Universität Stuttgart
KI – Institute for Artificial Intelligence

Analytic Computing

Machine Learning
10 Support Vector Machines

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https://www.ki.uni-stuttgart.de/

- based on slides by
  - Thomas Gottron, U. Koblenz-Landau, https://west.uni-koblenz.de/de/studying/courses/ws1718/machine-learning-and-data-mining-1
  - Andrew Zisserman, http://www.robots.ox.ac.uk/~az/lectures/ml/lect2.pdf

1 Perceptron Algorithm

Binary classification

- Given training data $\{(x_i, y_i)\}_{i=1}^{N}$ with $x_i \in \mathbb{R}^d$ and $y_i \in \{-1,1\}$,

- Learn a classifier

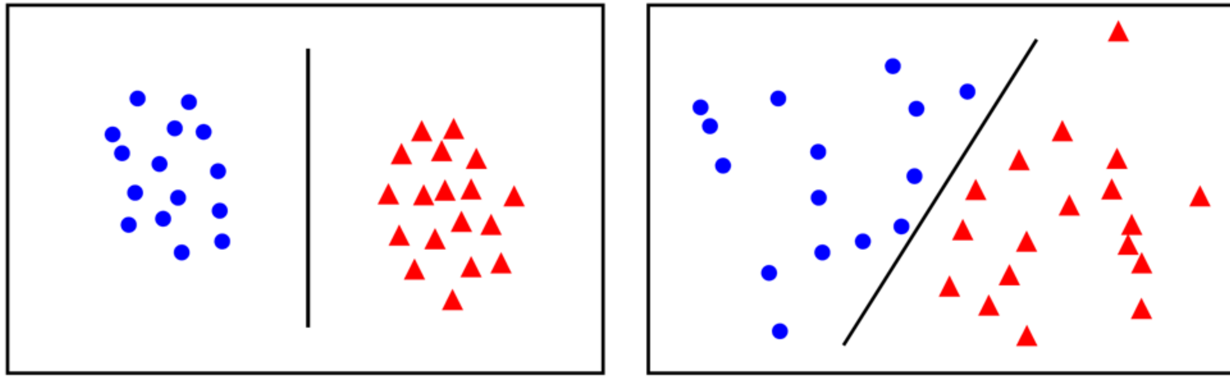
$$\hat{f}(x_i) = \begin{cases} > 0, \text{if } y_i = +1 \\ < 0, \text{if } y_i = -1 \end{cases}$$

- Correct classification:

$$\hat{f}(x_i) y_i > 0$$

Linear separability

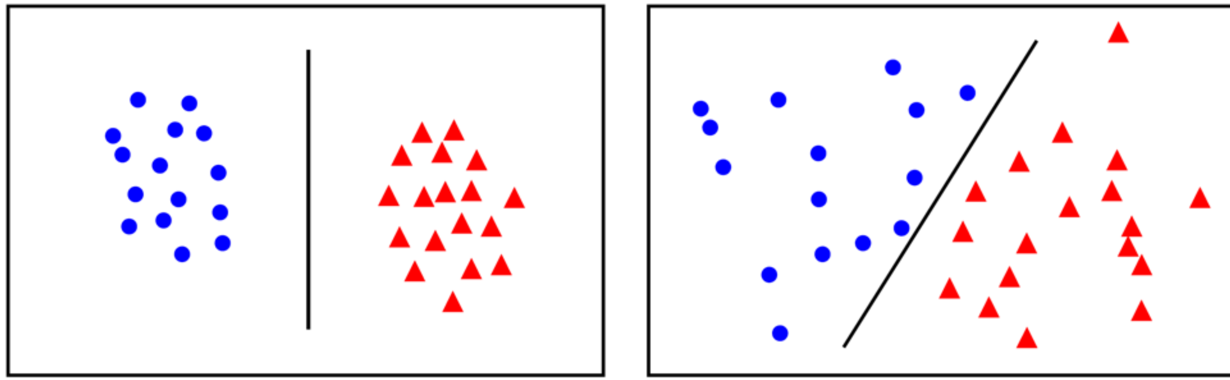
linearly separable



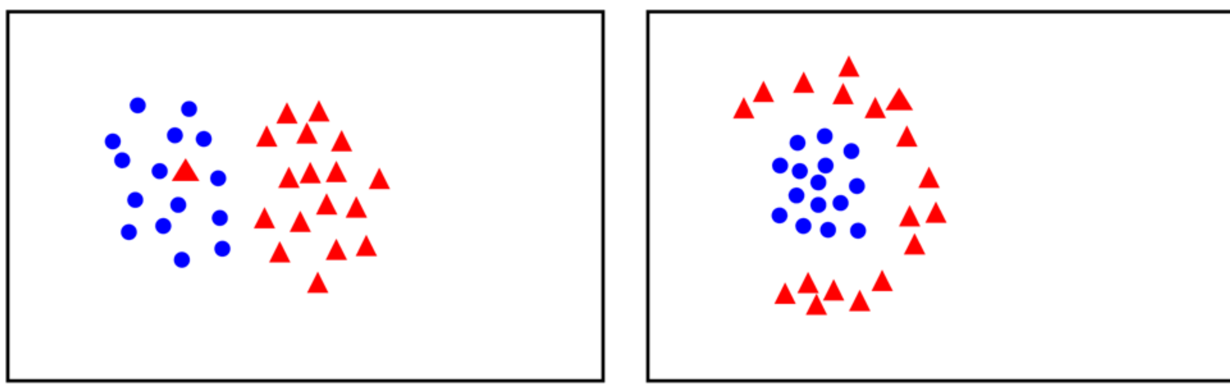
(Zisserman 2015)

Linear separability

linearly
separable



not
linearly
separable



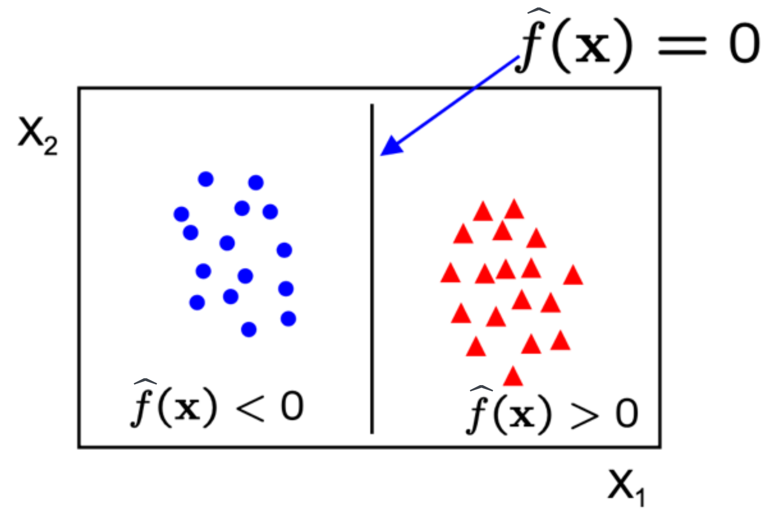
(Zisserman 2015)

Linear classifiers

- A linear classifier has the form

$$\hat{f}(x) = w^T x + b$$

- In 2D the discriminant is a line
- w is the normal to the line, and b the bias
- w is known as the weight vector

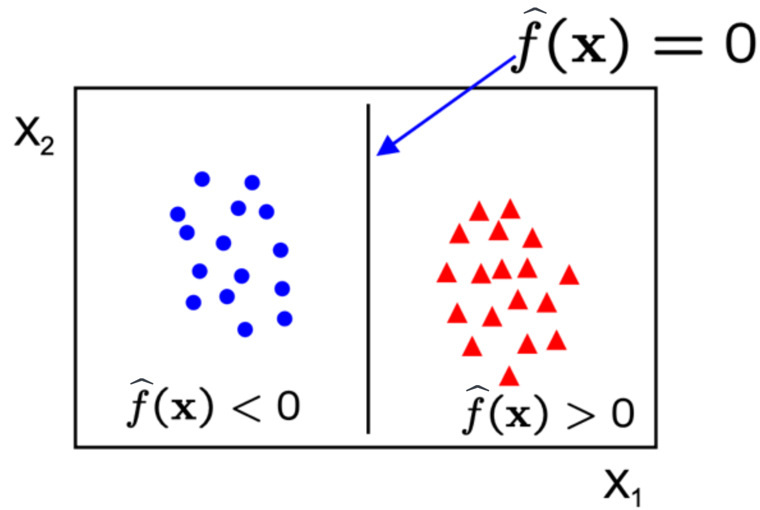


(Zisserman 2015)

Linear classifiers

- Let‘s assume $\boldsymbol{x_i = (1, x_{i,1}, \dots, x_{i,d})}$
  (as in linear regression)
- The we write

$$\hat{f}(x) = w^T x$$



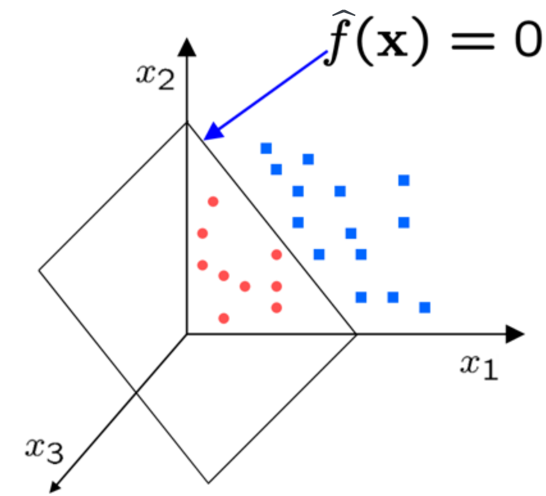
(Zisserman 2015) $x_0=1$

Linear classifiers

- A linear classifier has the form

$$\hat{f}(x) = w^T x$$

- In 3D the discriminant is a plane
- In n-dim the discriminant is a hyperplane
- Only w (including b) are needed to classify new data



(Zisserman 2015)

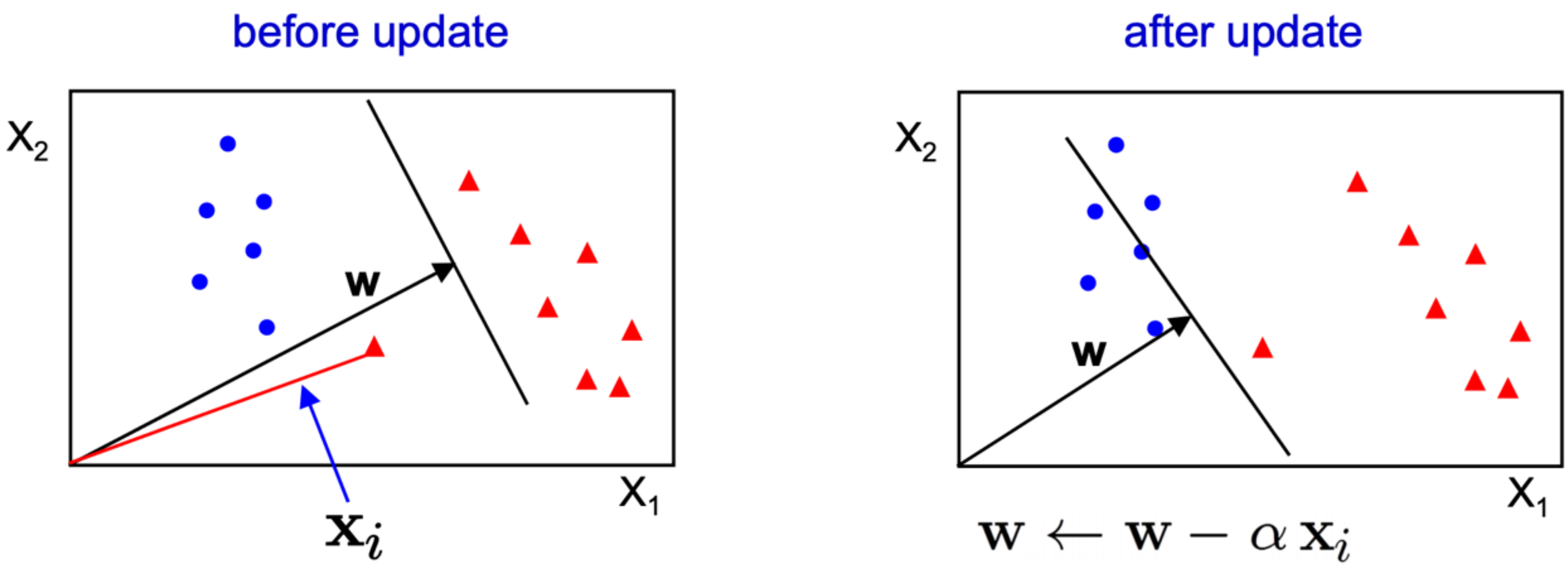
The perceptron classifier

- Given training data $\{(x_i, y_i)\}_{i=1}^{N}$ with $x_i \in \mathbb{R}^d$ and $y_i \in \{-1,1\}$,
- how to find a weight vector w, the *separating hyperplane*, such that the two categories are separated for the dataset?

