Physica C: Superconductivity and its applications 595 (2022) 1354030

Contents lists available at ScienceDirect

Physica C: Superconductivity and its applications

journal homepage: www.elsevier.com/locate/physc

Critical temperatures and critical currents of wide and narrow quasi-one-dimensional superconducting aluminum structures in zero magnetic field

V.I. Kuznetsov *, O.V. Trofimov

Institute of Microelectronics Technology and High Purity Materials, Russian Academy of Sciences, Chernogolovka, Moscow Region 142432, Russia

ARTICLE INFO

Keywords:
Quasi-one-dimensional superconducting aluminum structure
Critical superconducting temperature
Critical current density
Temperature-dependent switching critical current
Critical Josephson current
Josephson SNS junction

ABSTRACT

We measured the critical temperatures and critical switching and retrapping currents of wide and narrow thin-film quasi-one-dimensional superconducting aluminum structures of the same thickness in zero magnetic field. For the first time, we found that the narrower the structure, the lower the critical temperature and critical current density in the structure. Probably, the influence of depairing centers that are on dirty longitudinal boundaries of the structure, is the stronger than the narrower the structure. It is found for the first time that, in most cases, the temperature-dependent switching critical current in both structures is approximated by two functions. At temperatures below the temperature corresponding to the bottom of the resistive N-S transition of structures, the switching critical current is described by the Kupriyanov–Lukichev theory. At temperatures close to the top of the N-S transition, the switching current is linear with temperature and coincides with the critical Josephson current. At these temperatures, Josephson SNS junctions are formed in structures.

1. Introduction

Superconducting critical current is the most important characteristic of superconducting and hybrid devices. The critical current of superconducting quasi-one-dimensional (i.e., having transverse dimensions less than twice the superconducting temperature-dependent coherence length $2\xi(T)$) of wires and various structures with weak link has been widely studied [1–4].

In nanodevices, the critical current can have specific features due to thermally activated and quantum fluctuations of the superconducting order parameter, nonlocal and other effects. “Nonlocality” is the mutual influence of electronic transport in different parts of the structure.

In superconducting structures, nonlocal effects appear at a shorter length, close to the coherence length $\xi(T)$ and at a larger length, close to the relaxation length of the quasiparticle charge imbalance $\lambda_Q(T,B)$, which depends on temperature and magnetic field [5].

It is surprising that the critical current and nonlocal (equilibrium and nonequilibrium) phenomena in a long quasi-one-dimensional wire with a total length several times greater than twice the length of the quasiparticle imbalance $2\lambda_Q(T,B)$ and having a transverse narrowing with a length in the range from $\xi(T)$ to $\lambda_Q(T,B)$ are practically not studied. Also, the critical current and nonlocal effects in a long quasi-one-dimensional wire, consisting of wire segments of different widths, have not been studied. It should be expected that the critical current as a function of temperature and magnetic field, measured on a short section of the wire, will depend on the values of the superconducting order parameter in other parts of the current-carrying wire of variable width and in potential quasi-one-dimensional wires.

Quasi-one-dimensional aluminum wires are part of many superconducting nanodevices. It has long been known [6,7] that the thinner the aluminum film, the higher the critical temperature of this film. It is also considered an established fact that the smaller the diameter quasi-one-dimensional superconducting aluminum wire, the higher the critical temperature of the wire. It should be expected that the narrower the quasi-one-dimensional thin-film aluminum wire, the higher the critical temperature. Analyzing new effects in superconducting quasi-one-dimensional aluminum wires of different widths and structures consisting of such wires [8–15], we assumed that another statement is valid for the structures of these works, namely, that the smaller the wire width, the lower the critical temperature of the wire.

This assumption that the narrow parts of the structure have a critical temperature lower than the critical temperature of a wide part of the structure is a necessary condition for the appearance of negative

* Corresponding author.
E-mail address: kuznetcvova@mail.ru (V.I. Kuznetsov).

https://doi.org/10.1016/j.physc.2022.1354030
Received 28 November 2021; Accepted 2 February 2022
Available online 4 February 2022
0921-4534/

nonlocal and local voltages (resistances) in a superconducting quasi-one-dimensional dc-biased aluminum structure at temperatures close to the critical temperature [9].

We believe that the assumption of different critical temperatures of the wide and narrow parts of the circularly-asymmetric aluminum ring will help to remove the long-term intractable challenge, which consists in the unexplained mysterious phase shift in the magnetic field of critical currents of opposite polarities in circularly-asymmetric aluminum structures permeated with a magnetic flux [11,12].

This phase shift of the critical currents is the reason for the appearance of a rectified time-averaged direct voltage $V_{rec}(\Phi/\Phi_0)$, oscillating with a period equal to the superconducting quantum of the magnetic flux Φ_0, in a circularly-asymmetric superconducting aluminum ring (a structure of such rings in a series), permeated with the flux Φ, when an alternating current (with a zero dc component) with an amplitude close to the critical current is passed through this ring (structure) at temperatures slightly below the critical temperature T_c [10–14]. The circular asymmetry of a ring with a constant inner radius r_i is due to the fact that the ratio of the widths of the wide and narrow semirings was usually $w_w/w_n = 2$. Such a ring can be used in an asymmetric dc micro-SQUID [1,16]. Moreover, such a ring and a structure of such rings in series can operate as a highly efficient magnetic field-dependent ac voltage rectifier and as a highly sensitive detector of nonequilibrium electromagnetic noise [10–14].

The voltage $V_{rec}(\Phi/\Phi_0)$ appears due to the difference in critical currents for positive and negative alternating current half-waves. It was found [11,12] that the critical currents of opposite polarities $I_{c+}(\Phi/\Phi_0+\phi_l)$ and $I_{c-}(\Phi/\Phi_0-\phi_l)$ differ not in amplitude, but in phase shift relative to the zero of the flux in different directions by the phase difference $\phi_l = d\Phi/\Phi_0$. The mutual shift of these critical currents relative to each other $2\phi_l$ can reach 0.5. Any significant mutual phase shift relative to each other of critical currents of opposite polarities cannot be obtained from the known geometric and physical parameters of circularly-asymmetric aluminum structures [10–14]. The reason for the appearance of this phase shift has not yet been found. We proposed a model for the appearance of this shift, considering a circularly-asymmetric ring as an asymmetric quantum superconducting interferometer [1,16]. In this model, which will be presented elsewhere, the additional phase shift of the critical currents will be nonzero only if the critical current densities $j_{c1}(T)$ and $j_{c2}(T)$ are not the same in the wide and narrow arms of the interferometer. For a given geometry of circularly-asymmetric rings, this is possible only in a situation where the critical temperatures of the wide and narrow arms of the interferometer T_{cw} and T_{cn}, respectively, differ.

