6 Propositional Logic

6.1 Symbols and Translation

Earlier chapters showed that the validity of a deductive argument is purely a function of its form. By knowing the form of an argument, we can often tell immediately whether it is valid or invalid. Unfortunately, however, ordinary linguistic usage often obscures the form of an argument. To dispel this obscurity, logic introduces various simplifying procedures. In Chapter 5, letters were used to represent the terms in a syllogism, and techniques were developed to reduce syllogisms to what is called standard form. In this chapter, form recognition is facilitated through the introduction of special symbols called **operators,** or **connectives.** When arguments are expressed in terms of these symbols, determining validity often becomes a matter of mere visual inspection.

In the two previous chapters, the fundamental elements were terms. In **propositional logic,** however, the fundamental elements are whole statements (or propositions). Statements are represented by letters, and these letters are then combined by means of the operators to form more-complex symbolic representations.

To understand the symbolic representation used in propositional logic, we must distinguish simple statements from compound statements. A **simple statement** is one that does not contain any other statement as a component. Here are some examples:

> Fast foods tend to be unhealthy.
> James Joyce wrote *Ulysses*.
> Parakeets are colorful birds.
> The bluefin tuna is threatened with extinction.

Any convenient uppercase letter may be selected to represent each statement. Thus, *F* might be selected to represent the first, *J* the second, *P* the third, and *B* the fourth. As will be explained shortly, lowercase letters are reserved for use as statement variables.

A **compound statement** is one that contains at least one simple statement as a component. Here are some examples:

> It is not the case that Al Qaeda is a humanitarian organization.
> Dianne Reeves sings jazz, and Christina Aguilera sings pop.
> Either people get serious about conservation or energy prices will skyrocket.
> If nations spurn international law, then future wars are guaranteed.
> The Broncos will win if and only if they run the ball.

6

Using letters to stand for the simple statements, these compound statements may be represented as follows:

> It is not the case that *A*.
> *D* and *C*.
> Either *P* or *E*.
> If *N* then *F*.
> *B* if and only if *R*.

In the first example, note that the statement is compound even though it contains only a single component (*A*). In general, negative statements are interpreted as compound units consisting of an affirmative statement and the phrase "it is not the case that."

The expressions "it is not the case that," "and," "or," "if . . . then . . . ," and "if and only if" are translated by logical operators. The five logical operators are as follows:

Operator	Name	Logical function	Used to translate
~	tilde	negation	not, it is not the case that
•	dot	conjunction	and, also, moreover
∨	wedge	disjunction	or, unless
⊃	horseshoe	implication	if . . . then . . . , only if
≡	triple bar	equivalence	if and only if

Saying that logical operators are used to "translate" these English expressions does not mean that the expressions and the operators are identical. As in any translation (from English to French, for example), a certain distortion of meaning occurs.

The meaning of such English expressions as "and," "or," and "if and only if" is often vague and may vary with context, whereas the meaning of the logical operators is clear, precise, and invariable. Thus, when we say that the logical operators may be used to translate expressions in ordinary language, we mean that the operators capture a certain aspect of their correlative English expressions. The precise character of this aspect is spelled out in the next section of this chapter. The purpose of this section is to develop a familiarity with the logical operators through practice in translation.

When we use the operators to translate the previous examples of compound statements, the results are as follows:

It is not the case that *A*.	$\sim A$
D and *C*.	$D \bullet C$
Either *P* or *E*.	$P \vee E$
If *N* then *F*.	$N \supset F$
B if and only if *R*.	$B \equiv R$

The statement $\sim A$ is called a **negation.** The statement $D \bullet C$ is called a **conjunctive statement** (or a **conjunction**), and the statement $P \vee E$ is called a **disjunctive statement** (or a **disjunction**); in the conjunctive statement, the components *D* and *C* are called **conjuncts,** and in the disjunctive statement the components *P* and *E* are called **disjuncts**. The statement $W \supset F$ is called a **conditional statement** (or a **conditional**), and it expresses the relation of **material implication.** Its components are called **antecedent** (*W*) and **consequent** (*F*). Lastly, $B \equiv R$ is called a **biconditional statement** (or a **biconditional**), and it expresses the relation of **material equivalence.**

6

Let us now use the logical operators to translate additional English statements. The tilde symbol is used to translate any negated simple proposition:

Rolex does not make computers.	$\sim R$
It is not the case that Rolex makes computers.	$\sim R$
It is false that Rolex makes computers.	$\sim R$

As these examples show, the tilde is always placed *in front* of the proposition it negates. All of the other operators are placed *between* two propositions. Also, unlike the other operators, the tilde cannot be used to connect two propositions. Thus, $G \sim H$ is not a proper expression. But the tilde is the only operator that can immediately follow another operator. Thus, it would be proper to write $G \bullet \sim H$. In the Rolex examples, the tilde is used to negate a simple proposition, but it can also be used to negate a compound proposition—for example $\sim(G \bullet F)$. In this case the tilde negates the entire expression inside the parentheses.

These statements are all **negations.** The main operator is a tilde.

$\sim B$

$\sim(G \supset H)$

$\sim[(A \equiv F) \bullet (C \equiv G)]$

At this point we should define what is called the main operator in a compound statement. The **main operator** is the operator that has as its scope everything else in the statement. If there are no parentheses in the statement, the main operator will either be the only operator or, if there is more than one, it will be the operator that is not a tilde. If there are parentheses, brackets, or braces in the statement, the main operator will be the operator that lies outside all parentheses, brackets, and braces; if there is more than one such operator, the main operator will be the one that is not a tilde.

For example, in the statement $H \bullet (J \vee K)$, the main operator is the dot, because its scope extends to everything else in the statement, whereas the scope of the wedge extends only to the J and K. In the statement $\sim(K \bullet M)$, the main operator is the tilde because its scope extends to everything else. In the statement $K \supset \sim(L \bullet M)$, the main operator is the horseshoe, because, once again, its scope extends to everything else in the statement. Excluding the tilde, it is the only operator outside the parentheses.

The dot symbol is used to translate such conjunctions as "and," "also," "but," "however," "yet," "still," "moreover," "although," and "nevertheless":

Tiffany sells jewelry, and Gucci sells cologne.	$T \bullet G$
Tiffany sells jewelry, but Gucci sells cologne.	$T \bullet G$
Tiffany sells jewelry; however, Gucci sells cologne.	$T \bullet G$
Tiffany and Ben Bridge sell jewelry.	$T \bullet B$

6

Note that the last example is equivalent in meaning to "Tiffany sells jewelry, and Ben Bridge sells jewelry." To translate such a statement as a conjunction of two simple statements, the original statement must be equivalent to a compound statement in English. For example, the statement "Mary and Louise are friends" is *not* equivalent in meaning to "Mary is a friend, and Louise is a friend," so this statement cannot be translated as $M \bullet L$.

These statements are all **conjunctions.** The main operator is a dot.

$K \bullet \sim L$
$(E \vee F) \bullet \sim(G \vee H)$
$[(R \supset T) \vee (S \supset U)] \bullet [(W \equiv X) \vee (Y \equiv Z)]$

The wedge symbol is used to translate "or" and "unless." A previous chapter explained that "unless" is equivalent in meaning to "if not." This equivalence holds in propositional logic as well, but in propositional logic it is usually simpler to equate "unless" with "or." For example, the statement "You won't graduate unless you pass freshman English" is equivalent to "Either you pass freshman English or you won't graduate" and also to "If you don't pass freshman English, then you won't graduate." As the next section demonstrates, the wedge symbol has the meaning of "and/or"—that is, "or" in the inclusive sense. Although "or" and "unless" are sometimes used in an exclusive sense, the wedge is usually used to translate them as well.

The word "either,"which is often used to introduce disjunctive statements, has primarily a punctuational meaning. The placement of this word often tells us where parentheses and brackets must be introduced in the symbolic expression. If parentheses or brackets are not needed, "either" does not affect the translation. A similar point applies to the word "both," which is often used to introduce conjunctive statements. Here are some disjunctive statements:

Aspen allows snowboards or Telluride does.	$A \vee T$
Either Aspen allows snowboards or Telluride does.	$A \vee T$
Aspen allows snowboards unless Telluride does.	$A \vee T$
Unless Aspen allows snowboards, Telluride does.	$A \vee T$

From the English sense of these statements, it should be clear that $A \vee T$ is logically equivalent to $T \vee A$. Also $T \bullet G$ is logically equivalent to $G \bullet T$. Logically equivalent propositions necessarily have the same truth value.

6

These statements are all **disjunctions.** The main operator is a wedge.

$\sim C \vee \sim D$

$(F \bullet H) \vee (\sim K \bullet \sim L)$

$\sim [S \bullet (T \supset U)] \vee \sim [X \bullet (Y \equiv Z)]$

The horseshoe symbol is used to translate "if . . . then . . . ," "only if," and similar expressions that indicate a conditional statement. The expressions "in case," "provided that," "given that," and "on condition that" are usually translated in the same way as "if." By customary usage, the horseshoe symbol is also used to translate "implies." Although "implies" is used most properly to describe the relationship between the premises and conclusion of an argument, we may accept this translation as an alternate meaning for "implies."

The function of "only if" is, in a sense, just the reverse of "if." For example, the statement "You will catch a fish only if your hook is baited" does not mean "If your hook is baited, then you will catch a fish." If it meant this, then everyone with a baited hook would catch a fish. Rather, the statement means "If your hook is not baited, then you will not catch a fish," which is logically equivalent to "If you catch a fish, then your hook was baited." To avoid mistakes in translating "if" and "only if" remember this rule: The statement that follows "if" is always the antecedent, and the statement that follows "only if" is always the consequent. Thus, "C only if H" is translated $C \supset H$, whereas "C if H" is translated $H \supset C$. Additional examples:

If Purdue raises tuition, then so does Notre Dame.	$P \supset N$
Notre Dame raises tuition if Purdue does.	$P \supset N$
Purdue raises tuition only if Notre Dame does.	$P \supset N$
Cornell cuts enrollment provided that Brown does.	$B \supset C$

Cornell cuts enrollment on condition that Brown does. $B \supset C$

Brown's cutting enrollment implies that Cornell does. $B \supset C$

In translating conditional statements, it is essential not to confuse antecedent with consequent. The statement $A \supset B$ is not logically equivalent to $B \supset A$.

These statements are all **conditionals** (material implications). The main operator is a horseshoe.

$H \supset \sim J$
$(A \vee C) \supset \sim(D \cdot E)$
$[K \vee (S \cdot \sim T)] \supset [\sim F \vee (M \cdot O)]$

The horseshoe symbol is also used to translate statements phrased in terms of sufficient conditions and necessary conditions. Event *A* is said to be a **sufficient condition** for event *B* whenever the occurrence of *A* is all that is required for the occurrence of *B*. On the other hand, event *A* is said to be a **necessary condition** for event *B* whenever *B* cannot occur without the occurrence of *A*. For example, having the flu is a sufficient condition for feeling miserable, whereas having air to breathe is a necessary condition for survival. Other things besides having the flu might cause a person to feel miserable, but that by itself is sufficient; other things besides having air to breathe are required for survival, but without air survival is impossible. In other words, air is necessary.

To translate statements involving sufficient and necessary conditions into symbolic form, place the statement that names the sufficient condition in the antecedent of the conditional and the statement that names the necessary condition in the consequent. The mnemonic device "SUN" may be conveniently used to keep this rule in mind. Turning the U sideways creates $S \supset N$, wherein *S* and *N* designate sufficient and necessary conditions, respectively. Whatever is given as a sufficient condition goes in the place of the *S*, and whatever is given as a necessary condition goes in the place of the *N:*

Hilton's opening a new hotel is a sufficient condition for Marriott's doing so. $H \supset M$

Hilton's opening a new hotel is a necessary condition for Marriott's doing so. $M \supset H$

The triple bar symbol is used to translate the expressions "if and only if" and "is a sufficient and necessary condition for":

JFK tightens security if and only if O'Hare does. $J \equiv O$

JFK's tightening security is a sufficient and necessary condition for O'Hare's doing so. $J \equiv O$

Analysis of the first statement reveals that $J \equiv O$ is logically equivalent to $(J \supset O) \cdot (O \supset J)$. The statement "JFK tightens security only if O'Hare does" is translated $J \supset O$, and "JFK tightens security if O'Hare does" is translated $O \supset J$. Combining the two English statements, we have $(J \supset O) \bullet (O \supset J)$, which is just a longer way of writing $J \equiv O$. A similar analysis applies to the second statement. Because the order of the two conjuncts can be reversed, $J \equiv O$ is logically equivalent to $O \equiv J$. However, when translating such statements, we adopt the convention that the letter representing the first English statement is written to the left of the triple bar, and the letter representing the second English statement is written to the right of the triple bar. Thus, the examples above are translated $J \equiv O$, and not $O \equiv J$.

These statements are all **biconditionals** (material equivalences). The main operator is a triple bar.

$M \equiv \sim T$

$\sim(B \vee D) \equiv \sim(A \bullet C)$

$[K \vee (F \supset I)] \equiv [\sim L \bullet (G \vee H)]$

6

Whenever more than two letters appear in a translated statement, we must use parentheses, brackets, or braces to indicate the proper range of the operators. The statement $A \bullet B \vee C$, for example, is ambiguous. When parentheses are introduced, this statement becomes either $(A \bullet B) \vee C$ or $A \bullet (B \vee C)$. These two statements are not logically equivalent. Thus, with statements such as these, some clue must be found in the English statement that indicates the correct placement of the parentheses in the symbolic statement. Such clues are usually given by commas and semicolons, by such words as "either" and "both," and by the use of a single predicate in conjunction with two or more subjects. The following examples illustrate the correct placement of parentheses and brackets:

Prozac relieves depression and Allegra combats allergies, or Zocor lowers cholesterol.	$(P \bullet A) \vee Z$
Prozac relieves depression, and Allegra combats allergies or Zocor lowers cholesterol.	$P \bullet (A \vee Z)$
Either Prozac relieves depression and Allegra combats allergies or Zocor lowers cholesterol.	$(P \bullet A) \vee Z$
Prozac relieves depression and either Allegra combats allergies or Zocor lowers cholesterol.	$P \bullet (A \vee Z)$
Prozac relieves depression or both Allegra combats allergies and Zocor lowers cholesterol.	$P \vee (A \bullet Z)$
Prozac relieves depression and Allegra or Zocor lowers cholesterol.	$P \bullet (A \vee Z)$
If Merck changes its logo, then if Pfizer increases sales, then Lilly will reorganize.	$M \supset (P \supset L)$

If Merck's changing its logo implies that Pfizer increases sales, then Lilly will reorganize.	$(M \supset P) \supset L$
If Schering and Pfizer lower prices or Novartis downsizes, then Warner will expand production.	$[(S \cdot P) \vee N] \supset W$

Do not confuse these three statement forms:

A if *B*	$B \supset A$
A only if *B*	$A \supset B$
A if and only if *B*	$A \equiv B$

When a tilde appears in a symbolic expression, by convention it is considered to affect only the unit that immediately follows it. For example, in the expression $\sim K \vee M$ the tilde affects only the *K;* in the expression $\sim(K \vee M)$ it affects the entire expression inside the parentheses. In English, the expression "It is not the case that *K* or *M*" is ambiguous, because the range of the negating words is indefinite. To eliminate this ambiguity, we now adopt the convention that the negating words are considered to affect only the unit that follows them. Thus, "It is not the case that *K* or *M*" is translated $\sim K \vee M$.

The statement "Not both *S* and *T*" is translated $\sim(S \cdot T)$. By an important rule called *De Morgan's rule*, this statement is logically equivalent to $\sim S \vee \sim T$. For example, the statement "Not both Steven and Thomas were fired" is equivalent in meaning to "Either Stephen was not fired or Thomas was not fired." Because the former statement is *not* equivalent in meaning to "Stephen was not fired and Thomas was not fired," $\sim(S \cdot T)$ is *not* logically equivalent to $\sim S \cdot \sim T$. Analogously, the statement "Not either *S* or *T*" is translated $\sim(S \vee T)$, which by De Morgan's rule is logically equivalent to $\sim S \cdot \sim T$. For example, "Not either Steven or Thomas was fired" is equivalent in meaning to "Steven was not fired and Thomas was not fired." Thus, $\sim(S \vee T)$ is *not* logically equivalent to $\sim S \vee \sim T$. The following examples illustrate these points:

Megan is not a winner, but Kathy is.	$\sim M \cdot K$
Not both Megan and Kathy are winners.	$\sim (M \cdot K)$
Either Megan or Kathy is not a winner.	$\sim M \vee \sim K$
Both Megan and Kathy are not winners.	$\sim M \cdot \sim K$
Not either Megan or Kathy is a winner.	$\sim (M \vee K)$
Neither Megan nor Kathy is a winner.	$\sim (M \vee K)$

Notice the function of "either" and "both":

Not either *A* or *B*.	$\sim (A \vee B)$
Either not *A* or not *B*.	$\sim A \vee \sim B$
Not both *A* and *B*.	$\sim (A \cdot B)$
Both not *A* and not *B*.	$\sim A \cdot \sim B$

6

Eminent Logicians

Gottfried Wilhelm Leibniz 1646–1716


© Bildarchiv Preussischer Kulturbesitz/Art Resource, NY

Gottfried Wilhelm Leibniz was a polymath who knew virtually everything that could be known at the time about nearly every area of intellectual endeavor. He also made important contributions to many of them, including physics, engineering, philosophy, theology, history, law, politics, and philology. In mathematics Leibniz invented differential and integral calculus (independently of Newton) and the theory of differential equations. He also discovered the binary number system (used by all of today's digital computers), and he created the first calculating machine that could add, subtract, multiply, and divide. In metaphysics he created the famous theory of monads which, among other things, explained the relation between the soul and the body.

Leibniz was born in Leipzig to prominent parents. His father (who died when Leibniz was six) was a professor of moral philosophy at the city's university, and his mother was the daughter of a famous lawyer. As a child Leibniz proved himself a prodigy. By age twelve he was fluent in Latin and had a passing knowledge of Greek, both of which he had learned by himself. By thirteen he was deep into the works of Aristotle and scholastic philosophy, and at fourteen he entered the University of Leipzig, where he studied philosophy, mathematics, and law. After completing that program he began a doctoral program in law; however, when he applied for the degree at age twenty, he was refused because of his youth. Not to be outdone, Leibniz presented his thesis to the University of Altdorf, and the professors there were so impressed that they immediately awarded him the degree of Doctor of Laws and offered him a professorship.

As he grew older Leibniz developed a taste for the finer things in life, including expensive clothing, long, flowing wigs, fancy carriages, and luxurious accommodations. However, following the death of his mother when he was eighteen, an uncle received what should have been Leibniz's inheritance. When this happened Leibniz figured that the best way of satisfying his expensive tastes was to attach himself to the wealthy and the powerful, which he did with great success. He entered the employment of the elector of Mainz, and by the age of twenty-four he held the rank of privy counselor of justice, one of the highest positions in the government. His work as a diplomat allowed him the opportunity to travel widely and meet most of the prominent figures in Europe. Later he worked for the Duke of Hanover, which also allowed much time for travel and independent study.