- Perceptron algorithm
  1. Initialize $w = \bar{0}$
  2. While there is i such that $\hat{f}(x_i)y_i < 0$ do
     - $w \coloneqq w - \alpha x_i \operatorname{sign}\left(\hat{f}(x_i)\right) = w + \alpha x_i y_i$ (not for sign(...)=0)

Example for perceptron algorithm in 2D

1. Initialize $w = \bar{0}$
2. While there is i such that $\hat{f}(x_i)y_i < 0$ do
   - $w := w + \alpha x_i \operatorname{sign}(\hat{f}(x_i))$



At convergence: $w = \sum_{i=1}^{N} \alpha_i \, x_i$

(Zisserman 2015)

Example

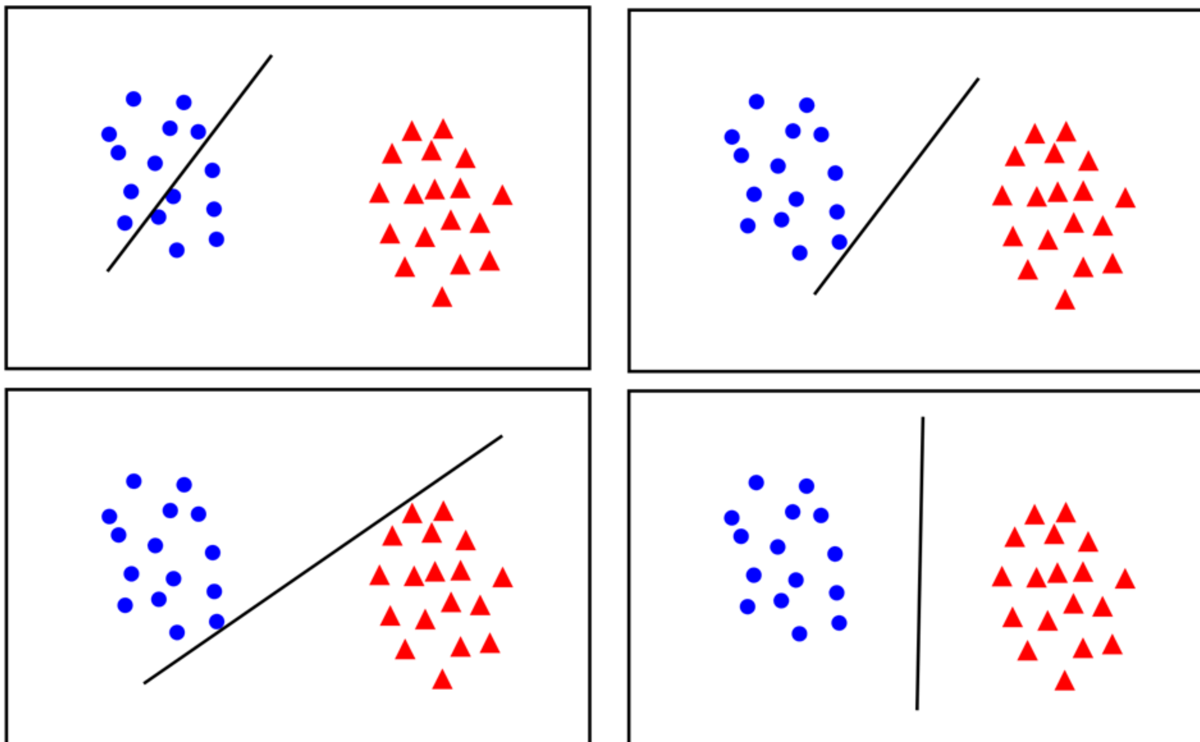
- if the data is linearly separable, then the algorithm will converge
- convergence can be slow …
- separating line close to training data

(Zisserman 2015)

2 Support Vectors

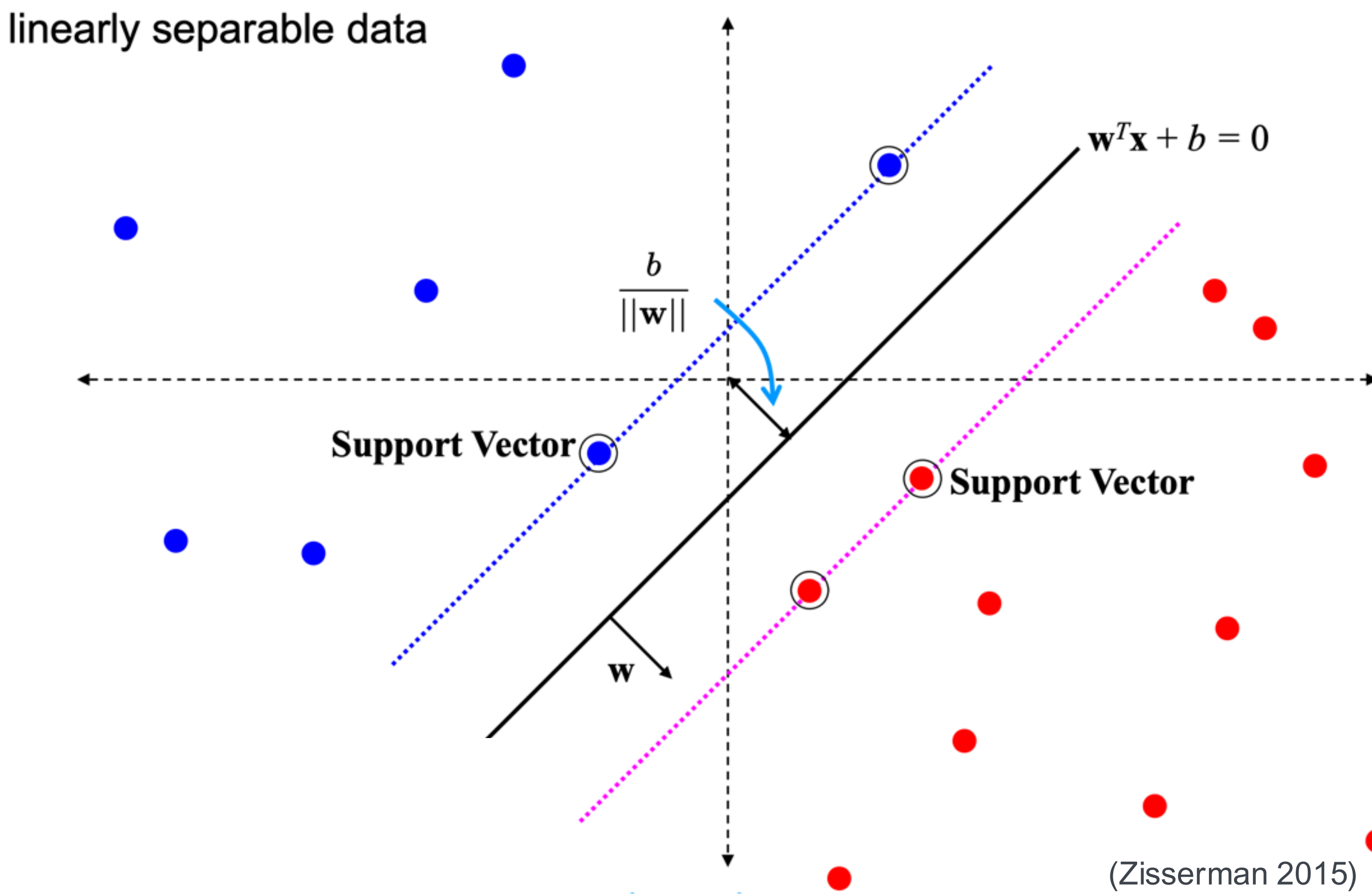
What is the best w?

- Idea: maximum margin solution is most stable under perturbations of the inputs



(Zisserman 2015)

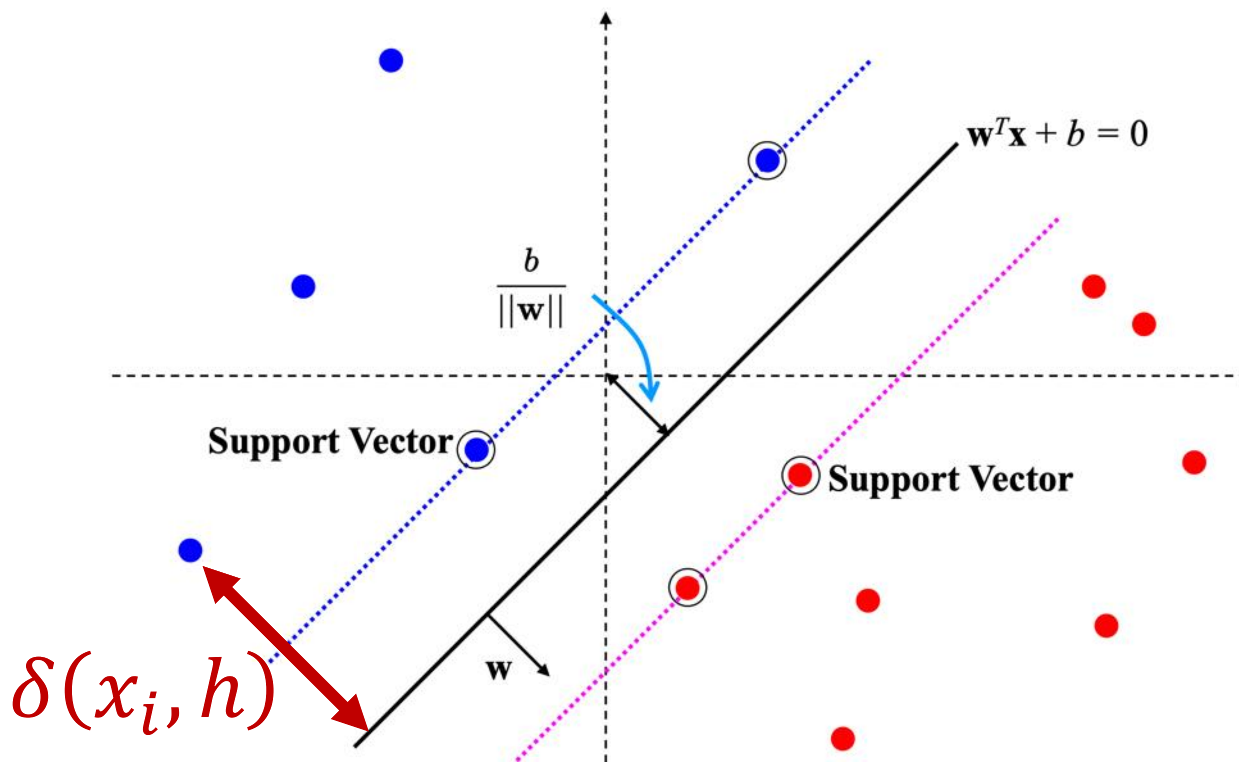
Support Vector Machine



SVM Optimization Problem (1)

