SamuellH12 - ICPC Library

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1 Data Structures

1.1 BIT

```
714 struct BIT {
116   vector<int> bit;
060   int N;
D41
FE2   BIT(){}
1FC   BIT(int n) : N(n+1), bit(n+1){}
D41
A69   void update(int pos, int val){
685     for(; pos < N; pos += pos&(-pos))
9D0       bit[pos] += val;
35A   }
D41
4D6   int query(int pos){
A93     int sum = 0;
5D0     for(; pos > 0; pos -= pos&(-pos))
6ED       sum += bit[pos];
E66     return sum;
FF7   }
2B4 };
```

1.2 BIT2DSparse

```
Sparse Binary Indexed Tree 2D

Recebe o conjunto de pontos que serao usados para fazer os updates e as queries e cria
    uma BIT 2D esparsa que independe do "tamanho do grid".

Build: O(N Log N) (N -> Quantidade de Pontos)
Query/Update: O(Log N)
IMPORTANTE! Offline!

BIT2D(pts);   // pts -> vecotor<pii> com todos os pontos em que serao feitas queries ou
    updates
```

```
E40 #define pii pair<ll, ll>
AA8 #define upper(v, x) (upper_bound(begin(v), end(v), x) - begin(v))
```

```
4BA struct BIT2D {
D54   vector<ll> ord;
302   vector<vector<ll>> bit, coord;
D41
8A4   BIT2D(vector<pii> pts){
B03     sort(begin(pts), end(pts));
D41
7D3     for(auto [x, y] : pts)
76B       if(ord.empty() || x != ord.back())
580         ord.push_back(x);
D41
261     bit.resize(ord.size() + 1);
3EB     coord.resize(ord.size() + 1);
D41
CC7     sort(begin(pts), end(pts), [&](pii &a, pii &b){ return a.second < b.second; });
D41
7D3     for(auto [x, y] : pts)
837       for(int i=upper(ord, x); i < bit.size(); i += i&-i)
3E1         if(coord[i].empty() || coord[i].back() != y)
739           coord[i].push_back(y);
D41
A22     for(int i=0; i<bit.size(); i++) bit[i].assign(coord[i].size()+1, 0);
461   }
D41
14A   void update(ll X, ll Y, ll v){
784     for(int i = upper(ord, X); i<bit.size(); i += i&-i)
609       for(int j = upper(coord[i], Y); j < bit[i].size(); j += j&-j)
9ED         bit[i][j] += v;
5E0   }
D41
258   ll query(ll X, ll Y){
5FF     ll sum = 0;
2C2     for(int i = upper(ord, X); i > 0; i -= i&-i)
40B       for(int j = upper(coord[i], Y); j > 0; j -= j&-j)
B03         sum += bit[i][j];
E66     return sum;
414   }
D41
867   ll queryArea(ll xi, ll yi, ll xf, ll yf){
ABD     return query(xf, yf) - query(xf, yi-1) - query(xi-1, yf) + query(xi-1, yi-1);
7D1   }
D41
6DB   void updateArea(ll xi, ll yi, ll xf, ll yf, ll val){ // OPTIONAL
C02     update(xi,   yi,    val);  // DOESN'T UPDATE AN AREA!!!
061     update(xf+1, yi,   -val);  // It is like: bit1d.update(l-1, -v), bit1d.update(r, +v)
2ED     update(xi,   yf+1, -val);  // so you can do like bit1d.query(i) to see the value "at" i
2BC     update(xf+1, yf+1,  val);  // in this case, call bit2d.query(X, Y)
A75   }
4F2 };
```

1.3 SegTree

```
CD5 template<typename T> struct SegTree {
130   vector<T> seg;
060   int N;
788   T NEUTRO = T(0);
F15   SegTree(int n) : N(n) { seg.assign(4*n, NEUTRO); }
136   SegTree(vector<T> &lista) : N(lista.size()) { seg.assign(4*N); build(1, 0, N-1, lista); }
493   T join(T lv, T rv){ return lv + rv; }
D41
07D   T query(int no, int l, int r, int a, int b){
83C     if(b <  l || r <  a) return NEUTRO;
83F     if(a <= l && r <= b) return seg[no];
A48     int m=(l+r)/2, e=no*2, d=e+1;
D41
703     return join(query(e, l, m, a, b), query(d, m+1, r, a, b));
2F0   }
692   void update(int no, int l, int r, int pos, T v){
085     if(pos < l || r < pos) return;
727     if(l == r){ seg[no] = v; return; }  // set value -> change to += if sum
A48     int m=(l+r)/2, e=no*2, d=e+1;
D41
618     update(e, l,   m, pos, v);
B39     update(d, m+1, r, pos, v);
D41
F93     seg[no] = join(seg[e], seg[d]);
186   }
230   void build(int no, int l, int r, vector<T> &lista){
5FB     if(l == r){ seg[no] = lista[l]; return; }
A48     int m=(l+r)/2, e=no*2, d=e+1;
91F     build(e, l,   m, lista);
415     build(d, m+1, r, lista);
F93     seg[no] = join(seg[e], seg[d]);
F00   }
D41
367   T query(int ls, int rs){ return query (1, 0, N-1, ls, rs); }
345   void update(int pos, T v){      update(1, 0, N-1, pos, v); }
9B6 };
```

1.4 SegTree Lazy

```
-> Segment Tree - Lazy Propagation com:
  - Query em Range
  - Update em Range
  - Closed interval & 0-indexed: [L, R] & [0, N-1]
Build:  O(N)
Query:  O(log N) | seg.query(l, r);
Update: O(log N) | seg.update(l, r, v);
Unlazy: O(1)
Update Join, NEUTRO, Update and Unlazy if needed
```

```
CD5 template<typename T> struct SegTree {
130   vector<T> seg;
22C   vector<T> lazy;
060   int N;
070   T NEUTRO = 0;
DF1   SegTree(int n) : N(n){ seg.assign(4*N, NEUTRO), lazy.assign(4*N, NEUTRO); }
A94   SegTree(vector<T> &lista) : N(lista.size()){
647     seg.assign(4*N), lazy.assign(4*N, NEUTRO);
575     build(1, 0, N-1, lista);
713   }
493   T join(T lv, T rv){ return lv + rv; }
6B5   void unlazy(int no, int l, int r){
1BB     if(lazy[no] == NEUTRO) return;
A48     int m=(l+r)/2, e=no*2, d=e+1;
D41
5A7     seg[no] += (r-l+1) * lazy[no]; /// Range Sum
D41
1EF     if(l != r) lazy[e] += lazy[no], lazy[d] += lazy[no];
47C     lazy[no] = NEUTRO;
9F0   }
D41
07D   T query(int no, int l, int r, int a, int b){
5C5     unlazy(no, l, r);
83C     if(b <  l || r <  a) return NEUTRO;
83F     if(a <= l && r <= b) return seg[no];
A48     int m=(l+r)/2, e=no*2, d=e+1;
D41
703     return join(query(e, l, m, a, b), query(d, m+1, r, a, b));
E4D   }
```

```
D41
DC1   void update(int no, int l, int r, int a, int b, T v){
5C5     unlazy(no, l, r);
2E6     if(b <  l || r <  a) return;
02B     if(a <= l && r <= b){
7CC       lazy[no] = join(lazy[no], v); // cumulative?
8DC       return unlazy(no, l, r);
13F     }
A48     int m=(l+r)/2, e=no*2, d=e+1;
D41
142     update(e, l,   m, a, b, v);
9D3     update(d, m+1, r, a, b, v);
D41
F93     seg[no] = join(seg[e], seg[d]);
B3A   }
D41
230   void build(int no, int l, int r, vector<T> &lista){
5FB     if(l == r){ seg[no] = lista[l]; return; }
A48     int m=(l+r)/2, e=no*2, d=e+1;
D41
91F     build(e, l,   m, lista);
415     build(d, m+1, r, lista);
D41
F93     seg[no] = join(seg[e], seg[d]);
F00   }
D41
367   T query(int ls, int rs){ return query (1, 0, N-1, ls, rs); }
62C   void update(int l, int r, T v){ update(1, 0, N-1, l, r, v); }
2DE };
```

1.5 SegTree Persistente

```
-> Segment Tree Persistente: (2x mais rapido que com ponteiro)
Build(1, N) -> Cria uma Seg Tree completa de tamanho N; RETORNA o NodeId da Raiz
Update(Root, pos, v) -> Soma +V em POS; RETORNA o NodeId da nova Raiz;
Query(Root, a, b) -> RETORNA o valor do range [a, b];
Kth(RootL, RootR, K) -> Faz uma Busca Binaria na Seg de diferenca entre as duas versoes.
[ Root -> No Raiz da Versao da Seg na qual se quer realizar a operacao ]

Build:  O(N)  !!! Sempre chame o Build
Query:  O(log N)
Update: O(log N)
Kth:  O(Log N)

Comportamento do K-th(SegL, SegR, 1, N, K):
  -> Retorna indice da primeira posicao i cuja soma de prefixos [1, i] e >= k na Seg
     resultante da subtracao dos valores da (Seg R) - (Seg L).
  -> Pode ser utilizada para consultar o K-esimo menor valor no intervalo [L, R] de um
     array.
  A Seg deve ser utilizada como um array de frequencias. Comece com a Seg zerada
     (Build).
  Para cada valor V do Array chame um update(roots.back(), 1, N, V, 1) e guarde o
     ponteiro da seg.
  Consultar o K-esimo menor valor de [L, R]: chame kth(roots[L-1], roots[R]);
```

```
80E const int MAXN = 1e5 + 5;
2D8 const int MAXLOG = 31 - __builtin_clz(MAXN) + 1;
4B4 typedef int NodeId;
6E2 typedef int STp;

EA9 const STp NEUTRO = 0;
B50 int IDN, LSEG, RSEG;
519 extern struct Node NODES[];

BF2 struct Node {
AEE   STp val;
1BC   NodeId L, R;
9DA   Node(STp v = NEUTRO) : val(v), L(-1), R(-1) {}
2F4   Node& l(){ return NODES[L]; }
F2E   Node& r(){ return NODES[R]; }
5A4 };

318 Node NODES[4*MAXN + MAXLOG*MAXN]; //!!!CUIDADO COM O TAMANHO (aumente se necessario)
1E7 pair<Node&, NodeId> newNode(STp v = NEUTRO){ return {NODES[IDN] = Node(v), IDN++}; }

C3F STp join(STp lv, STp rv){ return lv + rv; }

8B5 NodeId build(int l, int r, bool root=true){
85B   if(root) LSEG = l, RSEG = r;
844   if(l == r) return newNode().second;
D41
EE4   int m = (l+r)/2;
DC6   auto [node, id] = newNode();
D41
C12   node.L = build(l,   m, false);
373   node.R = build(m+1, r, false);
45D   node.val = join(node.l().val, node.r().val);
D41
648   return id;
9D5 }

2F1 NodeId update(NodeId node, int l, int r, int pos, int v){
703   if( pos < l || r < pos ) return node;
D99   if(l == r) return newNode(NODES[node].val + v).second;
D41
EE4   int m = (l+r)/2;
BE4   auto [nw, id] =newNode();
D41
E2C   nw.L = update(NODES[node].L, l,   m, pos, v);
D4A   nw.R = update(NODES[node].R, m+1, r, pos, v);
D41
6EC   nw.val = join(nw.l().val, nw.r().val);
D41
648   return id;
938 }
8C0 NodeId update(NodeId node, int pos, STp v){ return update(node, LSEG, RSEG, pos, v); }

BFA int query(Node& node, int l, int r, int a, int b){
83C   if(b <  l || r <  a) return NEUTRO;
65A   if(a <= l && r <= b) return node.val;
D41
EE4   int m = (l+r)/2;
D41
083   return join(query(node.l(), l, m, a, b), query(node.r(), m+1, r, a, b));
7B5 }
8B3 int query(NodeId node, int a, int b){ return query(NODES[node], LSEG, RSEG, a, b); }

D0A int kth(Node& Left, Node& Right, int l, int r, int k){
3CE   if(l == r) return l;
D41
A3B   int sum =Right.l().val - Left.l().val;
EE4   int m = (l+r)/2;
D41
BBB   if(sum >= k) return kth(Left.l(), Right.l(), l, m, k);
5D8   return kth(Left.r(), Right.r(), m+1, r, k - sum);
9D7 }
A8D int kth(NodeId Left, NodeId Right, int k){ return kth(NODES[Left], NODES[Right], LSEG,
    RSEG, k); }
```

1.6 SegTree Iterativa

```
CD5 template<typename T> struct SegTree {
1A8   int n;
130   vector<T> seg;
F93   T join(T&l, T&r){ return l + r; }
D41
5A8   SegTree(int n) : n(n), seg(2*n) {}
BD8   SegTree(){}
D5D   void init(vector<T>&base){
FC7     n = base.size();
A61     seg.resize(2*n);
8DB     for(int i=0; i<n; i++) seg[i+n] = base[i];
2E1     for(int i=n-1; i>0; i--) seg[i] = join(seg[i*2], seg[i*2+1]);
D60   }
D41
B7A   T query(int l, int r){ //[L, R] // [0, n-1]
7DE     T lp = 0, rp = 0; //NEUTRO
706     for(l+=n, r+=n+1; l<r; l/=2, r/=2){
8C0       if(l&1) lp = join(lp, seg[l++]);
A01       if(r&1) rp = join(seg[--r], rp);
FE5     }
757     return join(lp, rp);
7E8   }
D41
FB2   void update(int i, T v){ // Set Value seg[i+=n] = v // change to += v to sum
CBC     for(seg[i+=n] = v; i/=2;) seg[i] = join(seg[i*2], seg[i*2+1]);
5E8   }
406 };
```

1.7 SegTree Lazy Iterativa

```
CD5 template<typename T> struct SegTree {
D16   int n, h;
070   T NEUTRO = 0;
97F   vector<T> seg, lzy;
1DF   vector<int> sz;
F93   T join(T&l, T&r){ return l + r; }
D41
5C7   void init(int _n){
8FD     n = _n;
704     h = 32 - __builtin_clz(n);
A61     seg.resize(2*n);
A88     lzy.assign(n, NEUTRO);
528     sz.resize(2*n, 1);
E3F     for(int i=n-1; i; i--) sz[i] = sz[i*2] + sz[i*2+1];
D41     // for(int i=0; i<n; i++) seg[i+n] = base[i];
D41     // for(int i=n-1; i; i--) seg[i] = join(seg[i*2], seg[i*2+1]);
95C   }
D41
45B   void apply(int p, T v){
13A     seg[p] += v * sz[p]; // cumulative?
9F8     if(p < n) lzy[p] += v;
853   }
3B4   void push(int p){
835     for(int s=h, i=p>>s; s; s--, i=p>>s)
E15       if(lzy[i] != 0) {
561         apply(i*2,   lzy[i]);
1AD         apply(i*2+1, lzy[i]);
16B         lzy[i] = NEUTRO; //NEUTRO
227       }
3C7   }
F6E   void build(int p) {
5B2     for(p/=2; p; p/= 2){
F12       seg[p] = join(seg[p*2], seg[p*2+1]);
C3C       if(lzy[p] != 0) seg[p] += lzy[p] * sz[p];
D65     }
972   }
D41
B7A   T query(int l, int r){ //[L, R] & [0, n-1]
0ED     l+=n, r+=n+1;
F4B     push(l); push(r-1);
D41
821     T lp = NEUTRO, rp = NEUTRO; //NEUTRO
DC6     for(; l<r; l/=2, r/=2){
8C0       if(l&1) lp = join(lp, seg[l++]);
A01       if(r&1) rp = join(seg[--r], rp);
833     }
BA7     return ans;
F57   }
D41
FAB   void update(int l, int r, T v){
0ED     l+=n, r+=n+1;
F4B     push(l); push(r-1);
D41
98D     int l0 = l, r0 = r;
DC6     for(; l<r; l/=2, r/=2){
5D1       if(l&1) apply(l++, v);
E94       if(r&1) apply(--r, v);
55B     }
FE7     build(l0); build(r0-1);
E29   }
AEB };
```

1.8 SegTreeSparse

```
4C9 template<typename T> struct SparseSeg {
BF2   struct Node {
CEE     T val = 0; Node *L = NULL, *R = NULL;
942     Node(T v = 0) : val(v), L(NULL), R(NULL) {}
06D   };
223   int n; Node* root;
CA2   SparseSeg(int n) : n(n){ root = new Node(NEUTRO); }
CFB   T join(T lv, T rv){ return max(lv, rv); }
D2F   const T NEUTRO = 0;
D41
B2C   T query(Node*& no, int l, int r, int a, int b){
502     if(b <  l || r <  a || no == NULL) return NEUTRO;
984     if(a <= l && r <= b) return no->val;
EE4     int m=(l+r)/2;
D25     return join(query(no->L, l, m, a, b), query(no->R, m+1, r, a, b));
870   }
B80   void update(Node*& no, int l, int r, int pos, T v){
DAB     if(!no) no = new Node(NEUTRO);
085     if(pos < l || r < pos) return;
1D2     if(l == r) return (void)(no->val = v);
EE4     int m=(l+r)/2;
D41
687     update(no->L,  l,  m, pos, v);
712     update(no->R, m+1, r, pos, v);
D41
FEE     no->val = join(no->L->val, no->R->val);
8D6   }
D41
55D   void update(int pos, T v){ update(root, 0, n, pos, v); }
93C   T query(int a, int b){ return query(root, 0, n, a, b); }
AE6 };
```

1.9 SegTreeSparseLazy

```
4C9 template<typename T> struct SparseSeg {
BF2   struct Node {
C2E     T val = 0, lazy=0; Node *L = NULL, *R = NULL;
942     Node(T v = 0) : val(v), L(NULL), R(NULL) {}
481   };
223   int n; Node* root;
```

```
CA2   SparseSeg(int n) : n(n){ root = new Node(NEUTRO); }
F2A   T join(T lv, T rv){ return (lv + rv); }
D2F   const T NEUTRO = 0;
D41
B2C   T query(Node*& no, int l, int r, int a, int b){
163     if(!no) no = new Node(NEUTRO); unlazy(no, l, r);
83C     if(b <  l || r <  a) return NEUTRO;
984     if(a <= l && r <= b) return no->val;
EE4     int m=(l+r)/2;
D25     return join(query(no->L, l, m, a, b), query(no->R, m+1, r, a, b));
DAE   }
D1B   void update(Node*& no, int l, int r, int a, int b, T v){
163     if(!no) no = new Node(NEUTRO); unlazy(no, l, r);
2E6     if(b <  l || r <  a) return;
DBA     if(a <= l && r <= b){ no->lazy += v; return unlazy(no, l, r); }
EE4     int m=(l+r)/2;
D41
1F1     update(no->L,  l,  m, a, b, v);
E9D     update(no->R, m+1, r, a, b, v);
D41
FEE     no->val = join(no->L->val, no->R->val);
D9F   }
09A   void unlazy(Node* no, int l, int r){ //delete this if not lazy
525     if(no->lazy == 0) return;
579     if(l != r){
15E       if(!no->L) no->L = new Node(no->val);
C04       if(!no->R) no->R = new Node(no->val);
45D       no->L->lazy += no->lazy;
C12       no->R->lazy += no->lazy;
626     }
BB9     no->val += no->lazy * (r-l+1);
82A     no->lazy = 0;
1BA   }
D41
27D   void update(int a, int b, T v){ update(root, 0, n, a, b, v); }
199   void update(int pos, T v){ update(root, 0, n, pos, pos, v); }
93C   T query(int a, int b){ return query(root, 0, n, a, b); }
BB7 };
```

