NLTK Parsing Demos

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Contents

1 Recursive descent parsing (top-down, depth-first)

[**py1**]
```
>>> import nltk, re, pprint
>>> from __future__ import division
```

[**py2**]
```
>>> grammar1 = nltk.parse_cfg("""
...     S -> NP VP
...     VP -> V NP | V NP PP
...     PP -> P NP
...     V -> "saw" | "ate" | "walked"
```

[*]Based on the NLTK book (Bird et al. 2009) and created with the PythonTeX package (Poore 2013).

```
...     NP -> "John" | "Mary" | "Bob" | Det N | Det N PP
...     Det -> "a" | "an" | "the" | "my"
...     N -> "man" | "dog" | "cat" | "telescope" | "park"
...     P -> "in" | "on" | "by" | "with"
... """)
```

[**py3**]
```
>>> rd_parser = nltk.RecursiveDescentParser(grammar1)
>>> sent = 'Mary saw a dog'.split()
>>> sent
['Mary', 'saw', 'a', 'dog']
>>> trees = rd_parser.nbest_parse(sent)
>>> for tree in trees:
...     print tree, "\n\n"
...
(S (NP Mary) (VP (V saw) (NP (Det a) (N dog))))
```

[**py4**]
```
>>> rd_parser = nltk.RecursiveDescentParser(grammar1, trace=2)
>>> rd_parser.nbest_parse(sent)
Parsing 'Mary saw a dog'
    [ * S ]
  E [ * NP VP ]
  E [ * 'John' VP ]
  E [ * 'Mary' VP ]
  M [ 'Mary' * VP ]
  E [ 'Mary' * V NP ]
  E [ 'Mary' * 'saw' NP ]
  M [ 'Mary' 'saw' * NP ]
  E [ 'Mary' 'saw' * 'John' ]
  E [ 'Mary' 'saw' * 'Mary' ]
  E [ 'Mary' 'saw' * 'Bob' ]
  E [ 'Mary' 'saw' * Det N ]
  E [ 'Mary' 'saw' * 'a' N ]
  M [ 'Mary' 'saw' 'a' * N ]
  E [ 'Mary' 'saw' 'a' * 'man' ]
  E [ 'Mary' 'saw' 'a' * 'dog' ]
  M [ 'Mary' 'saw' 'a' 'dog' ]
  + [ 'Mary' 'saw' 'a' 'dog' ]
  E [ 'Mary' 'saw' 'a' * 'cat' ]
  E [ 'Mary' 'saw' 'a' * 'telescope' ]
  E [ 'Mary' 'saw' 'a' * 'park' ]
  E [ 'Mary' 'saw' * 'an' N ]
  E [ 'Mary' 'saw' * 'the' N ]
  E [ 'Mary' 'saw' * 'my' N ]
  E [ 'Mary' 'saw' * Det N PP ]
  E [ 'Mary' 'saw' * 'a' N PP ]
  M [ 'Mary' 'saw' 'a' * N PP ]
  E [ 'Mary' 'saw' 'a' * 'man' PP ]
```

```
E [ 'Mary' 'saw' 'a' * 'dog' PP ]
M [ 'Mary' 'saw' 'a' 'dog' * PP ]
E [ 'Mary' 'saw' 'a' 'dog' * P NP ]
E [ 'Mary' 'saw' 'a' 'dog' * 'in' NP ]
E [ 'Mary' 'saw' 'a' 'dog' * 'on' NP ]
E [ 'Mary' 'saw' 'a' 'dog' * 'by' NP ]
E [ 'Mary' 'saw' 'a' 'dog' * 'with' NP ]
E [ 'Mary' 'saw' 'a' * 'cat' PP ]
E [ 'Mary' 'saw' 'a' * 'telescope' PP ]
E [ 'Mary' 'saw' 'a' * 'park' PP ]
E [ 'Mary' 'saw' * 'an' N PP ]
E [ 'Mary' 'saw' * 'the' N PP ]
E [ 'Mary' 'saw' * 'my' N PP ]
E [ 'Mary' * 'ate' NP ]
E [ 'Mary' * 'walked' NP ]
E [ 'Mary' * V NP PP ]
E [ 'Mary' * 'saw' NP PP ]
M [ 'Mary' 'saw' * NP PP ]
E [ 'Mary' 'saw' * 'John' PP ]
E [ 'Mary' 'saw' * 'Mary' PP ]
E [ 'Mary' 'saw' * 'Bob' PP ]
E [ 'Mary' 'saw' * Det N PP ]
E [ 'Mary' 'saw' * 'a' N PP ]
M [ 'Mary' 'saw' 'a' * N PP ]
E [ 'Mary' 'saw' 'a' * 'man' PP ]
E [ 'Mary' 'saw' 'a' * 'dog' PP ]
M [ 'Mary' 'saw' 'a' 'dog' * PP ]
E [ 'Mary' 'saw' 'a' 'dog' * P NP ]
E [ 'Mary' 'saw' 'a' 'dog' * 'in' NP ]
E [ 'Mary' 'saw' 'a' 'dog' * 'on' NP ]
E [ 'Mary' 'saw' 'a' 'dog' * 'by' NP ]
E [ 'Mary' 'saw' 'a' 'dog' * 'with' NP ]
E [ 'Mary' 'saw' 'a' * 'cat' PP ]
E [ 'Mary' 'saw' 'a' * 'telescope' PP ]
E [ 'Mary' 'saw' 'a' * 'park' PP ]
E [ 'Mary' 'saw' * 'an' N PP ]
E [ 'Mary' 'saw' * 'the' N PP ]
E [ 'Mary' 'saw' * 'my' N PP ]
E [ 'Mary' 'saw' * Det N PP PP ]
E [ 'Mary' 'saw' * 'a' N PP PP ]
M [ 'Mary' 'saw' 'a' * N PP PP ]
E [ 'Mary' 'saw' 'a' * 'man' PP PP ]
E [ 'Mary' 'saw' 'a' * 'dog' PP PP ]
M [ 'Mary' 'saw' 'a' 'dog' * PP PP ]
E [ 'Mary' 'saw' 'a' 'dog' * P NP PP ]
E [ 'Mary' 'saw' 'a' 'dog' * 'in' NP PP ]
E [ 'Mary' 'saw' 'a' 'dog' * 'on' NP PP ]
```

```
  E [ 'Mary' 'saw' 'a' 'dog' * 'by' NP PP ]
  E [ 'Mary' 'saw' 'a' 'dog' * 'with' NP PP ]
  E [ 'Mary' 'saw' 'a' * 'cat' PP PP ]
  E [ 'Mary' 'saw' 'a' * 'telescope' PP PP ]
  E [ 'Mary' 'saw' 'a' * 'park' PP PP ]
  E [ 'Mary' 'saw' * 'an' N PP PP ]
  E [ 'Mary' 'saw' * 'the' N PP PP ]
  E [ 'Mary' 'saw' * 'my' N PP PP ]
  E [ 'Mary' * 'ate' NP PP ]
  E [ 'Mary' * 'walked' NP PP ]
  E [ * 'Bob' VP ]
  E [ * Det N VP ]
  E [ * 'a' N VP ]
  E [ * 'an' N VP ]
  E [ * 'the' N VP ]
  E [ * 'my' N VP ]
  E [ * Det N PP VP ]
  E [ * 'a' N PP VP ]
  E [ * 'an' N PP VP ]
  E [ * 'the' N PP VP ]
  E [ * 'my' N PP VP ]
[Tree('S', [Tree('NP', ['Mary']), Tree('VP', [Tree('V', ['saw']), Tree('NP', [Tree('De
```

2 Shift-reduce (bottom-up)

[**py5**]
```
>>> sr_parser = nltk.ShiftReduceParser(grammar1, trace=2)
>>> sr_parser.parse(sent)
Parsing 'Mary saw a dog'
    [ * Mary saw a dog]
  S [ 'Mary' * saw a dog]
  R [ NP * saw a dog]
  S [ NP 'saw' * a dog]
  R [ NP V * a dog]
  S [ NP V 'a' * dog]
  R [ NP V Det * dog]
  S [ NP V Det 'dog' * ]
  R [ NP V Det N * ]
  R [ NP V NP * ]
  R [ NP VP * ]
  R [ S * ]
Tree('S', [Tree('NP', ['Mary']), Tree('VP', [Tree('V', ['saw']), Tree('NP', [Tree('Det
```

3 Left-corner parser: top-down with bottom-up filtering

A left-corner parser preprocesses the context-free grammar to build a table where each row contains two cells, the first holding a non-terminal, and the second holding the collection of possible left corners of that non-terminal:

Category	Left-Corners (non-terminals)
S	NP
VP	V
PP	P
NP	Det

Each time a production is considered by the parser, the parser checks that the next input word is compatible with at least one of the non-terminal categories in the left-corner table.

