AI5030-Probability Assignment 3

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Question 10.13.3.23
Two dice are numbered 1, 2, 3, 4, 5, 6 and 1, 1, 2, 2, 3, 3, respectively. They are thrown and the sum of the number yuils on them is noted. Find the probability of getting each sum from 2 to 9 separately

Solution: **Solving using Convolution**

Let X be the discrete random variable corresponding to dice 1:

$$X \in \{1, 2, 3, 4, 5, 6\}$$

Let Y be the discrete random variable corresponding to dice 2:

$$Y \in \{1, 1, 2, 2, 3, 3\}$$

Let Z be the random variable that denotes the sum of the numbers when the above two dice are thrown.

$$Z \in \{2, 3, 4, 5, 6, 7, 8, 9\}$$

We need the sum z, hence we take $x \in X$ and sum over all possibilities of y where $y = z - x$ so that sum is retained as z. The theoretical Probability Mass Function(PMF) of Z can be generated using convolution operation as follows:

$$P_Z(z) = P(Z = z) = \sum_{x=1}^{6} P(X = x, Y = z - x) \tag{1}$$

Since X and Y are independent, equation 1 can be written as :

$$P(Z = z) = \sum_{x=1}^{6} P(X = x)P(Y = z - x)$$

Hence,

$$P_Z(z) = \sum_{x=1}^{6} P_X(x)P_Y(z - x) \tag{2}$$

The PMF of X and Y are plotted as shown in Figure 1:

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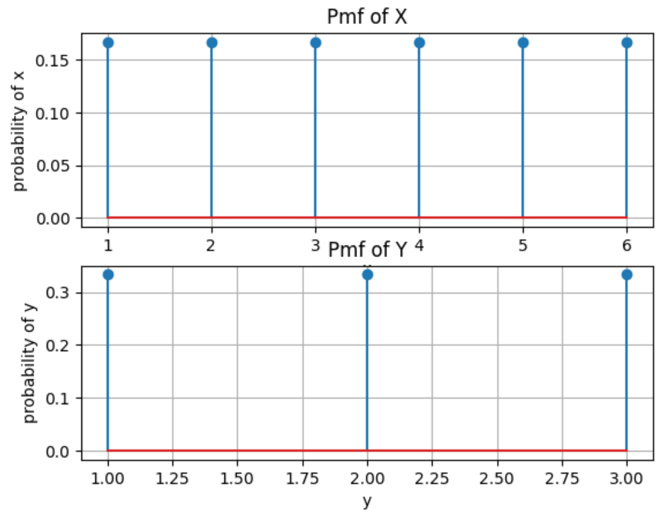

Fig. 1. PMFs of X and Y

When **sum is 2**, we have

$$P_Z(2) = \sum_{x=1}^{6} P(X = x)P(Y = 2 - x) = \frac{1}{18}$$

When **sum is 3**, we have

$$P_Z(3) = \sum_{x=1}^{6} P(X = x)P(Y = 3 - x) = \frac{1}{9}$$

When **sum is 4**, we have

$$P_Z(4) = \sum_{x=1}^{6} P(X = x)P(Y = 4 - x) = \frac{1}{6}$$

When **sum is 5**, we have

$$P_Z(5) = \sum_{x=1}^{6} P(X = x)P(Y = 5 - x) = \frac{1}{6}$$

When **sum is 6**, we have

$$P_Z(6) = \sum_{x=1}^{6} P(X = x)P(Y = 6 - x) = \frac{1}{6}$$

When **sum is 7**, we have

$$P_Z(7) = \sum_{x=1}^{6} P(X = x)P(Y = 7 - x) = \frac{1}{6}$$

When **sum is 8**, we have

$$P_Z(8) = \sum_{x=1}^{6} P(X = x)P(Y = 8 - x) = \frac{1}{9}$$

When **sum is 9**, we have

$$P_Z(9) = \sum_{x=1}^{6} P(X = x)P(Y = 9 - x) = \frac{1}{18}$$

The PMF of X,Y and Z are shown in equation 3,4 and 5 respectively,

$$p_X(x) = \frac{1}{6} \qquad for \quad 1 \le x \le 6 \tag{3}$$

$$p_Y(y) = \frac{1}{3} \qquad for \quad 1 \le< x \le 3 \tag{4}$$

$$p_Z(z) = \begin{cases} \frac{1}{18} & for \quad z = 2, 9 \\ \frac{1}{9} & for \quad z = 3, 8 \\ \frac{1}{6} & for \quad 4 \le z \le 7 \end{cases} \tag{5}$$

The pmf of Z is symmetrical and the distribution is plotted as shown in figure 2

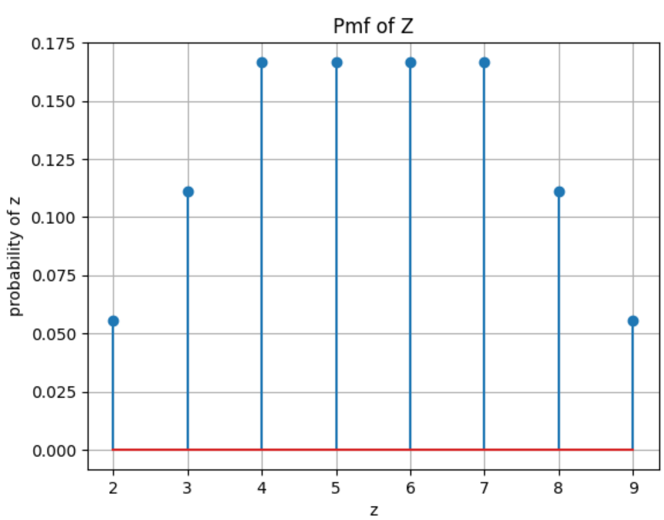

Fig. 2. PMF of Z