Leibniz is sometimes called the father of symbolic logic for his work in developing the *universal characteristic,* a symbolic language in which any item of information can be represented in a natural and systematic way, together with the *calculus ratiocinator,* a deductive system for manipulating the symbols and deriving consequences. Given the dispassionate nature of this enterprise, Leibniz thought that it would serve to resolve differences of opinion in religion, theology, and philosophy. However, at age seventy he died in Hanover before completing the project.

The symbolic expressions that we have used throughout this section to translate meaningful, unambiguous English statements are called **well-formed formulas (WFFs)**. "WFFs" is usually pronounced "woofs." A well-formed formula is a *syntactically* correct arrangement of symbols. In English, for example, the expression "there is a cat on the porch" is syntactically correct, but "Porch on the is cat a there" is not syntactically correct. Some examples of symbolic arrangements that are *not* well-formed formulas are "$A \supset \vee B$," "$A \bullet B (\vee C)$," and "$\sim\vee B \equiv \supset C$."

Summary	**Operator**
not, it is not the case that, it is false that	$\sim$
and, yet, but, however, moreover, nevertheless, still, also, although, both, additionally, furthermore	$\bullet$
or, unless	$\vee$
if . . . then, only if, implies, given that, in case, provided that, on condition that, sufficient condition for, necessary condition for (Note: Do not confuse antecedent with consequent!)	$\supset$
if and only if, is equivalent to, sufficient and necessary condition for	$\equiv$

6

Exercise 6.1

I. Translate the following statements into symbolic form using capital letters to represent affirmative English statements.

★1. Cartier does not make cheap watches.
2. Arizona has a national park but Nebraska does not.
3. Either Stanford or Tulane has an architecture school.
★4. Both Harvard and Baylor have medical schools.
5. If Chanel has a rosewood fragrance, then so does Lanvin.
6. Chanel has a rosewood fragrance if Lanvin does.
★7. Maureen Dowd writes incisive editorials if and only if Paul Krugman does.
8. Reese Witherspoon wins best actress only if Martin Scorsese wins best director.
9. Armani will launch a leather collection given that Gucci rejects skinny models.
★10. The Colts' winning most of their games implies that Peyton Manning is a great quarterback.
11. Bill Gates does not support malaria research unless Warren Buffet does.

12. Mercedes will introduce a hybrid model only if Lexus and BMW do.
★13. Mariah Carey sings pop and either Elton John sings rock or Diana Krall sings jazz.
14. Either Mariah Carey sings pop and Elton John sings rock or Diana Krall sings jazz.
15. Not both Jaguar and Porsche make motorcycles.
★16. Both Jaguar and Porsche do not make motorcycles.
17. Either Nokia or Seiko makes cell phones.
18. Not either Ferrari or Maserati makes economy cars.
★19. Neither Ferrari nor Maserati makes economy cars.
20. Either Ferrari or Maserati does not make economy cars.
21. If Glenn Beck spins the news, then if Keith Olberman fights back, then Rachel Maddow tells it straight.
★22. If Glenn Beck's spinning the news implies that Keith Olberman fights back, then Rachel Maddow tells it straight.
23. Tommy Hilfiger celebrates casual if and only if neither Ralph Lauren nor Calvin Klein offers street chic.
24. If Saks promotes gift cards, then either Macy's or Bloomingdale's puts on a fashion show.
★25. Either Rado does not make a sapphire watch or if Movado makes one then so does Pulsar.
26. If either Renée Zellweger or Michelle Pfeiffer accepts a dramatic role, then neither Charlie Sheen nor Ethan Hawke will make an action film.
27. If Kate Winslet and Jessica Biel do a comedy, then either Forest Whitaker will make a documentary or Paris Hilton will do a skin flick.
★28. Mercury is a planet given that both Pluto and Ceres are not.
29. Saturn has rings, and Neptune is windy or Jupiter is massive.
30. Saturn has rings and Neptune is windy, or Jupiter is massive.
★31. Tiffany and Ben Bridge will release an emerald collection unless Zales and Kay do not.
32. Brad Pitt will travel abroad provided that Angelina Jolie does, but Shiloh and Maddox will stay home.
33. Either Sonia Sotomayor or Antonin Scalia have a modern approach to the Constitution, but it is not the case that both do.
★34. Barack Obama emphasizes peaceful negotiation; but if North Korea starts a war, then either China or Japan will be involved.
35. It is not the case that both Iran gives up its nuclear program and Syria or Pakistan combats terrorism.
36. It is not the case that either Hezbollah will renounce violence or Al Qaeda and the Taliban will be defeated.

★37. If Spike Lee writes a screen play, then if Denzel Washington stars in the movie, then Paramount and MGM will compete to produce it.

38. If Maria Cantwell promotes alternative energy, then if Patty Murray supports wilderness areas, then Olympia Snowe's advocating gun control implies that Susan Collins does so, too.

39. It is not the case that either Tiger Woods and Maria Sharapova play professional football or Apolo Ohno and Lindsey Vonn play professional baseball.

★40. It is not the case that both Kobe Bryant or Shaquille O'Neal plays professional tennis and Chris Johnson or Philip Rivers plays professional basketball.

41. Israel's abandoning its settlements is a sufficient condition for the Palestinians' declaring an end to hostilities.

42. Israel's abandoning its settlements is a necessary condition for the Palestinian's declaring an end to hostilities.

★43. Israel's abandoning its settlements is a sufficient and necessary condition for the Palestinians' declaring an end to hostilities.

44. The Taliban's being defeated is a sufficient condition for Pakistan's winning the war on terror only if Afghanistan's securing its borders is a necessary condition for the UN's stopping the opium trade.

45. Katie Couric and Diane Sawyer report international news if and only if Robin Meade and Nora O'Donnell cover political developments.

★46. It is not the case that both Atari's releasing a stalker game implies that Nintendo does and Sega's releasing a molester game implies that Commodore does.

47. Cameron Diaz promotes environmental causes if Ben Affleck supports civil liberties, provided that Sean Penn opposes the death penalty.

48. The Dixie Chicks' opening the show implies that the Chili Peppers close it, given that the Black Eyed Peas' showing up implies that neither Gnarls Barkley nor Rascal Flatts will perform.

★49. If Christina Aguilera's singing soul and Justin Timberlake's singing pop are sufficient and necessary conditions for Kelly Clarkson's singing rock, then neither Beyoncé nor Shakira will sing rap.

50. Nassau's advertising sizzling night clubs is a necessary condition for Puerto Vallarta's offering luxury hotels; moreover, Cancun's having turquoise waters and Acapulco's promising powder-white beaches is a sufficient condition for Jamaica's offering reggae music.

II. Translate the following statements into symbolic form using capital letters to represent affirmative English statements.

★1. Unless we reduce the incidence of child abuse, future crime rates will increase.

2. If pharmaceutical makers conceal test results, they are subject to substantial fines.

3. African safaris are amazing, but they are also expensive.

6

★4. Cigarette manufacturers are neither honest nor socially responsible.

5. Psychologists and psychiatrists do not both prescribe antidepressant drugs.

6. If health maintenance organizations cut costs, then either preventive medicine is emphasized or the quality of care deteriorates.

★7. A necessary condition for a successful business venture is good planning.

8. If cocaine is legalized, then its use may increase but criminal activity will decline.

9. Ozone depletion in the atmosphere is a sufficient condition for increased cancer rates.

★10. If affirmative action programs are dropped, then if new programs are not created, then minority applicants will suffer.

11. If Internet use continues to grow, then more people will become cyberaddicts and normal human relations will deteriorate.

12. Human life will not perish unless either we poison ourselves with pollution or a large asteroid collides with the earth.

★13. Cooling a group of atoms to absolute zero and keeping them bunched together is a necessary and sufficient condition for producing a Bose-Einstein condensate.

14. If motion pictures contain subliminal sex messages or if they challenge the traditional family, then conservative politicians call for censorship.

15. Either clear-cutting in national forests is halted and old-growth trees are allowed to stand, or salmon runs will be destroyed and bird habitats obliterated.

★16. Three-strikes laws will be enforced and longer sentences imposed only if hundreds of new prisons are built, and that will happen only if taxes are increased.

17. The Ebola virus is deadly, but it will become a major threat to humanity if and only if it becomes airborne and a vaccine is not developed.

18. If evolutionary biology is correct, then higher life-forms arose by chance, and if that is so, then it is not the case that there is any design in nature and divine providence is a myth.

★19. If banks charge fees for teller-assisted transactions, then more people will use ATMs; and if that happens and ATM fees increase, then banks will close branches and profits will skyrocket.

20. If corporate welfare continues, then taxpayer interests will be ignored and billions of tax dollars will go to giant corporations; and if the latter occurs, then there will not be anything left for the poor and the budget will not be balanced.

III. Determine which of the following are *not* well-formed formulas.

1. $(S \bullet {\sim}T) \vee ({\sim}U \bullet W)$
2. ${\sim}(K \vee L) \bullet (\supset G \vee H)$
3. $(E{\sim}F) \vee (W \equiv X)$

4. $(B \supset \sim T) \equiv \sim(\sim C \supset U)$
5. $(F \equiv \sim Q) \bullet (A \supset E \vee T)$
6. $\sim D \vee \sim[(P \supset Q) \bullet (T \supset R)]$
7. $[(D \bullet \vee Q) \supset (P \vee E)] \vee [A \supset (\bullet H)]$
8. $M(N \supset Q) \vee (\sim C \bullet D)$
9. $\sim(F \vee \sim G) \supset [(A \equiv E) \bullet \sim H]$
10. $(R \equiv S \bullet T) \supset \sim(\sim W \bullet \sim X)$

6.2 Truth Functions

6

The truth value of a compound proposition expressed in terms of one or more logical operators is said to be a function of the truth values of its components. This means that the truth value of the compound proposition is completely determined by the truth values of its components. If the truth values of the components are known, then the truth value of the compound proposition can be calculated from the definitions of the logical operators. Accordingly, a **truth function** is any compound proposition whose truth value is completely determined by the truth values of its components.

Many compound propositions in ordinary language are not truth functions. For example, the statement "Mary believes that Paul is dishonest" is compound because it contains the statement "Paul is dishonest" as a component. Yet the truth value of the compound statement is not determined by the truth value of the component, because Mary's beliefs about Paul are not compelled by any attribute that Paul may or may not possess.

The first part of this section presents the definitions of the five logical operators, the second part shows how they are used to compute the truth values of more complicated propositions, and the third examines further the degree to which symbolized expressions match the meaning of expressions in ordinary language.

Definitions of the Logical Operators

The definitions of the logical operators are presented in terms of **statement variables,** which are lowercase letters (p, q, r, s) that can stand for any statement. For example, the statement variable p could stand for the statements A, $A \supset B$, $B \vee C$, and so on.

Statement variables are used to construct statement forms. A **statement form** is an arrangement of statement variables and operators such that the uniform substitution of statements in place of the variables results in a statement. For example, $\sim p$ and $p \supset q$ are statement forms because substituting the statements A and B in place of p and q, respectively, results in the statements $\sim A$ and $A \supset B$. A compound statement is said to have a certain form if it can be produced by substituting statements in place of the letters in that form. Thus, $\sim A$, $\sim(A \vee B)$, and $\sim[A \bullet (B \vee C)]$ are negations because they can be produced by substituting statements in place of p in the form $\sim p$.

Now let us consider the definition of the tilde operator (negation). This definition is given by a **truth table**, an arrangement of truth values that shows in every possible case how the truth value of a compound proposition is determined by the truth values of its simple components. The truth table for negation shows how any statement having the form of a negation ($\sim p$) is determined by the truth value of the statement that is negated (p):

Negation

p	$\sim p$
T	F
F	T

The truth table shows that $\sim p$ is false when p is true and that $\sim p$ is true when p is false. This is exactly what we would expect, because it perfectly matches ordinary English usage. Examples:

It is not the case that McDonald's makes hamburgers.	$\sim M$
It is not the case that Starbucks makes hamburgers.	$\sim S$

The first statement is false because M is true, and the second is true because S is false.

Let us now consider the definition of the dot operator (conjunction). The truth table that follows shows how any statement having the form of a conjunction ($p \bullet q$) is determined by the truth values of its conjuncts (p, q):

Conjunction

p	q	$p \bullet q$
T	T	T
T	F	F
F	T	F
F	F	F

This truth table shows that a conjunction is true when its two conjuncts are true and is false in all other cases. This definition reflects ordinary language usage almost as perfectly as negation. Consider the following conjunctive statements:

Ferrari and Maserati make sports cars.	$F \bullet M$
Ferrari and GMC make sports cars.	$F \bullet G$
GMC and Jeep make sports cars.	$G \bullet J$

The first statement is true, because both conjuncts are true; but the second and third statements are false, because at least one of their conjuncts is false.

Turning now to the definition of the wedge operator (disjunction), the truth table is as follows:

Disjunction

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

The truth table indicates that the disjunction is true when at least one of the disjuncts is true and that otherwise it is false. The truth-functional interpretation of "or" is that of *inclusive* disjunction: Cases in which the disjunction is true include the case when both disjuncts are true. This inclusive sense of "or" corresponds to many instances of ordinary usage, as the following examples illustrate:

Either Steven King or Cate Blanchett is a novelist. $S \vee C$

Either Steven King or Danielle Steel is a novelist. $S \vee D$

Either Kobe Bryant or Tiger Woods is a novelist. $K \vee T$

6

The first two statements are true, because in each case at least one of the disjuncts is true. The third is false, because both disjuncts are false.

The match between the truth-functional definition of disjunction and ordinary usage is not perfect, however. Sometimes the sense of a statement in ordinary language is that of *exclusive* disjunction. Examples:

The Orient Express is on either track A or track B.

You can have either soup or salad with this meal.

Tammy is either ten or eleven years old.

The sense of these statements excludes the possibility of both alternatives being true. Thus, if these statements were translated using the wedge, a portion of their ordinary meaning would be lost. If the exclusive aspect of these "either . . . or . . ." statements is essential, the symbolic equivalent of "but not both" can be attached to their translations. Thus the first statement could be translated $(A \vee B) \bullet \sim(A \bullet B)$.

Let us now consider the horseshoe operator (material implication, or conditional). Its truth table is as follows:

Conditional
(material implication)

p	q	$p \supset q$
T	T	T
T	F	F
F	T	T
F	F	T

The truth table shows that a conditional statement is false when the antecedent is true and the consequent false and is true in all other cases. This truth-functional interpretation of conditional statements conforms in part with the ordinary meaning of "if . . . then . . ." and in part it diverges. Consider the following examples:

If Nicole Kidman is an actor, then so is Meryl Streep.	$N \supset M$
If Nicole Kidman is an actor, then so is Wolf Blitzer.	$N \supset W$
If Wolf Blitzer is an actor, then so is Helen Hunt.	$W \supset H$
If Wolf Blitzer is an actor, then so is Roger Ebert.	$W \supset R$

In these statements *N, M,* and *H* are true and *W* and *R* are false. Thus, according to the truth-functional interpretation, the first statement is true and the second false. This result conforms in large measure to our expectations. But the truth-functional interpretation of the last two statements is true. Although this result may not conflict with our expectations, it is not at all clear why these statements should be considered true.

6

For an intuitive approach to this problem, imagine that your logic instructor made the following statement: "If you get an A on the final exam, then you will get an A for the course." Under what conditions would you say that your instructor had lied to you? Clearly, if you got an A on the final exam but did not get an A for the course, you would say that she had lied. This outcome corresponds to a true antecedent and a false consequent. On the other hand, if you got an A on the final exam and also got an A for the course, you would say that she had told the truth (true antecedent, true consequent). But what if you failed to get an A on the final exam? Two alternatives are then possible: Either you got an A for the course anyway (false antecedent, true consequent) or you did not get an A for the course (false antecedent, false consequent). In neither case, though, would you say that your instructor had lied to you. Giving her the benefit of the doubt, you would say that she had told the truth.

Lastly, let us consider the definition of the triple bar operator (material equivalence, or biconditional). Its truth table is as follows:

Biconditional
(material equivalence)

p	q	$p \equiv q$
T	T	T
T	F	F
F	T	F
F	F	T

The truth table shows that the biconditional is true when its two components have the same truth value and that otherwise it is false. These results conform reasonably well with our expectations. However, given that $p \equiv q$ is simply a shorter way of writing $(p \supset q) \bullet (q \supset p)$, the truth-table results are required by the definition of material implication. If p and q are either both true or both false, then $p \supset q$ and $q \supset p$ are both true, making their conjunction true. But if p is true and q is false, then $p \supset q$ is false, making the conjunction false. Similarly, if p is false and q is true, then $q \supset p$ is false, again making the conjunction false. Thus, $p \equiv q$ is true when p and q have the same truth value and false when they have opposite truth values.

The truth table definition of the triple bar symbol conforms quite closely with ordinary usage, as the following examples illustrate:

Bill Maher is a show host if and only if Jay Leno is.	$B \equiv J$
Bill Maher is a show host if and only if Meg Ryan is.	$B \equiv M$
Meg Ryan is a show host if and only if Al Pacino is.	$M \equiv A$

In these statements, B and J are true and M and A false. Thus, from the truth-functional standpoint, the first is true and the second false. This is what we would ordinarily expect. The third statement, however, turns out to be true because both of its components are false. While this result may not be what we would expect, it does not violate our expectations either. Other biconditional statements having false components are more obviously true. Example:

Al Gore was elected president if and only if he received a majority vote from the electoral college.

This statement asserts what is required for any candidate to be elected or not elected, and so it is clearly true.

In summary, the definitions of the five logical operators conform reasonably well with ordinary linguistic usage. However, as the last part of this section shows, the match is less than perfect. Before considering this question, though, let us use the operator definitions to compute the truth values of more-complicated statements.

Computing the Truth Value of Longer Propositions

To compute the truth value of a more complicated expression, use this procedure: Enter the truth values of the simple components directly beneath the letters. Then use these truth values to compute the truth values of the compound components. The truth value of a compound statement is written beneath the operator representing it. Let us suppose, for example, that we are told in advance that the simple propositions A, B, and C are true and D, E, and F are false. We may then compute the truth value of the following compound proposition:

$$(A \vee D) \supset E$$

First, we write the truth values of the simple propositions immediately below the respective letters and bring the operators and parentheses down:

$$(A \vee D) \supset E$$
$$(\mathrm{T} \vee \mathrm{F}) \supset \mathrm{F}$$

6

Next, we compute the truth value of the proposition in parentheses and write it beneath the operator to which it pertains:

$(A \vee D) \supset E$

$(\mathrm{T} \vee \mathrm{F}) \supset \mathrm{F}$

$\mathrm{T} \quad \supset \mathrm{F}$

Finally, we use the last-completed line to obtain the truth value of the conditional, which is the main operator in the proposition:

$(A \vee D) \supset E$

$(\mathrm{T} \vee \mathrm{F}) \supset \mathrm{F}$

$\mathrm{T} \quad \supset \mathrm{F}$

F

The final answer is circled. This is the truth value of the compound proposition given that *A* is true and *D* and *E* are false.

The general strategy is to build the truth values of the larger components from the truth values of the smaller ones. In general, the order to be followed in entering truth values is this:

1. Individual letters representing simple propositions
2. Tildes immediately preceding individual letters
3. Operators joining letters or negated letters
4. Tildes immediately preceding parentheses
5. And so on

Here are some additional examples. As before, let *A*, *B*, and *C* be true, *D*, *E*, and *F* false. Note that the computed truth values are written beneath the operators to which they pertain. The final answers, which are written beneath the main operators, are circled.