In this work, in order to confirm the assumption about the difference between T_{cw} and T_{cn}, and, consequently, the difference between $j_{c1}(T)$ and $j_{c2}(T)$ of the wide and narrow arms of the interferometer, we fabricated wide and narrow superconducting quasi-one-dimensional aluminum structures with dimensions close to the typical dimensions of a wide and narrow aluminum semirings. We measured the critical temperatures and plotted critical currents as functions of T in wide and narrow superconducting aluminum structures with different measurement circuits in zero magnetic field.

2. Results and discussion

To prove the statement about different critical temperatures of wide T_{cw} and narrow T_{cn} semirings of a circularly-asymmetric aluminum interferometer, the narrow and wide structures (upper and lower insets, Fig. 1) were fabricated on a single silicon chip in one cycle by thermal deposition of an aluminum film with a thickness of $d = 19$ nm using the lift-off process of electron beam lithography. The widths of the narrow and wide parts of both structures are approximately the same and equal to $w_n = 0.27$ and $w_w = 0.48$ μm, respectively. The total length of both narrow and wide quasi-one-dimensional wires constituting the structures reached 60 μm. Long wire length minimizes influence of wide current and potential contacts of the structure.

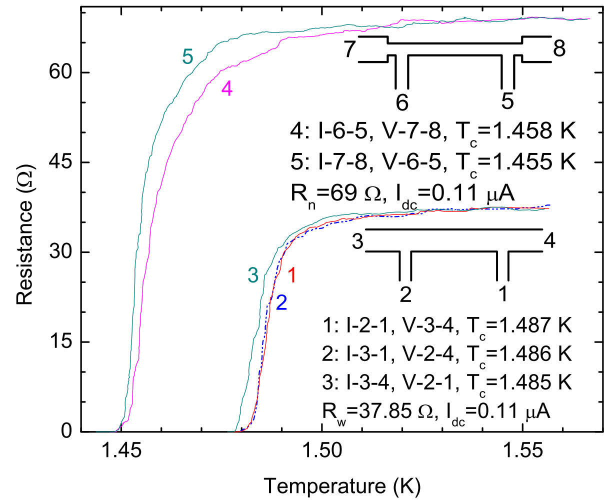

Fig. 1. (Color online) Lines 1, 2 (dash-dotted), and 3 - resistive $R_w(T)$ transitions of the wide structure (bottom inset) at $I_{dc} = 0.11$ μA. Lines 1, 2, and 3 correspond to the measurement circuits: (I-2-1, V-3-4), (I-3-1, V-2-4), and (I-3-4, V-2-1), respectively. Lines 4 and 5 are $R_n(T)$ transitions of the narrow structure (inset at the top) at $I_{dc} = 0.11$ μA. Lines 4 and 5 correspond to the measurement circuits: (I-6-5, V-7-8) and (I-7-8, V-6-5), respectively.

In this work, low voltage electrical signals versus temperature and applied dc were measured in a room shielded from high frequency electromagnetic interference. In order to reduce the influence of low-frequency mains interference and high-frequency noise on the structures under study, we used, in mainly analog devices, trying to exclude digital devices. In addition, homemade low-frequency current generator and dc amplifier powered by galvanic batteries were used. To minimize the effect of noise on the structure, one-kilo-ohm resistances were placed on the chip holder with the structure and connected in series to each current (potential) contact of the structure.

We measured resistive N-S $R_w(T)$ transitions (lines 1, 2, 3 in Fig. 1, below) of the wide structure and $R_n(T)$ transitions (lines 4, 5 in Fig. 1, above) of the narrow structure with different connection of current I and potential V wires. The measurement circuits are shown in Fig. 1. Functions $R_w(T)$ and $R_n(T)$ are recorded in short sections of the same length 6.69 μm at an applied dc $I_{dc} = 0.11$ μA. Note that the total length of the current-carrying parts of both structures for all measurement circuits was approximately 60 μm. The resistances in the normal state of the narrow and wide wires are $R_n = 69$ and $R_w = 37.85$ Ω, respectively.

We have determined the critical temperatures $T_{cl} = T_{cl}(0)$, $T_c = T_c(0.5)$, and $T_{ch}(0.96)$, corresponding to three levels of the N-S transition of the wide structure $R_w(T)/R_w = 0, 0.5$, and 0.96, respectively. The critical temperatures of the wide structure for different measurement circuits are equal for line 1 (I-2-1, V-3-4) - $T_{cl} = 1.480$, $T_c = 1.487$, $T_{ch}(0.96) = 1.521$ K, for line 2 (I-3-1, V-2-4) - $T_{cl} = 1.480$, $T_c = 1.486$, $T_{ch}(0.96) = 1.522$ K, for line 3 (I-3-4, V-2-1) - $T_{cl} = 1.478$, $T_c = 1.485$, $T_{ch}(0.96) = 1.506$ K.

The critical temperatures of the narrow structure are also determined by the three levels of the N-S transition. The critical temperatures for the narrow structure for different measurement circuits are equal for line 4 (I-6-5, V-7-8) - $T_{cl} = 1.449$, $T_c = 1.458$, $T_{ch}(0.96) = 1.500$ K, for line 5 (I-7-8, V-6-5) - $T_{cl} = 1.448$, $T_c = 1.455$, $T_{ch}(0.96) = 1.480$ K.

