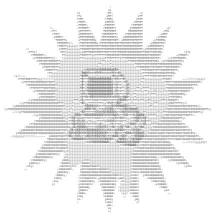
SamuellH12 - ICPC Library

Conteúdo



1 Data Structures

1.1 BIT

```cpp
struct BIT {
  vector<int> bit;
  int N;

  BIT(){}
  BIT(int n) : N(n+1), bit(n+1){}

  void update(int pos, int val){
    for(; pos < N; pos += pos&(-pos))
      bit[pos] += val;
  }

  int query(int pos){
    int sum = 0;
    for(; pos > 0; pos -= pos&(-pos))
      sum += bit[pos];
    return sum;
  }
};
```

1.2 BIT2D

```cpp
const int MAXN = 1e3 + 5;

struct BIT2D {
  int bit[MAXN][MAXN];

  void update(int X, int Y, int val){
    for(int x = X; x < MAXN; x += x&(-x))
      for(int y = Y; y < MAXN; y += y&(-y))
        bit[x][y] += val;
  }

  int query(int X, int Y){
    int sum = 0;
    for(int x = X; x > 0; x -= x&(-x))
      for(int y = Y; y > 0; y -= y&(-y))
        sum += bit[x][y];
    return sum;
  }

  void updateArea(int xi, int yi, int xf, int yf, int val){
    update(xi,   yi,    val);
    update(xf+1, yi,   -val);
    update(xi,   yf+1, -val);
    update(xf+1, yf+1,  val);
  }

  int queryArea(int xi, int yi, int xf, int yf){
    return query(xf, yf) - query(xf, yi-1) - query(xi-1, yf) +
        query(xi-1, yi-1);
  }
};

/* Complexity: O(Log^2 N)
Bit.update(x, y, v);  //Adiciona +v na posicao {x, y} da BIT
Bit.query(x, y);    //Retorna o somatorio do retangulo de
    inicio {1, 1} e fim {x, y}
Bit.queryArea(xi, yi, xf, yf);     //Retorna o somatorio do
    retangulo de inicio {xi, yi} e fim {xf, yf}
Bit.updateArea(xi, yi, xf, yf, v); //adiciona +v no retangulo
    de inicio {xi, yi} e fim {xf, yf}
IMPORTANTE! UpdateArea NAO atualiza o valor de todas as
    celulas no retangulo!!! Deve ser usado para Color Update
IMPORTANTE! Use query(x, y) Para acessar o valor da posicao (x
    , y) quando estiver usando UpdateArea
IMPORTANTE! Use queryArea(x, y, x, y) Para acessar o valor da
    posicao (x, y) quando estiver usando Update Padrao  */
```

1.3 BIT2D Sparse

```
Sparse Binary Indexed Tree 2D

Recebe o conjunto de pontos que serao usados para fazer os updates e
as queries e cria uma BIT 2D esparsa que independe do "tamanho do
    grid".

Build: O(N Log N)  (N -> Quantidade de Pontos)
Query/Update: O(Log N)

BIT2D(pts);   // pts -> vecotor<pii> com todos os pontos em que serao
    feitas queries ou updates
```

```cpp
#define upper(v, x) (upper_bound(begin(v), end(v), x) - begin(
    v))

struct BIT2D {
  vector<int> ord;
  vector<vector<int>> bit, coord;

  BIT2D(vector<pii> pts){
    sort(begin(pts), end(pts));

    for(auto [x, y] : pts)
      if(ord.empty() || x != ord.back())
        ord.push_back(x);

    bit.resize(ord.size() + 1);
    coord.resize(ord.size() + 1);

    sort(begin(pts), end(pts), [&](pii &a, pii &b){
      return a.second < b.second;
    });

    for(auto [x, y] : pts)
      for(int i=upper(ord, x); i < bit.size(); i += i&-i)
        if(coord[i].empty() || coord[i].back() != y)
          coord[i].push_back(y);

    for(int i=0; i<bit.size(); i++) bit[i].assign(coord[i].
        size()+1, 0);
  }

  void update(int X, int Y, int v){
    for(int i = upper(ord, X); i<bit.size(); i += i&-i)
      for(int j = upper(coord[i], Y); j < bit[i].size(); j +=
          j&-j)
        bit[i][j] += v;
  }

  int query(int X, int Y){
    int sum = 0;
    for(int i = upper(ord, X); i > 0; i -= i&-i)
      for(int j = upper(coord[i], Y); j > 0; j -= j&-j)
        sum += bit[i][j];
    return sum;
  }

  void updateArea(int xi, int yi, int xf, int yf, int val){
```

```
    update(xi,   yi,    val);
    update(xf+1, yi,   -val);
    update(xi,   yf+1, -val);
    update(xf+1, yf+1,  val);
  }

  int queryArea(int xi, int yi, int xf, int yf){
    return query(xf, yf) - query(xf, yi-1) - query(xi-1, yf) +
          query(xi-1, yi-1);
  }
};
```

1.4 Prefix Sum 2D

```
Complexidade:
-> Calcular:  O(N^2)
-> Queries:   O(1)
```

```
const int MAXN = 1e3 + 5;
int ps [MAXN][MAXN];

void calcPS2d(){
  for (int i = 1; i < MAXN; i++) ps[0][i] += ps[0][i - 1];  //
       inicializo a 1a linha
  for (int i = 1; i < MAXN; i++) ps[i][0] += ps[i - 1][0];  //
       inicializo a 1a coluna

  for (int i = 1; i < MAXN; i++)
    for (int j = 1; j < MAXN; j++)
      ps[i][j] += ps[i - 1][j] + ps[i][j - 1] - ps[i - 1][j -
           1];
}
int queryPS2d(int xi, int yi, int xf, int yf){ return ps[xf][
     yf] - ps[xf][yi-1] - ps[xi-1][yf] + ps[xi-1][yi-1]; }
```

1.5 SegTree

```
-> Segment Tree com:
  - Query em Range
  - Update em Ponto

build (1, 1, n, lista);
query (1, 1, n, a, b);
update(1, 1, n, i, x);

|   n    | tamanho
| [a, b] | intervalo da busca
|   i    | posicao a ser modificada
|   x    | novo valor da posicao i
| lista  | vector de elementos originais

Build:  O(N)
Query:  O(log N)
Update: O(log N)
```

```
const int MAXN = 1e6 + 5;
int seg[4*MAXN];

int query(int no, int l, int r, int a, int b){
  if(b <  l || r <  a) return 0;
  if(a <= l && r <= b) return seg[no];

  int m=(l+r)/2, e=no*2, d=no*2+1;

  return query(e, l, m, a, b) + query(d, m+1, r, a, b);
}

void update(int no, int l, int r, int pos, int v){
  if(pos < l || r < pos) return;
  if(l == r){seg[no] = v; return; }

  int m=(l+r)/2, e=no*2, d=no*2+1;

  update(e, l,   m, pos, v);
  update(d, m+1, r, pos, v);

  seg[no] = seg[e] + seg[d];
}

void build(int no, int l, int r, vector<int> &lista){
  if(l == r){ seg[no] = lista[l]; return; }

  int m=(l+r)/2, e=no*2, d=no*2+1;

  build(e, l,   m, lista);
  build(d, m+1, r, lista);

  seg[no] = seg[e] + seg[d];
}
```

1.6 SegTree Lazy

```
-> Segment Tree - Lazy Propagation com:
  - Query em Range
  - Update em Range

build (1, 1, n, lista);
query (1, 1, n, a, b);
update(1, 1, n, a, b, x);

|   n    | o tamanho maximo da lista
| [a, b] | o intervalo da busca ou update
|   x    | o novo valor a ser somada no intervalo [a, b]
| lista  | o array de elementos originais

Build:  O(N)
Query:  O(log N)
Update: O(log N)
Unlazy: O(1)
```

```
const int MAXN = 1e6 + 5;
int seg[4*MAXN];
int lazy[4*MAXN];

void unlazy(int no, int l, int r){
  if(lazy[no] == 0) return;

  int m=(l+r)/2, e=no*2, d=no*2+1;

  seg[no] += (r-l+1) * lazy[no];

  if(l != r){
    lazy[e] += lazy[no];
    lazy[d] += lazy[no];
  }

  lazy[no] = 0;
}

int query(int no, int l, int r, int a, int b){
  unlazy(no, l, r);
  if(b <  l || r <  a) return 0;
  if(a <= l && r <= b) return seg[no];

  int m=(l+r)/2, e=no*2, d=no*2+1;

  return query(e, l, m, a, b) + query(d, m+1, r, a, b);
}

void update(int no, int l, int r, int a, int b, int v){
  unlazy(no, l, r);
  if(b <  l || r <  a) return;
  if(a <= l && r <= b)
  {
    lazy[no]+= v;
    unlazy(no, l, r);
    return;
  }

  int m=(l+r)/2, e=no*2, d=no*2+1;

  update(e, l,   m, a, b, v);
  update(d, m+1, r, a, b, v);

  seg[no] = seg[e] + seg[d];
}

void build(int no, int l, int r, vector<int> &lista){
  if(l == r){ seg[no] = lista[l-1]; return; }

  int m=(l+r)/2, e=no*2, d=no*2+1;

  build(e, l,   m, lista);
  build(d, m+1, r, lista);

  seg[no] = seg[e] + seg[d];
}
```