1. $(B \bullet C) \supset (E \supset A)$

   $(\mathrm{T} \bullet \mathrm{T}) \supset (\mathrm{F} \supset \mathrm{T})$

   $\mathrm{T} \quad \supset \quad \mathrm{T}$

   T

2. $\sim (C \vee \sim A) \supset \sim B$

   $\sim (\mathrm{T} \vee \sim \mathrm{T}) \supset \sim \mathrm{T}$

   $\sim (\mathrm{T} \vee \mathrm{F}) \quad \supset \mathrm{F}$

   $\sim \quad \mathrm{T} \quad\quad \supset \mathrm{F}$

   $\mathrm{F} \quad\quad\quad \supset \mathrm{F}$

   T

3. $[\sim (D \vee F) \bullet (B \vee \sim A)] \supset \sim (F \supset \sim C)$

   $[\sim (\mathrm{F} \vee \mathrm{F}) \bullet (\mathrm{T} \vee \sim \mathrm{T})] \supset \sim (\mathrm{F} \supset \sim \mathrm{T})$

$$
\begin{array}{l}
[\sim (F \vee F) \bullet (T \vee F)] \supset \sim (F \supset F) \\
[\sim \ F \ \bullet \ T \ ] \supset \sim \ T \\
[T \ \bullet \ T \ ] \supset F \\
\quad T \quad \supset F \\
\quad\quad F
\end{array}
$$

If preferred, the truth values of the compound components may be entered directly beneath the operators, without using the line-by-line approach illustrated in these examples. The following examples illustrate this second approach, which is used in the next section:

1. $[(D \equiv \sim A) \bullet \sim (C \bullet \sim B)] \equiv \sim [(A \supset \sim D) \vee (C \equiv E)]$

   F T FT T T T F F T F F T T T F T T F F

2. $\sim \{[(C \bullet \sim E) \supset \sim (A \bullet \sim B)] \supset [\sim (B \vee D) \equiv (\sim C \vee E)]\}$

   F T T T F T T T F F T T F T T F T F T F F

Further Comparison with Ordinary Language

The first part of this section showed that the definitions of the five logical operators conform reasonably well with ordinary linguistic usage. This part further examines the extent of this match in meaning.

In regard to the dot operator, which is used to translate “and” and “but,” the match is often good; but it fails, at least in part, when the meaning of a conjunctive statement depends on the order of the conjuncts. Consider the following statements:

She got married and had a baby.	$M \bullet B$
She had a baby and got married.	$B \bullet M$

The first statement implies that the marriage occurred first, and the baby came later, while the second statement implies that the baby came first. This implied meaning is lost in the truth-functional interpretation, because $M \bullet B$ is logically equivalent to $B \bullet M$.

Another kind of mismatch between the truth functional meaning of conjunctive statements and their ordinary language meaning occurs when the order of the conjuncts implies a causal connection. Example:

He fell off a ladder and broke his arm.	$F \bullet B$
He broke his arm and fell off a ladder.	$B \bullet F$

The first statement implies that the fall caused the broken arm, but the second statement implies no such thing. However, the truth functional meanings of the two statements are logically equivalent.

For yet another example, consider the following:

This car is ugly, but it’s economical to drive.	$U \bullet E$
This car is economical to drive, but it’s ugly.	$E \bullet U$

The first statement subtly implies that we should buy this car, while the second implies that we should not buy it. But once again, the truth functional interpretations are logically equivalent.

Another instance where the truth-functional interpretation of "and" differs from the ordinary linguistic meaning is offered by slang statements like this one:

You go for that gun, and you'll regret it.

The sense of this statement is not that you will in fact go for the gun but rather that, *if* you go for that gun, then . . . Accordingly, if this statement were interpreted as a truth-functional conjunction, its meaning would be distorted.

In regard to the wedge operator, which is used to translate "or" and "unless," we saw that the wedge is defined as inclusive disjunction, but we observed that the English word "or" sometimes has the sense of exclusive disjunction. This same observation applies to "unless." In the following statements, "unless" has an inclusive sense:

You won't win the lottery unless you buy a ticket.

It will not rain unless there are clouds in the sky.

6

The meaning of the first statement includes the case of buying a ticket and not winning, and the meaning of the second includes the case of there being clouds and no rain. In statements like these, where "unless" has an inclusive sense, using the wedge symbol to translate "unless" results in no loss of meaning.

On the other hand, in the following statements "unless" is used in the exclusive sense:

Pork is not properly cooked unless the meat is white.

These logs will make a nice campfire unless they are wet.

The first statement suggests that the meat cannot be white and at the same time not be properly cooked, and the second suggests that the logs cannot be wet and at the same time be used to make a nice campfire. Thus, if these statements are translated using the wedge operator, part of the meaning will be left out. If this additional part is essential, it can be included by adding the symbolic equivalent of "but not both" to the translation.

In connection with the horseshoe operator, we saw that a question arose when the antecedent of a conditional statement turned out to be false. Why, under this circumstance, should the conditional statement be said to be true? For an example of some conditional statements that conform to the truth-functional interpretation, consider the following:

If the temperature rises above 32°F, then the snow will begin to melt.

If Figure A is a triangle, then Figure A has three sides.

If all *A* are *B* and all *B* are *C*, then all *A* are *C*.

In all three examples the statement remains true even though the antecedent might be false. In the first, even though the temperature does not rise above 32°F at any particular moment, the law that governs the melting point of snow holds true. In other words, the statement (which expresses this law) is true regardless of the truth value of the antecedent. In the second, the mere fact that Figure A might not be a triangle does not affect the fact that a triangle, by definition, has three sides. Thus, the statement (which expresses this fact) is true regardless of the truth value of the antecedent. The third statement expresses a logical relationship between statements. This logical relationship remains unchanged regardless of what the terms *A*, *B*, and *C* are taken to represent. Thus, if *A*, *B*, and *C* represent "dogs," "cats," and "birds," respectively, both antecedent and consequent turn out to be false, but the conditional statement remains true.

As these examples illustrate, the definition of the horseshoe operator matches the meaning of some conditional statements in ordinary language very well. However, in general, the match is far from perfect. The source of the mismatch stems from the fact that the horseshoe operator designates the *material* conditional, or *truth-functional* conditional. The material conditional is a kind of conditional statement whose truth value depends purely on the truth or falsity of the antecedent and consequent and not on any inferential connection *between* antecedent and consequent. Since many conditional statements in ordinary language express such an inferential connection, when the horseshoe operator is used to translate them, part of their meaning is left out. For example, compare the following two statements:

> If Shakespeare wrote *Hamlet*, then the sun rises in the east.
>
> If ice is lighter than water, then ice floats in water.

The first statement expresses no inferential connection between antecedent and consequent, so using the horseshoe operator to translate it results in no loss of meaning. However, the second statement does express such a connection. The fact that ice is lighter than water is the reason why it floats. Accordingly, when the horseshoe operator is used to translate the second statement, this special meaning is lost.

The fact that the material conditional ignores inferential connections between antecedent and consequent allows for conflicts between the truth-functional interpretation of a conditional statement and the ordinary interpretation. Consider, for example, the following:

> If Barbara Boxer advocates the use of cocaine, then she is a good senator.
>
> If Chicago is in Michigan, then Chicago is very close to Miami.

According to their ordinary language interpretation, both of these statements are false. Good senators do not advocate the use of cocaine, and Michigan is far from Miami. Yet, when these statements are interpreted as material conditionals, both turn out to be true, because their antecedents are false. In cases like these, when the truth-functional interpretation of a conditional statement conflicts with the ordinary language interpretation, using the horseshoe operator to translate it may not be appropriate.

While inferential relations between antecedent and consequent often play some role in conditionals expressed in the indicative mood (such as those we just considered), they play a dominant role in conditional statements expressed in the subjunctive mood. Consider, for example, the following:

> If I were Bill Gates, then I would be rich.
>
> If dolphins were fish, then they would be cold-blooded.
>
> If the Washington Monument were made of lead, then it would be lighter than air.
>
> If President Kennedy had committed suicide, then he would be alive today.

Subjunctive conditionals are often called counterfactual conditionals because their antecedents are typically false. As a result, the *only* way of determining their truth value in ordinary language is through some kind of inference. Thus, from our knowledge that Bill Gates is rich, we reason that if I were he, then I would be rich. Similarly, from our knowledge that all fish are cold-blooded, we conclude that if dolphins were fish, then they would be cold-blooded. On the other hand, we reason that the second two are false from our knowledge that lead is heavier than air and our knowledge that suicide results in death. Because the truth value of subjunctive conditionals is so closely tied to inferences like these and is so unrelated to the truth or falsity of the components, subjunctive conditionals are generally not considered to be truth functional at all, and the horseshoe operator is not used to translate them. But if they were interpreted truth-functionally, note that all four of these statements would turn out true, because they have false antecedents.

These observations about conditional statements apply equally to biconditionals. Just as the horseshoe operator expresses *material* implication, the triple bar operator expresses *material* equivalence. As such, it ignores any inferential connection between its component statements, and, as a result, conflicts can arise between the ordinary meaning of a biconditional and its truth-functional meaning. Here are two examples of biconditionals expressed in the indicative mood that illustrate such a conflict:

> Adolf Hitler was justified in killing millions of Jews if and only if he always confessed his sins to a priest.

> The Department of Defense building is a hexagon if and only if it has eight sides.

According to the ordinary interpretation, these statements are false. Confessing one's sins to a priest does not justify anything, and hexagons, by definition, have six sides, not eight. Yet, when these statements are interpreted as expressing material biconditionals, both are true, because in each case the component statements are false. In cases like these, when the ordinary meaning of a biconditional conflicts with the truth-functional meaning, using the triple bar operator to translate it may not be appropriate. Furthermore, as with subjunctive conditionals, subjunctive biconditionals are generally not considered to be truth-functional at all, so the triple bar operator is not used to translate them.

Exercise 6.2

I. Identify the main operator in the following propositions:

★1. $\sim(A \vee M) \bullet \sim(C \supset E)$

2. $(G \bullet \sim P) \supset \sim(H \vee \sim W)$

3. $\sim[P \bullet (S \equiv K)]$

★4. $\sim(K \bullet \sim O) \equiv \sim(R \vee \sim B)$

5. $(M \bullet B) \vee \sim[E \equiv \sim(C \vee I)]$

6. $\sim[(P \bullet \sim R) \supset (\sim E \vee F)]$

★7. $\sim[(S \vee L) \bullet M] \supset (C \vee N)$
8. $[\sim F \vee (N \bullet U)] \equiv \sim H$
9. $E \bullet [(F \supset A) \equiv (\sim G \vee H)]$
★10. $\sim[(X \vee T) \bullet (N \vee F)] \vee (K \supset L)$

II. Write the following compound statements in symbolic form, then use your knowledge of the historical events referred to by the simple statements to determine the truth value of the compound statements.

★1. It is not the case that Hitler ran the Third Reich.
2. Nixon resigned the presidency and Lincoln wrote the Gettysburg Address.
3. France bombed Pearl Harbor, or Lindbergh crossed the Atlantic.
★4. Hitler ran the Third Reich and Nixon did not resign the presidency.
5. Edison invented the telephone, or Custer was killed by the Indians.
6. Alexander the Great civilized America if Napoleon ruled France.
★7. Washington was assassinated only if Edison invented the telephone.
8. Lincoln wrote the Gettysburg Address if and only if France bombed Pearl Harbor.
9. It is not the case that either Alexander the Great civilized America or Washington was assassinated.
★10. If Hitler ran the Third Reich, then either Custer was killed by the Indians or Einstein discovered aspirin.
11. Either Lindbergh crossed the Atlantic and Edison invented the telephone or both Nixon resigned the presidency and it is false that Edison invented the telephone.
12. Lincoln's having written the Gettysburg Address is a sufficient condition for Alexander the Great's having civilized America if and only if Washington's being assassinated is a necessary condition for Custer's having been killed by the Indians.
★13. Both Hitler ran the Third Reich and Lindbergh crossed the Atlantic if neither Einstein discovered aspirin nor France bombed Pearl Harbor.
14. It is not the case that Custer was killed by the Indians unless both Nixon resigned the presidency and Edison invented the telephone.
15. Custer was killed by the Indians, and Lincoln wrote the Gettysburg Address only if either Washington was assassinated or Alexander the Great civilized America.

III. Determine the truth values of the following symbolized statements. Let *A*, *B*, and *C* be true and *X*, *Y*, and *Z* be false. Circle your answer.

★1. $A \bullet X$
2. $B \bullet \sim Y$
3. $X \vee \sim Y$
★4. $\sim C \vee Z$
5. $B \supset \sim Z$

6

6. $Y \supset {\sim}A$

★7. ${\sim}X \supset Z$

8. $B \equiv Y$

9. ${\sim}C \equiv Z$

★10. ${\sim}(A \bullet {\sim}Z)$

11. ${\sim}B \vee (Y \supset A)$

12. $A \supset {\sim}(Z \vee {\sim}Y)$

★13. $(A \bullet Y) \vee ({\sim}Z \bullet C)$

14. ${\sim}(X \vee {\sim}B) \bullet ({\sim}Y \vee A)$

15. $(Y \supset C) \bullet {\sim}(B \supset {\sim}X)$

★16. $(C \equiv {\sim}A) \vee (Y \equiv Z)$

17. ${\sim}(A \bullet {\sim}C) \supset ({\sim}X \supset B)$

18. ${\sim}[(B \vee {\sim}C) \bullet {\sim}(X \vee {\sim}Z)]$

★19. ${\sim}[{\sim}(X \supset C) \equiv {\sim}(B \supset Z)]$

20. $(X \supset Z) \supset [(B \equiv {\sim}X) \bullet {\sim}(C \vee {\sim}A)]$

21. $[({\sim}X \vee Z) \supset ({\sim}C \vee B)] \bullet [({\sim}X \bullet A) \supset ({\sim}Y \bullet Z)]$

★22. ${\sim}[(A \equiv X) \vee (Z \equiv Y)] \vee [({\sim}Y \supset B) \bullet (Z \supset C)]$

23. $[(B \bullet {\sim}C) \vee (X \bullet {\sim}Y)] \supset {\sim}[(Y \bullet {\sim}X) \vee (A \bullet {\sim}Z)]$

24. ${\sim}\{{\sim}[(C \vee {\sim}B) \bullet (Z \vee {\sim}A)] \bullet {\sim}[{\sim}(B \vee Y) \bullet ({\sim}X \vee Z)]\}$

★25. $(Z \supset C) \supset \{[({\sim}X \supset B) \supset (C \supset Y)] \equiv [(Z \supset X) \supset ({\sim}Y \supset Z)]\}$

IV. When possible, determine the truth values of the following symbolized statements. Let *A* and *B* be true, *Y* and *Z* false. *P* and *Q* have unknown truth value. If the truth value of the statement cannot be determined, write "undetermined."

★1. $A \vee P$

2. $Q \vee Z$

3. $Q \bullet Y$

★4. $Q \bullet A$

5. $P \supset B$

6. $Z \supset Q$

★7. $A \supset P$

8. $P \equiv {\sim}P$

9. $(P \supset A) \supset Z$

★10. $(P \supset A) \equiv (Q \supset B)$

11. $(Q \supset B) \supset (A \supset Y)$

12. ${\sim}(P \supset Y) \vee (Z \supset Q)$

★13. ${\sim}(Q \bullet Y) \equiv {\sim}(Q \vee A)$

14. $[(Z \supset P) \supset P] \supset P$

15. $[Q \supset (A \vee P)] \equiv [(Q \supset B) \supset Y]$

6.3 Truth Tables for Propositions

The previous section showed how the truth value of a compound proposition could be determined, given a *designated* truth value for each simple component. A truth table gives the truth value of a compound proposition for *every possible* truth value of its simple components. Each line in the truth table represents one such possible arrangement of truth values.

In constructing a truth table the first step is to determine the number of lines (or rows). Because each line represents one possible arrangement of truth values, the total number of lines is equal to the number of possible combinations of truth values for the simple propositions. As we saw in Section 6.2, if there is only one simple proposition, p, the number of lines in the truth table is two, because there are only two possible truth values for p: true, and false. If there are two simple propositions, p and q, there are four lines, because there are four combinations of truth values: p true and q true, p true and q false, p false and q true, and p false and q false. The relationship between the number of different simple propositions and the number of lines in the truth table is expressed as follows:

Number of different simple propositions	Number of lines in truth table
1	2
2	4
3	8
4	16
5	32
6	64

The relationship between these two columns of numbers is expressed by the formula,

$$L = 2^n$$

Where L designates the number of lines in the truth table, and n the number of *different* simple propositions.

Let us now construct a truth table for a compound proposition. We may begin with a fairly simple one:

$$(A \vee \sim B) \supset B$$

The number of different simple propositions is two. Thus the number of lines in the truth table is four. We draw these lines beneath the proposition as follows:

$$(A \vee \sim B) \supset B$$

The next step is to divide the number of lines in half. The result is 2. Then we go to the first letter on the left (*A*) and enter T on the first two lines and F on the remaining two lines.

$(A$	$\vee$	$\sim$	$B)$	$\supset$	B
T					
T					
F					
F					

Next we divide that number (two) in half and, since the result is one, write one T, one F, one T, and one F beneath the next letter (*B*):

$(A$	$\vee$	$\sim$	$B)$	$\supset$	B
T			T		
T			F		
F			T		
F			F		

6

Inspection of the truth table at this stage reveals that every possible combination of truth and falsity has now been assigned to *A* and *B*. In other words, the truth table exhausts the entire range of possibilities. The next step is to duplicate the *B* column under the second *B*.

$(A$	$\vee$	$\sim$	$B)$	$\supset$	B
T			T		T
T			F		F
F			T		T
F			F		F

This much has been automatic.

Now, using the principles developed in the previous section, we compute the remaining columns. First, the column under the tilde is computed from the column under *B*:

$(A$	$\vee$	$\sim$	$B)$	$\supset$	B
T		F	T		T
T		T	F		F
F		F	T		T
F		T	F		F

Next, the column under the wedge is computed from the column under *A* and the column under the tilde:

$(A$	$\vee$	$\sim$	$B)$	$\supset$	B
T	T	F	T		T
T	T	T	F		F
F	F	F	T		T
F	T	T	F		F

Last, the column under the horseshoe is computed from the column under the wedge and the column under *B*:

(*A*	∨	~	*B*)	⊃	*B*
T	T	F	T	T	T
T	T	T	F	F	F
F	F	F	T	T	T
F	T	T	F	F	F

The column under the main operator is outlined to indicate that it represents the entire compound proposition. Inspecting the completed truth table, we see that the truth value of the compound proposition is true when *B* is true and false when *B* is false, regardless of the truth value of *A*.