It can be seen that the critical temperatures of the wide and narrow structures depend on the measurement circuit. Thus, the critical temperatures of wide and narrow structures found in the middle of N-S transitions are 1.487 and 1.458 K, in the case when the measurement circuits (I-2-1, V-3-4) and (I-6-5, V-7-8) are used for wide and narrow structures, respectively (lines 1 and 4, Fig. 1). In this case, the voltage is recorded on both structures using wide wires.

Critical temperatures of the wide and narrow structures found in the middle of the N-S transitions, are equal to 1.485 and 1.455 K, in the case when the wide and narrow structures are measured according to the measurement circuits (I-3-4, V-2-1) and (I-7-8, V-6-5), respectively (lines 3 and 5, Fig. 1). In the case, the voltage is measured on both structures using narrow wires.

We have found that the critical temperature of the narrow structure is less than the critical temperature of the wide structure. We believe that critical temperature of narrow wires of the wide structure is close to the critical temperature 1.455 K of the narrow structure, recorded according to the measurement circuit: I-7-8, V-6-5 (line 5, Fig. 1). Similarly, the critical temperature of wide wires of the narrow structure is close to the critical temperature of 1.487 K of the wide structure, recorded according to the measurement circuit: I-2-1, V-3-4 (line 1, Fig. 1). Thus, at $1.455 < T < 1.487$ K, both wide and narrow structures, taken together with current and potential wires, are heterogeneous normal-superconducting (N-S) structures. This hybridity of structures can lead to new effects. We believe that the true critical temperatures (without taking into account the influence of the proximity effect [5]) have values less than 1.455 K for a narrow wire and values greater than 1.487 K, for a wide wire.

We briefly explain why critical temperatures depend on the measurement circuit. The critical temperatures of wide and narrow structures are higher when the voltage is measured on these structures using wide wires with a higher critical temperature T_{cw} than the critical temperature T_{cn} of a narrow wire. The high density of superconducting electrons n_s in parts of wide wires that do not carry current leads, due to the proximity effect, to an effective increase in n_s and the critical temperature of the structure.

So, we experimentally found that the difference between the critical temperatures of wide and narrow aluminum structures consisting of quasi-one-dimensional superconducting wires of different widths reaches a value greater than 30 mK when the wide and narrow wires of both structures have the same thickness $d = 19$ nm and widths equal to $w_w = 0.48$ and $w_n = 0.27$ μm, respectively.

Earlier, it was experimentally established [6] that critical temperature T_c of the thin aluminum film is much higher than the critical temperature $T_{cb} = 1.194$ K of the bulk superconductor. In [7], the critical temperature of narrow sections of submicron width in a superconducting thin-film aluminum structure of variable width reached 5.8 K. It was found in Chubov et al. [6] that the relative increase in the critical temperature $dT_c/T_{cb} \propto 1/d$ and practically does not depend on the width of w for wide aluminum films with a width of 0.5–1.5 mm. It should be expected that in a quasi-one-dimensional superconducting aluminum wire with a width satisfying the condition $d < w < 2\xi(T)$, the critical temperature will slightly increase with decreasing w. However, for our aluminum structures of the same thickness d, the critical temperature of the narrower structure is slightly lower than the critical temperature of the wider structure.

Here, we propose a mechanism to clarify such a difference in the critical temperatures of narrow and wide aluminum structures of the same thickness. The special feature to fabricate our quasi-one-dimensional superconducting aluminum structures leads to contamination of the longitudinal boundaries of structures. Dirty boundaries can contain magnetic atoms or vacancies that electrons can land on, creating uncompensated electron spin. As a result, the presence of these magnetic atoms and vacancies causes the destruction of superconducting pairs and a decrease in the effective critical temperature of the wire. In our case, the depairing effect of dirty boundaries is stronger in a narrower structure than in a wider structure. Therefore, the critical temperature of the narrow structure is lower than the critical temperature of the wide structure.

The resistances per square of the narrow and wide structures are $R_n^{sqr} = 2.8\ \Omega$ and $R_w^{sqr} = 2.7\ \Omega$, respectively. Resistivities of the narrow and wide structures $\rho_n^n = 5.3\times 10^{-8}\ \Omega$ m and $\rho_n^w = 5.2\times 10^{-8}\ \Omega$ m, respectively, are found from the expression $\rho = R^{sqr}d$. From the refined theoretical expression for aluminum $\rho l_{el} = 5.1\times 10^{-16}\ \Omega\ \mathrm{m}^2$ [17], we obtain the electron mean free paths in narrow and wide structures $l_{eln} = 9.6$ nm and $l_{elw} = 9.9$ nm, respectively.

Structures are in a dirty limit since $l_{el} << \xi_0 = 1.6$ μm. Near T_c, the Ginzburg–Landau coherence length is $\xi(T) = \xi(0)(1 - T/T_c)^{-1/2}$, where $\xi(0) = 0.85(l_{el}\xi_0)^{1/2}$ is the coherence length at $T = 0$ K [5]. The temperature-dependent penetration depth of the magnetic field is $\lambda_{GL}(T) = \lambda(0)(1 - T/T_c)^{-1/2}$, where $\lambda(0) = 0.615\lambda_L(\xi_0/l_{el})^{1/2}$ is the depth penetration of the field at $T = 0$ K, $\lambda_L = 16$ nm is the London penetration depth for aluminum [5]. Since l_{eln} and l_{elw} are narrow and wide structures are close, we get close coherence lengths $\xi_n(0) \approx \xi_w(0) \approx 0.11$ μm and the field penetration depth $\lambda_n(0) \approx \lambda_w(0) \approx 0.13$ μm. Indexes n and w refer to narrow and wide structures, respectively. Temperature-dependent density of the Ginzburg–Landau depairing current is equal to $j_{GL}(T) = j_{GL}(0)(1 - T/T_c)^{3/2}$ [5]. The critical current density at $T = 0$ is $j_{GL}^n(0) \approx j_{GL}^w(0) \approx 6.0\times 10^{10}$ A/m^2 for narrow and wide wires. Critical depairing currents at $T = 0$ for narrow ($w_n = 0.27$ μm, $d = 19$ nm) and wide wires ($w_w = 0.48$ μm, $d = 19$ nm) are equal to $I_{GL}^n(0) = 303$ μA and $I_{GL}^w(0) = 550$ μA, respectively.

