SamuellH12 - ICPC Library

Conteúdo

1 Data Structures

1.1 BIT

```
struct BIT {
  vector<int> bit;
  int N;

  BIT(){}
  BIT(int n) : N(n+1), bit(n+1){}

  void update(int pos, int val){
    for(; pos < N; pos += pos&(-pos))
      bit[pos] += val;
  }

  int query(int pos){
    int sum = 0;
    for(; pos > 0; pos -= pos&(-pos))
      sum += bit[pos];
    return sum;
  }
};
```

1.2 BIT2D

```
const int MAXN = 1e3 + 5;

struct BIT2D {
  int bit[MAXN][MAXN];

  void update(int X, int Y, int val){
    for(int x = X; x < MAXN; x += x&(-x))
      for(int y = Y; y < MAXN; y += y&(-y))
        bit[x][y] += val;
  }

  int query(int X, int Y){
    int sum = 0;
    for(int x = X; x > 0; x -= x&(-x))
      for(int y = Y; y > 0; y -= y&(-y))
        sum += bit[x][y];
    return sum;
  }

  void updateArea(int xi, int yi, int xf, int yf, int val){
    update(xi,   yi,    val);
    update(xf+1, yi,   -val);
    update(xi,   yf+1, -val);
    update(xf+1, yf+1,  val);
  }

  int queryArea(int xi, int yi, int xf, int yf){
    return query(xf, yf) - query(xf, yi-1) - query(xi-1, yf) +
        query(xi-1, yi-1);
  }
};

/* Complexity: O(Log^2 N)
Bit.update(x, y, v);  //Adiciona +v na posicao {x, y} da BIT
Bit.query(x, y);    //Retorna o somatorio do retangulo de
    inicio {1, 1} e fim {x, y}
Bit.queryArea(xi, yi, xf, yf);     //Retorna o somatorio do
    retangulo de inicio {xi, yi} e fim {xf, yf}
Bit.updateArea(xi, yi, xf, yf, v); //adiciona +v no retangulo
    de inicio {xi, yi} e fim {xf, yf}

IMPORTANTE! UpdateArea NAO atualiza o valor de todas as
    celulas no retangulo!!! Deve ser usado para Color Update
IMPORTANTE! Use query(x, y) Para acessar o valor da posicao (x
    , y) quando estiver usando UpdateArea
IMPORTANTE! Use queryArea(x, y, x, y) Para acessar o valor da
    posicao (x, y) quando estiver usando Update Padrao  */
```

1.3 BIT2DSparse

```
#define upper(v, x) (upper_bound(begin(v), end(v), x) - begin(
    v))

struct BIT2D {
  vector<int> ord;
  vector<vector<int>> bit, coord;

  BIT2D(vector<pii> pts){
    sort(begin(pts), end(pts));

    for(auto [x, y] : pts)
      if(ord.empty() || x != ord.back())
        ord.push_back(x);

    bit.resize(ord.size() + 1);
    coord.resize(ord.size() + 1);

    sort(begin(pts), end(pts), [&](pii &a, pii &b){
      return a.second < b.second;
    });

    for(auto [x, y] : pts)
      for(int i=upper(ord, x); i < bit.size(); i += i&-i)
        if(coord[i].empty() || coord[i].back() != y)
          coord[i].push_back(y);

    for(int i=0; i<bit.size(); i++) bit[i].assign(coord[i].
        size()+1, 0);
  }

  void update(int X, int Y, int v){
    for(int i = upper(ord, X); i<bit.size(); i += i&-i)
      for(int j = upper(coord[i], Y); j < bit[i].size(); j +=
          j&-j)
        bit[i][j] += v;
  }

  int query(int X, int Y){
    int sum = 0;
    for(int i = upper(ord, X); i > 0; i -= i&-i)
      for(int j = upper(coord[i], Y); j > 0; j -= j&-j)
        sum += bit[i][j];
    return sum;
  }

  void updateArea(int xi, int yi, int xf, int yf, int val){
    update(xi,   yi,    val);
    update(xf+1, yi,   -val);
    update(xi,   yf+1, -val);
    update(xf+1, yf+1,  val);
  }

  int queryArea(int xi, int yi, int xf, int yf){
    return query(xf, yf) - query(xf, yi-1) - query(xi-1, yf) +
        query(xi-1, yi-1);
  }
};

Sparse Binary Indexed Tree 2D

Recebe o conjunto de pontos que serao usados para fazer os
    updates e
```

```
as queries e cria uma BIT 2D esparsa que independe do "tamanho
      do grid".

Build: O(N Log N) (N -> Quantidade de Pontos)
Query/Update: O(Log N)

BIT2D(pts);   // pts -> vecotor<pii> com todos os pontos em
     que serao feitas queries ou updates
```

1.4 PrefixSum2D

```
const int MAXN = 1e3 + 5;
int ps [MAXN][MAXN];

void calcPS2d(){
  for (int i = 1; i < MAXN; i++) ps[0][i] += ps[0][i - 1];  //
       inicializo a la linha
  for (int i = 1; i < MAXN; i++) ps[i][0] += ps[i - 1][0];  //
       inicializo a la coluna

  for (int i = 1; i < MAXN; i++)
    for (int j = 1; j < MAXN; j++)
      ps[i][j] += ps[i - 1][j] + ps[i][j - 1] - ps[i - 1][j -
           1];
}
int queryPS2d(int xi, int yi, int xf, int yf){ return ps[xf][
     yf] - ps[xf][yi-1] - ps[xi-1][yf] + ps[xi-1][yi-1]; }

Complexidade:
-> Calcular:  O(N^2)
-> Queries:   O(1)
```

1.5 SegTree

```
const int MAXN = 1e6 + 5;
int seg[4*MAXN];

int query(int no, int l, int r, int a, int b){
  if(b <  l || r <  a) return 0;
  if(a <= l && r <= b) return seg[no];

  int m=(l+r)/2, e=no*2, d=no*2+1;

  return query(e, l, m, a, b) + query(d, m+1, r, a, b);
}

void update(int no, int l, int r, int pos, int v){
  if(pos < l || r < pos) return;
  if(l == r){seg[no] = v; return; }

  int m=(l+r)/2, e=no*2, d=no*2+1;

  update(e, l,   m, pos, v);
  update(d, m+1, r, pos, v);

  seg[no] = seg[e] + seg[d];
}

void build(int no, int l, int r, vector<int> &lista){
  if(l == r){ seg[no] = lista[l]; return; }

  int m=(l+r)/2, e=no*2, d=no*2+1;

  build(e, l,   m, lista);
  build(d, m+1, r, lista);

  seg[no] = seg[e] + seg[d];
}

-> Segment Tree com:
  - Query em Range
  - Update em Ponto

build (1, 1, n, lista);
query (1, 1, n, a, b);
update(1, 1, n, i, x);

|   n    | tamanho
| [a, b] | intervalo da busca
|   i    | posicao a ser modificada
|   x    | novo valor da posicao i
| lista  | vector de elementos originais

Build:  O(N)
Query:  O(log N)
Update: O(log N)
```

1.6 SegTreeIterativa

```
template<typename T> struct SegTree {
  int n;
  vector<T> seg;
  T join(T&l, T&r){ return l + r; }

  void init(vector<T>&base){
    n = base.size();
    seg.resize(2*n);
    for(int i=0; i<n; i++) seg[i+n] = base[i];
    for(int i=n-1; i>0; i--) seg[i] = join(seg[i*2], seg[i
         *2+1]);
  }

  T query(int l, int r){ //[L, R] & [0, n-1]
    T ans = 0; //NEUTRO //if order matters, change to l_ans,
         r_ans
    for(l+=n, r+=n+1; l<r; l/=2, r/=2){
      if(l&1) ans = join(ans, seg[l++]);
      if(r&1) ans = join(seg[--r], ans);
    }
    return ans;
  }

  void update(int i, T v){ // Set Value seg[i+=n] = v //
       change to += v to sum
    for(seg[i+=n] = v; i/=2;) seg[i] = join(seg[i*2], seg[i
         *2+1]);
  }
};
```

