

AI5030-Probability Assignment 1

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Question 10.13.3.23

Two dice are numbered 1, 2, 3, 4, 5, 6 and 1, 1, 2, 2, 3, 3, respectively. They are thrown and the sum of the numbers on them is noted. Find the probability of getting each sum from 2 to 9 separately

Solution: Solving using Z-Transform

Let X be the discrete random variable corresponding to dice 1:

$$X \in \{1, 2, 3, 4, 5, 6\}$$

The Z-transform of X is denoted as

$$X(z) = E[z^{-X}] = \sum_{x=-\infty}^{\infty} p_X(x)z^{-x}$$

Here,

$$X[z] = \frac{z^{-1}}{6} + \frac{z^{-2}}{6} + \frac{z^{-3}}{6} + \frac{z^{-4}}{6} + \frac{z^{-5}}{6} + \frac{z^{-6}}{6}$$

$$X[z] = \frac{z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5} + z^{-6}}{6} \quad (1)$$

Let Y be the discrete random variable corresponding to dice 2:

$$Y \in \{1, 1, 2, 2, 3, 3\}$$

The Z-transform of Y is denoted as

$$Y(z) = E[z^{-Y}] = \sum_{y=-\infty}^{\infty} p_Y(y)z^{-y}$$

Here,

$$Y[z] = \frac{z^{-1}}{6} + \frac{z^{-1}}{6} + \frac{z^{-2}}{6} + \frac{z^{-2}}{6} + \frac{z^{-3}}{6} + \frac{z^{-3}}{6}$$

$$Y[z] = \frac{z^{-1} + z^{-2} + z^{-3}}{3} \quad (2)$$

Let Z be the random variable that denotes the sum of the numbers when the above two dice are thrown.

$$Z \in \{2, 3, 4, 5, 6, 7, 8, 9\}$$

Since X and Y are independent events, the probability corresponding to Z can be obtained by finding the Z-Transform as follows:

$$Z(z) = E[Z^{-(X+Y)}] = E[Z^{-X}]E[Z^{-Y}]$$

$$Z[z] = X[z]Y[z] \quad (3)$$

Substituting (1) and (2) in (3) we get,

$$Z[z] = \left(\frac{z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5} + z^{-6}}{6} \right) \left(\frac{z^{-1} + z^{-2} + z^{-3}}{3} \right)$$

On solving we obtain,

$$Z[z] = \left(\frac{z^{-2}}{18} + \frac{z^{-3}}{9} + \frac{z^{-4} + z^{-5} + z^{-6} + z^{-7}}{6} + \frac{z^{-8}}{9} + \frac{z^{-9}}{18} \right)$$

Hence from the above expression we get the required probabilities as,

$$Pr[Z = 2] = \frac{1}{18}$$

$$Pr[Z = 3] = \frac{1}{9}$$

$$Pr[Z = 4] = \frac{1}{6}$$

$$Pr[Z = 5] = \frac{1}{6}$$

$$Pr[Z = 6] = \frac{1}{6}$$

$$Pr[Z = 7] = \frac{1}{6}$$

$$Pr[Z = 8] = \frac{1}{9}$$

$$Pr[Z = 9] = \frac{1}{18}$$