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# Bitcoin as a Digital Asset:

## Correlation and Optimal Portfolio Allocation


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# Objectives

The main goals of this work are the following:

1. Study the **correlation** of Bitcoin's returns with those of other standard assets by computing the *sample (Pearson's) correlation*, performing tests on its *significance* and computing the *rolling correlations*.
2. **Calibrate** three different continuous-time **models for the multivariate dynamics of the asset prices** (Merton, Heston and Bates) in order to obtain the levels of the correlation in a more sophisticated frameworks that includes *jumps* and *stochastic volatility*.
3. Study the **optimal allocation** for a portfolio that contains Bitcoin and the **diversification properties** of the inclusion of the digital asset.

# Introduction

Bitcoin was introduced in (Nakamoto, 2009) as a *peer-to-peer* electronic cash system. It started off as a system that was only used by a niche of people in online cryptography forums but year by year it quickly gained notoriety.

Through its distributed blockchain, Bitcoin is able to solve the problem of double spending without the need for a trusted third party. Transaction are quickly validated by the network, making the transfer of wealth as easy as online data sharing.

Bitcoin's monetary policy based on deterministic supply achieves *scarcity* in the digital realm and mimics the progressive scarcity of gold.

# Outline

Dataset

Correlation Analysis

Models Presentation and Calibration

Optimal Portfolio Allocation

# Dataset I

The dataset is composed of 2163 observations of the prices of *16 assets valued daily* (excluding holidays and weekends) from 19/07/2010 till 2/11/2018 (all data provided by Bloomberg).
The assets are grouped into five classes:

1. **Bitcoin** (btc): Value of a single bitcoin, quoted in dollars.
2. Stock indexes:
   - **S&P500** (sp500): American stock market index based on 500 large company with stock listed either on the NYSE or NASDAQ.
   - **EUROSTOXX 50** (eurostoxx): equity index of eurozone stocks, covering 50 stocks from 11 eurozone countries.
   - **MSCI BRIC** (bric): market cap weighted index designed to measure the equity market performance across the emerging country indexes of Brazil, Russia, India and China.
   - **NASDAQ**(nasdaq): market cap weighted index including all NASDAQ tiers: Global Select, Global Market and Capital Market.

# Dataset II

3. Bond indexes:
   - **BBG Pan European** (bond_europe): Bloomberg Barclays Pan-European Aggregate Index that tracks fixed-rate, investment-grade securities issued in different European currencies.
   - **BBG Pan US** (bond_us): BBG US Aggregate Bond Index, a benchmark that measures investment grade, US dollar-denominated, fixed-rate taxable bond market.
   - **BBG Pan EurAgg** (bond_eur): similar to the Pan European but it only considers securities issued in Euros.
4. Currencies:
   - **EUR/USD** (eur): spot value of one Euro in US dollars.
   - **GBP/USD** (gbp): spot value of one British Pound in US dollars.
   - **CHF/USD** (chf): spot value of one Swiss Franc in US dollars.
   - **JPY/USD** (jpy): spot value of one Japanese Yen in US dollars.

# Dataset III

5. Commodities:
   - **Gold** (gold): price of gold measured in USD/Oz.
   - **WTI** (wti): price of crude oil used as benchmark in oil pricing and as the underlying commodity in the NYMEX oil future contracts.
   - **Grain** (grain): S&P GPSCI index that measures the performance of the grain commodity market.
   - **Metals** (metal): S&P GSCI Industrial Metals index that measures the movements of industrial metal prices including aluminium, copper, zinc, nickel and lead.

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# Empirical Correlation

The empirical correlation is computed using Pearson's sample correlation formula on the daily log-returns obtained from the price dataset.