1.7 SegTree Iterativa

```
template<typename T> struct SegTree {
  int n;
  vector<T> seg;
  T join(T&l, T&r){ return l + r; }

  void init(vector<T>&base){
    n = base.size();
    seg.resize(2*n);
    for(int i=0; i<n; i++) seg[i+n] = base[i];
    for(int i=n-1; i>0; i--) seg[i] = join(seg[i*2], seg[i
         *2+1]);
  }

  T query(int l, int r){ //[L, R] & [0, n-1]
    T ans = 0; //NEUTRO //if order matters, change to l_ans,
         r_ans
    for(l+=n, r+=n+1; l<r; l/=2, r/=2){
      if(l&1) ans = join(ans, seg[l++]);
      if(r&1) ans = join(seg[--r], ans);
    }
    return ans;
  }
```

```
  void update(int i, T v){ // Set Value seg[i+=n] = v //
      change to += v to sum
    for(seg[i+=n] = v; i/=2;) seg[i] = join(seg[i*2], seg[i
        *2+1]);
  }
};
```

1.8 SegTree Lazy Iterativa

```
template<typename T> struct SegTree {
  int n, h;
  vector<T> seg, lzy;
  vector<int> sz;
  T join(T&l, T&r){ return l + r; }

  void init(int _n){
    n = _n;
    h = 32 - __builtin_clz(n);
    seg.resize(2*n);
    lzy.resize(n);
    sz.resize(2*n, 1);
    for(int i=n-1; i; i--) sz[i] = sz[i*2] + sz[i*2+1];
    // for(int i=0; i<n; i++) seg[i+n] = base[i];
    // for(int i=n-1; i; i--) seg[i] = join(seg[i*2], seg[i
        *2+1]);
  }

  void apply(int p, T v){
    seg[p] += v * sz[p];
    if(p < n) lzy[p] += v;
  }
  void push(int p){
    for(int s=h, i=p>>s; s; s--, i=p>>s)
      if(lzy[i] != 0) {
        apply(i*2,   lzy[i]);
        apply(i*2+1, lzy[i]);
        lzy[i] = 0; //NEUTRO
      }
  }
  void build(int p) {
    for(p/=2; p; p/= 2){
      seg[p] = join(seg[p*2], seg[p*2+1]);
      if(lzy[p] != 0) seg[p] += lzy[p] * sz[p];
    }
  }

  T query(int l, int r){ //[L, R] & [0, n-1]
    l+=n, r+=n+1;
    push(l); push(r-1);

    T ans = 0; //NEUTRO
    for(; l<r; l/=2, r/=2){
      if(l&1) ans = join(seg[l++], ans);
      if(r&1) ans = join(ans, seg[--r]);
    }
    return ans;
  }

  void update(int l, int r, T v){
    l+=n, r+=n+1;
    push(l); push(r-1);

    int l0 = l, r0 = r;
    for(; l<r; l/=2, r/=2){
      if(l&1) apply(l++, v);
      if(r&1) apply(--r, v);
    }
    build(l0); build(r0-1);
  }
};
```

1.9 SegTree Persistente

```
-> Segment Tree Persistente
Build(1, N) -> Cria uma Seg Tree completa de tamanho N; RETORNA um
      *Ponteiro pra Raiz
Update(Root, 1, N, pos, v)  -> Soma +V na posicao POS; RETORNA um
      *Ponteiro pra Raiz da nova versao;
Query(Root, 1, N, a, b)   -> RETORNA o valor calculado no range [a,
      b];
Kth(RootL, RootR, 1, N, K)  -> Faz uma Busca Binaria na Seg; Mais
      detalhes abaixo;
[ Root -> No Raiz da Versao da Seg na qual se quer realizar a
      operacao ]
Para guardar as Raizes, use: vector<Node*> roots

Build:  O(N) !!! Sempre chame o Build
Query:  O(log N)
Update: O(log N)
Kth:  O(Log N)

Comportamento do K-th(SegL, SegR, 1, N, K):
  -> Retorna indice da primeira posicao i cuja soma de prefixos [1,
      i] e >= k
  na Seg resultante da subtracao dos valores da (Seg R) - (Seg L).
  -> Pode ser utilizada para consultar o K-esimo menor valor no
      intervalo [L, R] de um array.
  Para isso a Seg deve ser utilizada como um array de frequencias.
      Comece com a Seg zerada (Build).
  Para cada valor V do Array chame um update(roots.back(), 1, N, V,
      1) e guarde o ponteiro da seg.
  Para consultar o K-esimo menor valor de [L, R] chame
      kth(roots[L-1], roots[R], 1, N, K);
```

```
struct Node {
  int val = 0;
  Node *L = NULL, *R = NULL;
  Node(int v = 0) : val(v), L(NULL), R(NULL) {}
};

Node* build(int l, int r){
  if(l == r) return new Node();

  int m = (l+r)/2;

  Node *node = new Node();

  node->L = build(l,   m);
  node->R = build(m+1, r);
  node->val = node->L->val + node->R->val;

  return node;
}

Node* update(Node *node, int l, int r, int pos, int v){
  if( pos < l || r < pos ) return node;
  if(l == r) return new Node(node->val + v);

  int m = (l+r)/2;

  Node *nw = new Node();

  nw->L = update(node->L, l,   m, pos, v);
  nw->R = update(node->R, m+1, r, pos, v);
  nw->val = nw->L->val + nw->R->val;

  return nw;
}

int query(Node *node, int l, int r, int a, int b){
  if(b <  l || r <  a) return 0;
  if(a <= l && r <= b) return node->val;

  int m = (l+r)/2;

  return query(node->L, l, m, a, b) + query(node->R, m+1, r, a
      , b);
}

int kth(Node *Left, Node *Right, int l, int r, int k){
  if(l == r) return l;

  int sum = Right->L->val - Left->L->val;
  int m = (l+r)/2;

  if(sum >= k) return kth(Left->L, Right->L, l, m, k);
  return kth(Left->R, Right->R, m+1, r, k - sum);
}
```

1.10 Sparse Table

```
Sparse Table for Range Minimum Query [L, R] [0, N)
build:  O(N log N)
Query:  O(1)
Value -> Original Array
```

```
const int MAXN = 1e5 + 5;
const int MAXLG = 31 - __builtin_clz(MAXN) + 1;

int value[MAXN], table[MAXLG][MAXN];

void build(int N){
  for(int i=0; i<N; i++) table[0][i] = value[i];

  for(int p=1; p < MAXLG; p++)
    for(int i=0; i + (1 << p)  <= N; i++)
      table[p][i] = min(table[p-1][i], table[p-1][i+(1 << (p
          -1))]);
}

int query(int l, int r){
  int p = 31 - __builtin_clz(r - l + 1);  //floor log
  return min(table[p][l], table[p][ r - (1<<p) + 1 ]);
}
```

2 dp

2.1 Digit DP

```
Digit DP - Sum of Digits

Solve(K)  ->  Retorna a soma dos digitos de todo numero X tal que: 0
      <= X <= K
dp[D][S][f]  ->  D: Quantidade de digitos; S: Soma dos digitos; f:
      Flag que indica o limite.
int limite[D] -> Guarda os digitos de K.
```

```
Complexity: O(D^2 * B^2)  (B = Base = 10)
```

```cpp
ll dp[2][19][170];

int limite[19];
ll digitDP(int idx, int sum, bool flag){
    if(idx < 0) return sum;
    if(~dp[flag][idx][sum]) return dp[flag][idx][sum];

    dp[flag][idx][sum] = 0;
  int lm = flag ? limite[idx] : 9;

    for(int i=0; i<=lm; i++)
        dp[flag][idx][sum] += digitDP(idx-1, sum+i, (flag && i
              == lm));

    return dp[flag][idx][sum];
}

ll solve(ll k){
    memset(dp, -1, sizeof dp);

  int sz=0;
  while(k){
    limite[sz++] = k % 10LL;
    k /= 10LL;
  }

  return digitDP(sz-1, 0, true);
}
```