[**py6**]
```
>>> lc_parser = nltk.LeftCornerChartParser(grammar1, trace=2)
>>> lc_parser.parse(sent)
|.   Mary   .   saw   .    a    .   dog   .|
Leaf Init Rule:
|[---------]          .         .         .| [0:1] 'Mary'
|.         [---------]          .         .| [1:2] 'saw'
|.         .         [---------]          .| [2:3] 'a'
|.         .         .         [---------]| [3:4] 'dog'
Filtered Bottom Up Predict Combine Rule:
|[---------]          .         .         .| [0:1] NP -> 'Mary' *
Filtered Bottom Up Predict Combine Rule:
|[--------->          .         .         .| [0:1] S  -> NP * VP
Filtered Bottom Up Predict Combine Rule:
|.         [---------]          .         .| [1:2] V  -> 'saw' *
Filtered Bottom Up Predict Combine Rule:
|.         [--------->          .         .| [1:2] VP -> V * NP
|.         [--------->          .         .| [1:2] VP -> V * NP PP
Filtered Bottom Up Predict Combine Rule:
|.         .         [---------]          .| [2:3] Det -> 'a' *
Filtered Bottom Up Predict Combine Rule:
|.         .         [--------->          .| [2:3] NP -> Det * N
|.         .         [--------->          .| [2:3] NP -> Det * N PP
Filtered Bottom Up Predict Combine Rule:
|.         .         .         [---------]| [3:4] N  -> 'dog' *
Filtered Single Edge Fundamental Rule:
|.         .         [-------------------]| [2:4] NP -> Det N *
Filtered Single Edge Fundamental Rule:
|.         [-----------------------------]| [1:4] VP -> V NP *
Filtered Single Edge Fundamental Rule:
|[=======================================]| [0:4] S  -> NP VP *
Tree('S', [Tree('NP', ['Mary']), Tree('VP', [Tree('V', ['saw']), Tree('NP', [Tree('Det
```

4 General Background: Memoization

This section provides a quick introduction to memoization (and dynamic programing).[1]

Put simply, memoization is the caching of results from previously called functions; it can sometimes provide dramatic improvements in speed. We're going to look at a calculation that would take many years if it were done without memoization. With memoization, it takes about 0.0005 seconds.

Let's say we have a simple function, one that calculates the cube of a number:

[**py7**]
```
>>> def cube(x):
...     return x**3
...
>>> cube(11)
1331
>>> cube(1111e12)
1.371330631e+45
```

Each time we invoke this function with an argument x , we calculate x^3. But if we record the result for any given value of x, we can just return that result if the function is called with that value again. A simple way to do this in Python:

[**py8**]
```
>>> results = {} # initialize memo table: a dictionary with function arguments as keys
>>> def cube(x):
...     if x not in results:
...         results[x] = x**3
...     return results[x]
...
>>> print results
{}
>>> cube(5)
125
>>> print results
{5: 125}
>>> cube(11)
1331
>>> print results
{11: 1331, 5: 125}
>>> cube(1111e12)
1.371330631e+45
>>> print results
{1111000000000000.0: 1.371330631e+45, 11: 1331, 5: 125}
```

We create a dictionary and store the results of previous function calls using the function argument as a keyword.

A more general way to doing this is to create a decorator – basically, a higher-order function invoked with a special `@` syntax – that can be applied to any function we want to memoize.

[1]This section is based on notes and code available here: `http://www.pycogsci.info/?p=221`.

[**py9**]
```
>>> def memoize(func):
...     S = {}
...     def wrappingfunction(*args):
...         if args not in S:
...             S[args] = func(*args)
...         print S # print memo table for edification; delete in final version
...         return S[args]
...     return wrappingfunction
...
```

With this we can do:

[**py10**]
```
>>> @memoize
... def cube(x):
...     return x**3
...
>>> cube(5)
{(5,): 125}
125
>>> cube(11)
{(5,): 125, (11,): 1331}
1331
>>> cube(1111e12)
{(5,): 125, (1111000000000000.0,): 1.371330631e+45, (11,): 1331}
1.371330631e+45
```

Memoization should not be used all the time: storing and retrieving the results might be more expensive than the function call. But if the function is of a particular kind – e.g., it is recursive and each recursion 'level' includes multiple function calls each of which can recursively branch into mutiple function calls – memoization improves speed dramatically.

A simple, classic case is the recursive definition of the *n*th Fibonacci number. A more familiar example to linguists is constructing a syntactic tree or doing compositional interpretation over a syntactic tree. At each level of the tree, the tree 'generation' function (i.e., the parser) is recursive and branches out into multiple function call to the same function, one for each daughter. The same applies to the interpretation function – interpreting a tree, i.e., calling the interpretation function on a tree, involves recursively calling the same interpretation function multiple times, once for each of the immediate subtrees.

Let's see this in action for the simple case of Fibonacci numbers. We save the `fibonacci` function in a separate module `fib` and import it. This is the definition of the function:

(1) File **fib.py**:

```
def fibonacci(n):
    if n == 0:
        return 0
    elif n == 1:
        return 1
    else:
        return fibonacci(n-1) + fibonacci(n-2)
```

[**py11**]
```
>>> import os
>>> import sys
>>> sys.path.append(os.getcwd())
>>> import fib
>>> fib.fibonacci(2)
1
>>> fib.fibonacci(3)
2
>>> fib.fibonacci(4)
3
>>> fib.fibonacci(5)
5
>>> fib.fibonacci(10)
55
```

The runtime of this function is exponential in the 'size' of the input. You can see this by timing the function for several different arguments:

[**py12**]
```
>>> import timeit
>>> for n in [5,10,15,20,25,30,35]:
...     t = timeit.Timer(stmt='fib.fibonacci(%d)' % n, setup='import fib')
...     print 'fibonacci(%d): %f seconds' % (n, min(t.repeat(5, 1)))
...
fibonacci(5): 0.000002 seconds
fibonacci(10): 0.000020 seconds
fibonacci(15): 0.000223 seconds
fibonacci(20): 0.002580 seconds
fibonacci(25): 0.027558 seconds
fibonacci(30): 0.304238 seconds
fibonacci(35): 3.367676 seconds
```

The unmemoized version of `fibonacci(100)` would have taken many years. We can estimate how long by extrapolating based on the above numbers: we fit a linear regression to the log of the above runtimes.

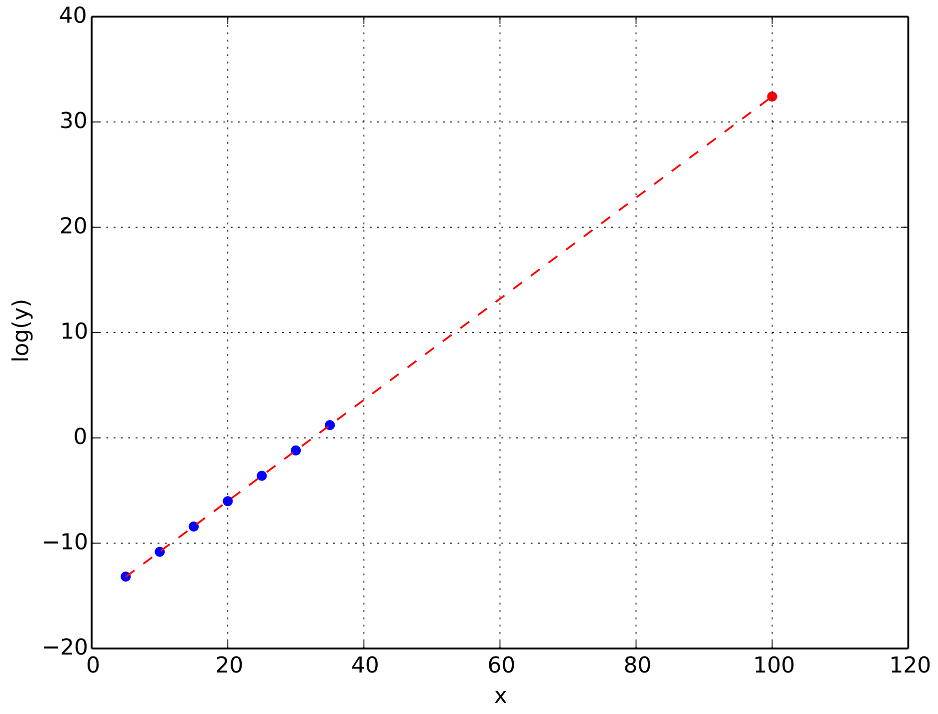
[**py13**]
```
>>> import numpy as np
>>> import matplotlib.pylab as plt
>>> from sklearn.linear_model import LinearRegression

>>> x = [5,10,15,20,25,30,35]
>>> y = [min(timeit.Timer(stmt='fib.fibonacci(%d)' % n, \
...                       setup='import fib').repeat(5, 1)) \
...      for n in x]
>>> log_y = list(np.log(y))

>>> for i in range(len(x)):
...     print x[i], '\t', y[i], '\t', log_y[i]
...
```

```
5     1.90734863281e-06     -13.1697964306
10     2.00271606445e-05     -10.8184211735
15     0.000221014022827     -8.41728440675
20     0.00245308876038     -6.01040732994
25     0.0273399353027     -3.59940681409
30     0.303564071655     -1.19216258152
35     3.36617398262     1.21377678267
>>> lm = LinearRegression()
>>> lm.fit(np.array(x)[:,np.newaxis], np.array(log_y))
LinearRegression(copy_X=True, fit_intercept=True, normalize=False)
>>> print 'Int: \t:', lm.intercept_, '\n', 'Coef: \t', lm.coef_
Int:     : -15.6021166243
Coef:     [ 0.48015082]

>>> plt.scatter(x, log_y,  color='blue')
<matplotlib.collections.PathCollection object at 0x51cc590>
>>> plt.scatter(100, lm.predict(100), color='red')
<matplotlib.collections.PathCollection object at 0x51cce50>
>>> plt.grid(True)
>>> plt.xlabel('x')
<matplotlib.text.Text object at 0x4d32110>
>>> plt.ylabel('log(y)')
<matplotlib.text.Text object at 0x4d37090>
>>> plt.plot(np.array(x + [100]), \
...          lm.predict(np.array(x + [100])[:,np.newaxis]), \
...          color='red', \
...          linewidth=1, \
...          linestyle='--')
[<matplotlib.lines.Line2D object at 0x7f37ace935d0>]
>>> plt.savefig("graph1.pdf", dpi=600)
>>> plt.close()
```



That is, computing the 100th Fibonacci number is expected to take millions of years:

[**py14**]
```
>>> print lm.predict(100)
[ 32.4129651]
>>> print np.exp(lm.predict(100))/(3600 * 24 * 365) # approx no of years
[ 3784123.30074697]
```

The reason for the exponential runtime is that each function call (recursively) makes two more function calls, and each one of these makes two more, and so on and so forth.

