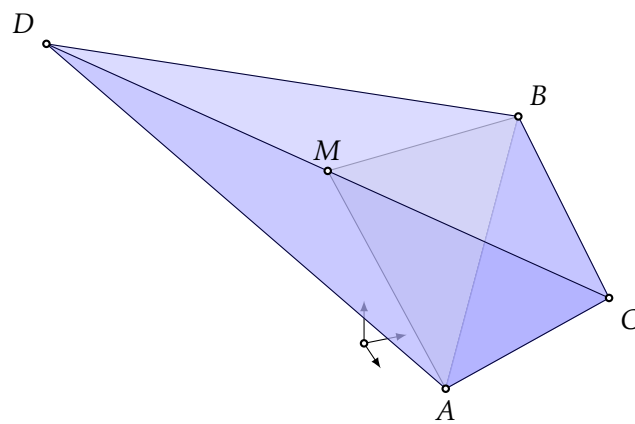


**Planes** in the Euclidean space  $\mathbb{E}^3$ .

1. Determine parametric equations for the plane  $\pi$  in the following cases:
  1.  $\pi$  contains the point  $M(1, 0, 2)$  and is parallel to the vectors  $\mathbf{a}_1(3, -1, 1)$  and  $\mathbf{a}_2(0, 3, 1)$ ,
  2.  $\pi$  contains the point  $A(1, 2, 1)$  and is parallel to  $\mathbf{i}$  and  $\mathbf{j}$ ,
  3.  $\pi$  contains the point  $M(1, 7, 1)$  and is parallel coordinate plane  $Oyz$ ,
  4.  $\pi$  contains the points  $M_1(5, 3, 4)$  and  $M_2(1, 0, 1)$ , and is parallel to the vector  $\mathbf{a}(1, 3, -3)$ ,
  5.  $\pi$  contains the point  $A(1, 5, 7)$  and the coordinate axis  $Ox$ .
2. Determine Cartesian equations for the plane  $\pi$  in the following cases:
  1.  $\pi : x = 2 + 3u - 4v, y = 4 - v, z = 2 + 3u$ ;
  2.  $\pi : x = u + v, y = u - v, z = 5 + 6u - 4v$ .
3. Determine parametric equations for the plane  $\pi$  in the following cases:
  1.  $3x - 6y + z = 0$ ;
  2.  $2x - y - z - 3 = 0$ ;
4. Determine an equation for each plane passing through  $P(3, 5, -7)$  and intersecting the coordinate axes in congruent segments.
5. Let  $A(2, 1, 0)$ ,  $B(1, 3, 5)$ ,  $C(6, 3, 4)$ ,  $D(0, -7, 8)$  be vertices of a tetrahedron. Determine a Cartesian equation of the plane containing  $[AB]$  and the midpoint of  $[CD]$ .



6. Show that a parallelepiped with faces in the planes  $2x + y - 2z + 6 = 0$ ,  $2x - 2y + z - 8 = 0$  and  $x + 2y + 2z + 1 = 0$  is rectangular.

7. Determine a Cartesian equation of the plane  $\pi$  if  $A(1, -1, 3)$  is the orthogonal projection of the origin on  $\pi$ .

8. Determine the distance between the planes  $x - 2y - 2z + 7 = 0$  and  $2x - 4y - 4z + 17 = 0$ .

**Lines** in the Euclidean space  $\mathbb{E}^3$ .

9. Determine parametric equations for the line  $\ell$  in the following cases:

1.  $\ell$  contains the point  $M_0(2, 0, 3)$  and is parallel to the vector  $\mathbf{a}(3, -2, -2)$ ,

2.  $\ell$  contains the point  $A(1, 2, 3)$  and is parallel to the  $Oz$ -axis,

3.  $\ell$  contains the points  $M_1(1, 2, 3)$  and  $M_2(4, 4, 4)$ .

10. Give Cartesian equations for the lines  $\ell$  in the previous exercise.

11. Determine parametric equations for the line contained in the planes  $x + y + 2z - 3 = 0$  and  $x - y + z - 1 = 0$ .

12. Determine the relative positions of the lines  $x = -3t, y = 2 + 3t, z = 1$  and  $x = 1 + 5s, y = 1 + 13s, z = 1 + 10s$ .

13. Let  $A(1, 2, -7)$ ,  $B(2, 2, -7)$  and  $C(3, 4, -5)$  be vertices of a triangle. Determine the equation of the internal angle bisector of  $\angle A$ .

14. Determine the parameter  $m$  for which the line  $x = -1 + 3t, y = 2 + mt, z = -3 - 2t$  doesn't intersect the plane  $x + 3y + 3z - 2 = 0$ .

15. Determine the values  $a$  and  $d$  for which the line  $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-3}{-2}$  is contained in the plane  $ax + y - 2z + d = 0$ .

16. Determine the values  $a$  and  $c$  for which the line  $3x - 2y + z + 3 = 0 \cap 4x - 3y + 4z + 1 = 0$  is perpendicular to the plane  $ax + 8y + cz + 2 = 0$ .

17. Determine the orthogonal projection of the point  $A(2, 11, -5)$  on the plane  $x + 4y - 3z + 7 = 0$ .

18. Determine the orthogonal reflection of the point  $P(6, -5, 5)$  in the plane  $2x - 3y + z - 4 = 0$ .

19. Determine the orthogonal projection of the point  $A(1, 3, 5)$  on the line  $2x + y + z - 1 = 0 \cap 3x + y + 2z - 3 = 0$ .