Let us consider another example: $(C \bullet {\sim}D) \supset E$. The number of different letters is three, so the number of lines is eight. Under *C* we make half this number true, half false (that is, four true, four false). Then, under *D* we make half *this* number true, half false, and so on (two true, two false, two true, two false). Finally, under *E* the truth value alternates on every line. The truth table thus exhausts every possible arrangement of truth values:

(*C*	•	~	*D*)	⊃	*E*
T			T		T
T			T		F
T			F		T
T			F		F
F			T		T
F			T		F
F			F		T
F			F		F

Now we compute the truth values for the remaining columns—first for the tilde, then for the dot, and finally for the horseshoe:

(*C*	•	~	*D*)	⊃	*E*
T	F	F	T	T	T
T	F	F	T	T	F
T	T	T	F	T	T
T	T	T	F	F	F
F	F	F	T	T	T
F	F	F	T	T	F
F	F	T	F	T	T
F	F	T	F	T	F

Inspecting the completed truth table, we see that the compound proposition is false only when *C* is true and *D* and *E* are false.

An alternate method for constructing truth tables, which turns out to be faster for certain compound propositions, replicates the type of truth table used to define the meaning of the five logical operators in Section 6.2. Suppose, for example, that we are given this proposition: $[(A \lor B) \bullet (B \supset A)] \supset B$. We would begin by constructing columns for the simple propositions A and B, writing them to the left of the given proposition:

A	B	$[(A$	$\lor$	$B)$	$\bullet$	$(B$	$\supset$	$A)]$	$\supset$	B
T	T									
T	F									
F	T									
F	F									

We would then use the columns on the left to derive the truth values of the compound propositions. We would compute first the truth values of the expressions in parentheses, then the dot, and finally the right-hand horseshoe:

A	B	$[(A$	$\lor$	$B)$	$\bullet$	$(B$	$\supset$	$A)]$	$\supset$	B
T	T		T		T		T		T	
T	F		T		T		T		F	
F	T		T		F		F		T	
F	F		F		F		T		T	

Classifying Statements

Truth tables may be used to determine whether the truth value of a compound statement depends solely on its form or whether it also depends on the specific truth values of its components. A compound statement is said to be a **logically true** or **tautologous statement** if it is true regardless of the truth values of its components. It is said to be a **logically false** or **self-contradictory statement** if it is false regardless of the truth values of its components. And it is said to be a **contingent statement** if its truth value varies depending on the truth values of its components. By inspecting the column of truth values under the main operator, we can determine how the compound proposition should be classified:

Column under main operator	Statement classification
all true	tautologous (logically true)
all false	self-contradictory (logically false)
at least one true, at least one false	contingent

As the truth table we developed earlier indicates, $(C \bullet {\sim}D) \supset E$ is a contingent proposition. The column under the main operator contains at least one T and at least one F. In other words, the truth value of the compound proposition is "contingent" on the truth values of its components. Sometimes it is true, sometimes false, depending on the truth values of the components.

On the other hand, consider the following truth tables:

[(G	$\supset$	H)	•	G]	$\supset$	H
T	T	T	T	T	T	T
T	F	F	F	T	T	F
F	T	T	F	F	T	T
F	T	F	F	F	T	F

(G	$\vee$	H)	$\equiv$	(~	G	•	~	H)
T	T	T	F	F	T	F	F	T
T	T	F	F	F	T	F	T	F
F	T	T	F	T	F	F	F	T
F	F	F	F	T	F	T	T	F

The proposition on the left is tautologous (logically true or a **tautology**) because the column under the main operator is all true. The one on the right is self-contradictory (logically false) because the main operator column is all false. In neither case is the truth value of the compound proposition contingent on the truth values of the components. The one on the left is true regardless of the truth values of its components—in other words, *necessarily* true. The one on the right is *necessarily* false.

If a proposition is either logically true or logically false, its truth value depends merely on its form and has nothing to do with its content. As a result, such statements do not make any genuine assertions about things in the world. For example, the tautologous statement "It is either raining or it is not raining" provides no information about the weather. Similarly, the self-contradictory statement "It is raining and it is not raining" provides no information about the weather. On the other hand, the contingent statement "It is raining in the mountains" does provide information about the weather.

Comparing Statements

Truth tables may also be used to determine how two propositions are related to each other. Two propositions are said to be **logically equivalent statements** if they have the same truth value on each line under their main operators, and they are **contradictory statements** if they have opposite truth values on each line under their main operators. If neither of these relations hold, the propositions are either consistent or inconsistent. Two (or more) propositions are **consistent statements** if there is at least one line on which both (or all) of them turn out to be true, and they are **inconsistent statements** if there is no line on which both (or all) of them turn out to be true. By comparing the main operator columns, one can determine which is the case. However, because the first two relations are stronger than (and may overlap) the second two, the first two relations should be considered first.

Columns under main operators	Relation
same truth value on each line	logically equivalent
opposite truth value on each line	contradictory
there is at least one line on which the truth values are both true	consistent
there is no line on which the truth values are both true	inconsistent

For example, the following two propositions are logically equivalent. The main operator columns of their respective truth tables are identical. Note that for proper comparison the columns under *K* must be identical and the columns under *L* must be identical.

K	⊃	*L*		~	*L*	⊃	~	*K*	
T	T	T		F	T	T	F	T	
T	F	F		T	F	F	F	T	
F	T	T		F	T	T	T	F	**Logically equivalent**
F	T	F		T	F	T	T	F	

For any two propositions that are logically equivalent, the biconditional statement formed by joining them with a triple bar is tautologous. Thus, $(K \supset L) \equiv ({\sim}L \supset {\sim}K)$ is tautologous. This is easy to see because the columns under the main operators of $K \supset L$ and ${\sim}L \supset {\sim}K$ are identical.

The next two propositions are contradictory:

K	⊃	*L*		*K*	•	~	*L*	
T	T	T		T	F	F	T	
T	F	F		T	T	T	F	
F	T	T		F	F	F	T	**Contradictory**
F	T	F		F	F	T	F	

The next two propositions are consistent. On the first line of each truth table the column under the main operator turns out true. This means that it is possible for both propositions to be true, which is the meaning of consistency:

K	∨	*L*		*K*	•	*L*	
T	T	T		T	T	T	
T	T	F		T	F	F	
F	T	T		F	F	T	**Consistent**
F	F	F		F	F	F	

Finally, the next two propositions are inconsistent. There is no line in the columns under the main operators where the truth values are both true:

K	≡	*L*		*K*	•	~	*L*	
T	T	T		T	F	F	T	
T	F	F		T	T	T	F	
F	F	T		F	F	F	T	**Inconsistent**
F	T	F		F	F	T	F	

Any pair of propositions is either consistent or inconsistent. Furthermore, some consistent propositions are also logically equivalent, and some inconsistent propositions are either contradictory or logically equivalent. Because of this partial overlap, pairs of propositions are usually first classified in terms of the stronger of these relations, which are logical equivalence and contradiction. If neither of these stronger relations applies, then the pair of propositions is classified in terms of the weaker relations, consistency and inconsistency.

Unlike logical equivalence and contradiction, which usually relate exactly two propositions, consistency and inconsistency often apply to larger groups of propositions. For consistency, the only requirement is that there be at least one line in the group of truth tables where all of the propositions are true, and for inconsistency the only requirement is that there be no such line. As a result of these requirements, the statement consisting of the conjunction of a group of inconsistent propositions will always be self-contradictory, whereas the statement consisting of the conjunction of a group of consistent propositions will never be self-contradictory.

Consistency and inconsistency are important because, among other things, they can be used to evaluate the overall rationality of a person's stated position on something. If the statements expressing such a position are consistent, then there is at least a possibility that the position makes sense. This is so because there will be at least one line in the group of truth tables where all of the person's statements are true. On the other hand, if the statements are inconsistent, then there is no possibility that the position makes sense. In this case there is no line in the truth tables where all of the statements are true. The group of statements, conjoined together, amounts to a self-contradiction.

The truth tables for consistency and logical equivalence also illustrate the important difference between two propositions being factually true and their being logically equivalent. For example, the statements "Water boils at 100°C" and "The current population of the United States is over 200 million" are both true in the present actual world. This real-world situation conforms to the one truth-table line on which both statements are true. As a result of this line, the two statements are consistent. However, they are not logically equivalent, because their truth values are not *necessarily* the same. The truth value of the second proposition might change in the future, while that of the first would remain the same. An analogous distinction, incidentally, holds between two statements having actually opposite truth values and their being contradictory.

6

Exercise 6.3

I. Use truth tables to determine whether the following symbolized statements are tautologous, self-contradictory, or contingent.

★**1.** $N \supset (N \supset N)$

2. $(G \supset G) \supset G$

3. $(S \supset R) \bullet (S \bullet {\sim}R)$

★**4.** $[(E \supset F) \supset F] \supset E$

5. $({\sim}K \supset H) \equiv {\sim}(H \vee K)$

6. $(M \supset P) \vee (P \supset M)$

★**7.** $[(Z \supset X) \bullet (X \vee Z)] \supset X$

8. $[(C \supset D) \bullet {\sim}C] \supset {\sim}D$

9. $[X \supset (R \supset F)] \equiv [(X \supset R) \supset F]$

★10. $[G \supset (N \supset \sim G)] \bullet [(N \equiv G) \bullet (N \vee G)]$
11. $[(Q \supset P) \bullet (\sim Q \supset R)] \bullet \sim(P \vee R)$
12. $[(H \supset N) \bullet (T \supset N)] \supset [(H \vee T) \supset N]$
★13. $[U \bullet (T \vee S)] \equiv [(\sim T \vee \sim U) \bullet (\sim S \vee \sim U)]$
14. $\{[(G \bullet N) \supset H] \bullet [(G \supset H) \supset P]\} \supset (N \supset P)$
15. $[(F \vee E) \bullet (G \vee H)] \equiv [(G \bullet E) \vee (F \bullet H)]$

II. Use truth tables to determine whether the following pairs of symbolized statements are logically equivalent, contradictory, consistent, or inconsistent. First, determine whether the pairs of propositions are logically equivalent or contradictory; then, if these relations do not apply, determine if they are consistent or inconsistent.

★1.	$\sim D \vee B$	$\sim(D \bullet \sim B)$
2.	$F \bullet M$	$\sim(F \vee M)$
3.	$\sim K \supset L$	$K \supset \sim L$
★4.	$R \vee \sim S$	$S \bullet \sim R$
5.	$\sim A \equiv X$	$(X \bullet \sim A) \vee (A \bullet \sim X)$
6.	$H \equiv \sim G$	$(G \bullet H) \vee (\sim G \bullet \sim H)$
★7.	$(E \supset C) \supset L$	$E \supset (C \supset L)$
8.	$N \bullet (A \vee \sim E)$	$\sim A \bullet (E \vee \sim N)$
9.	$M \supset (K \supset P)$	$(K \bullet M) \supset P$
★10.	$W \equiv (B \bullet T)$	$W \bullet (T \supset \sim B)$
11.	$G \bullet (E \vee P)$	$\sim(G \bullet E) \bullet \sim(G \bullet P)$
12.	$R \bullet (Q \vee S)$	$(S \vee R) \bullet (Q \vee R)$
★13.	$H \bullet (K \vee J)$	$(J \bullet H) \vee (H \bullet K)$
14.	$Z \bullet (C \equiv P)$	$C \equiv (Z \bullet \sim P)$
15.	$Q \supset \sim(K \vee F)$	$(K \bullet Q) \vee (F \bullet Q)$

III. Use truth tables to obtain the answers to the following exercises.

★1. Renowned economist Harold Carlson makes the following prediction: "The balance of payments will decrease if and only if interest rates remain steady; however, it is not the case that either interest rates will not remain steady or the balance of payments will decrease." What can we say about Carlson's prediction?

2. A high school principal made this statement to the school board: "Either music is not dropped from the curriculum or the students will become cultural philistines; furthermore, the students will not become cultural philistines if and only if music is dropped from the curriculum." Assuming the principal is correct, what has she told us about music and the students? (Hint: Construct a truth table for the principal's statement and examine the line on which the statement turns out true.)

3. Christina and Thomas are having a discussion about their plans for the evening. Christina: "If you don't love me, then I'm certainly not going to have sex

6

with you." Thomas: "Well, that means that if I do love you, then you will have sex with me, right?" Is Thomas correct? (Hint: Construct a truth table for each statement and compare them.)

★4. Two astronomers are discussing supernovas. Dr. Frank says, "Research has established that if a supernova occurs within ten light years of the earth, then life on earth will be destroyed." Dr. Harris says, "Research has also established that either a supernova will not occur within ten light years of the earth or life on earth will not be destroyed." Is it possible that both astronomers are correct? If so, what can we determine about the occurrence of a supernova?

5. Antonia Martinez, who is running for the state senate, makes this statement: "Either a tax reduction is feasible only if both educational costs do not increase and the welfare program is abolished, or a tax reduction is feasible and either the welfare program will not be abolished or educational costs will increase." What has Martinez told us about taxes, educational costs, and welfare?

6. Automotive expert Frank Goodbody has this to say about Japanese imports: "If Mitsubishi is the sportiest, then both Toyota is the most trouble-free and Isuzu is not the lowest priced. If Isuzu is the lowest priced, then both Toyota is not the most trouble-free and Mitsubishi is the sportiest." Is it possible that Goodbody is correct in his assessment? If so, what may we conclude about Mitsubishi, Toyota, and Isuzu?

★7. Two stockbrokers are having a discussion. One claims that Netmark will introduce a new product if and only if both Datapro cuts its work force and Compucel expands production. The other claims that Datapro will cut its work force, and Compucel will expand production if and only if Netmark introduces a new product. Is it possible that both stockbrokers are right? If so, what have they told us about these companies?

8. Eric Carson sums up his beliefs about God as follows: "God exists if and only if either life is meaningful or the soul is not immortal. God exists and the soul is immortal. If God exists, then life is not meaningful." Is it possible that Eric's beliefs make sense?

9. Cindy, Jane, and Amanda witnessed a bank robbery. At trial, Cindy testified that Lefty did not enter the bank, and if Howard pulled a gun, then Conrad collected the money. Jane testified that if Howard did not pull a gun, then Lefty entered the bank. Amanda testified that if Conrad collected the money, then Howard pulled a gun. Is it possible that all three witnesses told the truth? If so, what can we conclude about Lefty, Howard, and Conrad?

★10. Nicole Evans expresses her philosophy as follows: "If the mind is identical to the brain, then personal freedom does not exist and humans are not responsible for their actions. If personal freedom does not exist, then the mind is identical to the brain. Either humans are responsible for their actions or the mind is not identical to the brain. If personal freedom exists, then humans are responsible for their actions." Is it possible that Nicole's philosophy makes sense? If so, what does it say about the mind, personal freedom, and responsibility?

6.4 Truth Tables for Arguments

Truth tables provide the standard technique for testing the validity of arguments in propositional logic. To construct a truth table for an argument, follow these steps:

1. Symbolize the arguments using letters to represent the simple propositions.
2. Write out the symbolized argument, placing a single slash between the premises and a double slash between the last premise and the conclusion.
3. Draw a truth table for the symbolized argument as if it were a proposition broken into parts, outlining the columns representing the premises and conclusion.
4. Look for a line in which all of the premises are true and the conclusion is false. If such a line exists, the argument is invalid; if not, it is valid.

For example, let us test the following argument for validity:

If juvenile killers are as responsible for their crimes as adults, then execution is a justifiable punishment.
Juvenile killers are not as responsible for their crimes as adults.
Therefore, execution is not a justifiable punishment.

The first step is to symbolize the argument:

$$\begin{array}{l} J \supset E \\ \underline{\sim J} \\ \sim E \end{array}$$

Now a truth table may be constructed. Since the symbolized argument contains two different letters, the truth table has four lines. Make sure that identical letters have identical columns beneath them. Here are the columns for the individual letters:

J	$\supset$	E	/	$\sim$	J	//	$\sim$	E
T		T			T			T
T		F			T			F
F		T			F			T
F		F			F			F

The truth table is now completed, and the columns representing the premises and conclusion are outlined:

J	$\supset$	E	/	$\sim$	J	//	$\sim$	E
T	T	T		F	T		F	T
T	F	F		F	T		T	F
F	T	T		T	F		F	T
F	T	F		T	F		T	F

Inspection of the third line reveals that both of the premises are true and the conclusion is false. The argument is therefore invalid.

Another example:

If insider trading occurs, then investors will not trust the securities markets. If investors do not trust the securities markets, then business in general will suffer. Therefore, if insider trading occurs, then business in general will suffer.

The completed truth table is this:

O	⊃	~	*T*	/	~	*T*	⊃	*B*	//	*O*	⊃	*B*
T	F	F	T		F	T	T	T		T	T	T
T	F	F	T		F	T	T	F		T	F	F
T	T	T	F		T	F	T	T		T	T	T
T	T	T	F		T	F	F	F		T	F	F
F	T	F	T		F	T	T	T		F	T	T
F	T	F	T		F	T	T	F		F	T	F
F	T	T	F		T	F	T	T		F	T	T
F	T	T	F		T	F	F	F		F	T	F

Inspection of the truth table reveals that there is no line on which both premises are true and the conclusion is false. The argument is therefore valid.

The logic behind the method of truth tables is easy to understand. By definition, a valid argument is one in which it is not possible for the premises to be true and the conclusion false. A truth table presents every possible combination of truth values that the components of an argument may have. Therefore, if no line exists on which the premises are true and the conclusion false, then it is not possible for the premises to be true and the conclusion false, in which case the argument is valid. Conversely, if there *is* a line on which the premises are true and the conclusion false, then it *is* possible for the premises to be true and the conclusion false, and the argument is invalid. Accordingly, to test the validity of an argument, use this procedure:

Look for a line that has all true premises and a false conclusion. If you find such a line, the argument is invalid. If not, the argument is valid.

Truth tables provide a convenient illustration of the fact that any argument having inconsistent premises is valid regardless of what its conclusion may be, and any argument having a tautologous conclusion is valid regardless of what its premises may be. Example:

The sky is blue.
The sky is not blue.
Therefore, Paris is the capital of France.

S	/	~	*S*	//	*P*
T		F	T		T
T		F	T		F
F		T	F		T
F		T	F		F

Since the premises of this argument are inconsistent, there is no line on which the premises are both true. Accordingly, there is no line on which the premises are both

Ada Byron, Countess of Lovelace 1815–1852

Ada Byron, born in 1815, was the only child of the English poet George Gordon (Lord Byron) and Annabella Milbanke. By the time Ada was born, Lady Byron had grown to detest her husband, and she did everything in her power to ensure that Ada would grow up to be as much unlike him as possible. Lady Byron had a talent for mathematics, which her husband did not share in the least, so she hired a series of tutors to instruct young Ada in that discipline. One of those tutors was the famous mathematician and logician Augustus De Morgan, with whom Ada became a close friend.

In 1835 Ada Byron married William King, and when King was elevated to Earl of Lovelace three years later, Ada became Countess of Lovelace. The couple had three children, but the duties of wife and mother did not interrupt Ada's study of mathematics and logic. Two years prior to her marriage Ada met Charles Babbage, an early inventor of mechanical computers, and when she was first shown Babbage's Difference Engine, she immediately grasped all the intricacies of its operation. A few years later, when Babbage proposed a design for the Analytic Engine, Ada wrote a lengthy program for the new machine, and she envisioned how it could be used not only to solve problems in mathematics, but to produce music and graphics as well. Because of this work, Ada Byron is usually credited with being the first computer programmer.

Ada had suffered problems with her health ever since childhood, and as she grew older, these problems were aggravated by alcohol abuse. In 1852 she died from cancer at the relatively young age of 36.

true and the conclusion false, so the argument is valid. Of course, the argument is unsound, because it has a false premise. Another example:

Bern is the capital of Switzerland. Therefore, it is either raining or it is not raining.

