Identidades Trigonométricas Inversas

Definición 1. • *La* **función inversa del seno**, *denotada por* $\operatorname{arc\,sen}$ *o por* sen^{-1} *está definida por*

$$y = \operatorname{arc\,sen}(x) \iff x = \operatorname{sen}(y), \qquad \textit{para } -1 \le x \le 1 \quad \textit{y} \quad -\frac{\pi}{2} \le y \le \frac{\pi}{2}.$$

- *La* **función inversa del coseno**, *denotada por* $\operatorname{arc\,cos}$ *o por* $\cos^{-1}$ *está definida por*

$$y = \operatorname{arc\,cos}(x) \iff x = \cos(y), \qquad \textit{para } -1 \le x \le 1 \quad \textit{y} \quad 0 \le y \le \pi.$$

- *La* **función inversa de la tangente**, *denotada por* $\arctan$ *o por* $\tan^{-1}$ *está definida por*

$$y = \arctan(x) \iff x = \tan(y), \qquad \textit{para cualquier } x \in \mathbb{R} \quad \textit{y} \quad -\frac{\pi}{2} < y < \frac{\pi}{2}.$$

- *La* **función inversa de la cotangente**, *denotada por* arccot *o por* $\cot^{-1}$ *está definida por*

$$y = \operatorname{arccot}(x) \iff x = \cot(y), \qquad \textit{para cualquier } x \in \mathbb{R} \quad \textit{y} \quad 0 < y < \pi.$$

- *La* **función inversa de la secante**, *denotada por* arcsec *o por* $\sec^{-1}$ *está definida por*

$$y = \operatorname{arcsec}(x) \iff x = \sec(y), \qquad \textit{para cualquier } |x| \ge 1 \quad \textit{y} \quad \left[0, \frac{\pi}{2}\right) \cup \left[\pi, \frac{3\pi}{2}\right).$$

- *La* **función inversa de la cosecante**, *denotada por* arccsc *o por* $\csc^{-1}$ *está definida por*

$$y = \operatorname{arccsc}(x) \iff x = \csc(y), \qquad \textit{para cualquier } |x| \ge 1 \quad \textit{y} \quad \left(-\pi, -\frac{\pi}{2}\right] \cup \left(0, \frac{\pi}{2}\right].$$

Suma y diferencia de inversas

$$\operatorname{arc\,sen}(x) + \operatorname{arc\,cos}(x) = \frac{\pi}{2}$$

$$\arctan(x) + \operatorname{arccot}(x) = \frac{\pi}{2}$$

$$\arctan(x) + \arctan\left(\frac{1}{x}\right) = \begin{cases} \dfrac{\pi}{2} & \text{si} \quad x > 0 \\ -\dfrac{\pi}{2} & \text{si} \quad x < 0 \end{cases}$$

$$\operatorname{arc\,sen}(x) \pm \operatorname{arc\,sen}(y) = \operatorname{arc\,sen}\left(x\sqrt{1-y^2} \;\pm\; y\sqrt{1-x^2}\right)$$

$$\operatorname{arc\,cos}(x) \pm \operatorname{arc\,cos}(y) = \operatorname{arc\,cos}\left(xy \;\mp\; \sqrt{(1-x^2)(1-y^2)}\right)$$

$$\arctan(x) \pm \arctan(y) = \arctan\left(\frac{x \pm y}{1 \mp xy}\right)$$

Derivadas Trigonométricas Inversas

Las derivadas de las funciones trigonométricas inversas son:

1. $\frac{d}{dx}[\operatorname{arc\,sen}(u)] = \frac{1}{\sqrt{1-u^2}}\left(\frac{d[u]}{dx}\right) \qquad u \in (-1,1)$

2. $\frac{d}{dx}[\arccos(u)] = \frac{-1}{\sqrt{1-u^2}}\left(\frac{d[u]}{dx}\right) \qquad u \in (-1,1)$

3. $\frac{d}{dx}[\arctan(u)] = \frac{1}{u^2+1}\left(\frac{d[u]}{dx}\right) \qquad u \in \mathbb{R}$

4. $\frac{d}{dx}[\operatorname{arccot}(u)] = \frac{-1}{u^2+1}\left(\frac{d[u]}{dx}\right) \qquad u \in \mathbb{R}$

5. $\frac{d}{dx}[\operatorname{arcsec}(u)] = \frac{1}{u\sqrt{u^2-1}}\left(\frac{d[u]}{dx}\right) \qquad |u| > 1$

