Taller 2 - Puntos teóricos
Métodos Computacionales

1. Integración - Punto 1

$$p_1(x) = \frac{x-b}{a-b}f(a) + \frac{x-a}{b-a}f(b)$$

$$\begin{aligned}
I = \int_a^b f(x)dx \approx \int_a^b p_1(x)dx &= \int_a^b \left(\frac{x-b}{a-b}f(a) + \frac{x-a}{b-a}f(b)\right)dx \\
&= f(a)\int_a^b \frac{x-b}{a-b}dx + f(b)\int_a^b \frac{x-a}{b-a}dx \\
&= f(a)\left(\frac{1}{a-b}\int_a^b xdx - \frac{b}{a-b}\int_a^b dx\right) \\
&+ f(b)\left(\frac{1}{b-a}\int_a^b xdx - \frac{a}{b-a}\int_a^b dx\right) \\
&= f(a)\left(\frac{bx}{b-a}\bigg|_a^b - \frac{x^2}{2(b-a)}\bigg|_a^b\right) + f(b)\left(\frac{x^2}{2(b-a)}\bigg|_a^b - \frac{ax}{b-a}\bigg|_a^b\right) \\
&= f(a)\left(\frac{2b^2-2ab}{2(b-a)} - \frac{b^2-a^2}{2(b-a)}\right) + f(b)\left(\frac{b^2-a^2}{2(b-a)} - \frac{2ab-2a^2}{2(b-a)}\right) = \\
&= f(a)\left(\frac{b^2-2ab+a^2}{2(b-a)}\right) + f(b)\left(\frac{b^2-2ab+a^2}{2(b-a)}\right) \\
&= f(a)\left(\frac{(b-a)^2}{2(b-a)}\right) + f(b)\left(\frac{(b-a)^2}{2(b-a)}\right) \\
&= f(a)\left(\frac{(b-a)}{2}\right) + f(b)\left(\frac{(b-a)}{2}\right) = \left(\frac{(b-a)}{2}\right)(f(a)+f(b))
\end{aligned}$$

2. Integración - Punto 3

$$p_2(x) = \frac{(x-b)(x-x_m)}{(a-b)(a-x_m)}f(a) + \frac{(x-a)(x-b)}{(x_m-a)(x_m-b)}f(x_m) + \frac{(x-a)(x-x_m)}{(b-a)(b-x_m)}f(b)$$

$$I = \int_a^b f(x)dx \approx \int_a^b p_2(x)dx$$

$$= \int_a^b \left(\frac{(x-b)(x-x_m)}{(a-b)(a-x_m)} f(a) + \frac{(x-a)(x-b)}{(x_m-a)(x_m-b)} f(x_m) + \frac{(x-a)(x-x_m)}{(b-a)(b-x_m)} f(b) \right) dx$$

$$= f(a) \int_a^b \left(\frac{(x-b)(x-x_m)}{(a-b)(a-x_m)} \right) dx + f(x_m) \int_a^b \left(\frac{(x-a)(x-b)}{(x_m-a)(x_m-b)} \right) dx + f(b) \int_a^b \left(\frac{(x-a)(x-x_m)}{(b-a)(b-x_m)} \right) dx$$

$$= \frac{f(a)}{(a-b)(a-x_m)} \int_a^b (x^2 - xx_m - bx + bx_m) dx + \frac{f(x_m)}{(x_m-a)(x_m-b)} \int_a^b (x^2 - xb - xa + ab) dx + \frac{f(b)}{(b-a)(b-x_m)} \int_a^b (x^2 - xx_m - ax + ax_m) dx$$

$$= \frac{f(a)}{(a-b)(a-x_m)} \left(\frac{x^3}{3} - \frac{x^2}{2} x_m - \frac{x^2}{2} b + xbx_m\right)\Big|_a^b + \frac{f(x_m)}{(x_m-a)(x_m-b)} \left(\frac{x^3}{3} - \frac{x^2}{2} b - \frac{x^2}{2} a + xab\right)\Big|_a^b + \frac{f(b)}{(b-a)(b-x_m)} \left(\frac{x^3}{3} - \frac{x^2}{2} x_m - \frac{x^2}{2} a + xax_m\right)\Big|_a^b$$

$$= \frac{f(a)}{(a-b)(a-x_m)} \left(\frac{2b^3 - 2a^3}{6} - \frac{3b^2x_m - 3a^2x_m}{6} - \frac{3b^3 - 3a^2b}{6} + \frac{6b^2x_m - 6abx_m}{6}\right) + \frac{f(x_m)}{(x_m-a)(x_m-b)} \left(\frac{2b^3 - 2a^3}{6} - \frac{3b^3 - 3a^2b}{6} - \frac{3b^2a - 3a^3}{6} + \frac{6ab^2 - 6a^2b}{6}\right) + \frac{f(b)}{(b-a)(b-x_m)} \left(\frac{2b^3 - 2a^3}{6} - \frac{3b^2x_m - 3a^2x_m}{6} - \frac{3b^2a - 3a^3}{6} + \frac{6abx_m - 6a^2x_m}{6}\right) =$$

$$\frac{f(a)(b-a)}{(a-b)^2(2a-2x_m)} \left(\frac{2a^3 - 3a^2x_m - 3a^2b + 6abx_m + b^3 - 3b^2x_m}{3}\right)$$

$$= \frac{f(a)(b-a)}{(a-b)^3} \left(\frac{a^3 - 3a^2b + 3ab^2 - b^3}{3}\right) = \frac{f(a)(b-a)}{(a-b)^3} \left(\frac{(a-b)^3}{3}\right)$$

$$= \frac{f(a)(b-a)}{3} = \frac{hf(a)}{3}$$

$$\frac{4f(x_m)}{(2x_m-2a)(2x_m-2b)}\left(\frac{2b^3-2a^3-3b^3+3a^2b-3b^2a+3a^3+6ab^2-6a^2b}{6}\right)$$
$$=\frac{(b-a)4f(x_m)}{(b-a)^3}\left(\frac{b^3-3ab^2+3a^2b-a^3}{3}\right)=\frac{h4f(x_m)}{(b-a)^3}\frac{(b-a)^3}{3}$$
$$=\frac{h4f(x_m)}{3}$$

$$\frac{f(b)(b-a)}{(b-a)^2(2b-2x_m)}\left(\frac{2b^3-3b^2x_m-3b^2a+a^3+6abx_m-3a^2x_m}{3}\right)$$
$$=\frac{f(b)(b-a)}{(b-a)^3}\left(\frac{b^3-3ab^2+3a^2b-a^3}{3}\right)=\frac{f(b)(b-a)}{(b-a)^3}\left(\frac{(b-a)^3}{3}\right)$$
$$=\frac{f(b)(b-a)}{3}=\frac{hf(b)}{3}$$

Entonces

$$I=\frac{hf(a)}{3}+\frac{h4f(x_m)}{3}+\frac{hf(b)}{3}=\frac{h}{3}(f(a)+4f(x_m)+f(b))$$