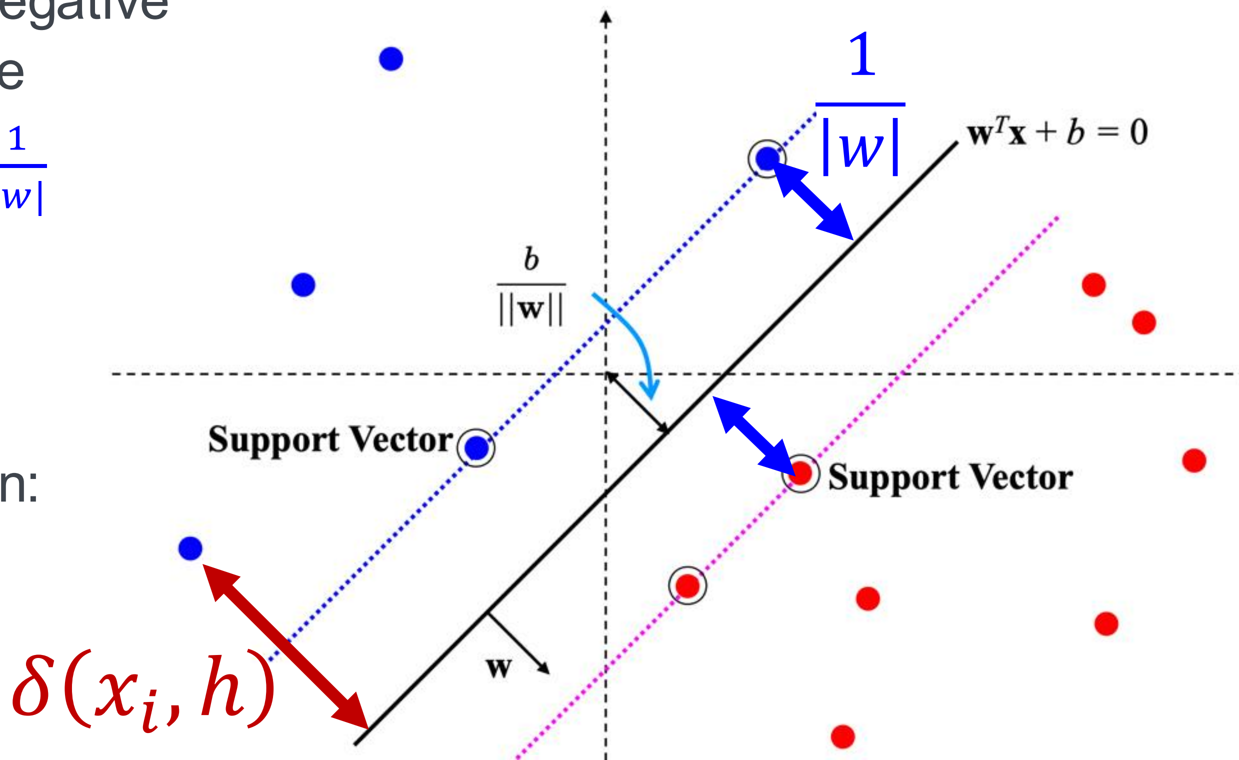
- Distance $\delta(x_i, h)$ of data point x_i from hyperplane h

$$\delta(x_i, h) = \frac{y_i \cdot (w^T x_i + b)}{|w|}$$

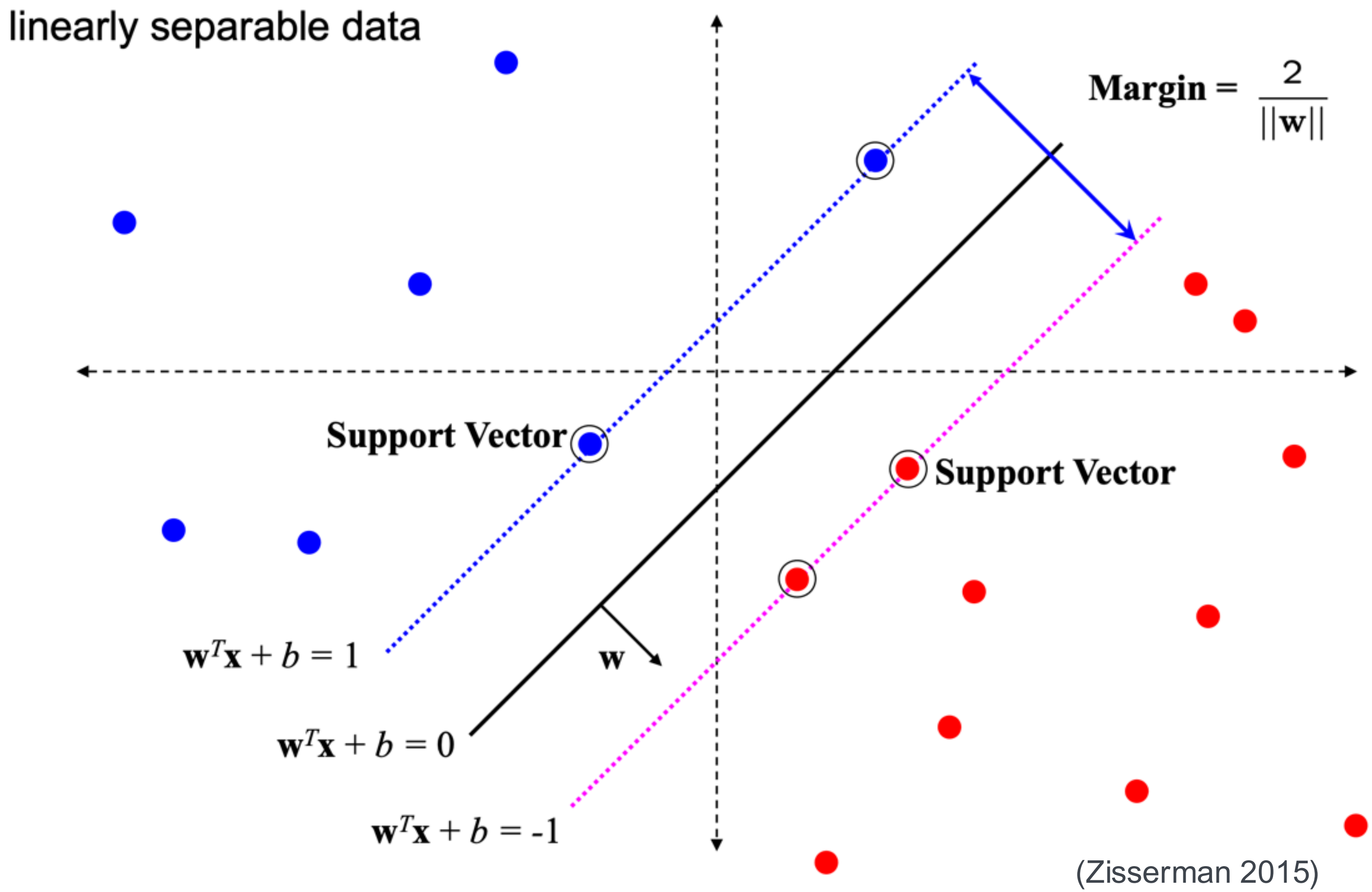


SVM Optimization Problem (1)

- In general, length of vector w does not matter
  - fix w such that support vectors x_j make $y_j \cdot (w^T x_j + b) = 1$
- Then positive and negative support vectors have distance $\delta(x_j, h) = \frac{1}{|w|}$ from hyperplane – which we want to maximize
- Standard formulation: Minimize $\frac{|w|}{2}$ (inverse margin)



Support Vector Machine



SVM Optimization Problem (2)

$$\max_{w} \frac{2}{||w||}$$

subject to

$$y_i\left(w^T x_i + b\right) \geq 1, \text{for } i = 1 \dots N$$

Or equivalently

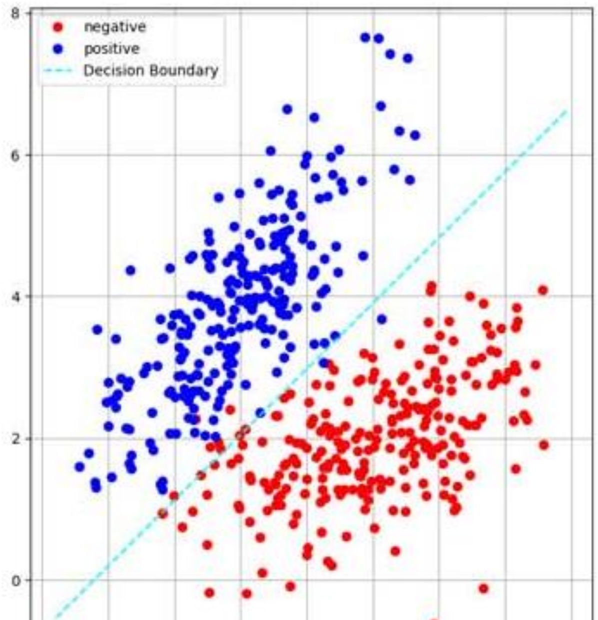
$$\min_{w} ||w||^2$$

subject to the same constraints

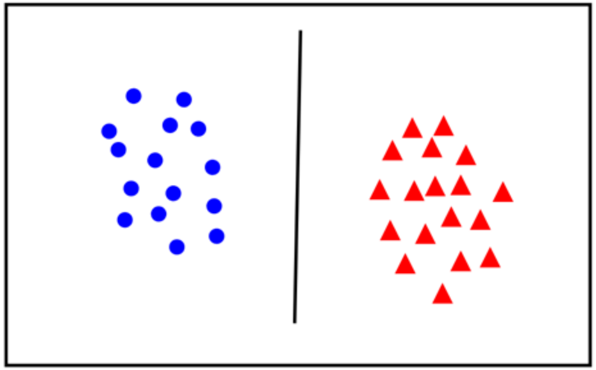
This is a quadratic optimization problem
subject to linear constraints and there is a unique minimum

Compare the two optimization criteria for classfication of linearly separable data by linear regression with classification by SVM

Linear classification using regression: decision line is average between regression lines; all data points are considered



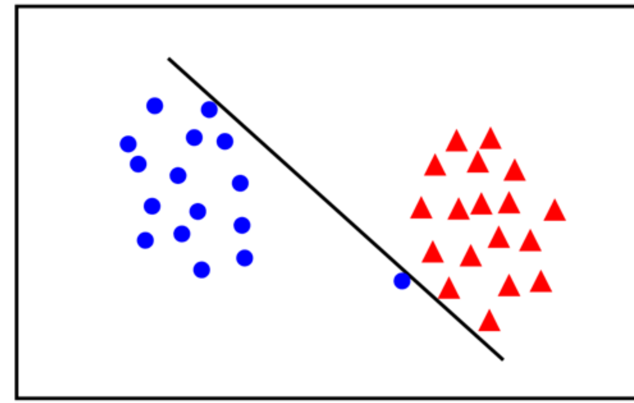
Linear classification using SVM: decision line maximizes margins between support vectors; far away data points are irrelevant



3 Soft Margin and Hinge Loss

Re-visiting linear separability

- Points can be linearly separated, but with very narrow margin



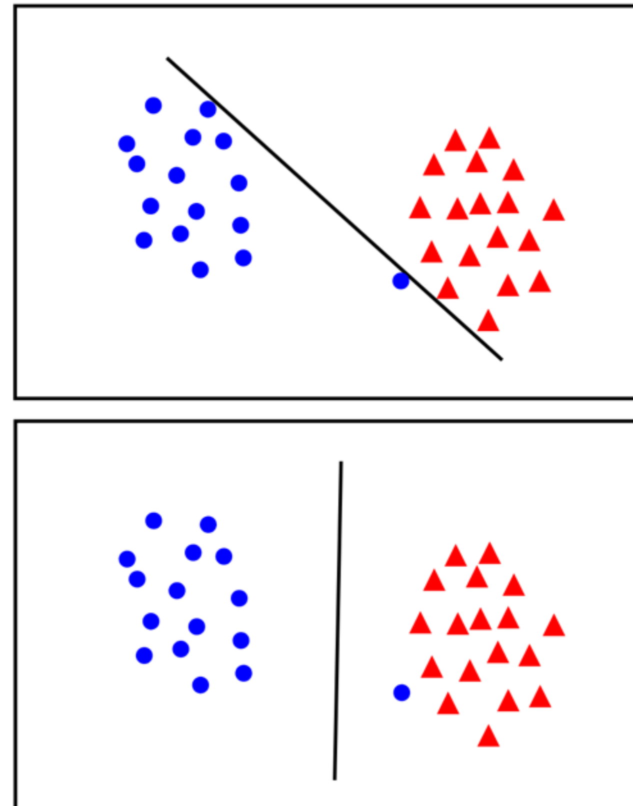
(Zisserman 2015)

Re-visiting linear separability

- Points can be linearly separated, but with very narrow margin

- Possibly the large margin solution is better, even though one constraint is violated