This is particularly wasteful because the same function values are being computed over and over again. If we simply cached the results of the functions, the unnecessary calculations could be avoided. We can do this by adding the `memoize` decorator. We do this in a new module `fibmemo`:

(2) File **fibmemo.py**:

```
def memoize(func):
    S = {}
    def wrappingfunction(*args):
        if args not in S:
            S[args] = func(*args)
        return S[args]
    return wrappingfunction

@memoize
def fibonacci(n):
    if n == 0:
        return 0
    elif n == 1:
        return 1
    else:
        return fibonacci(n-1) + fibonacci(n-2)
```

The function computes the same values as before:

[**py15**]
```
>>> import fibmemo
>>> fibmemo.fibonacci(2)
1
>>> fibmemo.fibonacci(3)
2
>>> fibmemo.fibonacci(4)
3
>>> fibmemo.fibonacci(5)
5
>>> fibmemo.fibonacci(10)
55
```

But the runtimes have dramatically improved:

[**py16**]
```
>>> for n in [5,10,15,20,25,30,35,60,100,200]:
...     t = timeit.Timer(stmt='fibmemo.fibonacci(%d)' % n, \
...                      setup='import fibmemo')
...     print 'fibonacci(%d): %f seconds' % (n, min(t.repeat(5, 1)))
...
fibonacci(5): 0.000000 seconds
fibonacci(10): 0.000001 seconds
fibonacci(15): 0.000000 seconds
fibonacci(20): 0.000000 seconds
fibonacci(25): 0.000000 seconds
fibonacci(30): 0.000000 seconds
fibonacci(35): 0.000000 seconds
fibonacci(60): 0.000000 seconds
fibonacci(100): 0.000000 seconds
fibonacci(200): 0.000000 seconds
```

5 Chart parsing

The key aspect of the memoized `fibonacci` function is that it uses a table to avoid recomputation. We can actually reorganize the entire computation around filling in such a table. The technique of organizing a computation around filling in a table that avoids recomputation is called dynamic programming.

Chart parsing is an example of dynamic programming – for example, the Left Corner (LC) parser above is not simply an LC parser, but an LC *chart* parser.

Chart parsers make use of dynamic programming in the sense that every time we parse a sub-part of our input, we store the intermediate results and re-use them when appropriate. That is, unlike the top-down algorithm above, we never do the same work multiple times.

The first time we build a structure, e.g., an NP, based on some sub-part of our input, we save it in a table, then we look it up when we need to use it as a subconstituent of higher phrases. This table – which is analogous to the memoization table we used to compute Fibonacci numbers efficiently – is known as a Well-Formed Substring Table (WFST).

In a WFST, we record the position of the words by filling in cells in a triangular matrix: the horizontal axis will denote the start position of a substring, while the vertical axis will denote the end position.

Recall our sentence:

```
[py17] >>> sent
       ['Mary', 'saw', 'a', 'dog']
```

For example, *saw* will appear in the cell with coordinates (1, 2).

To simplify this presentation, we will assume each word has a unique lexical category, and we will store this (not the word) in the matrix. So cell (1, 2) will contain the entry V because:

```
[py18] >>> grammar1.productions(rhs=sent[1])
       [V -> 'saw']
```

For every word in sent, we can look up in our grammar what category it belongs to and insert it in the WFST.

For our WFST, we create a matrix as a list of lists in Python, and initialize it with the lexical categories of each token:

```
[py19] >>> def init_wfst(tokens, grammar):
       ...     numtokens = len(tokens)
       ...     wfst = [[None for i in range(numtokens+1)] for j in range(numtokens+1)]
       ...     for i in range(numtokens):
       ...         productions = grammar.productions(rhs=tokens[i])
       ...         wfst[i][i+1] = productions[0].lhs()
       ...     return wfst
       ...
```

```
[py20] >>> wfst0 = init_wfst(sent, grammar1)
```

We also define a utility function `display()` to pretty-print the WFST for us. As expected, there is a V in cell (1, 2).

```
[py21] >>> def display(wfst, tokens):
       ...     print '\nWFST ' + ' '.join([("%-4d" % i) for i in range(1, len(wfst))])
       ...     for i in range(len(wfst)-1):
       ...         print "%d   " % i,
       ...         for j in range(1, len(wfst)):
       ...             print "%-4s" % (wfst[i][j] or '.'),
       ...         print
       ...
```

[**py22**]
```
>>> display(wfst0, sent)

WFST 1    2    3    4
0    NP   .    .    .
1    .    V    .    .
2    .    .    Det  .
3    .    .    .    N
```

We then define a function that completes the WFST based on our grammar:

[**py23**]
```
>>> def complete_wfst(wfst, tokens, grammar, trace=False):
...     index = dict((p.rhs(), p.lhs()) for p in grammar.productions())
...     numtokens = len(tokens)
...     for span in range(2, numtokens+1):
...         for start in range(numtokens+1-span):
...             end = start + span
...             for mid in range(start+1, end):
...                 nt1, nt2 = wfst[start][mid], wfst[mid][end]
...                 if nt1 and nt2 and (nt1,nt2) in index:
...                     wfst[start][end] = index[(nt1,nt2)]
...                     if trace:
...                         print "[%s] %3s [%s] %3s [%s] ==> [%s] %3s [%s]" % \
...                         (start, nt1, mid, nt2, end, \
...                          start, index[(nt1,nt2)], end)
...     return wfst
...
```

[**py24**]
```
>>> wfst1 = complete_wfst(wfst0, sent, grammar1)
>>> display(wfst1, sent)

WFST 1    2    3    4
0    NP   .    .    S
1    .    V    .    VP
2    .    .    Det  NP
3    .    .    .    N
```

And here's the initial WFST for ease of reference:

[**py25**]
```
>>> display(init_wfst(sent, grammar1), sent)

WFST 1    2    3    4
0    NP   .    .    .
1    .    V    .    .
2    .    .    Det  .
3    .    .    .    N
```

And here's the WFST completion step by step:

```
[py26] >>> complete_wfst(wfst0, sent, grammar1, trace=True)
       [2] Det [3]   N [4] ==> [2]  NP [4]
       [1]   V [2]  NP [4] ==> [1]  VP [4]
       [0]  NP [1]  VP [4] ==> [0]   S [4]
       [[None, NP, None, None, S], [None, None, V, None, VP], [None, None, None, Det, NP], [N
```

All of the edges that we've seen so far represent complete constituents. But it is helpful to record incomplete constituents to document the work already done by the parser. For example, when a top-down parser processes $VP \rightarrow VNP$, it may find V, but not NP. This work can be reused when processing $VP \rightarrow VNP$, so we should record the hypothesis that the V constituent *saw* is the beginning of a VP.

We can do this by adding a dot to the edge's right hand side. Material to the left of the dot records what has been found so far; material to the right of the dot specifies what still needs to be found in order to complete the constituent. This is very similar to LC parsing.

Types of edges:

- **self-loop edges**, e.g. an edge $[VP \rightarrow \bullet VNPPP, (i,i)]$ recording the hypothesis that a VP begins at location i, and that we anticipate finding a sequence V NP PP starting here
- **incomplete edges**, e.g., an edge $[VP \rightarrow V \bullet NPPP, (i,j)]$ recording the fact that we have discovered a V spanning (i, j), and hypothesize a following NP PP sequence to complete a VP beginning at i
- **complete edges**, e.g., an edge $[VP \rightarrow VNPPP\bullet, (i,k)]$ recording the discovery that a VP consisting of the sequence $VNPPP$ has been discovered for the span (i,j)
- **parse edges**: these are complete edges that span the entire sentence and have the grammar's start symbol as their left-hand side; they encode one or more parse trees for the sentence.

To parse a sentence, a chart parser first creates an empty chart spanning the sentence. It then finds edges that are licensed by its knowledge about the sentence, and adds them to the chart one at a time until one or more parse edges are found.

The edges that it adds can be licensed in one of three ways:

- the input can license an edge: each word w_i in the input licenses the complete edge $[w_i \rightarrow \bullet, (i, i+1)]$.
- the grammar can license an edge: each grammar production $A \rightarrow \alpha$ licenses the self-loop edge $[A \rightarrow \bullet\alpha, (i,i)]$ for every i, $0 \leq i \leq n$.
- the current chart contents can license an edge: a suitable pair of existing edges triggers the addition of a new edge

Chart parsers use a set of rules to heuristically decide when an edge should be added to a chart. This set of rules together with a specification of when they should be applied form a parsing strategy.

(3) **The Fundamental Rule ('Completion')**: if the chart contains the edges $[A \rightarrow \alpha \bullet B\beta, (i,j)]$ and $[B \rightarrow \gamma\bullet, (j,k)]$, then add a new edge $[A \rightarrow \alpha B \bullet \beta, (i,k)]$.

- this rule is used by every chart parser
- as per the usual conventions in formal language theory, $\alpha, \beta, \gamma, \cdots$ denote (possibly empty) sequences of terminals or non-terminals
- in the new edge, the dot has moved one place to the right
- the span of the new edge is the combined span of the original edges
- when we add this new edge, we do not remove the other two because they might be used again

6 Bottom-Up Chart Parsing

As we saw with the shift-reduce parser, bottom-up parsing starts from the input string and tries to find sequences of words and phrases that correspond to the right hand side of a grammar production.