B	/ /	R	$\vee$	$\sim$	R
T		T	T	F	T
T		F	T	T	F
F		T	T	F	T
F		F	T	T	F

The conclusion of this argument is a tautology. Accordingly, there is no line on which the premise is true and the conclusion false, and so the argument is valid. Incidentally, it is also sound, because the premise is true.

The conditional statement having the conjunction of an argument's premises as its antecedent and the conclusion as its consequent is called the argument's

corresponding conditional. For example, the corresponding conditional of the second argument tested in this section is $[(O \supset {\sim}T) \bullet ({\sim}T \supset B)] \supset (O \supset B)$. For any valid argument (such as this one), the corresponding conditional is a tautology. This is easy to see. In any valid argument, there is no line on which the premises are all true and the conclusion false. Thus, in the corresponding conditional, there is no line on which the antecedent is true and the consequent false, so the corresponding conditional is true on every line.

Exercise 6.4

I. Translate the following arguments into symbolic form. Then determine whether each is valid or invalid by constructing a truth table for each.

★**1.** If national elections deteriorate into TV popularity contests, then smooth-talking morons will get elected. Therefore, if national elections do not deteriorate into TV popularity contests, then smooth-talking morons will not get elected.

2. Brazil has a huge foreign debt. Therefore, either Brazil or Argentina has a huge foreign debt.

3. If fossil fuel combustion continues at its present rate, then a greenhouse effect will occur. If a greenhouse effect occurs, then world temperatures will rise. Therefore, if fossil fuel combustion continues at its present rate, then world temperatures will rise.

★**4.** If there are dried-up riverbeds on Mars, then water once flowed on the Martian surface. There are dried-up riverbeds on Mars. Therefore, water once flowed on the Martian surface.

5. If high school graduates are deficient in reading, they will not be able to compete in the modern world. If high school graduates are deficient in writing, they will not be able to compete in the modern world. Therefore, if high school graduates are deficient in reading, then they are deficient in writing.

6. The disparity between rich and poor is increasing. Therefore, political control over economic equality will be achieved only if restructuring the economic system along socialist lines implies that political control over economic equality will be achieved.

★**7.** Einstein won the Nobel Prize either for explaining the photoelectric effect or for the special theory of relativity. But he did win the Nobel Prize for explaining the photoelectric effect. Therefore, Einstein did not win the Nobel Prize for the special theory of relativity.

8. If microchips are made from diamond wafers, then computers will generate less heat. Computers will not generate less heat and microchips will be made from diamond wafers. Therefore, synthetic diamonds will be used for jewelry.

9. Either the USS *Arizona* or the USS *Missouri* was not sunk in the attack on Pearl Harbor. Therefore, it is not the case that either the USS *Arizona* or the USS *Missouri* was sunk in the attack on Pearl Harbor.

★10. If racial quotas are adopted for promoting employees, then qualified employees will be passed over; but if racial quotas are not adopted, then prior discrimination will go unaddressed. Either racial quotas will or will not be adopted for promoting employees. Therefore, either qualified employees will be passed over or prior discrimination will go unaddressed.

II. Determine whether the following symbolized arguments are valid or invalid by constructing a truth table for each.

★1. $K \supset {\sim}K$
$/ {\sim}K$

2. $R \supset R$
$/ R$

3. $P \equiv {\sim}N$
$/ N \vee P$

★4. ${\sim}(G \bullet M)$
$M \vee {\sim}G$
$/ {\sim}G$

5. $K \equiv {\sim}L$
${\sim}L \bullet {\sim}K)$
$/ K \supset L$

6. Z
$/ E \supset (Z \supset E)$

★7. ${\sim}(W \bullet {\sim}X)$
${\sim}(X \bullet {\sim}W)$
$/ X \vee W$

8. $C \equiv D$
$E \vee {\sim}D$
$/ E \supset C$

9. $A \equiv (B \vee C)$
${\sim}C \vee B$
$/ A \supset B$

★10. $J \supset (K \supset L)$
$K \supset (J \supset L)$
$/ (J \vee K) \supset L$

11. ${\sim}(K \equiv S)$
$S \supset {\sim}(R \vee K)$
$/ R \vee {\sim}S$

12. $E \supset (F \bullet G)$
$F \supset (G \supset H)$
$/ E \supset H$

★13. $A \supset (N \vee Q)$
${\sim}(N \vee {\sim}A)$
$/ A \supset Q$

14. $G \supset H$
$R \equiv G$
${\sim}H \vee G$
$/ R \equiv H$

15. $L \supset M$
$M \supset N$
$N \supset L$
$/ L \vee N$

★16. $S \supset T$
$S \supset {\sim}T$
${\sim}T \supset S$
$/ S \vee {\sim}T$

17. $W \supset X$
$X \supset W$
$X \supset Y$
$Y \supset X$
$/ W \equiv Y$

18. $K \equiv (L \vee M)$
$L \supset M$
$M \supset K$
$\underline{K \vee L}$
$K \supset L$

★19. $A \supset B$
$(A \bullet B) \supset C$
$\underline{A \supset (C \supset D)}$
$A \supset D$

20. $\sim A \vee R$
$\sim(N \bullet \sim C)$
$R \supset C$
$\underline{C \supset \sim N}$
$A \vee C$

III. The following dialogue contains eleven arguments. Translate each into symbolic form, and then use truth tables to determine whether each is valid or invalid.

Romance with an Android

"I just came from Professor Shaw's class in the Philosophy of Human Nature," Nick says to his friend Erin, as he meets her in the hallway outside the classroom. "We discussed the question of whether an android could be a person and whether we would ever consider going out on a date with an android—assuming it looked just like an attractive human."

"Sounds like an interesting class," Erin replies, "but I think it's just silly to think that an android could be a person."

"Why is that?" Nick asks.

"It's really quite simple," she says. "If an android is a person, then it's rational. But no android is rational, so it's not a person."

"But wait," Nick says. "Androids can solve problems, and they can also deliberate. And if they can either deliberate or solve problems, then they're rational. Wouldn't you agree? So androids are rational, after all."

"No they're not," Erin says with determination. "If an android is rational, then it's conscious, and if it's conscious, then it has reflective mental activity—it can reflect on its own act of thinking. But no android can do that, so it's not rational."

"How do you know that no android has reflective mental activity?" he asks.

"Because an android has reflective mental activity only if it has a soul," Erin says. "And it's ridiculous to think that an android could have a soul. Hence, it has no reflective mental activity."

"But consider this," Nick says, as he and Erin exit the building and walk down the steps. "Either a soul is a material entity or it's a nonmaterial entity. You would agree with that, wouldn't you?"

"Of course," Erin replies. "Your statement is a tautology."

"Okay," says Nick. "Now let me finish the argument. If a soul is a material entity, then if an android is material, it could easily have a soul. But if a soul is a nonmaterial entity, then if God could infuse a soul into it, then it could have a soul. Now an android is material and God could infuse a soul into an android—after all, God can do anything. Thus, an android could have a soul."

"Well, I know that Descartes considered humans to be machines with souls, but to me it's simply crazy to think that God would infuse a soul into a computer. He might as well infuse a soul into a pile of rocks. In any event, let me try another approach," Erin says, as she and Nick stroll across the grassy quad separating the buildings. "If an android is a person, then it has free will. But if androids are programmed, then they have no free will. Androids are just computers made to appear like humans, and every computer is programmed. Hence, once again, an android is not a person. What do you think of that?"

"By your reasoning," Nick replies, "even humans may not be free."

"How is that?" Erin asks.

"Well," he says, "whatever we do is caused by our biological makeup or by our social conditioning. But if it's caused by our biological makeup, then it's determined. Also, if it's caused by our social conditioning, then it's determined. And if it's determined, then it's not free. Thus, whatever we do is not free."

"Not so," Erin replies with a touch of exasperation. "Our actions may be influenced by our biological makeup and our social conditioning, but they are not strictly caused by them. And if they are not strictly caused by them, they are not determined by them, and if they are not determined by them, then they are free. Thus, our actions are free."

"Well, I don't know what it means for our actions to be influenced by something yet not be determined," Nick replies as he and Erin turn to avoid some students sitting on the grass. "If *X*, for example, is influenced by *Y*, then *X* is caused by *Y*, and if *X* is caused by *Y*, then *X* is determined by *Y*. Thus, if *X* is influenced by *Y*, then *X* is determined by *Y*."

6

"I think you're equivocating on the meaning of cause," Erin replies. "But let me try something else. If an android is a person, then it has feelings. And if it has feelings, then it has love or compassion. But no android loves anything. Just imagine two computers in love. The very thought is absurd. And it's equally foolish to think of one android feeling compassion for another. Thus, an android cannot be a person."

"Well, look at it this way," Nick replies. "Feelings are either mental or they are physical. If they are mental, then they are brain states, and if they are brain states, then androids could have them—because all brain states are mere arrangements of atoms. And if feelings are physical, then androids could have them—because, once again, all physical things are mere arrangements of atoms. Thus, androids can have feelings."

"I've never heard such flimsy reasoning in my life," Erin replies while trying to avoid outright laughter. "It may be the case that feelings are accompanied by physical states, but they're certainly not identical with them. But tell me this—before I have to head off to my biochem class. Do you really think that androids could be persons?"

"I think it's possible," Nick replies.

"So, would you go out on a date with an android?"

"That depends," he says.

"On what?" Erin asks, looking puzzled.

"On whether I think she'd be good in bed," he replies.

"Oh what a typically stupid male answer," she says with a sigh. "Well, I'm off to class. Bye."

6.5 Indirect Truth Tables

Indirect truth tables provide a shorter and faster method for testing the validity of arguments than do ordinary truth tables. This method is especially applicable to arguments that contain a large number of different simple propositions. For example, an argument containing five different simple propositions would require an ordinary truth table having thirty-two lines. The indirect truth table for such an argument, on the other hand, would usually require only a single line and could be constructed in a fraction of the time required for the ordinary truth table.

Indirect truth tables can also be used to test a series of statements for consistency. In Section 6.3 we showed how ordinary truth tables are used to test pairs of statements for consistency and we noted that consistency was a relation that applied to any group of propositions. In this section we use indirect truth tables to test groups of three, four, five, and six propositions for consistency. Given the abbreviated nature of indirect truth tables, this evaluation can usually be done much more quickly than it can with ordinary truth tables.

Preliminary Skills

Using indirect truth tables requires developing the skill to work backwards from the truth value of the main operator of a compound statement to the truth values of its simple components. Suppose, for example, that you are given a conjunctive statement that is true:

$$\begin{array}{ccc} A & \cdot & B \\ & \mathrm{T} & \end{array}$$

Because a conjunctive statement can be true in only one way, you know immediately that both *A* and *B* are true:

$$\begin{array}{ccc} A & \cdot & B \\ \mathrm{T} & \mathrm{T} & \mathrm{T} \end{array}$$

Suppose, on the other hand, that you are given a conditional statement that is false:

$$\begin{array}{ccc} A & \supset & B \\ & \mathrm{F} & \end{array}$$

Since a conditional statement can be false in only one way, you know that *A* is true and *B* is false:

$$\begin{array}{ccc} A & \supset & B \\ \mathrm{T} & \mathrm{F} & \mathrm{F} \end{array}$$

But suppose you are given a conditional statement that is true:

$$\begin{array}{ccc} A & \supset & B \\ & \mathrm{T} & \end{array}$$

Since a conditional statement can be true in three ways, you cannot compute the truth values of *A* and *B*. It could be the case that *A* is true and *B* is true, or *A* is false and *B* is true, or *A* is false and *B* is false. But let us suppose, in relation to this example, that you have one more piece of information. Suppose you know that *B* is false:

$$\begin{array}{ccc} A & \supset & B \\ & \mathrm{T} & \mathrm{F} \end{array}$$

Then you know immediately that *A* is false. If *A* were true, then the truth value under the horseshoe would have to be false. But since this truth value is given as true, *A* must be false:

$$\begin{array}{ccc} A & \supset & B \\ \text{F} & \text{T} & \text{F} \end{array}$$

Similarly, suppose you are given a disjunctive statement with these truth values:

$$\begin{array}{ccc} A & \vee & B \\ \text{F} & \text{T} & \end{array}$$

Then you know immediately that *B* is true, because if a disjunctive statement is true, at least one of the disjuncts must be true:

$$\begin{array}{ccc} A & \vee & B \\ \text{F} & \text{T} & \text{T} \end{array}$$

Computing the truth values for the simple components of a compound proposition, as we have just done, requires a thorough knowledge of the truth-table definitions of the five operators given in Section 6.2. But after a little practice with examples such as these, this skill becomes almost automatic.

6

Testing Arguments for Validity

To construct an indirect truth table for an argument, we begin by assuming that the argument is invalid. That is, we assume that it is possible for the premises to be true and the conclusion false. Truth values corresponding to true premises and false conclusion are entered beneath the main operators for the premises and conclusion. Then, working backward, the truth values of the separate components are derived. If no contradiction is obtained in the process, this means that it is indeed possible for the premises to be true and the conclusion false, as originally assumed, so the argument is therefore invalid. If, however, the attempt to make the premises true and the conclusion false necessarily leads to a contradiction, it is not possible for the premises to be true and the conclusion false, in which case the argument is valid. Consider the following symbolized argument:

$$\begin{array}{l} \sim A \supset (B \vee C) \\ \underline{\sim B \qquad\qquad} \\ C \supset A \end{array}$$

We begin as before by writing the symbolized argument on a single line, placing a single slash between the premises and a double slash between the last premise and the conclusion. Then we assign T to the premises and F to the conclusion:

$$\begin{array}{cccccccccccccc} \sim & A & \supset & (B & \vee & C) & / & \sim & B & // & C & \supset & A \\ & & \text{T} & & & & & \text{T} & & & & \text{F} & \end{array}$$

We can now derive the truth values of *B*, *C*, and *A*, as follows:

$$\begin{array}{cccccccccccccc} \sim & A & \supset & (B & \vee & C) & / & \sim & B & // & C & \supset & A \\ & & \text{T} & & & & & \text{T} & \text{F} & & \text{T} & \text{F} & \text{F} \end{array}$$

These truth values are now transferred to the first premise:

```
~  A  ⊃  (B  ∨  C)  /  ~  B  //  C  ⊃  A
T  F  T   F  T  T      T  F      T  F  F
```

We thus have a perfectly consistent assignment of truth values, which makes the premises true and the conclusion false. The argument is therefore invalid. If an ordinary truth table were constructed for this argument, it would be seen that the argument fails on the line on which *A* is false, *B* is false, and *C* is true. This is the exact arrangement presented in the indirect truth table just presented.

Here is another example. As always, we begin by assigning T to the premises and F to the conclusion:

```
A  ⊃  (B  ∨  C)  /  B  ⊃  D  /  A  //  ~  C  ⊃  D
   T                   T        T            F
```

From the conclusion we can now derive the truth values of *C* and *D*, which are then transferred to the first two premises:

```
A  ⊃  (B  ∨  C)  /  B  ⊃  D  /  A  //  ~  C  ⊃  D
   T         F         T  F     T      T  F  F  F
```

The truth value of *B* is now derived from the second premise and transferred, together with the truth value of *A*, to the first premise:

```
A  ⊃  (B  ∨  C)  /  B  ⊃  D  /  A  //  ~  C  ⊃  D
T  T   F  F  F      F  T  F     T      T  F  F  F
```

A contradiction now appears in the truth values assigned to the first premise, since T ⊃ F is F. The inconsistent truth values are circled. Because every step was strictly necessitated by some prior step, we have shown that it is impossible for the premises to be true and the conclusion false. The argument is therefore valid.

Sometimes a single row of truth values is not sufficient to prove an argument valid. Example:

```
~  A  ⊃  B  /  B  ⊃  A  /  A  ⊃  ~  B  //  A  •  ~  B
      T           T           T                F
```

Since a conditional statement can be true in any one of three ways, and a conjunctive statement can be false in any one of three ways, merely assigning truth to the premises and falsity to the conclusion of this argument is not sufficient to obtain the truth values of any of the component statements. When faced with a situation such as this, we must list all of the possible ways that one of the premises can be true or the conclusion false, and proceed from there. If we list all of the possible ways the conclusion may be false, we obtain the following:

```
~  A  ⊃  B  /  B  ⊃  A  /  A  ⊃  ~  B  //  A  •  ~  B
      T           T           T             T  F  F  T
      T           T           T             F  F  T  F
      T           T           T             F  F  F  T
```

Extending the truth values of *A* and *B* to the premises, we obtain the following result:

$\sim$	A	$\supset$	B	/	B	$\supset$	A	/	A	$\supset$	$\sim$	B	//	A	$\bullet$	$\sim$	B
		T				T			T	T	F	T		T	F	F	T
T	F	T	F			T				T				F	F	T	F
		T			T	T	F			T				F	F	F	T

Since each line necessarily leads to a contradiction, the argument is valid. If a contradiction had been avoided on some line, the argument would, of course, be invalid, because it would be possible for the premises to be true and the conclusion false. Note that in this argument it is not necessary to fill out all the truth values on any one line to be forced into a contradiction. On each line the contradiction is necessarily derived within the context of a single premise.

If an indirect truth table requires more than one line, the method to be followed is this. Either select one of the premises and compute all of the ways it can be made true, or select the conclusion and compute all of the ways it can be made false. This selection should be dictated by the requirement of simplicity. For example, if the conclusion can be made false in only two ways, while each of the premises can be made true in three ways, then select the conclusion. On the other hand, if one of the premises can be made true in only two ways while the conclusion can be made false in three ways, then select that premise. If neither of these situations prevails, then select the conclusion.

Having made your selection, proceed to compute the truth values of each line, beginning with the first. If no contradiction is derived on this line, stop! The argument has been proved invalid. If a contradiction *is* derived on the first line, proceed to the second line. If no contradiction is derived on this line, then, again, the argument has been proved invalid. If a contradiction *is* derived, proceed to the third line, and so on. Remember, the objective is to produce a line having no contradiction. Once such a line is produced, the argument has been proved invalid, and no further work need be done. If, on the other hand, each line necessarily leads to a contradiction, the argument is valid.

Three final points need to be made about indirect truth tables for arguments. First, if a contradiction is obtained in the assignment of truth values, every step leading to it must be logically implied by some prior step. In other words, the contradiction must be unavoidable. If a contradiction is obtained after truth values are assigned haphazardly or by guessing, then nothing has been proved. The objective is not to produce a contradiction but to *avoid* one (if possible).

For example, in the following indirect truth table a contradiction is apparent in the first premise:

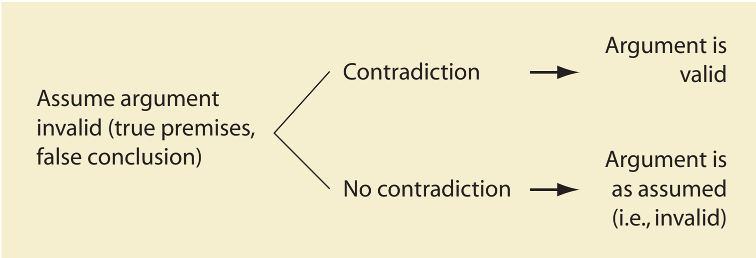
A	$\supset$	B	/	C	$\supset$	B	//	A	$\supset$	C
T	T	F		F	T	F		T	F	F

Yet the argument is invalid. The contradiction that appears is not *required* by the assignment of truth to the premises and falsity to the conclusion. The following indirect truth table, which is done correctly, proves the argument invalid:

A	$\supset$	B	/	C	$\supset$	B	//	A	$\supset$	C
T	T	T		F	T	T		T	F	F

The second point is that for valid arguments the order in which the truth values are assigned may affect where the contradiction is obtained. That is, depending on the order of assignment, the contradiction may appear in the first premise, second premise, third premise, and so on. But, of course, the order of assignment does not affect the final determination of validity.