1.7 SegTreeLazy

```
const int MAXN = 1e6 + 5;
int seg[4*MAXN];
int lazy[4*MAXN];

void unlazy(int no, int l, int r){
  if(lazy[no] == 0) return;

  int m=(l+r)/2, e=no*2, d=no*2+1;

  seg[no] += (r-l+1) * lazy[no];

  if(l != r){
    lazy[e] += lazy[no];
    lazy[d] += lazy[no];
  }

  lazy[no] = 0;
}

int query(int no, int l, int r, int a, int b){
  unlazy(no, l, r);
  if(b <  l || r <  a) return 0;
  if(a <= l && r <= b) return seg[no];

  int m=(l+r)/2, e=no*2, d=no*2+1;

  return query(e, l, m, a, b) + query(d, m+1, r, a, b);
}

void update(int no, int l, int r, int a, int b, int v){
  unlazy(no, l, r);
  if(b <  l || r <  a) return;
  if(a <= l && r <= b)
  {
    lazy[no]+= v;
    unlazy(no, l, r);
    return;
  }

  int m=(l+r)/2, e=no*2, d=no*2+1;

  update(e, l,   m, a, b, v);
  update(d, m+1, r, a, b, v);

  seg[no] = seg[e] + seg[d];
}

void build(int no, int l, int r, vector<int> &lista){
  if(l == r){ seg[no] = lista[l-1]; return; }

  int m=(l+r)/2, e=no*2, d=no*2+1;

  build(e, l,   m, lista);
  build(d, m+1, r, lista);

  seg[no] = seg[e] + seg[d];
}

-> Segment Tree - Lazy Propagation com:
  - Query em Range
  - Update em Range

build (1, 1, n, lista);
query (1, 1, n, a, b);
update(1, 1, n, a, b, x);

|   n    | o tamanho maximo da lista
| [a, b] | o intervalo da busca ou update
|   x    | o novo valor a ser somada no intervalo [a, b]
| lista  | o array de elementos originais

Build:  O(N)
Query:  O(log N)
Update: O(log N)
Unlazy: O(1)
```

1.8 SegTreeLazyIterativa

```
template<typename T> struct SegTree {
```

```
  int n, h;
  vector<T> seg, lzy;
  vector<int> sz;
  T join(T&l, T&r){ return l + r; }

  void init(int _n){
    n = _n;
    h = 32 - __builtin_clz(n);
    seg.resize(2*n);
    lzy.resize(n);
    sz.resize(2*n, 1);
    for(int i=n-1; i; i--) sz[i] = sz[i*2] + sz[i*2+1];
    // for(int i=0; i<n; i++) seg[i+n] = base[i];
    // for(int i=n-1; i; i--) seg[i] = join(seg[i*2], seg[i
        *2+1]);
  }

  void apply(int p, T v){
    seg[p] += v * sz[p];
    if(p < n) lzy[p] += v;
  }
  void push(int p){
    for(int s=h, i=p>>s; s; s--, i=p>>s)
      if(lzy[i] != 0) {
        apply(i*2,   lzy[i]);
        apply(i*2+1, lzy[i]);
        lzy[i] = 0; //NEUTRO
      }
  }
  void build(int p) {
    for(p/=2; p; p/= 2){
      seg[p] = join(seg[p*2], seg[p*2+1]);
      if(lzy[p] != 0) seg[p] += lzy[p] * sz[p];
    }
  }

  T query(int l, int r){ //[L, R] & [0, n-1]
    l+=n, r+=n+1;
    push(l); push(r-1);

    T ans = 0; //NEUTRO
    for(; l<r; l/=2, r/=2){
      if(l&1) ans = join(seg[l++], ans);
      if(r&1) ans = join(ans, seg[--r]);
    }
    return ans;
  }

  void update(int l, int r, T v){
    l+=n, r+=n+1;
    push(l); push(r-1);

    int l0 = l, r0 = r;
    for(; l<r; l/=2, r/=2){
      if(l&1) apply(l++, v);
      if(r&1) apply(--r, v);
    }
    build(l0); build(r0-1);
  }
};
```

1.9 SegTreePersistente2x

```
const int MAXN = 1e5 + 5;
const int MAXLOG = 31 - __builtin_clz(MAXN) + 1;
typedef int NodeId;
typedef int STp;

const STp NEUTRO = 0;
int IDN, LSEG, RSEG;
extern struct Node NODES[];

struct Node {
  STp val;
  NodeId L, R;
  Node(STp v = NEUTRO) : val(v), L(-1), R(-1) {}
  Node& l(){ return NODES[L]; }
  Node& r(){ return NODES[R]; }
};

Node NODES[4*MAXN + MAXLOG*MAXN]; //!!!CUIDADO COM O TAMANHO (
     aumente se necessario)
pair<Node&, NodeId> newNode(STp v = NEUTRO){ return {NODES[IDN
     ] = Node(v), IDN++}; }

STp join(STp lv, STp rv){ return lv + rv; }

NodeId build(int l, int r, bool root=true){
  if(root) LSEG = l, RSEG = r;
  if(l == r) return newNode().second;

  int m = (l+r)/2;
  auto [node, id] = newNode();

  node.L = build(l,   m, false);
  node.R = build(m+1, r, false);
  node.val = join(node.l().val, node.r().val);

  return id;
}

NodeId update(NodeId node, int l, int r, int pos, int v){
  if( pos < l || r < pos ) return node;
  if(l == r) return newNode(NODES[node].val + v).second;

  int m = (l+r)/2;
  auto [nw, id] =newNode();

  nw.L = update(NODES[node].L, l,   m, pos, v);
  nw.R = update(NODES[node].R, m+1, r, pos, v);

  nw.val = join(nw.l().val, nw.r().val);

  return id;
}
NodeId update(NodeId node, int pos, STp v){ return update(node
     , LSEG, RSEG, pos, v); }

int query(Node& node, int l, int r, int a, int b){
  if(b <  l || r <  a) return NEUTRO;
  if(a <= l && r <= b) return node.val;

  int m = (l+r)/2;

  return join(query(node.l(), l, m, a, b), query(node.r(), m
      +1, r, a, b));
}
int query(NodeId node, int a, int b){ return query(NODES[node
     ], LSEG, RSEG, a, b); }

int kth(Node& Left, Node& Right, int l, int r, int k){
  if(l == r) return l;

  int sum =Right.l().val - Left.l().val;
  int m = (l+r)/2;

  if(sum >= k) return kth(Left.l(), Right.l(), l, m, k);
  return kth(Left.r(), Right.r(), m+1, r, k - sum);
}
int kth(NodeId Left, NodeId Right, int k){ return kth(NODES[
     Left], NODES[Right], LSEG, RSEG, k); }

-> Segment Tree Persistente: (2x mais rapido que com ponteiro)
Build(1, N) -> Cria uma Seg Tree completa de tamanho N;
     RETORNA o NodeId da Raiz
Update(Root, pos, v) -> Soma +V em POS; RETORNA o NodeId da
     nova Raiz;
Query(Root, a, b) -> RETORNA o valor do range [a, b];
Kth(RootL, RootR, K) -> Faz uma Busca Binaria na Seg de
     diferenca entre as duas versoes.
[ Root -> No Raiz da Versao da Seg na qual se quer realizar a
     operacao ]

Build:  O(N)  !!! Sempre chame o Build
Query:  O(log N)
Update: O(log N)
Kth:  O(Log N)
```

1.10 SparseTable

```
const int MAXN = 1e5 + 5;
const int MAXLG = 31 - __builtin_clz(MAXN) + 1;

int value[MAXN], table[MAXLG][MAXN];

void build(int N){
  for(int i=0; i<N; i++) table[0][i] = value[i];

  for(int p=1; p < MAXLG; p++)
    for(int i=0; i + (1 << p) <= N; i++)
      table[p][i] = min(table[p-1][i], table[p-1][i+(1 << (p
          -1))]);
}

int query(int l, int r){
  int p = 31 - __builtin_clz(r - l + 1);  //floor log
  return min(table[p][l], table[p][ r - (1<<p) + 1 ]);
}
Sparse Table for Range Minimum Query [L, R] [0, N)
build:  O(N log N)
Query:  O(1)
Value -> Original Array
```

2 dp

2.1 Digit DP

```
ll dp[2][19][170];

int limite[19];
ll digitDP(int idx, int sum, bool flag){
    if(idx < 0) return sum;
    if(~dp[flag][idx][sum]) return dp[flag][idx][sum];

    dp[flag][idx][sum] = 0;
  int lm = flag ? limite[idx] : 9;

    for(int i=0; i<=lm; i++)
        dp[flag][idx][sum] += digitDP(idx-1, sum+i, (flag && i
            == lm));
```

```
    return dp[flag][idx][sum];
}

ll solve(ll k){
    memset(dp, -1, sizeof dp);

  int sz=0;
  while(k){
    limite[sz++] = k % 10LL;
    k /= 10LL;
  }

  return digitDP(sz-1, 0, true);
}

Digit DP - Sum of Digits

Solve(K)  ->  Retorna a soma dos digitos de todo numero X tal
     que: 0 <= X <= K
dp[D][S][f]  ->  D: Quantidade de digitos; S: Soma dos digitos
     ; f: Flag que indica o limite.
int limite[D] -> Guarda os digitos de K.

Complexity: O(D^2 * B^2)  (B = Base = 10)
```