Trade-off between the margin and the number of mistakes on training data

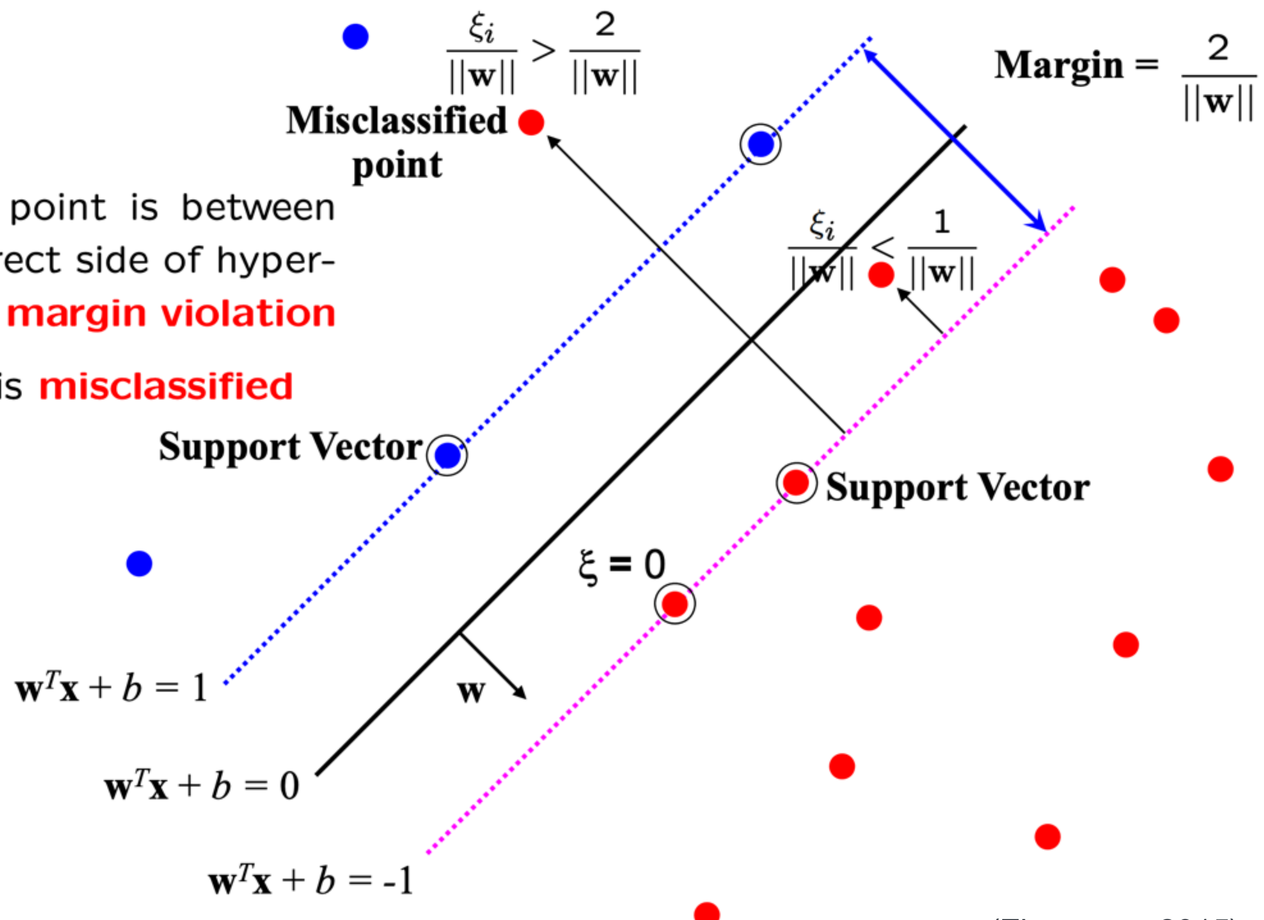


(Zisserman 2015)

Introduce „slack“ variables

$$\xi_i \geq 0$$

- for $0 < \xi \leq 1$ point is between margin and correct side of hyper-plane. This is a **margin violation**
- for $\xi > 1$ point is **misclassified**



(Zisserman 2015)

Soft margin solution

Revised optimization problem

$$\min_{w \in \mathbb{R}^d, \xi_i \in \mathbb{R}^+} ||w||^2 + C \sum_{i=1}^{N} \xi_i$$

subject to

$$y_i\left(w^T x_i + b\right) \geq 1 - \xi_i, \text{for } i = 1 \dots N$$

- Every constraint can be satisfied if ξ_i is sufficiently large
- C is a regularization parameter:
  - small C allows constraints to be easily ignored $\Longrightarrow$ large margin
  - large C makes constraints hard to ignore $\Longrightarrow$ narrow margin
  - $C = \infty$ enforces all constraints $\Longrightarrow$ hard margin
- Still a quadratic optimization problem with unique minimum
- One hyperparameter C

Loss function

- Given constraints:

$$y_i\left(w^T x_i + b\right) \geq 1 - \xi_i$$
$$\xi_i \geq 0$$

- We can rewrite ξ_i as:

$$\xi_i = \max(0,1 - y_i \hat{f}(x_i))$$

- Hence, we can optimize the unconstrained optimization problem over w:

$$\min_{w \in \mathbb{R}^d} \underbrace{\frac{1}{C}\|w\|^2}_{\text{regularization}} + \sum_{i=1}^{N} \underbrace{\max(0,1 - y_i \hat{f}(x_i))}_{\text{loss function}}$$

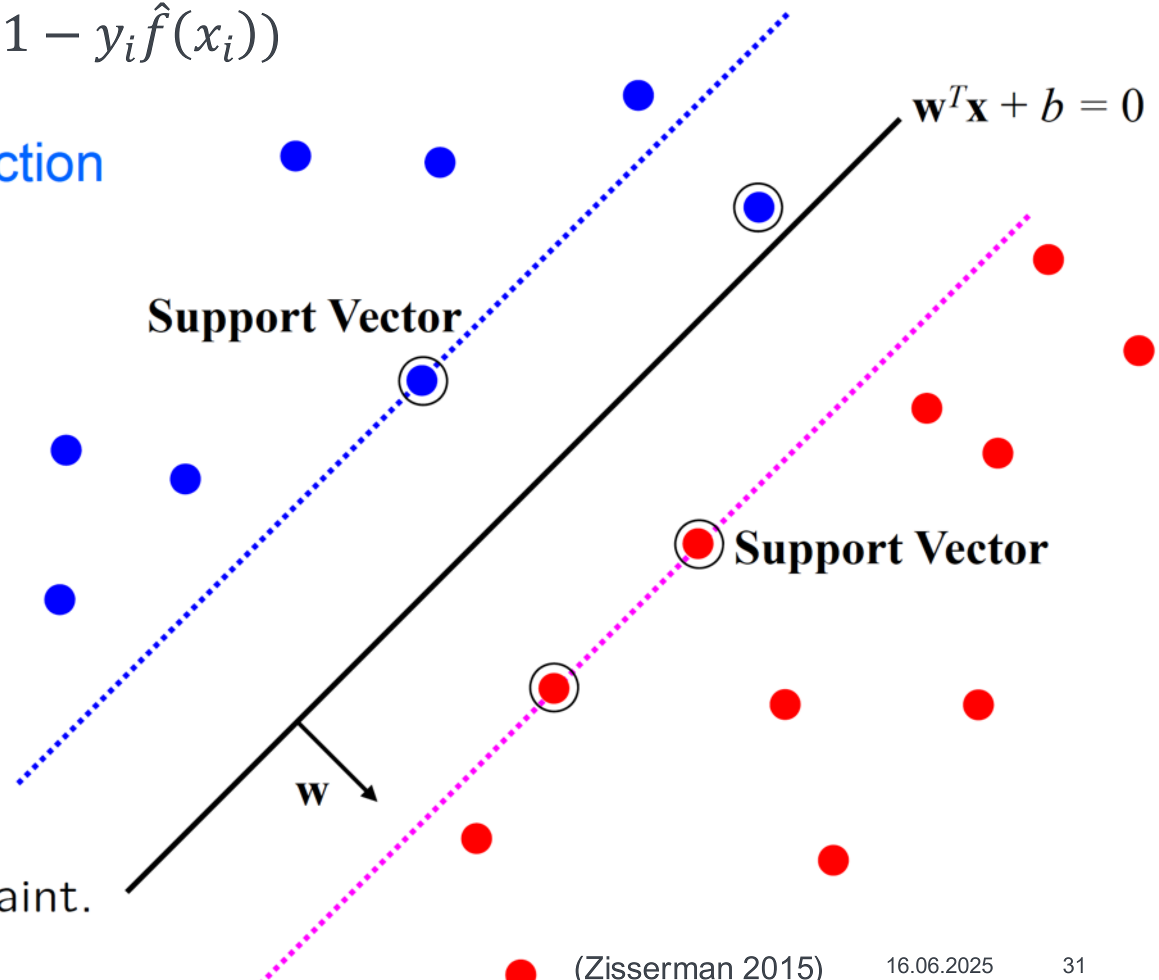
Loss function

$$\min_{w \in \mathbb{R}^d, \xi_i \in \mathbb{R}^+} \frac{1}{C} ||w||^2 + \sum_{i=1}^{N} \max(0, 1 - y_i \hat{f}(x_i))$$

loss function

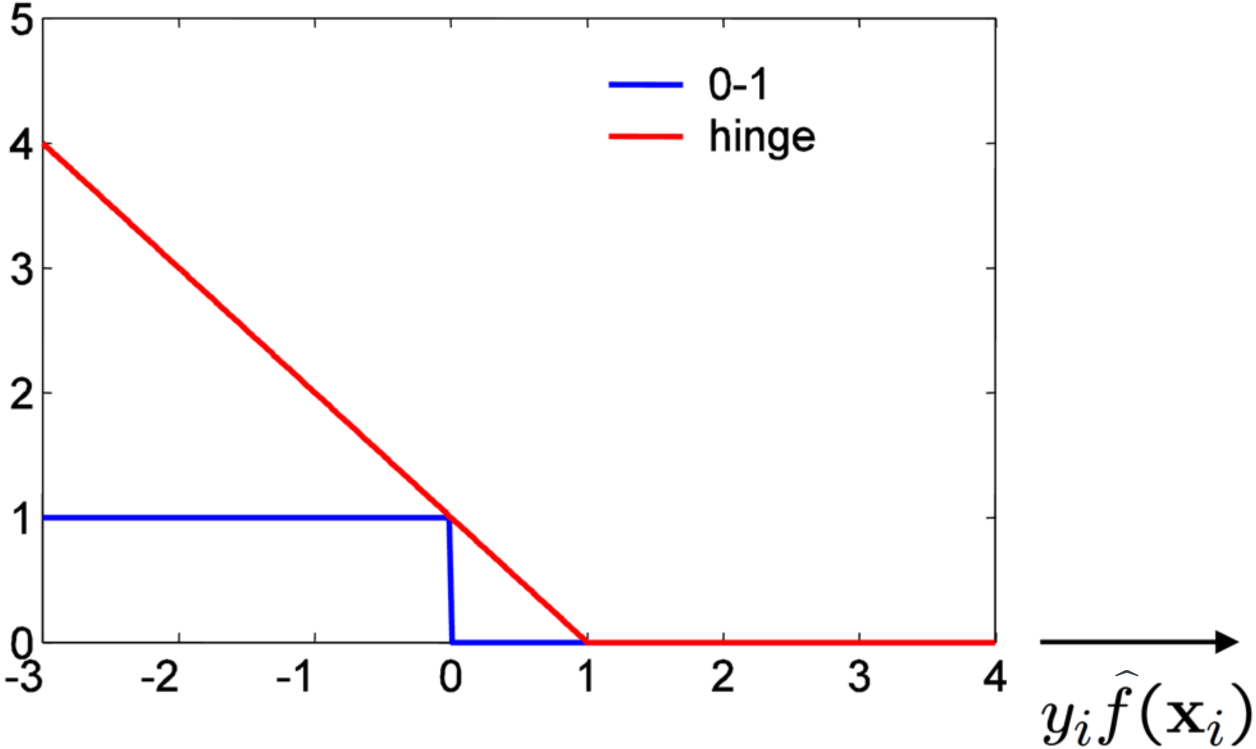
Points are in three categories:

1. $y_i f(x_i) > 1$
   Point is outside margin.
   No contribution to loss
2. $y_i f(x_i) = 1$
   Point is on margin.
   No contribution to loss.
   As in hard margin case.
3. $y_i f(x_i) < 1$
   Point violates margin constraint.
   Contributes to loss



(Zisserman 2015)

Hinge loss



- SVM uses "hinge" loss $\max\left(0, 1 - y_i \widehat{f}(\mathbf{x}_i)\right)$
- an approximation to the 0-1 loss

(Zisserman 2015)

4 Gradient descent over convex function

Gradient descent/ascent

Climb down a hill
Climb up a hill

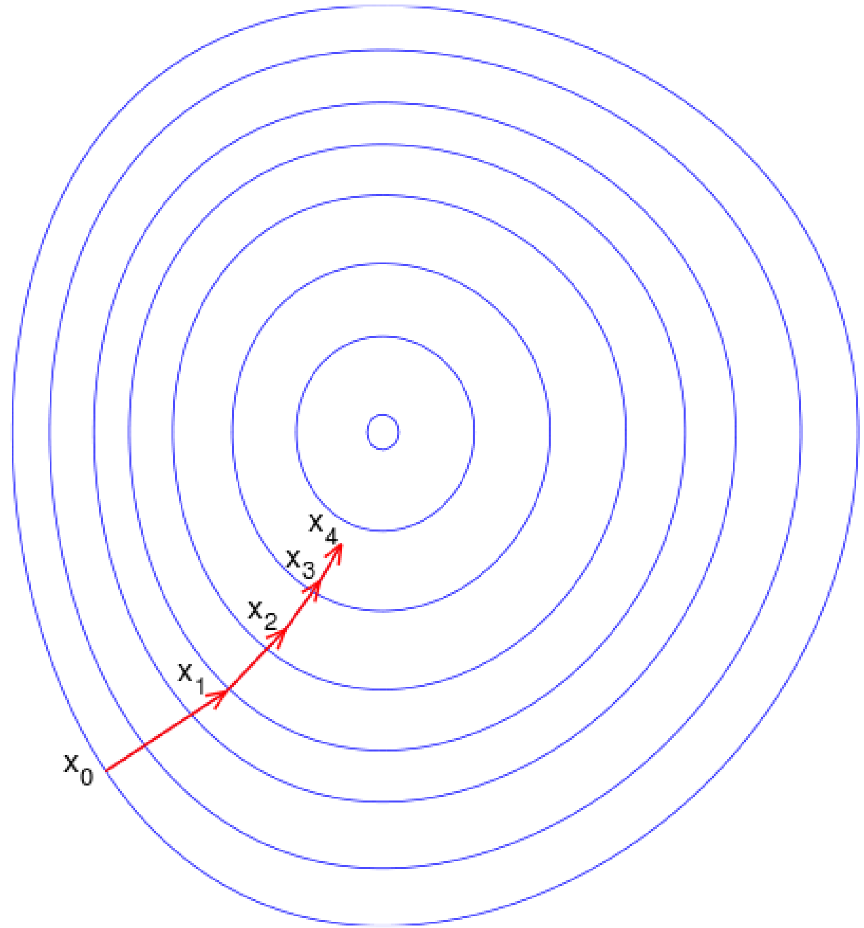
Given differentiable function describing height of hill at position $\boldsymbol{x} = (x_1, \dots, x_k)$ height of hill $f(\boldsymbol{x})$.

