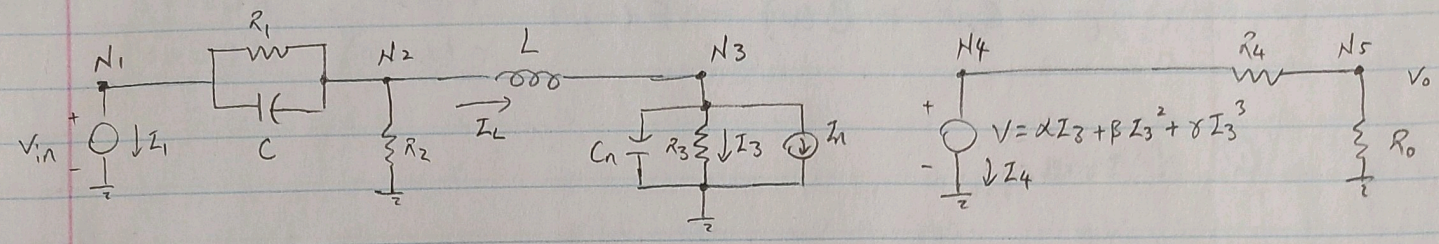
Appendix C Nonlinearity



node N_1 : $\boxed{N_1 = V_{in}}$

$$\boxed{I_1 + \frac{N_1 - N_2}{R_1} + C\frac{\partial(N_1 - N_2)}{\partial t} = 0}$$

node N_2 : $\boxed{\frac{N_2 - N_1}{R_1} + C\frac{\partial(N_2 - N_1)}{\partial t} + \frac{N_2}{R_2} + I_L = 0}$

$$\boxed{N_2 - N_3 = L\frac{\partial I_L}{\partial t}}$$

node N_3 : $\boxed{-I_L + I_3 + I_n + C_n\frac{\partial N_3}{\partial t} = 0}$

$$\boxed{I_3 = \frac{N_3}{R_3}} \qquad \boxed{I_n = I_n}$$

node N_4 : $\boxed{I_4 + \frac{N_4 - N_5}{R_4} = 0}$

$$\boxed{N_4 = \alpha I_3 + \beta I_3^2 + \gamma I_3^3}$$

node N_5 : $\boxed{\frac{N_5 - N_4}{R_4} + \frac{N_5}{R_o} = 0}$

$$\boxed{V_o = N_5}$$

$$C\frac{\partial \vec{V}}{\partial t} + G\vec{V} + \vec{B}(V) = \vec{F}(t)$$

$V(10\times1)$ $F(10\times1)$ $B(10\times1)$

$C(10\times10)$ $G(10\times10)$

① $N_1 = V_{in}$

② $C\frac{\partial N_1}{\partial t} - C\frac{\partial N_2}{\partial t} + \frac{1}{R_1}N_1 - \frac{1}{R_1}N_2 + I_1 = 0$

③ $-C\frac{\partial N_1}{\partial t} + C\frac{\partial N_2}{\partial t} + -\frac{1}{R_1}N_1 + \left(\frac{1}{R_1} + \frac{1}{R_2}\right)N_2 + I_L = 0$

④ $-L\frac{\partial I_L}{\partial t} + N_2 - N_3 = 0$

⑤ $C_n\frac{\partial N_3}{\partial t} - I_L + I_3 + I_n = 0$

⑥ $-\frac{1}{R_3}N_3 + I_3 = 0$

⑦ $\frac{1}{R_4}N_4 - \frac{1}{R_4}N_5 + I_4 = 0$

⑧ $N_4 - \alpha I_3 - \beta I_3^2 - \gamma I_3^3 = 0$

⑨ $-\frac{1}{R_4}N_4 + \left(\frac{1}{R_4} + \frac{1}{R_0}\right)N_5 = 0$

⑩ $I_n = I_n$

$$\vec{V} = \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \\ I_1 \\ I_L \\ I_3 \\ I_4 \\ I_n \end{bmatrix} \quad \vec{F} = \begin{bmatrix} V_{in} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ I_n \end{bmatrix}$$

$$C = \begin{array}{c} \begin{matrix} N_1 & N_2 & N_3 & N_4 & N_5 & I_1 & I_L & I_3 & I_4 & I_n \end{matrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ C & -C & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -C & C & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -L & 0 \\ 0 & 0 & C_n & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$G = \begin{array}{c} \begin{matrix} N_1 & N_2 & N_3 & N_4 & N_5 & I_1 & I_2 & I_3 & I_4 & I_n \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{R_1} & -\frac{1}{R_1} & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -\frac{1}{R_1} & \frac{1}{R_1}+\frac{1}{R_2} & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 1 \\ 0 & 0 & -\frac{1}{R_3} & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{R_4} & -\frac{1}{R_4} & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -\alpha & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{R_4} & \frac{1}{R_4}+\frac{1}{R_0} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$\vec{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\beta I_3^2 - \gamma I_3^3 \\ 0 \\ 0 \end{bmatrix}$$

$$J = \frac{\partial B}{\partial V} = \begin{array}{c} \begin{matrix} N_1 & N_2 & N_3 & N_4 & N_5 & I_1 & I_2 & I_3 & I_4 & I_n \end{matrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2\beta I_3 - 3\gamma I_3^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{matrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \\ B_6 \\ B_7 \\ B_8 \\ B_9 \\ B_{10} \end{matrix}$$

$$C\frac{\partial \vec{v}}{\partial t} + G\vec{v} + \vec{B}(v_n) = \vec{F}(t)$$

$$C\frac{\vec{v}_n - \vec{v}_{n-1}}{\Delta t} + G\vec{v}_n + \vec{B}(v_n) = \vec{F}(t)$$

$$\underbrace{\left(\frac{C}{\Delta t} + G\right)\vec{v}_n - \frac{C}{\Delta t}\vec{v}_{n-1} + \vec{B}(v_n) - \vec{F}(t)}_{f(\vec{v})} = 0$$

$$\because f(\vec{v} + d\vec{v}) \approx f(\vec{v}) + \frac{\partial f}{\partial v}dv = 0$$

$$\therefore dv = -\left[\frac{\partial f}{\partial v}\right]^{-1} f(\vec{v})$$

$$f(\vec{v}) = \left(\frac{C}{\Delta t} + G\right)\vec{v}_n - \frac{C}{\Delta t}\vec{v}_{n-1} + \vec{B}(v_n) - \vec{F}(t)$$

let $H = \frac{\partial f}{\partial v} = \frac{C}{\Delta t} + G + J$, where $J = \frac{\partial B}{\partial v}$

Iterate while $dv > tol$.

$$f(v_n) = \left(\frac{C}{\Delta t} + G\right)\vec{v}_n - \frac{C}{\Delta t}\vec{v}_{n-1} + \vec{B}(v_n) - \vec{F}(t)$$

$$\vec{H} = \frac{C}{\Delta t} + G + J$$

$$d\vec{v} = -H^{-1} f(v_n)$$

$$\vec{v}_n = \vec{v}_n + d\vec{v}$$