The parser then replaces these with the left-hand side of the production, until the whole sentence is reduced to an S.

Bottom-up chart parsing is an extension of this approach in which hypotheses about structure are recorded as edges on a chart. Bottom-up chart parsing can be seen as a parsing strategy: bottom-up is a particular choice of heuristics for adding new edges to a chart.

The general procedure for chart parsing is inductive: we start with a base case, and then show how we can move from a given state of the chart to a new state. Since we are working bottom-up:

- **the base case** for our induction will be determined by the words in the input string, so we add new edges for each word
- **the induction step**: suppose the chart contains an edge labeled with constituent A; since we are working bottom-up, we want to build constituents that can have an A as a child; in other words, we are going to look for productions of the form $B \rightarrow A\beta$ and use these to label new edges

More formally, to create a bottom-up chart parser, we add two new rules to the Fundamental Rule:

- the Bottom-Up Initialization Rule
- the Bottom-Up Predict Rule.

(4) **The Bottom-Up Initialization Rule** says to add all edges licensed by the input: for every word w_i, add the edge $[w_i \rightarrow \bullet, (i, i+1)]$

Now suppose the chart contains a complete edge e whose left hand category is A. Then the Bottom-Up Predict Rule requires the parser to add a self-loop edge at the left boundary of e for each grammar production whose right hand side begins with category A.

(5) **The Bottom-Up Predict Rule**: for each complete edge $[A \rightarrow \alpha\bullet, (i, j)]$ and each production $B \rightarrow A\beta$, add the self-loop edge $[B \rightarrow \bullet A\beta, (i, i)]$

The next step is to use the Fundamental Rule to add edges like $[NP \rightarrow Mary\textbf{.}, (0,1)]$ where we have "moved the dot" one position to the right.

We will now be able to add new self-loop edges such as $[S \rightarrow \textbf{.}NPVP, (0,0)]$ and $[VP \rightarrow \textbf{.}VPNP, (1,1)]$, and use these to build more complete edges.

(6) Bottom-Up Strategy (summary):
   a. create an empty chart spanning the sentence
   b. apply the Bottom-Up Initialization Rule to each word
   c. do the following until no more edges are added:
      i. apply the Bottom-Up Predict Rule everywhere it applies
      ii. apply the Fundamental Rule everywhere it applies
      iii. return all of the parse trees corresponding to the parse edges in the chart

[**py27**]
```
>>> bu_chart_parser = nltk.BottomUpChartParser(grammar1, trace=2)
>>> sent
['Mary', 'saw', 'a', 'dog']
>>> bu_chart_parser.nbest_parse(sent)
|.   Mary   .   saw    .    a     .   dog    .|
Leaf Init Rule:
|[---------]         .          .          .| [0:1] 'Mary'
|.         [---------]          .          .| [1:2] 'saw'
|.         .         [---------]          .| [2:3] 'a'
|.         .         .          [---------]| [3:4] 'dog'
Bottom Up Predict Rule:
|>         .         .          .          .| [0:0] NP -> * 'Mary'
Single Edge Fundamental Rule:
|[---------]         .          .          .| [0:1] NP -> 'Mary' *
Bottom Up Predict Rule:
|>         .         .          .          .| [0:0] S  -> * NP VP
Single Edge Fundamental Rule:
|[--------->         .          .          .| [0:1] S  -> NP * VP
Bottom Up Predict Rule:
|.         >         .          .          .| [1:1] V  -> * 'saw'
Single Edge Fundamental Rule:
|.         [---------]          .          .| [1:2] V  -> 'saw' *
Bottom Up Predict Rule:
|.         >         .          .          .| [1:1] VP -> * V NP
|.         >         .          .          .| [1:1] VP -> * V NP PP
Single Edge Fundamental Rule:
|.         [--------->          .          .| [1:2] VP -> V * NP
|.         [--------->          .          .| [1:2] VP -> V * NP PP
Bottom Up Predict Rule:
|.         .         >          .          .| [2:2] Det -> * 'a'
Single Edge Fundamental Rule:
|.         .         [---------]          .| [2:3] Det -> 'a' *
Bottom Up Predict Rule:
```

```
|.         .         >         .         .| [2:2] NP -> * Det N
|.         .         >         .         .| [2:2] NP -> * Det N PP
Single Edge Fundamental Rule:
|.         .         [--------->         .| [2:3] NP -> Det * N
|.         .         [--------->         .| [2:3] NP -> Det * N PP
Bottom Up Predict Rule:
|.         .         .         >         .| [3:3] N  -> * 'dog'
Single Edge Fundamental Rule:
|.         .         .         [---------]| [3:4] N  -> 'dog' *
Single Edge Fundamental Rule:
|.         .         [-------------------]| [2:4] NP -> Det N *
|.         .         [------------------->| [2:4] NP -> Det N * PP
Bottom Up Predict Rule:
|.         .         >         .         .| [2:2] S  -> * NP VP
Single Edge Fundamental Rule:
|.         [-----------------------------]| [1:4] VP -> V NP *
|.         [----------------------------->| [1:4] VP -> V NP * PP
|.         .         [------------------->| [2:4] S  -> NP * VP
Single Edge Fundamental Rule:
|[=======================================]| [0:4] S  -> NP VP *
[Tree('S', [Tree('NP', ['Mary']), Tree('VP', [Tree('V', ['saw']), Tree('NP', [Tree('De
```

And here's an ambiguous example:

```
[py28] >>> groucho_grammar = nltk.parse_cfg("""
       ...     S -> NP VP
       ...     PP -> P NP
       ...     NP -> Det N | Det N PP | 'I'
       ...     VP -> V NP | VP PP
       ...     Det -> 'an' | 'my'
       ...     N -> 'elephant' | 'pajamas'
       ...     V -> 'shot'
       ...     P -> 'in'
       ... """)
```

```
[py29] >>> sent2 = ['I', 'shot', 'an', 'elephant', 'in', 'my', 'pajamas']
       >>> bu_chart_parser2 = nltk.BottomUpChartParser(groucho_grammar, trace=2)
       >>> trees = bu_chart_parser2.nbest_parse(sent2)
       |.  I   . shot.  an  .eleph.  in  .  my  .pajam.|
       Leaf Init Rule:
       |[-----]     .     .     .     .     .     .| [0:1] 'I'
       |.     [-----]     .     .     .     .     .| [1:2] 'shot'
       |.     .     [-----]     .     .     .     .| [2:3] 'an'
       |.     .     .     [-----]     .     .     .| [3:4] 'elephant'
       |.     .     .     .     [-----]     .     .| [4:5] 'in'
       |.     .     .     .     .     [-----]     .| [5:6] 'my'
       |.     .     .     .     .     .     [-----]| [6:7] 'pajamas'
```