How to climb up/down fastest?

Go in direction where

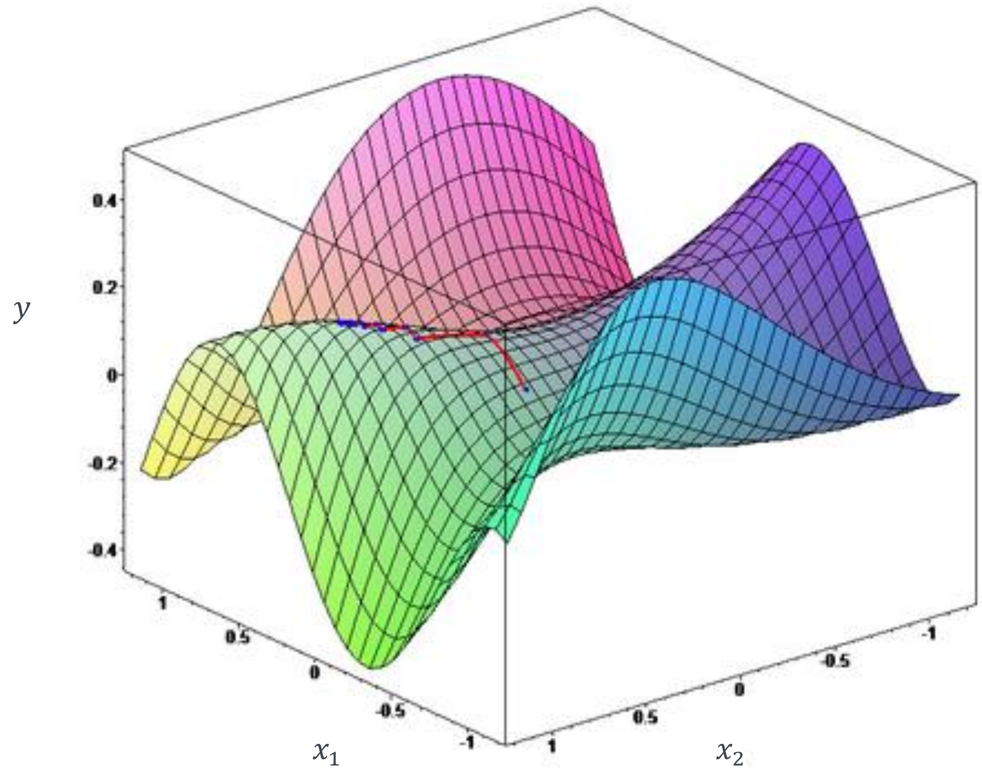
$$\frac{df(\boldsymbol{x})}{d\boldsymbol{x}} = \nabla_{\boldsymbol{x}} f(\boldsymbol{x})$$

is maximal/minimal



https://en.wikipedia.org/wiki/Gradient_descent#/media/File:Gradient_descent.svg

In general: challenge can be difficult



Gradient Descent (- but without posts)

https://goo.gl/images/JKN6zm

Optimization continued

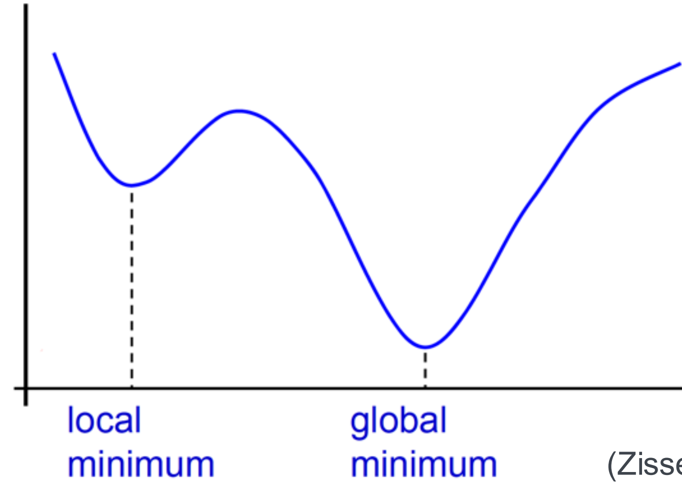
$$\min_{\mathbf{w}\in\mathbb{R}^d} C\sum_{i}^{N} \max\left(0, 1 - y_i\widehat{f}(\mathbf{x}_i)\right) + ||\mathbf{w}||^2$$

Questions

- Does this cost function have a unique solution?

Optimization continued

$$\min_{\mathbf{w}\in\mathbb{R}^d} C\sum_{i}^{N} \max\left(0, 1 - y_i f(\mathbf{x}_i)\right) + ||\mathbf{w}||^2$$



(Zisserman 2015)

Questions

- Does this cost function have a unique solution?
- Do we find it using gradient descent?
  Does the solution we find using gradient descent depend on the starting point?

To the rescue:

- If the cost function is convex, then a locally optimal point is globally optimal (provided the optimization is over constraints that form a convex set – given in our case)

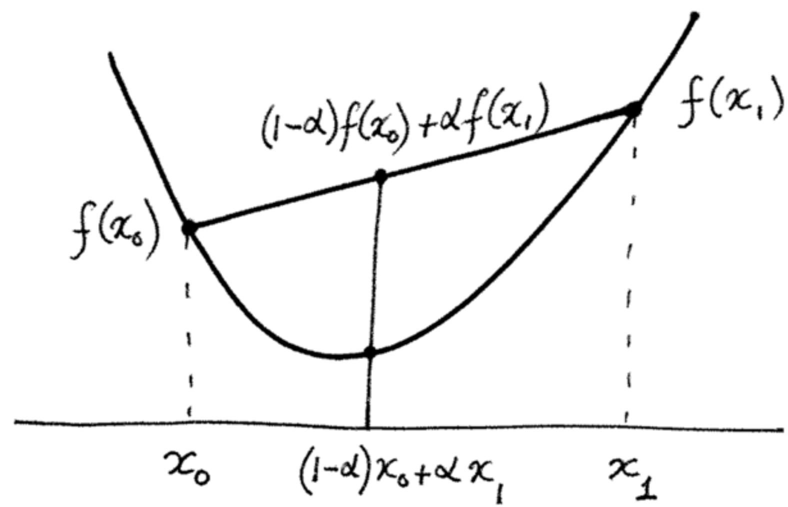
Convex functions

D – a domain in $\mathbb{R}^n$.

A convex function $f : D \to \mathbb{R}$ is one that satisfies, for any $\mathbf{x}_0$ and $\mathbf{x}_1$ in D:

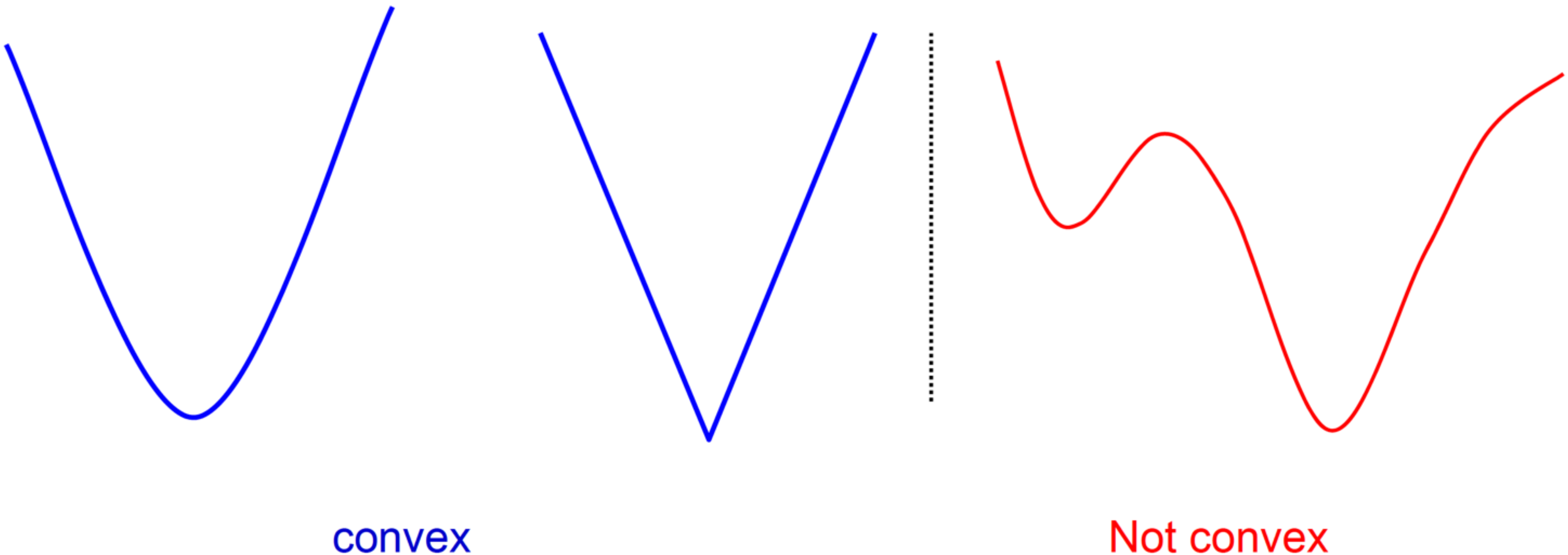
$$f((1-\alpha)\mathbf{x}_0 + \alpha\mathbf{x}_1) \leq (1-\alpha)f(\mathbf{x}_0) + \alpha f(\mathbf{x}_1) \quad .$$

Line joining $(\mathbf{x}_0, f(\mathbf{x}_0))$ and $(\mathbf{x}_1, f(\mathbf{x}_1))$ lies above the function graph.



(Zisserman 2015)

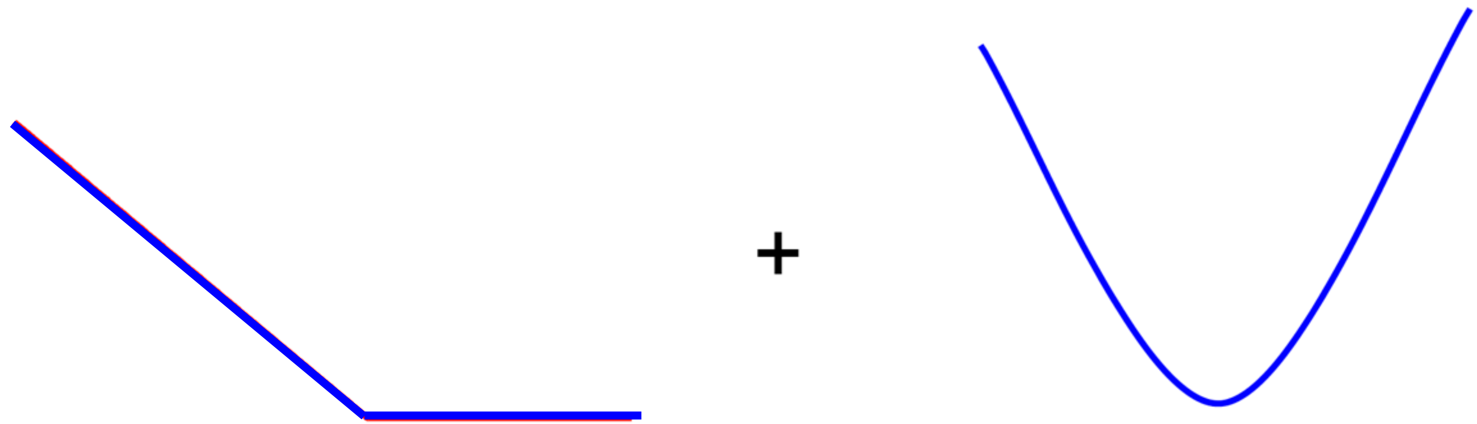
Convex function examples



- A non-negative sum of convex functions is convex

(Zisserman 2015)

Applied to hinge loss and regularization



SVM

$$\min_{\mathbf{w}\in\mathbb{R}^d} C \sum_{i}^{N} \max\left(0, 1 - y_i f(\mathbf{x}_i)\right) + ||\mathbf{w}||^2$$

convex

Gradient descent algorithm for SVM

To minimize a cost function $\mathcal{C}(w)$ use the iterative update

$$w_{t+1} \coloneqq w_t - \eta_t \nabla_{\mathrm{w}} \mathcal{C}(\mathrm{w_t})$$

where η is the learning rate.

