Punto 21

a)

$$f(x) = \sum_{n=0}^{N} c_n P_n$$

Multiplicando a ambos lados por $P_n{}'$ se tiene

$$P_n{}'\, f(x) = \sum_{n=0}^{N} c_n P_n P_n{}'$$

Integrando a ambos lados

$$\int_{-1}^{1} P_n{}'\, f(x)dx = \int_{-1}^{1} \sum_{n=0}^{N} c_n P_n P_n{}' dx$$

$$\int_{-1}^{1} \sum_{n=0}^{N} c_n P_n P_n{}' dx \ = c_n \int_{-1}^{1} P_n P_n{}' dx$$

$$\int_{-1}^{1} P_n{}'\, f(x)dx = c_n \int_{-1}^{1} P_n P_n{}' dx$$

Usando la relación de completez

$$\int_{-1}^{1} P_n P_n{}' dx = \frac{2}{2n+1}$$

$$\int_{-1}^{1} P_n{}'\, f(x)dx = c_n \frac{2}{2n+1}$$

Despejando

$$c_n = \frac{2n+1}{2} \int_{-1}^{1} P_n{}'\, f(x)dx$$