Part IIB Paper 4 Game Theory Supervision 3

Michaelmas 2019

1. In the All-Pay auction with two bidders and independent private values, both uniform on $[0, 1]$, show that there exists a unique symmetric Bayes-Nash equilibrium in which the bidders use a strictly increasing differentiable bidding function. [Assume that j uses such a strategy $b(v_j)$, with inverse function $g(.)$. Write down the first-order condition for i's best response, as a function of i's value v_i. Then assume that this optimal response is $b(.)$, i.e., the same as j's strategy, and so solve for the function b.]

2. In the model of question 1, assume that the seller does not know the valuations but each bidder knows the other bidder's valuation (as well as his own). Assume that a bidder, having bought the good, is able to sell the good, if he chooses, to the other bidder at a price equal to the latter's valuation. For each of the four standard auctions, find a pure strategy equilibrium. Is it the unique pure strategy equilibrium? What is the seller's expected revenue in equilibrium? What is the seller's expected revenue from the revenue-maximizing posted price? Would it make sense for the seller to set an entry fee followed by a first- or second-price auction (assuming the bidders use undominated strategies)?

3. Two players have a surplus of v to share. They play a finite-horizon complete-information discrete-time bargaining game in which player 1 makes all the offers. There is no discounting and payoffs are linear, i.e., they get $(x, v - x)$ if they agree at date t on the split $(x, v - x)$.

(a) Find a subgame-perfect Nash equilibrium (SPNE) in which there is no delay in agreement and show that it is a SPNE.

(b) Find another pure strategy SPNE.

(c) Find a Nash equilibrium in which the payoff pair is $(v/4, 3v/4)$.

4. Suppose that two bargainers play a game which is the same as the Rubin-

stein infinite-horizon alternating-offers game except that player 1 offers at dates $1, 2, 4, 5, 7, 8, ..$ and player 2 offers at dates $3, 6, 9, ...$ Payoffs from an agreement $(x, v - x)$ at date t are $(\delta^{t-1}x, \delta^{t-1}(v - x))$. Construct an SPNE.

5. (a) For the Rubinstein alternating-offers game with linear utility, size of surplus v and asymmetric discount factors ($\delta_1 \neq \delta_2$), derive the subgame-perfect equilibrium.

(b) Suppose instead that offers are made at dates $\Delta, 2\Delta, 3\Delta, ...$ Let the discount factors over a period of length Δ be $\delta_1(\Delta)$ and $\delta_2(\Delta)$ and let $\delta_1(\Delta) = exp(-r_1\Delta)$ and $\delta_2(\Delta) = exp(-r_2\Delta)$. Find the limit of the subgame-perfect equilibrium payoffs as the time between offers vanishes, i.e., $\Delta \to 0$.

(c) Show that for some $\alpha \in (0, 1)$ and disagreement payoff pair (d_1, d_2), this payoff pair maximizes the generalized Nash product $(u_1 - d_1)^\alpha(u_2 - d_2)^{1-\alpha}$ subject to feasibility, i.e., $u_1 + u_2 \leq v$.