Let‘s rewrite the minimization problem as an average with $\lambda = \frac{2}{C}$:

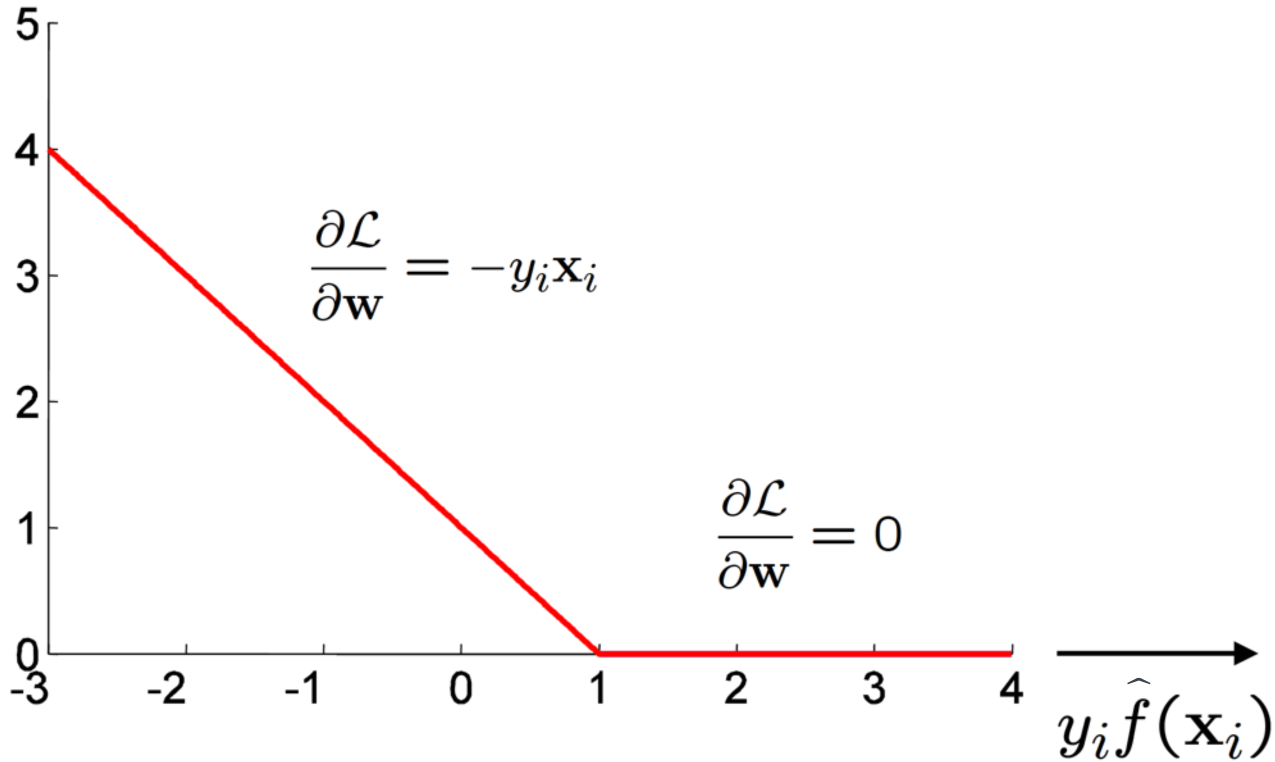
$$\mathcal{C}(w) = \frac{1}{NC} \|w\|^2 + \frac{1}{N} \sum_{i=1}^{N} \max(0,1 - y_i \hat{f}(x_i)) =$$

$$= \frac{1}{N} \sum_{i=1}^{N} \left(\frac{\lambda}{2} \|w\|^2 + \max(0,1 - y_i \hat{f}(x_i)) \right)$$

and $\hat{f}(x_i) = w^T x + b$

Sub-gradient for hinge loss

$\mathcal{L}(x_i, y_i; w) = \max(0, 1 - y_i \hat{f}(x_i))$, $\hat{f}(x_i) = w^T x_i + b$



(Zisserman 2015)

Sub-gradient descent algorithm for SVM

$$\mathcal{C}(w) = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{\lambda}{2}\|w\|^2 + \mathcal{L}(x_i, y_i; w)\right)$$

The iterative update is

$$w_{t+1} \coloneqq w_t - \eta \nabla_w \mathcal{C}(\mathrm{w_t}) \coloneqq$$

$$\coloneqq w_t - \eta \frac{1}{N}\sum_{i=1}^{N}(\lambda w_t + \nabla_w \mathcal{L}(x_i, y_i; w))$$

Then each iteration t involves cycling through the training data with the updates:

$$w_{t+1} \coloneqq \begin{cases} w_t - \eta(\lambda w_t - y_i x_i), & \text{if } \; y_i \hat{f}(x_i) < 1 \\ w_t - \eta \lambda w_t, & \text{otherwise} \end{cases}$$

Typical learning rate in Pegasos: $\eta_t = \frac{1}{\lambda t}$

5 The dual problem

Primal vs dual problem

- **SVM** is a linear classifier: $\hat{f}(x) = w^T x + b$
- **The primal problem**: an optimization problem over w:

$$\min_{w \in \mathbb{R}^d} \frac{1}{C} \|w\|^2 + \sum_{i=1}^{N} \max(0, 1 - y_i \hat{f}(x_i))$$

- **The dual problem**: Getting rid of the w for a slightly different representation of $\hat{f}(x)$ leads to the following representation

$$\hat{f}(x) = \sum_{i=1}^{N} \alpha_i \, y_i \left(x_i^T x \right) + b$$

and a new optimization problem with the same solution, but several advantages. Let us show this on following slides...

Revisit Optimization Problem for Hard Margin Case

- Minimize the quadratic form

$$\frac{||w||^2}{2} = \frac{w^T w}{2}$$

  - With constraints

$$y_i \cdot (w^T x_i + b) \geq 1 \;\; \forall i$$

- The constraints will reach a value of 1 for at least one instance.

- Include hard constraints into the loss function:

$$\mathcal{L}(w, b, \alpha) = \frac{||w||^2}{2} - \sum_{i=1}^{N} \alpha_i (y_i \cdot (w^T x_i + b) - 1)$$

- failed constraints “punish“ the objective function

Excursion: Lagrange Multiplier

- We want to maximize a function $f(x)$ under the constraints $g(x) = a$

Solution with Lagrange Multiplier

- Optimize the Lagrangian

$$f(x) - \lambda(g(x) - a)$$

instead!

Nicely visual explanation of Lagrange optimization at
https://www.svm-tutorial.com/2016/09/duality-lagrange-multipliers/

Algorithm for optimization with a Lagrange multiplier

1. Write down the Lagrangian $f(x) - \lambda \cdot (g(x) - a)$
2. Take derivative of Lagrangian wrt x,
   set it to 0
   to find estimate of x that depends on λ
3. Plug your estimate of x in the Lagrangian,
   take the derivative wrt λ,
   and set it to 0,
   to find the optimal value for the lagrange multiplier λ
4. Plug in the Lagrange multiplier in your estimate for x

Revisit Optimization Problem for Hard Margin Case

- Minimize the quadratic form

$$\frac{||w||^2}{2} = \frac{w^T w}{2}$$

  - With constraints

$$y_i \cdot (w^T x_i + b) \geq 1 \;\; \forall i$$

- The constraints will reach a value of 1 for at least one instance.

- Include hard constraints into the loss function:

$$\mathcal{L}(w, b, \alpha) = \frac{||w||^2}{2} - \sum_{i=1}^{N} \alpha_i (y_i \cdot (w^T x_i + b) - 1)$$

α_i are the Lagrange multipliers

  - failed constraints "punish" the objective function

Lagrangian primal problem

- Lagrangian primal problem is:

$$\min_{w,b} \max_{\alpha} \mathcal{L}(w, b, \alpha)$$

subject to $\forall i$: $\alpha_i \geq 0$

Finding the optimum

- Loss is a function of w, b, and α

$$\mathcal{L}(w,b,\alpha) = \frac{\|w\|^2}{2} - \sum_{i=1}^{N} \alpha_i(y_i \cdot (w^T x_i + b) - 1)$$

- Find optimum using derivatives:

w is a linear combination of the data instances!

$$\frac{\partial}{\partial b}\mathcal{L}(w,b,\alpha) = 0 \implies 0 = \sum_{i=1}^{N} \alpha_i y_i$$

$$\frac{\partial}{\partial w_j}\mathcal{L}(w,b,\alpha) = 0 \implies w_j = \sum_{i=1}^{N} \alpha_i y_i x_{i,j} \implies w = \sum_{i=1}^{N} \alpha_i y_i x_i$$

Substitution into $\mathcal{L}(w, b, \alpha)$

$$\mathcal{L}(w,b,\alpha)_{|w=\sum_{i=1}^{N}\alpha_i y_i x_i} = \frac{\|w\|^2}{2} - \sum_{i=1}^{N} \alpha_i \left(y_i \cdot \left(w^T x_i + b\right) - 1\right) =$$

$$= \frac{1}{2}\left\{\sum_{j=1}^{N} \alpha_j y_j x_j\right\}^T \left\{\sum_{k=1}^{N} \alpha_k y_k x_k\right\} - \sum_{i=1}^{N} \alpha_i \left(y_i \cdot \left(\left\{\sum_{j=1}^{N} \alpha_j y_j x_j\right\}^{\mathrm{T}} x_i + b\right) - 1 \right) =$$

$$= \underbrace{\frac{1}{2}\sum_{j,k} \alpha_j \alpha_k y_j y_k \left(x_j^T x_k\right) - \sum_{i,j} \alpha_i \alpha_j y_i y_j \left(x_i^T x_j\right)}_{=} - \underbrace{b \sum_{i=1}^{N} \alpha_i y_i}_{=0} + \sum_{i=1}^{N} \alpha_i =$$

$$= \mathcal{L}(\alpha) = \sum_{i=1}^{N} \alpha_i - \frac{1}{2}\sum_{j,k} \alpha_j \alpha_k y_j y_k \left(x_j^T x_k\right)$$

Wolfe dual problem

$$\max_{\alpha} \sum_{i=1}^{N} \alpha_i - \frac{1}{2} \sum_{j,k} \alpha_j \alpha_k y_j y_k \left(x_j^T x_k \right)$$

subject to $\forall i$: $\alpha_i \geq 0$, and $0 = \sum_{i=1}^{N} \alpha_i y_i$

- This problem is solvable with quadratic programming, because it fulfills the Karush-Kuhn-Tucker conditions on α_i that handle inequality constraints (≥1) in the Lagrange optimization (not given here!).
- It gives us the classification function:

$$\hat{f}(x) = \sum_{i=1}^{N} \alpha_i \cdot y_i \cdot x_i^T x + b$$

α_i is positive if x_i is a support vector

Non-separable Case (similar as before)

- Introduce (positive) „slack variables“ ξ_i to allow deviations from the minimum distance:

$$y_i(w^T x_i + b) \geq 1 - \xi_i$$

- Include a penalizing term in the optimization function:

$$C\left(\sum_{i=1}^{m} \xi_i\right)^k$$

- Transform to Lagrangian
  - **with additional Lagrange multipliers** for the slack variables being constrained to positive values ...

Summary: Primal and dual formulations

- **Primal** version of classifier

$$\hat{f}(x) = w^T x + b$$

- **Dual** version of classifier

$$\hat{f}(x) = \sum_{i=1}^{N} \alpha_i \cdot y_i \cdot x_i^T x + b$$

The dual form classifier seems to work like a kNN classifier, it requires the training data points x_i. However, many of the α_i are zero.
The ones that are non-zero define the support vectors x_i.