1.10 SparseTable

```
Sparse Table for Range Minimum Query [L, R] [0, N-1]
build:  O(N log N)   Query:  O(1)
build(v) -> v = Original Array
if you want a static array, do this: for(int i=0; i<N; i++) table[0][i] = v[i];
```

```
875 template<typename T> struct Sparse {
F9A   vector<vector<T>> table;
D41
985   void build(vector<T> &v){
128     int N = v.size(), MLOG = 32 - __builtin_clz(N);
554     table.assign(MLOG, v);
D41
DAD     for(int p=1; p < MLOG; p++)
13B       for(int i=0; i + (1 << p) <= N; i++)
67C         table[p][i] = min(table[p-1][i], table[p-1][i+(1<<(p-1))]);
215   }
D41
B7A   T query(int l, int r){
796     int p = 31 - __builtin_clz(r - l + 1);  //floor log
E56     return min(table[p][l], table[p][ r - (1<<p)+1 ]);
3C2   }
B78 };
```

1.11 orderedSet

```
30F #include <ext/pb_ds/tree_policy.hpp>
774 #include <ext/pb_ds/assoc_container.hpp>
0D7 using namespace __gnu_pbds;
7AF template <class T> using ordered_set = tree<T, null_type,  less<T>, rb_tree_tag,
     tree_order_statistics_node_update>;
816 template <class K, class V> using ordered_map = tree<K, V, less<K>, rb_tree_tag,
     tree_order_statistics_node_update>;

339 ordered_set<int> os;
1C6 int okey = os.order_of_key(k);  // Number of items strictly smaller than K
398 auto kth = os.find_by_order(k); // pointer to K-th element in set (0-index)
```

2 dp

2.1 LineContainer

```
72C struct Line {
3E2   mutable ll k, m, p;
CA5   bool operator<(const Line& o) const { return k < o.k; }
ABF   bool operator<(ll x) const { return p < x; }
7E3 };

781 struct LineContainer : multiset<Line, less<>> {
FD2   static const ll inf = LLONG_MAX; // Double: inf = 1/.0, div(a,b) = a/b
10F   ll div(ll a, ll b) { return a / b - ((a ^ b) < 0 && a % b); } //floored division
D41
A1C   bool isect(iterator x, iterator y) {
A95     if(y == end()) return x->p = inf, 0;
9CB     if(x->k == y->k) x->p = x->m > y->m ? inf : -inf;
591     else x->p = div(y->m - x->m, x->k - y->k);
870     return x->p >= y->p;
2FA   }
D41
141   void add_line(ll k, ll m){ // kx + m  //if minimum k*=-1, m*=-1, query*-1
116     auto z = insert({k, m, 0}), y = z++, x = y;
7B1     while(isect(y, z)) z = erase(z);
141     if(x != begin() && isect(--x, y)) isect(x, y = erase(y));
1A4     while((y = x) != begin() && (--x)->p >= y->p) isect(x, erase(y));
17C   }
D41
4AD   ll query(ll x) {
229     assert(!empty());
7D1     auto l = *lower_bound(x);
96A     return l.k * x + l.m;
D21   }
0B9 };
```

2.2 Digit DP

```
Digit DP - Sum of Digits - O(D^2 * B^2)  (B = Base = 10)

Solve(K)  ->  Retorna a soma dos digitos de todo numero X tal que: 0 <= X <= K
dp[D][S][f]  ->  D: Quantidade de digitos; S: Soma dos digitos; f: Flag que indica o
     limite.
int limite[D] -> Guarda os digitos de K.
```

```
EF8 ll dp[2][19][170];

EFF int limite[19];
67A ll digitDP(int idx, int sum, bool flag){
A56     if(idx < 0) return sum;
```

```
FA7     if(~dp[flag][idx][sum]) return dp[flag][idx][sum];
D41
6C1     dp[flag][idx][sum] = 0;
F61   int lm = flag ? limite[idx] : 9;
D41
8DA     for(int i=0; i<=lm; i++)
41E         dp[flag][idx][sum] += digitDP(idx-1, sum+i, (flag && i == lm));
D41
FCB     return dp[flag][idx][sum];
20C }

8E6 ll solve(ll k){
773     memset(dp, -1, sizeof dp);
D41
1FC   int sz=0;
95F   while(k){
BE0     limite[sz++] = k % 10LL;
9F1     k /= 10LL;
24A   }
D41
B58   return digitDP(sz-1, 0, true);
766 }
```

2.3 LCS

```
-> LCS - Longest Common Subsequence O(N^2)
* If recursive: memset(memo, -1, sizeof memo); LCS(0, 0);
* RecoverLCS O(N) Recover just one of all the possible LCS
```

```
A2C const int MAXN = 5*1e3 + 5;
DD0 int memo[MAXN][MAXN];

AC1 string s, t;

478 inline int LCS(int i, int j){
BF8   if(i == s.size() || j == t.size()) return 0;
B5D   if(memo[i][j] != -1) return memo[i][j];
D41
052   if(s[i] == t[j]) return memo[i][j] = 1 + LCS(i+1, j+1);
D41
A17   return memo[i][j] = max(LCS(i+1, j), LCS(i, j+1));
F66 }

406 int LCS_It(){
A17   for(int i=s.size()-1; i>=0; i--)
377     for(int j=t.size()-1; j>=0; j--)
1A9       if(s[i] == t[j])
23E         memo[i][j] = 1 + memo[i+1][j+1];
295       else
1EE         memo[i][j] = max( memo[i+1][j], memo[i][j+1] );
D41
0C2   return memo[0][0];
67C }

DBD string RecoverLCS(int i, int j){
F34   if(i == s.size() || j == t.size()) return "";
D41
134   if(s[i] == t[j]) return s[i] + RecoverLCS(i+1, j+1);
D41
495   if(memo[i+1][j] > memo[i][j+1]) return RecoverLCS(i+1, j);
D41
DCC   return RecoverLCS(i, j+1);
5E7 }
```

2.4 LIS

```
-> LIS - Longest Increasing Subsequence - O(N Log N)
* For ICREASING sequence, use lower_bound()
* For NON DECREASING sequence, use upper_bound()
```

```
7A6 int LIS(vector<int>& nums){
0FF   vector<int> lis;
D41
7F4   for(auto x : nums){
3D0     auto it = lower_bound(lis.begin(), lis.end(), x);
CDF     if(it == lis.end()) lis.push_back(x);
77C     else *it = x;
795   }
737   return (int) lis.size();
F27 }
```

2.5 SOS DP

```
-> SOS DP - Sum over Subsets

Dado que cada mask possui um valor inicial (iVal), computa
para cada mask a soma dos valores de todas as suas submasks.

N -> Numero Maximo de Bits
iVal[mask] -> initial Value / Valor Inicial da Mask
dp[mask]   -> Soma de todos os SubSets

Iterar por todas as submasks: for(int sub=mask; sub>0; sub=(sub-1)&mask)
```

```
F17 const int N = 20;
0A7 ll dp[1<<N], iVal[1<<N];

B70 void sosDP(){ // O(N * 2^N)
8CC     for(int i=0; i<(1<<N); i++)
0B3         dp[i] = iVal[i];
D41
972   for(int i=0; i<N; i++)
D57     for(int mask=0; mask<(1<<N); mask++)
281       if(mask&(1<<i))
E0E         dp[mask] += dp[mask^(1<<i)];
E5B }

7E1 void sosDPsub(){ // O(3^N) //suboptimal
EA1   for (int mask = 0, i; mask < (1<<N); mask++)
CC7     for(i = mask, dp[mask] = iVal[0]; i>0; i=(i-1) & mask) //iterate over all submasks
85B       dp[mask] += iVal[i];
986 }
```

3 Grafos

3.1 2-SAT

```
2 SAT - Two Satisfiability Problem
Retorna uma valoracao verdadeira se possivel ou um vetor vazio se impossivel;
inverso de u = ~u
```

A	B	OR	AND	NOR	NAND	XOR	XNOR	IMPLY
0	0	0	0	1	1	0	1	1
0	1	1	0	0	1	1	0	1
1	0	1	0	0	1	1	0	0
1	1	1	1	0	0	0	1	1

```
D9D struct TwoSat {
060   int N;
67E   vector<vector<int>> E;
D41
662   TwoSat(int N) : N(N), E(2 * N) {}
3E1   inline int eval(int u) const{ return u < 0 ? ((~u)+N)%(2*N) : u; }
D41
B0E   void add_or(int u, int v){
245     E[eval(~u)].push_back(eval(v));
F37     E[eval(~v)].push_back(eval(u));
30A     }
4B9   void add_nand(int u, int v) {
9FA     E[eval(u)].push_back(eval(~v));
CED     E[eval(v)].push_back(eval(~u));
D1C   }
CEB   void set_true (int u){ E[eval(~u)].push_back(eval(u)); }
5A5   void set_false(int u){ set_true(~u); }
286   void add_imply(int u, int v){ E[eval(u)].push_back(eval(v)); }
E81   void add_and  (int u, int v){ set_true(u);  set_true(v);    }
347   void add_nor  (int u, int v){ add_and(~u, ~v); }
A32   void add_xor  (int u, int v){ add_or(u, v); add_nand(u, v); }
F65   void add_xnor (int u, int v){ add_xor(u, ~v); }
D41
28E   vector<bool> solve() {
F18     vector<bool> ans(N);
F40     auto scc = tarjan();
D41
51F     for (int u = 0; u < N; u++)
FC2       if(scc[u] == scc[u+N]) return {}; //false
951       else ans[u] = scc[u+N] > scc[u];
D41
BA7     return ans; //true
166   }
BF2 private:
401   vector<int> tarjan() {
798     vector<int> low(2*N), pre(2*N, -1), scc(2*N, -1), st;
226     int clk = 0, ncomps = 0;
D41
214     function<void(int)> dfs = [&](int u){
FD2       pre[u] = low[u] = clk++;
2D9       st.push_back(u);
D41
7F2       for(auto v : E[u])
3C0         if(pre[v] == -1) dfs(v), low[u] = min(low[u], low[v]);
295         else
16E         if(scc[v] == -1) low[u] = min(low[u], pre[v]);
D41
8AD       if(low[u] == pre[u]){
78B         int v = -1;
931         while(v != u) scc[v = st.back()] = ncomps, st.pop_back();
9DF         ncomps++;
B25       }
601     };
D41
438     for(int u=0; u < 2*N; u++)
DC6       if(pre[u] == -1)
512         dfs(u);
D41
9AB     return scc; //tarjan SCCs order is the reverse of topoSort, so (u->v if scc[v] <=
      scc[u])
094   }
4BB };
```

3.2 BlockCutTree

Block Cut Tree - BiConnected Component

```
BlockCutTree bcc(n);
bcc.addEdge(u, v);
bcc.build();

bcc.tree    -> graph of BlockCutTree (tree.size() <= 2n)
bcc.id[u]   -> componet of u in the tree
bcc.cut[u]  -> 1 if u is a cut vertex; 0 otherwise
bcc.comp[i] -> vertex of comp i (cut are part of multiple comp)
```

```
142 struct BlockCutTree {
0AD     vector<vector<int>> g, tree, comp;
657     vector<int> id, cut;
40B     BlockCutTree(int n) : n(n), g(n), cut(n) {}
D41
FAE     void addEdge(int u, int v){
7EA         g[u].emplace_back(v);
4A3         g[v].emplace_back(u);
1DB     }
D41
0A8     void build(){
9AB         pre = low = id = vector<int>(n, -1);
D0A         for(int u=0; u<n; u++, chd=0) if(pre[u] == -1) //if graph is disconected
86E             tarjan(u, -1), makeComp(-1);                //find cut vertex and make
    components
D41
35C         for(int u=0; u<n; u++) if(cut[u]) comp.emplace_back(1, u); //create cut
    components
584         for(int i=0; i<comp.size(); i++)                           //mark id of each
    node
679             for(auto u : comp[i]) id[u] = i;
D41
6A6         tree.resize(comp.size());
584         for(int i=0; i<comp.size(); i++)
5AE             for(auto u : comp[i]) if(id[u] != i)
30E                 tree[i].push_back(id[u]),
D8D                 tree[id[u]].push_back(i);
1D5     }
BF2 private:
5D4     vector<int> pre, low;
EA9     vector<pair<int, int>> st;
226     int n, clk = 0, chd=0, ct, a, b;
D41
20D     void makeComp(int u){
DAB         comp.emplace_back();
016         do {
986             tie(a, b) = st.back();
D73             st.pop_back();
71A             comp.back().push_back(b);
203         } while(a != u);
7C1         if(~u) comp.back().push_back(u);
5CF     }
D41
701     void tarjan(int u, int p){
FD2         pre[u] = low[u] = clk++;
5C6         st.emplace_back(p, u);
D41
DD3         for(auto v : g[u]) if(v != p){
EE1             if(pre[v] == -1){
3D2                 tarjan(v, u);
AB6                 low[u]  =  min(low[u], low[v]);
30C                 cut[u] |= ct = (~p && low[v] >= pre[u]) || (p==-1 && ++chd >= 2);
10E                 if(ct) makeComp(u);
995             }
553             else low[u] = min(low[u], pre[v]);
AC4         }
0D9     }
```

```
D8F };
```

3.3 Centroid Decomposition

```
Centroid Decomposition - O(N*LogN)

dfsc()    -> para criar a centroid tree
rem[u]    -> True se U ja foi removido (pra dfsc)
szt[u]    -> Size da subarvore de U (pra dfsc)
parent[u] -> Pai de U na centroid tree *parent[ROOT] = -1
dist[u][i]-> Distancia na arvore original de u p i-esimo pai na centroid tree
     *distToAncestor[u][0] = 0

dfsc(u=node, p=parent(subtree), f=parent(centroid tree), sz=size of tree)
```

```
229 const int MAXN = 1e6 + 5;

282 vector<int> g[MAXN];
CAF deque<int> dist[MAXN];

C76 bool rem[MAXN];
BBD int szt[MAXN], parent[MAXN];

1B0 void getDist(int u, int p, int d=0){
384   for(auto v : g[u]) if(v != p && !rem[v])
334     getDist(v, u, d+1);
4BF   dist[u].emplace_front(d);
644 }

3A5 int getSz(int u, int p){
030   szt[u] = 1;
384   for(auto v : g[u]) if(v != p && !rem[v])
35F     szt[u] += getSz(v, u);
865   return szt[u];
CEC }

994 void dfsc(int u=0, int p=-1, int f=-1, int sz=-1){
C0F   if(sz < 0) sz = getSz(u, -1); //starting new tree
D41
70D   for(auto v : g[u])
E5C     if(v != p && !rem[v] && szt[v]*2 >= sz)
6F7       return dfsc(v, u, f, sz);
D41
2EA   rem[u] = true, parent[u] = f;
C5E   getDist(u, -1, 0); //get subtree dists to centroid
D41
C79   for(auto v : g[u]) if(!rem[v])
D8F     dfsc(v, u, u, -1);
16D }
```

3.4 Dijkstra

```
51C #define INF 0x3f3f3f3f3f3f3f3f
E40 #define pii pair<ll,ll>

161 vector<pii> g[MAXN];

F22 vector<ll> dijkstra(int s, int N){
187   vector<ll> dist (N, INF);
F37   priority_queue<pii, vector<pii>, greater<pii>> pq;
7BA   pq.push({0, s});
A93   dist[s] = 0;
D41
502   while(!pq.empty()){
2F9     auto [d, u] = pq.top();
716     pq.pop();
3E1     if(d > dist[u]) continue;
D41
706     for(auto [v, c] : g[u])
511       if( dist[v] > dist[u] + c ){
085         dist[v] = dist[u] + c;
BF3         pq.push({dist[v], v});
F86       }
BE3   }
8D7   return dist;
D99 }
```

3.5 DSU Rollback

```
Disjoint Set Union with Rollback - O(Log n)
checkpoint() -> salva o estado atual
rollback() -> restaura no ultimo checkpoint
save another var? +save in join & +line in pop
```

```
4EA struct DSUr {
ECD   vector<int> pai, sz, savept;
D35   stack<pair<int&, int>>  st;
EB0   DSUr(int n) : pai(n+1), sz(n+1, 1) {
51E     for(int i=0; i<=n; i++) pai[i] = i;
6CE   }
D41
43F   int find(int u){ return pai[u] == u ? u : find(pai[u]); }
D41
AF9   void join(int u, int v){
B80     u = find(u), v = find(v);
D41
360     if(u == v) return;
844     if(sz[v] > sz[u]) swap(u, v);
D41
A60     save(pai[v]); pai[v] = u;
5DA     save(sz[u]);  sz[u] += sz[v];
047   }
D41
2D0   void save(int &x){ st.emplace(x, x); }
42D   void pop(){
6A1     st.top().first = st.top().second; st.pop();
6A1     st.top().first = st.top().second; st.pop();
4DD   }
D41
6E6   void checkpoint(){ savept.push_back(st.size()); }
5CF   void rollback(){
8EB     while(st.size() > savept.back()) pop();
520     savept.pop_back();
BB2   }
9E2 };
```

3.6 DSU Persistente

```
SemiPersistent DSU - O(Log n)
find(u, q) -> Retorna o pai de U no tempo q
* tim -> tempo em que o pai de U foi alterado
```

```
2CE struct DSUp {
```

```
AE4   vector<int> pai, sz, tim;
258   int t=1;
910   DSUp(int n) : pai(n+1), sz(n+1, 1), tim(n+1) {
51E     for(int i=0; i<=n; i++) pai[i] = i;
50F   }
D41
7F9   int find(int u, int q = INT_MAX){
568     if( pai[u] == u || q < tim[u] ) return u;
8B3     return find(pai[u], q);
0A1   }
D41
AF9   void join(int u, int v){
B80     u = find(u), v = find(v);
D41
360     if(u == v) return;
844     if(sz[v] > sz[u]) swap(u, v);
D41
555     pai[v] = u;
36E     tim[v] = t++;
CC3     sz[u] += sz[v];
8D8   }
96D };
```