2.2 LCS

```
const int MAXN = 5*1e3 + 5;
int memo[MAXN][MAXN];

string s, t;

inline int LCS(int i, int j){
  if(i == s.size() || j == t.size()) return 0;
  if(memo[i][j] != -1) return memo[i][j];

  if(s[i] == t[j]) return memo[i][j] = 1 + LCS(i+1, j+1);

  return memo[i][j] = max(LCS(i+1, j), LCS(i, j+1));
}

int LCS_It(){
  for(int i=s.size()-1; i>=0; i--)
    for(int j=t.size()-1; j>=0; j--)
      if(s[i] == t[j])
        memo[i][j] = 1 + memo[i+1][j+1];
      else
        memo[i][j] = max( memo[i+1][j], memo[i][j+1] );

  return memo[0][0];
}

string RecoverLCS(int i, int j){
  if(i == s.size() || j == t.size()) return "";

  if(s[i] == t[j]) return s[i] + RecoverLCS(i+1, j+1);

  if(memo[i+1][j] > memo[i][j+1]) return RecoverLCS(i+1, j);

  return RecoverLCS(i, j+1);
}
LCS - Longest Common Subsequence

Complexity: O(N^2)

* Recursive:
memset(memo, -1, sizeof memo);
LCS(0, 0);

* Iterative:
LCS_It();

* RecoverLCS
  Complexity: O(N)
  Recover one of all the possible LCS
```

2.3 LIS

```
int LIS(vector<int>& nums){
  vector<int> lis;

  for(auto x : nums)
  {
    auto it = lower_bound(lis.begin(), lis.end(), x);

    if(it == lis.end()) lis.push_back(x);
    else *it = x;
  }

  return (int) lis.size();
}
LIS - Longest Increasing Subsequence

Complexity: O(N Log N)
* For ICREASING sequence, use lower_bound()
* For NON DECREASING sequence, use upper_bound()
```

2.4 SOS DP

```
const int N = 20;
ll dp[1<<N], iVal[1<<N];

void sosDP() // O(N * 2^N)
{
    for(int i=0; i<(1<<N); i++)
        dp[i] = iVal[i];

  for(int i=0; i<N; i++)
    for(int mask=0; mask<(1<<N); mask++)
      if(mask&(1<<i))
        dp[mask] += dp[mask^(1<<i)];
}

SOS DP - Sum over Subsets

Dado que cada mask possui um valor inicial (iVal), computa
para cada mask a soma dos valores de todas as suas submasks.

N -> Numero Maximo de Bits
iVal[mask] -> initial Value / Valor Inicial da Mask
dp[mask]   -> Soma de todos os SubSets

Iterar por todas as submasks: for(int sub=mask; sub>0; sub=(
     sub-1)&mask)
```

3 Geometry

3.1 ConvexHull

```
struct PT {
  ll x, y;
  PT(ll x=0, ll y=0) : x(x), y(y) {}

  PT operator- (const PT&a) const{ return PT(x-a.x, y-a.y); }
  ll operator% (const PT&a) const{ return  (x*a.y - y*a.x); }
       //Cross // Vector product

  bool operator==(const PT&a) const{ return x == a.x && y == a
       .y; }
  bool operator< (const PT&a) const{ return x != a.x ? x < a.x
        : y < a.y; }
};

// Colinear? Mude >= 0 para > 0 nos while
vector<PT> ConvexHull(vector<PT> pts, bool sorted=false) {
  if(!sorted) sort(begin(pts), end(pts));
  pts.resize(unique(begin(pts), end(pts)) - begin(pts));
  if(pts.size() <= 1) return pts;

  int s=0, n=pts.size();
  vector<PT> h (2*n+1);

  for(int i=0; i<n; h[s++] = pts[i++])
    while(s > 1 && (pts[i] - h[s-2]) % (h[s-1] - h[s-2]) >= 0
         )
      s--;

  for(int i=n-2, t=s; ~i; h[s++] = pts[i--])
    while(s > t && (pts[i] - h[s-2]) % (h[s-1] - h[s-2]) >= 0
         )
      s--;

  h.resize(s-1);
  return h;
}
// FOR DOUBLE POINT //
See Geometry - General
```

3.2 Geometry - General

```
#define ld long double

// !!! NOT TESTED !!! //

struct PT {
  ll x, y;
  PT(ll x=0, ll y=0) : x(x), y(y) {}

  PT operator+ (const PT&a) const{ return PT(x+a.x, y+a.y); }
  PT operator- (const PT&a) const{ return PT(x-a.x, y-a.y); }
  ll operator* (const PT&a) const{ return  (x*a.x + y*a.y); }
       //DOT product  // norm // lenght^2 // inner
  ll operator% (const PT&a) const{ return  (x*a.y - y*a.x); }
       //Cross // Vector product
  PT operator* (ll c)  const{ return PT(x*c, y*c); }
  PT operator/ (ll c)  const{ return PT(x/c, y/c); }

  bool operator==(const PT&a) const{ return x == a.x && y == a
       .y; }
  bool operator< (const PT&a) const{ return x != a.x ? x < a.x
        : y < a.y; }
  bool operator<<(const PT&a) const{ PT p=*this; return (p%a
       == 0) ? (p*p < a*a) : (p%a < 0);  } //angle(p) < angle(a
       )
};

// FOR DOUBLE POINT //
```

```cpp
const ld EPS = 1e-9;
bool eq(ld a, ld b){ return abs(a-b) < EPS; } // ==
bool lt(ld a, ld b){ return a + EPS < b;    } // <
bool gt(ld a, ld b){ return a > b + EPS;    } // >
bool le(ld a, ld b){ return a < b + EPS;    } // <=
bool ge(ld a, ld b){ return a + EPS > b;    } // >=
bool operator==(const PT&a) const{ return eq(x, a.x) && eq(y,
    a.y); }       // for double point
bool operator< (const PT&a) const{ return eq(x, a.x) ? lt(y, a
    .y) : lt(x, a.x); } // for double point
bool operator<<(PT&a){ PT&p=*this; return eq(p%a, 0) ? lt(p*p,
    a*a) : lt(p%a, 0);  } //angle(this) < angle(a)
//Change LL to LD and uncomment this
//Also, consider replacing comparisons with these functions

ld dist  (PT a, PT b){ return sqrtl((a-b)*(a-b)); }
                          // distance from A to B
ld angle (PT a, PT b){ return acos((a*b) / sqrtl(a*a) / sqrtl(
    b*b)); } //Angle between A and B
PT rotate(PT p, double ang){ return PT(p.x*cos(ang) - p.y*sin(
    ang), p.x*sin(ang) + p.y*cos(ang)); } //Left rotation.
    Angle in radian

ll Area(vector<PT>& p){
  ll area = 0;
  for(int i=2; i < p.size(); i++)
    area += (p[i]-p[0]) % (p[i-1]-p[0]);
  return abs(area) / 2LL;
}

PT intersect(PT a1, PT d1, PT a2, PT d2){
  return a1 + d1 * (((a2 - a1)%d2) / (d1%d2));
}

ld dist_pt_line(PT a, PT l1, PT l2){
  return abs( ((a-l1) % (l2-l1)) / dist(l1, l2)  );
}

ld dist_pt_segm(PT a, PT s1, PT s2){
  if(s1 == s2) return dist(s1, a);

  PT d = s2 - s1;
  ld t = max(0.0L, min(1.0L, ((a-s1)*d) / sqrtl(d*d)) );

  return dist(a, s1+(d*t));
}
```

3.3 LineContainer

```cpp
struct Line {
  mutable ll k, m, p;
  bool operator<(const Line& o) const { return k < o.k; }
  bool operator<(ll x) const { return p < x; }
};

struct LineContainer : multiset<Line, less<>> {
  static const ll inf = LLONG_MAX; // Double: inf = 1/.0, div(
      a,b) = a/b
  ll div(ll a, ll b) { return a / b - ((a ^ b) < 0 && a % b);
      } //floored division

  bool isect(iterator x, iterator y) {
    if(y == end()) return x->p = inf, 0;
    if(x->k == y->k) x->p = x->m > y->m ? inf : -inf;
    else x->p = div(y->m - x->m, x->k - y->k);
    return x->p >= y->p;
  }
  void add_line(ll k, ll m){ // kx + m  //if minimum k*=-1, m
      *=-1, query*-1
    auto z = insert({k, m, 0}), y = z++, x = y;
    while(isect(y, z)) z = erase(z);
    if(x != begin() && isect(--x, y)) isect(x, y = erase(y));
    while((y = x) != begin() && (--x)->p >= y->p) isect(x,
        erase(y));
  }

  ll query(ll x) {
    assert(!empty());
    auto l = *lower_bound(x);
    return l.k * x + l.m;
  }
};
```