6. $\frac{d}{dx}[\operatorname{arccsc}(u)] = \frac{-1}{u\sqrt{u^2-1}}\left(\frac{d[u]}{dx}\right) \qquad |u| > 1$

Integrales Trigonométricas Inversas

1. $\int \frac{du}{\sqrt{1-u^2}} = \operatorname{arc\,sen}(u) + C$

2. $\int \frac{du}{u^2+1} = \arctan(u) + C$

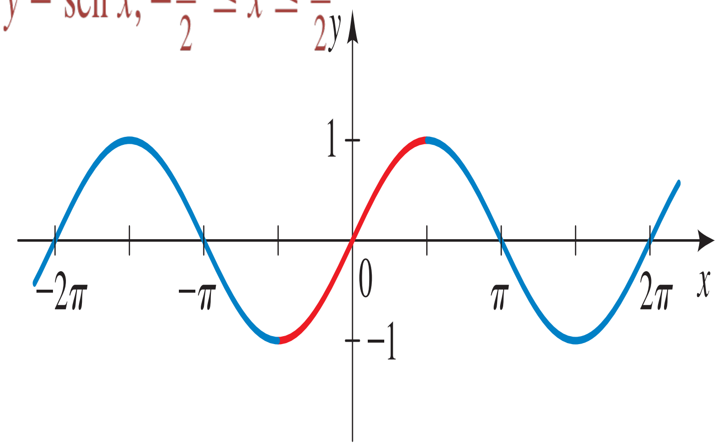
3. $\int \frac{du}{u\sqrt{u^2-1}} = \operatorname{arcsec}(u) + C$

4. $\int \frac{du}{\sqrt{a^2-u^2}} = \operatorname{arc\,sen}\left(\frac{u}{a}\right) + C,$ donde $a < 0$

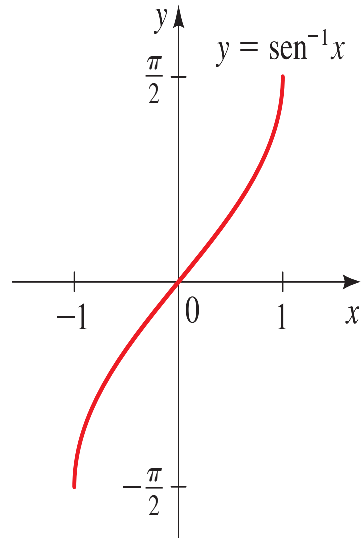
5. $\int \frac{du}{u^2+a^2} = \frac{1}{a}\arctan\left(\frac{u}{a}\right) + C,$ donde $a \neq 0$

6. $\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\left(\frac{u}{a}\right) + C,$ donde $a > 0$

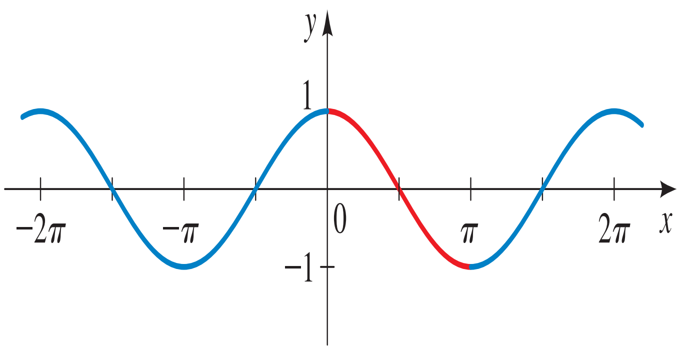
$y = \operatorname{sen} x, -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$