Summary: Primal and dual formulations

- Lagrangian **primal** problem is:

$$\min_{w,b} \max_{\alpha} \mathcal{L}(w, b, \alpha)$$

subject to $\forall i$: $\alpha_i \geq 0$

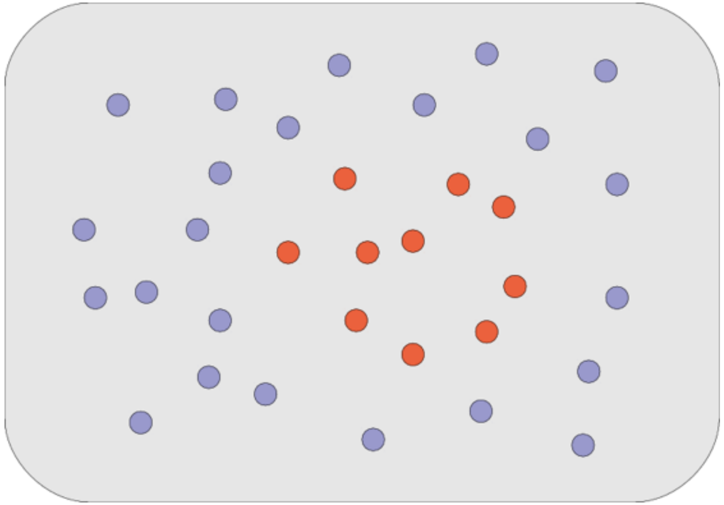
- Lagrangian **dual** problem is:

$$\max_{\alpha} \sum_{i=1}^{N} \alpha_i - \frac{1}{2} \sum_{j,k} \alpha_j \alpha_k y_j y_k \left(x_j^T x_k\right)$$

subject to $\forall i$: $\alpha_i \geq 0$, and $0 = \sum_{i=1}^{N} \alpha_i y_i$

6 Kernelization Tricks in SVMs

Non-linear Case



- Not all classes can be separated via a hyperplane
- Essential:
  - Dual representation uses only the product of data instances:

$$\hat{f}(x) = \sum_{i=1}^{N} \alpha_i \cdot y_i \cdot \boxed{x_i^T x} + b$$

- x_i : i-th training instance
- α_i : weight for i-th training instance

- Same for the Lagrangian...

Feature engineering using $\phi(x)$

Cf. lecture on regression. Chapter "beyond linear input"

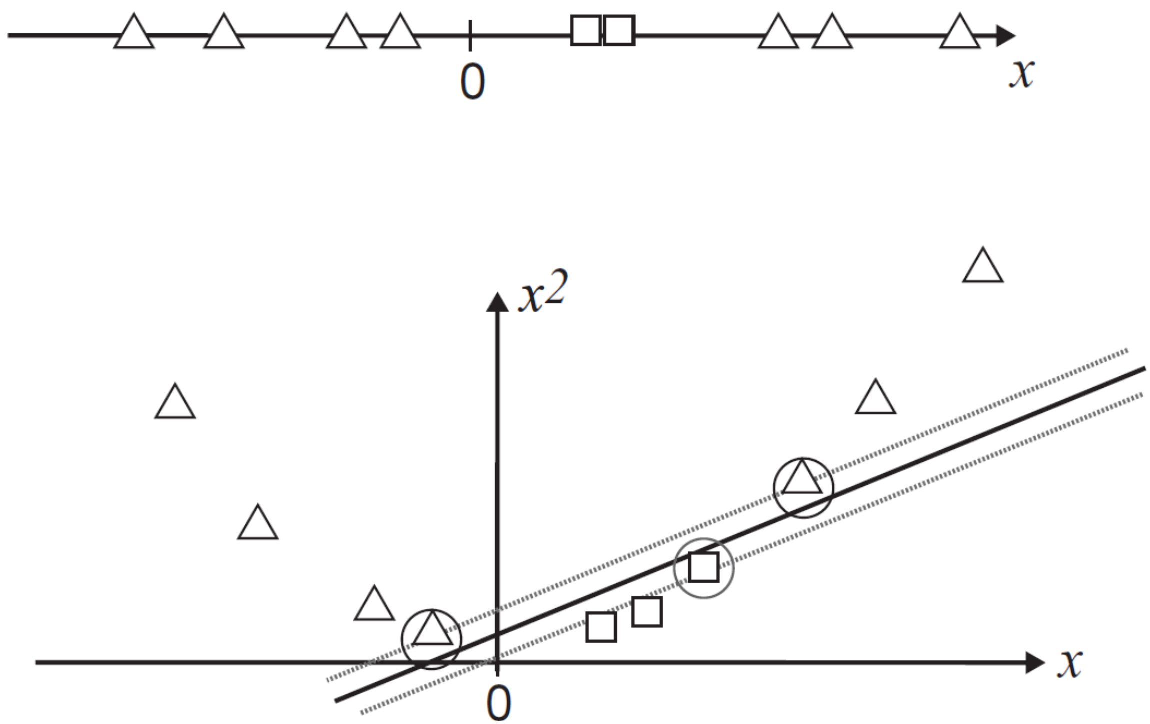
- Classifier: Given $x_i \in \mathbb{R}^d$, $\phi: \mathbb{R}^d \to \mathbb{R}^D, w \in \mathbb{R}^D$

$$\hat{f}(x) = w^T \phi(x) + b$$

- Learning:

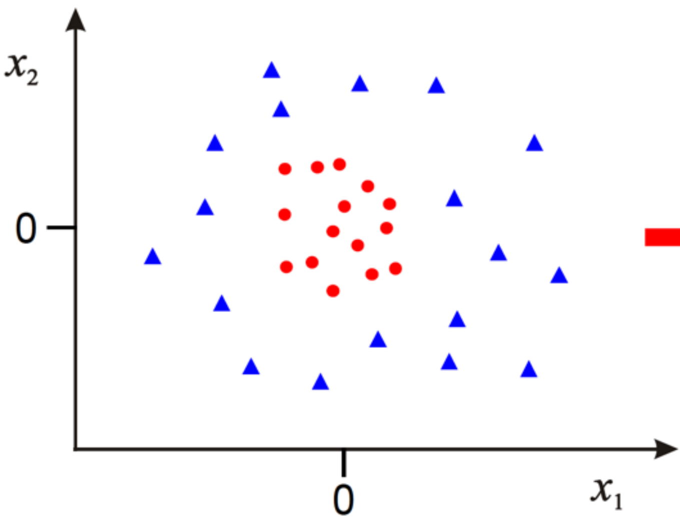
$$\min_{w \in \mathbb{R}^D} \frac{1}{C} \|w\|^2 + \sum_{i=1}^{N} \max(0, 1 - y_i \hat{f}(x_i))$$

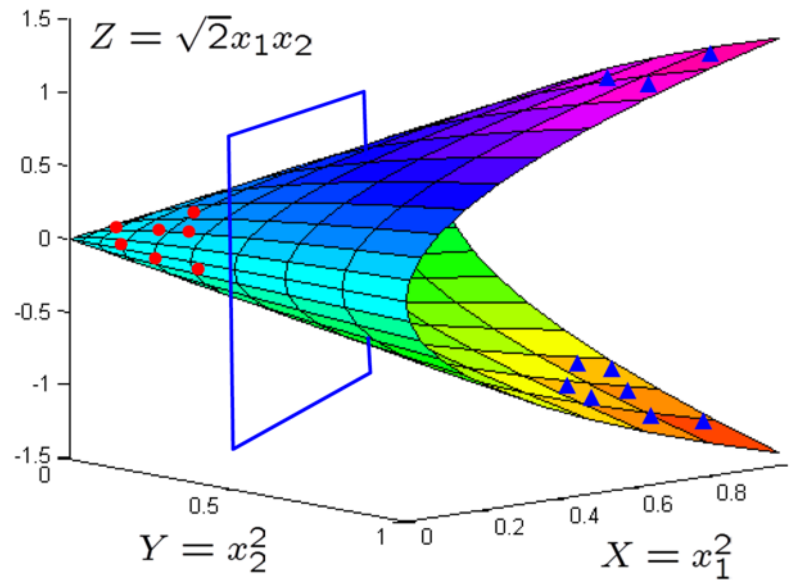
Example 1: From 1-dim to 2-dim



Example 2: From 2-dim to 3-dim

$$\Phi : \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \to \begin{pmatrix} x_1^2 \\ x_2^2 \\ \sqrt{2}x_1x_2 \end{pmatrix} \qquad \mathbb{R}^2 \to \mathbb{R}^3$$





- Data is linearly separable in 3D

(Zisserman 2015)

- This means that the problem can still be solved by a linear classifier

Feature engineering using $\phi(x)$

- Classifier: Given $x_i \in \mathbb{R}^d$, $\phi: \mathbb{R}^d \to \mathbb{R}^D, w \in \mathbb{R}^D$

$$\hat{f}(x) = w^T \phi(x) + b$$

- Learning:

$$\min_{w \in \mathbb{R}^D} \frac{1}{C} \|w\|^2 + \sum_{i=1}^{N} \max(0, 1 - y_i \hat{f}(x_i))$$

- $\phi(x)$ maps to high dimensional space $\mathbb{R}^D$ where data is separable
- If $D \gg d$ then there are many more parameters to learn for w

Dual classifier in transformed feature space

Classifier:

$$\hat{f}(x) = \sum_{i=1}^{N} \alpha_i \cdot y_i \cdot x_i^T x \; + b$$

$$\Longrightarrow \; \hat{f}(x) = \sum_{i=1}^{N} \alpha_i \cdot y_i \cdot \phi(x_i)^T \phi(x) + b$$

Learning:

$$\max_{\alpha} \sum_{i=1}^{N} \alpha_i - \frac{1}{2} \sum_{j,k} \alpha_j \alpha_k y_j y_k \left(x_j^T x_k\right)$$

$$\Longrightarrow \max_{\alpha} \sum_{i=1}^{N} \alpha_i - \frac{1}{2} \sum_{j,k} \alpha_j \alpha_k y_j y_k \left(\phi\left(x_j\right)^T \phi(x_k)\right)$$

subject to $\forall i$: $\alpha_i \geq 0$, and $0 = \sum_{i=1}^{N} \alpha_i y_i$

ϕ(x)
only occurs in pairs
$\phi\left(x_j\right)^T \phi(x_i)$

Kernels
$k\left(x_j, x_i\right) = \phi\left(x_j\right)^T \phi(x_i)$

Dual classifier using kernels

Classifier:

$$\hat{f}(x) = \sum_{i=1}^{N} \alpha_i \cdot y_i \cdot \phi(x_i)^T \phi(x) + b$$

$$\Longrightarrow \hat{f}(x) = \sum_{i=1}^{N} \alpha_i \cdot y_i \cdot k(x_i, x) + b$$

Learning:

$$\max_{\alpha} \sum_{i=1}^{N} \alpha_i - \frac{1}{2} \sum_{j,k} \alpha_j \alpha_k y_j y_k \left(\phi(x_j)^T \phi(x_k) \right)$$

$$\Longrightarrow \max_{\alpha} \sum_{i=1}^{N} \alpha_i - \frac{1}{2} \sum_{j,k} \alpha_j \alpha_k y_j y_k k(x_j, x_k)$$

subject to $\forall i$: $\alpha_i \geq 0$, and $0 = \sum_{i=1}^{N} \alpha_i y_i$

Example kernels

- Linear kernels: $k(x, x') = x^T x'$

- Polynomial kernels: $k(x, x') = (1 + x^T x')^d, \text{for any } d > 0$
  - Contains all polynomial terms up to degree

- Gaussian kernels: $k(x, x') = e^{-\frac{\|x - x'\|^2}{2\sigma^2}}$, for $\sigma > 0$
  - Infinite dimensional feature space
  - Also called Radial basis function kernel (RBF)
    - often works quite well!

- Graph kernels: random walk
- String kernels: ...
- **build your own kernel for your own problem!**

Summary on kernels

- „Instead of inventing funny non-linear features, we may directly invent funny kernels" (Toussaint 2019)
- Inventing a kernel is intuitive:
  - $k(x, x')$ expresses how correlated y and y' should be
  - it is a meassure of similarity, it compares x and x'.
- Specifying how 'comparable' x and x' are is often more intuitive than defining "features that might work".