4 Grafos

4.1 2-SAT

```cpp
struct TwoSat {
  int N;
  vector<vector<int>> E;

  TwoSat(int N) : N(N), E(2 * N) {}
  inline int eval(int u) const{ return u < 0 ? ((~u)+N)%(2*N)
      : u; }

  void add_or(int u, int v){
    E[eval(~u)].push_back(eval(v));
    E[eval(~v)].push_back(eval(u));
    }
  void add_nand(int u, int v) {
    E[eval(u)].push_back(eval(~v));
    E[eval(v)].push_back(eval(~u));
  }
  void set_true (int u){ E[eval(~u)].push_back(eval(u)); }
  void set_false(int u){ set_true(~u); }
  void add_imply(int u, int v){ E[eval(u)].push_back(eval(v));
      }
  void add_and  (int u, int v){ set_true(u);  set_true(v);
      }
  void add_nor  (int u, int v){ add_and(~u, ~v); }
  void add_xor  (int u, int v){ add_or(u, v); add_nand(u, v);
      }
  void add_xnor (int u, int v){ add_xor(u, ~v); }

  vector<bool> solve() {
    vector<bool> ans(N);
    auto scc = tarjan();

    for (int u = 0; u < N; u++)
      if(scc[u] == scc[u+N]) return {}; //false
      else ans[u] = scc[u+N] > scc[u];

    return ans; //true
  }
private:
  vector<int> tarjan() {
    vector<int> low(2*N), pre(2*N, -1), scc(2*N, -1);
    stack<int> st;
    int clk = 0, ncomps = 0;

    auto dfs = [&](auto&& dfs, int u) -> void {
      pre[u] = low[u] = clk++;
      st.push(u);

      for(auto v : E[u])
        if(pre[v] == -1) dfs(dfs, v), low[u] = min(low[u], low
            [v]);
        else
        if(scc[v] == -1) low[u] = min(low[u], pre[v]);

      if(low[u] == pre[u]){
        int v = -1;
        while(v != u) scc[v = st.top()] = ncomps, st.pop();
        ncomps++;
      }
    };

    for(int u=0; u < 2*N; u++)
      if(pre[u] == -1)
        dfs(dfs, u);

    return scc; //tarjan SCCs order is the reverse of topoSort
        , so (u->v if scc[v] <= scc[u])
  }
};
```

```
  2 SAT - Two Satisfiability Problem

IMPORTANTE! o grafo deve estar 0-indexado!

inverso de u = ~u

Retorna uma valoracao verdadeira se possivel
Ou um vetor vazio se impossivel;
```

4.2 BlockCut Tree

```cpp
const int MAXN = 1e6 + 5;
const int MAXM = 1e6 + 5;//Cuidado

vector<pii> grafo [MAXN];
int pre[MAXN], low[MAXN], clk=0, C=0;

vector<pii> edge;
bool visEdge[MAXM];
int edgeComponent[MAXM];
int vertexComponent[MAXN];

int cut[MAXN];
stack<int> s;

vector<int> tree [2*MAXN];
int componentSize[2*MAXN]; //vertex - cutPoints

void reset(int n){
  for(int i=0; i<=edge.size(); i++)
    visEdge[i] = edgeComponent[i] = 0;

  edge.clear();

  for(int i=0; i<=n; i++){
    pre[i] = low[i] = -1;
    cut[i] = false;
    vertexComponent[i] = 0;
    grafo[i].clear();
  }

  for(int i=0; i<=C; i++){
    componentSize[i] = 0;
    tree[i].clear();
  }
```

```
  while(!s.empty()) s.pop();

  clk = C = 0;
}

void newComponent(int i){
  C++;
  int j;

  do {
    j = s.top(); s.pop();
    edgeComponent[j] = C;

    auto [u, v]  = edge[j];
    if(!cut[u] && !vertexComponent[u]) componentSize[C]++,
        vertexComponent[u] = C;
    if(!cut[v] && !vertexComponent[v]) componentSize[C]++,
        vertexComponent[v] = C;

  } while(!s.empty() && j != i);
}

void tarjan(int u, bool root = true){
  pre[u] = low[u] = clk++;

  bool any = false;
  int chd = 0;

  for(auto [v, i] : grafo[u]){
    if(visEdge[i]) continue;
    visEdge[i] = true;

    s.emplace(i);

    if(pre[v] == -1)
    {
      tarjan(v, false);

      low[u] = min(low[v], low[u]);
      chd++;

      if(!root && low[v] >= pre[u]) cut[u] = true,
          newComponent(i);
      if( root && chd >= 2)       cut[u] = true, newComponent(
          i);
    }
    else
      low[u] = min(low[u], pre[v]);
  }

  if(root) newComponent(-1);
}

//ATENCAO: ESTA 1-INDEXADO
void buildBCC(int n){
  vector<bool> marc(C+1, false);

  for(int u=1; u<=n; u++)
  {
    if(!cut[u]) continue;

    C++;
    cut[u] = C;

    for(auto [v, i] : grafo[u])
    {
      int ec = edgeComponent[i];
      if(!marc[ec])
      {
        marc[ec] = true;
        tree[cut[u]].emplace_back(ec);
        tree[ec].emplace_back(cut[u]);
      }
    }

    for(auto [v, i] : grafo[u])
      marc[edgeComponent[i]] = false;
  }
}

void addEdge(int u, int v){
  int i = edge.size();
  grafo[u].emplace_back(v, i);
  grafo[v].emplace_back(u, i);
  edge.emplace_back(u, v);
}
```

Block Cut Tree - BiConnected Component

```
reset(n);
addEdge(u, v);
tarjan(Root);
buildBCC(n);
```

No fim o grafo da Block Cut Tree estara em _vector<int> tree
[]_

4.3 CentroidDecomposition

```
const int MAXN = 1e6 + 5;

vector<int> grafo[MAXN];
deque<int> distToAncestor[MAXN];

bool rem[MAXN];
int szt[MAXN], parent[MAXN];

void getDist(int u, int p, int d=0){
  for(auto v : grafo[u])
    if(v != p && !rem[v])
      getDist(v, u, d+1);
  distToAncestor[u].emplace_front(d);
}

int getSz(int u, int p){
  szt[u] = 1;
  for(auto v : grafo[u])
    if(v != p && !rem[v])
      szt[u] += getSz(v, u);
  return szt[u];
}

void dfsc(int u=0, int p=-1, int f=-1, int sz=-1){
  if(sz < 0) sz = getSz(u, -1); //starting new tree

  for(auto v : grafo[u])
    if(v != p && !rem[v] && szt[v]*2 >= sz)
      return dfsc(v, u, f, sz);

  rem[u] = true, parent[u] = f;
  getDist(u, -1, 0); //get subtree dists to centroid

  for(auto v : grafo[u])
    if(!rem[v])
      dfsc(v, u, u, -1);
}
```

Centroid Decomposition

dfsc() -> para criar a centroid tree

rem[u]   -> True se U ja foi removido (pra dfsc)
szt[u]   -> Size da subarvore de U (pra dfsc)
parent[u] -> Pai de U na centroid tree *parent[ROOT] = -1
distToAncestor[u][i] -> Distancia na arvore original de u para seu i-esimo pai na centroid tree *distToAncestor[u][0] = 0

dfsc(u=node, p=parent(subtree), f=parent(centroid tree), sz=size of tree)

4.4 Dijkstra

```
const int MAXN = 1e6 + 5;
#define INF 0x3f3f3f3f
#define vi vector<int>

vector<pii> grafo [MAXN];

vi dijkstra(int s){
  vi dist (MAXN, INF); // !!! Change MAXN to N

  priority_queue<pii, vector<pii>, greater<pii>> fila;
  fila.push({0, s});
  dist[s] = 0;

  while(!fila.empty())
  {
    auto [d, u] = fila.top();
    fila.pop();

    if(d > dist[u]) continue;

    for(auto [v, c] : grafo[u])
      if( dist[v] > dist[u] + c )
      {
        dist[v] = dist[u] + c;
        fila.push({dist[v], v});
      }
  }

  return dist;
}
```

Dijkstra - Shortest Paths from Source

caminho minimo de um vertice u para todos os outros vertices de um grafo ponderado

Complexity: O(N Log N)

dijkstra(s)   -> s : Source, Origem. As distancias serao calculadas com base no vertice s
grafo[u] = {v, c};   -> u : Vertice inicial, v : Vertice final, c : Custo da aresta
priority_queue<pii, vector<pii>, greater<pii>> -> Ordena pelo menor custo -> {d, v} -> d : Distancia, v : Vertice