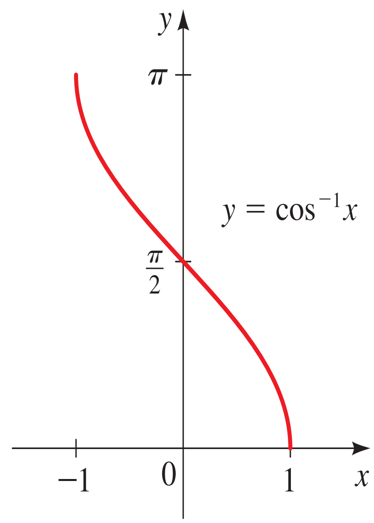
$y = \operatorname{sen} x$

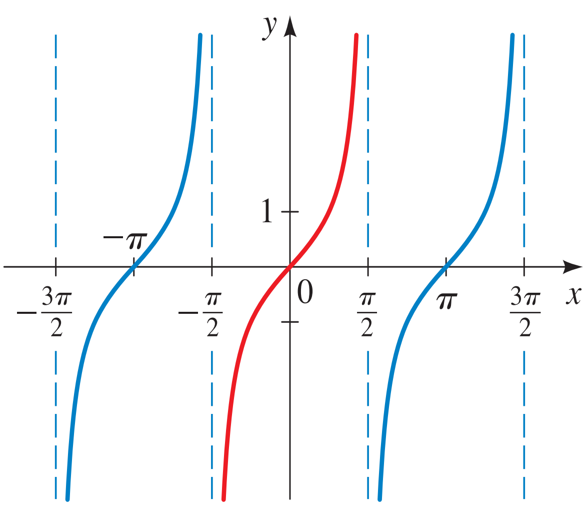


$y = \cos x, \; 0 \leq x \leq \pi$

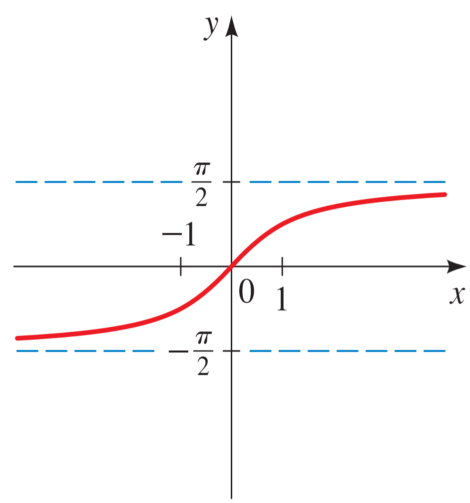


$y = \cos x$

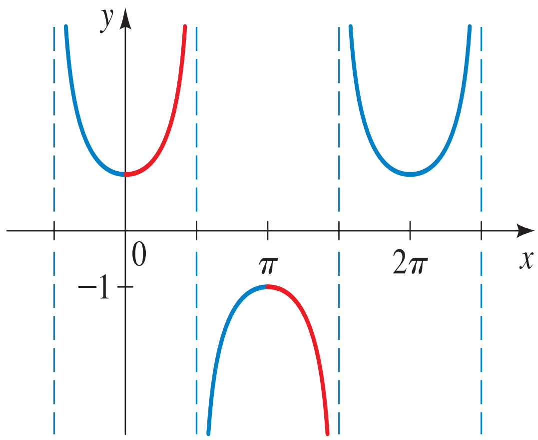




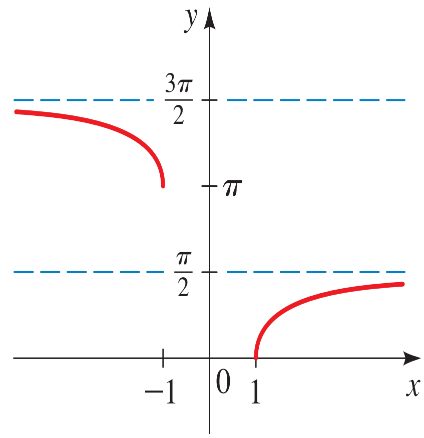
$y = \tan x, -\frac{\pi}{2} < x < \frac{\pi}{2}$