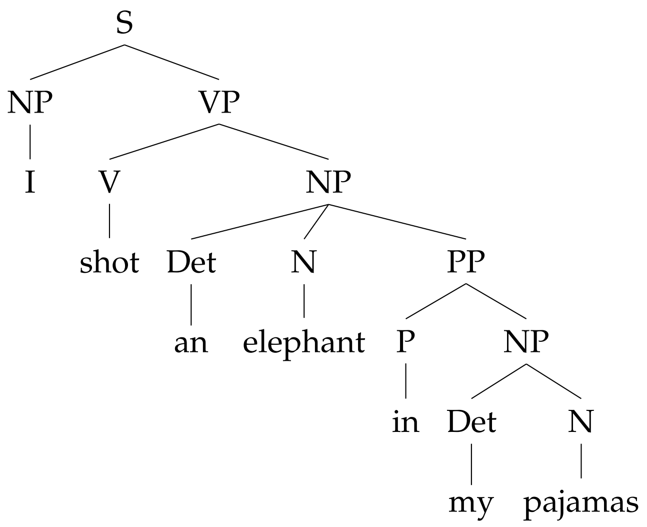
```
Bottom Up Predict Rule:
|>     .     .     .     .     .     .     .| [0:0] NP -> * 'I'
Single Edge Fundamental Rule:
|[-----]     .     .     .     .     .     .| [0:1] NP -> 'I' *
Bottom Up Predict Rule:
|>     .     .     .     .     .     .     .| [0:0] S  -> * NP VP
Single Edge Fundamental Rule:
|[----->     .     .     .     .     .     .| [0:1] S  -> NP * VP
Bottom Up Predict Rule:
|.     >     .     .     .     .     .     .| [1:1] V  -> * 'shot'
Single Edge Fundamental Rule:
|.     [-----]     .     .     .     .     .| [1:2] V  -> 'shot' *
Bottom Up Predict Rule:
|.     >     .     .     .     .     .     .| [1:1] VP -> * V NP
Single Edge Fundamental Rule:
|.     [----->     .     .     .     .     .| [1:2] VP -> V * NP
Bottom Up Predict Rule:
|.     .     >     .     .     .     .     .| [2:2] Det -> * 'an'
Single Edge Fundamental Rule:
|.     .     [-----]     .     .     .     .| [2:3] Det -> 'an' *
Bottom Up Predict Rule:
|.     .     >     .     .     .     .     .| [2:2] NP -> * Det N
|.     .     >     .     .     .     .     .| [2:2] NP -> * Det N PP
Single Edge Fundamental Rule:
|.     .     [----->     .     .     .     .| [2:3] NP -> Det * N
|.     .     [----->     .     .     .     .| [2:3] NP -> Det * N PP
Bottom Up Predict Rule:
|.     .     .     >     .     .     .     .| [3:3] N  -> * 'elephant'
Single Edge Fundamental Rule:
|.     .     .     [-----]     .     .     .| [3:4] N  -> 'elephant' *
Single Edge Fundamental Rule:
|.     .     [-----------]     .     .     .| [2:4] NP -> Det N *
|.     .     [----------->     .     .     .| [2:4] NP -> Det N * PP
Bottom Up Predict Rule:
|.     .     >     .     .     .     .     .| [2:2] S  -> * NP VP
Single Edge Fundamental Rule:
|.     [-----------------]     .     .     .| [1:4] VP -> V NP *
|.     .     [----------->     .     .     .| [2:4] S  -> NP * VP
Bottom Up Predict Rule:
|.     >     .     .     .     .     .     .| [1:1] VP -> * VP PP
Single Edge Fundamental Rule:
|[-----------------------]     .     .     .| [0:4] S  -> NP VP *
|.     [----------------->     .     .     .| [1:4] VP -> VP * PP
Bottom Up Predict Rule:
|.     .     .     .     >     .     .     .| [4:4] P  -> * 'in'
Single Edge Fundamental Rule:
|.     .     .     .     [-----]     .     .| [4:5] P  -> 'in' *
```

```
Bottom Up Predict Rule:
|.      .      .      .      >      .      .      .| [4:4] PP -> * P NP
Single Edge Fundamental Rule:
|.      .      .      .      [----->      .      .| [4:5] PP -> P * NP
Bottom Up Predict Rule:
|.      .      .      .      .      >      .      .| [5:5] Det -> * 'my'
Single Edge Fundamental Rule:
|.      .      .      .      .      [-----]      .| [5:6] Det -> 'my' *
Bottom Up Predict Rule:
|.      .      .      .      .      >      .      .| [5:5] NP -> * Det N
|.      .      .      .      .      >      .      .| [5:5] NP -> * Det N PP
Single Edge Fundamental Rule:
|.      .      .      .      .      [----->      .| [5:6] NP -> Det * N
|.      .      .      .      .      [----->      .| [5:6] NP -> Det * N PP
Bottom Up Predict Rule:
|.      .      .      .      .      .      >      .| [6:6] N  -> * 'pajamas'
Single Edge Fundamental Rule:
|.      .      .      .      .      .      [-----]| [6:7] N  -> 'pajamas' *
Single Edge Fundamental Rule:
|.      .      .      .      .      [-----------]| [5:7] NP -> Det N *
|.      .      .      .      .      [----------->| [5:7] NP -> Det N * PP
Bottom Up Predict Rule:
|.      .      .      .      .      >      .      .| [5:5] S  -> * NP VP
Single Edge Fundamental Rule:
|.      .      .      .      [-----------------]| [4:7] PP -> P NP *
|.      .      .      .      .      [----------->| [5:7] S  -> NP * VP
Single Edge Fundamental Rule:
|.      .      [-----------------------------]| [2:7] NP -> Det N PP *
|.      [-----------------------------------]| [1:7] VP -> VP PP *
Single Edge Fundamental Rule:
|[=========================================]| [0:7] S  -> NP VP *
|.      [----------------------------------->| [1:7] VP -> VP * PP
Single Edge Fundamental Rule:
|.      [-----------------------------------]| [1:7] VP -> V NP *
|.      .      [----------------------------->| [2:7] S  -> NP * VP
Single Edge Fundamental Rule:
|[=========================================]| [0:7] S  -> NP VP *
|.      [----------------------------------->| [1:7] VP -> VP * PP
>>> len(trees)
2
>>> for tree in trees:
...     print tree, "\n\n"
...
(S
  (NP I)
  (VP
    (V shot)
    (NP (Det an) (N elephant) (PP (P in) (NP (Det my) (N pajamas))))))
```

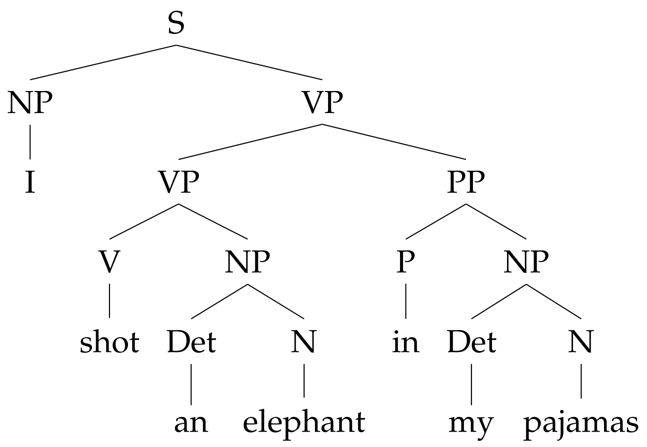
```
(S
  (NP I)
  (VP
    (VP (V shot) (NP (Det an) (N elephant)))
    (PP (P in) (NP (Det my) (N pajamas)))))
```

```
>>> with open("tree0.txt", "w") as file:
...     file.write(trees[0].pprint_latex_qtree())
...
>>> #trees[0].draw()
>>> with open("tree1.txt", "w") as file:
...     file.write(trees[1].pprint_latex_qtree())
...
>>> #trees[1].draw()
```

(7)



(8)



7 Top-down Chart Parsing

Top-down chart parsing works in a similar way to the recursive descent parser:

- it starts off with the top-level goal of finding an S
- this goal is broken down into the subgoals of trying to find constituents such as NP and VP predicted by the grammar

To create a top-down chart parser, we use the Fundamental Rule as before plus three other rules:

- the Top-Down Initialization Rule
- the Top-Down Expand Rule
- the Top-Down Match Rule

(9) **The Top-Down Initialization Rule** captures the fact that the root of any parse must be the start symbol S: for each production $S \rightarrow \alpha$, add the self-loop edge $[S \rightarrow \bullet\alpha, (0, 0)]$

In our running example, we are predicting that we will be able to find an NP and a VP starting at 0, but have not yet satisfied these subgoals.

In order to find an NP we need to invoke a production that has NP on its left hand side. This work is done by the Top-Down Expand Rule.

(10) **The Top-Down Expand Rule** tells us that if our chart contains an incomplete edge whose dot is followed by a nonterminal B, then the parser should add any self-loop edges licensed by the grammar whose left-hand side is B: for each incomplete edge $[A \rightarrow \alpha \bullet B\beta, (i, j)]$ and for each grammar production $B \rightarrow \gamma$, add the edge $[B \rightarrow \bullet\gamma, (j, j)]$

Finally:

(11) **The Top-Down Match Rule** allows the predictions of the grammar to be matched against the input string – if the chart contains an incomplete edge whose dot is followed by a terminal w, then the parser should add an edge if the terminal corresponds to the current input symbol: for each incomplete edge $[A \rightarrow \alpha \bullet w_j\beta, (i, j)]$, where w_j is the jth word of the input (counting from 0), add a new complete edge $[w_j \rightarrow \bullet, (j, j+1)]$.

(12) Top-Down Strategy (summary):

a. create an empty chart spanning the sentence
b. apply the Top-Down Initialization Rule (at node 0)
c. do until no more edges are added:
   i. apply the Top-Down Expand Rule everywhere it applies
   ii. apply the Top-Down Match Rule everywhere it applies
   iii. apply the Fundamental Rule everywhere it applies
d. return all of the parse trees corresponding to the parse edges in the chart

[**py30**]
```
>>> td_chart_parser = nltk.TopDownChartParser(grammar1, trace=2)
>>> td_chart_parser.nbest_parse(sent)
|.   Mary  .   saw   .    a    .   dog   .|
Leaf Init Rule:
|[---------]         .         .         .| [0:1] 'Mary'
|.         [---------]         .         .| [1:2] 'saw'
|.         .         [---------]         .| [2:3] 'a'
|.         .         .         [---------]| [3:4] 'dog'
Top Down Init Rule:
|>         .         .         .         .| [0:0] S  -> * NP VP
Cached Top Down Predict Rule:
|>         .         .         .         .| [0:0] NP -> * 'Mary'
|>         .         .         .         .| [0:0] NP -> * Det N
|>         .         .         .         .| [0:0] NP -> * Det N PP
Single Edge Fundamental Rule:
|[---------]         .         .         .| [0:1] NP -> 'Mary' *
Single Edge Fundamental Rule:
|[--------->         .         .         .| [0:1] S  -> NP * VP
Cached Top Down Predict Rule:
|.         >         .         .         .| [1:1] VP -> * V NP
|.         >         .         .         .| [1:1] VP -> * V NP PP
Cached Top Down Predict Rule:
|.         >         .         .         .| [1:1] V  -> * 'saw'
Single Edge Fundamental Rule:
|.         [---------]         .         .| [1:2] V  -> 'saw' *
Single Edge Fundamental Rule:
|.         [--------->         .         .| [1:2] VP -> V * NP
|.         [--------->         .         .| [1:2] VP -> V * NP PP
Cached Top Down Predict Rule:
|.         .         >         .         .| [2:2] NP -> * Det N
|.         .         >         .         .| [2:2] NP -> * Det N PP
Cached Top Down Predict Rule:
|.         .         >         .         .| [2:2] Det -> * 'a'
Single Edge Fundamental Rule:
|.         .         [---------]         .| [2:3] Det -> 'a' *
Single Edge Fundamental Rule:
|.         .         [--------->         .| [2:3] NP -> Det * N
|.         .         [--------->         .| [2:3] NP -> Det * N PP
Cached Top Down Predict Rule:
|.         .         .         >         .| [3:3] N  -> * 'dog'
Single Edge Fundamental Rule:
|.         .         .         [---------]| [3:4] N  -> 'dog' *
Single Edge Fundamental Rule:
|.         .         [-------------------]| [2:4] NP -> Det N *
|.         .         [------------------->| [2:4] NP -> Det N * PP
Cached Top Down Predict Rule:
|.         .         .         .         >| [4:4] PP -> * P NP
```

```
Single Edge Fundamental Rule:
|.         [-----------------------------]| [1:4] VP -> V NP *
|.         [----------------------------->| [1:4] VP -> V NP * PP
Single Edge Fundamental Rule:
|[=======================================]| [0:4] S  -> NP VP *
[Tree('S', [Tree('NP', ['Mary']), Tree('VP', [Tree('V', ['saw']), Tree('NP', [Tree('De
```