4.5 Dinic

```
struct Aresta {
  int u, v; ll cap;
  Aresta(int u, int v, ll cap) : u(u), v(v), cap(cap) {}
};
```

```
struct Dinic {

  int n, source, sink;
  vector<vector<int>> adj;
  vector<Aresta> arestas;
  vector<int> level, ptr; //pointer para a proxima aresta nao
       saturada de cada vertice

  Dinic(int n, int source, int sink) : n(n), source(source),
       sink(sink) { adj.resize(n); }

  void addAresta(int u, int v, ll cap)
  {
    adj[u].push_back(arestas.size());
    arestas.emplace_back(u, v, cap);

    adj[v].push_back(arestas.size());
    arestas.emplace_back(v, u, 0);
  }

  ll dfs(int u, ll flow = 1e9){
    if(flow == 0) return 0;
    if(u == sink) return flow;

    for(int &i = ptr[u];  i < adj[u].size();  i++)
    {
      int atual = adj[u][i];
      int v = arestas[atual].v;

      if(level[u] + 1 != level[v]) continue;

      if(ll got = dfs(v, min(flow, arestas[atual].cap)) )
      {
        arestas[atual].cap -= got;
        arestas[atual^1].cap += got;
        return got;
      }
    }

    return 0;
  }

  bool bfs(){
    level = vector<int> (n, n);
    level[source] = 0;

    queue<int> fila;
    fila.push(source);

    while(!fila.empty())
    {
      int u = fila.front();
      fila.pop();

      for(auto i : adj[u]){
        int v = arestas[i].v;

        if(arestas[i].cap == 0 || level[v] <= level[u] + 1 )
             continue;

        level[v] = level[u] + 1;
        fila.push(v);
      }
    }

    return level[sink] < n;
  }

  bool inCut(int u){ return level[u] < n; }

  ll maxFlow(){
    ll ans = 0;

    while( bfs() ){
      ptr = vector<int> (n+1, 0);

      while(ll got = dfs(source)) ans += got;
    }

    return ans;
  }
};
    Dinic - Max Flow Min Cut
Algoritmo de Dinitz para encontrar o Fluxo Maximo
IMPORTANTE! O algoritmo esta 0-indexado

Complexity:
O( V^2 * E )      ->  caso geral
O( sqrt(V) * E )  ->  grafos com cap = 1 para toda aresta //
     matching bipartido

* Informacoes:
  Crie o Dinic:
    Dinic dinic(n, source, sink);
  Adicione as Arestas:
    dinic.addAresta(u, v, capacity);
  Para calcular o Fluxo Maximo:
    dinic.maxFlow()
  Para saber se um vertice U esta no Corte Minimo:
    dinic.inCut(u)

* Sobre o Codigo:
  vector<Aresta> arestas;  -> Guarda todas as arestas do grafo
       e do grafo residual
  vector<vector<int>> adj; -> Guarda em adj[u] os indices de
       todas as arestas saindo de u
  vector<int> ptr;   ->  Pointer para a proxima aresta ainda
       nao visitada de cada vertice
  vector<int> level; -> Distancia em vertices a partir do
       Source. Se igual a N o vertice nao foi visitado.
  A BFS retorna se Sink e alcancavel de Source. Se nao e
       porque foi atingido o Fluxo Maximo
  A DFS retorna um possivel aumento do Fluxo
* Use Cases of Flow

+ Minimum cut: the minimum cut is equal to maximum flow.
  i.e. to split the graph in two parts, one on the source side
       and another on sink side.
  The capacity of each edge is it weight.

+ Edge-disjoint paths: maximum number of edge-disjoint paths
     equals maximum flow of the
  graph, assuming that the capacity of each edge is one. (
       paths can be found greedily)

+ Node-disjoint paths: can be reduced to maximum flow. each
     node should appear in at most one
  path, so limit the flow through a node dividing each node in
       two. One with incoming edges,
  other with outgoing edges and a new edge from the first to
       the second with capacity 1.

+ Maximum matching (bipartite): maximum matching is equal to
     maximum flow. Add a source and
  a sink, edges from the source to every node at one partition
       and from each node of the
  other partition to the sink.

+ Minimum node cover (bipartite): minimum set of nodes such
     each edge has at least one
  endpoint. The size of minimum node cover is equal to maximum
       matching (Konig's theorem).

+ Maximum independent set (bipartite): largest set of nodes
     such that no two nodes are
  connected with an edge. Contain the nodes that aren't in "
       Min node cover" (N - MAXFLOW).

+ Minimum path cover (DAG): set of paths such that each node
     belongs to at least one path.
  - Node-disjoint: construc a matching where each node is
       represented by two nodes, a left and
    a right at the matching and add the edges (from l to r).
         Each edge in the matching
    corresponds to an edge in the path cover. The number of
         paths in the cover is (N - MAXFLOW).
  - General: almost like a minimum node-disjoint. Just add
       edges to the matching whenever there
    is an path from U to V in the graph (possibly through
         several edges).
  - Antichain: a set of nodes such that there is no path from
       any node to another. In a DAG, the
    size of min general path cover equals the size of maximum
         antichain (Dilworth's theorem).
```

4.6 DSU Persistente

```
struct DSUp {
  vector<int> pai, sz, tim;
  int t=1;
  DSUp(int n) : pai(n+1), sz(n+1, 1), tim(n+1) {
    for(int i=0; i<=n; i++) pai[i] = i;
  }

  int find(int u, int q = INT_MAX){
    if( pai[u] == u || q < tim[u] ) return u;
    return find(pai[u], q);
  }

  void join(int u, int v){
    u = find(u), v = find(v);

    if(u == v) return;
    if(sz[v] > sz[u]) swap(u, v);

    pai[v] = u;
    tim[v] = t++;
    sz[u] += sz[v];
  }
};
SemiPersistent Disjoint Set Union - O(Log n)
find(u, q) -> Retorna o pai de U no tempo q
* tim -> tempo em que o pai de U foi alterado
```

4.7 DSU Rollback

```
struct DSUr {
  vector<int> pai, sz, savept;
  stack<pair<int&, int>>  st;
  DSUr(int n) : pai(n+1), sz(n+1, 1) {
    for(int i=0; i<=n; i++) pai[i] = i;
  }

  int find(int u){ return pai[u] == u ? u : find(pai[u]); }

  void join(int u, int v){
```

```
    u = find(u), v = find(v);

    if(u == v) return;
    if(sz[v] > sz[u]) swap(u, v);

    save(pai[v]); pai[v] = u;
    save(sz[u]);  sz[u] += sz[v];
  }

  void save(int &x){ st.emplace(x, x); }
  void pop(){
    st.top().first = st.top().second; st.pop();
    st.top().first = st.top().second; st.pop();
  }

  void checkpoint(){ savept.push_back(st.size()); }
  void rollback(){
    while(st.size() > savept.back()) pop();
    savept.pop_back();
  }
};
Disjoint Set Union with Rollback - O(Log n)
checkpoint() -> salva o estado atual
rollback() -> restaura no ultimo checkpoint
save another var? +save in join & +line in pop
```

4.8 DSU

```
struct DSU {
  vector<int> pai, sz;
  DSU(int n) : pai(n+1), sz(n+1, 1) {
    for(int i=0; i<=n; i++) pai[i] = i;
  }

  int find(int u){ return pai[u] == u ? u : pai[u] = find(pai[
      u]); }

  void join(int u, int v){
    u = find(u), v = find(v);

    if(u == v) return;
    if(sz[v] > sz[u]) swap(u, v);

    pai[v] = u;
    sz[u] += sz[v];
  }
};
Disjoint Set Union - Union Find
Find: O( a(n) ) -> Inverse Ackermann function
Join: O( a(n) ) -> a(1e6) <= 5
```

4.9 Euler Path

```
#define vi vector<int>

const int MAXN = 1e6 + 5;
const bool BIDIRECIONAL = true;

vector<pii> grafo[MAXN];
vector<bool> used;

void addEdge(int u, int v){
  grafo[u].emplace_back(v, used.size()); if(BIDIRECIONAL && u
      != v)
  grafo[v].emplace_back(u, used.size());
  used.emplace_back(false);
}

pair<vi, vi> EulerPath(int n, int src=0){
  int s=-1, t=-1;
  vector<int> selfLoop(n*BIDIRECIONAL, 0);

  if(BIDIRECIONAL)
  {
    for(int u=0; u<n; u++) for(auto&[v, id] : grafo[u]) if(u==
        v) selfLoop[u]++;
    for(int u=0; u<n; u++)
      if((grafo[u].size() - selfLoop[u])%2)
        if(t != -1) return {vi(), vi()};   // mais que 2 com
            grau impar
        else t = s, s = u;

    if(t == -1 && t != s) return {vi(), vi()}; // so 1 com
        grau impar
    if(s == -1 || t == src) s = src;           // se possivel,
         seta start como src
  }
  else
  {
    vector<int> in(n, 0), out(n, 0);

    for(int u=0; u<n; u++)
      for(auto [v, edg] : grafo[u])
        in[v]++, out[u]++;

    for(int u=0; u<n; u++)
      if(in[u] - out[u] == -1 && s == -1) s = u;  else
      if(in[u] - out[u] ==  1 && t == -1) t = u;  else
      if(in[u] !=out[u]) return {vi(), vi()};

    if(s == -1 && t == -1) s = t = src;          // se possivel
        , seta s como src
    if(s == -1 && t != -1) return {vi(), vi()}; // Existe S
        mas nao T
    if(s != -1 && t == -1) return {vi(), vi()}; // Existe T
        mas nao S
  }

  for(int i=0; grafo[s].empty() && i<n; i++) s =(s+1)%n; //
      evita s ser vertice isolado

  /////// DFS ///////
  vector<int> path, pathId, idx(n, 0);
  stack<pii> st; // {Vertex, EdgeId}
  st.push({s, -1});

  while(!st.empty())
  {
    auto [u, edg] = st.top();
    while(idx[u] < grafo[u].size() && used[grafo[u][idx[u]].
        second]) idx[u]++;

    if(idx[u] < grafo[u].size())
    {
      auto [v, id] = grafo[u][idx[u]];
      used[id] = true;
      st.push({v, id});
      continue;
    }

    path.push_back(u);
    pathId.push_back(edg);
    st.pop();
  }

  pathId.pop_back();
  reverse(begin(path),   end(path));
  reverse(begin(pathId), end(pathId));

  ///  Grafo conexo ? ///
  int edgesTotal = 0;
  for(int u=0; u<n; u++) edgesTotal += grafo[u].size() + (
      BIDIRECIONAL ? selfLoop[u] : 0);
  if(BIDIRECIONAL) edgesTotal /= 2;
  if(pathId.size() != edgesTotal) return {vi(), vi()};
  //////////////////////

  return {path, pathId};
}
Euler Path - Algoritmo de Hierholzer para caminho Euleriano

Complexity: O(V + E)

IMPORTANTE! O algoritmo esta 0-indexado

* Informacoes
  addEdge(u, v) -> Adiciona uma aresta de U para V
  EulerPath(n) -> Retorna o Euler Path, ou um vetor vazio se
      impossivel
  vi path -> vertices do Euler Path na ordem
  vi pathId -> id das Arestas do Euler Path na ordem

Euler em Undirected graph:
  - Cada vertice tem um numero par de arestas (circuito); OU
  - Exatamente dois vertices tem um numero impar de arestas (
      caminho);
Euler em Directed graph:
  - Cada vertice tem quantidade de arestas |entrada| == |saida
      | (circuito); OU
  - Exatamente 1 tem |entrada|+1 == |saida| && exatamente 1
      tem |entrada| == |saida|+1 (caminho);
* Circuito -> U e o primeiro e ultimo
* Caminho -> U e o primeiro e V o ultimo
```

