1. Beta$(1,1)$ is actually a uniform distribution.

Normal distribution assumptions can be violated either because of narrow tails or because of skewedness.

Weibull distribution.

$Exp(\lambda) = Gamma(1, \lambda)$

Poisson related to negative binomial distribution.

$$\chi^2_K = Gamma\left(\frac{K}{2}, \frac{1}{2}\right)$$

2. Joint densities and variable transformations:

$$g_{U,V}(u,v) = f_{X,Y}(x(u,v), y(u,v)) \cdot |J|$$

Then independence of U and V follows from $g_{U,V}(u,v)$ factorisation.

U and V distributions can be obtained by integrating out variables from $g_{U,V}(u,v)$.

9. First establish posterior distribution. Only then deal with the loss function.

Proportionality argument for posterior distribution.

For $f_{\theta|X}(\theta|x)$ only ~~$\theta^{\alpha-1} e^{-\lambda\theta}$~~

$$\theta^{n+\alpha-1} \cdot e^{-\lambda\theta} \cdot \left(\prod_{i=1}^{n} x_i\right)^{\theta-1}$$

terms are not constant with respect to θ.

Try to see which distribution function is fully defined by these non-constant terms.

$$\theta|X \sim \text{Gamma}\left(n+\alpha, \lambda - \sum_{i=1}^{n} \ln(x_i)\right)$$

Otherwise get a Gamma function integral in the denominator.

For loss functions:

$$h(a) = \int (a-\theta)^2 \pi(\theta|X)\, d\theta$$

$$h'(a) = \int 2(a-\theta)\, \pi(\theta|X)\, d\theta$$

$h'(a) = 0$ if

$$a\int \pi(\theta|X)\, d\theta = \int \theta\, \pi(\theta|X)\, d\theta$$

$$\Rightarrow \hat{\theta} = \int \theta\, \pi(\theta|X)\, d\theta$$ minimises the loss function

Either obtain posterior mean from a clear distribution or do integration.

Here, $\hat{\theta} = \dfrac{n+\alpha}{\lambda - \sum_{i=1}^{n} \ln(x_i)}$

Look up means and variances of ~~tra~~ traditional distributions.

11. Think about the question as a variable transformation: $(X_i) \to (\mathcal{E}_i)$. Justifies taking joint density for $(\mathcal{E}_i)$.