2.2 LIS

```
LIS - Longest Increasing Subsequence

Complexity: O(N Log N)
* For ICREASING sequence, use lower_bound()
* For NON DECREASING sequence, use upper_bound()
```

```cpp
int LIS(vector<int>& nums){
  vector<int> lis;

  for(auto x : nums)
  {
    auto it = lower_bound(lis.begin(), lis.end(), x);

    if(it == lis.end()) lis.push_back(x);
    else *it = x;
  }

  return (int) lis.size();
}
```

2.3 SOS DP

```
SOS DP - Sum over Subsets

Dado que cada mask possui um valor inicial (iVal), computa

para cada mask a soma dos valores de todas as suas submasks.

N -> Numero Maximo de Bits
iVal[mask] -> initial Value / Valor Inicial da Mask
dp[mask]   -> Soma de todos os SubSets

Iterar por todas as submasks: for(int sub=mask; sub>0;
      sub=(sub-1)&mask)
```

```cpp
const int N = 20;
ll dp[1<<N], iVal[1<<N];

void sosDP() // O(N * 2^N)
{
    for(int i=0; i<(1<<N); i++)
        dp[i] = iVal[i];

  for(int i=0; i<N; i++)
    for(int mask=0; mask<(1<<N); mask++)
      if(mask&(1<<i))
        dp[mask] += dp[mask^(1<<i)];
}
```

3 Geometry

3.1 ConvexHull

```
// FOR DOUBLE POINT //
See Geometry - General
```

```cpp
struct PT {
  ll x, y;
  PT(ll x=0, ll y=0) : x(x), y(y) {}

  PT operator- (const PT&a) const{ return PT(x-a.x, y-a.y); }
  ll operator% (const PT&a) const{ return  (x*a.y - y*a.x); }
       //Cross // Vector product

  bool operator==(const PT&a) const{ return x == a.x && y == a
       .y; }
  bool operator< (const PT&a) const{ return x != a.x ? x < a.x
       : y < a.y; }
};

// Colinear? Mude >= 0 para > 0 nos while
vector<PT> ConvexHull(vector<PT> pts, bool sorted=false) {
  if(!sorted) sort(begin(pts), end(pts));
  pts.resize(unique(begin(pts), end(pts)) - begin(pts));
  if(pts.size() <= 1) return pts;

  int s=0, n=pts.size();
  vector<PT> h (2*n+1);

  for(int i=0; i<n; h[s++] = pts[i++])
    while(s > 1 && (pts[i] - h[s-2]) % (h[s-1] - h[s-2]) >= 0
        )
      s--;

  for(int i=n-2, t=s; ~i; h[s++] = pts[i--])
    while(s > t && (pts[i] - h[s-2]) % (h[s-1] - h[s-2]) >= 0
        )
      s--;

  h.resize(s-1);
  return h;
}
```

3.2 Geometry - General

```
// FOR DOUBLE POINT //
const ld EPS = 1e-9;
bool eq(ld a, ld b){ return abs(a-b) < EPS; } // ==
bool lt(ld a, ld b){ return a + EPS < b;    } // <
bool gt(ld a, ld b){ return a > b + EPS;    } // >
bool le(ld a, ld b){ return a < b + EPS;    } // <=
bool ge(ld a, ld b){ return a + EPS > b;    } // >=
bool operator==(const PT&a) const{ return eq(x, a.x) && eq(y, a.y); }
          // for double point
bool operator< (const PT&a) const{ return eq(x, a.x) ? lt(y, a.y) :
     lt(x, a.x); } // for double point
bool operator<<(PT&a){ PT&p=*this; return eq(p%a, 0) ? lt(p*p, a*a) :
      lt(p%a, 0);  } //angle(this) < angle(a)
//Change LL to LD and uncomment this
//Also, consider replacing comparisons with these functions
```

```cpp
#define ld long double

// !!! NOT TESTED !!! //

struct PT {
  ll x, y;
  PT(ll x=0, ll y=0) : x(x), y(y) {}

  PT operator+ (const PT&a) const{ return PT(x+a.x, y+a.y); }
  PT operator- (const PT&a) const{ return PT(x-a.x, y-a.y); }
  ll operator* (const PT&a) const{ return  (x*a.x + y*a.y); }
     //DOT product  // norm // lenght^2 // inner
  ll operator% (const PT&a) const{ return  (x*a.y - y*a.x); }
     //Cross // Vector product
  PT operator* (ll c)  const{ return PT(x*c, y*c); }
  PT operator/ (ll c)  const{ return PT(x/c, y/c); }

  bool operator==(const PT&a)  const{ return x == a.x && y == a
       .y; }
  bool operator< (const PT&a)  const{ return x != a.x ? x < a.x
       : y < a.y; }
  bool operator<<(const PT&a)  const{ PT p=*this; return (p%a
       == 0) ? (p*p < a*a) : (p%a < 0);  } //angle(p) < angle(a
       )
};


ld dist  (PT a, PT b){ return sqrtl((a-b)*(a-b)); }
                            // distance from A to B
ld angle (PT a, PT b){ return acos((a*b) / sqrtl(a*a) / sqrtl(
     b*b)); } //Angle between A and B
PT rotate(PT p, double ang){ return PT(p.x*cos(ang) - p.y*sin(
     ang), p.x*sin(ang) + p.y*cos(ang)); } //Left rotation.
     Angle in radian
```

```
ll Area(vector<PT>& p){
  ll area = 0;
  for(int i=2; i < p.size(); i++)
    area += (p[i]-p[0]) % (p[i-1]-p[0]);
  return abs(area) / 2LL;
}

PT intersect(PT a1, PT d1, PT a2, PT d2){
  return a1 + d1 * (((a2 - a1)%d2) / (d1%d2));
}

ld dist_pt_line(PT a, PT l1, PT l2){
  return abs( ((a-l1) % (l2-l1)) / dist(l1, l2)  );
}

ld dist_pt_segm(PT a, PT s1, PT s2){
  if(s1 == s2) return dist(s1, a);

  PT d = s2 - s1;
  ld t = max(0.0L, min(1.0L, ((a-s1)*d) / sqrtl(d*d)) );

  return dist(a, s1+(d*t));
}
```

3.3 LineContainer

```
struct Line {
  mutable ll k, m, p;
  bool operator<(const Line& o) const { return k < o.k; }
  bool operator<(ll x) const { return p < x; }
};

struct LineContainer : multiset<Line, less<>> {
  static const ll inf = LLONG_MAX; // Double: inf = 1/.0, div(
      a,b) = a/b
  ll div(ll a, ll b) { return a / b - ((a ^ b) < 0 && a % b);
      } //floored division

  bool isect(iterator x, iterator y) {
    if(y == end()) return x->p = inf, 0;
    if(x->k == y->k) x->p = x->m > y->m ? inf : -inf;
    else x->p = div(y->m - x->m, x->k - y->k);
    return x->p >= y->p;
  }

  void add_line(ll k, ll m){ // kx + m  //if minimum k*=-1, m
      *=-1, query*-1
    auto z = insert({k, m, 0}), y = z++, x = y;
    while(isect(y, z)) z = erase(z);
    if(x != begin() && isect(--x, y)) isect(x, y = erase(y));
    while((y = x) != begin() && (--x)->p >= y->p) isect(x,
        erase(y));
  }

  ll query(ll x) {
    assert(!empty());
    auto l = *lower_bound(x);
    return l.k * x + l.m;
  }
};
```