4.10 HLD

```
const bool EDGE = false;
struct HLD {
public:
  vector<vector<int>> g; //grafo
  vector<int> sz, parent, tin, nxt;
  HLD(){}
  HLD(int n){ init(n); }
  void init(int n){
    t = 0;
    g.resize(n); tin.resize(n);
    sz.resize(n);nxt.resize(n);
    parent.resize(n);
  }
  void addEdge(int u, int v){
    g[u].emplace_back(v);
    g[v].emplace_back(u);
  }
  void build(int root=0){
    nxt[root]=root;
    dfs(root, root);
    hld(root, root);
  }

  ll query_path(int u, int v){
    if(tin[u] < tin[v]) swap(u, v);
    if(nxt[u] == nxt[v]) return qry(tin[v]+EDGE, tin[u]);
    return qry(tin[nxt[u]], tin[u]) + query_path(parent[nxt[u
        ]], v);
```

```
  }

  void update_path(int u, int v, ll x){
    if(tin[u] < tin[v]) swap(u, v);
    if(nxt[u] == nxt[v]) return updt(tin[v]+EDGE, tin[u], x);
    updt(tin[nxt[u]], tin[u], x); update_path(parent[nxt[u]],
        v, x);
  }

private:
  ll qry(int l, int r){ if(EDGE && l>r) return 0;/*NEUTRO*/ }
      //call Seg, BIT, etc
  void updt(int l, int r, ll x){ if(EDGE && l>r) return; }
      //call Seg, BIT, etc

  void dfs(int u, int p){
    sz[u] = 1, parent[u] = p;
    for(auto &v : g[u]) if(v != p) {
      dfs(v, u); sz[u] += sz[v];

      if(sz[v] > sz[g[u][0]] || g[u][0] == p)
        swap(v, g[u][0]);
    }
  }

  int t=0;
  void hld(int u, int p){
    tin[u] = t++;
    for(auto &v : g[u]) if(v != p)
      nxt[v] = (v == g[u][0] ? nxt[u] : v),
      hld(v, u);
  }

  /// OPTIONAL ///
  int lca(int u, int v){
    while(!inSubtree(nxt[u], v)) u = parent[nxt[u]];
    while(!inSubtree(nxt[v], u)) v = parent[nxt[v]];
    return tin[u] < tin[v] ? u : v;
  }
  bool inSubtree(int u, int v){ return tin[u] <= tin[v] && tin
      [v] < tin[u] + sz[u]; }
  //query/update_subtree[tin[u]+EDGE, tin[u]+sz[u]-1];
};

Heavy-Light Decomposition

Complexity: #Query_path: O(LogN*qry) #Update_path: O(LogN*updt
    )
Nodes: 0 <= u, v < N

Change qry(l, r) and updt(l, r) to call a query and update
structure of your will

HLD hld(n); //call init
hld.add_edges(u, v); //add all edges
hld.build(); //Build everthing for HLD

tin[u] -> Pos in the structure (Seg, Bit, ...)
nxt[u] -> Head/Endpoint
```

4.11 LCA

```
const int MAXN = 1e4 + 5;
const int MAXLG = 16;

vector<int> grafo[MAXN];

int bl[MAXLG][MAXN], lvl[MAXN];

void dfs(int u, int p, int l=0){
  lvl[u] = l;
  bl[0][u] = p;

  for(auto v : grafo[u])
    if(v != p)
      dfs(v, u, l+1);
}

void buildBL(int N){
  for(int i=1; i<MAXLG; i++)
    for(int u=0; u<N; u++)
      bl[i][u] = bl[i-1][bl[i-1][u]];
}

int lca(int u, int v){
  if(lvl[u] < lvl[v]) swap(u, v);

  for(int i=MAXLG-1; i>=0; i--)
    if(lvl[u] - (1<<i) >= lvl[v])
      u = bl[i][u];

  if(u == v) return u;

  for(int i=MAXLG-1; i>=0; i--)
    if(bl[i][u] != bl[i][v])
      u = bl[i][u],
      v = bl[i][v];

  return bl[0][u];
}
  LCA - Lowest Common Ancestor - Binary Lifting
Algoritmo para encontrar o menor ancestral comum
entre dois vertices em uma arvore enraizada

IMPORTANTE! O algoritmo esta 0-indexado

Complexity:
buildBL() ->  O(N Log N)
lca()     ->  O(Log N)

* Informacoes
  -> Monte o grafo na lista de adjacencias
  -> chame dfs(root, root) para calcular o pai e a altura de
      cada vertice
  -> chame buildBL() para criar a matriz do Binary Lifting
  -> chame lca(u, v) para encontrar o menor ancestral comum
  bl[i][u] -> Binary Lifting com o (2^i)-esimo pai de u
  lvl[u]   -> Altura ou level de U na arvore

* Em LCA o primeiro FOR iguala a altura de U e V
* E o segundo anda ate o primeiro vertice de U que nao e
    ancestral de V
* A resposta e o pai desse vertice
```

4.12 MinCost MaxFlow

```
struct Aresta {
  int u, v; ll cap, cost;
  Aresta(int u, int v, ll cap, ll cost) : u(u), v(v), cap(cap)
      , cost(cost) {}
};

struct MCMF {
  const ll INF = numeric_limits<ll>::max();
  int n, source, sink;
  vector<vector<int>> adj;
  vector<Aresta> edges;
  vector<ll> dist, pot;
  vector<int> from;

  MCMF(int n, int source, int sink) : n(n), source(source),
      sink(sink) { adj.resize(n); pot.resize(n); }

  void addAresta(int u, int v, ll cap, ll cost){
    adj[u].push_back(edges.size());
    edges.emplace_back(u, v, cap, cost);

    adj[v].push_back(edges.size());
    edges.emplace_back(v, u, 0, -cost);
  }

  queue<int> q;
  vector<bool> vis;
  bool SPFA(){
    dist.assign(n, INF);
    from.assign(n, -1);
    vis.assign(n, false);

    q.push(source);
    dist[source] = 0;

    while(!q.empty()){
      int u = q.front();
      q.pop();

      vis[u] = false;

      for(auto i : adj[u]){
        if(edges[i].cap == 0) continue;
        int v = edges[i].v;
        ll cost = edges[i].cost;

        if(dist[v] > dist[u] + cost + pot[u] - pot[v]){
          dist[v] = dist[u] + cost + pot[u] - pot[v];
          from[v] = i;
          if(!vis[v]) q.push(v), vis[v] = true;
        }
      }
    }

    for(int u=0; u<n; u++)  //fix pot
      if(dist[u] < INF)
        pot[u] += dist[u];

    return dist[sink] < INF;
  }

  pair<ll, ll> augment(){
    ll flow = edges[from[sink]].cap, cost = 0; //fixed flow:
        flow = min(flow, remainder)

    for(int v=sink; v != source; v = edges[from[v]].u)
      flow = min(flow, edges[from[v]].cap),
      cost += edges[from[v]].cost;

    for(int v=sink; v != source; v = edges[from[v]].u)
      edges[from[v]].cap   -= flow,
      edges[from[v]^1].cap += flow;

    return {flow, cost};
  }

  bool inCut(int u){ return dist[u] < INF; }

  pair<ll, ll> maxFlow(){
    ll flow = 0, cost = 0;
```

```
    while( SPFA() ){
      auto [f, c] = augment();
      flow += f;
      cost += f*c;
    }
    return {flow, cost};
  }
};
```

