Supervision Questions 2

Economic Growth

Macroeconomics Part IIB, Paper 2

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Michaelmas 2019

1 Problems

1. Based on question 8 of the 2007 ECONOMICS TRIPOS Part IIB

   Consider the following three sector model of the economy developed by Romer (1990). Competitive firms (indexed by i) in the final goods sector maximise profits by producing output Y; by hiring labour L_Y and a number of different capital goods x_j taking the wage w and rental prices p_j as given:

$$\max_{Y_i, L_{Y,i}, x_{j,i}} Y_i - wL_{Y,i} - \sum_{j=1}^{A} p_j x_{j,i}, \tag{1}$$

   subject to

$$Y_i = L_{Y,i}^{1-\alpha} \sum_{j=1}^{A} x_{j,i}^{\alpha}, \ \alpha \in (0,1). \tag{2}$$

   A is the number of capital goods that are available to the final goods sector at any point in time. Note that the price of final output is normalised to 1 and capital goods depreciate fully after one period. Producers in the intermediate capital goods sector own the monopoly right (patent) to produce capital goods, x_j. Intermediate producers convert

one unit of existing capital (rented at interest rate r) into intermediate capital goods. They maximise profits according to:

$$\max_{x_j} \pi_j = p_j(x_j)x_j - rx_j. \tag{3}$$

where $p_j(x_j)$ is the demand function for the capital good in the final goods sector. Scientists in the research sector produce new patents using labour, L_A, and existing technology, A:

$$\dot{A} = L_A A. \tag{4}$$

Assume that labour markets clear, so that $L_Y + L_A = L = 1$

(a) Give an economic interpretation of the production function of final output (2). Derive the first-order conditions characterising the wage w and rental price of intermediate capital goods p_j paid by each final goods producer.

(b) Solve for the optimal rental price p_j charged by intermediate firms producing capital goods x_j. Solve for profits π and aggregate output Y as a function of the aggregate capital stock $K = \sum_{j=1}^{A} x_j$. (Hint: each intermediate producer produces the same amount $x_j = x$). Provide a brief economic interpretation of the solutions.

(c) Using an arbitrage equation for returns in the research sector and the interest rate r, solve for the optimal price of a patent P_A. Give an intuitive explanation.

(d) Derive the equilibrium interest rate r as function of parameters and the output/capital ratio Y/K. Give an intuitive explanation of the result.

2. Consider a modified version of the continuous-time Solow growth model where the aggregate production function depends also on land, which is a fixed factor. Precisely,

$$F(K, L, Q) = K^{\alpha} L^{\beta} Q^{1-\alpha-\beta}, \quad \alpha, \beta \in (0, 1), \text{ and } \alpha + \beta < 1,$$

where K represents the capital stock, which depreciates at rate δ; L is labour; and Q is land, available in fixed inelastic supply, $Q \leq \bar{Q}$. Households save a constant fraction of income, $s \in (0, 1)$. The economy is closed which implies that investment equals savings.

Suppose that there is no population growth.

(a) Find the steady-state capital per capita. Does the economy converge to this steady-state equilibrium for any initial capital per capita $k_0 > 0$? What is the speed of convergence around the steady-state? Does it depend on the stock of land? Explain.

Next, suppose that there is population growth at rate n, that is, $\frac{\dot{L}}{L} = n$.

b. What happens to the capital-labour ratio and to output per capita as $t \to \infty$?

c. What happens to returns to land and the wage rate as $t \to \infty$? Would you expect the population growth rate n to change over time in this economy? Explain.

2 Essay

1. What are the short and long run growth effects on an economy of raising the fraction of inputs devoted to domestic R&D? Why and how could other economies be affected by such a policy?

(Question 5, TRIPOS 2008)

Main Readings

- Jones (2002), *Introduction to Economic Growth*. Norton. 2nd edition.
- Jones (2004), "Growth and Ideas." *Handbook of Economic Growth*. Available at: `http://elsa.berkeley.edu/users/chad/handbook200.pdf`
- Symposium on Endogenous Growth, *Journal of Economic Perspectives*, Winter 1994 issue.

Further Readings

- Jones (1995), "R&D-Based Models of Economic Growth." *Journal of Politcal Economy* 103 (Aug): 759-84.

- Romer (1990), "Endogenous Technological Change." *Journal of Political Economy* 98 (Oct): S71-S102.

-Sorensen, P. & H. Whitta-Jacobsen (2005). *Introducing Advanced Macroeconomics: Growth and Business Cycles*, McGraw-Hill.