3.7 Euler Path

```
Euler Path - Algoritmo de Hierholzer para caminho Euleriano - O(V + E)
IMPORTANTE! O algoritmo esta 0-indexado

* Informacoes
  addEdge(u, v) -> Adiciona uma aresta de U para V
  EulerPath(n) -> Retorna o Euler Path, ou um vetor vazio se impossivel
  vi path -> vertices do Euler Path na ordem
  vi pathId -> id das Arestas do Euler Path na ordem

Euler em Undirected graph:
  - Cada vertice tem um numero par de arestas (circuito); OU
  - Exatamente dois vertices tem um numero impar de arestas (caminho);
Euler em Directed graph:
  - Cada vertice tem quantidade de arestas |entrada| == |saida| (circuito); OU
  - Exatamente 1 tem |entrada|+1 == |saida| && exatamente 1 tem |entrada| == |saida|+1
     (caminho);
* Circuito -> U e o primeiro e ultimo
* Caminho  -> U e o primeiro e V o ultimo
```

```
0C1 #define vi vector<int>

210 const bool BIDIRECIONAL = true;
161 vector<pii> g [MAXN];
CBD vector<bool> used;

FAE void addEdge(int u, int v){
F07   g[u].emplace_back(v, used.size()); if(BIDIRECIONAL && u != v)
F57   g[v].emplace_back(u, used.size());
EDA   used.emplace_back(false);
A16 }

EFB pair<vi, vi> EulerPath(int n, int src=0){
79C   int s=-1, t=-1;
E4D   vector<int> selfLoop(n*BIDIRECIONAL, 0);
D41
C30   if(BIDIRECIONAL)
F95   {
BC5     for(int u=0; u<n; u++) for(auto&[v, id] : g[u]) if(u==v) selfLoop[u]++;
19E     for(int u=0; u<n; u++)
181       if((g[u].size() - selfLoop[u])%2)
A4F         if(t != -1) return {vi(), vi()};   // mais que 2 com grau impar
F8A         else t = s, s = u;
D41
C0E     if(t == -1 && t != s) return {vi(), vi()}; // so 1 com grau impar
E78     if(s == -1 || t == src) s = src;           // se possivel, seta start como src
0D3   }
295   else
F95   {
8E2     vector<int> in(n, 0), out(n, 0);
D41
19E     for(int u=0; u<n; u++)
006       for(auto [v, edg] : g[u])
8C0         in[v]++, out[u]++;
D41
19E     for(int u=0; u<n; u++)
074       if(in[u] - out[u] == -1 && s == -1) s = u;  else
3C0       if(in[u] - out[u] ==  1 && t == -1) t = u;  else
825       if(in[u] !=out[u]) return {vi(), vi()};
D41
755     if(s == -1 && t == -1) s = t = src;          // se possivel, seta s como src
A6E     if(s == -1 && t != -1) return {vi(), vi()}; // Existe S mas nao T
1E2     if(s != -1 && t == -1) return {vi(), vi()}; // Existe T mas nao S
9D3   }
D41
460   for(int i=0; g[s].empty() && i<n; i++) s =(s+1)%n; //evita s ser vertice isolado
D41
D41   /////// DFS ///////
66A   vector<int> path, pathId, idx(n, 0);
982   stack<pii> st; // {Vertex, EdgeId}
D1E   st.push({s, -1});
D41
6F2   while(!st.empty()){
723     auto [u, edg] = st.top();
E44     while(idx[u] < g[u].size() && used[g[u][idx[u]].second]) idx[u]++;
D41
F22     if(idx[u] < g[u].size()){
EED       auto [v, id] = g[u][idx[u]];
3C1       used[id] = true;
F26       st.push({v, id});
5E2       continue;
C50     }
D41
960     path.push_back(u);
E1A     pathId.push_back(edg);
25A     st.pop();
CFD   }
D41
301   pathId.pop_back();
023   reverse(begin(path),   end(path));
6FF   reverse(begin(pathId), end(pathId));
D41
D41   ///  Grafo conexo ? ///
ADC   int edgesTotal = 0;
B9F   for(int u=0; u<n; u++) edgesTotal += g[u].size() + (BIDIRECIONAL ? selfLoop[u] : 0);
0A8   if(BIDIRECIONAL) edgesTotal /= 2;
934   if(pathId.size() != edgesTotal) return {vi(), vi()};
D41
438   return {path, pathId};
861 }
```

3.8 LCA

```
  LCA - Lowest Common Ancestor - Binary Lifting - O(Log N) - Build O(N Log N)
Encontrar o menor ancestral comum entre dois vertices em uma arvore enraizada
IMPORTANTE! O algoritmo esta 0-indexado
  -> chame dfs(root, root) para calcular o pai e a altura de cada vertice
```

```
    -> chame buildBL() para criar a matriz do Binary Lifting
    -> chame lca(u, v) para encontrar o menor ancestral comum
    bl[i][u] -> Binary Lifting com o (2^i)-esimo pai de u
    lvl[u]   -> Altura ou level de U na arvore
```

```
9EC const int MAXN = 5e5 + 5;
256 const int MAXLG = 20;

282 vector<int> g[MAXN];
A87 int bl[MAXLG][MAXN], lvl[MAXN];

80E void dfs(int u, int p, int l=0){
34C   lvl[u] = l;
4FB   bl[0][u] = p;
D41
E8B   for(auto v : g[u]) if(v != p)
0C5     dfs(v, u, l+1);
671 }

555 void buildBL(int N){
977   for(int i=1; i<MAXLG; i++)
51F     for(int u=0; u<N; u++)
69C       bl[i][u] = bl[i-1][bl[i-1][u]];
59A }

310 int lca(int u, int v){
DC4   if(lvl[u] < lvl[v]) swap(u, v);
D41
D07   for(int i=MAXLG-1; i>=0; i--)
179     if(lvl[u] - (1<<i) >= lvl[v])
319       u = bl[i][u];
D41
60E   if(u == v) return u;
D41
D07   for(int i=MAXLG-1; i>=0; i--)
BFA     if(bl[i][u] != bl[i][v])
E01       u = bl[i][u],
4BC       v = bl[i][v];
D41
68E   return bl[0][u];
381 }
```

3.9 HLD

```
Heavy-Light Decomposition

Complexity: O(LogN * (qry || updt))

Change qry(l, r) and updt(l, r) to call a query and update structure of your will

HLD hld(n); //call init
hld.add_edges(u, v); //add all edges
hld.build(); //Build everthing for HLD

tin[u] -> Pos in the structure (Seg, Bit, ...)
nxt[u] -> Head/Endpoint
IMPORTANTE! o grafo deve estar 0-indexado!
```

```
EAA const bool EDGE = false;
403 struct HLD {
673 public:
789   vector<vector<int>> g;
575   vector<int> sz, parent, tin, nxt;
011   SegTree<int> seg;
C05   HLD() : seg(1) {}
08E   HLD(int n) : seg(n) { init(n); }
940   void init(int n){
341     t = 0; seg = SegTree<int>(n);
8F5     g.resize(n); tin.resize(n);
7BA     sz.resize(n);nxt.resize(n);
62B     parent.resize(n);
94C   }
FAE   void addEdge(int u, int v){
7EA     g[u].emplace_back(v);
4A3     g[v].emplace_back(u);
1DB   }
1F8   void build(int root=0){
E4A     nxt[root]=root;
043     dfs(root, root);
7D9     hld(root, root);
F40   }
3D1   ll query_path(int u, int v){
0E8     if(tin[u] < tin[v]) swap(u, v);
D63     if(nxt[u] == nxt[v]) return qry(tin[v]+EDGE, tin[u]);
7C8     return qry(tin[nxt[u]], tin[u]) + query_path(parent[nxt[u]], v);
C6B   }
2F3   void update_path(int u, int v, ll x){
0E8     if(tin[u] < tin[v]) swap(u, v);
D55     if(nxt[u] == nxt[v]) return updt(tin[v]+EDGE, tin[u], x);
0A7     updt(tin[nxt[u]], tin[u], x); update_path(parent[nxt[u]], v, x);
177   }
D41
BF2 private:
2E7   ll qry(int l, int r){ if(EDGE && l>r) return seg.NEUTRO; return seg.query(l, r); }
CB5   void updt(int l, int r, ll x){ if(EDGE && l>r) return; seg.update(l, r, x); }
D41
FB6   void dfs(int u, int p){
573     sz[u] = 1, parent[u] = p;
E69     for(auto &v : g[u]) if(v != p) {
1FB       dfs(v, u); sz[u] += sz[v];
D41
14A       if(sz[v] > sz[g[u][0]] || g[u][0] == p)
06F         swap(v, g[u][0]);
7E2     }
53F   }
D41
6BB   int t=0;
11E   void hld(int u, int p){
2C6     tin[u] = t++;
BF0     for(auto &v : g[u]) if(v != p)
B18       nxt[v] = (v == g[u][0] ? nxt[u] : v),
42C       hld(v, u);
36C   }
D41
D41   /// OPTIONAL ///
310   int lca(int u, int v){
582     while(!inSubtree(nxt[u], v)) u = parent[nxt[u]];
E1D     while(!inSubtree(nxt[v], u)) v = parent[nxt[v]];
40A     return tin[u] < tin[v] ? u : v;
AEB   }
65E   bool inSubtree(int u, int v){ return tin[u] <= tin[v] && tin[v] < tin[u] + sz[u]; }
D41   //query/update_subtree[tin[u]+EDGE, tin[u]+sz[u]-1];
095   vector<pair<int, int>> pathToAncestor(int u, int a){
F77         vector<pair<int, int>> ans;
7F3         while(nxt[u] != nxt[a])
FCA             ans.emplace_back(tin[nxt[u]], tin[u]),
5C3             u = parent[nxt[u]];
B35         ans.emplace_back(tin[a], tin[u]);
BA7         return ans;
52A     }
7CA };
```

3.10 Dinic

```
    Dinic - Max Flow Min Cut
Algoritmo de Dinitz para encontrar o Fluxo Maximo.
IMPORTANTE! O algoritmo esta 0-indexado

Complexity:
O( V^2 * E )      ->  caso geral
O( sqrt(V) * E )  ->  grafos com cap = 1 para toda Edge // matching bipartido

* Informacoes:
  Crie o Dinic: Dinic dinic(n, src, sink);
  Adicione as edges: dinic.addEdge(u, v, capacity);
  Para calcular o Fluxo Maximo: dinic.maxFlow()
  Para saber se um vertice U esta no Corte Minimo: dinic.inCut(u)

* Sobre o Codigo:
  vector<Edge> edges;  -> Guarda todas as edges do grafo e do grafo residual
  vector<vector<int>> adj; -> Guarda em adj[u] os indices de todas as edges saindo de u
  vector<int> ptr;   ->  Pointer para a proxima Edge ainda nao visitada de cada vertice
  vector<int> lvl; -> Distancia em vertices a partir do Source. Se igual a N o vertice
     nao foi visitado.
  A BFS retorna se Sink e alcancavel de Source. Se nao e porque foi atingido o Fluxo
     Maximo
  A DFS retorna um possivel aumento do Fluxo


 Use Cases of Flow

+ Minimum cut: the minimum cut is equal to maximum flow.
  i.e. to split the graph in two parts, one on the src side and another on sink side.
     The capacity of each edge is it weight.

+ Edge-disjoint-paths: maximum number of edge-disjoint paths equals maximum flow of the
     graph, assuming that the capacity of each edge is one. (paths can be found
     greedily)

+ Node-disjoint-paths: can be reduced to maximum flow. each node should appear in at
     most one path, so limit the flow through a node dividing each node in two. One
     with incoming edges, other with outgoing edges and a new edge from the first to
     the second with capacity 1.

+ Maximum matching (bipartite): maximum matching is equal to maximum flow. Add a src
     and a sink, edges from the src to every node at one partition and from each node
     of the other partition to the sink.

+ Minimum node cover
      (bipartite) : minimum set of nodes such each edge has at least one endpoint. The
     size of minimum node cover is equal to maximum matching (Konig's theorem).

+ Maximum-independent-set (bipartite): largest set of nodes such that no two nodes are
     connected with an edge. Contain the nodes that aren't in "Min node cover" (N -
     MAXFLOW).

+ Minimum path
     cover (DAG): set of paths such that each node belongs to at least one path.
  - Node-disjoint: construc a matching where each node is represented by two nodes, a
     left and a right at the matching and add the edges (from l to r). Each edge in the
     matching corresponds to an edge in the path cover. The number of paths in the
     cover is (N - MAXFLOW).
  - General: almost like a minimum node-disjoint. Just add edges to the matching
     whenever there is an path from U to V in the graph (possibly through several
     edges).
  - Antichain: a set of nodes such that there is no path from any node to another. In a
     DAG, the size of min general path cover equals the size of maximum antichain
     (Dilworth's theorem).

+ Project selection: Given N projects, each w profit pi, and M machines, each w cost ci.
  A project requires a set of machines (can be shared).
  Choose a set that maximizes value of the profit(projects) - the cost(machines). Add
     an edge (cap pi) from Source to project.
  An edge (cap ci) from machine to Sink. An edge (cap INF) from a project to each
     machine it requires.
  ans = SUM(pi) - MAXFLOW. If the edge of a machine is saturated, buy it.

+ Closure Problem (directed graph): Each node has a weight w (+ or -). choose a closure
     with maximum sum.
  A closure is a set of nodes such that there is no edge from a node inside the set to
     a node outside.
  Is a general case of project selection. Original edges with cap INF. Add edges from
     Source to nodes with W > 0; and from nodes with W < 0 to Sink (cap |W|).
```

```
E9B struct Edge {
37D   int u, v; ll cap;
525   Edge(int u, int v, ll cap) : u(u), v(v), cap(cap) {}
15B };

14D struct Dinic {
B82   int n, src, sink;
903   vector<vector<int>> adj;
321   vector<Edge> edges;
B4A   vector<int> lvl, ptr; //pointer para a proxima Edge nao saturada de cada vertice
D41
4A1   Dinic(int n, int src, int sink) : n(n), src(src), sink(sink) { adj.resize(n); }
D41
F82   void addEdge(int u, int v, ll cap){
471     adj[u].push_back(edges.size());
497     edges.emplace_back(u, v, cap);
D41
282     adj[v].push_back(edges.size());
659     edges.emplace_back(v, u, 0);
7B9   }
D41
AD2   ll dfs(int u, ll flow = 1e9){
87D     if(flow == 0) return 0;
B2A     if(u == sink) return flow;
D41
F73     for(int &i = ptr[u];  i < adj[u].size();  i++){
023       int at = adj[u][i];
C99       int v = edges[at].v;
D41
6A0       if(lvl[u] + 1 != lvl[v]) continue;
D41
A80       if(ll got = dfs(v, min(flow, edges[at].cap)) ){
6FA         edges[at].cap -= got;
E39         edges[at^1].cap += got;
529         return got;
B9F       }
5A7     }
D41
BB3     return 0;
95A   }
D41
838   bool bfs(){
26B     lvl = vector<int> (n, n);
91E     lvl[src] = 0;
D41
26A     queue<int> q;
8A7     q.push(src);
D41
14D     while(!q.empty()){
E4A       int u = q.front();
833       q.pop();
D41
E20       for(auto i : adj[u]){
628         int v = edges[i].v;
D41
1B2         if(edges[i].cap == 0 || lvl[v] <= lvl[u] + 1 ) continue;
D41
```

```
97B         lvl[v] = lvl[u] + 1;
2A1         q.push(v);
714       }
3AF     }
710     return lvl[sink] < n;
752   }
D41
D6E   bool inCut(int u){ return lvl[u] < n; }
D41
FE4   ll maxFlow(){
04B     ll ans = 0;
6D4     while( bfs() ){
11B       ptr = vector<int> (n+1, 0);
CF2       while(ll got = dfs(src)) ans += got;
815     }
BA7     return ans;
E9E   }
36C };
```

3.11 MinCostMaxFlow

```
E9B struct Edge {
F0B   int u, v; ll cap, cost;
DC4   Edge(int u, int v, ll cap, ll cost) : u(u), v(v), cap(cap), cost(cost) {}
49B };

6F3 struct MCMF {
878   const ll INF = numeric_limits<ll>::max();
DA6   int n, src, snk;
903   vector<vector<int>> adj;
321   vector<Edge> edges;
39D   vector<ll> dist, pot;
E3B   vector<int> from;
D41
00D   MCMF(int n, int src, int snk) : n(n), src(src), snk(snk) { adj.resize(n); pot.resize(
     n); }
D41
461   void addEdge(int u, int v, ll cap, ll cost){
471     adj[u].push_back(edges.size());
986     edges.emplace_back(u, v, cap, cost);
D41
282     adj[v].push_back(edges.size());
29F     edges.emplace_back(v, u, 0, -cost);
CA1   }
D41
26A   queue<int> q;
B57   vector<bool> vis;
791   bool SPFA(){
EF2     dist.assign(n, INF);
0B5     from.assign(n, -1);
543     vis.assign(n, false);
D41
8A7     q.push(src);
E13     dist[src] = 0;
D41
14D     while(!q.empty()){
E4A       int u = q.front();
833       q.pop();
D41
776       vis[u] = false;
D41
E20       for(auto i : adj[u]){
F42         if(edges[i].cap == 0) continue;
628         int v = edges[i].v;
99A         ll cost = edges[i].cost;
D41
148         if(dist[v] > dist[u] + cost + pot[u] - pot[v]){
DEC           dist[v] = dist[u] + cost + pot[u] - pot[v];
203           from[v] = i;
A1A           if(!vis[v]) q.push(v), vis[v] = true;
888         }
652       }
344     }
D41
19E     for(int u=0; u<n; u++)  //fix pot
067       if(dist[u] < INF)
AB7         pot[u] += dist[u];
D41
071     return dist[snk] < INF;
532   }
D41
B84   pair<ll, ll> augment(){
20B     ll flow = edges[from[snk]].cap, cost = 0; //fixed flow: flow = min(flow, remainder)
D41
473     for(int v=snk; v != src; v = edges[from[v]].u)
73D       flow = min(flow, edges[from[v]].cap),
871       cost += edges[from[v]].cost;
D41
473     for(int v=snk; v != src; v = edges[from[v]].u)
86A       edges[from[v]].cap   -= flow,
674       edges[from[v]^1].cap += flow;
D41
884     return {flow, cost};
890   }
D41
164   bool inCut(int u){ return dist[u] < INF; }
D41
6DC   pair<ll, ll> maxFlow(){
D7D     ll flow = 0, cost = 0;
D41
4EB     while( SPFA() ){
274       auto [f, c] = augment();
C87       flow += f;
BFC       cost += f*c;
35C     }
884     return {flow, cost};
D37   }
586 };
```

