Punto 4 de Algebra Lineal

$$x_i = b - \sum_{j=0}^{i-1} A_{ij} x_j$$

Supongo $A = M(n \times n)$ tal que $Ax = b$

$$\Rightarrow \begin{bmatrix} A_{11} & 0 & \cdots & 0 \\ A_{21} & A_{22} & \cdots & 0 \\ A_{m1} & A_{m2} & \cdots & A_{nm} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$$

$$\Rightarrow A_{11} \cdot x_1 = b_1$$

$$A_{21} x_1 + A_{22} x_2 = b_2$$

$$\vdots \quad \vdots$$

$$A_{m1} x_1 + A_{m2} x_2 \cdots A_{nm} x_n = b_n$$

$\Rightarrow$ si $m = n$ $A_{ij} = 1$

$$\Rightarrow x_1 = b_1$$

$$A_{21} x_1 + x_2 = b_2$$

$$A_{n1} x_1 + A_{n2} x_2 \cdots x_p = b_n$$

$\Rightarrow$ Despejamos x_n

$$A_{n1} x_1 + A_{n2} x_2 \cdots x_p = b_n$$

$$\sum_{n=0}^{n-1} A_{mn} x_n + x_n = b_n$$

$$x_n = \sum_{n=0}^{n-1} A_{mn} x_n - b_n$$

$\blacksquare$

Punto 5 de Algebra Lineal

$$x_i = b - \sum_{j=i+1}^{n} A_{ij} x_j \qquad i = n, n-1, \ldots, 0$$

Suponga $A = M(n \times m)$, x un vector de n filas y b uno de m filas

$\Rightarrow$ tal que $Ax = b$

$$\begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ 0 & A_{22} & \cdots & A_{2n} \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & A_{nn} \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$$

$$A_{11} x_1 + A_{12} x_2 \cdots A_{1n} x_n = b_1$$

$$A_{22} x_2 \cdots A_{2n} x_n = b_2$$

$(m = n)$

$$A_{nm} x_n = b_n$$

$\Rightarrow$ Despejamos x_n

$$A_{11} x_1 + A_{12} x_2 \cdots A_{1n} x_n = b_1$$

$$\sum_{n=0}^{n-1} A_{mn} x_n + A_{nn} x_n = b_n$$

$$x_n = \frac{\sum_{n=n+1}^{k} A_{mn} x_n - b_n}{A_{nn}}$$

$\Rightarrow$ tal que $n = k, k-1, \ldots, 0$

$\blacksquare$

Punto 6 de Minimos Cuadrados

$$\frac{\partial \chi^2}{\partial a_0} = \sum_i \frac{\partial}{\partial a_0} \left[(y_i - (a_i x_i + a_0))^2 \right]$$

$$\Rightarrow \sum_i (y_i - (a_i x_i + a_0)) = 0$$

$$\Rightarrow \sum_i y_i - \sum_i a_i x_i - \sum_i^{N} a_0 = 0$$

$$\Rightarrow \sum y_i - a_i \sum_i x_i - N a_0 = 0$$

$$\Rightarrow \bar{y} - a_i \bar{x} \cdot a_0 = 0$$

$$a_0 = \bar{y} - a_1 \bar{x}$$

$$\Rightarrow \frac{\partial \chi^2}{\partial a_i} = \sum_i \frac{\partial}{\partial a_i} \left[(y_i - (a_i x_i + a_0))^2 \right] = 0$$

$$\Rightarrow \sum_i 2(y_i - (a_i x_i + a_0))(x_i) = 0$$

$$\Rightarrow \sum_i y_i x_i \, a_i \sum_i x_i^2 + a_0 \sum_i x_i = 0$$

$$\Rightarrow -2 \sum x_i y_i + a_i \sum_i x_i^2 + \left[\frac{1}{N} \sum_i y_i + \frac{a_i}{N} \sum x_i \right] \sum_i x_i = 0$$

$$\Rightarrow a_i \left[\sum_i x_i^2 + \frac{1}{N} \left(\sum_i x_i \right)^2 \right] = 2 \sum_i x_i y_i + \frac{1}{N} \sum_i x_i \sum_i y_i$$

$$\Rightarrow a_i = \frac{2 \sum x_i y_i + \frac{1}{N} \sum_i x_i \sum_i y_i}{\sum_i x_i^2 + \frac{1}{N} \left(\sum x_i \right)^2}$$

$$\Rightarrow \chi^2(a_0, a_1, a_2) = \sum_{i=1}^{N} (y_i - (a_0 + a_1 x_i + a_2 x_i^2))^2$$

$$\Rightarrow \frac{\partial \chi^2}{\partial a_0} = \sum \left[\frac{\partial}{\partial a_0} \left[(y_i - (a_0 + a_1 x_i + a_2 x_i^2))^2 \right] \right]$$

$$\Rightarrow \sum \left[2(y_i - (a_0 + a_1 x_i + a_2 x_i^2)) - 1 \right] = 0$$

$$\Rightarrow \sum_{i=1}^{N} a_0 + a_1 x_1 + a_2 x_i^2 = y_i$$

$$x_i \quad \sum_{i=1}^{N} a_0 x_i + a_1 x_1^2 + a_2 x_i^3 = x_1 y_i$$

$$x_1^2 \quad \sum_{i=1}^{N} a_0 x_i^2 + a_1 x_1^3 + a_2 x_i^4 = x_1^2 y_i$$

$\blacksquare$