Tarea 4, clase 5 y 6

Maicol Becerra Ardila -506231720

Juan Sebastián Suárez Gutiérrez- 506231053

Samuel Alejandro Monsalve Sarmiento-506231009

Fundación Universitaria Konrad Lorenz

Facultad de ingeniería y matemáticas

Bogotá D.C

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Tarea 4

clase

Actividad 1

1. $\operatorname{senh}(5\ln(x))$

$$\operatorname{senh}(5\ln(x)) = \frac{e^{5\ln(x)} - e^{-5\ln(x)}}{2}$$

$$\operatorname{senh}(5\ln(x)) = \frac{x^5 - \frac{1}{x^5}}{2}$$

$$\operatorname{senh}(5\ln(x)) = \frac{\frac{(x^5+1)(x^5-1)}{x^5}}{2}$$

$$\operatorname{senh}(5\ln(x)) = \frac{(x^5+1)(x^5-1)}{2x^5}$$

2.

$$\operatorname{senh}(x) = \frac{e^x - e^{-x}}{2} = \frac{3}{4}$$

$$e^x - e^{-x} = \frac{3}{4} \cdot 2$$

$$e^x - e^{-x} = \frac{6}{4}$$

$$e^x - e^{-x} = \frac{3}{2} \qquad e^x = t$$

$$t - t^{-1} - \frac{3}{2} = 0$$

$$(t - 1/t - 3/2) \cdot 2t = 0 \cdot 2t$$

$$2t^2 - 2t - 3t = 0$$

$$(t-2)(2t+1) = 0$$

$t-2=0$ $\quad 2t+1=$

$t=2$ $\quad t=-\frac{1}{2}$

$e^x=2$ $\quad e^x=-\frac{1}{2}$

$\ln(e^x)=\ln(2)$

$x=\ln 2$

$$\cosh(\ln(2))=\frac{e^{\ln(2)}+e^{-\ln(2)}}{2}$$

$$\cosh(\ln(2))=\frac{2+\frac{1}{2}}{2}$$

$$\cosh(\ln(2))=\frac{\frac{5}{2}}{\frac{2}{1}}$$

$$\cosh(\ln(2))=\frac{5}{4}$$

$$\tanh(\ln(2))=\frac{\operatorname{senh}(\ln(2))}{\cosh(\ln(2))}=\frac{\frac{3}{4}}{\frac{5}{4}}=\frac{12}{20}=\frac{\cancel{6}}{\cancel{10}}=\frac{3}{5}$$

$$\coth(\ln(2))=\frac{1}{\tanh(\ln(2))}=\frac{\frac{1}{1}}{\frac{3}{5}}=\frac{5}{3}$$

$$\operatorname{sech}(\ln(2))=\frac{1}{\cosh(\ln(2))}=\frac{\frac{1}{1}}{\frac{5}{4}}=\frac{4}{5}$$

$$\operatorname{csch}(\ln(2))=\frac{1}{\operatorname{senh}(\ln(2))}=\frac{1}{\frac{3}{4}}=\frac{4}{3}$$

3. $\cos h(2\ln(x))$ $\qquad$ $\cos h(\quad (x)) = \dfrac{e^{x}+e^{-x}}{2}$

$$\cos h(2\ln(x) = \frac{e^{2\ln(x)}+e^{-2\ln(x)}}{2}$$

$$\cos h(2\ln(x)) = \frac{x^2+\frac{1}{x^2}}{2}$$

$$\cos h(2\ln(x)) = \frac{\frac{x^4+1}{x^2}}{2}$$

$$\cos h(2\ln(x)) = \frac{x^4+1}{2x^2}$$

4. $\operatorname{sen} h(\ln[x+\sqrt{1+x^2}])$

$$= \frac{e^{(\ln[x+\sqrt{1+x^2}])} - e^{-(\ln[x+\sqrt{1+x^2}])}}{2}$$

$$= \frac{x+\sqrt{1+x^2} - (-x+\sqrt{1+x^2})}{2}$$

$$= \frac{2x}{2}$$

$$\operatorname{sen} h(\ln[x+\sqrt{1+x^2}]) = x$$

$\operatorname{sen} h(x) = \frac{e^x - e^{-x}}{2}$ $\cosh(x) = \frac{e^x + e^{-x}}{2}$

