[redacted]

[redacted]

Note: This assignment requires that you know the lectures 7-8 from CS2810. You can rewatch the lectures at : https://www.youtube.com/playlist?list=PLdk2fd27CQzSoVKlPDFsjvRfDdjMpIMhR
You can also find the slides to the videos at:
https://drive.google.com/drive/folders/17iOdrT2TLHLQTUncR4atLMr7lX8KIamv?usp=sharing

Question 70pts. Given the following basis vectors and a target point y. For each problem, do it both by hand and by Python.

1) Identify if the target y is within the span of the provided basis vectors.
2) Calculate the normalized version of the basis.
3) Use the original basis as the column of a matrix A. Identify the matrix A.
   Note: You can stack vectors horizontally and vertically in python via

```
np.vstack((v1.T, v2.T))
np.hstack((v1, v2))
```

4) Use the original basis as the rows of a matrix B. Identify the matrix B.
5) Identify the null space of matrix A and B.
6) Using the columns of A as basis vectors, identify if these basis vectors are independent or dependent.
7) Using the columns of B as basis vectors, identify if these basis vectors are independent or dependent.
8) What is the dimension of the null space for A and B.
9) Identify the rank of matrix A and B.
   Note: The rank is the number of leading variables once you have identified the RREF.

$$a = \begin{bmatrix}7\\1\end{bmatrix}, b = \begin{bmatrix}0\\1\end{bmatrix} \xrightarrow{?} y = \begin{bmatrix}1\\8\end{bmatrix} \tag{1}$$

$$v_1 = \begin{bmatrix}0\\8\\0\\3\end{bmatrix} v_2 = \begin{bmatrix}0\\8\\1\\0\end{bmatrix} \xrightarrow{?} y = \begin{bmatrix}1\\0\\2\\2\end{bmatrix} \tag{2}$$

$$v_1 = \begin{bmatrix}1\\0\\0\end{bmatrix}, v_2 = \begin{bmatrix}1\\1\\0\end{bmatrix}, v_3 = \begin{bmatrix}1\\1\\1\end{bmatrix} \xrightarrow{?} y = \begin{bmatrix}1\\4\\2\end{bmatrix} \tag{3}$$

$$x_1 = \begin{bmatrix}1\\0\\0\end{bmatrix}, x_2 = \begin{bmatrix}3\\0\\0\end{bmatrix}, x_3 = \begin{bmatrix}0\\1\\0\end{bmatrix}, x_4 = \begin{bmatrix}1\\0\\1\end{bmatrix} x_5 = \begin{bmatrix}1\\1\\1\end{bmatrix} \xrightarrow{?} y = \begin{bmatrix}1\\2\\3\end{bmatrix} \tag{4}$$

$$x_1 = \begin{bmatrix}1\\0\\0\\0\\3\end{bmatrix}, x_2 = \begin{bmatrix}1\\0\\0\\1\\3\end{bmatrix} x_3 = \begin{bmatrix}0\\1\\0\\1\\3\end{bmatrix} x_4 = \begin{bmatrix}0\\0\\1\\1\\0\end{bmatrix} \xrightarrow{?} y = \begin{bmatrix}1\\0\\0\\1\\6\end{bmatrix} \tag{5}$$

$$x_1 = \begin{bmatrix}0\\1\\0\\0\\3\end{bmatrix}, x_2 = \begin{bmatrix}3\\0\\0\\2\\0\end{bmatrix} \xrightarrow{?} y = \begin{bmatrix}0\\0\\1\\1\\6\end{bmatrix} \tag{6}$$

Question 10pts. From the previous question, study the relationship between

1) The number of basis vectors.
2) The dimension of the null space.
3) The rank.

What is the equation that links all 3 concepts together?

Question 10pts. Given vectors $v_1, v_2, v_3, ...,$

- Write down the definition of their span.
- Describe in your own words what it means.
- Identify something in your life that the concept of span can describe.

Question 10pts.

- Write out the definition of linear independence.
- Describe in your own words what it means.
- Identify something in your life that the concept of linear dependence can describe.