In order to prove the statement about different critical current densities of wide $j_{c1}(T)$ and narrow $j_{c2}(T)$ semirings of a circularly-asymmetric aluminum interferometer, we measured the voltage $V(I)$ as a function of the applied dc I in wide and narrow structures at T slightly below T_c in zero magnetic field with different measurement circuits. Note that the voltage was recorded on a short section of a rather long superconducting quasi-one-dimensional aluminum wire. $V(I)$ curves have thermal hysteresis depending on the direction of the current sweep. Using the $V(I)$ curves, we plotted the temperature dependences of the switching and retrapping critical currents in wide and narrow structures. The switching current switches the structure from a superconducting state to a resistive state. The retrapping current is the current at which structure returns from the resistive state into the superconducting state.

For a more accurate comparison of the measured critical switching current with the theory over a wider temperature range, we used the phenomenological expression for temperature-dependent critical current density of the dirty quasi-one-dimensional superconducting wire $j_c(T) = (j_{KL}(0)/4)(1 - (T/T_c)^2)(1 - (T/T_c)^4)^{1/2}$, where $j_{KL}(0) = \left((8\pi^2\sqrt{2\pi})/(21\zeta(3)e)\right)\sqrt{(kT_c)^3/(\hbar v_F \rho_n \rho_n l_{el})}$ is critical current density in the dirty limit at $T = 0$, calculated in Romijn et al. [18] within the framework of the Kupriyanov–Lukichev theory [19]. The expression for $j_c(T)$ is obtained using the phenomenological expressions $\lambda(T) = \lambda(0)/\sqrt{1 - (T/T_c)^4}$ and $H_c(T) = H_c(0)(1 - (T/T_c)^2)$ [5,20].

The critical current density at $T = 0$ K, calculated within the framework of the Kupriyanov–Lukichev theory, is equal to $j_{KL}^n(0) = 7.2\times 10^{10}$ A/m^2 for a narrow wire ($\rho_n^n = 5.3\times 10^{-8}\ \Omega$ m, $T_c = 1.455$ K). For wide wire ($\rho_n^w = 5.2\times 10^{-8}\ \Omega$m, $T_{cw} = 1.486$ K) this critical current density is equal to $j_{KL}^w(0) = 7.6\times 10^{10}$ A/m ^{2}.

The values of the Kupriyanov–Lukichev critical currents at $T = 0$ for a narrow wire ($w_n = 0.27$ μm, $d = 19$ nm) and a wide wire ($w_w = 0.48$ μm, $d = 19$ nm) are equal to $I_{KL}^n(0) = 371$ μA and $I_{KL}^w(0) = 690$ μA, respectively. Then $I_{KL}^n(0)/4 = 93$ μA and $I_{KL}^w(0)/4 = 173$ μA. The Kupriyanov–Lukichev critical currents differ from the corresponding Ginzburg–Landau critical currents, equal to $I_{GL}^n(0) = 303$ μA and $I_{GL}^w(0) = 550$ μA.

Fig. 2 shows the critical switching current (squares) and the retrapping current (triangles) as functions of T measured over several cycles in wide structure (I-3-4, V-2-1) and corresponding adjustable lines 1 and 2. Inset of Fig. 2 represents the sketch of the structure. Line 1 shows the approximation of the experimental switching current at $T = 1.25 -$ 1.466 K using the expression $I_{c1}(T) = I_{cf1}(0)(1 -$

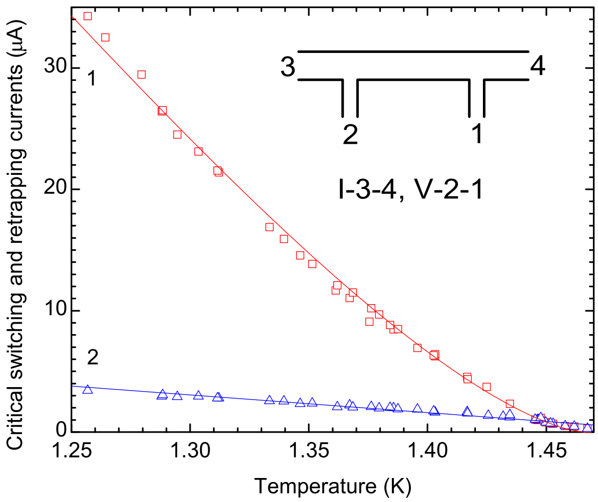

Fig. 2. (Color online) Squares and triangles are critical switching and retrapping currents as functions of the temperature, respectively, recorded in the wide structure according to the measurement circuit: *I*-3-4, *V*-2-1. Lines 1 and 2 are adjustable theoretical functions for the switching current $I_{c1}(T)$ and the retrapping current $I_{ret2}(T)$, respectively. Inset is the sketch of the structure.

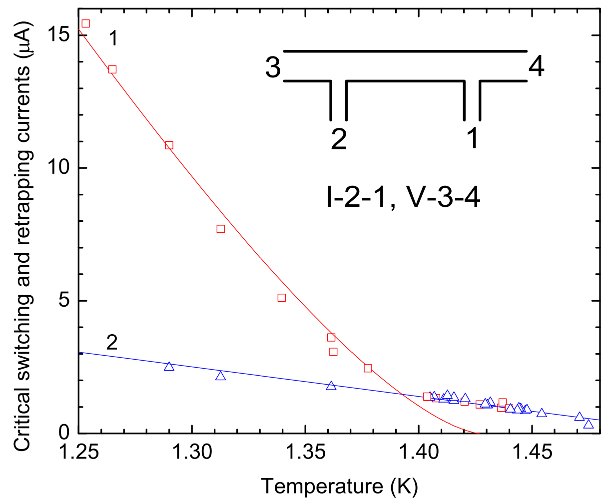

Fig. 3. (Color online) Squares and triangles - switching and retrapping currents as functions of the temperature, respectively, recorded in a wide structure according to the measurement circuit: *I*-2-1, *V*-3-4. Line 1 is the adjustable curve $I_{c1}(T)$ for the experimental switching current in the range $T = 1.25 - 1.4$ K. Line 2 is the adjustable curve $I_{ret2}(T)$ for the switching current in the interval $T = 1.404 - 1.475$ K and for the retrapping current in the interval $T = 1.25 - 1.475$ K. Inset is a sketch of the structure.

