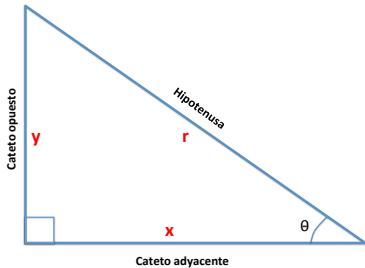


Identidades Trigonométricas

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$$\begin{aligned}\operatorname{sen}(\theta) &= \frac{y}{r} & \cos(\theta) &= \frac{x}{r} & \tan(\theta) &= \frac{y}{x} \\ \csc(\theta) &= \frac{r}{y} & \sec(\theta) &= \frac{r}{x} & \cot(\theta) &= \frac{x}{y}\end{aligned}$$

Identidades Trigonométricas Fundamentales

$$\tan(x) = \frac{\operatorname{sen}(x)}{\cos(x)} \quad \sec(x) = \frac{1}{\cos(x)} \quad \csc(x) = \frac{1}{\operatorname{sen}(x)} \quad \cot(x) = \frac{\cos(x)}{\operatorname{sen}(x)} = \frac{1}{\tan(x)}$$

Identidades Pitagóricas

$$\operatorname{sen}^2(x) + \cos^2(x) = 1 \quad \sec^2(x) - \tan^2(x) = 1 \quad \csc^2(x) - \cot^2(x) = 1$$

Identidades de Doble Ángulo

$$\begin{aligned}\operatorname{sen}(2x) &= \frac{2 \tan(x)}{1 + \tan^2(x)} = 2 \operatorname{sen}(x) \cos(x) \\ \cos(2x) &= \frac{1 - \tan^2(x)}{1 + \tan^2(x)} = \cos^2(x) - \operatorname{sen}^2(x) = 2 \cos^2(x) - 1 = 1 - 2 \operatorname{sen}^2(x) \\ \tan(2x) &= \frac{2 \tan(x)}{1 - \tan^2(x)} \\ \cot(2x) &= \frac{\cot^2(x) - 1}{2 \cot(x)}\end{aligned}$$

Identidades de Mitad del Ángulo

$$\begin{aligned}\operatorname{sen}\left(\frac{x}{2}\right) &= \pm \sqrt{\frac{1 - \cos(x)}{2}} \\ \cos\left(\frac{x}{2}\right) &= \pm \sqrt{\frac{1 + \cos(x)}{2}} \\ \tan\left(\frac{x}{2}\right) &= \pm \sqrt{\frac{1 - \cos(x)}{1 + \cos(x)}} = \csc(x) - \cot(x) = \frac{\operatorname{sen}(x)}{1 + \cos(x)} \\ \cot\left(\frac{x}{2}\right) &= \pm \sqrt{\frac{1 + \cos(x)}{1 - \cos(x)}} = \csc(x) + \cot(x) = \frac{\operatorname{sen}(x)}{1 - \cos(x)} = \frac{1 + \cos(x)}{\operatorname{sen}(x)} \\ \tan\left(\frac{x+y}{2}\right) &= \frac{\operatorname{sen}(x) + \operatorname{sen}(y)}{\cos(x) + \cos(y)} = -\frac{\cos(x) - \cos(y)}{\operatorname{sen}(x) - \operatorname{sen}(y)}\end{aligned}$$

Identidades de Productos

$$\begin{aligned}
 \operatorname{sen}^2(x) &= \frac{1 - \cos(2x)}{2} \\
 \cos^2(x) &= \frac{1 + \cos(2x)}{2} \\
 \operatorname{sen}(x)\cos(x) &= \frac{\operatorname{sen}(2x)}{2} \\
 \operatorname{sen}(x)\operatorname{sen}(y) &= \frac{1}{2}[\cos(x - y) - \cos(x + y)] \\
 \operatorname{sen}(x)\cos(y) &= \frac{1}{2}[\operatorname{sen}(x - y) + \operatorname{sen}(x + y)] \\
 \cos(x)\cos(y) &= \frac{1}{2}[\cos(x - y) + \cos(x + y)]
 \end{aligned}$$

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Suma y Diferencia de Ángulos

$$\begin{aligned}
 \operatorname{sen}(x \pm y) &= \operatorname{sen}(x)\cos(y) \pm \cos(x)\operatorname{sen}(y) \\
 \cos(x \pm y) &= \cos(x)\cos(y) \mp \operatorname{sen}(x)\operatorname{sen}(y) \\
 \tan(x \pm y) &= \frac{\tan(x) \pm \tan(y)}{1 \mp \tan(x)\tan(y)} \\
 \csc(x \pm y) &= \frac{1}{\operatorname{sen}(x \pm y)} \\
 \sec(x \pm y) &= \frac{1}{\cos(x \pm y)} \\
 \cot(x \pm y) &= \frac{\cot(x)\cot(y) \mp 1}{\cot(y) \pm \cot(x)}
 \end{aligned}$$

Otras Identidades

$$\begin{aligned}
 \operatorname{sen}(-x) &= -\operatorname{sen}(x) & \cos(-x) &= \cos(x) \\
 \tan(-x) &= -\tan(x) & \operatorname{sen}\left(\frac{\pi}{2} - x\right) &= \cos(x) \\
 \cos\left(\frac{\pi}{2} - x\right) &= \operatorname{sen}(x) & \tan\left(\frac{\pi}{2} - x\right) &= \cot(x)
 \end{aligned}$$

Algunos Valores Importantes

Función	$0(0^\circ)$	$\frac{\pi}{6}(30^\circ)$	$\frac{\pi}{4}(45^\circ)$	$\frac{\pi}{3}(60^\circ)$	$\frac{\pi}{2}(90^\circ)$	$\frac{2\pi}{3}(120^\circ)$	$\frac{3\pi}{4}(135^\circ)$	$\frac{5\pi}{6}(150^\circ)$	$\pi(180^\circ)$	$\frac{3\pi}{2}(270^\circ)$	$2\pi(360^\circ)$
$\operatorname{sen}(x)$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0
$\cos(x)$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1	0	1
$\tan(x)$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0	$-\infty$	0
$\csc(x)$	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	$-\infty$	-1	∞
$\sec(x)$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞	-2	$-\sqrt{2}$	$-\frac{2}{\sqrt{3}}$	-1	$-\infty$	1
$\cot(x)$	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	$-\frac{1}{\sqrt{3}}$	-1	$-\sqrt{3}$	$-\infty$	0	∞