Identidades Hiperbólicas

Identidades Hiperbólicas Fundamentales

$$\operatorname{senh}(x) = \frac{e^x - e^{-x}}{2} \qquad \cosh(x) = \frac{e^x + e^{-x}}{2} \qquad \tanh(x) = \frac{\operatorname{senh}(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$csch(x) = \frac{1}{\operatorname{senh}(x)} = \frac{2}{e^x - e^{-x}} \qquad sech(x) = \frac{1}{\cosh(x)} = \frac{2}{e^x + e^{-x}} \qquad \coth(x) = \frac{1}{\tanh(x)} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

Identidades Pitagóricas

$$\cosh^2(x) - \operatorname{senh}^2(x) = 1 \qquad sech^2(x) + \tanh^2(x) = 1 \qquad \coth^2(x) - csch^2(x) = 1$$

$$\begin{aligned} \operatorname{senh}(-x) &= -\operatorname{senh}(x) & \cosh(-x) &= \cosh(x) \\ \tanh(-x) &= -\tanh(x) & \coth(-x) &= -\coth(x) \\ sech(-x) &= sech(x) & csch(-x) &= -csch(x) \end{aligned}$$

$$\cosh(x) + \operatorname{senh}(x) = e^x \qquad \cosh(x) - \operatorname{senh}(x) = e^{-x}$$

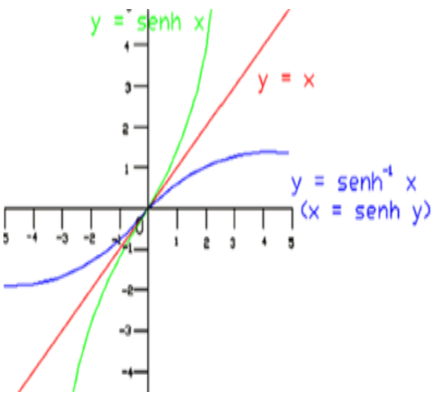
Otras Identidades

$$\begin{aligned}
\operatorname{senh}(2x) &= 2\operatorname{senh}(x)\cosh(x) \\
\cosh(2x) &= \cosh^2(x) + \operatorname{senh}^2(x) = 2\operatorname{senh}(x) + 1 = 2\cosh(x) - 1 \\
\operatorname{senh}(x) &= \frac{2\tanh(\frac{x}{2})}{1 - \tanh^2(\frac{x}{2})} \\
\cosh(x) &= \frac{1 + \tanh^2(\frac{x}{2})}{1 - \tanh^2(\frac{x}{2})} \\
\operatorname{senh}^2(x) &= \frac{\cosh(2x) - 1}{2} = \frac{\tanh^2(x)}{1 - \tanh^2(x)} \\
\cosh^2(x) &= \frac{\cosh(2x) + 1}{2} \\
\operatorname{senh}(x \pm y) &= \operatorname{senh}(x)\cosh(y) \pm \cosh(x)\operatorname{senh}(y) \\
\cosh(x \pm y) &= \cosh(x)\cosh(y) \pm \operatorname{senh}(x)\operatorname{senh}(y) \\
\tanh(x \pm y) &= \frac{\tanh(x) \pm \tanh(y)}{1 \pm \tanh(x)\tanh(y)}
\end{aligned}$$