$(T/T_{cf1})^2)(1-(T/T_{cf1})^4)^{1/2}$, where the adjustable critical current $I_{cf1}(0) = 183\,\mu$A and the adjustable critical temperature $T_{cf1} = 1.466$ K. The value of $I_{cf1}(0)$ is close to the expected theoretical value $I^{w}_{KL}(0)\ /4 = 173\,\mu$A. We measured, for the first time, that the adjustable critical temperature T_{cf1}, determined from the temperature dependence of the switching current $I_{c1}(T)$, less than the critical temperature $T_c = 1.486$ K, found in the middle of the N-S transition of the structure. Moreover, the adjustable critical temperature T_{cf1} less than the critical temperature $T_{cl} = 1.478$ K, corresponding to the bottom of the N-S transition, by 12 mK. We experimentally checked that the adjustable critical temperature T_{cf1} and the critical temperature T_c coincide for the control quasi-one-dimensional aluminum structure with the same width of the current and potential wires.

Line 2 represents the linear fit of the experimental retrapping current at $T = 1.25 - 1.445$ K by the expression $I_{ret2}(T) = I_{retf2}(0)(1 - T/T_{cf2})$, where the adjustable critical current $I_{retf2}(0) = 22\,\mu$A and the adjustable critical temperature $T_{cf2} = 1.510$ K. The adjustable critical temperature $T_{cf2} = 1.510$ K is close to the critical temperature $T_{ch}(0.96) = 1.506$ K, that corresponds to the top of the N-S transition of the wide structure. We believe that the adjustable critical temperature T_{cf2} is close to the true critical temperature of a wide wire without taking into account the effect of narrow parts of the structure. We assume that the linear dependence of the retrapping current at $T = 1.25 - 1.445$ K is determined by the Joule (or quasiparticle) overheating of the structure.

At $T = 1.445 - 1.466$ K, the measured switching and retrapping critical currents coincide in experimental error (no hysteresis) and lie on line 1, that is below the linear function $I_{ret2}(T)$.

Fig. 3 shows the critical switching (squares) and retrapping (triangles) currents as functions of the temperature, recorded over several cycles in the wide structure according to the measurement circuit (*I*-2-1, *V*-3-4) and adjustable lines 1 and 2. The inset in Fig. 3 demonstrates the structure sketch. Note that in the case of the measurement circuit *I*-2-1, *V*-3-4, the switching current is mainly determined by the narrowest part of the structure. We found, for the first time, that the measured switching current as a function of the temperature is approximated by two theoretical curves (lines 1 and 2). We draw attention that for this structure, recorded by another measurement circuit: *I*-3-4, *V*-2-1 (Fig. 2) and for the control quasi-one-dimensional aluminum structure with one width, the experimental switching current is approximated by a single nonlinear temperature dependence.

Line 1 is the fit of the experimental switching current at $T = 1.25 - 1.4$ K by the expression $I_{c1}(T) = I_{cf1}(0)(1 - (T/T_{cf1})^2)(1-(T/T_{cf1})^4)^{1/2}$, here the adjustable critical current $I_{cf1}(0) = 102\,\mu$A and the adjustable critical temperature $T_{cf1} = 1.427$ K. The value of $I_{cf1}(0)$ is close to the expected theoretical value for a narrow structure $I^{n}_{KL}(0)/4 = 93\,\mu$A. We found that the same as for the switching current (Fig. 2), the adjustable critical temperature T_{cf1} and the critical temperature $T_c = 1.487$ K, determined from the middle N-S transition of the structure, do not match. In addition, the value T_{cf1} is less than the critical temperature $T_{cl} = 1.480$ K, corresponding to the bottom of the N-S transition of the wide structure by 53 mK. We assume that the value $T_{cf1} = 1.427$ K is close to the true critical temperature of a narrow wire in the structure (without taking into account the proximity effect). In the interval $T = 1.404 - 1.475$ K, the experimental switching current, which coincides with the retrapping current, is approximated by line 2, which is given by the linear dependence $I_{ret2}(T) = I_{retf2}(0)(1 - T/T_{cf2})$, where the adjustable critical current $I_{retf2}(0) = 17\,\mu$A and the adjustable critical temperature $T_{cf2} = 1.525$ K. The adjustable critical temperature $T_{cf2} = 1.525$ K is close to the critical temperature $T_{ch}(0.96) = 1.521$ K, which corresponds to the top of the N-S transition of the wide structure. We believe that the value of T_{cf2} is close to the true critical temperature of a wide wire, without taking into account the effect of narrow parts of the structure. Thus, the difference in the assumed true critical temperatures of the wide and narrow wires is equal to $T_{cf2} - T_{cf1} = 98$ mK.

We believe that at $T = 1.404 - 1.475$ K, a wide structure measured according to the measurement circuit *I*-2-1, *V*-3-4 (Fig. 3), is a hybrid structure, consisting of two SNS junctions formed at the points of connection of narrow current wires. We assume that in the interval $T = 1.404 - 1.475$ K, the function $I_{ret2}(T)$ coincides with the critical current of the Josephson structure $I_J(T)$. Near T_c, $I_J(T) = \pi\Delta^2(T)/(4ekTR_J)$, where $\Delta(T) = 1.74\Delta(0)(1 - T/T_c)^{1/2}$ is a temperature-dependent energy superconducting gap in zero magnetic field, $\Delta(0) = 1.764kT_c$ is a gap at $T = 0$, R_J is the Josephson resistance [2]. The expression $I_J(T) = I_J(0)(1 - T/T_c)$ can be written, where $I_J(0)$ is the Josephson critical current at $T = 0$ K. In our case, $I_J(0) = I_{retf2}(0) = 17$ μA, $T_c = T_{cf2} = 1.525$ K. In this case, the value of $I_J(0)$ corresponds to the Josephson resistance $R_J = 57.2$ Ω, that is close to the resistance of the narrow structure $R_n(T) = 69$ Ω.

