AI5030-Probability Assignment 2

Samuktha V. (AI23MTECH02004)

Question 10.13.3.23
Two dice are numbered 1, 2, 3, 4, 5, 6 and 1, 1, 2, 2, 3, 3, respectively. They are thrown and the sum of the numbers on them is noted. Find the probability of getting each sum from 2 to 9 separately

Solution: **Solving using Z-Transform**

Let X be the discrete random variable corresponding to dice 1:

$$X \in \{1, 2, 3, 4, 5, 6\}$$

The Z-transform of X is denoted as

$$X(z) = E[z^{-X}] = \sum_{x=1}^{6} p_X(x) z^{-x}$$

Here,

$$X[z] = \frac{z^{-1}}{6} + \frac{z^{-2}}{6} + \frac{z^{-3}}{6} + \frac{z^{-4}}{6} + \frac{z^{-5}}{6} + \frac{z^{-6}}{6}$$

$$\boldsymbol{X[z] = \left(\frac{z^{-1} - z^{-7}}{1 - z^{-1}}\right)\frac{1}{6}} \tag{1}$$

Let Y be the discrete random variable corresponding to dice 2:

$$Y \in \{1, 1, 2, 2, 3, 3\}$$

The Z-transform of Y is denoted as

$$Y(z) = E[z^{-Y}] = \sum_{y=1}^{3} p_Y(y) z^{-y}$$

Here,

$$Y[z] = \frac{z^{-1}}{6} + \frac{z^{-1}}{6} + \frac{z^{-2}}{6} + \frac{z^{-2}}{6} + \frac{z^{-3}}{6} + \frac{z^{-3}}{6}$$

$$\boldsymbol{Y[z] = \left(\frac{z^{-1} - z^{-4}}{1 - z^{-1}}\right)\frac{1}{3}} \tag{2}$$

*The author is with the Department of Dept. of AI, Indian Institute of Technology, Hyderabad 502285 India e-mail: ai23mtech02004@iith.ac.in.

Let Z be the random variable that denotes the sum of the numbers when the above two dice are thrown.

$$Z \in \{2, 3, 4, 5, 6, 7, 8, 9\}$$

Since X and Y are independent events, the probability corresponding to Z can be obtained by fining the Z-Transform as follows:

$$Z(z) = E[Z^{-(X+Y)}] = E[Z^{-X}]E[Z^{-Y}]$$

$$\boldsymbol{Z[z] = X[z]Y[z]} \tag{3}$$

Substituting (1) and (2) in (3) we get,

$$Z[z] = \left(\frac{z^{-1} - z^{-7}}{1 - z^{-1}}\right)\frac{1}{6}\left(\frac{z^{-1} - z^{-4}}{1 - z^{-1}}\right)\frac{1}{3}$$

On solving we obtain,

$$\boldsymbol{Z[z] = \left(\frac{z^{-11} - z^{-8} - z^{-5} + z^{-2}}{z^{-2} - 2z^{-1} + 1}\right)\frac{1}{18}}$$

On dividing the numerator and denominator we obtain Z[z] as,

$$\boldsymbol{\left(\frac{z^{-9} + 2z^{-8} + 3z^{-7} + 3z^{-6} + 3z^{-5} + 3z^{-4} + 2z^{-3} + z^{-2}}{18}\right)}$$

Hence from the above expression we get the required probabilities as,

$$\boldsymbol{Pr[Z = 2] = \frac{1}{18}}$$

$$\boldsymbol{Pr[Z = 3] = \frac{1}{9}}$$

$$\boldsymbol{Pr[Z = 4] = \frac{1}{6}}$$

$$\boldsymbol{Pr[Z = 5] = \frac{1}{6}}$$

$$\boldsymbol{Pr[Z = 6] = \frac{1}{6}}$$

$$\boldsymbol{Pr[Z = 7] = \frac{1}{6}}$$

$$\boldsymbol{Pr[Z = 8] = \frac{1}{9}}$$

$$\boldsymbol{Pr[Z = 9] = \frac{1}{18}}$$