4 Grafos

4.1 2SAT

```
 2 SAT - Two Satisfiability Problem

IMPORTANTE! o grafo deve estar 0-indexado!

inverso de u = ~u

Retorna uma valoracao verdadeira se possivel
Ou um vetor vazio se impossivel;
```

```
struct TwoSat {
  int N;
  vector<vector<int>> E;

  TwoSat(int N) : N(N), E(2 * N) {}
  inline int eval(int u) const{ return u < 0 ? ((~u)+N)%(2*N)
      : u; }

  void add_or(int u, int v){
    E[eval(~u)].push_back(eval(v));
    E[eval(~v)].push_back(eval(u));
  }
  void add_nand(int u, int v) {
    E[eval(u)].push_back(eval(~v));
    E[eval(v)].push_back(eval(~u));
  }
  void set_true (int u){ E[eval(~u)].push_back(eval(u)); }
  void set_false(int u){ set_true(~u); }
  void add_imply(int u, int v){ E[eval(u)].push_back(eval(v));
      }
  void add_and  (int u, int v){ set_true(u);  set_true(v);
      }
  void add_nor  (int u, int v){ add_and(~u, ~v); }
  void add_xor  (int u, int v){ add_or(u, v); add_nand(u, v);
      }
  void add_xnor (int u, int v){ add_xor(u, ~v); }

  vector<bool> solve() {
    vector<bool> ans(N);
    auto scc = tarjan();

    for (int u = 0; u < N; u++)
      if(scc[u] == scc[u+N]) return {}; //false
      else ans[u] = scc[u+N] > scc[u];

    return ans; //true
  }
private:
  vector<int> tarjan() {
    vector<int> low(2*N), pre(2*N, -1), scc(2*N, -1);
    stack<int> st;
    int clk = 0, ncomps = 0;

    auto dfs = [&](auto&& dfs, int u) -> void {
      pre[u] = low[u] = clk++;
      st.push(u);

      for(auto v : E[u])
        if(pre[v] == -1) dfs(dfs, v), low[u] = min(low[u], low
            [v]);
        else
        if(scc[v] == -1) low[u] = min(low[u], pre[v]);

      if(low[u] == pre[u]){
        int v = -1;
        while(v != u) scc[v = st.top()] = ncomps, st.pop();
        ncomps++;
      }
    };

    for(int u=0; u < 2*N; u++)
      if(pre[u] == -1)
        dfs(dfs, u);

    return scc; //tarjan SCCs order is the reverse of topoSort
        , so (u->v if scc[v] <= scc[u])
  }
};
```

4.2 BlockCutTree

```
Block Cut Tree - BiConnected Component

reset(n);
addEdge(u, v);
tarjan(Root);
buildBCC(n);

No fim o grafo da Block Cut Tree estara em _vector<int> tree []_
```

```
const int MAXN = 1e6 + 5;
const int MAXM = 1e6 + 5;//Cuidado

vector<pii> grafo [MAXN];
int pre[MAXN], low[MAXN], clk=0, C=0;

vector<pii> edge;
bool visEdge[MAXM];
int edgeComponent[MAXM];
int vertexComponent[MAXN];

int cut[MAXN];
stack<int> s;

vector<int> tree [2*MAXN];
int componentSize[2*MAXN]; //vertex - cutPoints

void reset(int n){
  for(int i=0; i<=edge.size(); i++)
    visEdge[i] = edgeComponent[i] = 0;

  edge.clear();

  for(int i=0; i<=n; i++){
    pre[i] = low[i] = -1;
    cut[i] = false;
    vertexComponent[i] = 0;
    grafo[i].clear();
  }

  for(int i=0; i<=C; i++){
    componentSize[i] = 0;
    tree[i].clear();
  }

  while(!s.empty()) s.pop();

  clk = C = 0;
}
```

```cpp
void newComponent(int i){
  C++;
  int j;

  do {
    j = s.top(); s.pop();
    edgeComponent[j] = C;

    auto [u, v]  = edge[j];
    if(!cut[u] && !vertexComponent[u]) componentSize[C]++,
        vertexComponent[u] = C;
    if(!cut[v] && !vertexComponent[v]) componentSize[C]++,
        vertexComponent[v] = C;

  } while(!s.empty() && j != i);
}

void tarjan(int u, bool root = true){
  pre[u] = low[u] = clk++;

  bool any = false;
  int chd = 0;

  for(auto [v, i] : grafo[u]){
    if(visEdge[i]) continue;
    visEdge[i] = true;

    s.emplace(i);

    if(pre[v] == -1)
    {
      tarjan(v, false);

      low[u] = min(low[v], low[u]);
      chd++;

      if(!root && low[v] >= pre[u]) cut[u] = true,
          newComponent(i);
      if( root && chd >= 2)       cut[u] = true, newComponent(
          i);
    }
    else
      low[u] = min(low[u], pre[v]);
  }

  if(root) newComponent(-1);
}

//ATENCAO: ESTA 1-INDEXADO
void buildBCC(int n){
  vector<bool> marc(C+1, false);

  for(int u=1; u<=n; u++)
  {
    if(!cut[u]) continue;

    C++;
    cut[u] = C;

    for(auto [v, i] : grafo[u])
    {
      int ec = edgeComponent[i];
      if(!marc[ec])
      {
        marc[ec] = true;
        tree[cut[u]].emplace_back(ec);
        tree[ec].emplace_back(cut[u]);
      }
    }

    for(auto [v, i] : grafo[u])
      marc[edgeComponent[i]] = false;
  }
}

void addEdge(int u, int v){
  int i = edge.size();
  grafo[u].emplace_back(v, i);
  grafo[v].emplace_back(u, i);
  edge.emplace_back(u, v);
}
```

4.3 Centroid Decomposition

```
Centroid Decomposition

dfsc() -> para criar a centroid tree

rem[u]    -> True se U ja foi removido (pra dfsc)
szt[u]    -> Size da subarvore de U (pra dfsc)
parent[u] -> Pai de U na centroid tree *parent[ROOT] = -1
distToAncestor[u][i] -> Distancia na arvore original de u para
  seu i-esimo pai na centroid tree *distToAncestor[u][0] = 0

dfsc(u=node, p=parent(subtree), f=parent(centroid tree), sz=size of
     tree)
```

```cpp
const int MAXN = 1e6 + 5;

vector<int> grafo[MAXN];
deque<int> distToAncestor[MAXN];

bool rem[MAXN];
int szt[MAXN], parent[MAXN];

void getDist(int u, int p, int d=0){
  for(auto v : grafo[u])
    if(v != p && !rem[v])
      getDist(v, u, d+1);
  distToAncestor[u].emplace_front(d);
}

int getSz(int u, int p){
  szt[u] = 1;
  for(auto v : grafo[u])
    if(v != p && !rem[v])
      szt[u] += getSz(v, u);
  return szt[u];
}

void dfsc(int u=0, int p=-1, int f=-1, int sz=-1){
  if(sz < 0) sz = getSz(u, -1); //starting new tree

  for(auto v : grafo[u])
    if(v != p && !rem[v] && szt[v]*2 >= sz)
      return dfsc(v, u, f, sz);

  rem[u] = true, parent[u] = f;
  getDist(u, -1, 0); //get subtree dists to centroid

  for(auto v : grafo[u])
    if(!rem[v])
      dfsc(v, u, u, -1);
}
```

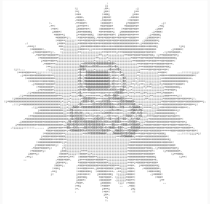
4.4 Dijkstra

```
Dijkstra - Shortest Paths from Source

caminho minimo de um vertice u para todos os
outros vertices de um grafo ponderado

Complexity: O(N Log N)

dijkstra(s)        ->  s : Source, Origem. As distancias serao
      calculadas com base no vertice s
grafo[u] = {v, c};     ->  u : Vertice inicial, v : Vertice final, c :
      Custo da aresta
priority_queue<pii, vector<pii>, greater<pii>> ->  Ordena pelo menor
      custo -> {d, v} -> d : Distancia, v : Vertice
```

```cpp
const int MAXN = 1e6 + 5;
#define INF 0x3f3f3f3f
#define vi vector<int>

vector<pii> grafo [MAXN];

vi dijkstra(int s){
  vi dist (MAXN, INF); // !!! Change MAXN to N

  priority_queue<pii, vector<pii>, greater<pii>> fila;
  fila.push({0, s});
  dist[s] = 0;

  while(!fila.empty())
  {
    auto [d, u] = fila.top();
    fila.pop();

    if(d > dist[u]) continue;

    for(auto [v, c] : grafo[u])
      if( dist[v] > dist[u] + c )
      {
        dist[v] = dist[u] + c;
        fila.push({dist[v], v});
      }
  }

  return dist;
}
```

4.5 Dinic

```
    Dinic - Max Flow Min Cut

[!NOTE]
Dinic - Max Flow Min Cut
Algoritmo de Dinitz para encontrar o Fluxo Maximo
IMPORTANTE! O algoritmo esta 0-indexado
[!/NOTE]
```



Complexidade:
- O(V^2 * E) caso geral
- O(\sqrt(V) * E) grafos com cap = 1 para toda aresta (matching bipartido)

Uso:
```cpp
Dinic dinic(n, source, sink);  // Criacao
dinic.addAresta(u, v, cap);   // Adiciona aresta
dinic.maxFlow();               // Calcula fluxo
dinic.inCut(u);                // Verifica corte
```

Algoritmo de Dinitz para encontrar o Fluxo Maximo
IMPORTANTE! O algoritmo esta 0-indexado

Complexity:
O(V^2 * E_i)   -> caso geral
O(\sqrt(V) * E) -> grafos com cap = 1 para toda aresta // matching bipartido

* Informacoes:
  Crie o Dinic:
    Dinic dinic(n, source, sink);
  Adicione as Arestas:
    dinic.addAresta(u, v, capacity);
  Para calcular o Fluxo Maximo:
    dinic.maxFlow()
  Para saber se um vertice U esta no Corte Minimo:
    dinic.inCut(u)

* Sobre o Codigo:
  vector<Aresta> arestas; -> Guarda todas as arestas do grafo e do grafo residual
  vector<vector<int>> adj; -> Guarda em adj[u] os indices de todas as arestas saindo de u
  vector<int> ptr;   -> Pointer para a proxima aresta ainda nao visitada de cada vertice
  vector<int> level; -> Distancia em vertices a partir do Source. Se igual a N o vertice nao foi visitado.
  A BFS retorna se Sink e alcancavel de Source. Se nao e porque foi atingido o Fluxo Maximo
  A DFS retorna um possivel aumento do Fluxo
* Use Cases of Flow

+ **Minimum cut**: the minimum cut is equal to maximum flow. i.e. to split the graph in two parts, one on the source side and another on sink side. The capacity of each edge is it weight.