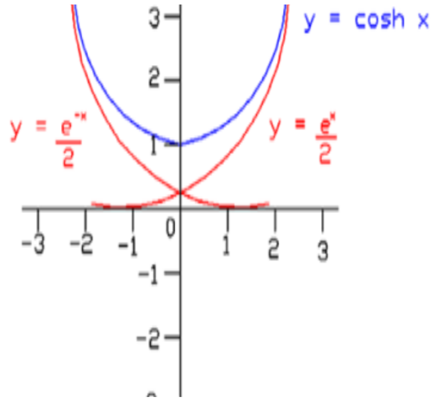
$$
\begin{aligned}
\operatorname{senh}(x)+\operatorname{senh}(y) &= 2\operatorname{senh}\left(\frac{x+y}{2}\right)\cosh\left(\frac{x-y}{2}\right)\\
\cosh(x)+\cosh(y) &= 2\cosh\left(\frac{x+y}{2}\right)\cosh\left(\frac{x-y}{2}\right)\\
(\operatorname{senh}(x)+\cosh(x))^n &= \operatorname{senh}(nx)+\cosh(nx)\\
\operatorname{senh}(3x) &= 3\operatorname{senh}(x)+4\operatorname{senh}^3(x)\\
\cosh(3x) &= 4\cosh^3(x)-3\cosh(x)\\
\tanh(3x) &= \frac{3\tanh(x)+\tanh^3(x)}{1+3\tanh^2(x)}\\
\operatorname{senh}\left(\frac{x}{2}\right) &= \pm\sqrt{\frac{\cosh(x)-1}{2}}\\
\cosh\left(\frac{x}{2}\right) &= \pm\sqrt{\frac{\cosh(x)+1}{2}}\\
\operatorname{senh}(x)\operatorname{senh}(y) &= \frac{1}{2}[\cosh(x+y)-\cosh(x-y)]\\
\operatorname{senh}(x)\cosh(y) &= \frac{1}{2}[\operatorname{senh}(x+y)-\operatorname{senh}(x-y)]\\
\cosh(x)\cosh(y) &= \frac{1}{2}[\cosh(x+y)+\cosh(x-y)]
\end{aligned}
$$

Hiperbólicas Inversas

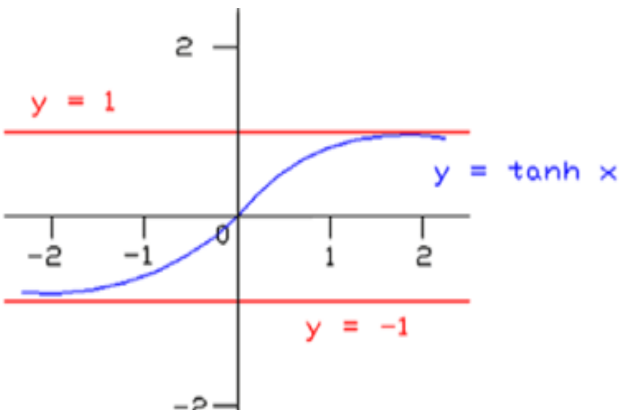
$$
\begin{aligned}
\operatorname{senh}^{-1}(x) &= \ln(x+\sqrt{x^2+1}) \ \text{ para cualquier } x\in\mathbb{R}\\
\cosh^{-1}(x) &= \ln(x+\sqrt{x^2-1}),\quad x\geq 1\\
\tanh^{-1}(x) &= \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right),\quad |x|<1\\
\coth^{-1}(x) &= \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right),\quad |x|>1\\
sech^{-1}(x) &= \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right),\\
csch^{-1}(x) &= \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right)
\end{aligned}
$$



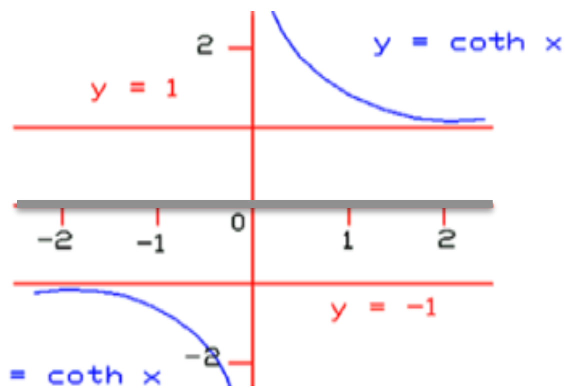
$$\text{senh} : \mathbb{R} \longrightarrow \mathbb{R}$$
$$Dom(\text{senh}) = \mathbb{R}$$
$$Im(\text{senh}) = \mathbb{R}$$