e^x

$$\frac{e^x - e^{-x}}{2} + \frac{e^x + e^{-x}}{2} = e^x$$

$$\frac{e^x - e^{-x} + e^x + e^{-x}}{2} = e^x$$

$$\frac{2e^x}{2} = e^x$$

$$e^x = e^x$$

e^{-x}

$$\frac{e^x - e^{-x}}{2} - \frac{e^x + e^{-x}}{2} = -e^{-x}$$

$$\frac{e^x - e^{-x} - (e^x + e^{-x})}{2} = -e^{-x}$$

$$\frac{e^x - e^{-x} - e^x - e^{-x}}{2}$$

$$\frac{-2e^{-x}}{2} = -e^{-x}$$

$$(-1)\ -e^{-x} = -e^{-x}\ (-1)$$

$$e^{-x} = e^{-x}$$

clase 6

$f(x) = \operatorname{sech}(2x-3)+4$

$\operatorname{sech}^{-1}(x) = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right)$

$y = \operatorname{sech}(2x-3)+4$

$y-4 = \operatorname{sech}(2x-3)$

$\operatorname{sech}(2x-3) = y-4$

$2x-3 = \operatorname{sech}^{-1}(y-4)$

$2x = \operatorname{sech}^{-1}(y-4)+3$

$x = \frac{\operatorname{sech}^{-1}(y-4)+3}{2}$

$x = \frac{\ln\left(\frac{1+\sqrt{1-(y-4)^2}}{(y-4)}\right)+3}{2}$

$x = \frac{\ln\left(\frac{1+\sqrt{(y-3)(-y+5)}}{(y-4)}\right)+3}{2}$

$f^{-1}(x) = \frac{\ln\left(\frac{1+\sqrt{(x-3)(-x+5)}}{(x-4)}\right)+3}{2}$

$\operatorname{senh}^{-1}(x) = \ln(x + \sqrt{x^2+1})$

2. $\operatorname{senh}^{-1}(0) = \ln(0 + \sqrt{0^2+1})$

$\operatorname{senh}^{-1}(0) = \ln(0 + \sqrt{1})$

$\operatorname{senh}^{-1}(0) = \ln(1)$

$\operatorname{senh}^{-1}(0) = 0$

3. $\tanh^{-1}(x) = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right)$

$\tanh^{-1}(0) = \frac{1}{2} \ln\left(\frac{1+0}{1-0}\right)$

$\tanh^{-1}(0) = \frac{1}{2} \ln\left(\frac{1}{1}\right)$

$\tanh^{-1}(0) = \frac{1}{2} \cdot 0$

$\tanh^{-1}(0) = 0$

4. $\cosh^{-1}(x) = \ln(x + \sqrt{x^2-1})$

$\cosh^{-1}(1) = \ln(1 + \sqrt{1-1})$

$\cosh^{-1}(1) = \ln(1)$

$\cosh^{-1}(1) = 0$

5. $2\sinh(x)+\cosh(x)=0$

$$\cancel{2}\left(\frac{e^{x}-e^{-x}}{\cancel{2}}\right)+\frac{e^{x}+e^{-x}}{2}=0$$

$$e^{x}-e^{-x}+\frac{e^{x}+e^{-x}}{2}=0$$

$$\frac{2(e^{x}-e^{-x})+e^{x}+e^{-x}}{2}=0$$

$$\frac{2e^{x}-2e^{-x}+e^{x}+e^{-x}}{2}=0$$

$$\frac{3e^{x}-e^{-x}}{2}=0$$

$$3e^{x}-e^{-x}=0\cdot 2$$

$$3e^{x}-e^{-x}=0$$

$$3e^{x}=e^{-x}$$

$$\ln(3)+x=-x$$

$$\ln(3)=-x-x$$

$$-\frac{\ln(3)}{2}=-2x$$

$$-\frac{\ln(3)}{2}=x$$

6. $\cosh^{-1}(x) = \ln\left(x + \sqrt{x^2 - 1}\right)$ $x \geq 1$

$f: [1, \infty) \to [0, \infty)$

La funcion $\cosh^{-1}(x)$ acepta valores mayores o iguales a 1, dada que si se ingresa menores a 1, en el radical quedaria números negativos, lo cual, da un resultado que no hace parte de los números reales. Ademas, el ln de numeros negativos no es posible.