| | btc | bric | sp500 | eurostoxx | nasdaq | bond_europe | bond_us | bond_eur | eur | gbp | chf | jpy | gold | wti | grain | metal |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| btc | 100.0 | 1.4 | 4.4 | 4.1 | 3.6 | 1.4 | -1.8 | 1.9 | 2.3 | 0.7 | 2.5 | -1.1 | -0.2 | 0.8 | 3.5 | 2.7 |
| bric | 1.4 | 100.0 | 48.4 | 57.1 | 47.4 | 19.6 | -15.1 | 19.8 | 20.1 | 24.2 | 8.1 | -16.2 | 12.9 | 30.3 | 15.2 | 43.2 |
| sp500 | 4.4 | 48.4 | 100.0 | 62.0 | 94.9 | 13.3 | -34.1 | 15.0 | 18.4 | 21.1 | 0.1 | -22.2 | -0.6 | 35.1 | 15.2 | 34.9 |
| eurostoxx | 4.1 | 57.1 | 62.0 | 100.0 | 56.2 | 42.0 | -27.8 | 44.7 | 48.6 | 41.9 | 21.2 | -16.5 | 9.3 | 32.7 | 15.2 | 46.5 |
| nasdaq | 3.6 | 47.4 | 94.9 | 56.2 | 100.0 | 10.9 | -31.4 | 12.3 | 15.1 | 18.6 | -1.8 | -21.4 | -1.0 | 29.3 | 14.1 | 32.4 |
| bond_europe | 1.4 | 19.6 | 13.3 | 42.0 | 10.9 | 100.0 | 19.4 | 98.4 | 91.9 | 61.8 | 60.1 | 39.9 | 42.8 | 14.5 | 11.6 | 26.8 |
| bond_us | -1.8 | -15.1 | -34.1 | -27.8 | -31.4 | 19.4 | 100.0 | 14.5 | -0.4 | -5.0 | 14.0 | 38.6 | 21.3 | -21.1 | -6.8 | -17.1 |
| bond_eur | 1.9 | 19.8 | 15.0 | 44.7 | 12.3 | 98.4 | 14.5 | 100.0 | 94.5 | 53.6 | 58.4 | 37.0 | 40.7 | 14.3 | 11.3 | 28.0 |
| eur | 2.3 | 20.1 | 18.4 | 48.6 | 15.1 | 91.9 | -0.4 | 94.5 | 100.0 | 57.1 | 59.4 | 31.2 | 36.7 | 18.2 | 13.5 | 31.0 |
| gbp | 0.7 | 24.2 | 21.1 | 41.9 | 18.6 | 61.8 | -5.0 | 53.6 | 57.1 | 100.0 | 35.5 | 14.2 | 24.7 | 21.9 | 11.9 | 26.1 |
| chf | 2.5 | 8.1 | 0.1 | 21.2 | -1.8 | 60.1 | 14.0 | 58.4 | 59.4 | 35.5 | 100.0 | 36.7 | 37.1 | 6.7 | 7.5 | 20.8 |
| jpy | -1.1 | -16.2 | -22.2 | -16.5 | -21.4 | 39.9 | 38.6 | 37.0 | 31.2 | 14.2 | 36.7 | 100.0 | 39.5 | -6.5 | 2.1 | -3.1 |
| gold | -0.2 | 12.9 | -0.6 | 9.3 | -1.0 | 42.8 | 21.3 | 40.7 | 36.7 | 24.7 | 37.1 | 39.5 | 100.0 | 14.7 | 13.7 | 32.0 |
| wti | 0.8 | 30.3 | 35.1 | 32.7 | 29.3 | 14.5 | -21.1 | 14.3 | 18.2 | 21.9 | 6.7 | -6.5 | 14.7 | 100.0 | 17.8 | 36.0 |
| grain | 3.5 | 15.2 | 15.2 | 15.2 | 14.1 | 11.6 | -6.8 | 11.3 | 13.5 | 11.9 | 7.5 | 2.1 | 13.7 | 17.8 | 100.0 | 20.7 |
| metal | 2.7 | 43.2 | 34.9 | 46.5 | 32.4 | 26.8 | -17.1 | 28.0 | 31.0 | 26.1 | 20.8 | -3.1 | 32.0 | 36.0 | 20.7 | 100.0 |

The results clearly show that:

- Bitcoin has **low correlation** with every asset.
- Assets in the same class usually have a **high correlation** among them.

# Correlation Significance

We perform two tests on the significance of the correlation between Bitcoin and each of the other assets, both of which investigate the following hypotheses:

$$\mathbf{H_0}: \quad \rho = 0 \quad \textit{vs.} \quad \mathbf{H_1}: \quad \rho \neq 0.$$

**Pearson's Test**: by computing *Pearson's t-statistic*, which under the null hypothesis is distributed as a Student's $t$, we can obtain the *p-value* of the test and compare it to the confidence level of 95%.

**Permutation test**: by permuting the sample of a pair of asset and computing Pearson's correlation on the permuted data a large number of times, we can reconstruct an empirical distribution for the possible correlations. Once we obtained this distribution, we can compute the *p-value* of the test and compare it to the chosen confidence level.

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# References I

Nakamoto, S. (2009). Bitcoin: A peer-to-peer electronic cash system.