3.12 SCC - Kosaraju

```
Kosaraju - Strongly Connected Component
Algoritmo de Kosaraju para encontrar Componentes Fortemente Conexas

Complexity: O(V + E)
IMPORTANTE! O algoritmo esta 0-indexado

* Variaveis e explicacoes *
int C   -> C e a quantidade de Componetes Conexas. As componetes estao numeradas de 0 a
     C-1
dag     -> Apos rodar o Kosaraju, o grafo das componentes conexas sera criado aqui
comp[u] -> Diz a qual componente conexa U faz parte
g   -> grafo direcionado
gr  -> grafo reverso (que deve ser construido junto ao grafo normal) !!!

NOTA: A ordem que o Kosaraju descobre as componentes e uma Ordenacao Topologica do SCC
em que o dag[0] nao possui grau de entrada e o dag[C-1] nao possui grau de saida
```

```
229 const int MAXN = 1e6 + 5;

C3B vector<int> g[MAXN],  gr[MAXN], dag[MAXN];
A87 vector<int> comp, order;
B57 vector<bool> vis;
```

```
1EC int C = 0;

315 void dfs(int u){
B9C   vis[u] = true;
B41   for(auto v : g[u]) if(!vis[v])
6B4     dfs(v);
C75   order.push_back(u);
9C4 }

DD1 void dfsr(int u){
361   comp[u] = C;
D4E   for(auto v : gr[u]) if(comp[v] == -1)
AEF     dfsr(v);
595 }

955 void kosaraju(int n){
070   order.clear();
E28   comp.assign(n, -1);
543   vis.assign(n, false);
796   C = 0;
D41
54F   for(int v=0; v<n; v++) if(!vis[v])
6B4     dfs(v);
D41
3B9   reverse(begin(order), end(order));
D41
E3F   for(auto v : order) if(comp[v] == -1)
CA8     dfsr(v), C++;
D41
D41   //// Montar DAG  ////
78F   vector<bool> marc(C, false);
D41
687   for(int u=0; u<n; u++){
7B9     for(auto v : g[u]){
264       if(comp[v] == comp[u] || marc[comp[v]]) continue;
D41
812       marc[comp[v]] = true;
F26       dag[comp[u]].emplace_back(comp[v]);
661     }
C7B     for(auto v : g[u]) marc[comp[v]] = false;
556   }
013 }
```

3.13 Tarjan

```
Tarjan - Pontes e Pontos de Articulacao - O(V + E)
Algoritmo para encontrar pontes e pontos de articulacao.

pre[u]= "Altura", ou, x-esimo elemento visitado na DFS. Usado para saber a posicao de
     um vertice na arvore de DFS
low[u]= Low Link de U, ou a menor aresta de retorno (mais proxima da raiz) que U
     alcanca entre seus filhos
chd   = Children. Quantidade de componentes filhos de U. Usado para saber se a Raiz e
     Ponto de Articulacao.
any   = Marca se alguma aresta de retorno em qualquer dos componentes filhos de U nao
     ultrapassa U. Se isso for verdade, U e Ponto de Articulacao.

if(low[v] >  pre[u]) pontes.emplace_back(u, v); -> se a mais alta aresta de retorno de
     V (ou o menor low) estiver abaixo de U, entao U-V e ponte
if(low[v] >= pre[u]) any = true;        -> se a mais alta aresta de retorno de V (ou o
     menor low) estiver abaixo de U ou igual a U, entao U e Ponto de Articulacao
```

```
229 const int MAXN = 1e6 + 5;
F4C int pre[MAXN], low[MAXN], clk=0;
282 vector<int> g[MAXN];
A2B vector<pair<int, int>> pontes;
252 vector<int> cut;

CF2 void tarjan(int u, int p = -1){
FF7   if(p == -1) memset(pre, -1, sizeof pre); //so chama na root
FD2   pre[u] = low[u] = clk++;
034   int any = false, chd = 0;
D41
DD3   for(auto v : g[u]) if(v != p){
EE1     if(pre[v] == -1){
3D2       tarjan(v, u);
D41
E7F       low[u] = min(low[v], low[u]);
D41
334       if(low[v] >  pre[u]) pontes.emplace_back(u, v);
23A       if(low[v] >= pre[u]) any = true;
87D       chd++;
F1C     }
553     else low[u] = min(low[u], pre[v]);
E15   }
D41
B82   if(p == -1 && chd >= 2) cut.push_back(u);
5F3   if(p != -1 && any)      cut.push_back(u);
ECF }
```

4 Strings

4.1 Hash

```
String Hash - Double Hash
precalc()    -> O(N)
StringHash() -> O(|S|)
gethash()    -> O(1)

StringHash hash(s); -> Cria o Hash da string s
hash.gethash(l, r); -> Hash [L,R] (0-Indexado)
```

```
229 const int MAXN = 1e6 + 5;

E8E const ll MOD1 = 131'807'699;
D5D const ll MOD2 = 1e9 + 9;
145 const ll base = 157;

DB4 ll expb1[MAXN], expb2[MAXN];

921 #warning "Call precalc() before use StringHash"
FE8 void precalc(){
6D8     expb1[0] = expb2[0] = 1;
D41
7E4   for(int i=1;i<MAXN;i++)
E0E         expb1[i] = expb1[i-1]*base % MOD1,
C4B         expb2[i] = expb2[i-1]*base % MOD2;
A02 }

3CE struct StringHash{
0DD     vector<pair<ll,ll>> hsh;
AC0     string s; // comment S if you dont need it
D41
6F2     StringHash(string& s) : s(s){
63F         hsh.assign(s.size()+1, {0,0});
D41
724         for (int i=0;i<s.size();i++)
```

```
B7A             hsh[i+1].first  = ( hsh[i].first *base % MOD1 + s[i] ) % MOD1,
08F             hsh[i+1].second = ( hsh[i].second*base % MOD2 + s[i] ) % MOD2;
5A6     }
D41
2F0     ll gethash(int a,int b){
F96         ll h1 = (MOD1+ hsh[b+1].first  - hsh[a].first *expb1[b-a+1] % MOD1) % MOD1;
F4A         ll h2 = (MOD2+ hsh[b+1].second - hsh[a].second*expb2[b-a+1] % MOD2) % MOD2;
D23         return (h1<<32) | h2;
C77     }
1D3 };

D41 // OPTIONAL
0FB int firstDiff(StringHash& a, int la, int ra, StringHash& b, int lb, int rb){
7E5   int l=0, r=min(ra-la, rb-lb), diff=r+1;
3D5   while(l <= r){
EE4     int m = (l+r)/2;
065     if(a.gethash(la, la+m) == b.gethash(lb, lb+m)) l = m+1;
72D     else r = m-1, diff = m;
BAD   }
2B1   return diff;
C88 }

03D int hshComp(StringHash& a, int la, int ra, StringHash& b, int lb, int rb){
E85   int diff = firstDiff(a, la, ra, b, lb, rb);
23E   if(diff > ra-la && ra-la == rb-lb) return 0; //equal
D15   if(diff > ra-la || diff  >  rb-lb) return ra-la < rb-lb ? -2 : +2; //prefix of the other
626   return a.s[la+diff] < b.s[lb+diff] ? -1 : +1;
8C4 }
```

4.2 KMP

```
692 vector<int> Pi(string &t){
82B   vector<int> p(t.size(), 0);
D41
6F4   for(int i=1, j=0; i<t.size(); i++){
90B     while(j > 0  && t[j] != t[i]) j = p[j-1];
3C7     if(t[j] == t[i]) j++;
F8C     p[i] = j;
9E8   }
74E   return p;
85D }

2AD vector<int> kmp(string &s, string &t){
D9E   vector<int> p = Pi(t), occ;
D41
1EF   for(int i=0, j=0; i<s.size(); i++){
705     while( j > 0 && s[i] != t[j]) j = p[j-1];
566     if(s[i]==t[j]) j++;
2F0     if(j == t.size()) occ.push_back(i-j+1), j = p[j-1];
6C4   }
FB0   return occ;
087 }
```

Optional: KMP Automato. j = state atual [root=j=0]

```
3E3 struct Automato {
632   vector<int> p;
78F     string t;
119   Automato(string &t) : t(t), p(Pi(t)){}
6DD     int next(int j, char c){ //return nxt state
E60         if(final(j)) j = p[j-1];
28D         while(j && c != t[j]) j = p[j-1];
5B4         return j + (c == t[j]);
26F     }
DFA     bool final(int j){ return j == t.size(); }
8C2 };
```

```
25D **KMP** - Knuth-Morris-Pratt Pattern Searching
05C Complexity: O(|S|+|T|)
DB8 kmp(s, t) -> returns all occurences of t in s
020 p = Pi(t) -> p[i] = biggest prefix that is a sufix of t[0,i]
```

4.3 Aho-Corasick

Aho-Corasick: Trie automaton to search multiple patterns in a text
O(SUM|P| + |S|) * ALPHA

```
for(auto p: patterns) aho.add(p);
aho.buildSufixLink();
auto ans = aho.findPattern(s);
```

parent(p), sufixLink(sl), outputLink(ol), patternID(idw)
outputLink -> edge to other pattern end (when p is a sufix of it)
ALPHA -> Size of the alphabet. If big, consider changing nxt to map

To find ALL occurrences of all patterns, don't delete ol in findPattern. But it can be slow (at number of occ), so consider using DP on the automaton.
If you need a **nextState** function, create it using the while in findPattern.
if you need to **store node indexes** add int i to Node, and in Aho add this and change the new Node() to it:

```
vector<trie> nodes;
trie new_Node(trie p, char c){
    nodes.push_back(new Node(p, c));
    nodes.back()->i = nodes.size()-1;
    return nodes.back();
}
```

```
322 const int ALPHA = 26, off = 'a';
BF2 struct Node {
E05     Node* p  = NULL;
A26     Node* sl = NULL;
C3A     Node* ol = NULL;
CB8     array<Node*, ALPHA> nxt;
D41
F02     int idw = -1, i; char c;
D41
212     Node(){ nxt.fill(NULL); }
B04     Node(Node* p, char c) : p(p), c(c) { nxt.fill(NULL); }
969 };
2CA typedef Node* trie;
C99 struct Aho {
ACD     trie root;
EAA     int nwords = 0;
F78     vector<trie> nodes;
E6D     Aho(){ root = new_Node(NULL, 0); }
D41
22D     void add(string &s){
346         trie t = root;
242         for(auto c : s){ c -= off;
508             if(!t->nxt[c])
02F                 t->nxt[c] = new Node(t, c);
4F8             t = t->nxt[c];
E9A         }
71E         t->idw = nwords++; //cuidado com strings iguais! use vector
625     }
34A     void buildSufixLink(){
A2F         deque<trie> q(1, root);
D41
14D         while(!q.empty()){
81D             trie t = q.front();
CED             q.pop_front();
D41
630             if(trie w = t->p){
```

```
29D                 do w = w->sl; while(w && !w->nxt[t->c]);
619                 t->sl = w ? w->nxt[t->c] : root;
D7B                 t->ol = t->sl->idw == -1 ? t->sl->ol : t->sl;
8DB             }
D41
806             for(int c=0; c<ALPHA; c++)
F72                 if(t->nxt[c])
78D                     q.push_back(t->nxt[c]);
693         }
09C     }
66F     vector<bool> findPattern(string &s){
BFD         vector<bool> ans(nwords, 0);
82D         trie w = root;
242         for(auto c : s){ c -= off;
A7A             while(w && !w->nxt[c]) w = w->sl;  // trie next(w, c)
AEA             w = w ? w->nxt[c] : root;
D41
5BE             for(trie z=w, nl; z; nl=z->ol, z->ol=NULL, z=nl)
972                 if(z->idw != -1) //get ALL occ: dont delete ol (may slow)
31E                     ans[z->idw] = true;
B04         }
BA7         return ans;
C8E     }
410     trie new_Node(trie p, char c){
0B8         nodes.push_back(new Node(p, c));
996         nodes.back()->i = nodes.size()-1;
13C         return nodes.back();
4E9     }
74B };
```

4.4 Suffix Array

```
sf = suffixArray(s) -> O(N log N)
LCP(s, sf) -> O(N)

SuffixArray -> index of suffix in lexicographic order
LCP[i] -> LargestCommonPrefix of sufix at sf[i] and sf[i-1]
LCP(i,j) = min(lcp[i+1...j])

To better understand, print: lcp[i] sf[i] s.substr(sf[i])
```

```
B6C vector<int> suffixArray(string s){
92A     int n = (s += "!").size();//if vector, s.push_back(-INF);
6B4     vector<int> sf(n), ord(n), aux(n), cnt(n);
CE4     iota(begin(sf), end(sf), 0);
30A     sort(begin(sf), end(sf), [&](int i, int j){ return s[i] < s[j]; });
D41
104     int cur = ord[sf[0]] = 0;
AA4     for(int i=1; i<n; i++)
0BB         ord[sf[i]] = s[sf[i]] == s[sf[i-1]] ? cur : ++cur;
D41
C1E     for(int k=1; cur+1 < n && k < n; k<<=1){
727         cnt.assign(n, 0);
8FF         for(auto &i : sf)          i = (i-k+n)%n, cnt[ord[i]]++;
DC5         for(int i=1; i<n; i++)     cnt[i] += cnt[i-1];
0A4         for(int i=n-1; i>=0; i--) aux[--cnt[ord[sf[i]]]] = sf[i];
71C         sf.swap(aux);
D41
662         aux[sf[0]] = cur = 0;
AA4         for(int i=1; i<n; i++)
AEB             aux[sf[i]] = ord[sf[i]] == ord[sf[i-1]] &&
E19             ord[(sf[i]+k)%n] == ord[(sf[i-1]+k)%n] ? cur : ++cur;
43A         ord.swap(aux);
52E     }
61D     return vector<int>(begin(sf)+1, end(sf));
968 }

B1D vector<int> LCP(string &s, vector<int> &sf){
163     int n = s.size();
BF1     vector<int> lcp(n), pof(n);
E51     for(int i=0; i<n; i++) pof[sf[i]] = i;
D41
9A7     for(int i=0, j, k=0; i<n; k?--k:k, i++){
76D         if(!pof[i]) continue;
D5B         j = sf[pof[i]-1];
329         while(i+k<n && j+k<n && s[i+k]==s[j+k]) k++;
F12         lcp[pof[i]] = k;
1D0     }
5ED     return lcp;
EC1 }
```

4.5 Trie

```
Trie - Arvore de Prefixos
insert(P) - O(|P|)
count(P)  - O(|P|)
sigma - Tamanho do alfabeto
off   - primeiro simbolo do alfabeto (offset)
```

```
322 const int ALPHA = 26, off = 'a';
BF2 struct Node {
CB8     array<Node*, ALPHA> nxt;
34A     int terminal = 0;
212     Node() { nxt.fill(NULL); }
E65 };

71A struct Trie {
95F   Node* root;
814   Trie(){ root = new Node(); }
D41
22D   void add(string &s){
D66     Node* t = root;
242     for(auto c : s){ c -= off;
508       if(!t->nxt[c])
957         t->nxt[c] = new Node();
4F8       t = t->nxt[c];
BE3     }
6FF     t->terminal++;
F94   }
D41
912   int count(string &s){
D66     Node* t = root;
242     for(auto c : s){ c -= off;
F63       if(!t->nxt[c]) return 0;
4F8       t = t->nxt[c];
0A5     }
515     return t->terminal;
B99   }
942 };
```

4.6 Manacher

```
Manacher Algorithm - O(N)
Find every palindrome in string
```

```
DC6 vector<int> manacher(string &st){
E13   string s = "$_";
821   for(char c : st){ s += c; s += "_"; }
095   s += "#";
D41
7AB   int n = s.size()-2, l=1, r=1;
BD7   vector<int> p(n+2, 0);
D41
E68   for(int i=1, j; i<=n; i++){
DAF     p[i] = max(0, min(r-i, p[l+r-i]) ); //atualizo o valor atual para o valor do palindromo espelho na string ou para o total que esta contido
A5F     while( s[i-p[i]] == s[i+p[i]] ) p[i]++;
39C     if( i+p[i] > r ) l = i-p[i], r = i+p[i];
E75   }
D41
6AE   for(auto &x : p) x--; //o valor de p[i] era o tamanho do palindromo + 1
74E   return p; //agora e o tamanho real
781 }
```

4.7 Z-Function

```
403 vector<int> Zfunction(string &s){ // O(N)
163   int n = s.size();
2B1   vector<int> z (n, 0);
D41
A5C   for(int i=1, l=0, r=0; i<n;  i++){
76D     if(i <= r) z[i] = min(z[i-l], r-i+1);
D41
F61     while(z[i] + i < n && s[z[i]] == s[i+z[i]]) z[i]++;
D41
EAF     if(r < i+z[i]-1) l = i, r = i+z[i]-1;
0CD   }
070   return z;
D58 }
```

5 others

5.1 MO

Algoritmo de MO para query em range

Complexity: O((N + Q) * SQRT(N) * F) | F e a complexidade do Add e Remove

IMPORTANTE! Queries devem ter seus indices (Idx) 0-indexados!