+ **Edge-disjoint paths**: maximum number of *edge-disjoint* paths equals maximum flow of the graph, assuming that the capacity of each edge is one. (paths can be found greedily)

+ Node-disjoint paths: can be reduced to maximum flow. each node should appear in at most one path, so limit the flow through a node dividing each node in two. One with incoming edges, other with outgoing edges and a new edge from the first to the second with capacity 1.

+ Maximum matching (bipartite): maximum matching is equal to maximum flow. Add a source and a sink, edges from the source to every node at one partition and from each node of the other partition to the sink.

+ Minimum node cover (bipartite): minimum set of nodes such each edge has at least one endpoint. The size of minimum node cover is equal to maximum matching (Konig's theorem).

+ Maximum independent set (bipartite): largest set of nodes such that no two nodes are connected with an edge. Contain the nodes that aren't in "Min node cover" (N - MAXFLOW).

+ Minimum path cover (DAG): set of paths such that each node belongs to at least one path.
- Node-disjoint: construc a matching where each node is represented by two nodes, a left and a right at the matching and add the edges (from l to r). Each edge in the matching corresponds to an edge in the path cover. The number of paths in the cover is (N - MAXFLOW).
- General: almost like a minimum node-disjoint. Just add edges to the matching whenever there is an path from U to V in the graph (possibly through several edges).
- Antichain: a set of nodes such that there is no path from any node to another. In a DAG, the size of min general path cover equals the size of maximum antichain (Dilworth's theorem).

```cpp
struct Aresta {
  int u, v; ll cap;
  Aresta(int u, int v, ll cap) : u(u), v(v), cap(cap) {}
};

struct Dinic {

  int n, source, sink;
  vector<vector<int>> adj;
  vector<Aresta> arestas;
  vector<int> level, ptr; //pointer para a proxima aresta nao saturada de cada vertice

  Dinic(int n, int source, int sink) : n(n), source(source), sink(sink) { adj.resize(n); }

  void addAresta(int u, int v, ll cap)
  {
    adj[u].push_back(arestas.size());
    arestas.emplace_back(u, v, cap);

    adj[v].push_back(arestas.size());
    arestas.emplace_back(v, u, 0);
  }

  ll dfs(int u, ll flow = 1e9){
    if(flow == 0) return 0;
    if(u == sink) return flow;

    for(int &i = ptr[u];  i < adj[u].size();  i++)
    {
      int atual = adj[u][i];
      int v = arestas[atual].v;

      if(level[u] + 1 != level[v]) continue;

      if(ll got = dfs(v, min(flow, arestas[atual].cap)) )
      {
        arestas[atual].cap -= got;
        arestas[atual^1].cap += got;
        return got;
      }
    }

    return 0;
  }

  bool bfs(){
    level = vector<int> (n, n);
    level[source] = 0;

    queue<int> fila;
    fila.push(source);

    while(!fila.empty())
    {
      int u = fila.front();
      fila.pop();

      for(auto i : adj[u]){
        int v = arestas[i].v;

        if(arestas[i].cap == 0 || level[v] <= level[u] + 1 )
          continue;

        level[v] = level[u] + 1;
        fila.push(v);
      }
    }

    return level[sink] < n;
  }

  bool inCut(int u){ return level[u] < n; }

  ll maxFlow(){
    ll ans = 0;

    while( bfs() ){
      ptr = vector<int> (n+1, 0);

      while(ll got = dfs(source)) ans += got;
    }

    return ans;
  }
};
```

4.6 DSU Persistente

SemiPersistent Disjoint Set Union - O(Log n)
find(u, q) -> Retorna o pai de U no tempo q
* tim -> tempo em que o pai de U foi alterado

```cpp
struct DSUp {
  vector<int> pai, sz, tim;
  int t=1;
  DSUp(int n) : pai(n+1), sz(n+1, 1), tim(n+1) {
    for(int i=0; i<=n; i++) pai[i] = i;
  }

  int find(int u, int q = INT_MAX){
    if( pai[u] == u || q < tim[u] ) return u;
    return find(pai[u], q);
  }

  void join(int u, int v){
    u = find(u), v = find(v);

    if(u == v) return;
    if(sz[v] > sz[u]) swap(u, v);

    pai[v] = u;
    tim[v] = t++;
```

```
    sz[u] += sz[v];
  }
};
```

4.7 DSU

```
Disjoint Set Union - Union Find
Find: O( a(n) ) -> Inverse Ackermann function
Join: O( a(n) ) -> a(1e6) <= 5
```

```
struct DSU {
  vector<int> pai, sz;
  DSU(int n) : pai(n+1), sz(n+1, 1) {
    for(int i=0; i<=n; i++) pai[i] = i;
  }

  int find(int u){ return pai[u] == u ? u : pai[u] = find(pai[
      u]); }

  void join(int u, int v){
    u = find(u), v = find(v);

    if(u == v) return;
    if(sz[v] > sz[u]) swap(u, v);

    pai[v] = u;
    sz[u] += sz[v];
  }
};
```

4.8 Euler Path

```
Euler Path - Algoritmo de Hierholzer para caminho Euleriano

Complexity: O(V + E)

IMPORTANTE! O algoritmo esta 0-indexado

* Informacoes
  addEdge(u, v) -> Adiciona uma aresta de U para V
  EulerPath(n) -> Retorna o Euler Path, ou um vetor vazio se
       impossivel
  vi path -> vertices do Euler Path na ordem
  vi pathId -> id das Arestas do Euler Path na ordem

Euler em Undirected graph:
  - Cada vertice tem um numero par de arestas (circuito); OU
  - Exatamente dois vertices tem um numero impar de arestas (caminho);
Euler em Directed graph:
  - Cada vertice tem quantidade de arestas |entrada| == |saida|
       (circuito); OU
  - Exatamente 1 tem |entrada|+1 == |saida| && exatamente 1 tem
       |entrada| == |saida|+1 (caminho);
* Circuito -> U e o primeiro e ultimo
* Caminho -> U e o primeiro e V o ultimo
```

```
#define vi vector<int>

const int MAXN = 1e6 + 5;
const bool BIDIRECIONAL = true;

vector<pii> grafo[MAXN];
vector<bool> used;

void addEdge(int u, int v){
  grafo[u].emplace_back(v, used.size()); if(BIDIRECIONAL && u
       != v)
  grafo[v].emplace_back(u, used.size());
  used.emplace_back(false);
}

pair<vi, vi> EulerPath(int n, int src=0){
  int s=-1, t=-1;
  vector<int> selfLoop(n*BIDIRECIONAL, 0);

  if(BIDIRECIONAL)
  {
    for(int u=0; u<n; u++) for(auto& [v, id] : grafo[u]) if(u==
        v) selfLoop[u]++;
    for(int u=0; u<n; u++)
      if((grafo[u].size() - selfLoop[u])%2)
        if(t != -1) return {vi(), vi()};   // mais que 2 com
            grau impar
        else t = s, s = u;

    if(t == -1 && t != s) return {vi(), vi()}; // so 1 com
        grau impar
    if(s == -1 || t == src) s = src;           // se possivel,
         seta start como src
  }
  else
  {
    vector<int> in(n, 0), out(n, 0);

    for(int u=0; u<n; u++)
      for(auto [v, edg] : grafo[u])
        in[v]++, out[u]++;

    for(int u=0; u<n; u++)
      if(in[u] - out[u] == -1 && s == -1) s = u;  else
      if(in[u] - out[u] ==  1 && t == -1) t = u;  else
      if(in[u] !=out[u]) return {vi(), vi()};

    if(s == -1 && t == -1) s = t = src;          // se possivel
        , seta s como src
    if(s == -1 && t != -1) return {vi(), vi()}; // Existe S
        mas nao T
    if(s != -1 && t == -1) return {vi(), vi()}; // Existe T
        mas nao S
  }

  for(int i=0; grafo[s].empty() && i<n; i++) s =(s+1)%n; //
      evita s ser vertice isolado

  /////// DFS ///////
  vector<int> path, pathId, idx(n, 0);
  stack<pii> st; // {Vertex, EdgeId}
  st.push({s, -1});

  while(!st.empty())
  {
    auto [u, edg] = st.top();
    while(idx[u] < grafo[u].size() && used[grafo[u][idx[u]].
        second]) idx[u]++;

    if(idx[u] < grafo[u].size())
    {
      auto [v, id] = grafo[u][idx[u]];
      used[id] = true;
      st.push({v, id});
      continue;
    }

    path.push_back(u);
    pathId.push_back(edg);
    st.pop();
  }

  pathId.pop_back();
  reverse(begin(path),   end(path));
  reverse(begin(pathId), end(pathId));

  ///  Grafo conexo ? ///
  int edgesTotal = 0;
  for(int u=0; u<n; u++) edgesTotal += grafo[u].size() + (
       BIDIRECIONAL ? selfLoop[u] : 0);
  if(BIDIRECIONAL) edgesTotal /= 2;
  if(pathId.size() != edgesTotal) return {vi(), vi()};
  //////////////////////

  return {path, pathId};
}
```