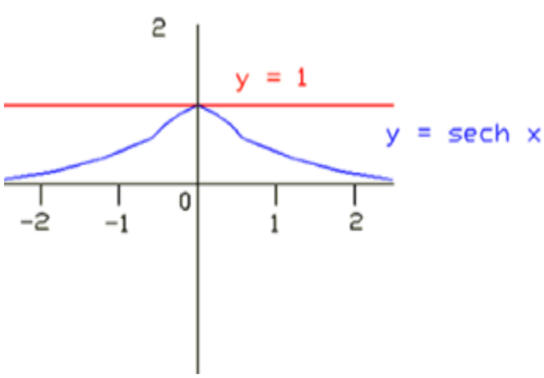
$$\cosh : \mathbb{R} \longrightarrow \mathbb{R}$$
$$Dom(\cosh) = \mathbb{R}$$
$$Im(\cosh) = \mathbb{R}$$



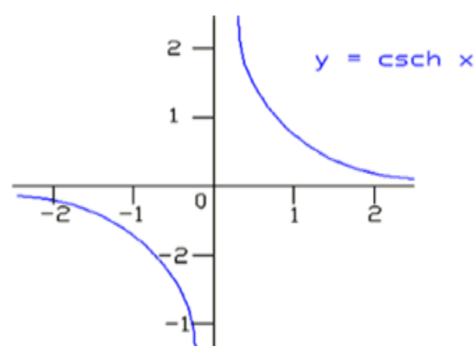
$$\tanh : \mathbb{R} \longrightarrow \mathbb{R}$$
$$Dom(\tanh) = \mathbb{R}$$
$$Im(\tanh) = (-1, 1)$$



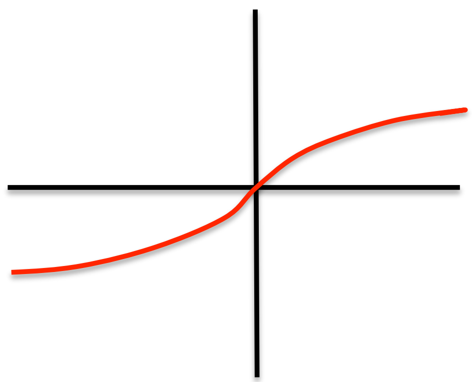
$$\coth : \mathbb{R} \longrightarrow \mathbb{R}$$
$$Dom(\coth) = \mathbb{R} \setminus \{0\}$$
$$Im(\coth) = (-\infty, -1) \cup (1, \infty)$$



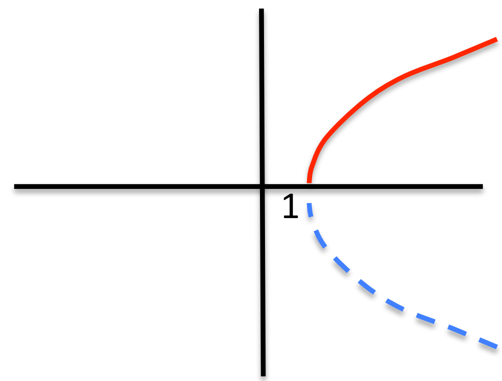
$$sech : \mathbb{R} \longrightarrow \mathbb{R}$$
$$Dom(sech) = \mathbb{R}$$
$$Im(sech) = (0, 1]$$



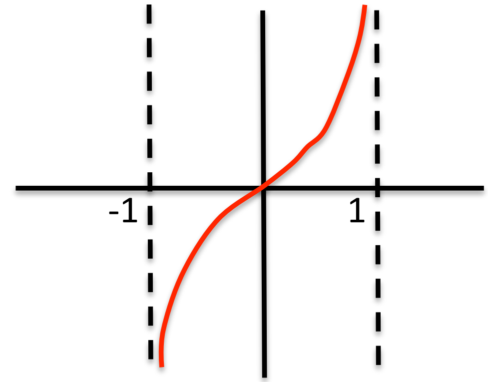
$$csch : \mathbb{R} \longrightarrow \mathbb{R}$$
$$Dom(csch) = \mathbb{R} \setminus \{0\}$$
$$Im(csch) = \mathbb{R} \setminus \{0\}$$



