Punto 1

Sistema definido por:

$\dot{q} = f(q, p)$

$\dot{p} = g(q, p)$

puntos fijos: $f(q_0, p_0) = 0$

$$\dot{q} = f(q_0, p_0) + \frac{\partial f}{\partial q}(q_0, p_0)(q - q_0) + \frac{\partial F}{\partial p}(q_0, p_0)(p - p_0)$$

$$\dot{p} = g(q_0, p_0) + \frac{\partial g}{\partial q}(q_0, p_0)(q - q_0) + \frac{\partial g}{\partial p}(q_0, p_0)(p - p_0)$$

$\Rightarrow f(q_0, p_0) = 0$

$$\dot{q} = \frac{\partial f}{\partial q}(q_0, p_0)(q - q_0) + \frac{\partial F}{\partial p}(q_0, p_0)(p - p_0)$$

$$\dot{p} = \frac{\partial g}{\partial q}(q_0, p_0)(q - q_0) + \frac{\partial g}{\partial p}(q_0, p_0)(p - p_0)$$

$$\dot{\lambda} = \lambda \frac{\partial f(q_0, p_0)}{\partial q} + \lambda' \frac{\partial f(q_0, p_0)}{\partial p}$$

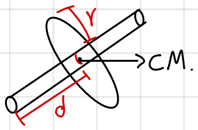
$$\dot{\lambda}' = \lambda \frac{\partial f(q_0, p_0)}{\partial q} + \lambda' \frac{\partial f(q_0, p_0)}{\partial p}$$

$$V = \begin{pmatrix} \lambda \\ \lambda' \end{pmatrix} \quad \frac{\partial V}{\partial t} = \begin{pmatrix} \frac{\partial f(q_0, p_0)}{\partial q} & \frac{\partial f(q_0, p_0)}{\partial p} \\ \frac{\partial g(q_0, p_0)}{\partial q} & \frac{\partial g(q_0, p_0)}{\partial p} \end{pmatrix} = M.$$

$\blacksquare$

Punto 3

a) Muestre que $I_0 = \frac{1}{4}Mr^2 + Md^2$



$I_0 = \frac{1}{4}Mr^2$

Ejes paralelos, CM

$I_{total} = \frac{1}{4}Mr^2 + Md^2$

b) Muestre que $P_0 = \frac{1}{2}Mr^2$

$I = \int r^2 dm$

$I = \frac{M}{\pi R^2}\int r^2 d(\pi r^2)$

$I = \frac{2M}{R^2}\int r^{(2+1)}\, dr$

$I = \frac{2M}{R^2} \cdot \frac{MR^4}{4} = \frac{MR^2}{2}$

c)

Euler - Lagrange $\rightarrow$ $\frac{\partial L}{\partial \phi} = \dot\phi(I_0\sin^2\theta + I_2\cos^2\theta) + I_2\dot\psi\cos\theta = P_\phi$

$\frac{\partial L}{\partial \psi} = I_2(\dot\psi + \dot\phi\cos\theta) = P_\psi$

$I_0\ddot\theta = \dot\psi^2\sin\theta\cos\theta(I_0 - I_2) - \dot\phi\dot\psi I_2\sin\theta + Mgd\sin\theta$

Lagrangiano en Coordenadas.

$L = \frac{1}{2}L_0(\dot\theta^2 + \dot\phi^2\sin^2\theta) + \frac{1}{2}I_2(\dot\phi\cos\theta + \dot\psi)^2 - Mgd\cos\theta$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot\phi}\right) = 0$

$\frac{\partial L}{\partial \phi} = cte.$

$\Rightarrow$

$\frac{\partial L}{\partial \psi} = I_2(\dot\phi\cos\theta + \dot\psi) = P_\psi$

$\frac{\partial L}{\partial \theta} = I_0\dot\phi^2\sin\theta\cos\theta - I_2(\dot\phi\cos\theta + \dot\psi) - \sin\theta + Mgd\sin\theta$

$\Rightarrow \dot\phi^2\sin\theta\cos\theta(I_0 - I_2) - I_2\dot\phi\dot\psi\sin\theta + Mgd\sin\theta$

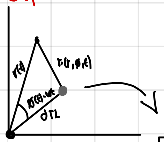
$\Rightarrow$ $\frac{\partial L}{\partial \phi} = \dot\phi(I_0\sin^2\theta + I_2\cos^2\theta) + I_2\dot\psi\cos\theta = P_\phi$

$\frac{\partial L}{\partial \psi} = I_2(\dot\psi + \dot\phi\cos\theta) = P_\psi$

$I_0\ddot\theta = \dot\psi^2\sin\theta\cos\theta(I_0 - I_2) - \dot\phi\dot\psi I_2\sin\theta + Mgd\sin\theta$

Punto 4

a)



$$r_1(r, \emptyset, t) = \sqrt{r(t)^2 + d^2 - 2r(t)d\cos(\emptyset - \omega t)}$$