4.9 HLD

```
Heavy-Light Decomposition

Complexity: #Query_path: O(LogN*qry) #Update_path: O(LogN*updt)
Nodes: 0 <= u, v < N

Change qry(l, r) and updt(l, r) to call a query and update
structure of your will

HLD hld(n); //call init
hld.add_edges(u, v); //add all edges
hld.build(); //Build everthing for HLD

tin[u] -> Pos in the structure (Seg, Bit, ...)
nxt[u] -> Head/Endpoint
```

```
const bool EDGE = false;
struct HLD {
public:
  vector<vector<int>> g; //grafo
  vector<int> sz, parent, tin, nxt;
  HLD(){}
  HLD(int n){ init(n); }
  void init(int n){
    t = 0;
    g.resize(n); tin.resize(n);
    sz.resize(n);nxt.resize(n);
    parent.resize(n);
  }
  void addEdge(int u, int v){
    g[u].emplace_back(v);
    g[v].emplace_back(u);
  }
  void build(int root=0){
    nxt[root]=root;
    dfs(root, root);
    hld(root, root);
  }

  ll query_path(int u, int v){
    if(tin[u] < tin[v]) swap(u, v);
    if(nxt[u] == nxt[v]) return qry(tin[v]+EDGE, tin[u]);
    return qry(tin[nxt[u]], tin[u]) + query_path(parent[nxt[u
        ]], v);
```

```
  }

  void update_path(int u, int v, ll x){
    if(tin[u] < tin[v]) swap(u, v);
    if(nxt[u] == nxt[v]) return updt(tin[v]+EDGE, tin[u], x);
    updt(tin[nxt[u]], tin[u], x); update_path(parent[nxt[u]],
         v, x);
  }

private:
  ll qry(int l, int r){ if(EDGE && l>r) return 0;/*NEUTRO*/ }
       //call Seg, BIT, etc
  void updt(int l, int r, ll x){ if(EDGE && l>r) return; }
       //call Seg, BIT, etc

  void dfs(int u, int p){
    sz[u] = 1, parent[u] = p;
    for(auto &v : g[u]) if(v != p) {
      dfs(v, u); sz[u] += sz[v];

      if(sz[v] > sz[g[u][0]] || g[u][0] == p)
        swap(v, g[u][0]);
    }
  }

  int t=0;
  void hld(int u, int p){
    tin[u] = t++;
    for(auto &v : g[u]) if(v != p)
      nxt[v] = (v == g[u][0] ? nxt[u] : v),
      hld(v, u);
  }

  /// OPTIONAL ///
  int lca(int u, int v){
    while(!inSubtree(nxt[u], v)) u = parent[nxt[u]];
    while(!inSubtree(nxt[v], u)) v = parent[nxt[v]];
    return tin[u] < tin[v] ? u : v;
  }
  bool inSubtree(int u, int v){ return tin[u] <= tin[v] && tin
       [v] < tin[u] + sz[u]; }
  //query/update_subtree[tin[u]+EDGE, tin[u]+sz[u]-1];
};
```

4.10 Kruskal

```
  Kruskal - Minimum Spanning Tree
Algoritmo para encontrar a Arvore Geradora Minima (MST)
-> Complexity: O(E log E)
E : Numero de Arestas
```

```
/*Create a DSU*/
void join(int u, int v); int find(int u);

const int MAXN = 1e6 + 5;
struct Aresta{ int u, v, c; };
bool compAresta(Aresta a, Aresta b){ return a.c < b.c; }

vector<Aresta> arestas;        //Lista de Arestas

int kruskal(){
  sort(begin(arestas), end(arestas), compAresta); //Ordena
       pelo custo
  int resp = 0;          //Custo total da MST

  for(auto a : arestas)
    if( find(a.u) != find(a.v) )
    {
      resp += a.c;
      join(a.u, a.v);
    }
  return resp;
}
```

4.11 LCA

```
  LCA - Lowest Common Ancestor - Binary Lifting
Algoritmo para encontrar o menor ancestral comum
entre dois vertices em uma arvore enraizada

IMPORTANTE! O algoritmo esta 0-indexado

Complexity:
buildBL() ->  O(N Log N)
lca()     ->  O(Log N)

* Informacoes
  -> Monte o grafo na lista de adjacencias
  -> chame dfs(root, root) para calcular o pai e a altura de cada
       vertice
  -> chame buildBL() para criar a matriz do Binary Lifting
  -> chame lca(u, v) para encontrar o menor ancestral comum
  bl[i][u] -> Binary Lifting com o (2^i)-esimo pai de u
  lvl[u]   -> Altura ou level de U na arvore

* Em LCA o primeiro FOR iguala a altura de U e V
* E o segundo anda ate o primeiro vertice de U que nao e ancestral de
     V
* A resposta e o pai desse vertice
```

```
const int MAXN = 1e4 + 5;
const int MAXLG = 16;

vector<int> grafo[MAXN];

int bl[MAXLG][MAXN], lvl[MAXN];

void dfs(int u, int p, int l=0){
  lvl[u] = l;
  bl[0][u] = p;

  for(auto v : grafo[u])
    if(v != p)
      dfs(v, u, l+1);
}

void buildBL(int N){
  for(int i=1; i<MAXLG; i++)
    for(int u=0; u<N; u++)
      bl[i][u] = bl[i-1][bl[i-1][u]];
}

int lca(int u, int v){
  if(lvl[u] < lvl[v]) swap(u, v);

  for(int i=MAXLG-1; i>=0; i--)
    if(lvl[u] - (1<<i) >= lvl[v])
      u = bl[i][u];

  if(u == v) return u;

  for(int i=MAXLG-1; i>=0; i--)
    if(bl[i][u] != bl[i][v])
      u = bl[i][u],
      v = bl[i][v];

  return bl[0][u];
}
```

4.12 MinCostMaxFlow - MCMF

```
struct Aresta {
  int u, v; ll cap, cost;
  Aresta(int u, int v, ll cap, ll cost) : u(u), v(v), cap(cap)
       , cost(cost) {}
};

struct MCMF {
  const ll INF = numeric_limits<ll>::max();
  int n, source, sink;
  vector<vector<int>> adj;
  vector<Aresta> edges;
  vector<ll> dist, pot;
  vector<int> from;

  MCMF(int n, int source, int sink) : n(n), source(source),
       sink(sink) { adj.resize(n); pot.resize(n); }

  void addAresta(int u, int v, ll cap, ll cost){
    adj[u].push_back(edges.size());
    edges.emplace_back(u, v, cap, cost);

    adj[v].push_back(edges.size());
    edges.emplace_back(v, u, 0, -cost);
  }

  queue<int> q;
  vector<bool> vis;
  bool SPFA(){
    dist.assign(n, INF);
    from.assign(n, -1);
    vis.assign(n, false);

    q.push(source);
    dist[source] = 0;

    while(!q.empty()){
      int u = q.front();
      q.pop();

      vis[u] = false;

      for(auto i : adj[u]){
        if(edges[i].cap == 0) continue;
        int v = edges[i].v;
        ll cost = edges[i].cost;

        if(dist[v] > dist[u] + cost + pot[u] - pot[v]){
          dist[v] = dist[u] + cost + pot[u] - pot[v];
          from[v] = i;
          if(!vis[v]) q.push(v), vis[v] = true;
        }
      }
    }

    for(int u=0; u<n; u++)  //fix pot
      if(dist[u] < INF)
        pot[u] += dist[u];
```

```
    return dist[sink] < INF;
  }

  pair<ll, ll> augment(){
    ll flow = edges[from[sink]].cap, cost = 0; //fixed flow:
         flow = min(flow, remainder)

    for(int v=sink; v != source; v = edges[from[v]].u)
      flow = min(flow, edges[from[v]].cap),
      cost += edges[from[v]].cost;

    for(int v=sink; v != source; v = edges[from[v]].u)
      edges[from[v]].cap   -= flow,
      edges[from[v]^1].cap += flow;

    return {flow, cost};
  }

  bool inCut(int u){ return dist[u] < INF; }

  pair<ll, ll> maxFlow(){
    ll flow = 0, cost = 0;

    while( SPFA() ){
      auto [f, c] = augment();
      flow += f;
      cost += f*c;
    }
    return {flow, cost};
  }
};
```