And here's the ambiguous example:

[**py31**]
```
>>> td_chart_parser2 = nltk.TopDownChartParser(groucho_grammar, trace=2)
>>> trees = td_chart_parser2.nbest_parse(sent2)
|.  I   . shot.  an  .eleph.  in  .  my  .pajam.|
Leaf Init Rule:
|[-----]     .     .     .     .     .     .| [0:1] 'I'
|.     [-----]     .     .     .     .     .| [1:2] 'shot'
|.     .     [-----]     .     .     .     .| [2:3] 'an'
|.     .     .     [-----]     .     .     .| [3:4] 'elephant'
|.     .     .     .     [-----]     .     .| [4:5] 'in'
|.     .     .     .     .     [-----]     .| [5:6] 'my'
|.     .     .     .     .     .     [-----]| [6:7] 'pajamas'
Top Down Init Rule:
|>     .     .     .     .     .     .     .| [0:0] S  -> * NP VP
Cached Top Down Predict Rule:
|>     .     .     .     .     .     .     .| [0:0] NP -> * Det N
|>     .     .     .     .     .     .     .| [0:0] NP -> * Det N PP
|>     .     .     .     .     .     .     .| [0:0] NP -> * 'I'
Single Edge Fundamental Rule:
|[-----]     .     .     .     .     .     .| [0:1] NP -> 'I' *
Single Edge Fundamental Rule:
|[----->     .     .     .     .     .     .| [0:1] S  -> NP * VP
Cached Top Down Predict Rule:
|.     >     .     .     .     .     .     .| [1:1] VP -> * V NP
|.     >     .     .     .     .     .     .| [1:1] VP -> * VP PP
Cached Top Down Predict Rule:
|.     >     .     .     .     .     .     .| [1:1] V  -> * 'shot'
Single Edge Fundamental Rule:
|.     [-----]     .     .     .     .     .| [1:2] V  -> 'shot' *
Single Edge Fundamental Rule:
|.     [----->     .     .     .     .     .| [1:2] VP -> V * NP
Cached Top Down Predict Rule:
|.     .     >     .     .     .     .     .| [2:2] NP -> * Det N
|.     .     >     .     .     .     .     .| [2:2] NP -> * Det N PP
Cached Top Down Predict Rule:
|.     .     >     .     .     .     .     .| [2:2] Det -> * 'an'
Single Edge Fundamental Rule:
|.     .     [-----]     .     .     .     .| [2:3] Det -> 'an' *
Single Edge Fundamental Rule:
```

```
|.     .     [----->     .     .     .     .| [2:3] NP -> Det * N
|.     .     [----->     .     .     .     .| [2:3] NP -> Det * N PP
Cached Top Down Predict Rule:
|.     .     .     >     .     .     .     .| [3:3] N  -> * 'elephant'
Single Edge Fundamental Rule:
|.     .     .     [-----]     .     .     .| [3:4] N  -> 'elephant' *
Single Edge Fundamental Rule:
|.     .     [-----------]     .     .     .| [2:4] NP -> Det N *
|.     .     [----------->     .     .     .| [2:4] NP -> Det N * PP
Cached Top Down Predict Rule:
|.     .     .     .     >     .     .     .| [4:4] PP -> * P NP
Cached Top Down Predict Rule:
|.     .     .     .     >     .     .     .| [4:4] P  -> * 'in'
Single Edge Fundamental Rule:
|.     .     .     .     [-----]     .     .| [4:5] P  -> 'in' *
Single Edge Fundamental Rule:
|.     .     .     .     [----->     .     .| [4:5] PP -> P * NP
Cached Top Down Predict Rule:
|.     .     .     .     .     >     .     .| [5:5] NP -> * Det N
|.     .     .     .     .     >     .     .| [5:5] NP -> * Det N PP
Cached Top Down Predict Rule:
|.     .     .     .     .     >     .     .| [5:5] Det -> * 'my'
Single Edge Fundamental Rule:
|.     .     .     .     .     [-----]     .| [5:6] Det -> 'my' *
Single Edge Fundamental Rule:
|.     .     .     .     .     [----->     .| [5:6] NP -> Det * N
|.     .     .     .     .     [----->     .| [5:6] NP -> Det * N PP
Cached Top Down Predict Rule:
|.     .     .     .     .     .     >     .| [6:6] N  -> * 'pajamas'
Single Edge Fundamental Rule:
|.     .     .     .     .     .     [-----]| [6:7] N  -> 'pajamas' *
Single Edge Fundamental Rule:
|.     .     .     .     .     [-----------]| [5:7] NP -> Det N *
|.     .     .     .     .     [----------->| [5:7] NP -> Det N * PP
Cached Top Down Predict Rule:
|.     .     .     .     .     .     .     >| [7:7] PP -> * P NP
Single Edge Fundamental Rule:
|.     .     .     .     [-----------------]| [4:7] PP -> P NP *
Single Edge Fundamental Rule:
|.     .     [-----------------------------]| [2:7] NP -> Det N PP *
Single Edge Fundamental Rule:
|.     [-----------------------------------]| [1:7] VP -> V NP *
Single Edge Fundamental Rule:
|[=========================================]| [0:7] S  -> NP VP *
|.     [----------------------------------->| [1:7] VP -> VP * PP
Single Edge Fundamental Rule:
|.     [-----------------]     .     .     .| [1:4] VP -> V NP *
```

```
Single Edge Fundamental Rule:
|[-----------------------]     .      .      .| [0:4] S  -> NP VP *
|.      [------------------>     .      .      .| [1:4] VP -> VP * PP
Single Edge Fundamental Rule:
|.      [-----------------------------------]| [1:7] VP -> VP PP *
Single Edge Fundamental Rule:
|[==========================================]| [0:7] S  -> NP VP *
|.      [----------------------------------->| [1:7] VP -> VP * PP
>>> len(trees)
2
>>> for tree in trees:
...     print tree, "\n\n"
...
(S
  (NP I)
  (VP
    (V shot)
    (NP (Det an) (N elephant) (PP (P in) (NP (Det my) (N pajamas))))))


(S
  (NP I)
  (VP
    (VP (V shot) (NP (Det an) (N elephant)))
    (PP (P in) (NP (Det my) (N pajamas)))))
```

8 The Earley Algorithm

The Earley algorithm (Earley 1970) is a parsing strategy that resembles the Top-Down Strategy, but deals more efficiently with matching against the input string. Here's the correspondence between the parsing rules introduced above and the rules used by the Earley algorithm:

Top-Down/Bottom-Up	Earley
Top-Down Initialization Rule, Top-Down Expand Rule	Predictor Rule
Top-Down/Bottom-Up Match Rule	Scanner Rule
Fundamental Rule	Completer Rule

[py32]
```
>>> e_chart_parser = nltk.EarleyChartParser(grammar1, trace=2)
>>> e_chart_parser.nbest_parse(sent)
|.   Mary  .   saw   .    a    .   dog   .|
Leaf Init Rule:
|[---------]         .         .         .| [0:1] 'Mary'
|.         [---------]         .         .| [1:2] 'saw'
|.         .         [---------]         .| [2:3] 'a'
|.         .         .         [---------]| [3:4] 'dog'
Top Down Init Rule:
```

```
|>         .         .         .         .| [0:0] S  -> * NP VP

* Processing queue: 0

Predictor Rule:
|>         .         .         .         .| [0:0] NP -> * 'Mary'
|>         .         .         .         .| [0:0] NP -> * Det N
|>         .         .         .         .| [0:0] NP -> * Det N PP

* Processing queue: 1

Scanner Rule:
|[---------]         .         .         .| [0:1] NP -> 'Mary' *
Completer Rule:
|[--------->         .         .         .| [0:1] S  -> NP * VP
Predictor Rule:
|.         >         .         .         .| [1:1] VP -> * V NP
|.         >         .         .         .| [1:1] VP -> * V NP PP
Predictor Rule:
|.         >         .         .         .| [1:1] V  -> * 'saw'

* Processing queue: 2

Scanner Rule:
|.         [---------]         .         .| [1:2] V  -> 'saw' *
Completer Rule:
|.         [--------->         .         .| [1:2] VP -> V * NP
|.         [--------->         .         .| [1:2] VP -> V * NP PP
Predictor Rule:
|.         .         >         .         .| [2:2] NP -> * Det N
|.         .         >         .         .| [2:2] NP -> * Det N PP
Predictor Rule:
|.         .         >         .         .| [2:2] Det -> * 'a'

* Processing queue: 3

Scanner Rule:
|.         .         [---------]         .| [2:3] Det -> 'a' *
Completer Rule:
|.         .         [--------->         .| [2:3] NP -> Det * N
|.         .         [--------->         .| [2:3] NP -> Det * N PP
Predictor Rule:
|.         .         .         >         .| [3:3] N  -> * 'dog'

* Processing queue: 4

Scanner Rule:
```

```
|.         .         .         [---------]| [3:4] N  -> 'dog' *
Completer Rule:
|.         .         [-------------------]| [2:4] NP -> Det N *
|.         .         [------------------->| [2:4] NP -> Det N * PP
Predictor Rule:
|.         .         .         .         >| [4:4] PP -> * P NP
Completer Rule:
|.         [-----------------------------]| [1:4] VP -> V NP *
|.         [----------------------------->| [1:4] VP -> V NP * PP
Completer Rule:
|[=======================================]| [0:4] S  -> NP VP *
[Tree('S', [Tree('NP', ['Mary']), Tree('VP', [Tree('V', ['saw']), Tree('NP', [Tree('De
```