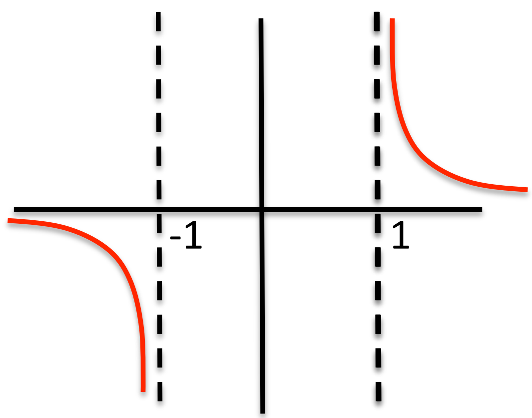
$\mathrm{senh}^{-1} : \mathbb{R} \longrightarrow \mathbb{R}$
$Dom(\mathrm{senh}^{-1}) = \mathbb{R}$
$Im(\mathrm{senh}^{-1}) = \mathbb{R}$



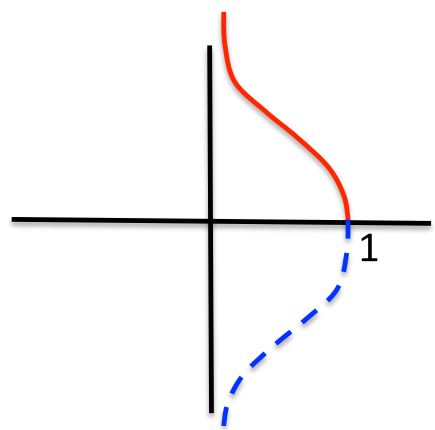
$\cosh^{-1} : \mathbb{R} \longrightarrow \mathbb{R}$
$Dom(\cosh^{-1}) = [1, \infty)$
$Im(\cosh^{-1}) = [0, \infty)$



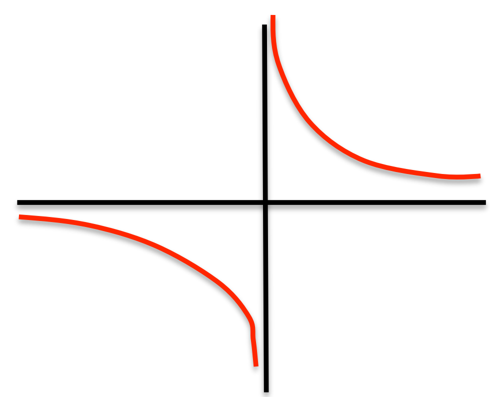
$\tanh^{-1} : \mathbb{R} \longrightarrow \mathbb{R}$
$Dom(\tanh^{-1}) = (-1, 1)$
$Im(\tanh^{-1}) = \mathbb{R}$



$\coth^{-1} : \mathbb{R} \longrightarrow \mathbb{R}$
$Dom(\coth^{-1}) = (-\infty, -1) \cup (1, \infty)$
$Im(\coth^{-1}) = \mathbb{R} \setminus \{0\}$



$sech^{-1} : \mathbb{R} \longrightarrow \mathbb{R}$
$Dom(sech^{-1}) = (0, 1]$
$Im(sech^{-1}) = [0, \infty)$



$csch^{-1} : \mathbb{R} \longrightarrow \mathbb{R}$
$Dom(csch^{-1}) = \mathbb{R} \setminus \{0\}$
$Im(csch^{-1}) = \mathbb{R} \setminus \{0\}$