University of Stuttgart | ANALYTIC COMPUTING

Foundations of Machine Learning - Exercise (SS 25)

Assignment 6: Logistic Regression and Bayes Error

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Submit your theoretical solution in ILIAS as a single PDF file.[1] Make sure to list the full names of all participants, matriculation number, study program, and B.Sc. or M.Sc on the first page. Optionally, you can *additionally* upload source files (e.g., PPTX files). Submit your programming task in ILIAS as a single Jupyter notebook. If you have any questions, feel free to ask them in the exercise forum in ILIAS.

Submission is open until Monday, 2nd of June, 12:00 noon.

[1] Your drawing software probably allows exporting as PDF. An alternative option is to use a PDF printer. If you create multiple PDF files, use a merging tool (like pdfarranger) to combine the PDFs into a single file.

Task 1: Optimal Bayes Classifier & Bayes Error in 2D

Two class-conditional Gaussian densities are given

$$\text{Class A: } X \sim \mathcal{N}(\mu_A, \Sigma_A), \quad \mu_A = [-1, -1]^\top, \ \Sigma_A = I,$$
$$\text{Class B: } X \sim \mathcal{N}(\mu_B, \Sigma_B), \quad \mu_B = [\ 1, \ \ 1]^\top, \ \Sigma_B = I.$$

1. **Task** Derive the Bayes decision rule and its decision boundary.
2. **Task** Compute the Bayes classification error (you may use Python).
3. **Task** Plot of the two densities, the decision line, and shade the Bayes-error region (you may use Python).
4. **Task** If we keep $\Sigma_A = I$ but set $\Sigma_B = 2I$. Derive the Bayes decision rule and the decision boundary in this case.
5. **Task** Compute the Bayes classification error for this case (you may use Python).
6. **Task** Plot of the two densities, the decision line, and shade the Bayes-error region (you may use Python).

Task 2: Bayes optimal classifier for Gaussian Mixture Models

The plot in Figure 1 shows the class-conditional densities of two One-dimensional classes. Class A (red) is a *mixture* of two Gaussians, while Class B (blue) is a single Gaussian.

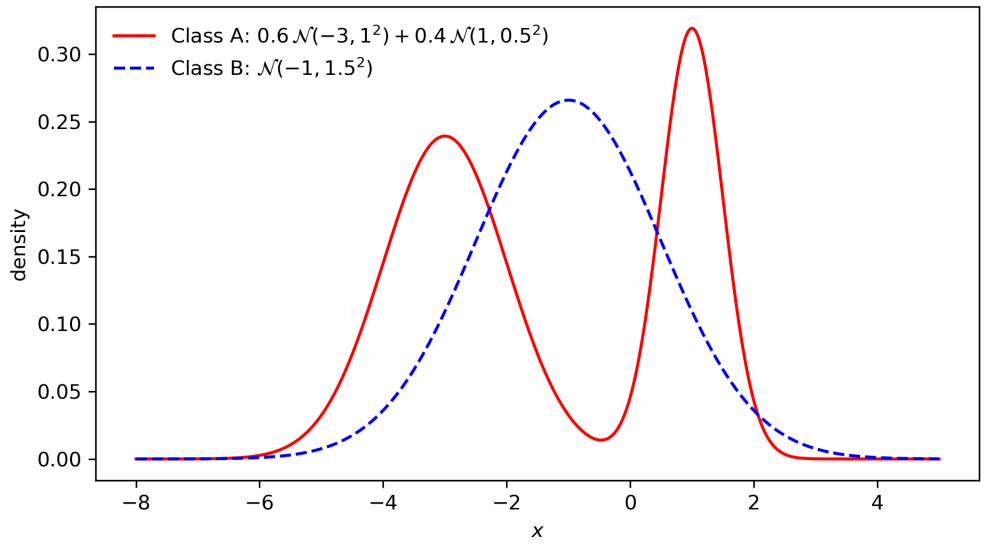

Figure 1 Multi-modal vs uni-modal class densities.

1. **Task** In the figure 1 above (assume the two classes are equally likely), mark on the horizontal axis where you expect the Bayes decision point(s) to lie and give a short justification.
2. **Task** Given Figure 1 assume the prior of Class B is increased to $P(B) = \frac{2}{3}$ (so $P(A) = \frac{1}{3}$). Describe qualitatively how each Bayes boundary will shift and how the total Bayes error will change.

Task 3: Logisitc Regression

Follow the instructions in the jupyter notebook.

Task 4: Linear Regression for Classification

Follow the instructions in the jupyter notebook.