The last point is that it is essential that identical letters be assigned identical truth values. For example, if the letter *A* appears three times in a certain symbolized argument and the truth value T is assigned to it in one occurrence, then the same truth value must be assigned to it in the other occurrences as well. After the truth table has been completed, each letter should be rechecked to ensure that one and the same truth value has been assigned to its various occurrences.



Testing Statements for Consistency

The method for testing a series of statements for consistency is similar to the method for testing arguments. We begin by writing the statements on a line, separating each with a single slash mark. (Since we have no conclusion, we use no double slash marks.) Then we assume that the statements are consistent. We assign a T to the main operator of each, and we then compute the truth values of the components. If this computation leads necessarily to a contradiction, the statements are not as we assumed them to be. That is, they are inconsistent. But if no contradiction is reached, the statements are consistent. Here is an example:

$$\begin{array}{l} A \vee B \\ B \supset (C \vee A) \\ C \supset {\sim} B \\ {\sim}A \end{array}$$

First, we write the statements on a single line separated by a single slash mark; we then assign T to each of the main operators:

$$\begin{array}{ccccccccccccccccc} A & \vee & B & / & B & \supset & (C & \vee & A) & / & C & \supset & \sim & B & / & \sim & A \\ & \text{T} & & & & \text{T} & & & & & & \text{T} & & & & \text{T} & \end{array}$$

Next, we compute the truth values of the components. First we compute the truth value of *A*. Next, we enter this truth value in the first statement and compute the truth value of *B*. Next, we enter the truth value of *B* in the second statement and

Eminent Logicians

Augustus De Morgan 1806–1871

© Hulton Archive/Getty Images

The English logician and mathematician Augustus De Morgan is famous for the development of predicate quantification and the invention of relational algebra—an algebra fundamental to the work of Russell and Whitehead in their *Principia Mathematica*. He is known to all students of symbolic logic owing to his formulation of what came to be known as De Morgan's rule of inference.

De Morgan was born in Madura, India, where his father was employed by the East India Company. He became blind in one eye not long after birth, and after his family returned to England his fellow students often taunted him and played cruel tricks on him because of his disability. When he was ten, his father died coming home from another trip to India. This left the boy under the influence of his mother, a devout Anglican who wanted her son to become an Anglican priest.

De Morgan obtained a B.A. from Trinity College, Cambridge, and might have received a Masters degree but for the fact that Cambridge then required all candidates to take a theological test. Because of his commitment to the ideal of religious neutrality, De Morgan refused to take this test. Perhaps in rebellion against his mother's influence, he developed a lifelong habit of avoiding churches, claiming that he could not bear hearing sermons. Following his refusal to take the exam, he continued his studies at University College London, a new institution founded on the principle of religious neutrality. At age twenty-two, he became the first professor of mathematics there.

Three years into his professorship, De Morgan became involved in a disagreement with the administration regarding its treatment of a colleague. De Morgan led a faculty protest, and in the end he resigned his position on the faculty. Five years later, he returned to his former position after his replacement accidentally drowned. He remained there for thirty years, until at age sixty he became involved in another administrative dispute—this time over a decision that De Morgan considered to be in violation of the university's stated policy of religious neutrality. He again resigned in protest, bringing his academic career to an end. Though active in academic politics, he curiously abstained from all involvement in national politics. An acquaintance once remarked that "he never voted in an election, and never visited the House of Commons."

De Morgan was proud of his son, George, who became a distinguished mathematician in his own right, and he was disconsolate at his son's untimely death, which occurred when De Morgan was sixty-two. The death of a daughter, during that same period, compounded his grief, and De Morgan died three years later, possibly owing to grief and stress.

De Morgan was known for his sense of humor and his interest in odd facts. He produced an almanac of full moons spanning a four-thousand-year period. A lunar crater is named after him. He liked to point out that he was x years old in the year x^2 (43 in 1849). He also enjoyed composing bits of verse in imitation of famous poets—for example, "Great fleas have little fleas upon their backs to bite 'em, and little fleas have lesser fleas, and so *ad infinitum*" (after Jonathan Swift).

compute the truth value of *C*. Finally, the truth values of *C* and *B* are carried to the third statement:

$$\begin{array}{l} A \vee B \ / \ B \supset (C \vee A) \ / \ C \supset {\sim} B \ / \ {\sim} A \\ \text{F T T} \quad \text{T T} \quad \text{T T F} \quad \text{T T F T} \quad \text{T F} \end{array}$$

Since this computation leads necessarily to a contradiction (third statement), the group of statements is inconsistent.

Assume statements consistent (assume all of them true)

- Contradiction ⟶ Statements are inconsistent
- No contradiction ⟶ Statements are as assumed (i.e., consistent)

Here is another example. The statements are written on a single line, and a T is assigned to each of the main operators:

$$\begin{array}{l} A \supset (B \bullet C) \ / \ C \supset {\sim} A \ / \ B \vee A \ / \ B \supset C \\ \quad \text{T} \qquad\qquad\quad \text{T} \qquad\qquad \text{T} \qquad\quad \text{T} \end{array}$$

Since all of the statements can be true in three ways, we select one of them (the fourth) and figure all of the ways it can be true:

$$\begin{array}{l} A \supset (B \bullet C) \ / \ C \supset {\sim} A \ / \ B \vee A \ / \ B \supset C \\ \quad \text{T} \qquad\qquad\quad \text{T} \qquad\qquad \text{T} \qquad\quad \text{T T T} \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad\ \text{F T T} \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad\ \text{F T F} \end{array}$$

Filling out the first line leads to no contradiction, so the statements are consistent:

$$\begin{array}{l} A \supset (B \bullet C) \ / \ C \supset {\sim} A \ / \ B \vee A \ / \ B \supset C \\ \text{F T} \quad \text{T T T} \quad \text{T T T F} \quad \text{T T F} \quad \text{T T T} \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad\ \text{F T T} \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad\ \text{F T F} \end{array}$$

As with testing arguments, the objective is to *avoid* a contradiction. As soon as no contradiction is reached, we stop. The statements are consistent. Only if all three lines had led to a contradiction would these statements be inconsistent.

Rule for all multiline indirect truth tables

Contradiction is derived ⟶ Go to next line (if there is one).

No contradiction is derived ⟶ Stop. Argument is invalid/ Statements are consistent.

Exercise 6.5

I. When possible, compute the truth values of the simple components in the following compound propositions. If no truth value can be computed, write a question mark (?) under the letter or letters with unknown truth value.

★1. $\begin{array}{ccc} K & \vee & D \\ \text{F} & \text{T} & \end{array}$

2. $\begin{array}{ccc} Q & \supset & N \\ & \text{T} & \end{array}$

3. $\begin{array}{ccc} B & \equiv & D \\ & \text{T} & \text{F} \end{array}$

★4. $\begin{array}{ccc} N & \supset & G \\ & \text{T} & \text{T} \end{array}$

5. $\begin{array}{ccc} S & \supset & B \\ & \text{T} & \text{F} \end{array}$

6. $\begin{array}{ccc} K & \bullet & B \\ & \text{T} & \end{array}$

★7. $\begin{array}{ccc} C & \vee & A \\ & \text{T} & \text{F} \end{array}$

8. $\begin{array}{ccc} S & \equiv & E \\ & \text{T} & \end{array}$

9. $\begin{array}{ccc} M & \supset & R \\ \text{T} & \text{T} & \end{array}$

★10. $\begin{array}{ccc} H & \vee & J \\ \text{T} & \text{T} & \end{array}$

11. $\begin{array}{ccc} E & \equiv & P \\ \text{T} & \text{F} & \end{array}$

12. $\begin{array}{ccc} H & \vee & S \\ & \text{T} & \end{array}$

★13. $\begin{array}{ccc} C & \supset & P \\ & \text{F} & \end{array}$

14. $\begin{array}{ccc} G & \bullet & B \\ & \text{F} & \end{array}$

15. $\begin{array}{ccc} S & \equiv & Q \\ & \text{T} & \text{F} \end{array}$

★16. $\begin{array}{ccc} G & \vee & K \\ & \text{F} & \end{array}$

17. $\begin{array}{cccc} N & \supset & \sim & P \\ \text{T} & \text{T} & & \end{array}$

18. $\begin{array}{cccc} \sim & A & \equiv & D \\ & & \text{F} & \text{T} \end{array}$

★19. $\begin{array}{cccc} \sim & L & \supset & M \\ & & \text{T} & \text{F} \end{array}$

20. $\begin{array}{cccc} E & \supset & \sim & M \\ & \text{T} & \text{F} & \end{array}$

21. $\begin{array}{cccc} \sim & N & \equiv & R \\ & & \text{F} & \text{F} \end{array}$

★22. $\begin{array}{cccccc} \sim & (& H & \vee & B &) \\ \text{T} & & & & & \end{array}$

23. $\begin{array}{ccccccc} Q & \supset & (& R & \bullet & S &) \\ & \text{T} & & & \text{F} & & \end{array}$

24. $\begin{array}{cccccccc} K & \supset & \sim & (& S & \equiv & M &) \\ \text{T} & \text{F} & & & & \text{F} & & \end{array}$

★25. $\begin{array}{ccccccccc} A & \vee & \sim & (& C & \bullet & \sim & R &) \\ \text{F} & \text{T} & & & & & & & \end{array}$

II. Use indirect truth tables to determine whether the following arguments are valid or invalid.

★1. $\begin{array}{l} B \supset C \\ \underline{\sim C} \\ \sim B \end{array}$

2. $\begin{array}{l} \sim E \vee F \\ \underline{\sim E} \\ \sim F \end{array}$

3. $\begin{array}{l} P \supset (Q \bullet R) \\ \underline{R \supset S} \\ P \supset S \end{array}$

★4. $\begin{array}{l} \underline{\sim(I \equiv J)} \\ \sim(I \supset J) \end{array}$

5. $W \supset (X \supset Y)$
$X \supset (Y \supset Z)$
$W \supset (X \supset Z)$

6. $A \supset (B \vee C)$
$C \supset (D \bullet E)$
$\sim B$
$A \supset \sim E$

★7. $G \supset H$
$H \supset I$
$\sim J \supset G$
$\sim I$
J

8. $J \supset (\sim L \supset \sim K)$
$K \supset (\sim L \supset M)$
$(L \vee M) \supset N$
$J \supset N$

9. $P \bullet (Q \vee R)$
$(P \bullet R) \supset \sim(S \vee T)$
$(\sim S \vee \sim T) \supset \sim(P \bullet Q)$
$S \equiv T$

★10. $(M \vee N) \supset O$
$O \supset (N \vee P)$
$M \supset (\sim Q \supset N)$
$(Q \supset M) \supset \sim P$
$N \equiv O$

11. $(A \vee B) \supset (C \bullet D)$
$(\sim A \vee \sim B) \supset E$
$(\sim C \vee \sim D) \supset E$

12. $F \supset G$
$\sim H \vee I$
$(G \vee I) \supset J$
$\sim J$
$\sim(F \vee H)$

★13. $(A \vee B) \supset (C \bullet D)$
$(X \vee \sim Y) \supset (\sim C \bullet \sim W)$
$(X \vee Z) \supset (A \bullet E)$
$\sim X$

14. $\sim G \supset (\sim H \bullet \sim I)$
$J \supset H$
$K \supset (L \bullet M)$
$K \vee J$
$L \bullet G$

15. $N \vee \sim O$
$P \vee O$
$P \supset Q$
$(N \vee Q) \supset (R \bullet S)$
$S \supset (R \supset T)$
$O \supset (T \supset U)$
U

III. Use indirect truth tables to determine whether the following groups of statements are consistent or inconsistent.

★1. $K \equiv (R \vee M)$
$K \bullet \sim R$
$M \supset \sim K$

2. $F \equiv (A \bullet \sim P)$
$A \supset (P \bullet S)$
$S \supset \sim F$
$A \bullet \sim F$

3. $(G \vee \sim Q) \supset (F \vee B)$
$\sim(F \vee Q)$
$B \supset N$
$(F \vee N) \supset Q$

★4. $(N \vee C) \equiv E$
$N \supset \sim(C \vee H)$
$H \supset E$
$C \supset H$

5. $P \vee \sim S$
$S \vee \sim T$
$T \vee \sim X$
$X \vee \sim J$
$J \vee \sim P$

6. $(Q \vee K) \supset C$
$(C \bullet F) \supset (N \vee L)$
$C \supset (F \bullet \sim L)$
$Q \bullet \sim N$

★7. $S \supset (R \equiv A)$
$A \supset (W \bullet \sim R)$
$R \equiv (W \vee T)$
$S \bullet U$
$U \supset T$

8. $(E \vee H) \supset (K \bullet D)$
$D \supset (M \bullet B)$
$B \supset \sim E$
$\sim(H \vee K)$
$D \supset B$

9. $G \supset P$
$P \supset (A \bullet \sim G)$
$(R \vee T) \supset G$
$Y \supset R$
$B \supset T$
$Y \vee B$

★10. $A \vee Z$
$A \supset (T \bullet F)$
$Z \supset (M \bullet Q)$
$Q \supset \sim F$
$T \supset \sim M$
$M \supset A$

6

6.6 Argument Forms and Fallacies

Many of the arguments that occur in propositional logic have forms that bear specific names and can be immediately recognized as either valid or invalid. The first part of this section presents some of the more common ones and explains how they are recognized. The second part discusses ways of refuting two of these forms, constructive and destructive dilemmas. The third part presents a word of caution relating to invalid forms. Finally, the fourth part applies to real-life arguments some of the principles developed in the first part.

Common Argument Forms

An **argument form** is an arrangement of statement variables and operators such that the uniform replacement of the variables by statements results in an argument. A *valid* argument form is any argument form that satisfies the truth-table test.

The first valid argument form to consider is **disjunctive syllogism,** which is defined as follows:

Disjunctive syllogism (DS):

$$\begin{array}{l} p \vee q \\ \underline{\sim p} \\ q \end{array}$$

The validity of this form can be easily checked by a truth table. Now, given that validity is purely a function of the form of an argument, any argument produced by uniformly substituting statements in place of the variables in this argument form is a valid argument. Such an argument is said to *have* the form of a disjunctive syllogism. The following argument was produced in this way and is therefore valid:

Either Harvard or Princeton is in New Jersey.	$H \vee P$
Harvard is not in New Jersey.	$\underline{\sim H}$
Therefore, Princeton is in New Jersey.	P

The validity of a disjunctive syllogism arises from the fact that one of the premises presents two alternatives and the other premise eliminates one of those alternatives, leaving the other as the conclusion. This so-called "method of elimination" is essential to the validity of a disjunctive syllogism. If one premise should present two alternatives and the other premise should *affirm* one of those alternatives, the argument is invalid (unless the conclusion is a tautology). Example:

Either Harvard or Amherst is in Massachusetts.	$H \vee A$
Harvard is in Massachusetts.	$\underline{H}$
Therefore, Amherst is not in Massachusetts.	$\sim A$

Since both Harvard and Amherst are in Massachusetts, the premises are true and the conclusion is false. Thus, the argument is invalid. Because the wedge symbol designates inclusive disjunction, the disjunctive premise includes the possibility of both disjuncts being true. Thus, for the argument to be valid, the other premise must eliminate one of the disjuncts.

6

The next valid argument form we consider is **pure hypothetical syllogism**. It consists of two premises and one conclusion, all of which are hypothetical (conditional) statements, and is defined as follows:

Pure hypothetical syllogism (HS):

$$\begin{array}{l} p \supset q \\ \underline{q \supset r} \\ p \supset r \end{array}$$

Any argument that has the form of a pure hypothetical syllogism (that is, any argument that can be produced by uniformly substituting statements in place of the variables in the form) is a valid argument. Example:

If world population continues to grow, then cities will become hopelessly overcrowded.	$W \supset C$
If cities become hopelessly overcrowded, then pollution will become intolerable.	$\underline{C \supset P}$
Therefore, if world population continues to grow, then pollution will become intolerable.	$W \supset P$

The validity of a pure hypothetical syllogism is grounded in the fact that the premises link together like a chain. In the population argument, the consequent of the first premise is identical to the antecedent of the second. If the premises fail to link together in this way, the argument may be invalid. Example:

If Bill Gates is a man, then Bill Gates is a human being.	$M \supset H$
If Bill Gates is a woman, then Bill Gates is a human being.	$\underline{W \supset H}$
Therefore, if Bill Gates is a man, then Bill Gates is a woman.	$M \supset W$

The premises of this argument are true, and the conclusion is false. Thus, the argument is invalid.

Another important valid argument form is called ***modus ponens*** ("asserting mode"). It consists of a conditional premise, a second premise that asserts the antecedent of the conditional premise, and a conclusion that asserts the consequent:

Modus ponens (MP):

$$\begin{array}{l} p \supset q \\ \underline{p \qquad} \\ q \end{array}$$

Any argument having the form of *modus ponens* is a valid argument. Example:

If twelve million children die yearly from starvation, then something is wrong with food distribution.	$T \supset S$
Twelve million children die yearly from starvation.	$\underline{T \qquad}$
Therefore, something is wrong with food distribution.	S

Closely associated with *modus ponens* is ***modus tollens*** ("denying mode"). *Modus tollens* is a valid argument form consisting of one conditional premise, a second premise that denies the consequent of the conditional premise, and a conclusion that denies the antecedent. It is defined as follows:

Modus tollens (MT):

$$\begin{array}{l} p \supset q \\ \underline{\sim q \qquad} \\ \sim p \end{array}$$

Although a little harder to understand than *modus ponens*, *modus tollens* can be understood by the following reasoning process: The conclusion states that we do not have *p*, because if we did have *p*, then (by the first premise) we would have *q*, and we do not have *q* (by the second premise). Any argument that has the form of *modus tollens* is a valid argument. Example:

If Japan cares about endangered species, then it has stopped killing whales.	$C \supset S$
Japan has not stopped killing whales.	$\underline{\sim S \qquad}$
Therefore, Japan does not care about endangered species.	$\sim C$

Two invalid forms are closely associated with *modus ponens* and *modus tollens*. These are **affirming the consequent** and **denying the antecedent.** Affirming the consequent consists of one conditional premise, a second premise that asserts the consequent of the conditional, and a conclusion that asserts the antecedent:

Affirming the consequent (AC):

$$\begin{array}{l} p \supset q \\ \underline{q \qquad} \\ p \end{array}$$

Any argument that has the form of affirming the consequent is an invalid argument.*
The following argument has this form and is therefore invalid:

If Napoleon was killed in a plane crash, then Napoleon is dead.	$K \supset D$
Napoleon is dead.	D
Therefore, Napoleon was killed in a plane crash.	K

Given that this argument has true premises and a false conclusion, it is clearly invalid.