Modifique as operacoes de Add, Remove e GetAnswer de acordo com o problema.
BLOCK_SZ pode ser alterado para aproximadamente SQRT(MAX_N)

```
861 const int BLOCK_SZ = 700;

670 struct Query{
738   int l, r, idx;
991   Query(int l, int r, int idx) : l(l), r(r), idx(idx) {}
406   bool operator < (Query q) const {
6EB     if(l / BLOCK_SZ != q.l / BLOCK_SZ) return l < q.l;
387     return (l / BLOCK_SZ &1) ? ( r < q.r ) : (r > q.r );
667   }
F51 };

543 void add(int idx);
F8A void remove(int idx);
AD7 int getAnswer();

73F vector<int> MO(vector<Query> &queries){
51F   vector<int> ans(queries.size());
D41
BFA   sort(queries.begin(), queries.end()); // to use hilbert curves, call sortQueries instead
D41
32D   int L = 0, R = 0;
49E   add(0);
D41
FE9   for(auto [l, r, idx] : queries){
128     while(l < L) add(--L);
C4A     while(r > R) add(++R);
684     while(l > L) remove(L++);
B50     while(r < R) remove(R--);
D41
830     ans[idx] = getAnswer();
08D   }
D41
BA7   return ans;
ACF }

D41 //OPTIONAL
E5B void sortQueries(vector<Query> &qr){
1FC   vector<ll> h(qr.size());
489   for(int i=0; i<qr.size(); i++) h[i] = hilbert(qr[i].l, qr[i].r);
35E   sort(qr.begin(), qr.end(), [&](Query&a, Query&b) { return h[a.idx] < h[b.idx]; });
308 }

E51 inline ll hilbert(int x, int y){ //OPTIONAL
C85   static int N = 1 << (__builtin_clz(0) - __builtin_clz(MAXN));
B69   int rx, ry, s; ll d = 0;
43B   for(s = N/2; s > 0; s /= 2){
C95     rx = (x & s) > 0, ry = (y & s) > 0;
F15     d += s * (ll)(s) * ((3 * rx) ^ ry);
E2D     if(ry == 0) { if(rx == 1) x = N-1 - x, y = N-1 - y; swap(x, y); }
200   }
BE2   return d;
038 }
```

5.2 MOTree

Algoritmo de MO para query de **caminho em arvore**
Complexity: O((N + Q) * SQRT(N) * F) | F e a complexidade do Add e Remove
IMPORTANTE! 0-indexado!

```
80E const int MAXN = 1e5+5;
F5A const int BLOCK_SZ = 500;
304 struct Query{int l, r, idx;}; //same of MO. Copy operator <

282 vector<int> g[MAXN];
212 int tin[MAXN], tout[MAXN];
03B int pai[MAXN], order[MAXN];

179 void remove(int u);
C8B void add(int u);
AD7 int getAnswer();

C0A void go_to(int ti, int tp, int otp){
B21   int u = order[ti], v, to;
61E   to = tout[u];
AA5   while(!(ti <= tp && tp <= to)){ //subo com U (ti) ate ser ancestral de W
E7C     v = pai[u];
D41
```

```
BAF     if(ti <= otp && otp <= to) add(v);
96E     else remove(u);
D41
A68     u = v;
363     ti = tin[u];
61E     to = tout[u];
462   }
D41
915   int w = order[tp];
D88   to = tout[w];
082   while(ti < tp){ //subo com W (tp) ate U
80E     v = pai[w];
D41
F19     if(tp <= otp && otp <= to) remove(v);
7AC     else add(w);
D41
9A1     w = v;
FCA     tp = tin[w];
D88     to = tout[w];
34D   }
B15 }

1D4 int TIME = 0;
FB6 void dfs(int u, int p){
49E   pai[u] = p;
6FD   tin[u] = TIME++;
A2B   order[tin[u]] = u;
D41
70D   for(auto v : g[u])
F6B     if(v != p)
95E       dfs(v, u);
916   tout[u] = TIME-1;
686 }

73F vector<int> MO(vector<Query> &queries){
51F   vector<int> ans(queries.size());
564   dfs(0, 0);
D41
C89   for(auto &[u, v, i] : queries)
563     tie(u, v) = minmax(tin[u], tin[v]);
BFA   sort(queries.begin(), queries.end());
D41
49E   add(0);
7AC   int Lm = 0, Rm = 0;
FE9   for(auto [l, r, idx] : queries){
9D4     if(l < Lm) go_to(Lm, l, Rm), Lm = l;
0E8     if(r > Rm) go_to(Rm, r, Lm), Rm = r;
A5C     if(l > Lm) go_to(Lm, l, Rm), Lm = l;
035     if(r < Rm) go_to(Rm, r, Lm), Rm = r;
830     ans[idx] = getAnswer();
30A   }
D41
BA7   return ans;
64A }
```

5.3 Hungarian

```
Hungarian Algorithm - Assignment Problem
Algoritmo para o problema de atribuicao minima.

Complexity: O(N^2 * M)

hungarian(int n, int m);  ->  Retorna o valor do custo minimo
getAssignment(int m)      ->  Retorna a lista de pares <linha, Coluna> do Minimum
     Assignment

n -> Numero de Linhas // m -> Numero de Colunas

IMPORTANTE! O algoritmo e 1-indexado
IMPORTANTE! O tipo padrao esta como int, para mudar para outro tipo altere | typedef
     <TIPO> TP; |
Extra: Para o problema da atribuicao maxima, apenas multiplique os elementos da matriz
     por -1
```

```
941 typedef int TP;

3CE const int MAXN = 1e3 + 5;
657 const TP INF = 0x3f3f3f3f;

F31 TP matrix[MAXN][MAXN];
F10 TP row[MAXN], col[MAXN];
E1F int match[MAXN], way[MAXN];

E5E TP hungarian(int n, int m){
715   memset(row, 0, sizeof row);
CD2   memset(col, 0, sizeof col);
187   memset(match, 0, sizeof match);
D41
78A   for(int i=1; i<=n; i++){
96C     match[0] = i;
23B     int j0 = 0, j1, i0;
76E     TP delta;
D41
693     vector<TP> minv (m+1, INF);
C04     vector<bool> used (m+1, false);
D41
016     do {
472       used[j0] = true;
F81       i0 = match[j0];
B27       j1 = -1;
7DA       delta = INF;
D41
2E2       for(int j=1; j<=m; j++)
F92         if(!used[j]){
76D           TP cur = matrix[i0][j] - row[i0] - col[j];
D41
9F2           if( cur < minv[j] ) minv[j] = cur, way[j] = j0;
821           if(minv[j] < delta) delta = minv[j], j1 = j;
6FD         }
D41
FC9       for(int j=0; j<=m; j++)
E48         if(used[j]){
7AC           row[match[j]] += delta,
429           col[j] -= delta;
23B         }
6EC         else minv[j] -= delta;
D41
6D4       j0 = j1;
A95     } while(match[j0]);
D41
016     do {
B8C       j1 = way[j0];
77A       match[j0] = match[j1];
6D4       j0 = j1;
196     } while(j0);
799   }
D41
A33   return -col[0];
7FF }

3B4 vector<pair<int, int>> getAssignment(int m){
F77   vector<pair<int, int>> ans;
8EA   for(int i=1; i<=m; i++)
843     ans.push_back(make_pair(match[i], i));
BA7   return ans;
01D }
```

```
E6E };
```

5.4 Date

```
D41 //
```

converts Gregorian date to integer (Julian day number)

```
B37 int dateToInt (int m, int d, int y){ return
B8C     + 1461 * (y + 4800 + (m - 14) / 12) / 4
CAD     + 367  * (m - 2     - (m - 14) / 12  * 12)  / 12
47F     - 3    *((y + 4900 + (m - 14) / 12) / 100) / 4
6BC     + d - 32075;
C1B }
```

converts integer (Julian day number) to Gregorian date: day/month/year

```
32D tuple<int, int, int> intToDate(int jd){
402     int x, n, i, j, d, m, y;
33A     x = jd + 68569;
403     n = 4 * x / 146097;
33E     x -= (146097 * n + 3) / 4;
6FC     i = (4000 * (x + 1)) / 1461001;
B1D     x -= 1461 * i / 4 - 31;
FC9     j = 80 * x / 2447;
C8D     d = x - 2447 * j / 80;
179     x = j / 11;
335     m = j + 2 - 12 * x;
23D     y = 100 * (n - 49) + i + x;
B86     return {d, m, y};
4AC }
```

converts integer (Julian day number) to day of week

```
58B string dayOfWeek[] = {"Mon", "Tue", "Wed", "Thu", "Fri", "Sat", "Sun"};
264 string intToWeek (int jd){ return dayOfWeek[jd % 7]; }
```

5.5 Dice

```
   Dadinho de 6 lados     |   |4|       ^ norte
24 estados possiveis      | |2|6|5| w < + > leste
Assume que a soma de lados|   |3|       v
opostos e 7.              |   |1|      sul
Se precisar de labels diferentes, crie um
 array<int, 7> face = {0, 1, 2, 3, 4, 5, 6}
e acesse a posicao da face retornada.
```

```
0E0 struct Dice {
F35     int topo=6, norte=4, leste=5;
D41
3AA     int top()    { return topo; }
895     int north() { return norte; }
BAF     int east()   { return leste; }
513     int bottom(){ return 7 - topo; }
247     int south() { return 7 - norte; }
A93     int west()   { return 7 - leste; }
D41
AAC     void roll_north(){ tie(topo, norte) = pair(south(), top());   }
210     void roll_east( ){ tie(topo, leste) = pair(west(),  top());   }
373     void rotate_ccw(){ tie(norte,leste) = pair(east(),south());   }
A3B     void roll_south(){ roll_north(); roll_north(); roll_north(); }
A4A     void roll_west( ){ roll_east();  roll_east();  roll_east();   }
FC0     void rotate_cw  (){ rotate_ccw(); rotate_ccw(); rotate_ccw(); }
D41
2D7     int get_id(){ //[0, 23]
07E         int id = (topo-1)^(norte-1);
EF3         id %= topo==3||topo==4 ? 6 : 4;
1E9         return topo*4 + id - 4;
44B     }
```

5.6 Busca Binaria Paralela

```
Busca Binaria Paralela - O(Q+K log K)

Dado K updates ordenados pelo tempo, e Q queries da forma:
- Qual o primeiro momento entre [0, K-1] em que * e verdade?
De forma que:
- A resposta e monotonica (se e verdadeiro para t=x, e verdade para t'=x+1)
- E possivel responder em O(N^2) iterando pelos updates e verificando a cada passo
    todas as queries nao satisfeitas ainda.

* No caso do range de busca ser [0, 1e9]: recursivo OU veja se e possivel olhar apenas
    os valores que aparecem no input(?)
```

```
1C8 void reset(){};
6F8 void update(int mid){};
DFF bool check(int i){}

D41 //q = #queries | [0, k-1] = intervalo de busca
019 vector<int> pbs(int q, int k){
194     bool LACK = true;
4D9     vector<int> L(q, 0), R(q, k-1), ans(q, -1);
25A     vector<vector<int>> atMid(k);
D41
234     while(LACK){
BD0         reset(); LACK = false;
D41
EDF         for(int i=0; i<q; i++)
4F2             if(L[i] <= R[i])
4CE                 atMid[(L[i]+R[i])/2].push_back(i), LACK = true;
D41
254         for(int mid=0; mid<k; mid++){
B5B             update(mid);
7A2             for(auto i : atMid[mid])
5AA                 if(check(i)) R[i] = mid-1, ans[i] = mid;
D64                 else L[i] = mid+1;
BF1             atMid[mid].clear();
F9D         }
593     }
BA7     return ans;
C64 }

DAB vector<int> ans; // recursive version
B6C void pbsR(vector<int> &qidx, int l, int r){
DE6   if(l > r) return;
1C3   int mid = (l+r)/2; update(mid);
8B1   vector<int> L, R;
225   for(auto i : qidx){
CF8     if(check(i)) L.push_back(i), ans[i] = mid;
FB4     else R.push_back(i);
74B   }
E69   if(!L.empty()) pbsR(L, l, mid-1);
724   if(!R.empty()) pbsR(R, mid+1, r);
5BB }
```

5.7 BuscaTernaria

```
Ternary search and Golden section search

if(f1 < f2) to find minimum
if(f1 > f2) to find maximum

Golden Search faz menos chamadas a funcao;
Para busca em inteiros, cuidado, mantenha uma margem do L e R para evitar looping
     infinito.
```

```
D40 #define ld long double

BF0 ld func(ld x){ return 2*x*x - 4*x + 2; }

C14 ld ternary_search(ld l, ld r) {
311     for(int it=250; it--;){
26A         ld m1 = l + (r-l)/3, m2 = r - (r-l)/3;
78D         ld f1 = func(m1), f2 = func(m2);
2C3         if(f1 < f2) r = m2;
E14         else l = m1;
DE0     }
792     return l;
CE1 }

90D ld golden_search(ld l, ld r) {
2A6     const ld iphi = (sqrt(5)-1)/2;
D41
6F1     ld m1 = r - iphi*(r-l);
34D     ld m2 = l + iphi*(r-l);
78D     ld f1 = func(m1), f2 = func(m2);
D41
311     for(int it=250; it--;){
F4D         if(f1 < f2){
A3B             r = m2;
DD9             tie(m2, f2) = tie(m1, f1);
B2C             f1 = func( m1 = r - iphi*(r-l) );
03B         } else {
6C9             l = m1;
894             tie(m1, f1) = tie(m2, f2);
FB8             f2 = func( m2 = l + iphi*(r-l) );
0F6         }
FA1     }
792     return l;
32F }

D7C int iternary_search(int l, int r) {
218     while(r-l > 5){  //margem de seguranca
898         int m1 = l + (r-l)/3, m2 = r - (r-l)/3;
C8F         int f1 = func(m1), f2 = func(m2);
2C3         if(f1 < f2) r = m2;
E14         else l = m1;
7E6     }
7AF     int ans = r, fa = func(ans);
6B0     for(int f1; l<r; l++){
356         f1 = func(l);
611         if(f1 < fa)
2BB             ans = l, fa = f1;
6CE     }
BA7     return ans;
EEF }
```

6 Math

6.1 fexp

```
FF4 ll mod = 1e9 + 7;

4A0 ll fexp(ll b, ll p){
D54   ll ans = 1;
D08   while(p){
F66     if(p&1) ans = ans * b % mod;
8B5     b = b * b % mod;
8B8     p >>= 1;
486   }
BA7   return ans;
1AE }
D41 // O(Log P) // b - Base // p - Potencia
```

6.2 Combinatoria

```
42D vector<ll> fat, finv;

AEC void Combin(int n){  // precalc
7FD   fat.assign(n+1, 1);
6AD   for(int i=2; i<=n; i++) fat[i] = fat[i-1]*i % mod;
0EB   finv.assign(n+1, fexp(fat.back(), mod-2));
4DB   for(int i=n; i>0; i--) finv[i-1] = finv[i]*i % mod;
169 }

15E ll choose(ll n, ll k){ if(k>n||k<0) return 0;
404   return fat[n] * finv[k] % mod * finv[n-k] % mod;
3B6 }

86B ll chooseLinear(ll n, ll k){ //O(k) || min(k, n-k);
63A   k = min(k, n-k);
506   ll ans = 1, inv=1;
4D1   for(int i=n; i>k; i--) ans = ans*i % mod;
B7C   for(int i=1; i<=n-k; i++) inv = inv*i % mod;
891   return ans * fexp(inv, mod-2) % mod;
427 }

58B ll permRepetition(const vector<int> &cnt){
60B   ll n = accumulate(begin(cnt), end(cnt), 0ll), ans = fat[n];
C87   for(int x : cnt) ans = ans * finv[x] % mod;
BA7   return ans;
09A }

75D ll pascal[5001][5001]; // pascal[n][k] = choose(n, k);
B39 void Pascal(int N){
A4F   pascal[0][0] = 1;
B49   for(int n=1; n<=N; n++){
E6B     pascal[n][0] = pascal[n][n] = 1;
DEA     for(int k=1; k<n; k++)
6ED       pascal[n][k] = (pascal[n-1][k-1] + pascal[n-1][k]) % mod;
2C1   }
C90 }


D41 ///////////////////////////
D41 ////// Stars and Bars //////
D41 ///////////////////////////

327 ll starsBars(ll n, ll k){ //O(choose)
BA7   return choose(n+k-1, n);
484 }
9B9 ll starsLowerBound(ll n, const vector<ll> &lw){ //O(k)
3D8   for(auto x : lw) n -= x;
```

```
6E7   return starsBars(n, lw.size());
981 }
2FF ll starsUpperBound(ll n, ll k, ll up){ //O(k)
04B   ll ans = 0;
238   for(int i=0; i<=k; i++)
1CC     ans += choose(k, i) * choose(n+k-1-(up+1)*i, k-1) % mod * (i&1? -1:+1);
BA7   return ans;
98D }
293 ll starsUpperBound(ll M, const vector<ll> &up){ //O(N*M)
652   int N = up.size();
D2A   vector dp(up.size()+1, vector<ll>(N+1));
624   for(int m=0; m<=M; m++) dp[0][m] = choose(N+m-1, m);
61C   for(int n=1; n<=N; n++)
655     for(int m=0; m<=M; m++)
163       dp[n][m] = dp[n-1][m] - (m-up[n-1]-1 < 0 ? 0 : dp[n-1][m-up[n-1]-1]);
11B   return dp[N][M];
789 }
5B3 ll starsLowerUpperBound(ll n, const vector<ll> &lw, const vector<ll> &up){ //O(N*M)
3D8   for(auto x : lw) n -= x;
229   return starsUpperBound(n, up);
41E }

D41 ///// Propriedades /////
777 ll sumNci (ll n){ return fexp(2, n); }            //for(i=0; i<=n) sum+=choose(n, i);
3F6 ll sumicK (ll n, ll k){ return choose(n+1, k+1); } //for(i=0; i<=n) sum+=choose(i, k);
E80 ll sumNKcK(ll n, ll k){ return choose(n+k+1, k); } //for(i=0; i<=k) sum+=choose(n+i, i)
    ;
1D8 ll sumNsqr(ll n){ return choose(n+n, n); }         //for(i=0; i<=n) sum+=choose(n, i)
    **2;
FC2 ll catalan(ll n){ return choose(2*n, n) * fexp(n+1, mod-2) % mod; }
```

6.3 Crivo

```
3E7 vector<int> calc_prime(int n){ // O(n log n)
781     vector<int> prime(n+1, 1);
D18     for(int i=2; i<=n; i++) if(prime[i] == i)
5A3         for(int j=i+i; j<=n; j+=i)
2F9             prime[j] = false;
AB1   return prime;
97D }

C08 vector<int> calc_phi(int n){ // O(n log n)
340     vector<int> phi(n+1);
606     for(int i=0; i<=n; i++) phi[i] = i;
301     for(int i=2; i<=n; i++) if(phi[i] == i)
B77         for(int j=i; j<=n; j+=i)
A9B             phi[j] -= phi[j] / i;
970   return phi;
2E1 }

8BB vector<int> calc_mobius(int n){ // O(n log n)
5C9     vector<int> mobius(n+1, 1), prime(n+1, 1);
10A     for(int i=2, j; i<=n; i++) if(prime[i])
7CD         for(mobius[i]=-1, j=i+i; j<=n; j+=i){
601             if((j/i)%i) mobius[j] *= -1;
4CD             else mobius[j] = 0;
2F9             prime[j] = false;
798         }
D78     return mobius;
621 }
```

6.4 CRT

```
D40 #define ld long double
593 ll modinv(ll a, ll b, ll s0=1, ll s1=0){ return b == 0 ? s0 : modinv(b, a%b, s1, s0 -
     s1 * (a/b)); }
D8B ll mul(ll a, ll b, ll m){
C95     ll q = (ld)a*(ld)b / (ld)m;
1A8     ll r = a*b - q*m;
B8B     return (r + m) % m;
154 }

28D struct Equation {
4C5     ll mod, ans;
08F     bool valid;
0FC     Equation() { valid = false; }
5E2     Equation(ll a, ll m) { mod = m, ans = (a % m + m) % m, valid = true; }
4D3     Equation(Equation a, Equation b){
355         if(!a.valid || !b.valid){ valid = false; return; }
85C         ll g = gcd(a.mod, b.mod);
DBE         if((a.ans - b.ans) % g != 0){ valid = false; return; }
D41
AF0         valid = true;
B98         mod = a.mod * (b.mod / g);
2F6         ans = a.ans;
5E0         ans += mul( mul(a.mod,  modinv(a.mod, b.mod), mod),   (b.ans - a.ans) / g, mod)
    ;
D41
C4C         ans = (ans % mod + mod) % mod;
2DB     }
634     Equation operator+(const Equation& b) const { return Equation(*this, b); }
E15 };
D41 // Equation eq1(2, 3); // x = 2 mod 3
D41 // Equation eq2(3, 5); // x = 3 mod 5
D41 // Equation ans = eq1 + eq2;
```

6.5 Eliminacao Gaussiana

Eliminacao Gaussiana - O(N*N*M)

Resolve um sistema de equacoes lineares, escalonando a matriz;

```
Entrada       || Saida         || interpretacao
1  4  6   80  || 1  0  0  76   || X0 = 76
2  0  6  140  || 0  1  0  4    || X1 = 4
1  2  12  60  || 0  0  1  -2   || X2 = -2

Entrada       || Saida         || interpretacao
1  4  6  18   || 1  4  0  -6   || X0 = -4 * X1  - 6
0  0  6  24   || 0  0  1   4   || X2 =  4
1  4  3   6   || 0  0  0   0   || 0  =  0
```

* Se 0 != 0, o sistema nao tem solucao
* Variaveis do lado direito da equacao tem valor livre

```
913 template<typename T> struct Gauss {
72C   int n, m = 0;
C24   vector<vector<T>> mat;
E7E   Gauss(int n) : n(n){}
D41
45B   void addLine(vector<T> l = vector<T>()){
8E6     l.resize(n, T(0));
D5C     mat.push_back(l); m++;
E7D   }
63D   void solve(){
7BA     for(int c=0, i=0, g; c<n && i < m; c++, i+=!g){
EF1       for(int j=i; j<m && mat[i][c] == T(0); j++)
DF2         swap(mat[j], mat[i]);
D41
CBA       if(g = mat[i][c] == T(0)) continue;
820       for(int j=n-1; j>=0; j--) mat[i][j] /= mat[i][c];
```

```
D41
F05       for(int j=0; j<m; j++) if(i != j && mat[j][c] != T(0)){
2B5         T alpha = mat[j][c];
90F         for(int k=i; k<n; k++)
792           mat[j][k] -= mat[i][k] * alpha;
01D       }
6DB     }
8B4   }
BC1   int isValid(){
444         int pivos = 0;
7C7         for(int i=0, c; i<m; i++){
3D3       for(c=0; c<n-2; c++) if(mat[i][c] != T(0)) break;
D35             if(mat[i][c] != T(0)){ pivos++;
03F                 for(int j=c+1; j<n-1; j++)
1B4                     if(mat[i][j] != T(0))
DD1                         pivos = -2*n;
552             }
461       else if(mat[i][n-1] != T(0)) return 0; // 0 = 1
C6D     }
EF6         return pivos == n-1 ? +1 : -1; // 1 - Solucao unica // 0 - Sem solucao // -1 - Infinitas solucoes
14F     }
6DC   void print(){ for(auto v : mat){ for(auto x:v) cout<<x<<"\t"; cout<<"\n"; } }
50A   void printSolution(){ // OPTIONAL / FOR DEBUG
7C7     for(int i=0, c; i<m; i++){
3D3       for(c=0; c<n-2; c++) if(mat[i][c] != T(0)) break;
F01       if(mat[i][c] != T(0)) cout << "X" << c << " = ";
A9D       else cout << "0  = ";
8A2       for(int j=c+1; j<n-1; j++) if(mat[i][j] != T(0))
581         cout << mat[i][j]*T(-1) << " * X" << j << "   +   ";
226       cout << mat[i][n-1] << endl;
F07     }
202   }
AFF };
```

6.6 FFT

Fast Fourier Transform for polynomials multiplication

conv(a, b) = c, where $c[x] = \sum a[i]b[x-i]$.

fft(a) computes $\hat{f}(k) = \sum_x a[x]\exp(2\pi i \cdot kx/N)$ for all k. N must be a power of 2.