And here's the ambiguous example:

[**py33**]
```
>>> e_chart_parser2 = nltk.EarleyChartParser(groucho_grammar, trace=2)
>>> trees = e_chart_parser2.nbest_parse(sent2)
|.  I  . shot.  an .eleph.  in .  my .pajam.|
Leaf Init Rule:
|[-----]     .     .     .     .     .     .| [0:1] 'I'
|.     [-----]     .     .     .     .     .| [1:2] 'shot'
|.     .     [-----]     .     .     .     .| [2:3] 'an'
|.     .     .     [-----]     .     .     .| [3:4] 'elephant'
|.     .     .     .     [-----]     .     .| [4:5] 'in'
|.     .     .     .     .     [-----]     .| [5:6] 'my'
|.     .     .     .     .     .     [-----]| [6:7] 'pajamas'
Top Down Init Rule:
|>     .     .     .     .     .     .     .| [0:0] S  -> * NP VP

* Processing queue: 0

Predictor Rule:
|>     .     .     .     .     .     .     .| [0:0] NP -> * Det N
|>     .     .     .     .     .     .     .| [0:0] NP -> * Det N PP
|>     .     .     .     .     .     .     .| [0:0] NP -> * 'I'

* Processing queue: 1

Scanner Rule:
|[-----]     .     .     .     .     .     .| [0:1] NP -> 'I' *
Completer Rule:
|[----->     .     .     .     .     .     .| [0:1] S  -> NP * VP
Predictor Rule:
|.     >     .     .     .     .     .     .| [1:1] VP -> * V NP
|.     >     .     .     .     .     .     .| [1:1] VP -> * VP PP
Predictor Rule:
|.     >     .     .     .     .     .     .| [1:1] V  -> * 'shot'
```

```
* Processing queue: 2

Scanner Rule:
|.     [-----]     .     .     .     .     .| [1:2] V  -> 'shot' *
Completer Rule:
|.     [----->     .     .     .     .     .| [1:2] VP -> V * NP
Predictor Rule:
|.     .     >     .     .     .     .     .| [2:2] NP -> * Det N
|.     .     >     .     .     .     .     .| [2:2] NP -> * Det N PP
Predictor Rule:
|.     .     >     .     .     .     .     .| [2:2] Det -> * 'an'

* Processing queue: 3

Scanner Rule:
|.     .     [-----]     .     .     .     .| [2:3] Det -> 'an' *
Completer Rule:
|.     .     [----->     .     .     .     .| [2:3] NP -> Det * N
|.     .     [----->     .     .     .     .| [2:3] NP -> Det * N PP
Predictor Rule:
|.     .     .     >     .     .     .     .| [3:3] N  -> * 'elephant'

* Processing queue: 4

Scanner Rule:
|.     .     .     [-----]     .     .     .| [3:4] N  -> 'elephant' *
Completer Rule:
|.     .     [-----------]     .     .     .| [2:4] NP -> Det N *
|.     .     [----------->     .     .     .| [2:4] NP -> Det N * PP
Predictor Rule:
|.     .     .     .     >     .     .     .| [4:4] PP -> * P NP
Predictor Rule:
|.     .     .     .     >     .     .     .| [4:4] P  -> * 'in'
Completer Rule:
|.     [-----------------]     .     .     .| [1:4] VP -> V NP *
Completer Rule:
|[-----------------------]     .     .     .| [0:4] S  -> NP VP *
|.     [----------------->     .     .     .| [1:4] VP -> VP * PP

* Processing queue: 5

Scanner Rule:
|.     .     .     .     [-----]     .     .| [4:5] P  -> 'in' *
Completer Rule:
|.     .     .     .     [----->     .     .| [4:5] PP -> P * NP
Predictor Rule:
```

```
|.     .     .     .     .     >     .     .| [5:5] NP -> * Det N
|.     .     .     .     .     >     .     .| [5:5] NP -> * Det N PP
Predictor Rule:
|.     .     .     .     .     >     .     .| [5:5] Det -> * 'my'

* Processing queue: 6

Scanner Rule:
|.     .     .     .     .     [-----]     .| [5:6] Det -> 'my' *
Completer Rule:
|.     .     .     .     .     [----->     .| [5:6] NP -> Det * N
|.     .     .     .     .     [----->     .| [5:6] NP -> Det * N PP
Predictor Rule:
|.     .     .     .     .     .     >     .| [6:6] N  -> * 'pajamas'

* Processing queue: 7

Scanner Rule:
|.     .     .     .     .     .     [-----]| [6:7] N  -> 'pajamas' *
Completer Rule:
|.     .     .     .     .     [-----------]| [5:7] NP -> Det N *
|.     .     .     .     .     [----------->| [5:7] NP -> Det N * PP
Predictor Rule:
|.     .     .     .     .     .     .     >| [7:7] PP -> * P NP
Completer Rule:
|.     .     .     .     [-----------------]| [4:7] PP -> P NP *
Completer Rule:
|.     .     [-----------------------------]| [2:7] NP -> Det N PP *
|.     [-----------------------------------]| [1:7] VP -> VP PP *
Completer Rule:
|[=========================================]| [0:7] S  -> NP VP *
|.     [----------------------------------->| [1:7] VP -> VP * PP
Completer Rule:
|.     [-----------------------------------]| [1:7] VP -> V NP *
Completer Rule:
|[=========================================]| [0:7] S  -> NP VP *
|.     [----------------------------------->| [1:7] VP -> VP * PP
>>> len(trees)
2
>>> for tree in trees:
...     print tree, "\n\n"
...
(S
  (NP I)
  (VP
    (V shot)
    (NP (Det an) (N elephant) (PP (P in) (NP (Det my) (N pajamas))))))
```

```
(S
  (NP I)
  (VP
    (VP (V shot) (NP (Det an) (N elephant)))
    (PP (P in) (NP (Det my) (N pajamas)))))
```

9 Back to Left-Corner chart parsing

[py34]
```
>>> lc_chart_parser = nltk.LeftCornerChartParser(grammar1, trace=2)
>>> lc_chart_parser.nbest_parse(sent)
|.   Mary   .   saw   .    a    .   dog   .|
Leaf Init Rule:
|[---------]         .         .         .| [0:1] 'Mary'
|.         [---------]         .         .| [1:2] 'saw'
|.         .         [---------]         .| [2:3] 'a'
|.         .         .         [---------]| [3:4] 'dog'
Filtered Bottom Up Predict Combine Rule:
|[---------]         .         .         .| [0:1] NP -> 'Mary' *
Filtered Bottom Up Predict Combine Rule:
|[--------->         .         .         .| [0:1] S  -> NP * VP
Filtered Bottom Up Predict Combine Rule:
|.         [---------]         .         .| [1:2] V  -> 'saw' *
Filtered Bottom Up Predict Combine Rule:
|.         [--------->         .         .| [1:2] VP -> V * NP
|.         [--------->         .         .| [1:2] VP -> V * NP PP
Filtered Bottom Up Predict Combine Rule:
|.         .         [---------]         .| [2:3] Det -> 'a' *
Filtered Bottom Up Predict Combine Rule:
|.         .         [--------->         .| [2:3] NP -> Det * N
|.         .         [--------->         .| [2:3] NP -> Det * N PP
Filtered Bottom Up Predict Combine Rule:
|.         .         .         [---------]| [3:4] N  -> 'dog' *
Filtered Single Edge Fundamental Rule:
|.         .         [-------------------]| [2:4] NP -> Det N *
Filtered Single Edge Fundamental Rule:
|.         [-----------------------------]| [1:4] VP -> V NP *
Filtered Single Edge Fundamental Rule:
|[=======================================]| [0:4] S  -> NP VP *
[Tree('S', [Tree('NP', ['Mary']), Tree('VP', [Tree('V', ['saw']), Tree('NP', [Tree('De
```

[py35]
```
>>> bulc_chart_parser = nltk.BottomUpLeftCornerChartParser(grammar1, trace=2)
>>> bulc_chart_parser.nbest_parse(sent)
|.   Mary   .   saw   .    a    .   dog   .|
Leaf Init Rule:
```

```
|[---------]         .         .         .| [0:1] 'Mary'
|.         [---------]         .         .| [1:2] 'saw'
|.         .         [---------]         .| [2:3] 'a'
|.         .         .         [---------]| [3:4] 'dog'
Bottom Up Predict Combine Rule:
|[---------]         .         .         .| [0:1] NP -> 'Mary' *
Bottom Up Predict Combine Rule:
|[--------->         .         .         .| [0:1] S  -> NP * VP
Bottom Up Predict Combine Rule:
|.         [---------]         .         .| [1:2] V  -> 'saw' *
Bottom Up Predict Combine Rule:
|.         [--------->         .         .| [1:2] VP -> V * NP
|.         [--------->         .         .| [1:2] VP -> V * NP PP
Bottom Up Predict Combine Rule:
|.         .         [---------]         .| [2:3] Det -> 'a' *
Bottom Up Predict Combine Rule:
|.         .         [--------->         .| [2:3] NP -> Det * N
|.         .         [--------->         .| [2:3] NP -> Det * N PP
Bottom Up Predict Combine Rule:
|.         .         .         [---------]| [3:4] N  -> 'dog' *
Single Edge Fundamental Rule:
|.         .         [-------------------]| [2:4] NP -> Det N *
|.         .         [------------------->| [2:4] NP -> Det N * PP
Bottom Up Predict Combine Rule:
|.         .         [------------------->| [2:4] S  -> NP * VP
Single Edge Fundamental Rule:
|.         [-----------------------------]| [1:4] VP -> V NP *
|.         [----------------------------->| [1:4] VP -> V NP * PP
Single Edge Fundamental Rule:
|[=======================================]| [0:4] S  -> NP VP *
[Tree('S', [Tree('NP', ['Mary']), Tree('VP', [Tree('V', ['saw']), Tree('NP', [Tree('De
```