Line 2 is the fit of the experimental retrapping current at $T = 1.25 - 1.475$ K using the same expression $I_{ret2}(T) = I_{retf2}(0)(1 - T/T_{cf2})$, as for the switching current at $T = 1.404 - 1.475$ K. The adjustable critical current $I_{retf2}(0) = 17$ μA and the adjustable critical temperature $T_{cf2} = 1.525$ K. In the interval $T = 1.25 - 1.4$ K, the whole structure is superconducting and linear dependence of the retrapping current $I_{ret2}(T)$ is determined by another reason, namely, Joule (or quasiparticle) overheating of the structure.

Fig. 4 shows the critical switching (squares) and retrapping (triangles) currents as functions of the temperature, measured in several cycles in a narrow structure (*I*-7-8, *V*-6-5) and adjustable lines 1 and 2. The inset in Fig. 4 represents a sketch of the structure. We found that for this narrow structure, as well as for the wide structure measured according to the measurement circuit *I*-2-1, *V*-3-4 (Fig. 3), the experimental switching current as a function of *T* is approximated with using two theoretical curves (lines 1 and 2).

Line 1 is the fit of the measured switching current at $T = 1.25 - 1.41$ K using the expression $I_{c1}(T) = I_{cf1}(0)(1 - (T/T_{cf1})^2)(1 - (T/T_{cf1})^4)^{1/2}$, here the adjustable critical current $I_{cf1}(0) = 102$ μA and the adjustable critical temperature $T_{cf1} = 1.427$ K. The value of $I_{cf1}(0)$ is close to the expected theoretical value $I^n_{KL}(0)/4 = 93$ μA. Note that for this switching current, as well as for switching currents (Figs. 2, 3), the adjustable critical temperature T_{cf1} and the critical temperature $T_c = 1.455$ K found in the middle of the N-S transition of the structure are different. Moreover, the value $T_{cf1} = 1.427$ K is less than the critical temperature $T_{cl} = 1.448$ K, corresponding to the bottom of the N-S transition of the narrow structure at 21 mK. We believe that the value of T_{cf1} is close to the true critical temperature of a narrow wire of the structure (without taking into account the proximity effect). Expressions for line 1 (Fig. 3) and for line 1 (Fig. 4) coincide.

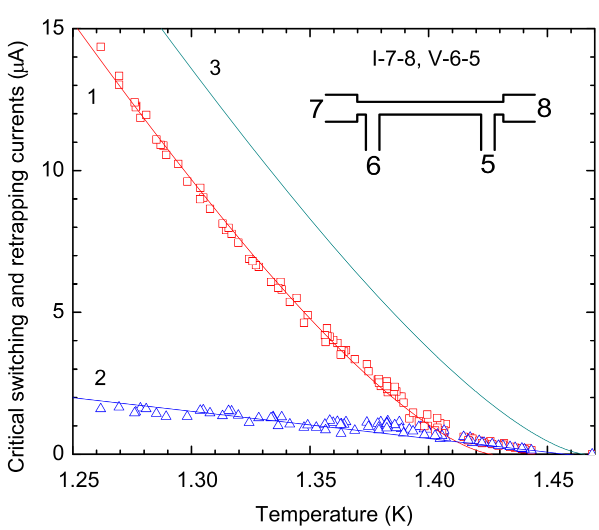

Fig. 4. (Color online) Squares and triangles are the switching and retrapping currents as functions of the temperature, respectively, recorded in a narrow structure according to the measurement circuit: *I*-7-8, *V*-6-5. Line 1 is the adjustable curve $I_{c1}(T)$ for the experimental switching current in the range $T = 1.25 - 1.41$ K. Line 2 is the adjustable curve $I_{ret2}(T)$ for the switching current at $T = 1.415 - 1.444$ K and for the retrapping current at $T = 1.25 - 1.45$ K. Line 3 is the expected switching current of a narrow structure if critical temperatures of narrow and wide wires making up a narrow and wide structure, are the same. Inset: a sketch of the structure.

In the interval $T = 1.415 - 1.444$ K, the measured switching current coincides with the retrapping current, and is given by line 2, that is described by the linear dependence $I_{ret2}(T) = I_{retf2}(0)(1 - T/T_{cf2})$, where the adjustable critical current $I_{retf2}(0) = 14$ μA and the adjustable critical temperature $T_{cf2} = 1.457$ K. The adjustable critical temperature T_{cf2} is the effective critical temperature of a narrow wire with allowance for the effect of wide parts of the structure. The adjustable critical temperature T_{cf2} is less than the critical temperature close to the top of the N-S transition $T_{ch}(0.96) = 1.480$ K.

We believe that at $T = 1.415 - 1.444$ K, the narrow structure, taken together with the wide current wires, behaves like a hybrid SNS structure. Note that at $T = 1.404 - 1.475$ K, a wide structure, measured according to the measurement circuit *I*-2-1, *V*-3-4 (Fig. 3), also represents a hybrid SNS structure. We believe that at $T = 1.415 - 1.444$ K, the function $I_{ret2}(T)$ coincides with the critical current of the Josephson structure $I_J(T) = I_J(0)(1 - T/T_c)$. In our case, $I_J(0) = I_{retf2}(0) = 14$ μA, $T_c = T_{cf2} = 1.457$ K. The critical Josephson current at zero temperature $I_J(0)$ corresponds to the Josephson resistance $R_J = 66.35$ Ω, which is close to the structure resistance $R_n(T) = 69$ Ω.

Line 2 is the fit of measured retrapping current at $T = 1.25 - 1.45$ K by the same expression $I_{ret2}(T) = I_{retf2}(0)(1 - T/T_{cf2})$, as for the switching current at $T = 1.415 - 1.444$ K. The adjustable critical current $I_{retf2}(0) = 14$ μA and the adjustable critical temperature $T_{cf2} = 1.457$ K. At $T = 1.25 - 1.41$ K, the entire structure is superconducting and linear dependence of the retrapping current $I_{ret2}(T)$ is due to the Joule (or quasiparticle) overheating of the structure.