Background reading and more

- Smooth readong about SVMs: Alexandre Kowalczyk, Support vector machines succinctly. Syncfusion. Free ebook: https://www.syncfusion.com/ebooks/support_vector_machines_succinctly
  - **Also talks about most efficient algorithms to be used for finding support vectors (it is neither of the two presented here!)**

7 Transductive Classification

Transductive learning characteristics

Characteristics

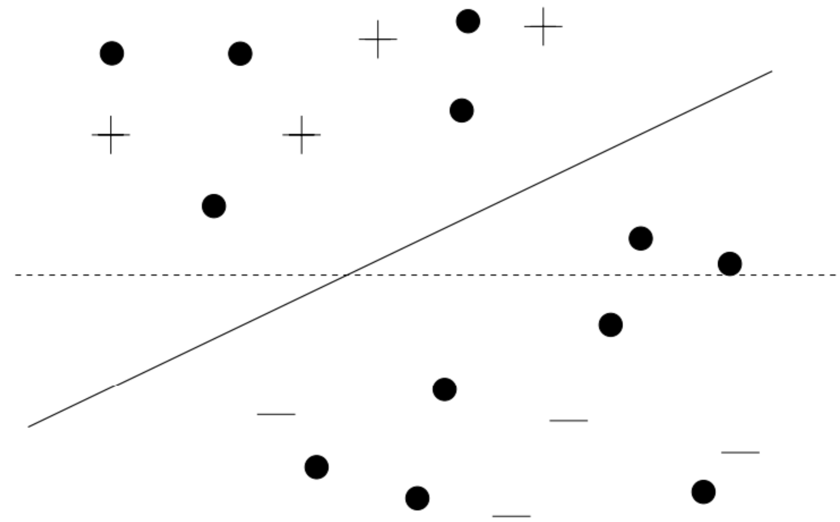
- Training data AND test data known at learning time
- Learning happens specifically for the given test cases

Use cases

- news recommender
- spam classifier
- document reorganization

Thorsten Joachims:
Transductive Inference for Text Classification using Support Vector Machines. *ICML 1999: 200-209*

Maximum margin hyperplane



Training data $\{\ldots, (\vec{x}_i, y_i), \ldots\}$

Test data $\{\ldots \vec{x}_j^* \ldots\}$

Loss function

$$\frac{1}{2} \parallel \vec{w} \parallel^2 + C \sum_{i=0}^{n} \xi_i + C^* \sum_{j=0}^{k} \xi_j^*$$

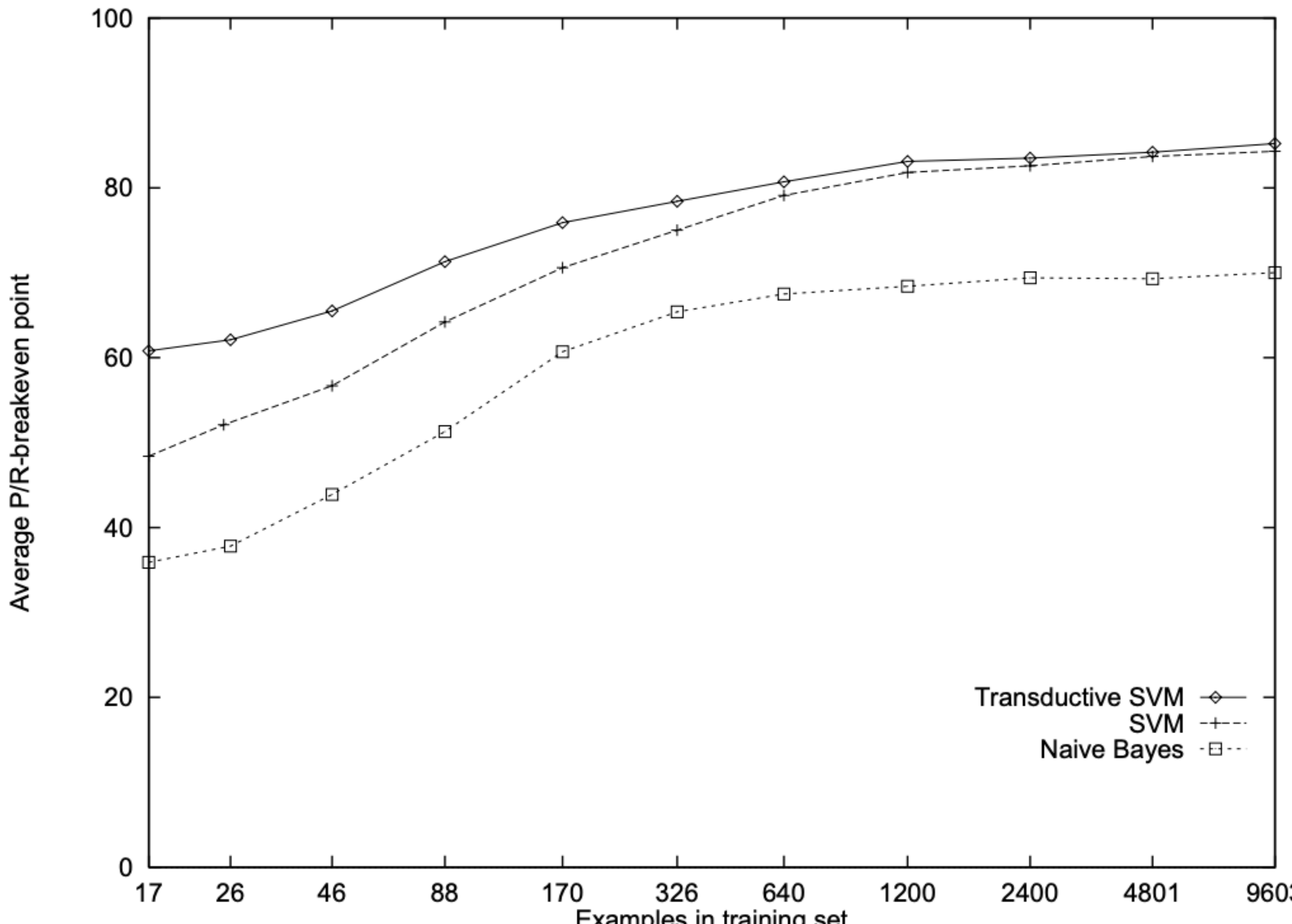
subject to:

$$\forall_{i=1}^{n}: y_i[\vec{w} \cdot \vec{x}_i + b] \geq 1 - \xi_i$$
$$\forall_{j=1}^{k}: y_j^*[\vec{w} \cdot \vec{x}_j^* + b] \geq 1 - \xi_j^*$$
$$\forall_{i=1}^{n}: \xi_i > 0$$
$$\forall_{j=1}^{k}: \xi_j^* > 0$$

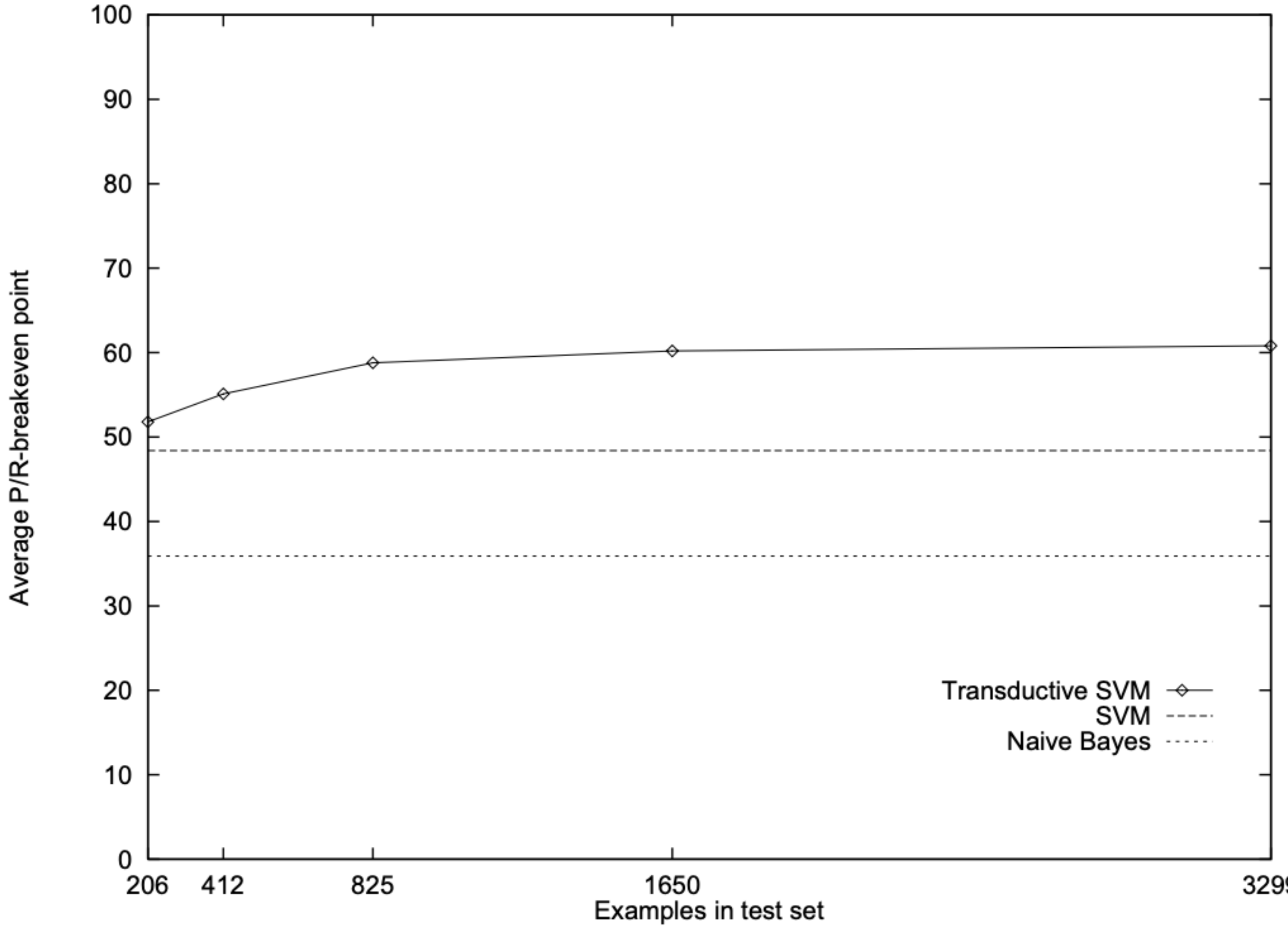
Naive, intractable approach:

- for every hyperplane:
  - classify $\vec{x}_j^*$
  - compute loss

Reuters data set experiments (3299 test documents)



Reuters data set experiments (17 training documents)



Current research at Analytic Computing:

Multi-label classification with hyperbolic hyperplanes

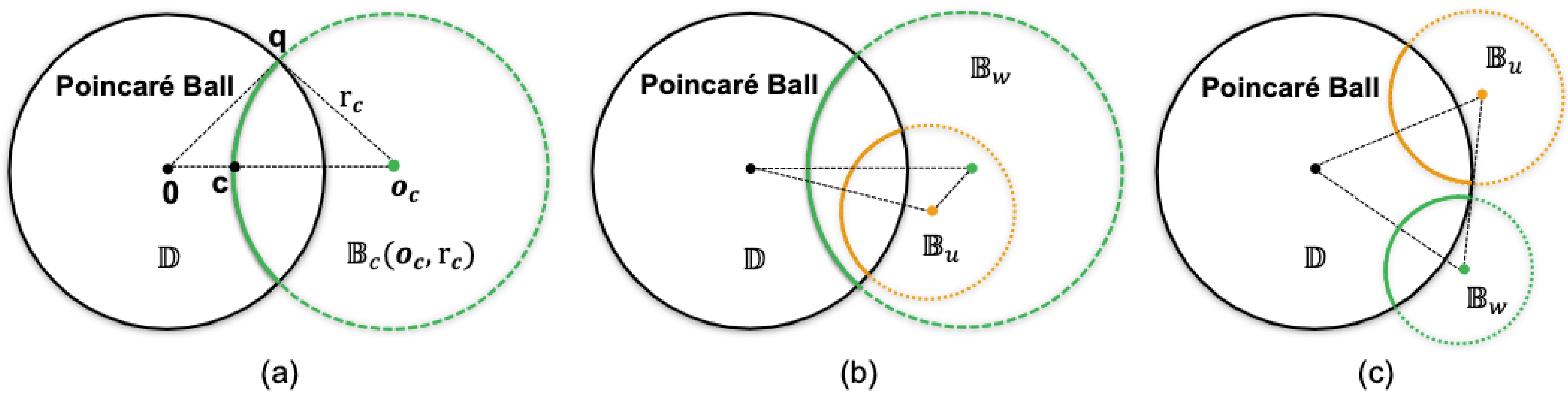


Figure 2: (a) A Poincaré hyperplane is defined as the intersection between the Poincaré ball $\mathbb{D}$ and the boundary of an n-ball $\mathbb{B}_\mathbf{c}$. The Poincaré hyperplane is uniquely parameterized by a center point $\mathbf{c}$, and the corresponding n-ball (its radius and center) can be uniquely determined by Proposition 1. (b) Label implication is interpreted as n-ball insideness. (c) Mutual exclusion is interpreted as n-ball disjointedness.

So far with hyperbolic logistic regression but
doing it with hyperbolic SVM could be a project or a bachelor thesis

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Thank you!

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