And here's the ambiguous example:

[**py36**]
```
>>> lc_chart_parser2 = nltk.LeftCornerChartParser(groucho_grammar, trace=2)
>>> trees = lc_chart_parser2.nbest_parse(sent2)
|.  I   . shot.  an  .eleph.  in  .  my  .pajam.|
Leaf Init Rule:
|[-----]     .     .     .     .     .     .| [0:1] 'I'
|.     [-----]     .     .     .     .     .| [1:2] 'shot'
|.     .     [-----]     .     .     .     .| [2:3] 'an'
|.     .     .     [-----]     .     .     .| [3:4] 'elephant'
|.     .     .     .     [-----]     .     .| [4:5] 'in'
|.     .     .     .     .     [-----]     .| [5:6] 'my'
|.     .     .     .     .     .     [-----]| [6:7] 'pajamas'
Filtered Bottom Up Predict Combine Rule:
|[-----]     .     .     .     .     .     .| [0:1] NP -> 'I' *
```

```
Filtered Bottom Up Predict Combine Rule:
|[----->     .     .     .     .     .     .| [0:1] S  -> NP * VP
Filtered Bottom Up Predict Combine Rule:
|.     [-----]     .     .     .     .     .| [1:2] V  -> 'shot' *
Filtered Bottom Up Predict Combine Rule:
|.     [----->     .     .     .     .     .| [1:2] VP -> V * NP
Filtered Bottom Up Predict Combine Rule:
|.     .     [-----]     .     .     .     .| [2:3] Det -> 'an' *
Filtered Bottom Up Predict Combine Rule:
|.     .     [----->     .     .     .     .| [2:3] NP -> Det * N
|.     .     [----->     .     .     .     .| [2:3] NP -> Det * N PP
Filtered Bottom Up Predict Combine Rule:
|.     .     .     [-----]     .     .     .| [3:4] N  -> 'elephant' *
Filtered Single Edge Fundamental Rule:
|.     .     [-----------]     .     .     .| [2:4] NP -> Det N *
|.     .     [----------->     .     .     .| [2:4] NP -> Det N * PP
Filtered Single Edge Fundamental Rule:
|.     [-----------------]     .     .     .| [1:4] VP -> V NP *
Filtered Bottom Up Predict Combine Rule:
|.     [----------------->     .     .     .| [1:4] VP -> VP * PP
Filtered Single Edge Fundamental Rule:
|[-----------------------]     .     .     .| [0:4] S  -> NP VP *
Filtered Bottom Up Predict Combine Rule:
|.     .     .     .     [-----]     .     .| [4:5] P  -> 'in' *
Filtered Bottom Up Predict Combine Rule:
|.     .     .     .     [----->     .     .| [4:5] PP -> P * NP
Filtered Bottom Up Predict Combine Rule:
|.     .     .     .     .     [-----]     .| [5:6] Det -> 'my' *
Filtered Bottom Up Predict Combine Rule:
|.     .     .     .     .     [----->     .| [5:6] NP -> Det * N
|.     .     .     .     .     [----->     .| [5:6] NP -> Det * N PP
Filtered Bottom Up Predict Combine Rule:
|.     .     .     .     .     .     [-----]| [6:7] N  -> 'pajamas' *
Filtered Single Edge Fundamental Rule:
|.     .     .     .     .     [-----------]| [5:7] NP -> Det N *
Filtered Single Edge Fundamental Rule:
|.     .     .     .     [-----------------]| [4:7] PP -> P NP *
Filtered Single Edge Fundamental Rule:
|.     .     [-----------------------------]| [2:7] NP -> Det N PP *
|.     [-----------------------------------]| [1:7] VP -> VP PP *
Filtered Single Edge Fundamental Rule:
|[=========================================]| [0:7] S  -> NP VP *
Filtered Single Edge Fundamental Rule:
|.     [-----------------------------------]| [1:7] VP -> V NP *
Filtered Single Edge Fundamental Rule:
|[=========================================]| [0:7] S  -> NP VP *
>>> len(trees)
```

```
2
>>> for tree in trees:
...     print tree, "\n\n"
...
(S
  (NP I)
  (VP
    (V shot)
    (NP (Det an) (N elephant) (PP (P in) (NP (Det my) (N pajamas))))))


(S
  (NP I)
  (VP
    (VP (V shot) (NP (Det an) (N elephant)))
    (PP (P in) (NP (Det my) (N pajamas)))))
```

[**py37**]
```
>>> bulc_chart_parser2 = nltk.BottomUpLeftCornerChartParser(groucho_grammar, trace=2)
>>> trees = bulc_chart_parser2.nbest_parse(sent2)
|.  I   . shot.  an  .eleph.  in  .  my  .pajam.|
Leaf Init Rule:
|[-----]     .     .     .     .     .     .| [0:1] 'I'
|.     [-----]     .     .     .     .     .| [1:2] 'shot'
|.     .     [-----]     .     .     .     .| [2:3] 'an'
|.     .     .     [-----]     .     .     .| [3:4] 'elephant'
|.     .     .     .     [-----]     .     .| [4:5] 'in'
|.     .     .     .     .     [-----]     .| [5:6] 'my'
|.     .     .     .     .     .     [-----]| [6:7] 'pajamas'
Bottom Up Predict Combine Rule:
|[-----]     .     .     .     .     .     .| [0:1] NP -> 'I' *
Bottom Up Predict Combine Rule:
|[----->     .     .     .     .     .     .| [0:1] S  -> NP * VP
Bottom Up Predict Combine Rule:
|.     [-----]     .     .     .     .     .| [1:2] V  -> 'shot' *
Bottom Up Predict Combine Rule:
|.     [----->     .     .     .     .     .| [1:2] VP -> V * NP
Bottom Up Predict Combine Rule:
|.     .     [-----]     .     .     .     .| [2:3] Det -> 'an' *
Bottom Up Predict Combine Rule:
|.     .     [----->     .     .     .     .| [2:3] NP -> Det * N
|.     .     [----->     .     .     .     .| [2:3] NP -> Det * N PP
Bottom Up Predict Combine Rule:
|.     .     .     [-----]     .     .     .| [3:4] N  -> 'elephant' *
Single Edge Fundamental Rule:
|.     .     [-----------]     .     .     .| [2:4] NP -> Det N *
|.     .     [----------->     .     .     .| [2:4] NP -> Det N * PP
Bottom Up Predict Combine Rule:
```

```
|.      .     [----------->     .     .     .| [2:4] S  -> NP * VP
Single Edge Fundamental Rule:
|.     [-----------------]     .     .     .| [1:4] VP -> V NP *
Bottom Up Predict Combine Rule:
|.     [----------------->     .     .     .| [1:4] VP -> VP * PP
Single Edge Fundamental Rule:
|[-----------------------]     .     .     .| [0:4] S  -> NP VP *
Bottom Up Predict Combine Rule:
|.      .     .     .     [-----]     .     .| [4:5] P  -> 'in' *
Bottom Up Predict Combine Rule:
|.      .     .     .     [----->     .     .| [4:5] PP -> P * NP
Bottom Up Predict Combine Rule:
|.      .     .     .     .     [-----]     .| [5:6] Det -> 'my' *
Bottom Up Predict Combine Rule:
|.      .     .     .     .     [----->     .| [5:6] NP -> Det * N
|.      .     .     .     .     [----->     .| [5:6] NP -> Det * N PP
Bottom Up Predict Combine Rule:
|.      .     .     .     .     .     [-----]| [6:7] N  -> 'pajamas' *
Single Edge Fundamental Rule:
|.      .     .     .     .     [-----------]| [5:7] NP -> Det N *
|.      .     .     .     .     [----------->| [5:7] NP -> Det N * PP
Bottom Up Predict Combine Rule:
|.      .     .     .     .     [----------->| [5:7] S  -> NP * VP
Single Edge Fundamental Rule:
|.      .     .     .     [-----------------]| [4:7] PP -> P NP *
Single Edge Fundamental Rule:
|.      .     [-----------------------------]| [2:7] NP -> Det N PP *
|.     [-----------------------------------]| [1:7] VP -> VP PP *
Bottom Up Predict Combine Rule:
|.     [----------------------------------->| [1:7] VP -> VP * PP
Single Edge Fundamental Rule:
|[=========================================]| [0:7] S  -> NP VP *
Bottom Up Predict Combine Rule:
|.      .     [----------------------------->| [2:7] S  -> NP * VP
Single Edge Fundamental Rule:
|.     [-----------------------------------]| [1:7] VP -> V NP *
Bottom Up Predict Combine Rule:
|.     [----------------------------------->| [1:7] VP -> VP * PP
Single Edge Fundamental Rule:
|[=========================================]| [0:7] S  -> NP VP *
>>> len(trees)
2
>>> for tree in trees:
...     print tree, "\n\n"
...
(S
  (NP I)
```

```
    (VP
      (V shot)
      (NP (Det an) (N elephant) (PP (P in) (NP (Det my) (N pajamas))))))


  (S
    (NP I)
    (VP
      (VP (V shot) (NP (Det an) (N elephant)))
      (PP (P in) (NP (Det my) (N pajamas)))))
```

References

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