We found that the switching and retrapping critical currents as functions of the temperature of the investigated narrow structure measured using other measurement circuit (*I*-6-5, *V*-7-8) are approximated with the same expressions $I_{c1}(T)$ and $I_{ret2}(T)$. This data are not listed here.

Line 3 (Fig. 4) shows the expected switching critical current of a narrow structure if the critical temperatures of the narrow and wide parts of the structures are equal. The expression for line 3 is the product of the ratio w_n/w_w and the expression for adjustable switching critical current of wide structure (line 1, Fig. 2).

Thus, we measured that for the same thickness $d = 19$, the critical current density in a narrow quasi-one-dimensional superconducting aluminum structure is less than the critical current density in a wide aluminum structure.

In addition, our results indicate that the temperature-dependent critical current in cases where the voltage is measured over a short section of long quasi-one-dimensional superconducting aluminum structures is a non-local value. This current depends on the temperature-dependent superconducting order parameter and the critical temperature of the sections of current and potential wires located outside this short section.

Additionally, to confirm the values of the true critical temperatures of wide and narrow aluminum wires, we measured control wide and narrow aluminum structures, consisting of current and potential wires of the same width. These structures were fabricated on a single chip and had a thickness and widths, close to the thickness and widths of wide and narrow wires that make up the wide and narrow structures studied here. The distances between potential contacts were the same for both control structures and were close to the distances between potential contacts of the structures presented here.

We found the critical temperatures determined by the middle of the N-S transition for the control wide and narrow structures are close to the assumed true critical temperatures of wide and narrow wires making up the investigated wide and narrow structures. In addition, we found that the critical switching currents of the control wide and narrow structures in the entire investigated temperature range near T_c are well approximated by similar theoretical expressions $I_c(T) = I_{cf}(0)(1 - (T/T_{cf})^2)(1-(T/T_{cf})^4)^{1/2}$, where the adjustable critical currents $I_{cf}(0)$ are close to the corresponding theoretical values $I_{KL}(0)/4$ and the adjustable critical temperatures T_{cf} are close to the corresponding critical temperatures T_c found in the middle of the N-S transition. Note that the linear temperature dependence of the switching current at T very close to T_c, inherent in hybrid S-N-S structure, is not observed on the control wide and narrow structures.

3. Conclusion

Thus, we have shown experimentally that the critical temperatures T_{cw} and T_{cn} of wide and narrow quasi-one-dimensional superconducting aluminum structures with typical dimensions close to the sizes of a wide and narrow semirings of the circularly-asymmetric interferometer are not the same.

For the first time, we found that a decrease in the width w of quasi-one-dimensional superconducting aluminum wires of the same thickness d leads to decrease in the critical temperature of the wire. At a time when a slight increase in the critical temperature is expected with decreasing w. We believe that the narrower the wire the higher the influence of depairing centers located on dirty longitudinal boundaries of the wire and, therefore, the lower the critical temperature.

In order to prove that the difference in the densities of the critical switching current $j_{c1}(T)$ and $j_{c2}(T)$ of wide and narrow aluminum structures with sizes close to those of wide and narrow arms of a circularly-asymmetric aluminum interferometer, we measured the switching and retrapping critical currents in these structures. The values of critical currents were obtained from the hysteresis $V(I)$ curves recorded at an applied dc in zero magnetic field at temperatures close to T_c. The $V(I)$ curves are recorded using different electrical measurement circuits.

We found that at T lower T_{cf1}, the switching current as a function of T in wide and narrow structures for different measurement circuits is well described by the expression $I_{c1}(T) = I_{cf1}(0)(1-(T/T_{cf1})^2)(1-(T/T_{cf1})^4)^{1/2}$, where $I_{cf1}(0)$ and T_{cf1} are the adjustable switching current at $T = 0$ and the adjustable critical temperature. The adjustable switching current $I_{cf1}(0)$ is close to the value $I_{KL}(0)\ /4$ calculated within the framework of the Kupriyanov–Lukichev theory [19]. First, it was found that the adjustable critical temperature T_{cf1} is lower than the critical temperature T_{cl} corresponding to the bottom of the N-S transition and is significantly lower than the critical temperature T_c determined from the middle of the N-S transition of the structure. Then, as for the control quasi-one-dimensional aluminum structure with the same width of current and potential wires, the critical temperatures T_{cf1} and T_c are the same.

We found that the switching current for a wide structure, recorded according to the measurement circuit: I-3-4, V-2-1 (Fig. 2), is approximated in the entire experimental temperature range by one dependence $I_{c1}(T)$, as well as for the control quasi-one-dimensional aluminum wire of the same width. It was found for the first time for cases (Figs. 3 and 4) that the switching temperature-dependent current of wide and narrow structures is described by two functions. At lower temperatures, this switching current is given by the $I_{c1}(T)$ function. At higher temperatures, Josephson SNS junctions are formed in structures. In the range of higher temperature, the switching current equal to the retrapping current is described by the linear function $I_{ret2}(T) = I_{retf2}(0)(1 - T/T_{cf2})$, which coincides with the Josephson critical current $I_J(T) = I_J(0)(1 - T/T_c)$, where the adjustable critical current $I_J(0) = I_{retf2}(0)$ and the adjustable critical temperature $T_c = T_{cf2}$. It was found that the adjustable critical temperature T_{cf2} is close to the critical temperature $T_{ch}(0.96)$, which corresponds to the top of the N-S transition in the structures.

For the entire experimental temperature range, except for the case (Fig. 2), the retrapping current as a function of T in wide and narrow structures with different connections of current and potential wires is approximated by the same expression $I_{ret2}(T) = I_{retf2}(0)(1 - T/T_{cf2})$ with the same adjustable critical current $I_{retf2}(0)$ and the adjustable critical temperature T_{cf2} as for the switching current at higher temperatures. The linear dependence of the retrapping current $I_{ret2}(T)$ is due to the Joule (or quasiparticle) overheating of the structure at lower temperatures and the linear function $I_J(T)$ at higher temperatures.