4.13 SCC - Kosaraju

```
#define vi vector<int>

const int MAXN = 1e6 + 5;

vi grafo[MAXN];
vi greve[MAXN];
vi dag[MAXN];
vi comp, order;
vector<bool> vis;
int C;

void dfs(int u){
  vis[u] = true;
  for(auto v : grafo[u])
    if(!vis[v])
      dfs(v);
  order.push_back(u);
}

void dfs2(int u){
  comp[u] = C;
  for(auto v : greve[u])
    if(comp[v] == -1)
      dfs2(v);
}

void kosaraju(int n){
  order.clear();
  comp.assign(n, -1);
  vis.assign(n, false);

  for(int v=0; v<n; v++)
    if(!vis[v])
      dfs(v);

  C = 0;
  reverse(begin(order), end(order));

  for(auto v : order)
    if(comp[v] == -1)
      dfs2(v), C++;

  //// Montar DAG  ////
  vector<bool> marc(C, false);

  for(int u=0; u<n; u++){
    for(auto v : grafo[u])
    {
      if(comp[v] == comp[u] || marc[comp[v]]) continue;

      marc[comp[v]] = true;
      dag[comp[u]].emplace_back(comp[v]);
    }

    for(auto v : grafo[u]) marc[comp[v]] = false;
  }
}
Kosaraju - Strongly Connected Component
Algoritmo de Kosaraju para encontrar Componentes Fortemente
     Conexas

Complexity: O(V + E)
IMPORTANTE! O algoritmo esta 0-indexado

*** Variaveis e explicacoes ***
int C   -> C e a quantidade de Componetes Conexas. As
     componetes estao numeradas de 0 a C-1
dag     -> Apos rodar o Kosaraju, o grafo das componentes
     conexas sera criado aqui
comp[u] -> Diz a qual componente conexa U faz parte
order   -> Ordem de saida dos vertices. Necessario para o
     Kosaraju
grafo   -> grafo direcionado
greve -> grafo reverso (que deve ser construido junto ao grafo
      normal) !!!

NOTA: A ordem que o Kosaraju descobre as componentes e uma
     Ordenacao Topologica do SCC
em que o dag[0] nao possui grau de entrada e o dag[C-1] nao
     possui grau de saida
```

4.14 Tarjan

```
const int MAXN = 1e6 + 5;
int pre[MAXN], low[MAXN], clk=0;
vector<int> grafo [MAXN];

vector<pair<int, int>> pontes;
vector<int> cut;

// lembrar do memset(pre, -1, sizeof pre);
void tarjan(int u, int p = -1){
  pre[u] = low[u] = clk++;

  bool any = false;
  int chd = 0;

  for(auto v : grafo[u]){
    if(v == p) continue;

    if(pre[v] == -1)
    {
      tarjan(v, u);

      low[u] = min(low[v], low[u]);

      if(low[v] >  pre[u]) pontes.emplace_back(u, v);
      if(low[v] >= pre[u]) any = true;

      chd++;
    }
    else
      low[u] = min(low[u], pre[v]);
  }

  if(p == -1 && chd >= 2) cut.push_back(u);
  if(p != -1 && any)      cut.push_back(u);
}

  Tarjan - Pontes e Pontos de Articulacao
Algoritmo para encontrar pontes e pontos de articulacao.

Complexity: O(V + E)
IMPORTANTE! Lembre do memset(pre, -1, sizeof pre);

*** Variaveis e explicacoes ***
pre[u] = "Altura", ou, x-esimo elemento visitado na DFS. Usado
      para saber a posicao de um vertice na arvore de DFS
low[u] = Low Link de U, ou a menor aresta de retorno (mais
     proxima da raiz) que U alcanca entre seus filhos

chd = Children. Quantidade de componentes filhos de U. Usado
     para saber se a Raiz e Ponto de Articulacao.
any = Marca se alguma aresta de retorno em qualquer dos
     componentes filhos de U nao ultrapassa U. Se isso for
     verdade, U e Ponto de Articulacao.

if(low[v] >  pre[u]) pontes.emplace_back(u, v); -> se a mais
     alta aresta de retorno de V (ou o menor low) estiver
     abaixo de U, entao U-V e ponte
if(low[v] >= pre[u]) any = true;        -> se a mais alta
     aresta de retorno de V (ou o menor low) estiver abaixo de
     U ou igual a U, entao U e Ponto de Articulacao
```

5 Math

5.1 fexp

```
ll MOD = 1e9 + 7;

ll fexp(ll b, ll p){
  ll ans = 1;

  while(p){
    if(p&1) ans = (ans*b) % MOD;
    b = b * b % MOD;
    p >>= 1;
  }

  return ans % MOD;
}
// O(Log P) // b - Base // p - Potencia
```

6 others

6.1 Hungarian

```
typedef int TP;

const int MAXN = 1e3 + 5;
const TP INF = 0x3f3f3f3f;

TP matrix[MAXN][MAXN];
TP row[MAXN], col[MAXN];
int match[MAXN], way[MAXN];

TP hungarian(int n, int m){
  memset(row, 0, sizeof row);
  memset(col, 0, sizeof col);
  memset(match, 0, sizeof match);

  for(int i=1; i<=n; i++)
  {
    match[0] = i;
    int j0 = 0, j1, i0;
    TP delta;
```

```
    vector<TP> minv (m+1, INF);
    vector<bool> used (m+1, false);

    do {
      used[j0] = true;
      i0 = match[j0];
      j1 = -1;
      delta = INF;

      for(int j=1; j<=m; j++)
        if(!used[j]){
          TP cur = matrix[i0][j] - row[i0] - col[j];

          if( cur < minv[j] ) minv[j] = cur, way[j] = j0;
          if(minv[j] < delta) delta = minv[j], j1 = j;
        }

      for(int j=0; j<=m; j++)
        if(used[j]){
          row[match[j]] += delta,
          col[j] -= delta;
        }else
          minv[j] -= delta;

      j0 = j1;
    } while(match[j0]);


    do {
      j1 = way[j0];
      match[j0] = match[j1];
      j0 = j1;
    } while(j0);
  }

  return -col[0];
}

vector<pair<int, int>> getAssignment(int m){
  vector<pair<int, int>> ans;

  for(int i=1; i<=m; i++)
    ans.push_back(make_pair(match[i], i));

  return ans;
}

  Hungarian Algorithm - Assignment Problem
Algoritmo para o problema de atribuicao minima.

Complexity: O(N^2 * M)

hungarian(int n, int m);  ->  Retorna o valor do custo minimo
getAssignment(int m)      ->  Retorna a lista de pares <linha,
      Coluna> do Minimum Assignment

n -> Numero de Linhas // m -> Numero de Colunas

IMPORTANTE! O algoritmo e 1-indexado
IMPORTANTE! O tipo padrao esta como int, para mudar para outro
      tipo altere | typedef <TIPO> TP; |
Extra: Para o problema da atribuicao maxima, apenas
     multiplique os elementos da matriz por -1
```

6.2 MO

```
const int BLOCK_SZ = 700;

struct Query{
  int l, r, idx;

  Query(int l, int r, int idx) : l(l), r(r), idx(idx) {}

  bool operator < (Query q) const {
    if(l / BLOCK_SZ != q.l / BLOCK_SZ) return l < q.l;
    return (l / BLOCK_SZ &1) ? ( r < q.r ) : (r > q.r );
  }
};

void add(int idx);
void remove(int idx);
int getAnswer();

vector<int> MO(vector<Query> &queries){
  vector<int> ans(queries.size());

  sort(queries.begin(), queries.end());

  int L = 0, R = 0;
  add(0);

  for(auto [l, r, idx] : queries){
    while(l < L) add(--L);
    while(r > R) add(++R);
    while(l > L) remove(L++);
    while(r < R) remove(R--);

    ans[idx] = getAnswer();
  }

  return ans;
}

Algoritmo de MO para query em range

Complexity: O( (N + Q) * SQRT(N) * F ) | F e a complexidade do
      Add e Remove

IMPORTANTE! Queries devem ter seus indices (Idx) 0-indexados!

Modifique as operacoes de Add, Remove e GetAnswer de acordo
     com o problema.
BLOCK_SZ pode ser alterado para aproximadamente SQRT(MAX_N)

IF you want to use hilbert curves on MO
vector<ll> h(ans.size());
for (int i = 0; i < ans.size(); i++) h[i] = hilbert(queries[i
     ].l, queries[i].r);
sort(queries.begin(), queries.end(), [&](Query&a, Query&b) {
     return h[a.idx] < h[b.idx]; });

inline ll hilbert(int x, int y) {
  static int N = 1 << (__builtin_clz(0) - __builtin_clz(MAXN))
      ;
  int rx, ry, s; ll d = 0;
  for (s = N/2; s > 0; s /= 2) {
    rx = (x & s) > 0, ry = (y & s) > 0;
    d += s * (ll)(s) * ((3 * rx) ^ ry);
    if (ry == 0) { if (rx == 1) x = N-1 - x, y = N-1 - y; swap
        (x, y); }
  }
  return d;
}
```