Rounding is safe if $(\sum a_i^2 + \sum b_i^2)\log_2 N < 9 \cdot 10^{14}$ (in practice 10^{16}; higher for random inputs).

```
O(N log N) // N=|A|+|B| (1s N <= 2^22)

||      +++ Four Sum i<j<k<l +++        ||     +++ Tree Sum i<j<k +++        ||
|| iiii = vx4;                          || iii = vx3; // vx3[i*3] = v[i]    ||
|| iiij = conv(vx3, v);                 || iij = conv(vx2, v);              ||
|| iijj = conv(vx2, vx2);               || ijk = conv(conv(v, v), v);       ||
|| iijk = conv(vx2, conv(v, v));        || ans = (ijk - 3*iij + 2*iii) / 6; ||
|| ijkl = conv(conv(v, v), conv(v, v)); ||
|| ans = (ijkl -6*iijk +3*iijj +8*iiij -6*iiii) / 24;
* similar pra FWHT, mas vx3 vira V^V^V ou V|V|V e etc...
```

```
8E9 #define ld double //(10% slower if long double)
A18 typedef complex<ld> CD;

B4C void fft(vector<CD>& a){
A5B   int n = a.size(), L = 31 - __builtin_clz(n);
D41
F82   static vector<complex<long double>> R(2, 1);
6B4   static vector<CD> rt(2, 1);
D41
AD8     for(static int k = 2; k < n; k *= 2){
411         auto x = polar(1.0L, acos(-1.0L)/k);
E92         R.resize(n); rt.resize(n);
D41
1D3     for(int i=k; i<2*k; i++)
CD4             rt[i] = R[i] = i&1 ? R[i/2] * x : R[i/2];
040   }
D41
808   vector<int> rev(n);
5EB   for(int i=0; i<n; i++) rev[i] = (rev[i/2] | (i&1)<<L)/2;
EE4   for(int i=0; i<n; i++) if(i<rev[i]) swap(a[i], a[rev[i]]);
D41
657     for(int k=1; k<n; k*=2)
1E5     for(int i=0; i<n; i+=2*k)
0C2             for(int j=0; j<k; j++){
CD2                 auto x=(ld*)&rt[j+k], y=(ld*)&a[i+j+k];
219                 CD z (x[0]*y[0] - x[1]*y[1], x[0]*y[1] + x[1]*y[0]);
D41             //  CD z = rt[j+k] * a[i+j+k]; //(~25% slower,but less code. Delete 2lines above)
20A                 a[i+j+k] = a[i+j] - z;
1B0                 a[i+j] += z;
707         }
F60 }

17B vector<ld> conv(const vector<ld>& a, const vector<ld>& b){
F88   if(a.empty() || b.empty()) return {};
BBB     vector<ld> res(a.size() + b.size() - 1);
E9A     int n = 1<<(32 - __builtin_clz(res.size()));
D41
576     vector<CD> in(n), out(n);
F83     copy(begin(a), end(a), begin(in));
234     for(int i=0; i<b.size(); i++) in[i].imag(b[i]);
D41
21A     fft(in);
11C     for(auto& x : in) x *= x;
2FC     for(int i=0; i<n; i++) out[i] = in[-i&(n-1)] - conj(in[i]);
3D7     fft(out);
D41
E35     for(int i=0; i<res.size(); i++) res[i] = imag(out[i]) / (4*n);
B50     return res;
733 }
```

6.7 FFT MOD

Fast Fourier Transform for polynomials multiplication with **MOD**
FFT com **ALTA PRECISAO** (nao precisa do mod, so coloque cut=1<<15)
Can be used for convolutions modulo arbitrary integers.
as long as $N \log_2 N \cdot \text{mod} < 8.6 \cdot 10^{14}$ (in practice 10^{16} or higher).

!!! Inputs must be in [0, mod). !!!
Get the fft function from fft section.
O(N log N) // (2x slower than NTT or FFT)

```
7A4 #include "FFT.cpp"

6D7 template<const int mod> vector<ll> convMod(const vector<ll> &a, const vector<ll> &b){
F88   if (a.empty() || b.empty()) return {};
290   vector<ll> res(a.size() + b.size() - 1);
A04   int B=32-__builtin_clz(res.size()), n=1<<B, cut=int(sqrt(mod));
584   vector<CD> L(n), R(n), outs(n), outl(n);
D41
FCF     for(int i=0; i<a.size(); i++) L[i] = CD((int)a[i] / cut, (int)a[i] % cut);
71C   for(int i=0; i<b.size(); i++) R[i] = CD((int)b[i] / cut, (int)b[i] % cut);
5D5     fft(L), fft(R);
```

```
D41
603   for(int i=0; i<n; i++){
39D     int j = -i&(n-1);
65E     outl[j] = (L[i] + conj(L[j])) * R[i] / (2.0 * n);
91A     outs[j] = (L[i] - conj(L[j])) * R[i] / (2.0 * n) / 1i;
20D   }
D08     fft(outl), fft(outs);
D41
2C0     for(int i=0; i<res.size(); i++){
54F     ll av = (ll)(real(outl[i])+.5) % mod;
FA2     ll bv = (ll)(imag(outl[i])+.5) + (ll)(real(outs[i])+.5);
A36         ll cv = (ll)(imag(outs[i])+.5);
557     res[i] = ((av * cut + bv) % mod * cut + cv) % mod;
6B2   }
B50   return res;
F58 }
```

6.8 NTT

Number Theoretic Transform for polynomials multiplication MOD

conv(a, b) = c, where $c[x] = \sum a[i]b[x-i]$.

!!! Inputs must be in [0, mod). !!!

For manual convolution: NTT the inputs, multiply pointwise, divide by n,
reverse(start+1, end), NTT back.
Consider using template<const ll mod, const ll root> in conv and ntt if you need more than one mod.
Mod primes must be of the form $2^a b + 1$,
Consider using CRT (Chinese Remainder Theorem) or FFTmod if you need a different MOD.
ntt(a) computes $\hat{f}(k) = \sum_x a[x] g^{xk}$ for all k, where $g = \text{root}^{(mod-1)/N}$.

O(N log N)

```
A6B const ll mod = 998244353, root = 62; //// 9e8 < mod1 < 1e9

15A void ntt(vector<ll> &a){
A5B   int n = a.size(), L = 31 - __builtin_clz(n);
D41
D51     static vector<ll> rt(2, 1);
8EE   for(static int k=2, s=2; k<n; k*=2, s++){
335     rt.resize(n);
8AA     ll z[] = {1, fexp(root, mod >> s)};
631     for(int i=k; i<2*k; i++) rt[i] = rt[i/2] * z[i&1] % mod;
E44   }
D41
808   vector<int> rev(n);
5EB   for(int i=0; i<n; i++) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
EE4   for(int i=0; i<n; i++) if (i < rev[i]) swap(a[i], a[rev[i]]);
D41
657     for(int k=1; k<n; k*=2)
1E5     for(int i=0; i<n; i+=2*k)
0C2             for(int j=0; j<k; j++){
86E                 ll z = rt[j+k] * a[i+j+k] % mod, &ai =a[i+j];
598                 a[i+j+k] = ai - z + (z>ai? mod:0);
4B8                 ai += z - (ai+z>=mod? mod:0);
D6A             }
FB7 }

CCC vector<ll> conv(const vector<ll> &a, const vector<ll> &b) {
F88   if (a.empty() || b.empty()) return {};
919   int s = a.size()+b.size()-1, B = 32 - __builtin_clz(s), n = 1<<B;
D41
F94     vector<ll> L(a), R(b), out(n);
6B4   L.resize(n), R.resize(n);
D9E     ntt(L), ntt(R);
D41
649     int inv = fexp(n, mod - 2);
9CD     for(int i=0; i<n; i++) out[-i&(n-1)] = L[i]*R[i] % mod * inv % mod;
D41
EC9     ntt(out);
C20     return {out.begin(), out.begin() + s};
4BF }

A01 const ll mod2 = 918552577, root2 = 63; // 9e8 < mod2 < 1e9 //also valid mods
551 const ll mod3 = 7340033,   root3 = 25; // 7e6 < mod3 < 1e7
```

Computes the first LIM terms of $P(x)^K$ in O(n*limit)

```
9E3 vector<ll> power(vector<ll> &p, int k, int limit=-1){ //O(n*limit)
884     while(p.back() == 0) p.pop_back();
EF6     if(p.empty() || limit == 0) return {};
AAF     if(limit == -1) limit = (p.size()-1) * k;
D41
AB5     vector<ll> ans(limit+1, 0);
1F5     ans[0] = fexp(p[0], k);
D41
D36     for(int i=1; i<=limit; i++){
B60         for(int j=1; j <= i && j < p.size(); j++){
FE0             ans[i] += p[j] * ans[i-j] % mod * (k*j - (i-j)) % mod;
30D             ans[i] %= mod;
3E4         }
8A6         ans[i] = ans[i] * fexp(i * p[0] % mod, mod-2) % mod;
C37     }
BA7     return ans;
213 }
```

6.9 FWHT

Fast Walsh Hadamard Transform - **Convolucao de XOR, OR e AND**
O(N log N)

```
0E4 template<const char op>
8A6 vector<ll> FWHT(vector<ll> a, const bool inv = false){
94D     int n = a.size();
1E0     for(int len=1; len<n; len+=len)
EBC         for(int i=0; i<n; i += 2*len)
7AB             for(int j=0; j<len; j++){
032                 ll u = a[i+j], v = a[i+j+len];
FBD                 if(op == '^') a[i+j]   = u+v, a[i+j+len] = u-v;
02F                 if(op == '|') a[i+j+len] = v + (inv ? -u : +u);
729                 if(op == '&') a[i+j]     = u + (inv ? -v : +v);
1C4             }
D41
163     if(op=='^'&&inv) for(auto &x : a) x /= n;
3F5     return a;
5FE }

0E4 template<const char op>
C36 vector<ll> multiply(vector<ll> a, vector<ll> b){
1C9   int n=1; while(n < max(a.size(), b.size())) n*=2;
067   a.resize(n, 0);  b.resize(n, 0);
FAE   a = FWHT<op>(a); b = FWHT<op>(b);
D41
3A6   vector<ll> ans(n);
224   for(int i=0; i<n; i++) ans[i] = a[i]*b[i] % mod;
90C   ans = FWHT<op>(ans, true);
BA7   return ans;
7BC }
```

```
A2A const int mxlog = 17;
FBF vector<ll> subset_multiply(vector<ll> a, vector<ll> b){ //OPTIONAL
21C     int n = 1; while(n < max(a.size(), b.size())) n <<= 1;
067     a.resize(n, 0);  b.resize(n, 0);
87C     vector<ll> ans(n, 0LL); vector A(mxlog+1, vector<ll>(n)), B = A;
06A     for(int i=0; i<n; i++) A[__builtin_popcount(i)][i]=a[i], B[__builtin_popcount(i)][i
     ]=b[i];
554     for(int i=0; i<=mxlog; i++) A[i] = FWHT<'|'>(A[i]), B[i] = FWHT<'|'>(B[i]);
811     for(int i=0; i<=mxlog; i++){
E71         vector<ll> C(n);
F7D         for(int x=0; x<=i; x++)
F90             for(int j=0; j<n; j++)
0C3                 C[j] += A[x][j] * B[i-x][j];
D41
E1C         C = FWHT<'|'>(C, true);
F90         for(int j=0; j < n; j++)
256             if(__builtin_popcount(j) == i)
FCA                 ans[j] += C[j];
B12     }
BA7     return ans;
1C3 }
```

6.10 Matrix

```
92B template<typename T> struct Matrix {
C24   vector<vector<T>> mat;
14E   int n, m;
D41
690   Matrix(int N, int M=0) : n(N), m(M?M:N){ mat.assign(n, vector<T>(m, 0)); }
D41
D77   friend Matrix operator* (const Matrix &a, const Matrix &b){
C00     assert(a.m == b.n);
014     Matrix ans(a.n, b.m);
B60     for(int i=0; i<a.n; i++)
A09       for(int j=0; j<b.m; j++)
7AF         for(int k=0; k<a.m; k++)
247           ans.mat[i][j] += a.mat[i][k] * b.mat[k][j];
BA7     return ans;
52E   }
3CC };

F4D template<typename T> Matrix<T> inverse(Matrix<T> mat){
C8E   int n = mat.n; assert(n == mat.m);
A65   Gauss<T> g(n+n);
603   for(int i=0; i<n; i++){
CEF     vector<T> line = mat.mat[i];
D7F     line.resize(n+n, 0); line[n+i] = 1;
841     g.addLine(line);
8A6   }
5A4   g.solve();
830   for(int i=0; i<n; i++)
B1F     mat.mat[i] = {begin(g.mat[i])+n, end(g.mat[i])};
B49   return mat;
F98 }
```

6.11 mint

```
E54 struct mint {
60E   ll v = 0;
279   mint(ll x=0) : v((x%mod+mod)%mod){}
2D0   mint operator+ (const mint &b) const { ll a = v+b.v; return a < mod ? a : a-mod; }
348   mint operator- (const mint &b) const { ll a = v-b.v; return a <  0  ? a+mod : a; }
AE3   mint operator* (const mint &b) const { return v * b.v % mod; }
834   mint operator/ (const mint &b) const { return v * fexp(b.v, mod-2) % mod;  }
6C4 };
D41 // Extra Operators - Bool Operators
A7B bool operator==(const mint&a, const mint &b){ return a.v == b.v; }
2F6 bool operator!=(const mint&a, const mint &b){ return a.v != b.v; }
6AB bool operator< (const mint&a, const mint &b){ return a.v <  b.v; }
D41 // Assignment Operators
D49 mint operator+= (mint&a, const mint &b){ return a = a + b; }
373 mint operator-= (mint&a, const mint &b){ return a = a - b; }
005 mint operator*= (mint&a, const mint &b){ return a = a * b; }
74C mint operator/= (mint&a, const mint &b){ return a = a / b; }
```

6.12 random

```
C8A mt19937 rng(chrono::steady_clock::now().time_since_epoch().count());
D41 //int x = rng();

463 int uniform(int l, int r){
A7F   uniform_int_distribution<int> uid(l, r);
F54   return uid(rng);
D9E }
```

6.13 StirlingNumbers

```
81D #include "Combinatorics.cpp"

0E0 ll st1[MAXN][MAXN];
55D void StirlingFirst_nn(){ // O(n^2)
55C     st1[0][0] = 1;
D41
E5E     for(int n=1; n<MAXN; n++)
871         for(int k=1; k<=n; k++)
3AA             st1[n][k] = (n-1) * st1[n-1][k] + st1[n-1][k-1];
C68 }

882 ll st2[MAXN][MAXN];
E95 void StirlingSecond_nn(){ // O(n^2)
524     st2[0][0] = 1;
D41
E5E     for(int n=1; n<MAXN; n++)
871         for(int k=1; k<=n; k++)
B3F             st2[n][k] = k * st2[n-1][k] + st2[n-1][k-1];
2BB }

D41 //// Fixed N ////
887 #include "NTT.cpp"

C6C vector<ll> shift(vector<ll> pol, ll n){ //para StirlingFirst
05B         ll s = pol.size(), k = 1;
5BF         vector<ll> a(s), b(s);
22C         for(int i=0; i<s; i++, k=k*n%mod){
377             a[i] = pol[i] * fat[i] % mod;
3F8             b[s-i-1] = k * finv[i] % mod;
F44         }
90A         a = conv(a, b);
10C         for(int i=0; i<s; i++) pol[i] = a[s-1+i] * finv[i] % mod;
B22         return pol;
7B6 }

8BD vector<ll> StirlingFirst(ll n, bool sign = false){  //O(n log^2 n) //lembrar do fat/
     finv
AD0     if(n == 0) return {1};
108     vector<ll> pol(n+1), q = n&1 ? StirlingFirst(n-1) : StirlingFirst(n/2);
D41
BE3   if(n&1){
891         for(int i=0; i<n-1; i++)
```

```
F2F             pol[i+1] = (q[i] + q[i+1] * (n-1)) % mod;
04C         pol[n] = 1;
8D4     }
884   else pol = conv(q, shift(q, n/2));
D41
B2C     if(sign&&n&1) for(int i=0; i<n/2+1; i++) pol[i*2]   = (mod - pol[i*2])   % mod;
FDF     if(sign>(n&1))for(int i=0; i<n/2;   i++) pol[i*2+1] = (mod - pol[i*2+1]) % mod;
D41
B22     return pol;
3E8 }

920 vector<ll> StirlingSecond(int n){ //O(n log^2 n) //lembrar do fat/finv
4F7     vector<ll> a(n+1), b(n+1);
690     for(int i=0; i<n+1; i++){
5D6         a[i] = fexp(i, n) * finv[i] % mod;
E3C         b[i] = i&1 ? mod - finv[i] : finv[i];
2F4     }
90A     a = conv(a, b);
6E9     return {begin(a), begin(a)+n+1};
D42 }
```

7 Geometry

7.1 Point

Dot product p*q $= p \cdot q$ | inner product | norm | lenght^2
$u \cdot v = x_1x_2 + y_1y_2 = \|u\|\,\|v\|\cos\theta$
$u \cdot v > 0 \Rightarrow$ angle $\theta < 90^\circ$ (acute);
$u \cdot v = 0 \Rightarrow$ angle $\theta = 90^\circ$ (perpendicular);
$u \cdot v < 0 \Rightarrow$ angle $\theta > 90^\circ$ (obtuse);

Cross product p % q $= p \times q$: | Vector product | Determinant
$u \times v = x_1y_2 - y_1x_2 = \|u\|\,\|v\|\sin\theta$.
$u \times v > 0 \Rightarrow v$ is to the *left* of u
$u \times v = 0 \Rightarrow u$ and v are collinear.
$u \times v < 0 \Rightarrow v$ is to the *right* of u
It equals the signed area of the parallelogram spanned by u and v.