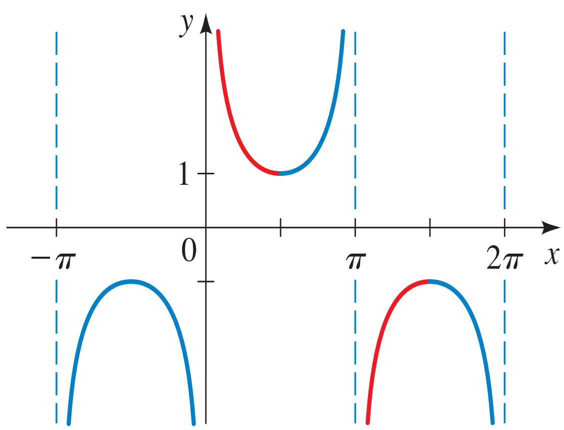
$y = \tan^{-1} x$



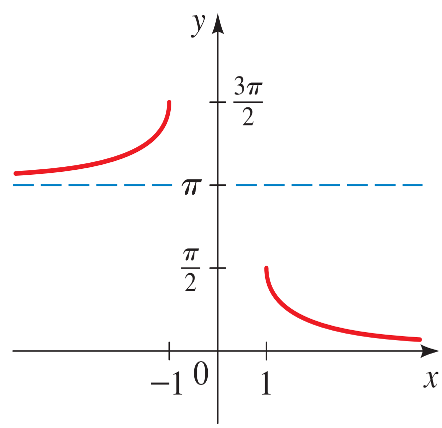
$y = \sec x,\ 0 \le x < \frac{\pi}{2}, \pi \le x < \frac{3\pi}{2}$



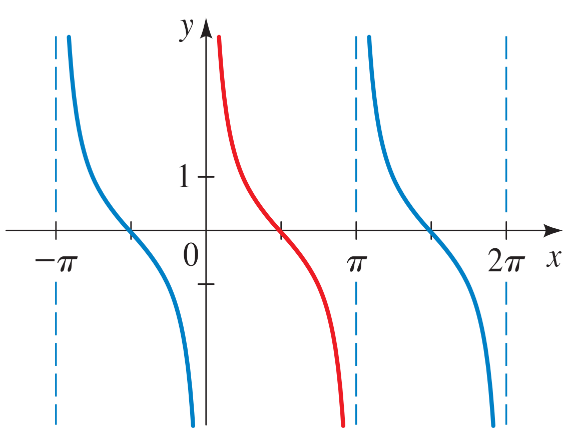
$y = \sec^{-1} x$



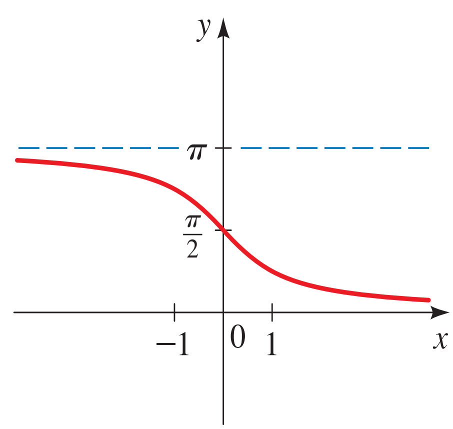
$y = \csc x,\ 0 < x \le \frac{\pi}{2}, \pi < x \le \frac{3\pi}{2}$



$y = \csc^{-1} x$



$y = \cot x,\ 0 < x < \pi$



$y = \cot^{-1} x$

Ecuación	Enunciado equivalente	Solución
$y = \operatorname{sen}^{-1}\left(\frac{1}{2}\right)$	$\operatorname{sen} y = \frac{1}{2}$ y $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$	$y = \frac{\pi}{6}$
$y = \operatorname{sen}^{-1}\left(-\frac{1}{2}\right)$	$\operatorname{sen} y = -\frac{1}{2}$ y $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$	$y = -\frac{\pi}{6}$
$y = \operatorname{sen}^{-1}(1)$	$\operatorname{sen} y = 1$ y $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$	$y = \frac{\pi}{2}$
$y = \operatorname{arcsen}(0)$	$\operatorname{sen} y = 0$ y $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$	$y = 0$
$y = \operatorname{arcsen}\left(-\frac{\sqrt{3}}{2}\right)$	$\operatorname{sen} y = -\frac{\sqrt{3}}{2}$ y $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$	$y = -\frac{\pi}{3}$

Ecuación	Enunciado equivalente	Solución
$y = \cos^{-1}\left(\frac{1}{2}\right)$	$\cos y = \frac{1}{2}$ y $0 \le y \le \pi$	$y = \frac{\pi}{3}$
$y = \cos^{-1}\left(-\frac{1}{2}\right)$	$\cos y = -\frac{1}{2}$ y $0 \le y \le \pi$	$y = \frac{2\pi}{3}$
$y = \cos^{-1}(1)$	$\cos y = 1$ y $0 \le y \le \pi$	$y = 0$
$y = \arccos(0)$	$\cos y = 0$ y $0 \le y \le \pi$	$y = \frac{\pi}{2}$
$y = \arccos\left(-\frac{\sqrt{3}}{2}\right)$	$\cos y = -\frac{\sqrt{3}}{2}$ y $0 \le y \le \pi$	$y = \frac{5\pi}{6}$