I'm basically curious about the following set and associated size function:

$$S(n,k,z) = \{(x,y) \in (\mathbb{Z}_m^k)^2 \mid \langle x,y \rangle = z\}$$
$$f(n,k,z) = |S(n,k,z)|$$

It's clear that there is a recursive definition of the set S(n, k, z), recurring on k:

$$S(n, k, z) = \bigoplus_{(x_1, y_1) \in \mathbb{Z}_m^2} append(x_1, y_1, S(n, k - 1, z - x_1 y_1))$$

Where append will place x_1, y_1 on the front of each x, y respectively. The reason the above is true is because $\sum_i x_i y_i = z$ which implies $x_1 y_1 + \sum_{i>1} x_i y_i = z$ which implies $\sum_{i>1} x_i y_i = z - x_1 y_1$. Clearly this is a disjoint union, as each set of vectors will have "different heads". We thus can write the function f as:

$$f(n,k,z) = \sum_{(x_1,y_1) \in \mathbb{Z}_m^2} f(n,k-1,z-x_1y_1)$$

Using the fact that $ax \equiv b \pmod{n}$ has exactly (a, n) solutions if $(a, n) \mid (b, n)$ and has no solutions otherwise, we have that:

$$\begin{split} f(n,1,z) &= \sum_{y \in \mathbb{Z}_n: (y,n) \mid (z,n)} (y,n) = \sum_{d \mid (z,n)} d\varphi(\frac{n}{d}) \\ f(n,1,z) &= n \sum_{d \mid (z,n)} \prod_{p \mid (n/d)} (1 - \frac{1}{p}) \end{split}$$

At this point, all I can come to is that this function is quite difficult to compute and essentially has no obvious and convenient closed form.