Paper 10 Time Series Models 2019-2020

Supervision Questions

Supervision 4. Multivariate Time Series Models. VAR models. Engle-Granger and ECMs.

Note Critical values for cointegration tests were given in a paper by Engle and Yoo then updated by Mackinnon. These will be provided in the exam but can be found in Hamilton's text book as well as on the web or in the original articles

(1) Write a VAR(1) in two variables as

$$\begin{pmatrix} a_{11}^0 & a_{12}^0 \\ a_{21}^0 & a_{22}^0 \end{pmatrix} \begin{pmatrix} x_{1t} \\ x_{2t} \end{pmatrix} = \begin{pmatrix} \gamma_1 \\ \gamma_2 \end{pmatrix} + \begin{pmatrix} a_{11}^1 & a_{12}^1 \\ a_{21}^1 & a_{22}^1 \end{pmatrix} \begin{pmatrix} x_{1t-1} \\ x_{2t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix}$$

where $\begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix} \sim N(0, \Sigma)$ with $\Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix}$

How many restrictions are needed to identify this model?

How you would estimate the model given the following sets of identifying assumptions

(i) $a_{21}^0 = a_{12}^0 = 0$, $a_{11}^0 = a_{22}^0 = 1$

(ii) $a_{11}^0 = a_{22}^0 = 1$, $a_{21}^0 = 0$ and $\sigma_{12} = 0$

How would you interpret the dynamic relationship between the variables in these two models?

(2) Write a structural form for a VAR in two variables as

$$A_0 x_t = A_1 x_{t-1} + \varepsilon_t$$

where A_0, A_1 are matrices and $x_t = (x_{1t}, x_{2t})'$ and ε_t is vector white noise. What problems occur in trying to estimate the structural form? Show how the structural form can be recovered from estimation of the reduced form by imposing restrictions on $\Sigma = E(\varepsilon_t \varepsilon_t')$ and A_0. If a Sims type causal ordering is used explain why the impulse response functions are not invariant to the ordering of elements of the vector x_t.

(3) Time series variables x_{1t} and x_{2t} satisfy the VAR equation

$$\begin{pmatrix} x_{1t} \\ x_{2t} \end{pmatrix} = \begin{pmatrix} 0.5 & -1.0 \\ -0.25 & 0.5 \end{pmatrix} \begin{pmatrix} x_{1t-1} \\ x_{2t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix}$$

where $\varepsilon_t = \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix}$ is a serially uncorrelated stationary process.

Define the matrices $A = \begin{pmatrix} 0.5 & -1.0 \\ -0.25 & 0.5 \end{pmatrix}$ and $L = \begin{pmatrix} 1 & -2 \\ 0.5 & 1 \end{pmatrix}$

(a) Show that $LAL^{-1} = \Lambda = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

(b) Show that if we define $\begin{pmatrix} y_{1t} \\ y_{2t} \end{pmatrix} = L \begin{pmatrix} x_{1t} \\ x_{2t} \end{pmatrix}$ then $\begin{pmatrix} y_{1t} \\ y_{2t} \end{pmatrix}$ satisfies the VAR

$$\begin{pmatrix} y_{1t} \\ y_{2t} \end{pmatrix} = \Lambda \begin{pmatrix} y_{1t-1} \\ y_{2t-1} \end{pmatrix} + L \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix}$$

and hence show that x_{1t} and x_{2t} are $I(1)$ variables.

(c) Define $\Pi = A - I$ (where I is a 2×2 identity matrix). Show that $det(\Pi) = 0$ and further that we may write

$$\Pi = \alpha\beta^{'}$$

where $\alpha = \begin{pmatrix} -0.5 \\ -0.25 \end{pmatrix}$ and $\beta = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

(d) Show that if we write $x_t = \begin{pmatrix} x_{1t} \\ x_{2t} \end{pmatrix}$ then $\beta^{'} x_t$ satisfies the equation

$$\beta^{'} x_t = \beta^{'} \varepsilon_t$$

(e) Explain what this means for the variables x_{1t} and x_{2t}

(4) A researcher has annual data on stock market returns (ret_t) and dividend yields (div_t) and estimates the following VAR over the period 1900-2002 (ie 103 observations)

$$\begin{aligned} ret_t &= \alpha_1 + \alpha_2 div_{t-1} + \alpha_3 ret_{t-1} + u_t^1 \\ div_t &= \alpha_4 + \alpha_5 div_{t-1} + \alpha_6 ret_{t-1} + u_t^2 \end{aligned}$$

and obtains the following estimates:

Coefficient	Estimate	Standard Error
α_1	-0.0663	0.062
α_2	0.0274	0.013
α_3	0.1671	0.109
α_4	0.8954	0.344
α_5	0.7790	0.073
α_6	0.6224	0.599

(i) Use these results to test for Granger Causality between ret_t and div_t.

On the basis of these result she decides the VAR model can be reduced to the following system

$$\begin{aligned} ret_t &= \beta_1 + \beta_2 div_{t-1} + v_t^1 \\ div_t &= \beta_3 + \beta_4 div_{t-1} + v_t^2 \end{aligned}$$

(ii) Show that this system implies a univariate ARMA(1,1) representation for ret_t.

She then estimates a univariate ARMA(1,1) process for ret_t over the same period and obtains the following results

$$\begin{aligned}
\widehat{ret}_t &= \underset{(0.006)}{0.0649} + \underset{(0.053)}{0.854} ret_{t-1} + e_t - \underset{(0.014)}{0.977} e_{t-1} \qquad (*) \\
R^2 &= 0.0499 \quad SBC = -0.336 \quad Obs = 103 \quad log: L = 24.3
\end{aligned}$$

where SBC is Schwarz's Bayesian Criterion defined as $SBC = -\frac{2\log(L)}{T} + k\frac{logT}{T}$ where L is the equation likelihood, k the number of coefficients estimated and T the number of observations.

For comparison she also estimates an ARMA(0,0) for ret_t over the same period and obtains

$$\begin{aligned}
ret_t &= \underset{(0.019)}{0.0621} \\
R^2 &= 0.000 \quad SBC = -0.375 \quad Obs = 103 \quad logl = 21.6
\end{aligned}$$

(iii) Test by likelihood ratio (at the 5% level) the hypothesis that the coefficients in the ARMA(1,1) (apart from the constant) are zero. What does this imply about return predictability?

(iv) Explain how the Schwarz Bayesian Criterion may be used here as a method of model selection. Which ARMA model is preferred? What does this imply about return predictability?

(v) By noting that the AR and MA coefficients in (*) are of comparable size but opposite signs explain how the unpredictability of the return series implied by the univariate ARMA analysis is consistent with your conclusions about Granger Causality.

(5) 2007 Paper 10 A5

(6) 2009 Paper 10 A1