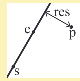
+ p.cross(a, b) $= (a-p) \times (b-p)$
 - > 0: CCW (left); $\curvearrowleft$
 - $= 0$: collinear; $\Rightarrow$
 - < 0: CW (right); $\curvearrowright$

```
8E9 #define ld double

C19 struct PT {
0BE   ll x, y;
0A5   PT(ll x=0, ll y=0) : x(x), y(y) {}
D41
006   PT operator+(const PT&a)const{return PT(x+a.x, y+a.y);}
0DC   PT operator-(const PT&a)const{return PT(x-a.x, y-a.y);}
954   ll operator*(const PT&a)const{return  (x*a.x + y*a.y);} //DOT
A68   ll operator%(const PT&a)const{return  (x*a.y - y*a.x);} //Cross
B54   PT operator*(ll c)  const{ return PT(x*c, y*c); }
B25   PT operator/(ll c)  const{ return PT(x/c, y/c); }
5C7   bool operator==(const PT&a) const{ return x == a.x && y == a.y; }
539   bool operator< (const PT&a) const{ return tie(x, y) < tie(a.x, a.y); }
D41
652   ld len() const  { return hypot(x,y); } // sqrt(p*p)
3FC   ll cross(const PT&a, const PT&b) const{ return (a-*this) % (b-*this); } // (a-p) % (b
    -p)
950   int quad() { return (x<0)^3*(y<0); } //cartesian plane quadrant |0++|1-+|2--|3+-|
94A   bool ccw(PT q, PT r){ return (q-*this) % (r-q) > 0;}
17A };

33E ld dist(PT p, PT q){ return sqrtl((p-q)*(p-q)); }
0FB ld proj(PT p, PT q){ return p*q / q.len(); }
D41 //Projection size from A to B

C4F const ld PI = acos(-1.0L);
50C ld angle(PT p, PT q){ return atan2(p%q, p*q); } // Angle between vectors p and q [-pi,
    pi] | acos(a*b/a.len()/b.len()))
E07 ld polarAngle(PT p){  return atan2(p.y, p.x); } // Angle to x-axis [-pi, pi]
AF5 bool cmp_ang(PT p, PT q){ return p.quad() != q.quad() ? p.quad()<q.quad() : q.ccw(PT
    (0,0), p); }

874 PT rotateCCW90(PT p){ return PT(-p.y, p.x); } // perp
222 PT rotateCW90(PT p){ return PT(p.y, -p.x); }
96F PT rotateCCW(PT p, ld t){
E8C   ld c = cos(t), s = sin(t);
D80   return PT(p.x*c - p.y*s, p.x*s + p.y*c);
93E }
```

7.2 Line

```
D41 //if p is on line s to e
77D bool onLine(PT s, PT e, PT p){ return p.cross(s, e) == 0;}
```

Returns the **signed dist** from p and the **line** of a and b. Positive value on left side and negative on right as seen from a->b. (a!=b)




```
41B ld lineDist(PT& a, PT& b, PT& p){ return (b-a) % (p-a) / (b-a).len(); }
```

Intersection between two lines
Unique -> {+1, pt}
No inter -> { 0, pt}
Infinity -> {-1, pt}May be rounded if inter isn't integer; Watch out for overflow if long long.

```
5E1 pair<int, PT> lineInter(PT a, PT b, PT e, PT f){
8B1   auto d = (b-a) % (f-e);
FC7   if(d == 0) return {-(a.cross(b, e) == 0), PT()}; //parallel
F29   auto p = e.cross(b, f), q = e.cross(f, a);
336   return {1, (a * p + b * q) / d};
F59 }
```

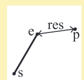
Projects point p onto line ab. Set refl=true to get reflection of point p across line ab instead.

```
4E5 PT lineProj(PT a, PT b, PT p, bool refl=false) {
493   PT v = b-a;
7A4   return p - rotateCCW90(v) * (1+refl) * (v%(p-a)) / (v*v);
7E1 }
```

7.3 Segment

```
D41 //if p is on segment s to e
C39 bool onSegment(PT s, PT e, PT p){
6A6   return p.cross(s, e) == 0 && (s-p) * (e-p) <= 0;
960 }
```

Returns the shortest **distance** between point p and the **segment** s->e.



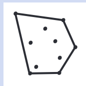

```
95D ld segmentDist(PT& s, PT& e, PT& p){
BD2   if (s==e) return (p-s).len();
4B2   ld d = (e-s)*(e-s);
385   ld t = min(d, max<ld>(0, (p-s)*(e-s)));
9E6   return ((p-s)*d - (e-s)*t).len() / d;
A45 }
```

Segment intersection
Unique -> {p}
No inter -> { }
Infinity -> {a, b}, the endpoints of the common segment.
May be rounded if inter isn't integer; Watch out for overflow if long long.

```
3DA int sgn(ll x){ return (x>0) - (x<0); }
FFB vector<PT> segInter(PT a, PT b, PT c, PT d){
E62   auto oa = c.cross(d, a), ob = c.cross(d, b);
473   auto oc = a.cross(b, c), od = a.cross(b, d);
914   if(sgn(oa)*sgn(ob) < 0 && sgn(oc)*sgn(od) < 0)
E5B     return {(a*ob - b*oa) / (ob-oa)};
529   set<PT> s;
CCB   if(onSegment(c, d, a)) s.insert(a);
0AD   if(onSegment(c, d, b)) s.insert(b);
3D8   if(onSegment(a, b, c)) s.insert(c);
2FA   if(onSegment(a, b, d)) s.insert(d);
C2C   return {begin(s), end(s)};
276 }
```

7.4 ConvexHull

Given a vector of points, return the convex hull in CCW order.
A convex hull is the smallest convex polygon that contains all the points.



If you want colinear points in border, change the >=0 to >0 in the while's.
WARNING: if collinear and all input PT are collinear, may have duplicated points (the round trip)

```
CD7 vector<PT> ConvexHull(vector<PT> pts, bool sorted=false){
EC1   if(!sorted) sort(begin(pts), end(pts));
6E7   pts.resize(unique(begin(pts), end(pts)) - begin(pts));
64B   if(pts.size() <= 1) return pts;
D41
B4E   int s=0, n=pts.size();
988   vector<PT> h(2*n+1);
D41
AA9   for(int i=0; i<n; h[s++] = pts[i++])
43F     while(s > 1 && h[s-2].cross(pts[i], h[s-1]) >= 0 )
351       s--;
D41
61B   for(int i=n-2, t=s; ~i; h[s++] = pts[i--])
BB6     while(s > t && h[s-2].cross(pts[i], h[s-1]) >= 0 )
351       s--;
D41
CBB   h.resize(s-1);
81C   return h;
CBB } //PT operators needed: {- % == <}
```

Check **if a point is inside convex hull** (CCW, no collinear).If strict == true, then pt on boundary return falseO(log N)

```
3D7 bool isInside(const vector<PT>& h, PT p, bool strict = true){
579   int a = 1, b = h.size() - 1, r = !strict;
795   if(h.size() < 3) return r && onSegment(h[0], h.back(), p);
59E   if(h[0].cross(h[a], h[b]) > 0) swap(a, b);
317   if(h[0].cross(h[a], p) >= r || h[0].cross(h[b], p) <= -r) return false;
48A   while(abs(a-b) > 1){
4F7     int c = (a + b) / 2;
142     if(h[0].cross(h[c], p) > 0) b = c;
1B9     else a = c;
7E3   }
B11   return h[a].cross(h[b], p) < r;
EB9 }
```

Check **if a point is inside convex hull**
O(log N)

```
E13 bool isInside(const vector<PT> &h, PT p){
66D   if(h[0].cross(p, h[1]) > 0 || h[0].cross(p, h.back()) < 0) return false;
B28     int n = h.size(), l=1, r = n-1;
E55     while(l != r){
264         int mid = (l+r+1)/2;
B64         if(h[0].cross(p, h[mid]) < 0) l = mid;
943         else r = mid - 1;
D3D     }
0F2     return h[l].cross(h[(l+1)%n], p) >= 0;
CBC }
```

Given a convex hull h and a point p, returns the indice of h **where the dot product is maximized.** This code assumes that there are NO 3 colinear points!

```
DD1 int maximizeScalarProduct(const vector<PT> &h, PT v) {
A75     int ans = 0, n = h.size();
F37     if(n < 20){
830     for(int i=0; i<n; i++)
070             if(v*h[ans] < v*h[i])
C46                 ans = i;
BA7     return ans;
E80     }
D41
866   for(int rep=0; rep<2; rep++){
D47     int l = 2, r = n-1;
E55     while(l != r){
264       int mid = (l+r+1)/2;
9E8       int f = v*h[mid] >= v*h[mid-1];
D41
FCF       if(rep) f |= v*h[mid-1] < v*h[0];
622       else    f &= v*h[mid]  >= v*h[0];
D41
109       if(f) l = mid;
943       else  r = mid - 1;
9A3     }
48D     if(v*h[ans] < v*h[l]) ans = l;
6A2   }
3D0   if(v*h[ans] < v*h[1] ) ans = 1;
BA7   return ans;
E80 }
```

7.5 Poligons

Returns **twice area of a simple polygon.** area*2 (Shoelace Formula: signed cross product sum)

```
5AB ll Area2x(vector<PT>& p){
604   ll area = 0;
37F   for(int i=2; i < p.size(); i++)
20A     area += (p[i]-p[0]) % (p[i-1]-p[0]);
199   return abs(area);
64B }
```

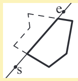
Returns if a point is **inside a triangle** (or in the border).

```
5CA bool ptInsideTriangle(PT p, PT a, PT b, PT c){
58B   if((b-a) % (c-b) < 0) swap(a, b);
805   if(onSegment(a,b,p)) return 1;
1A3   if(onSegment(b,c,p)) return 1;
1DB   if(onSegment(c,a,p)) return 1;
13A   bool x = (b-a) % (p-b) < 0;
B85   bool y = (c-b) % (p-c) < 0;
CE5   bool z = (a-c) % (p-a) < 0;
4B5   return x == y && y == z;
9C6 }
```

Returns the **center of mass for a polygon.** O(n)

```
303 PT polygonCenter(const vector<PT>& v){
313   PT res(0, 0); double A = 0;
E3C   for(int i=0, j=v.size()-1; i<v.size(); j=i++){
FF1     res = res + (v[i]+v[j]) * (v[j]%v[i]);
587     A += v[j] % v[i];
D4F   }
33C   return res / A / 3;
CD0 }
```

PolygonCut: Returns the vertices of the polygon cut away at the left of the line s->e.polygonCut(p, PT(0,0), PT(1,0));



```
767 vector<PT> polygonCut(const vector<PT>& poly, PT s, PT e){
81A   vector<PT> res;
6F1   for(int i=0; i<poly.size(); i++){
431     PT cur = poly[i], prev = i ? poly[i-1] : poly.back();
C5F     auto a = s.cross(e, cur), b = s.cross(e, prev);
498     if((a < 0) != (b < 0)) res.push_back(cur + (prev - cur) * (a / (a - b)));
DDB     if(a < 0) res.push_back(cur);
1E0   }
B50   return res;
D6D }
```

Pick's theorem for **lattice points** in a simple polygon. (lattice points = integer points)Area = insidePts + boundPts/2 - 12A - b + 2 = 2i

```
CDC ll cntInsidePts(ll area_db, ll bound){ return (area_db + 2LL - bound)/2; }
ED9 ll latticePointsInSeg(PT a, PT b){
FA7   ll dx = abs(a.x - b.x);
97A   ll dy = abs(a.y - b.y);
695   return gcd(dx, dy) + 1;
FA7 }
```

7.6 Circles

The **circumcirle of a triangle** is the circle intersecting all three vertices.

```
8BC double ccRadius(PT& A, PT& B, PT& C) {
F6D   return (B-A).len()*(C-B).len()*(A-C).len() / abs(A.cross(B, C))/2;
BEA }
660 PT ccCenter(PT& A, PT& B, PT& C) {
0BF   PT b = C-A, c = B-A;
D0F   return A + rotateCCW90(b*(c*c) - c*(b*b)) / (b%c) / 2;
311 }
```

Return the points at **two circles intersection.** If none or infinity, returns empty

```
240 vector<PT> circleCircleInter(PT a, ld r1, PT b, ld r2){
AC5   if (a == b) return {}; //r1==r2? infinity : none
493   PT v = b-a;
D41
95B   ld d2 = v*v, sum = r1+r2, dif = r1-r2;
102   ld p = (d2 + r1*r1 - r2*r2) / (d2+d2), h2 = r1*r1 - p*p*d2;
D41
975   if(sum*sum < d2 || dif*dif > d2) return {};
D41
56B   PT mid=a+v*p, per=rotateCCW90(v)*sqrt(fmax(0, h2) / d2);
D41
677   set<PT> ans = {mid + per, mid - per};
C85   return {begin(ans), end(ans)};
8C4 }
```

Return the **circle line intersection.** Return a vector of 0,1 or 2 PTs

```
CD6 vector<PT> circleLineInter(PT c, ld r, PT a, PT b){
C12   PT ab = b-a;
288   PT p = a + ab * ((c-a)*ab) / (ab*ab);
A8D   ld s = a.cross(b, c);
90B   ld h2 = r*r - s*s / (ab*ab);
3E4   if(h2 < 0) return {};
071   if(h2 == 0) return {p};
99E   PT h = ab/ab.len() * sqrt(h2);
D65   return {p - h, p + h};
8BF }
```

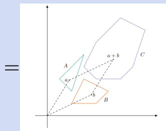
Returns the **minimum enclosing circle** for a set of points. Expected O(n)

```
839 pair<PT, ld> minEnclose(vector<PT> ps) {
504   shuffle(begin(ps), end(ps), mt19937(time(0)));
11E   PT o = ps[0];
F92   ld r=0, EPS = 1 + 1e-8;
D41
860   for(int i=0; i<ps.size(); i++) if(dist(o, ps[i]) > r*EPS){
5CC     o = ps[i], r = 0;
D41
373     for(int j=0; j<i; j++) if(dist(o, ps[j]) > r*EPS){
A30       o = (ps[i] + ps[j]) / 2;
FD2       r = dist(o, ps[i]);
D41
A09       for(int k=0; k<j; k++) if(dist(o, ps[k]) > r*EPS){
FA9         o = ccCenter(ps[i], ps[j], ps[k]);
ED2         r = (o - ps[i]).len();
8BA       }
A2E     }
277   }
D41
645   return {o, r};
AC9 }
```

7.7 Minkowski

Minkowski Sum of convex polygons - O(N)

Returns a convex hull of two polygons minkowski sum.
The minkowski sum of polygons A and B is a polygon such that every vectorinside it is the sum of a vector in A and a vector in B. $A + B = C = \{a + b \mid a \in A,\ b \in B\}$
min(a.size(), b.size()) >= 2



```
D41 // rotate the polygon such that the (bottom, left)-most point is at the first position
C16 void reorder_polygon(vector<PT> &p){
BEC     int pos = 0;
BAA     for(int i = 1; i < p.size(); i++)
8EE         if(pair(p[i].y, p[i].x) < pair(p[pos].y, p[pos].x))  //if(p[i].y < p[pos].y ||
    (p[i].y == p[pos].y && p[i].x < p[pos].x))
E4C             pos = i;
D3C     rotate(p.begin(), p.begin() + pos, p.end());
E7B }

809 vector<PT> minkowski(vector<PT> a, vector<PT> b){
83C     int n = a.size(), m = b.size(), i=0, j=0;
490     reorder_polygon(a); reorder_polygon(b);
5CA     a.push_back(a[0]); a.push_back(a[1]);
258     b.push_back(b[0]); b.push_back(b[1]);
D41
649     vector<PT> c;
59B     while(i < n || j < m){
018         c.push_back(a[i] + b[j]);
47E         auto p = (a[i+1] - a[i]) % (b[j+1] - b[j]);
46D         if(p >= 0) i++;
7D0         if(p <= 0) j++;
266     }
807     return c;
```

8 Theorems

8.1 Propriedades Matemáticas

- **Lagrange:** Todo número inteiro pode ser representado como soma de 4 quadrados.
- **Zeckendorf:** Todo número pode ser representado como soma de números de Fibonacci diferentes e não consecutivos.
- **Tripla de Pitágoras (Euclides):** Toda tripla pitagórica primitiva pode ser gerada por $(n^2 - m^2, 2nm, n^2 + m^2)$ onde n e m são coprimos e um deles é par.
- **Progressão Geométrica:** $S_n = a_1 \cdot \frac{q^n - 1}{q - 1}$
- **Soma de quadrados e cubos:** $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$, $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$
- **Chicken McNugget:** Para dois coprimos x e y, o número de inteiros que não podem ser expressos como $ax + by$ é $(x-1)(y-1)/2$. O maior inteiro não representável é $xy - x - y$.
- **Primos até 200:** 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199
- **Fibonacci** $[0, 20]$: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765.
- **Fatorial** $[0, 15]$: 1, 1, 2, 6, 24, 120, 720, 5040, 40320, 362880, 3628800, 39916800, 479001600, 6227020800, 87178291200, 1307674368000.
- **Potencias**
  2^n $[1, 16]$: 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096, 8192, 16384, 32768, 65536
  3^n $[1, 13]$: 3, 9, 27, 81, 243, 729, 2187, 6561, 19683, 59049, 177147, 531441, 1594323
  5^n $[1, 12]$: 5, 25, 125, 625, 3125, 15625, 78125, 390625, 1953125, 9765625, 48828125, 244140625
  7^n $[1, 9]$: 7, 49, 343, 2401, 16807, 117649, 823543, 5764801, 40353607, 282475249
- **Totiente de Euler:** $\varphi(n)$ conta quantos números entre 1 e n são coprimos com n.
  Seq.$[1, 32]$: 1, 1, 2, 2, 4, 2, 6, 4, 6, 4, 10, 4, 12, 6, 8, 8, 16, 6, 18, 8, 12, 10, 22, 8, 20, 12, 18, 12, 28, 8, 30

Primos

- **Goldbach:** Todo número par $n > 2$ pode ser representado como $n = a + b$, onde a e b são primos.
- **Primos Gêmeos:** Existem infinitos pares de primos p, $p + 2$.
- **Legendre:** Sempre existe um primo entre n^2 e $(n+1)^2$.
- **Wilson:** n é primo se e somente se $(n-1)! \mod n = n - 1$.