4.13 SCC - Kosaraju

```
Kosaraju - Strongly Connected Component
Algoritmo de Kosaraju para encontrar Componentes Fortemente Conexas

Complexity: O(V + E)
IMPORTANTE! O algoritmo esta 0-indexado

* Variaveis e explicacoes *
int C   -> C e a quantidade de Componetes Conexas. As componetes
     estao numeradas de 0 a C-1
dag     -> Apos rodar o Kosaraju, o grafo das componentes conexas
     sera criado aqui
comp[u] -> Diz a qual componente conexa U faz parte
order   -> Ordem de saida dos vertices. Necessario para o Kosaraju
grafo   -> grafo direcionado
greve -> grafo reverso (que deve ser construido junto ao grafo
     normal) !!!

NOTA: A ordem que o Kosaraju descobre as componentes e uma Ordenacao
     Topologica do SCC
em que o dag[0] nao possui grau de entrada e o dag[C-1] nao possui
     grau de saida
```

```
#define vi vector<int>

const int MAXN = 1e6 + 5;

vi grafo[MAXN];
vi greve[MAXN];
vi dag[MAXN];
vi comp, order;
vector<bool> vis;
int C;

void dfs(int u){
  vis[u] = true;
  for(auto v : grafo[u])
    if(!vis[v])
      dfs(v);
  order.push_back(u);
}

void dfs2(int u){
  comp[u] = C;
  for(auto v : greve[u])
    if(comp[v] == -1)
      dfs2(v);
}

void kosaraju(int n){
  order.clear();
  comp.assign(n, -1);
  vis.assign(n, false);

  for(int v=0; v<n; v++)
    if(!vis[v])
      dfs(v);

  C = 0;
  reverse(begin(order), end(order));

  for(auto v : order)
    if(comp[v] == -1)
      dfs2(v), C++;

  //// Montar DAG  ////
  vector<bool> marc(C, false);

  for(int u=0; u<n; u++){
    for(auto v : grafo[u])
    {
      if(comp[v] == comp[u] || marc[comp[v]]) continue;

      marc[comp[v]] = true;
      dag[comp[u]].emplace_back(comp[v]);
    }

    for(auto v : grafo[u]) marc[comp[v]] = false;
  }
}
```

4.14 Tarjan

```
  Tarjan - Pontes e Pontos de Articulacao
Algoritmo para encontrar pontes e pontos de articulacao.

Complexity: O(V + E)
IMPORTANTE! Lembre do memset(pre, -1, sizeof pre);

* Variaveis e explicacoes *
pre[u] = "Altura", ou, x-esimo elemento visitado na DFS. Usado para
      saber a posicao de um vertice na arvore de DFS
low[u] = Low Link de U, ou a menor aresta de retorno (mais proxima da
      raiz) que U alcanca entre seus filhos

chd = Children. Quantidade de componentes filhos de U. Usado para
      saber se a Raiz e Ponto de Articulacao.
any = Marca se alguma aresta de retorno em qualquer dos componentes
      filhos de U nao ultrapassa U. Se isso for verdade, U e Ponto de
      Articulacao.

if(low[v] >  pre[u]) pontes.emplace_back(u, v); -> se a mais alta
      aresta de retorno de V (ou o menor low) estiver abaixo de U,
      entao U-V e ponte
if(low[v] >= pre[u]) any = true;        -> se a mais alta aresta de
      retorno de V (ou o menor low) estiver abaixo de U ou igual a U,
      entao U e Ponto de Articulacao
```

```
const int MAXN = 1e6 + 5;
int pre[MAXN], low[MAXN], clk=0;
vector<int> grafo [MAXN];

vector<pair<int, int>> pontes;
vector<int> cut;

// lembrar do memset(pre, -1, sizeof pre);
void tarjan(int u, int p = -1){
  pre[u] = low[u] = clk++;

  bool any = false;
  int chd = 0;

  for(auto v : grafo[u]){
    if(v == p) continue;

    if(pre[v] == -1)
    {
      tarjan(v, u);

      low[u] = min(low[v], low[u]);

      if(low[v] >  pre[u]) pontes.emplace_back(u, v);
      if(low[v] >= pre[u]) any = true;

      chd++;
    }
    else
      low[u] = min(low[u], pre[v]);
  }

  if(p == -1 && chd >= 2) cut.push_back(u);
  if(p != -1 && any)      cut.push_back(u);
}
```

5 Math

5.1 fexp

```
ll MOD = 1e9 + 7;

ll fexp(ll b, ll p){
  ll ans = 1;

  while(p){
    if(p&1) ans = (ans*b) % MOD;
    b = b * b % MOD;
    p >>= 1;
  }

  return ans % MOD;
}
// O(Log P) // b - Base // p - Potencia
```

6 others

6.1 Hungarian

```
  Hungarian Algorithm - Assignment Problem
Algoritmo para o problema de atribuicao minima.

Complexity: O(N^2 * M)

hungarian(int n, int m);  ->  Retorna o valor do custo minimo
getAssignment(int m)      ->  Retorna a lista de pares <linha,
     Coluna> do Minimum Assignment

n -> Numero de Linhas // m -> Numero de Colunas

IMPORTANTE! O algoritmo e 1-indexado
IMPORTANTE! O tipo padrao esta como int, para mudar para outro tipo
     altere | typedef <TIPO> TP; |
Extra: Para o problema da atribuicao maxima, apenas multiplique os
     elementos da matriz por -1
```

```cpp
typedef int TP;

const int MAXN = 1e3 + 5;
const TP INF = 0x3f3f3f3f;

TP matrix[MAXN][MAXN];
TP row[MAXN], col[MAXN];
int match[MAXN], way[MAXN];

TP hungarian(int n, int m){
  memset(row, 0, sizeof row);
  memset(col, 0, sizeof col);
  memset(match, 0, sizeof match);

  for(int i=1; i<=n; i++)
  {
    match[0] = i;
    int j0 = 0, j1, i0;
    TP delta;

    vector<TP> minv (m+1, INF);
    vector<bool> used (m+1, false);

    do {
      used[j0] = true;
      i0 = match[j0];
      j1 = -1;
      delta = INF;

      for(int j=1; j<=m; j++)
        if(!used[j]){
          TP cur = matrix[i0][j] - row[i0] - col[j];

          if( cur < minv[j] ) minv[j] = cur, way[j] = j0;
          if(minv[j] < delta) delta = minv[j], j1 = j;
        }

      for(int j=0; j<=m; j++)
        if(used[j]){
          row[match[j]] += delta,
          col[j] -= delta;
        }else
          minv[j] -= delta;

      j0 = j1;
    } while(match[j0]);

    do {
      j1 = way[j0];
      match[j0] = match[j1];
      j0 = j1;
    } while(j0);
  }

  return -col[0];
}

vector<pair<int, int>> getAssignment(int m){
  vector<pair<int, int>> ans;

  for(int i=1; i<=m; i++)
    ans.push_back(make_pair(match[i], i));

  return ans;
}
```