Ej: $x = -1$

$\ln\left(-1 + \sqrt{(-1)^2 - 1}\right)$

$\ln\left(-1 + \sqrt{1 - 1}\right)$

$\ln(-1 + 0)$

$\ln(-1)$

Math Error

2. coseno hiperbolico inversa

$f: [0, \infty) \to [1, \infty)$ dada por

$f(x) = \cosh(x) = \frac{e^x + e^{-x}}{2}$ es invertible

$$y = \frac{e^x + e^{-x}}{2}$$

$$x = \frac{e^y + e^{-y}}{2}$$

$$2x = e^y + e^{-y}$$

$$(e^y)\ e^y - 2x + e^{-y} = 0\ (e^y)$$

$$e^{2y} - 2xe^y + 1 = 0$$

$a = 1$
$b = -2x$
$c = 1$

$$e^y = \frac{-(-2x) \pm \sqrt{(-2x)^2 - 4(1)(1)}}{2(1)}$$

$$e^y = \frac{2x \pm \sqrt{4x^2 - 4}}{2}$$

$$e^y = \frac{2x \pm 2\sqrt{x^2 - 1}}{2}$$

$$e^y = x \pm \sqrt{x^2 - 1}$$

$$\ln(e^y) = \ln(x \pm \sqrt{x^2 - 1})$$

$$y = \ln(x + \sqrt{x^2 - 1})$$

Primavera

Disney 100

$f^{-1}(x) = \cosh^{-1}(x) = \ln(x + \sqrt{x^2-1})$

$f: [1, \infty) \to [0, \infty)$

$f:(-1,1)\to\mathbb{R}$

4. cotangente hiperbolica inversa

$f:(-\infty,0)\cup(0,\infty)\to(-\infty,-1)\cup(1,\infty)$ dada por

$\coth^{-1}(x)=\dfrac{e^{x}+e^{-x}}{e^{x}-e^{-x}}$ es invertible

$$y=\frac{e^{x}+e^{-x}}{e^{x}-e^{-x}}$$

$$x=\frac{e^{y}+e^{-y}}{e^{y}-e^{-y}}$$

$$x(e^{y}-e^{-y})=e^{y}+e^{-y}$$

$$xe^{y}-xe^{-y}=e^{y}+e^{-y}$$

$$xe^{y}-e^{y}=xe^{-y}+e^{-y}$$

$$e^{y}(x-1)=e^{-y}(x+1)$$

$$e^{y}\cdot e^{y}(x-1)=e^{y}\cdot e^{-y}(x+1)$$

$$e^{2y}(x-1)=(x+1)$$

$$e^{2y}=\frac{x+1}{x-1}$$

$$2y=\ln\left(\frac{x+1}{x-1}\right)$$

$$y=\frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)$$

$$f^{-1}(x)=\coth^{-1}(x)=\frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)$$

$f:(-\infty,-1)\cup(1,\infty)\to(-\infty,0)\cup(0,\infty)$

G cosecante hiperbolica inversa

$f : (-\infty, 0) \cup (0, \infty) \to (-\infty, 0) \cup (0, \infty)$

dada por $f(x) = \operatorname{csch}(x) = \frac{2}{e^x - e^{-x}}$, es invertible

$$y = \frac{2}{e^x - e^{-x}}$$

$$x = \frac{2}{e^y - e^{-y}}$$

$$x(e^y - e^{-y}) = 2$$

$$xe^y - xe^{-y} = 2$$

$$xe^y - 2 - xe^{-y} = 0$$

$$(e^y)(xe^y - 2 - xe^{-y}) = 0$$

$$xe^{2y} - 2e^y - x = 0$$

$a = x$
$b = -2$
$c = -x$

$$e^y = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(x)(-x)}}{2(x)}$$

$$e^y = \frac{2 \pm \sqrt{4 + 4x^2}}{2x}$$

$$e^y = \frac{2 \pm 2\sqrt{1 + x^2}}{2x}$$

$$e^y = \frac{1 \pm \sqrt{1 + x^2}}{x} \to y = \ln\left(\frac{1 + \sqrt{1 + x^2}}{x}\right)$$

$$f^{-1}(x) = \operatorname{csch}^{-1}(x) = \ln\left(\frac{1+\sqrt{1+x^2}}{x}\right)$$

$$f: (-\infty, 0) \cup (0, \infty) \to (-\infty, 0) \cup (0, \infty)$$