Combinatória

- **Propriedades de Coeficientes Binomiais:**

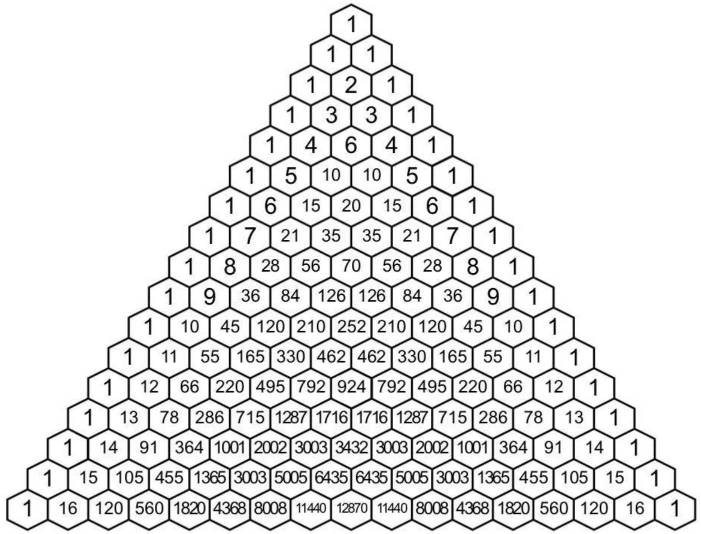
$$\binom{n}{k} = \binom{n}{n-k} = \frac{n}{k}\binom{n-1}{k-1},$$

$$\sum_{i=0}^{k}\binom{m}{i}\binom{n}{k-i} = \binom{m+n}{k}, \sum_{i=0}^{m}\binom{n}{i}(-1)^i = \binom{n-1}{m}(-1)^m$$

$$\sum_{i=0}^{n}\binom{n}{i} = 2^n, \qquad \sum_{i=0}^{n}\binom{n}{i} i = n \cdot 2^{n-1}, \qquad \sum_{i=0}^{n}\binom{i}{k} = \binom{n+1}{k+1},$$

$$\sum_{i=0}^{n}\binom{n-i}{i} = F_{n+1}, \qquad \sum_{i=0}^{m}\binom{n+i}{i} = \binom{n+m+1}{m}, \quad \sum_{i=0}^{n}\binom{n}{i}^2 = \binom{2n}{n}$$

- **Triângulo de Pascal**



- **Números de Catalan:** expressões de parênteses bem formadas, #binary trees com n+1 nodes, etc. $C_0 = 1$, e:

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \sum_{i=0}^{n-1} C_i C_{n-1-i}$$

Seq.$[0, 20]$: 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, 208012, 742900, 2674440, 9694845, 35357670, 129644790, 477638700, 1767263190, 6564120420.

- **Burnside Lemma:** Número de colares diferentes (sem contar rotações), com m cores e comprimento n:

$$\frac{1}{n}\left(m^n + \sum_{i=1}^{n-1} m^{\gcd(i,n)}\right)$$

- **Inversão de Möbius:** Útil para inclusão e exclusão nos primos. Seq.$[1,30]$: 1, -1, -1, 0, -1, 1, -1, 0, 0, 1, -1, 0, 1, 1, 0, -1, 1, 0, 0, 1, -1, 1, 0, -1, -1, 0, 1, 0, 0, 1. $\sum_{d|n} \mu(d) = 0$ if $n > 1$ else 1
- **Lindström-Gessel-Viennot:** A quantidade de caminhos disjuntos em um grid pode ser computada como o determinante da matriz do número de caminhos.
- **Numeros de Stirling:**
  - **First Type:** $|s(n,m)|$ é definido como a quantidade de permutações de n elementos que contém exatamente m ciclos de permutação. Tem aplicações em inclusão e exclusão.

$$\begin{bmatrix} n \\ k \end{bmatrix} = (n-1)\begin{bmatrix} n-1 \\ k \end{bmatrix} + \begin{bmatrix} n-1 \\ k-1 \end{bmatrix} = (-1)^{n-k} s(n,k)$$

$$\begin{bmatrix} n \\ 1 \end{bmatrix} = (n-1)! \quad , \quad \begin{bmatrix} n \\ n \end{bmatrix} = 1 \quad , \quad \begin{bmatrix} n+1 \\ m+1 \end{bmatrix} = \sum_i \begin{bmatrix} n \\ i \end{bmatrix} \binom{i}{m}$$

$n \vert k$	0	1	2	3	4	5	6
0	1						
1		1					
2		-1	1				
3		2	-3	1			
4		-6	11	-6	1		
5		24	-50	35	-10	1	
6		-120	274	-225	85	-15	1

  - **Second Type:** O Stirling set $\left\{ {n \atop k} \right\}$ conta maneiras de particionar um conjunto de n elementos em k sub-conjuntos não vazios. Equivalente a Rhyming Schemes de n posições e k símbolos. (exemplo n=3: k=1 {aaa}; k=2 {aab, aba, abb}; k=3 {abc})

$$\left\{ {n \atop k} \right\} = k\left\{ {n-1 \atop k} \right\} + \left\{ {n-1 \atop k-1} \right\}$$

$$\left\{ {n \atop 1} \right\} = \left\{ {n \atop n} \right\} = 1 \quad , \quad \left\{ {n+1 \atop k+1} \right\} = \sum_i \left\{ {i \atop k} \right\} \binom{n}{i}$$

$n \vert k$	0	1	2	3	4	5	6
0	1						
1		1					
2		1	1				
3		1	3	1			
4		1	7	6	1		
5		1	15	25	10	1	
6		1	31	90	65	15	1

- **Números de Bell B_n:** numero de partições de um conjunto de n elementos. $B_n = \sum_i \left\{ {n \atop i} \right\} = \sum_i \binom{n-1}{i} B_i$. $[0,12]$: 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, 115975, 678570, 4213597.
- **Eulerian numbers (first order):**
  O Eulerian number $\left\langle {n \atop k} \right\rangle$ ou $A(n,k)$ conta o número de permutações de tamanho n com exatamente k *ascents* $(p_i < p_{i+1})$.

$$\left\langle {n \atop k} \right\rangle = (k+1)\left\langle {n-1 \atop k} \right\rangle + (n-k)\left\langle {n-1 \atop k-1} \right\rangle = \sum_{i=0}^{k} (-1)^i \binom{n+1}{i} (k+1-i)^n$$

$$\left\langle {1 \atop 0} \right\rangle = 1, \qquad \left\langle {n \atop 1} \right\rangle = 2^n - n - 1, \qquad \sum_{k=0}^{n-1} \left\langle {n \atop k} \right\rangle = n!$$

$n \vert k$	0	1	2	3	4	5	6
1	1						
2	1	1					
3	1	4	1				
4	1	11	11	1			
5	1	26	66	26	1		
6	1	57	302	302	57	1	
7	1	120	1191	2416	1191	120	1

Modular

- **Fermat:** Se p é primo, então $a^{p-1} \equiv 1 \mod p$. Se x e m são coprimos e m primo, então $x^k \equiv x^{k \bmod (m-1)} \mod m$. *Euler:* $x^{\varphi(m)} \equiv 1 \mod m$. $\varphi(m)$ é o totiente de Euler.
- **Teorema Chinês do Resto:** Dado um sistema de congruências: $x \equiv a_1 \mod m_1, \quad \ldots, \quad x \equiv a_n \mod m_n$ com m_i coprimos dois a dois. E seja $M_i = \frac{m_1 m_2 \cdots m_n}{m_i}$ e $N_i = M_i^{-1} \mod m_i$. Então a solução é dada por $x = \sum_{i=1}^{n} a_i M_i N_i$ Outras soluções são obtidas somando $m_1 m_2 \cdots m_n$.

Probabilidade

- **Bertrand Ballot:** Com $p > q$ votos, a probabilidade de sempre haver mais votos do tipo A do que B até o fim é: $\frac{p-q}{p+q}$ Permitindo empates: $\frac{p+1-q}{p+1}$. Multiplicando pela combinação total $\binom{p+q}{q}$, obtém-se o número de possibilidades.
- **Linearidade da Esperança:** $E[aX + bY] = aE[X] + bE[Y]$
- **Variância:** $\mathrm{Var}(X) = E[(X-\mu)^2] = E[X^2] - E[X]^2$
- **Uniforme:** $X \in \{a, a+1, \ldots, b\}$, $E[X] = \frac{a+b}{2}$
- **Binomial:** n tentativas com probabilidade p de sucesso: $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, $\quad E[X] = np$
- **Geométrica:** Número de tentativas até o primeiro sucesso: $P(X = x) = (1-p)^{x-1} p$, $\quad E[X] = \frac{1}{p}$

8.2 Geometria

- **Fórmula de Euler:** Em um grafo planar ou poliedro convexo, temos: $V - E + F = 2$ onde V é o número de vértices, E o número de arestas e F o número de faces.
- **Teorema de Pick:** Para polígonos com vértices em coordenadas inteiras:

$$\text{Área} = i + \frac{b}{2} - 1$$

  onde i é o número de pontos interiores e b o número de pontos sobre o perímetro.
- **Teorema das Duas Orelhas (Two Ears Theorem):** Todo polígono simples com mais de três vértices possui pelo menos duas "orelhas"— vértices que podem ser removidos sem gerar interseções. A remoção repetida das orelhas resulta em uma triangulação do polígono.

- **Incentro de um Triângulo:** É o ponto de interseção das bissetrizes internas e centro da circunferência inscrita. Se a, b e c são os comprimentos dos lados opostos aos vértices $A(X_a, Y_a)$, $B(X_b, Y_b)$ e $C(X_c, Y_c)$, então o incentro (X, Y) é dado por:

$$X = \frac{aX_a + bX_b + cX_c}{a+b+c}, \quad Y = \frac{aY_a + bY_b + cY_c}{a+b+c}$$

- **Triangulação de Delaunay:** Uma triangulação de um conjunto de pontos no plano tal que nenhum ponto está dentro do círculo circunscrito de qualquer triângulo. Essa triangulação:
  - Maximiza o menor ângulo entre todos os triângulos.
  - Contém a árvore geradora mínima (MST) euclidiana como subconjunto.
- **Fórmula de Brahmagupta:** Para calcular a área de um quadrilátero cíclico (todos os vértices sobre uma circunferência), com lados a, b, c e d:

$$s = \frac{a+b+c+d}{2}, \quad \text{Área} = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

Se $d = 0$ (ou seja, um triângulo), ela se reduz à fórmula de Heron:

$$\text{Área} = \sqrt{(s-a)(s-b)(s-c)s}$$

8.3 Grafos

- **Fórmula de Euler (para grafos planares):**

$$V - E + F = 2$$

onde V é o número de vértices, E o número de arestas e F o número de faces.
- **Handshaking Lemma:** O número de vértices com grau ímpar em um grafo é par.
- **Teorema de Kirchhoff (contagem de árvores geradoras):** Monte a matriz M tal que:

$$M_{i,i} = \deg(i), \quad M_{i,j} = \begin{cases} -1 & \text{se existe aresta } i - j \\ 0 & \text{caso contrário} \end{cases}$$

O número de árvores geradoras (spanning trees) é o determinante de qualquer co-fator de M (remova uma linha e uma coluna).
- **Condições para Caminho Hamiltoniano:**
  - **Teorema de Dirac:** Se todos os vértices têm grau $\geq n/2$, o grafo contém um caminho Hamiltoniano.
  - **Teorema de Ore:** Se para todo par de vértices não adjacentes u e v, temos $\deg(u) + \deg(v) \geq n$, então o grafo possui caminho Hamiltoniano.
- **Algoritmo de Borůvka:** Enquanto o grafo não estiver conexo, para cada componente conexa escolha a aresta de menor custo que sai dela. Essa técnica constrói a árvore geradora mínima (MST).
- **Árvores:**
  - Existem C_n árvores binárias com n vértices (C_n é o n-ésimo número de Catalan).
  - Existem C_{n-1} árvores enraizadas com n vértices.
  - **Fórmula de Cayley:** Existem n^{n-2} árvores com vértices rotulados de 1 a n.
  - **Código de Prüfer:** Remova iterativamente a folha com menor rótulo e adicione o rótulo do vizinho ao código até restarem dois vértices.
- **Fluxo em Redes:**
  - **Corte Mínimo:** Após execução do algoritmo de fluxo máximo, um vértice u está do lado da fonte se level$[u] \neq -1$.
  - **Máximo de Caminhos Disjuntos:**
    * **Arestas disjuntas:** Use fluxo máximo com capacidades iguais a 1 em todas as arestas.
    * **Vértices disjuntos:** Divida cada vértice v em v_{in} e v_{out}, conectados por aresta de capacidade 1. As arestas que entram vão para v_{in} e as que saem saem de v_{out}.
  - **Teorema de König:** Em um grafo bipartido:

$$\text{Cobertura mínima de vértices} = \text{Matching máximo}$$

O complemento da cobertura mínima de vértices é o conjunto independente máximo.
  - **Coberturas:**
    * **Vertex Cover mínimo:** Os vértices da partição X que **não** estão do lado da fonte no corte mínimo, e os vértices da partição Y que **estão** do lado da fonte.
    * **Independent Set máximo:** Complementar da cobertura mínima de vértices.
    * **Edge Cover mínimo:** É $N -$matching, pegando as arestas do matching e mais quaisquer arestas restantes para cobrir os vértices descobertos.
  - **Path Cover:**
    * **Node-disjoint path cover mínimo:** Duplicar vértices em tipo A e tipo B e criar grafo bipartido com arestas de $A \to B$. O path cover é $N -$ matching.
    * **General path cover mínimo:** Criar arestas de $A \to B$ sempre que houver caminho de A para B no grafo. O resultado também é $N -$ matching.
  - **Teorema de Dilworth:** O path cover mínimo em um grafo dirigido acíclico é igual à **antichain máxima** (conjunto de vértices sem caminhos entre eles).
  - **Teorema do Casamento de Hall:** Um grafo bipartido possui um matching completo do lado X se:

$$\forall W \subseteq X, \quad |W| \leq |\text{vizinhos}(W)|$$

  - **Fluxo Viável com Capacidades Inferiores e Superiores:** Para rede sem fonte e sumidouro:
    * Substituir a capacidade de cada aresta por $c_{\text{upper}} - c_{\text{lower}}$
    * Criar nova fonte S e sumidouro T
    * Para cada vértice v, compute:

$$M[v] = \sum_{\text{arestas entrando}} c_{\text{lower}} - \sum_{\text{arestas saindo}} c_{\text{lower}}$$

    * Se $M[v] > 0$, adicione aresta (S, v) com capacidade $M[v]$; se $M[v] < 0$, adicione (v, T) com capacidade $-M[v]$.
    * Se todas as arestas de S estão saturadas no fluxo máximo, então um fluxo viável existe. O fluxo viável final é o fluxo computado mais os valores de c_{lower}.

8.4 DP

- **Divide and Conquer Optimization:** Utilizada em problemas do tipo:

$$dp[i][j] = \min_{k<j}\{dp[i-1][k] + C[k][j]\}$$

  onde o objetivo é dividir o subsegmento até j em i segmentos com algum custo. A otimização é válida se:

$$A[i][j] \leq A[i][j+1]$$

  onde $A[i][j]$ é o valor de k que minimiza a transição.

- **Knuth Optimization:** Aplicável quando:

$$dp[i][j] = \min_{i<k<j}\{dp[i][k] + dp[k][j]\} + C[i][j]$$

  e a condição de monotonicidade é satisfeita:

$$A[i][j-1] \leq A[i][j] \leq A[i+1][j]$$

  com $A[i][j]$ sendo o índice k que minimiza a transição.

- **Slope Trick:** Técnica usada para lidar com funções lineares por partes e convexas. A função é representada por pontos onde a derivada muda, que podem ser manipulados com `multiset` ou `heap`. Útil para manter o mínimo de funções acumuladas em forma de envelopes convexos.

- **Outras Técnicas e Truques Importantes:**
  - **FFT (Fast Fourier Transform):** Convolução eficiente de vetores.
  - **CHT (Convex Hull Trick):** Otimização para DP com funções lineares e monotonicidade.
  - **Aliens Trick:** Técnica para binarizar o custo em problemas de otimização paramétrica (geralmente em problemas com limite no número de grupos/segmentos).
  - **Bitset:** Utilizado para otimizações de espaço e tempo em DP de subconjuntos ou somas parciais, especialmente em problemas de mochila.

9 Extra

9.1 Stress Test

```
P=code   #mude pro filename do codigo
Q=brute  #mude pro filename do brute [correto]
g++ ${P}.cpp -o sol -O2 || exit 1
g++ ${Q}.cpp -o brt -O2 || exit 1
g++ gen.cpp  -o gen -O2 || exit 1
for ((i = 1; ; i++)) do
  echo $i
  ./gen $i > in
  ./sol < in > out
  ./brt < in > out2
  if (! cmp -s out out2) then
    echo "--> entrada:"
    cat in
    echo "--> saida code:"
    cat out
    echo "--> saida brute:"
    cat out2
    break;
  fi
done
```

9.2 Hash Function

```
Call
g++ hash.cpp -o hash
./hash < code.cpp
to get the hash of the code.

The hash ignores comments and whitespaces.
The hash of a line whith } is the hash of all the code since the { that opens it. (is
    the hash of that context)

(Optional) To make letters upperCase: for(auto&c:s)if('a'<=c) c^=32;
```

```
DE3 string getHash(string s){
909   ofstream ip("temp.cpp"); ip << s; ip.close();
EE9   system("g++ -E -P -dD -fpreprocessed ./temp.cpp | tr -d '[:space:]' | md5sum > hsh.
    temp");
CEF   ifstream fo("hsh.temp"); fo >> s; fo.close();
A15   return s.substr(0, 3);
17A }

E8D int main(){
973   string l, t;
3DA   vector<string> st(10);
C61   while(getline(cin, l)){
54F     t = l;
242     for(auto c : l)
F11       if(c == '{') st.push_back(""); else
2F0       if(c == '}') t = st.back() + l, st.pop_back();
C33     cout << getHash(t) + " " + l + "\n";
1ED     st.back() += t + "\n";
D1B   }
B65 }
```