Denying the antecedent consists of a conditional premise, a second premise that denies the antecedent of the conditional, and a conclusion that denies the consequent:

Denying the antecedent (DA):

$$\begin{array}{l} p \supset q \\ \underline{\sim p\quad\quad} \\ \sim q \end{array}$$

Any argument that has the form of denying the antecedent is an invalid argument. Example:

If Napoleon was killed in a plane crash, then Napoleon is dead.	$K \supset D$
Napoleon was not killed in a plane crash.	$\sim K$
Therefore, Napoleon is not dead.	$\sim D$

6

Again, this argument has true premises and a false conclusion, so it is clearly invalid.

A **constructive dilemma** is a valid argument form that consists of a conjunctive premise made up of two conditional statements, a disjunctive premise that asserts the antecedents in the conjunctive premise (like *modus ponens*), and a disjunctive conclusion that asserts the consequents of the conjunctive premise. It is defined as follows:

Constructive dilemma (CD):

$$\begin{array}{l} (p \supset q) \cdot (r \supset s) \\ \underline{p \vee r\quad\quad\quad\quad\quad} \\ q \vee s \end{array}$$

Any argument that has the form of a constructive dilemma is a valid argument. Example:

If we choose nuclear power, then we increase the risk of a nuclear accident; but if we choose conventional power, then we add to the greenhouse effect.	$(N \supset I) \cdot (C \supset A)$
We must choose either nuclear power or conventional power.	$N \vee C$
Therefore, we either increase the risk of nuclear accident or add to the greenhouse effect.	$I \vee A$

The **destructive dilemma** is also a valid argument form. It is similar to the constructive dilemma in that it includes a conjunctive premise made up of two conditional statements and a disjunctive premise. However, the disjunctive premise denies

* See "Note on Invalid Forms" later in this section.

the consequents of the conditionals (like *modus tollens*), and the conclusion denies the antecedents:

Destructive dilemma (DD):

$$\frac{\begin{array}{l}(p \supset q) \cdot (r \supset s) \\ \sim q \lor \sim s\end{array}}{\sim p \lor \sim r}$$

Any argument that has the form of a destructive dilemma is a valid argument. Example:

If we are to reverse the greenhouse effect, then we must choose nuclear power; but if we are to lower the risk of a nuclear accident, then we must choose conventional power.	$(R \supset N) \cdot (L \supset C)$
We will either not choose nuclear power or not choose conventional power.	$\sim N \lor \sim C$
Therefore, we will either not reverse the greenhouse effect or not lower the risk of a nuclear accident.	$\sim R \lor \sim L$

6

Refuting Constructive and Destructive Dilemmas

Now that we are familiar with several argument forms in propositional logic, we may return for a closer look at two of them, constructive and destructive dilemmas. Arguments having these forms occur frequently in public debate, where an arguer may use them to trap an opponent. Since both forms are valid, the only direct mode of defense available to the opponent is to prove the dilemma unsound. This can be done by proving at least one of the premises false. If the conjunctive premise (otherwise

Grasping by the horns: Prove the conjunctive premise false by proving either conjunct false	e.g.: $(p \supset q) \cdot (r \supset s)$ T F F F
Escaping between the horns: Prove the disjunctive premise false	e.g.: $p \lor r$ F F F

called the "horns of the dilemma") is proven false, the opponent is said to have "grasped the dilemma by the horns." This, of course, may be done by proving either one of the conditional statements false. If, on the other hand, the disjunctive premise is proven false, the opponent is said to have "escaped between the horns of the dilemma." The latter strategy often involves finding a third alternative that excludes the two that are given in the disjunctive premise. If such a third alternative can be found, both of the given disjuncts will be proved false. Consider the following constructive dilemma:

If taxes increase, the economy will suffer, and if taxes decrease, needed government services will be curtailed. Since taxes must either increase or decrease, it follows that the economy will suffer or that needed government services will be curtailed.

It is easy to escape between the horns of this dilemma by arguing that taxes could be kept as they are, in which case they would neither increase nor decrease.

Some dilemmas, however, do not allow for the possibility of escaping between the horns. Consider the following constructive dilemma:

If we encourage competition, we will have no peace, and if we do not encourage competition, we will make no progress. Since we must either encourage competition or not encourage it, we will either have no peace or make no progress.

Since the disjunctive premise of this dilemma is a tautology, it cannot be proven false. This leaves the strategy of grasping the dilemma by the horns, which may be done by proving either of the conditional statements in the conjunctive premise false. One debater might want to attack the first conditional and argue that competition and peace can coexist, while another might want to attack the second and argue that progress can be achieved through some means other than encouraging competition.

6

The strategy to be followed in refuting a dilemma is therefore this: Examine the disjunctive premise. If this premise is a tautology, attempt to grasp the dilemma by the horns by attacking one or the other of the conditional statements in the conjunctive premise. If the disjunctive premise is not a tautology, then either escape between the horns by, perhaps, finding a third alternative, or grasp the dilemma by the horns—whichever is easier.

A third, indirect strategy for refuting a dilemma involves constructing a counterdilemma. This is typically done by changing either the antecedents or the consequents of the conjunctive premise while leaving the disjunctive premise as it is, so as to obtain a different conclusion. If the dilemma in question is a constructive dilemma, the consequents of the conjunctive premise are changed. Here are possible counterdilemmas for the two dilemmas just presented:

If taxes increase, needed government services will be extended, and if taxes decrease, the economy will improve. Since taxes must either increase or decrease, it follows that needed government services will be extended or the economy will improve.

If we encourage competition, we will make progress, and if we do not encourage competition, we will have peace. Since we must either encourage competition or not encourage it, we will either make progress or have peace.

Constructing a counterdilemma falls short of a refutation of a given dilemma because it merely shows that a different approach can be taken to a certain problem. It does not cast any doubt on the soundness of the original dilemma. Yet the strategy is often effective because it testifies to the cleverness of the debater who can accomplish it successfully. In the heat of debate the attending audience is often persuaded that the original argument has been thoroughly demolished.

Note on Invalid Forms

Throughout this book we have seen that any substitution instance of a valid argument form is a valid argument. For example, consider *modus ponens:*

$$\begin{array}{l} p \supset q \\ \underline{p\quad\;} \\ q \end{array}$$

Literally any two statements uniformly substituted in the place of p and q will result in a valid argument. Thus, the following symbolized arguments both have the form of *modus ponens,* and are accordingly valid:

$$\begin{array}{l} S \supset T \\ \underline{S\quad\;} \\ T \end{array} \qquad\qquad \begin{array}{l} (K \vee B) \supset (N \bullet R) \\ \underline{K \vee B\qquad\qquad} \\ N \bullet R \end{array}$$

In the first argument S and T are uniformly substituted in the place of p and q, and in the second argument $K \vee B$ and $N \bullet R$ are uniformly substituted in the place of p and q.

However, this result does not extend to invalid argument forms. Consider, for example, affirming the consequent:

$$\begin{array}{l} p \supset q \\ \underline{q\quad\;} \\ p \end{array}$$

Sometimes the uniform substitution of statements in the place of p and q results in an invalid argument, and sometimes it does not. Both of the following symbolized arguments are substitution instances of affirming the consequent, but the one on the left is invalid while the one on the right is valid:

$$\begin{array}{l} G \supset N \\ \underline{N\quad\;\;} \\ G \end{array} \qquad\qquad \begin{array}{l} (F \vee D) \supset (F \bullet D) \\ \underline{F \bullet D\qquad\qquad} \\ F \vee D \end{array}$$

To deal with this problem we adopt a convention about when an argument will be said to *have* an invalid form. We will say that an argument has an invalid form if it is a substitution instance of that form *and* it is not a substitution instance of any valid form. According to this convention only the argument on the left has the form of affirming the consequent. The argument on the right does not have this form because it is a substitution instance of the following valid form:

$$\begin{array}{l} (p \vee q) \supset (p \bullet q) \\ \underline{p \bullet q\qquad\qquad} \\ p \vee q \end{array}$$

The validity of this form results from the fact that the conclusion follows from the second premise alone, without any involvement of the first premise. This fact may be easily checked with a truth table.

Here is another invalid form:

$$\begin{array}{l} p \supset q \\ \underline{r \supset q} \\ p \supset r \end{array}$$

Both of the following symbolized arguments are substitution instances of this form, but only the one on the left is invalid:

$$\begin{array}{l} K \supset L \\ \underline{R \supset L} \\ K \supset R \end{array} \qquad \begin{array}{l} {\sim}C \supset A \\ \underline{(C \supset E) \supset A} \\ {\sim}C \supset (C \supset E) \end{array}$$

The argument on the right is valid because its conclusion is a tautology. Accordingly, only the argument on the left will be said to have the invalid form in question.

The point of this discussion is that when we attempt to determine the validity of arguments through mere inspection, we have to exert caution with invalid forms. The mere fact that an argument is a substitution instance of an invalid form does not guarantee that it is invalid. Before judging it invalid we must make sure that it is not valid for some other reason, such as its conclusion being a tautology. However, as concerns the exercises at the end of this section, all of the arguments that are substitution instances of invalid forms are invalid. In other words, none of them is like either of the right-hand examples considered in these paragraphs.

6

Summary and Application

Any argument having one of the following forms is valid:

$$\begin{array}{l} p \vee q \\ \underline{{\sim}p} \\ q \end{array}$$ disjunctive syllogism (DS)

$$\begin{array}{l} p \supset q \\ \underline{q \supset r} \\ p \supset r \end{array}$$ pure hypothetical syllogism (HS)

$$\begin{array}{l} p \supset q \\ \underline{p} \\ q \end{array}$$ *modus ponens* (MP)

$$\begin{array}{l} p \supset q \\ \underline{{\sim}q} \\ {\sim}p \end{array}$$ *modus tollens* (MT)

$$\begin{array}{l} (p \supset q) \cdot (r \supset s) \\ \underline{p \vee r} \\ q \vee s \end{array}$$ constructive dilemma (CD)

$$\begin{array}{l} (p \supset q) \cdot (r \supset s) \\ \underline{{\sim}q \vee {\sim}s} \\ {\sim}p \vee {\sim}r \end{array}$$ destructive dilemma (DD)

Any argument having either of the following forms is invalid:

$$\begin{array}{l} p \supset q \\ \underline{q} \\ p \end{array}$$ affirming the consequent (AC)

$$\begin{array}{l} p \supset q \\ \underline{{\sim}p} \\ {\sim}q \end{array}$$ denying the antecedent (DA)

In identifying arguments as having these argument forms, use the following procedure. First symbolize the argument, using uppercase letters for the simple propositions. Then see whether the symbolized argument fits the pattern of one of these forms. For example, the following symbolized argument has the form of *modus ponens,* and is therefore valid:

$$\begin{array}{l} K \supset R \\ \underline{K \qquad} \\ R \end{array}$$

If *K* and *R* are substituted respectively in place of *p* and *q* in the *modus ponens* form, we obtain the symbolized argument in question.

However, not every attempt at argument recognition is as simple as this. For more complicated cases it helps to keep two points in mind:

The order of the premises never affects the argument's form.
Negated letters can be substituted in place of the *p*, *q*, *r*, and *s* of an argument form just as can non-negated letters.

In regard to the first point, consider these symbolized arguments:

$$\begin{array}{l} N \\ \underline{N \supset B} \\ B \end{array} \qquad \begin{array}{l} {\sim}S \\ \underline{S \vee F} \\ F \end{array}$$

The argument on the left is *modus ponens,* and the one on the right is a disjunctive syllogism. To see this more clearly, simply switch the order of the premises.

In regard to the second point (involving negated letters), consider these examples:

$$\begin{array}{l} {\sim}G \supset {\sim}H \\ \underline{{\sim}G \qquad\quad} \\ {\sim}H \end{array} \qquad \begin{array}{l} {\sim}K \supset {\sim}M \\ \underline{{\sim}{\sim}M \qquad\quad} \\ {\sim}{\sim}K \end{array}$$

The argument on the left is *modus ponens,* and the one on the right is *modus tollens.* To produce the argument on the left, substitute ~*G* in the place of *p* in the *modus ponens* form, and ~*H* in the place of *q*. For the argument on the right, substitute ~*K* in the place of *p* in the *modus tollens* form, and ~*M* in the place of *q.*

Another problem that complicates the task of argument recognition arises from the fact that many arguments can be translated in alternate ways. Consider, for example, this argument:

Either the witness lied or Bob is guilty.
The witness told the truth.
Therefore, Bob is guilty.

If we select *L* to represent "The witness lied," then the argument can be translated into symbols as follows:

$$\begin{array}{l} L \vee B \\ \underline{{\sim}L \qquad} \\ B \end{array}$$

This symbolized argument is clearly an instance of disjunctive syllogism.

On the other hand, if we select *T* to represent "The witness told the truth," then we have this translation:

$$\begin{array}{l} {\sim}T \vee B \\ \underline{T \qquad\quad} \\ B \end{array}$$

Technically this is not an instance of disjunctive syllogism, because the second premise, *T*, is not preceded by a tilde. To avoid this kind of difficulty in connection with alternative translations, we introduce two rules. They should be obvious, but if there is any doubt about them they can be proved using truth tables. The rules are as follows:

p is logically equivalent to $\sim\sim p$.	(Double Negation)
$p \vee q$ is logically equivalent to $q \vee p$.	(Commutativity)

According to the first rule, double tildes may be either inserted or deleted prior to any statement, and according to the second rule the order of the components in a disjunctive statement may be reversed. Applying the double negation rule to the second premise of the symbolized argument above, we have

$$\begin{array}{l} \sim T \vee B \\ \underline{\sim\sim T} \\ B \end{array}$$

After this change, the argument is now an instance of disjunctive syllogism.

For examples of how the commutativity rule is applied, consider these symbolized arguments:

$$\begin{array}{l} M \vee E \\ \underline{\sim E} \\ M \end{array} \qquad \begin{array}{l} (R \supset L) \bullet (T \supset K) \\ \underline{T \vee R \qquad\qquad} \\ L \vee K \end{array}$$

Technically the argument on the left is not an instance of disjunctive syllogism because the letters in the first premise are in the wrong order, and the argument on the right is not an instance of constructive dilemma because the letters in the second premise are in the wrong order. We can reverse the order of these letters by applying the commutativity rule:

$$\begin{array}{l} E \vee M \\ \underline{\sim E} \\ M \end{array} \qquad \begin{array}{l} (R \supset L) \bullet (T \supset K) \\ \underline{R \vee T \qquad\qquad} \\ L \vee K \end{array}$$

After these changes, the argument on the left is now clearly an instance of disjunctive syllogism, and the one on the right is an instance of constructive dilemma.

Here are some additional examples. In some cases the symbolized argument must be rewritten using double negation or commutativity before it fits the pattern of the argument form indicated.

$$\begin{array}{l} \sim A \supset \sim B \\ \underline{\sim B \supset C} \\ \sim A \supset C \end{array} \quad \text{HS—valid} \qquad \begin{array}{l} A \supset \sim B \\ \underline{B \supset \sim C} \\ A \supset \sim C \end{array} \quad \text{invalid}$$

$$\begin{array}{l} \sim A \supset \sim B \\ \underline{B \qquad\quad} \\ A \end{array} \quad \text{MT—valid} \qquad \begin{array}{l} \sim A \supset B \\ \underline{A \qquad\quad} \\ \sim B \end{array} \quad \text{DA—invalid}$$

$$\begin{array}{l} \sim A \vee \sim B \\ \underline{A \qquad\quad} \\ \sim B \end{array} \quad \text{DS—valid} \qquad \begin{array}{l} \sim A \vee B \\ \underline{\sim A \qquad} \\ B \end{array} \quad \text{invalid}$$

6

$$\begin{array}{l}(A \supset \sim B) \cdot (\sim C \supset D) \\ A \vee \sim C \\ \hline \sim B \vee D\end{array}\qquad \text{CD—valid}$$

$$\begin{array}{l}(\sim A \supset B) \cdot (C \supset \sim D) \\ B \vee \sim D \\ \hline A \vee \sim C\end{array}\qquad \text{invalid}$$

$$\begin{array}{l}A \vee \sim B \\ B \\ \hline A\end{array}\qquad \text{DS—valid}$$

$$\begin{array}{l}A \supset \sim B \\ \sim B \\ \hline A\end{array}\qquad \text{AC—invalid}$$

$$\begin{array}{l}A \\ A \supset B \\ \hline B\end{array}\qquad \text{MP—valid}$$

$$\begin{array}{l}A \vee C \\ (A \supset B) \cdot (C \supset D) \\ \hline B \vee D\end{array}\qquad \text{CD—valid}$$

Let us now see how the argument forms presented in this section can be used to interpret the structure of some real-life arguments. Consider the following letter to the editor of a newspaper:

> If U.S. servicemen are indeed being held in Southeast Asia, what is the motivation of their captors? No government there has asked for anything in return, as might be expected if they were deliberately holding Americans.
>
> (Norm Oshrin)

This argument is enthymematic; in other words, it is missing certain parts. The author intends to prove that U.S. servicemen are not being held in Southeast Asia—because if they were, their captors would be demanding something for their return. The argument can thus be structured as a *modus tollens:*

> If U.S. servicemen are being held in Southeast Asia, then their captors have demanded something for their return.
> Their captors have not demanded something for their return.
> Therefore, U.S. servicemen are not being held in Southeast Asia.

Here is another example:

> In a time when an entire nation believes in Murphy's law (that if anything can go wrong, it surely will) and has witnessed serious accidents in the highly regulated, supposedly fail-safe nuclear industry, it's fascinating that people can persist in the fantasy that an error will not occur in the area of nuclear weaponry.
>
> (Burk Gossom, *Newsweek*)

Although this argument allows for more than one analysis, clearly the arguer presents two main reasons why we can expect an accident in the area of nuclear weaponry: "Murphy's law" (which everyone believes to be true) dictates it, and accidents have occurred in the area of nuclear power (which is presumed fail-safe). Thus, at the very least, we can extract two *modus ponens* arguments from this selection:

> If everyone believes Murphy's law, then we can expect accidents in nuclear weaponry.
> Everyone believes Murphy's law.
> Therefore, we can expect accidents in nuclear weaponry.

If accidents have occurred in nuclear power, then we can expect accidents in nuclear weaponry.
Accidents have occurred in nuclear power.
Therefore, we can expect accidents in nuclear weaponry.

Many arguments that we encounter in ordinary life can be interpreted as instances of valid argument forms. After being so interpreted, however, not all will turn out sound. The invalid forms (denying the antecedent and affirming the consequent) should be reserved for the relatively few arguments that are clearly invalid as originally expressed.

Exercise 6.6

I. Interpret the following symbolized arguments in light of the eight argument forms presented in this section. In some cases a symbolized argument must be rewritten using commutativity or double negation before it becomes an instance of one of these forms. Those not having a named form are invalid.