6.2 MO

```
Algoritmo de MO para query em range

Complexity: O( (N + Q) * SQRT(N) * F ) | F e a complexidade do Add e
     Remove

IMPORTANTE! Queries devem ter seus indices (Idx) 0-indexados!

Modifique as operacoes de Add, Remove e GetAnswer de acordo com o
     problema.
BLOCK_SZ pode ser alterado para aproximadamente SQRT(MAX_N)

IF you want to use hilbert curves on MO
vector<ll> h(ans.size());
for (int i = 0; i < ans.size(); i++) h[i] = hilbert(queries[i].l,
     queries[i].r);
sort(queries.begin(), queries.end(), [&](Query&a, Query&b) { return
     h[a.idx] < h[b.idx]; });

inline ll hilbert(int x, int y) {
  static int N = 1 << (__builtin_clz(0) - __builtin_clz(MAXN));
  int rx, ry, s; ll d = 0;
  for (s = N/2; s > 0; s /= 2) {
    rx = (x & s) > 0, ry = (y & s) > 0;
    d += s * (ll)(s) * ((3 * rx) ^ ry);
    if (ry == 0) { if (rx == 1) x = N-1 - x, y = N-1 - y; swap(x, y);
         }
  }
  return d;
}
```

```cpp
const int BLOCK_SZ = 700;

struct Query{
  int l, r, idx;

  Query(int l, int r, int idx) : l(l), r(r), idx(idx) {}

  bool operator < (Query q) const {
    if(l / BLOCK_SZ != q.l / BLOCK_SZ) return l < q.l;
    return (l / BLOCK_SZ &1) ? ( r < q.r ) : (r > q.r );
  }
};

void add(int idx);
void remove(int idx);
int getAnswer();

vector<int> MO(vector<Query> &queries){
  vector<int> ans(queries.size());

  sort(queries.begin(), queries.end());

  int L = 0, R = 0;
  add(0);

  for(auto [l, r, idx] : queries){
    while(l < L) add(--L);
    while(r > R) add(++R);
    while(l > L) remove(L++);
    while(r < R) remove(R--);

    ans[idx] = getAnswer();
  }

  return ans;
}
```

7 Strings

7.1 Hash

```
String Hash
precalc()    -> O(N)
StringHash() -> O(|S|)
gethash()    -> O(1)

StringHash hash(s); -> Cria uma struct de StringHash para a string s
hash.gethash(l, r); -> Retorna o hash do intervalo L R da string
     (0-Indexado)

IMPORTANTE! Chamar precalc() no inicio do codigo

const ll MOD  = 131'807'699; -> Big Prime Number
const ll base = 127;         -> Random number larger than the Alphabet
```

```cpp
const int MAXN = 1e6 + 5;

const ll MOD = 1e9 + 7; //WA? Muda o MOD e a base
const ll base = 153;

ll expb[MAXN];

void precalc(){
  expb[0] = 1;
  for(int i=1; i<MAXN; i++)
    expb[i] = (expb[i-1]*base)%MOD;
}

struct StringHash{
  vector<ll> hsh;

  StringHash(string &s){
    hsh.assign(s.size()+1, 0);
    for(int i=0; i<s.size(); i++)
      hsh[i+1] = (hsh[i] * base % MOD + s[i]) % MOD;
  }

  ll gethash(int l, int r){
    return (MOD + hsh[r+1] - hsh[l]*expb[r-l+1] % MOD ) % MOD;
  }
```

```
};
```

7.2 Hash2

```
String Hash - Double Hash
precalc()    -> O(N)
StringHash() -> O(|S|)
gethash()    -> O(1)

StringHash hash(s); -> Cria o Hash da string s
hash.gethash(l, r); -> Hash [L,R] (0-Indexado)
```

```
const int MAXN = 1e6 + 5;

const ll MOD1 = 131'807'699;
const ll MOD2 = 1e9 + 9;
const ll base = 157;

ll expb1[MAXN], expb2[MAXN];

#warning "Call precalc() before use StringHash"
void precalc(){
    expb1[0] = expb2[0] = 1;

  for(int i=1;i<MAXN;i++)
        expb1[i] = expb1[i-1]*base % MOD1,
        expb2[i] = expb2[i-1]*base % MOD2;
}

struct StringHash{
    vector<pair<ll,ll>> hsh;
    string s; // comment S if you dont need it

    StringHash(string& s) : s(s){
        hsh.assign(s.size()+1, {0,0});

        for (int i=0;i<s.size();i++)
            hsh[i+1].first  = ( hsh[i].first *base % MOD1 + s[
                i] ) % MOD1,
            hsh[i+1].second = ( hsh[i].second*base % MOD2 + s[
                i] ) % MOD2;
    }

    ll gethash(int a,int b){
        ll h1 = (MOD1+ hsh[b+1].first  - hsh[a].first *expb1[b
            -a+1] % MOD1) % MOD1;
        ll h2 = (MOD2+ hsh[b+1].second - hsh[a].second*expb2[b
            -a+1] % MOD2) % MOD2;
        return (h1<<32) | h2;
    }
};

int firstDiff(StringHash& a, int la, int ra, StringHash& b,
     int lb, int rb)
{
  int l=0, r=min(ra-la, rb-lb), diff=r+1;
  while(l <= r){
    int m = (l+r)/2;
    if(a.gethash(la, la+m) == b.gethash(lb, lb+m)) l = m+1;
    else r = m-1, diff = m;
  }
  return diff;
}

int hshComp(StringHash& a, int la, int ra, StringHash& b, int
     lb, int rb){
  int diff = firstDiff(a, la, ra, b, lb, rb);
  if(diff > ra-la && ra-la == rb-lb) return 0; //equal
  if(diff > ra-la || diff  >  rb-lb) return ra-la < rb-lb ? -2
      : +2; //prefix of the other
  return a.s[la+diff] < b.s[lb+diff] ? -1 : +1;
}
```

7.3 KMP

```
KMP - Knuth-Morris-Pratt Pattern Searching

Complexity: O(|S|+|T|)

S -> String
T -> Pattern
```

```
vector<int> pi(string &t){
  vector<int> p(t.size(), 0);

  for(int i=1, j=0; i<t.size(); i++)
  {
    while(j > 0  && t[j] != t[i]) j = p[j-1];

    if(t[j] == t[i]) j++;

    p[i] = j;
  }

  return p;
}

vector<int> kmp(string &s, string &t){
  vector<int> p = pi(t), occ;

  for(int i=0, j=0; i<s.size(); i++)
  {
    while( j > 0 && s[i] != t[j]) j = p[j-1];

    if(s[i]==t[j]) j++;

    if(j == t.size()) occ.push_back(i-j+1), j = p[j-1];
  }

  return occ;
}
```

7.4 Manacher

```
Manacher Algorithm
Find every palindrome in string
Complexidade: O(N)
```

```
vector<int> manacher(string &st){
  string s = "$_";
  for(char c : st){ s += c; s += "_"; }
  s += "#";

  int n = s.size()-2;
  vector<int> p(n+2, 0);
  int l=1, r=1;

  for(int i=1, j; i<=n; i++)
  {
    p[i] = max(0, min(r-i, p[l+r-i]) ); //atualizo o valor
        atual para o valor do palindromo espelho na string ou
        para o total que esta contido

    while( s[i-p[i]] == s[i+p[i]] ) p[i]++;

    if( i+p[i] > r ) l = i-p[i], r = i+p[i];
  }

  for(auto &x : p) x--; //o valor de p[i] e igual ao tamanho
      do palindromo + 1

  return p;
}
```

7.5 trie

```
Trie - Arvore de Prefixos
insert(P) - O(|P|)
count(P)  - O(|P|)
MAXS  - Soma do tamanho de todas as Strings
sigma - Tamanho do alfabeto
```

```
const int MAXS = 1e5 + 10;
const int sigma = 26;

int trie[MAXS][sigma], terminal[MAXS], z = 1;

void insert(string &p){
  int cur = 0;

  for(int i=0; i<p.size(); i++){
    int id = p[i] - 'a';

    if(trie[cur][id] == -1 ){
      memset(trie[z], -1, sizeof trie[z]);
      trie[cur][id] = z++;
    }

    cur = trie[cur][id];
  }

  terminal[cur]++;
}

int count(string &p){
  int cur = 0;

  for(int i=0; i<p.size(); i++){
    int id = (p[i] - 'a');

    if(trie[cur][id] == -1) return 0;

    cur = trie[cur][id];
  }
  return terminal[cur];
}

void init(){
```

```
  memset(trie[0], -1, sizeof trie[0]);
  z = 1;
}
```

7.6 Z-Function

```
vector<int> Zfunction(string &s){ // O(N)
  int n = s.size();
  vector<int> z (n, 0);

  for(int i=1, l=0, r=0; i<n;  i++){
    if(i <= r) z[i] = min(z[i-l], r-i+1);

    while(z[i] + i < n && s[z[i]] == s[i+z[i]]) z[i]++;

    if(r < i+z[i]-1) l = i, r = i+z[i]-1;
  }

  return z;
}
```