
A decrease in unemployment benefits shifts the curve bounding the no-shirking area down by Δb, as such:



In equilibrium, wages fall from $\hat{w}_0$ to $\hat{w}_1$ while unemployment falls from $\hat{u}_0$ to $\hat{u}_1$. Essentially the lack of benefits makes shirking more dangerous, and wages no longer have to be as high to ensure there is no shirking. Firms are then willing to hire more workers given that they do not have to pay as much for a given level of effort.

(b)

A negative productivity shock reduces the marginal productivity of labour, that is, $F'(\bar{e}L)$ decreases for all values of L. Given that the firms seek to meet the condition $\bar{e}F'(\bar{e}L) = w$, it means that at every level of L they are now only willing to pay a lower wage w, and $L_d(w)$ shifts down.



The recession leads to a fall in wages from $\hat{w}_0$ to $\hat{w}_1$, and unemployment rises from $\hat{u}_0$ to $\hat{u}_1$, which is expected in a recession.

(c)

An increase in the probability that a shirker is detected has the exact same effects as (a) in all but magnitude. When π increases, the expected benefit of shirking decreases, and this shifts the curve bounding the no-shirking area down by $\frac{\bar{e}}{\pi+\Delta\pi} - \frac{\bar{e}}{\pi}$.

Question 2

(a)

Let the output price be normalized to 1. The firm seeks to maximize

$$\begin{aligned} \pi &= Y - wL \\ &= K^{\frac{1}{3}} L^{\frac{2}{3}} - wL \end{aligned}$$

and the first-order condition is

$$w = \frac{2}{3}\left(\frac{K}{L}\right)^{\frac{1}{3}}$$

Therefore the labour demand in this economy is

$$L_d = \frac{8K}{27w^3}$$

(b)

The real wage is where $L_d = L_s = 1000$ where $K = 1000$. Therefore

$$\frac{8000}{27w^3} = 1000$$
$$w^3 = \frac{8}{27}$$
$$w = \frac{2}{3}$$

There is full employment by definition of the equilibrium, output is $Y = K^{\frac{1}{3}}L^{\frac{2}{3}} = 1000$, and the total amount earned by workers is $\frac{2000}{3}$ or $\frac{2}{3}$ of the total output since this is a Cobb-Douglas production function.

(c)

Since prices are normalized to 1 the minimum wage is now $w = 1$. This is higher than the real wage in equilibrium which was $\frac{2}{3}$.

(d)

Now it is L_d which is endogenous and w which is exogenously given as 1. Hence

$$L_d = \left\lfloor \frac{8000}{27} \right\rfloor = 296$$

Employment is now 296, output is $Y = 1000^{\frac{1}{3}} \cdot 296^{\frac{2}{3}} \approx 444$, and the total amount earned by workers is just $w \cdot L_d = 296$.

(e)

At any one point in time the government's policy will have raised the standards of living of 296 of the working class while impoverishing the other 704.

(f)

No. Firstly, there are enough case studies reporting a negligible effect on employment with modest increases in the minimum wage, so there is at least some reason to be skeptical of the theory's applicability to all cases. Secondly, wages are taken to be independent of labour demand, which is inaccurate in firms with substantial market power. Standard analyses of monopsony power can show that there are levels of the minimum wage which would increase both employment and wages. Lastly, labour is taken to be inelastically supplied in this model and there is no intertemporal consumption/leisure tradeoff. In reality those earning lower wages (who are also likely to be liquidity constrained) are likely to have a higher marginal propensity to consume. It seems plausible to think of some model which could incorporate different classes of workers and owners of capital, and establish a link between the wages of low-wage workers and aggregate demand, such that a minimum wage would not result in much change in employment due to the boost to aggregate demand.

Question 3

The agent faces the following problem

$$\max_{C,\ell}\left\{\frac{C^\gamma-1}{\gamma}+\frac{\ell^\gamma-1}{\gamma}\right\}\text{ subject to }C=wL(1-\tau)-T$$
$$\ell+L=1$$

The two constraints can be combined into one, yielding $C=w(1-\ell)(1-\tau)-T$. Therefore, the Lagrangian for this problem is

$$\mathcal{L}=\frac{C^\gamma-1}{\gamma}+\frac{\ell^\gamma-1}{\gamma}-\lambda[C-w(1-\ell)(1-\tau)+T]$$

and the first order conditions are

$$\frac{\partial\mathcal{L}}{\partial C}=C^{\gamma-1}-\lambda=0 \tag{1}$$
$$\frac{\partial\mathcal{L}}{\partial \ell}=\ell^{\gamma-1}-\lambda w(1-\tau)=0 \tag{2}$$
$$\frac{\partial\mathcal{L}}{\partial \lambda}=C-w(1-\ell)(1-\tau)+T=0 \tag{3}$$

Dividing (1) by (2),

$$\left(\frac{C}{\ell}\right)^{\gamma-1}=\frac{1}{w(1-\tau)}$$
$$C=\frac{\ell}{[w(1-\tau)]^{\frac{1}{\gamma-1}}} \tag{4}$$

and substituting (4) into (3),

$$\frac{\ell}{[w(1-\tau)]^{\frac{1}{\gamma-1}}}-w(1-\ell)(1-\tau)+T=0$$
$$\frac{\ell}{[w(1-\tau)]^{\frac{1}{\gamma-1}}}+w(1-\tau)\ell-w(1-\tau)+T=0$$
$$\left\{\frac{1}{[w(1-\tau)]^{\frac{1}{\gamma-1}}}+w(1-\tau)\right\}\ell-w(1-\tau)+T=0$$
$$\frac{1+[w(1-\tau)]^{\frac{\gamma}{\gamma-1}}}{[w(1-\tau)]^{\frac{1}{\gamma-1}}}\ell=w(1-\tau)-T$$
$$\ell=\frac{[w(1-\tau)]^{\frac{\gamma}{\gamma-1}}}{1+[w(1-\tau)]^{\frac{\gamma}{\gamma-1}}}-\frac{[w(1-\tau)]^{\frac{1}{\gamma-1}}}{1+[w(1-\tau)]^{\frac{\gamma}{\gamma-1}}}T$$

and finally with $\ell+L=1$,

$$L=1+\frac{[w(1-\tau)]^{\frac{1}{\gamma-1}}}{1+[w(1-\tau)]^{\frac{\gamma}{\gamma-1}}}T-\frac{[w(1-\tau)]^{\frac{\gamma}{\gamma-1}}}{1+[w(1-\tau)]^{\frac{\gamma}{\gamma-1}}}$$

(a)

With $tau = 0$ and $\gamma = \frac{1}{2}$, the labour supply reduces to

$$L = 1 + \frac{w^{-2}}{1 + w^{-2}} T - \frac{w^{-1}}{1 + w^{-1}}$$

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