**1.** Let $A_0, \ldots, A_n$ be the vertices of a polygon. Determine $\overrightarrow{A_0A_1} + \overrightarrow{A_1A_2} + \cdots + \overrightarrow{A_{n-1}A_n} + \overrightarrow{A_nA_0}$.

**2.** In each of the following cases, decide if the indicated vectors $\mathbf{u}, \mathbf{v}, \mathbf{w}$ can be represented with the vertices of a triangle:

1. $\mathbf{u}(7,3)$, $\mathbf{v}(-2,-8)$, $\mathbf{w}(-5,5)$.
2. $\mathbf{u}(7,3)$, $\mathbf{v}(2,8)$, $\mathbf{w}(-5,5)$.
3. $\|\mathbf{u}\| = 7$, $\|\mathbf{v}\| = 3$, $\|\mathbf{w}\| = 11$.
4. $\mathbf{u}(1,0,1)$, $\mathbf{v}(0,1,0)$, $\mathbf{w}(2,2,2)$.

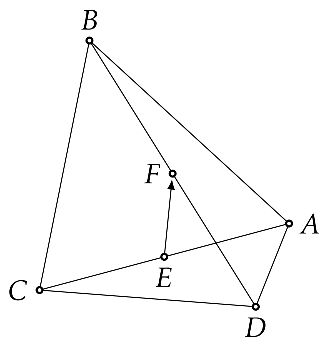
**3.** Let $ABCDEF$ be a regular hexagon centered at $O$.

1. Express the vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$, $\overrightarrow{OC}$, $\overrightarrow{OD}$ in terms of $\overrightarrow{OE}$ and $\overrightarrow{OF}$.
2. Show that $\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF} = 3\overrightarrow{AD}$.

**4.** Let $ABCD$ be a quadrilateral. Let $M$, $N$, $P$, $Q$ be the midpoints of $[AB]$, $[BC]$, $[CD]$ and $[DA]$ respectively. Show that $\overrightarrow{MN} + \overrightarrow{PQ} = 0$. Deduce that the midpoints of the sides of an arbitrary quadrilateral form a parallelogram.

**5.** Let $ABCD$ be a quadrilateral. Let $E$ be the midpoint of $[AC]$ and let $F$ be the midpoint of $[BD]$. Show that

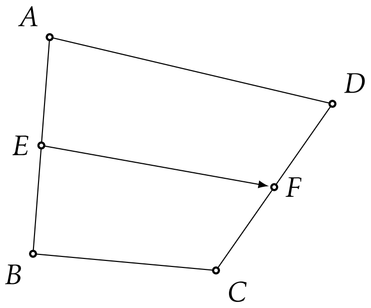
$$\overrightarrow{EF} = \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{CD}) = \frac{1}{2}(\overrightarrow{AD} + \overrightarrow{CB}).$$



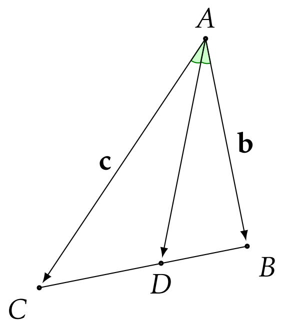

**6.** Let $ABCD$ be a quadrilateral. Let $E$ be the midpoint of $[AB]$ and let $F$ be the midpoint of $[CD]$. Show that

$$\overrightarrow{EF} = \frac{1}{2}(\overrightarrow{AD} + \overrightarrow{BC}).$$

Deduce that the length of the midsegment in a trapezoid is the arithmetic mean of the lengths of the bases.

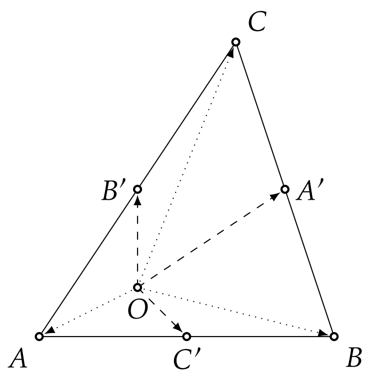



**7.** Let $ABC$ be a triangle and let $D \in [BC]$ be such that $AD$ is an angle bisector. Express $\overrightarrow{AD}$ in terms of $\mathbf{b} = \overrightarrow{AB}$ und $\mathbf{c} = \overrightarrow{AC}$.

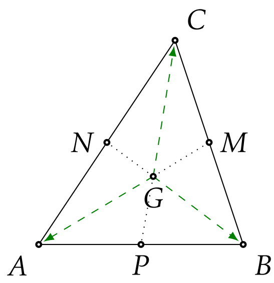



**8.** Let $A'$, $B'$ and $C'$ be midpoints of the sides of a triangle $ABC$. Show that for any point $O$ we have

$$\overrightarrow{OA'} + \overrightarrow{OB'} + \overrightarrow{OC'} = \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC}$$




**9.** Show that the medians in a triangle intersect in one point.




**10.** Let $ABCD$ be a tetrahedron. Determine the sums

1. $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}$,
2. $\overrightarrow{AD} + \overrightarrow{BC} + \overrightarrow{DB}$,
3. $\overrightarrow{AB} + \overrightarrow{CD} + \overrightarrow{BC} + \overrightarrow{DA}$.

**11.** Let $ABCD$ be a tetrahedron. Show that $\overrightarrow{AD} + \overrightarrow{BC} = \overrightarrow{BD} + \overrightarrow{AC}$.

**12.** Let $SABCD$ be a pyramid with apex $S$ and base the parallelogram $ABCD$. Show that

$$\overrightarrow{SA} + \overrightarrow{SB} + \overrightarrow{SC} + \overrightarrow{SD} = 4\overrightarrow{SO}$$

where $O$ is the center of the parallelogram.

**13.** In $\mathbb{E}^3$ consider the parallelograms $A_1A_2A_3A_4$ and $B_1B_2B_3B_4$. Show that the midpoints of the segments $[A_1B_1]$, $[A_2B_2]$, $[A_3B_3]$ and $[A_4B_4]$ are the vertices of a parallelogram.