In addition, we plotted a theoretical temperature dependence of the switching critical current of a narrow structure with one width and a critical temperature equal to the critical temperature of a wide structure. It turned out that the calculated switching current is much higher than the experimental switching current of the narrow structure under study.

Thus, for the first time, we have shown experimentally for thin-film quasi-one-dimensional superconducting aluminum structures of the same thickness $d = 19$ nm, that the critical current density in a narrow structure under study is lower than the critical current density in a wide structure under study.

In addition, we found that the critical current of quasi-one-dimensional superconducting aluminum structures is a nonlocal quantity.

This finding that critical temperatures and critical current density of wide and narrow quasi-one-dimensional superconducting aluminum wires of the same thickness differ, can help clarify the effects observed in thin-film aluminum wires and structures consisting of wires with different widths [8,10–15].

In addition, the results of this work allows explaining the negative local and nonlocal voltage (resistance) in a quasi-one-dimensional superconducting aluminum wire of variable width [9] and a mysterious phase shift in the magnetic field of critical currents of different polarity in opposite directions in circularly-asymmetric aluminum structures [11,12].

Funding

None.

CRediT authorship contribution statement

V.I. Kuznetsov: Conceptualization, Methodology, Validation, Formal analysis, Investigation, Resources, Writing – original draft, Writing – review & editing, Visualization. **O.V. Trofimov:** Formal analysis, Writing – original draft, Writing – review & editing, Visualization.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

We are grateful to A. Firsov for the fabrication of aluminum structures and M. Skvortsov for fruitful discussions. This work was carried out and financially supported within the framework of STATE TASK no. 075-00355-21-00.

References

[1] V. Barone, G. Paterno, Physics and Application of the Josephson Effect, John Willey and Sons, New York, 1982.
[2] K.K. Likharev, Rev. Mod. Phys. 51 (1979) 101.
[3] A.A. Golubov, M.Y. Kupriyanov, E. Il'ichev, Rev. Mod. Phys. 76 (2004) 411.
[4] K.Y. Arutyunov, D.S. Golubev, A.D. Zaikin, Phys. Rep. 464 (2008) 1.

[5] V.V. Schmidt, The Physics of Superconductors, Springer-Verlag, Berlin-Heidelberg, 1997.
[6] P.N. Chubov, V.V. Eremenko, Y.A. Pilipenko, Sov. Phys. JETP 28 (1969) 389. (Original Russian text: P.N. Chubov, V. V. Eremenko, and Yu. A. Pilipenko, Zh. Eksp. Teor. Fiz. **55**, 752 (1968))
[7] I.Y. Borisenko, V.I. Kuznetsov, V.A. Tulin, D. Esteve, J. Low Temp. Phys. 119 (2000) 483, https://doi.org/10.1023/A:1004646625994.
[8] V.I. Kuznetsov, A.A. Firsov, JETP Lett. 104 (2016) 709, https://doi.org/10.1134/S0021364016220100.(Original Russian text: V.I. Kuznetsov, A.A. Firsov, Pis'ma v Zhurnal Eksperimental'noi i Teoreticheskoi Fiziki **104**, 721 (2016))
[9] V.I. Kuznetsov, O.V. Trofimov, Phys. C 581 (2021) 1353811, https://doi.org/10.1016/j.physc.2020.1353811.
[10] S.V. Dubonos, V.I. Kuznetsov, I.N. Zhilyaev, A.V. Nikulov, A.A. Firsov, JETP Lett. 77 (2003) 371, https://doi.org/10.1134/1.1581963. arXiv:cond-mat/0303.538. (Original Russian text: S.V. Dubonos, V.I. Kuznetsov, I.N. Zhilyaev, A.V. Nikulov, A.A. Firsov, Pis'ma v Zhurnal Eksperimental'noi i Teoreticheskoi Fiziki **77**, 439 (2003))
[11] V.L. Gurtovoi, S.V. Dubonos, S.V. Karpii, A.V. Nikulov, V.A. Tulin, JETP 105 (2007) 262.(Original Russian text: V. L. Gurtovoi, S.V. Dubonos, S.V. Karpii, A.V. Nikulov, V.A. Tulin, Zhurnal Eksperimental'noi I Teoreticheskoi Fiziki **132**, 297 (2007))
[12] V.L. Gurtovoi, S.V. Dubonos, A.V. Nikulov, N.N. Osipov, V.A. Tulin, JETP 105 (2007) 1157.(Original Russian text: V. L. Gurtovoi, S.V. Dubonos, A.V. Nikulov, N. N. Osipov, V.A. Tulin, Zhurnal Eksperimental'noi I Teoreticheskoi Fiziki **132**, 1320 (2007))
[13] V.I. Kuznetsov, A.A. Firsov, S.V. Dubonos, Phys. Rev. B 77 (2008) 094521, https://doi.org/10.1103/PhysRevB.77.094521.
[14] V.I. Kuznetsov, A.A. Firsov, Phys. C 492 (2013) 11, https://doi.org/10.1016/j.physc.2013.05.003.
[15] V.I. Kuznetsov, O.V. Trofimov, JETP Lett. 110 (2019) 54, https://doi.org/10.1134/S0021364019130113.(Original Russian text: V.I. Kuznetsov, O.V. Trofimov, Pis'ma v Zhurnal Eksperimental'noi i Teoreticheskoi Fiziki **110**, 47 (2019))
[16] J. Clarke, A.I. Braginski (Eds.), The SQUID Handbook: Vol. 1 Fundamentals and Technology of SQUIDs and SQUID Systems, WILEY-VCH Verlag GmbH and Co. KgaA, Weinheim, 2004.
[17] M. Gershenson, W.L. McLean, J. Low Temp. Phys. 47 (1982) 123.
[18] J. Romijn, T.M. Klapwijk, M.J. Renne, J.E. Mooij, Phys. Rev. B 26 (1982) 3648.
[19] M.Y. Kupriyanov, V.F. Lukichev, Sov. J. Low Temp. Phys. 6 (4) (1980) 210.
[20] M. Tinkham, Introduction to Superconductivity, second ed., McGraw-Hill Book Co., New York, 1996.