6.3 MOTree

```
const int MAXN = 1e5+5;
const int BLOCK_SZ = 500;
struct Query{int l, r, idx;}; //same of MO. Copy operator <

vector<int> g[MAXN];
int tin[MAXN], tout[MAXN];
int pai[MAXN], order[MAXN];

void remove(int u);
void add(int u);
int getAnswer();

void go_to(int ti, int tp, int otp){
  int u = order[ti], v, to;
  to = tout[u];
  while(!(ti <= tp && tp <= to)){ //subo com U (ti) ate ser
        ancestral de W
    v = pai[u];

    if(ti <= otp && otp <= to) add(v);
    else remove(u);

    u = v;
    ti = tin[u];
    to = tout[u];
  }

  int w = order[tp];
  to = tout[w];
  while(ti < tp){ //subo com W (tp) ate U
    v = pai[w];

    if(tp <= otp && otp <= to) remove(v);
    else add(w);

    w = v;
    tp = tin[w];
    to = tout[w];
  }
}

int TIME = 0;
void dfs(int u, int p){
  pai[u] = p;
  tin[u] = TIME++;
  order[tin[u]] = u;

  for(auto v : g[u])
    if(v != p)
      dfs(v, u);
  tout[u] = TIME-1;
}

vector<int> MO(vector<Query> &queries){
  vector<int> ans(queries.size());
  dfs(0, 0);

  for(auto &[u, v, i] : queries)
    tie(u, v) = minmax(tin[u], tin[v]);
  sort(queries.begin(), queries.end());

  add(0);
  int Lm = 0, Rm = 0;
  for(auto [l, r, idx] : queries){
    if(l < Lm) go_to(Lm, l, Rm), Lm = l;
    if(r > Rm) go_to(Rm, r, Lm), Rm = r;
    if(l > Lm) go_to(Lm, l, Rm), Lm = l;
    if(r < Rm) go_to(Rm, r, Lm), Rm = r;
```

```
    ans[idx] = getAnswer();
  }

  return ans;
}
Algoritmo de MO para query de caminho em arvore
Complexity: O((N + Q) * SQRT(N) * F) | F e a complexidade do
     Add e Remove
IMPORTANTE! 0-indexado!
```

7 Strings

7.1 hash

```
const int MAXN = 1e6 + 5;

const ll MOD = 1e9 + 7; //WA? Muda o MOD e a base
const ll base = 153;

ll expb[MAXN];

void precalc(){
  expb[0] = 1;
  for(int i=1; i<MAXN; i++)
    expb[i] = (expb[i-1]*base)%MOD;
}

struct StringHash{
  vector<ll> hsh;

  StringHash(string &s){
    hsh.assign(s.size()+1, 0);
    for(int i=0; i<s.size(); i++)
      hsh[i+1] = (hsh[i] * base % MOD + s[i]) % MOD;
  }

  ll gethash(int l, int r){
    return (MOD + hsh[r+1] - hsh[l]*expb[r-l+1] % MOD ) % MOD;
  }
};
String Hash
precalc()     -> O(N)
StringHash() -> O(|S|)
gethash()     -> O(1)

StringHash hash(s); -> Cria uma struct de StringHash para a
     string s
hash.gethash(l, r); -> Retorna o hash do intervalo L R da
     string (0-Indexado)

IMPORTANTE! Chamar precalc() no inicio do codigo

const ll MOD  = 131'807'699; -> Big Prime Number
const ll base = 127;         -> Random number larger than the
     Alphabet
```

7.2 hash2

```
const int MAXN = 1e6 + 5;

const ll MOD1 = 131'807'699;
const ll MOD2 = 1e9 + 9;
const ll base = 157;

ll expb1[MAXN], expb2[MAXN];

#warning "Call precalc() before use StringHash"
void precalc(){
    expb1[0] = expb2[0] = 1;

  for(int i=1;i<MAXN;i++)
        expb1[i] = expb1[i-1]*base % MOD1,
        expb2[i] = expb2[i-1]*base % MOD2;
}

struct StringHash{
    vector<pair<ll,ll>> hsh;
    string s; // comment S if you dont need it

    StringHash(string& s) : s(s){
        hsh.assign(s.size()+1, {0,0});

        for (int i=0;i<s.size();i++)
            hsh[i+1].first  = ( hsh[i].first *base % MOD1 + s[
                 i] ) % MOD1,
            hsh[i+1].second = ( hsh[i].second*base % MOD2 + s[
                 i] ) % MOD2;
    }

    ll gethash(int a,int b){
        ll h1 = (MOD1+ hsh[b+1].first  - hsh[a].first *expb1[b
             -a+1] % MOD1) % MOD1;
        ll h2 = (MOD2+ hsh[b+1].second - hsh[a].second*expb2[b
             -a+1] % MOD2) % MOD2;
        return (h1<<32) | h2;
    }
};

int firstDiff(StringHash& a, int la, int ra, StringHash& b,
     int lb, int rb)
{
  int l=0, r=min(ra-la, rb-lb), diff=r+1;
  while(l <= r){
    int m = (l+r)/2;
    if(a.gethash(la, la+m) == b.gethash(lb, lb+m)) l = m+1;
    else r = m-1, diff = m;
  }
  return diff;
}

int hshComp(StringHash& a, int la, int ra, StringHash& b, int
     lb, int rb){
  int diff = firstDiff(a, la, ra, b, lb, rb);
  if(diff > ra-la && ra-la == rb-lb) return 0; //equal
  if(diff > ra-la || diff  >  rb-lb) return ra-la < rb-lb ? -2
       : +2; //prefix of the other
  return a.s[la+diff] < b.s[lb+diff] ? -1 : +1;
}

String Hash - Double Hash
precalc()     -> O(N)
StringHash() -> O(|S|)
gethash()     -> O(1)

StringHash hash(s); -> Cria o Hash da string s
hash.gethash(l, r); -> Hash [L,R] (0-Indexado)
```

7.3 KMP

```
vector<int> pi(string &t){
  vector<int> p(t.size(), 0);

  for(int i=1, j=0; i<t.size(); i++)
  {
    while(j > 0  && t[j] != t[i]) j = p[j-1];

    if(t[j] == t[i]) j++;

    p[i] = j;
  }

  return p;
}

vector<int> kmp(string &s, string &t){
  vector<int> p = pi(t), occ;

  for(int i=0, j=0; i<s.size(); i++)
  {
    while( j > 0 && s[i] != t[j]) j = p[j-1];

    if(s[i]==t[j]) j++;

    if(j == t.size()) occ.push_back(i-j+1), j = p[j-1];
  }

  return occ;
}
KMP - Knuth-Morris-Pratt Pattern Searching

Complexity: O(|S|+|T|)

S -> String
T -> Pattern
```

7.4 Manacher

```
vector<int> manacher(string &st){
  string s = "$_";
  for(char c : st){ s += c; s += "_"; }
  s += "#";

  int n = s.size()-2;

  vector<int> p(n+2, 0);
  int l=1, r=1;

  for(int i=1, j; i<=n; i++)
  {
    p[i] = max(0, min(r-i, p[l+r-i]) ); //atualizo o valor
         atual para o valor do palindromo espelho na string ou
         para o total que esta contido

    while( s[i-p[i]] == s[i+p[i]] ) p[i]++;

    if( i+p[i] > r ) l = i-p[i], r = i+p[i];
  }

  for(auto &x : p) x--; //o valor de p[i] e igual ao tamanho
       do palindromo + 1

  return p;
}

Manacher Algorithm
Find every palindrome in string
Complexidade: O(N)
```

7.5 trie

```
const int MAXS = 1e5 + 10;
const int sigma = 26;

int trie[MAXS][sigma], terminal[MAXS], z = 1;

void insert(string &p){
  int cur = 0;

  for(int i=0; i<p.size(); i++){
    int id = p[i] - 'a';

    if(trie[cur][id] == -1 ){
      memset(trie[z], -1, sizeof trie[z]);
      trie[cur][id] = z++;
    }

    cur = trie[cur][id];
  }

  terminal[cur]++;
}

int count(string &p){
  int cur = 0;

  for(int i=0; i<p.size(); i++){
    int id = (p[i] - 'a');

    if(trie[cur][id] == -1) return 0;

    cur = trie[cur][id];
  }
  return terminal[cur];
}

void init(){
  memset(trie[0], -1, sizeof trie[0]);
  z = 1;
}
Trie - Arvore de Prefixos
insert(P) - O(|P|)
count(P)  - O(|P|)
MAXS  - Soma do tamanho de todas as Strings
sigma - Tamanho do alfabeto
```

7.6 Z-Function

```
vector<int> Zfunction(string &s){ // O(N)
  int n = s.size();
  vector<int> z (n, 0);

  for(int i=1, l=0, r=0; i<n;  i++){
    if(i <= r) z[i] = min(z[i-l], r-i+1);

    while(z[i] + i < n && s[z[i]] == s[i+z[i]]) z[i]++;

    if(r < i+z[i]-1) l = i, r = i+z[i]-1;
  }

  return z;
}
```