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using PyPlot

function em_algorithm(y::Vector,  $\theta_0$ ::Vector, nbr_iter::Int)

    # set obs data
    y_A, y_B, y_AB, y_00 = y
    N = sum(y)

    # pre-allocate matrix to store param ests
     $\theta_{\text{matrix}}$  = zeros(length( $\theta_0$ ), nbr_iter)

    # set start values
     $\theta_{\text{matrix}}[:,1] = \theta_0$ 

    # loop over nbr_iter
    for i in 2:nbr_iter

        # set old parameter values
        p_i, q_i, r_i =  $\theta_{\text{matrix}}[:,i-1]$ 

        # E step
        E_AA =  $y_A * p_i^2 / (p_i^2 + 2 * p_i * r_i)$ 
        E_A0 =  $y_A * 2 * p_i * r_i / (p_i^2 + 2 * p_i * r_i)$ 
        E_AB = y_AB
        E_00 = y_00
        E_B0 =  $y_B * 2 * q_i * r_i / (2 * q_i * r_i + q_i^2)$ 
        E_BB =  $y_B * q_i^2 / (2 * q_i * r_i + q_i^2)$ 

        E_A = 2 * E_AA + E_A0 + E_AB
        E_B = 2 * E_BB + E_B0 + E_AB
        E_0 = 2 * E_00 + E_A0 + E_B0

        # M step
        p_hat = E_A / (2 * N)
        q_hat = E_B / (2 * N)
        r_hat = E_0 / (2 * N)

        # store new param ests

         $\theta_{\text{matrix}}[:,i] = [p\_hat; q\_hat; r\_hat]$ 

    end

    return  $\theta_{\text{matrix}}[:,\text{end}]$ ,  $\theta_{\text{matrix}}$ 

end

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```

# run em algorithm
y = [40; 27; 24; 9] # data
θ_0 = [1/3; 1/3; 1/3] # start value for parameters
N = 10 # number iterations of the EM algorithm

θ_hat, θ_matrix = em_alorithm(y, θ_0, N)

round.(θ_hat, digits= 2)

# plot em trace

θ_1 = [1/3; 1/3; 1/3]
θ_2 = [0.2; 0.2; 0.6]
θ_3 = [0.8; 0.1; 0.1]

θ_hat_1, θ_matrix_1 = em_alorithm(y, θ_1, N)
θ_hat_2, θ_matrix_2 = em_alorithm(y, θ_2, N)
θ_hat_3, θ_matrix_3 = em_alorithm(y, θ_3, N)

PyPlot.figure()
PyPlot.scatter3D(θ_matrix_1[1,:], θ_matrix_1[2,:],θ_matrix_1[3:], color="blue", "*")
PyPlot.scatter3D(θ_matrix_2[1,:], θ_matrix_2[2,:],θ_matrix_2[3:], color="red", "*")
PyPlot.scatter3D(θ_matrix_3[1,:], θ_matrix_3[2,:],θ_matrix_3[3:], color="green", "*")
PyPlot.xlabel("p", fontsize=12)
PyPlot.ylabel("q", fontsize=12)
PyPlot.zlabel("r", fontsize=12)
PyPlot.savefig("fig_em_trace.pdf")

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