

Triplos, Part 2A, Paper 3
Supervision 3

1. An investigator analysing the relationship between food expenditure, disposable income and prices across a random sample of 25 counties in the UK estimates the relationship (Figures in parentheses are standard errors)

$$\log(\text{FOOD}) = \begin{matrix} 4.7377 + \\ (0.6805) \end{matrix} \begin{matrix} 0.3506\log(\text{PDI}) - \\ (0.0899) \end{matrix} \begin{matrix} 0.5086\log(\text{PRICE}) \\ (0.1010) \end{matrix}$$

where

FOOD Average household expenditure on food

PDI Average Personal disposable income

PRICE The Average price of food deflated by a general price index

Suppose that the Gauss-markov assumptions hold.

- (i) Give an economic interpretation of the coefficients on $\log(\text{PDI})$ and $\log(\text{PRICE})$
- (ii) Test the hypothesis (using a 5% significance level) that the coefficient of $\log(\text{PRICE})$ is equal to zero against the alternative that it is nonzero.
- (iii) Test the hypothesis (using a 5% significance level) that the coefficient of $\log(\text{INCOME})$ is equal to 1 against the alternative that is significantly different from 1.

You are now given the following extra information

$$\text{SST} = \sum_{i=1}^n (y_i - \bar{y})^2 = 0.52876$$

$$\text{SSR} = \sum_{i=1}^n e_i^2 = 0.0046276$$

- (iv) Compute SSE and R^2 for the above regression
- (v) Test the joint hypothesis (at the 5% level) that the two 'slope' coefficients are all equal to zero against the alternative that at least one 'slope' coefficient is non-zero.

2. Download the dataset wage2.dta

Use the "des" and "summ" commands to understand the structure of the data and the meaning of the variable labels. Now answer the following questions relating to performance on the IQ test for this sample of working age women. The IQ test was taken as an adult after the woman had completed her formal education.

- (a) Run a regression of IQ test score on parents' education. What do you conclude?
- (b) Suppose I want to know whether the only way parents' education increases their daughter's test score is through the daughter's own education. How would you test that hypothesis?
- (c) What do you conclude from your test of part (b)?
- (d) Is there any policy recommendation from your results?
- (e) Suppose I wish to know whether the impact of own and parents' education on IQ test score varies by race. Conduct such a test of hypothesis and report your finding.

3. (Consistency) Let us revisit the following problem you did in the previous supervision sheet. Consider the regression model $Y_i = \beta_0 + \beta_1 X_i + U_i$ for an I.I.D. sample with $N = 1000$ observations. Suppose $U_i \sim \text{I.I.D. } (0, \sigma^2)$ and the X_i are I.I.D. for $i = 1, 2, \dots, 1000$, and that

X_i is independent of U_i . Let $\hat{\beta}_1$ denote the OLS estimator of β_1 , and consider another estimator $\tilde{\beta}_1$ of β_1 , constructed in the following way:

$$\tilde{\beta}_1 = \frac{Y_3 + Y_1 - 2Y_2}{X_3 + X_1 - 2X_2}.$$

You can assume that X_i are continuously distributed and that $X_3 + X_1 - 2X_2$ never takes the value 0.

Is $\tilde{\beta}_1$ a consistent estimator of β_1 ? Why?