★**1.** $\begin{array}{l} N \supset C \\ \sim C \\ \hline \sim N \end{array}$

2. $\begin{array}{l} S \supset F \\ F \supset \sim L \\ \hline S \supset \sim L \end{array}$

3. $\begin{array}{l} A \vee \sim Z \\ \sim Z \\ \hline A \end{array}$

★**4.** $\begin{array}{l} (S \supset \sim P) \bullet (\sim S \supset D) \\ S \vee \sim S \\ \hline \sim P \vee D \end{array}$

5. $\begin{array}{l} \sim N \\ \sim N \supset T \\ \hline T \end{array}$

6. $\begin{array}{l} M \vee \sim B \\ \sim M \\ \hline \sim B \end{array}$

★**7.** $\begin{array}{l} (E \supset N) \bullet (\sim L \supset \sim K) \\ \sim N \vee K \\ \hline \sim E \vee L \end{array}$

8. $\begin{array}{l} W \supset \sim M \\ \sim M \\ \hline W \end{array}$

9. $\begin{array}{l} \sim B \supset \sim L \\ G \supset \sim B \\ \hline G \supset \sim L \end{array}$

★**10.** $\begin{array}{l} F \supset O \\ \sim F \\ \hline \sim O \end{array}$

11. $\begin{array}{l} (K \vee B) \bullet (N \vee Q) \\ K \vee N \\ \hline B \vee Q \end{array}$

12. $\begin{array}{l} X \\ X \supset \sim E \\ \hline \sim E \end{array}$

★**13.** $\begin{array}{l} P \vee \sim S \\ S \\ \hline P \end{array}$

14. $\begin{array}{l} B \bullet T \\ T \\ \hline \sim B \end{array}$

15. $\begin{array}{l} \sim R \vee \sim Q \\ (G \supset Q) \bullet (H \supset R) \\ \hline \sim G \vee \sim H \end{array}$

★**16.** $\begin{array}{l} \sim G \supset H \\ H \\ \hline \sim G \end{array}$

17. $K \supset \sim C$
$\underline{C}$
$\sim K$

18. $(I \supset M) \bullet (\sim O \supset A)$
$\underline{\sim O \vee I}$
$M \vee A$

★19. $X \supset \sim F$
$\underline{W \supset \sim F}$
$W \supset X$

20. $\sim L \supset U$
$\underline{L}$
$\sim U$

II. Translate the following arguments into symbolic notation and then interpret them in light of the eight argument forms presented in this section. In some cases a symbolized argument must be rewritten using commutativity or double negation before it becomes an instance of one of these forms. Those not having a named form are invalid.

★**1.** A Boeing 757 crashed into the Pentagon on 9/11 only if two giant engines were found outside the building. It is not the case that two giant engines were found outside the building. Therefore, a Boeing 757 did not crash into the Pentagon on 9/11.

2. If Michelangelo painted the ceiling of the Sistine Chapel, then he was familiar with stories from the Old Testament. Michelangelo was familiar with stories from the Old Testament. Therefore, Michelangelo painted the ceiling of the Sistine Chapel.

3. If you enter the teaching profession, you will have no money for vacations; and if you do not enter the teaching profession, you will have no time for vacations. Since you must either enter or not enter the teaching profession, it follows that either you will have no money or no time for vacations.

★**4.** Either the wealthiest people are the happiest, or it is not the case that money can buy everything. The wealthiest people are not the happiest. Therefore, money cannot buy everything.

5. Either drivers are forbidden to send text messages, or the highways will not become safer. Drivers are forbidden to send text messages. Therefore, the highways will become safer.

6. If the sun is a variable star, then its energy will drop drastically at some point in the future. If the sun's energy drops drastically at some point in the future, then the earth will become a giant ice-ball. Therefore, if the sun is a variable star, then the earth will become a giant ice-ball.

★**7.** Nano-thermite is present in the debris from the World Trade Center. But if that is so, then the buildings were brought down by controlled demolition. Therefore, the buildings were brought down by controlled demolition.

8. If TV viewing provides genuine relaxation, then TV enhances the quality of life. But TV viewing does not provide genuine relaxation. Therefore, TV does not enhance the quality of life.

9. If high school clinics are to stem the tide of teenage pregnancy, then they must dispense birth control devices; but if they want to discourage illicit sex, then they must not dispense these devices. Since high school clinics must either

6

dispense or not dispense birth control devices, either they will not stem the tide of teenage pregnancy, or they will not discourage illicit sex.

★**10.** If limits are imposed on medical malpractice suits, then patients will not be adequately compensated for their injuries; but if the cost of malpractice insurance continues to rise, then physicians will be forced out of business. Limits will not be imposed, and the cost of malpractice insurance will not continue to rise. Therefore, patients will be adequately compensated and physicians will not be forced out of business.

11. If Prohibition succeeded in the 1920s, then the war on drugs will succeed today. But Prohibition did not succeed in the 1920s. Therefore, the war on drugs will not succeed today.

12. If life is always better than death, then people do not commit suicide. People do commit suicide. Therefore, life is not always better than death.

★**13.** If we want to arrest criminals, then police must engage in high-speed chases; but if we want to protect motorists, then police must not engage in high-speed chases. Since police must either engage or not engage in high-speed chases, either we will not arrest criminals or not protect motorists.

14. Either industrial pollutants will be more stringently controlled, or acid rain will continue to fall. Industrial pollutants will be more stringently controlled. Therefore, acid rain will not continue to fall.

15. Insurance companies contribute millions of dollars to political campaigns. But if that is so, then meaningful insurance reform is impossible. Therefore, meaningful insurance reform is impossible.

★**16.** If Mexico does not get its population growth under control, then its unemployment problem will never be solved. Mexico's unemployment problem will never be solved. Therefore, Mexico will not get its population growth under control.

17. Either the dinosaurs were not cold-blooded or they were not the ancestors of modern birds. The dinosaurs were the ancestors of modern birds. Therefore, the dinosaurs were not cold-blooded.

18. If coal burning continues, then heavy metals will be released into the atmosphere. If heavy metals are not released into the atmosphere, then nervous system damage will decrease. Therefore, if coal burning does not continue, then nervous system damage will decrease.

★**19.** If sea levels rise twenty feet worldwide, then coastal cities from New York to Sydney will be inundated. If the ice sheets on Antarctica slip into the sea, then sea levels will rise twenty feet worldwide. Therefore, if the ice sheets on Antarctica slip into the sea, then coastal cities from New York to Sydney will be inundated.

20. If tax credits are given for private education, then the government will be supporting religion; but if tax credits are not given for private education, then some parents will end up paying double tuition. Either tax credits will or will not be given for private education. Therefore, either the government will be supporting religion, or some parents will end up paying double tuition.

III. Identify the following dilemmas as either constructive or destructive. Then suggest a refutation for each by escaping between the horns, grasping by the horns, or constructing a counterdilemma.

★1. If Melinda spends the night studying, she will miss the party; but if she does not spend the night studying, she will fail the test tomorrow. Melinda must either spend the night studying or not studying. Therefore, she will either miss the party or fail the test.

2. If we build our home in the valley, it will be struck by floods; and if we build it on the hilltop, it will be hit by lightning. Since we must either build it in the valley or on the hilltop, our home will either be struck by floods or hit by lightning.

3. If psychotherapists respect their clients' right to confidentiality, then they will not report child abusers to the authorities; but if they have any concern for the welfare of children, then they will report them. Psychotherapists must either report or not report child abusers to the authorities. Therefore, psychotherapists either have no respect for their clients' right to confidentiality or no concern for the welfare of children.

6

★4. If corporations are to remain competitive, then they must not spend money to neutralize their toxic waste; but if the environment is to be preserved, then corporations must spend money to neutralize their toxic waste. Corporations either will or will not spend money to neutralize their toxic waste. Therefore, either they will not remain competitive, or the environment will be destroyed.

5. If physicians pull the plug on terminally ill patients, then they risk being charged with murder; but if they do not pull the plug, they prolong their patients' pain and suffering. Since physicians with terminally ill patients must do one or the other, either they risk being charged with murder or they prolong their patients' pain and suffering.

6. If the Mitchells get a divorce, they will live separately in poverty; but if they stay married, they will live together in misery. Since they must either get a divorce or stay married, they will either live separately in poverty or together in misery.

★7. If college students want courses that are interesting and rewarding, then they must major in liberal arts; but if they want a job when they graduate, then they must major in business. College students will either not major in liberal arts, or they will not major in business. Therefore, either they will not take courses that are interesting and rewarding, or they will not have a job when they graduate.

8. If merchants arrest suspected shoplifters, then they risk false imprisonment; but if they do not arrest them, they risk loss of merchandise. Merchants must either arrest or not arrest suspected shoplifters. Therefore, they will either risk false imprisonment or loss of merchandise.

9. If women threatened with rape want to avoid being maimed or killed, then they must not resist their assaulter; but if they want to ensure successful

prosecution of the assailant, they must resist him. Since women threatened with rape must do one or the other, either they will risk being maimed or killed or they will jeopardize successful prosecution of the assailant.

★**10.** If we prosecute suspected terrorists, then we risk retaliation by other terrorists; but if we release them, then we encourage terrorism. Since we must either prosecute or release suspected terrorists, we either risk retaliation by other terrorists or we encourage terrorism.

IV. The following dialogue contains at least fifteen arguments. Translate each into symbolic notation, and then interpret them in light of the eight argument forms presented in this section.

A Little Help from a Friend

"I can talk for only a minute," Liz says to her friend Amy. "I have this bio mid-term tomorrow, and I'm terribly afraid I'm going to fail it," she says, as she sits at her dorm room desk, biology textbook in hand.

"Okay," Amy replies. "I'll cut out after a minute or two. But why are you so afraid?"

"Because I really haven't studied at all," her friend replies. "I figure if I don't pull an all-nighter, I'll fail the test. But I can't fail the test, so I must pull the all-nighter."

"I don't envy you," says Amy, as she reaches for a cookie from the package on Liz's desk. "But I have this little tab of Adderall that might get you through. As I see it, either you take the tab or you'll fall asleep by midnight. But you can't fall asleep, so you must take the tab."

"Gee," replies Liz. "Adderall is for attention deficit disorder. You don't have that, do you?"

"No," says Amy. "If I had ADD, I'd have a prescription for this drug. But I don't have a prescription, so it's clear I don't have ADD. I got the tab from my boyfriend, Zach, who talked the health clinic out of a whole bottle by faking ADD."

"Wow," says Liz, with a look of amazement. "Do you think it's safe for me to take it?"

"Of course," replies Amy. "Zach takes it all the time when he needs an extra spurt of energy, and he has had no adverse reactions. Even I have tried it once or twice. If it's safe for him, then it's safe for me, and if it's safe for me, then it's safe for you. I wouldn't worry about it."

"And do you really think the Adderall will help me pass the test?" asks Liz.

"Absolutely," says Amy. "If you take it, you'll be totally focused for the test, and you must be focused. Hence, you take it, girl."

"But don't you think there are ethical considerations behind taking this drug?" Liz asks, as she takes a closer look at the little orange pill. "If I take the drug, then I'll put myself at an unfair advantage over the other students. I don't want to do that, so maybe I shouldn't take it."

"And now I see this terrible quandary looming before me," she continues. "Either I take the Adderall or I don't. If I take it, I'll feel like I cheated, but if I don't take it, I'll fail the test. Thus, I'll feel like I cheated, or I'll fail the test. Either way, the outcome is bad. So what should I do?"

"Well," replies Amy, "there is another way of looking at it. Either you take the Adderall or you don't. If you take it, you'll pass the test, and if you don't take it, you'll have a clear conscience. Thus, you'll either pass the test or you'll have a clear conscience."

"Very clever," says Liz, "but that really doesn't solve my problem."

"Okay," says Amy. "But maybe your problem isn't as bad as you think. Consider this. Older people take drugs all the time to restore memory loss and sexual function. If it's ethically permissible for them to take those drugs, and it is, then it's permissible for you to take the Adderall. Thus, you shouldn't sweat it."

"That's true," says Liz, "but those older people suffer from a medical condition. If I had such a condition, I'd be justified in taking the Adderall, but I have no such condition. Hence, I'm not justified."

"Allright," says Amy, as she helps herself to a second cookie. "Let's look at it another way. You could get through the night with lots of coffee and Red Bull. But if it's morally permissible to drink Red Bull, and it is, then it's morally permissible to take the Adderall. The conclusion is clear."

"Not quite," says Liz. "Coffee and Red Bull are not really comparable to Adderall—at least not if it's as good as you say it is. Suppose that I'm faced with this option: Either I drink lots of Red Bull, or I take the Adderall. I would say no to the Red Bull, because it's less effective, and it leaves me frazzled. Thus, I would take the Adderall. See, the two are not the same."

6

"Let's try another approach, then," says Amy. "We avail ourselves of technological advances every day without giving a second thought to them. We use cell phones instead of landlines because they're more convenient. We use lightbulbs instead of candles because we see better with them. And we use Adderall instead of coffee because it makes us sharper. If it's ethically okay to use cell phones, then it's okay to use Adderall; and it certainly is okay to use cell phones—just as it's okay to use lightbulbs. Hence, it's okay to use Adderall."

"The problem with that line of reasoning," Liz observes, "is that using lightbulbs and cell phones do not put anyone at a competitive advantage. Everyone uses them, so we're all on an equal plane. But not every student uses Adderall to pass a test. If everyone used it, I would have no problem with it. But not everyone does use it, so I do have a problem."

"I can see your point," Amy says. "At fifteen or twenty bucks a pop on the underground market, not every student can afford Adderall. If it were cheap, then everyone would use it. But it's not cheap, so many students don't."

"It is indeed a messy issue," Amy continues. "So, what do you think you'll do?"

"I just don't know," Liz replies as she puts her face in her hands. "But leave that tab on my desk. I'll see how it goes from now till midnight."

V. The following selections were taken from letters to the editor of newspapers. Each contains one or more arguments, but the exact form of the argument may be hidden or ambiguous. Use the argument forms presented in this section to structure the selections as specifically named arguments.

★**1.** There is a simple way to put a big dent in the national human organ shortage: allocate organs first to the people who have agreed to donate their own. Giving organs first to registered donors would persuade more people to register, and that would make the allocation system fairer. People who aren't willing to share the gift of life should go to the end of the waiting list.

(David J. Undis)

2. OK, I've tried it for a week again this year, but I still don't like daylight-saving time. My grass is brown enough already—it doesn't need another hour of daylight each day. Let's turn the clocks back to the way God intended—standard time.

(Jim Orr)

3. The religious right, in its impassioned fervor to correct our alleged moral wrongs and protect the rights of our unborn "children," may one day realize its ultimate goal of a constitutional amendment banning abortion. And what will the punishment be for those caught performing or receiving an abortion? The death penalty, of course.

(David Fisher)

★4. Most educators believe math instructors ought to emphasize group problem solving. If *group* problem solving is so important (and I think it is), why do we place such emphasis on individual testing? The national math test is a mistake.

(Frederick C. Thayer)

5. If voluntary school prayer for our children is going to make them more moral, then just think what mandatory church attendance on Sunday could do for the rest of us.

(Roderick M. Boyes)

6. A country that replaces the diseased hearts of old white men but refuses to feed schoolchildren, pay women adequately, educate adolescents, or care for the elderly—that country is doomed. We are acting as if there is no tomorrow. Where is our shame?

(Robert Birch)

★7. We cannot afford to close the library at Central Juvenile Hall. These young people in particular need to have access to ideas, dreams, and alternative ways of living. It can make the difference for many students who might become interested in reading for the first time in their lives while in Juvenile Hall.

(Natalie S. Field)

8. If the death penalty deters one person from becoming a murderer, it is effective. There are also some other important reasons for having the death penalty. First, the families and friends of innocent victims have the right to see effective retribution. Second, terminating the life of a killer is more economical than keeping him in jail at the taxpayer's expense. Third, everyone will have greater respect for the judicial system when justice is carried out.

(Doug Kroker)

9. Regarding the bill to require parental consent for a minor's abortion, I would like to point out that the pious platitudes about parental authority quickly fall by the wayside when the minor wants to keep the baby and the parents say, "Don't be silly! You have an abortion and finish your education." If the

parents can veto a minor's abortion, shouldn't they also be able to require one? Better the choice, either pro or con, be left to the girl/woman herself.

(Jane Roberts)

★**10.** More than a million adult videocassettes are rented each week. Nor, as the propagandists would have you believe, does viewing such material lead to violent sex crimes. If it did, there would be over one million such crimes per week.

(Lybrand P. Smith)

Summary

6

Propositional Logic:

- The fundamental units are whole statements (propositions).
- Simple statements are represented by capital letters (*A, B, C,* etc.).
- These are combined via logical operators to form compound statements.
- The logical operators:
  - Tilde (~) forms negations ("not," "it is not the case that").
  - Dot (•) forms conjunctions ("and," "also," "moreover," etc.).
  - Wedge (∨) forms disjunctions ("or," "unless").
  - Horseshoe (⊃) forms conditionals ("if . . . then," "only if," etc.).
  - Triple bar (≡) forms biconditionals ("if and only if," etc.).

Truth Table:

- An arrangement of truth values that shows in every possible case how the truth value of a compound statement is determined by the truth values of its components.
- Used to define the meaning of the five logical operators:
  - $\sim p$ is true only when p is false.
  - $p \cdot q$ is true only when both p and q are true.
  - $p \vee q$ is false only when both p and q are false.
  - $p \supset q$ is false only when p is true and q is false.
  - $p \equiv q$ is true only when p and q have the same truth value.
- Used to classify individual compound statements:
  - Tautologous: Truth values under main operator are all true.
  - Self-contradictory: Truth values under main operator are all false.
  - Contingent: Under main operator: at least one true, at least one false.

- Used to compare one compound statement with another:
  - Logically equivalent: Truth values under main operators are the same on each line.
  - Contradictory: Truth values under main operators are opposite on each line.
  - Consistent: There is at least one line under main operators where all of the truth values are true.
  - Inconsistent: There is no line under main operators where all of the truth values are true.
- Used to test arguments for validity:
  - Invalid: There is a line on which all the premises are true and the conclusion false.
  - Valid: There is no such line.

Indirect Truth Table: A usually shorter truth table constructed by first assigning truth values to the main operators and then working backwards to the simple components.

- Used to test arguments for validity:
  - Begin by assuming the premises true and the conclusion false.
  - Valid: Assumption necessarily leads to a contradiction.
  - Invalid: Assumption does not necessarily lead to a contradiction.
- Used to test a series of statements for consistency:
  - Begin by assuming all of the statements true.
  - Inconsistent: Assumption necessarily leads to a contradiction.
  - Consistent: Assumption does not necessarily lead to a contradiction.

Argument Forms and Fallacies:

- Valid Forms:
  - Disjunctive syllogism: $p \vee q$ / $\sim p$ // q
  - Pure hypothetical syllogism: $p \supset q$ / $q \supset r$ // $p \supset r$
  - *Modus ponens:* $p \supset q$ / p // q
  - *Modus tollens:* $p \supset q$ / $\sim q$ // $\sim p$
  - Constructive dilemma: $(p \supset q) \bullet (r \supset s)$ / $p \vee r$ // $q \vee s$
  - Destructive dilemma $(p \supset q) \bullet (r \supset s)$ / $\sim q \vee \sim s$ // $\sim p \vee \sim r$
- Invalid Forms (Fallacies):
  - Affirming the consequent: $p \supset q$ / q // p
  - Denying the antecedent: $p \supset q$ / $\sim p$ // $\sim q$
- Logical Equivalencies:
  - p is logically equivalent to $\sim\sim p$
  - $p \vee q$ is logically equivalent to $q \vee p$

6