[redacted]

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Question 1. (10 points) In the data folder, find and load the 2 files

```
stock_prediction_data.csv
stock_price.csv
```

This data predicts tomorrow's stock price difference given the previous day's data.

1) Use Preprocessing on the data
2) Perform regression on this dataset
   (a) Solve it with Sklearn library for linear regression.
   (b) Solve it by writing your own gradient descent algorithm in Python.
   (c) Solve it by writing your own closed-form solution in Python.
   (d) Print out the final total error for each method.
   (e) No need to include this portion in the homework, but do it for your own sanity check.
      - Print out the prediction of your function for every sample next to the true label
      - Compare them against the true labels. How good is your prediction?

Question 2. (10 points) Repeat the previous question but this time use Polynomial regression (2nd order).

Question 3. (10 points) I recorded this video to help you with this question.

`https://youtu.be/7FSIMC2xP2E?si=vt2nX0lak8E97zFf`

Given the data

$$\begin{bmatrix} x & y \\ 0 & 1 \\ 1 & 0 \\ 2 & 2 \\ 3 & -2 \end{bmatrix}.$$

If we assume that the function to predict y from x is a linear function then the function would like.

$$f(x) = ax + b$$

The goal is to use the data to identify the best a and b using the **Closed-Form solution**. Solve the question by hand as well as using Python.

Question 4. (20 points)

Let's say we have some $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, and the feature maps are

1) $\phi_1(x) = x_1$
2) $\phi_2(x) = x_2$
3) $\phi_3(x) = x_1 x_2^2$
4) $\phi_4(x) = x_1^3$
5) $\phi_5(x) = x_2^2$
6) $\phi_6(x) = 1$

Given the following data

$$\begin{bmatrix} x_1 & x_2 \\ 0 & 2 \\ 2 & -1 \\ -2 & 1 \\ 3 & 3 \\ 4 & 1 \end{bmatrix}.$$

Transform the data using the feature map

Question 5. (10 points) Given the probability table below where X represents the probability of you having a good or bad date and Y represents the potential topics you could talk about during a date.

	$x = good$	$x = bad$
$y = ex$	0	0.1
$y = food$	0.4	0.1
$y = travel$	0.2	0.1
$y = weather$	0.0	0.1

1) What is the probability of having a good date? bad date?
2) What is the probability that you would talk about food and have a good date?
3) What is the probability of you talking about food or travel?
4) What is the probability of you talking about travel **or** having a good date?
5) What is the probability of you having a bad date given you talk about your ex the whole time?
6) What is the probability of you having a good date given that you talk about the weather?
   - Solve this problem with conditional probability.
   - Solve this problem using Bayes' Rule.
   - Note: The solution should be identical.
7) Which topic should you talk about?

Question 6. (10 points) Go to the data folder under "probability distributions" and find the 3 csv file

```
SAT.csv
Lunch_wait_time.csv
Student_age.csv
```

For each file

1) Generate the histogram
2) Calculate the average value
3) Calculate the probability that someone scored higher than 1200 on SAT
4) Calculate the probability you have to wait more than 5 min.
5) Calculate the probability that someone is younger than 19.

Question 7. (20pts) Given the following distribution

$$p(x) = \begin{cases} x^2 - x + 1 & \forall\, 0 \leq x \leq b \\ 0 & \text{everywhere else} \end{cases}$$

Note: This expression says that between the interval of 0 and b, the probability distribution is $p(x) = x^2 - x + 1$ and $p(x) = 0$ everywhere else.

Given that the total probability is always 1, it implies that the area between 0 and b must add up to 1, implying that the integral between 0 and b must therefore also add up to 1 where

$$\int_0^b x^2 - x + 1 \, dx = 1.$$

1) Use Numpy to find the value of b?
   - Hints on how to solve this problem.
   - If you don't remember how to take integrals, make sure you watch my video on calculus refreshers.
     `https://youtu.be/N62Hdt5TV9g?si=Phynx6U_UeJRPEcS`
   - Note that if you have an expression $x^2 - x - 2 = 0$ (which we know the solution as $(x+1)(x-2) = 0$), we can solve a more complex problem with with numpy.root. Try to understand the following command.

- ```
  np.roots([1, -1, -2])
  >>>  array([ 2., -1.]
  ```

2) Solve the integral by hand and find the probability $p(0 \leq x \leq 0.3)$.

Question 8. (10 pts) Write out the entire derivation to show that

1)

$$\frac{1}{n}\sum_{i=1}^{n}(\phi(x_i)^\top w - y_i)^2 = \frac{1}{n}(\Phi w - y)^2 \quad \text{where} \quad \Phi = \begin{bmatrix} \phi(x_1)^\top \\ \phi(x_2)^\top \\ \cdots \end{bmatrix}, y = \begin{bmatrix} y_1 \\ y_2 \\ \cdots \end{bmatrix}.$$

2)

$$\frac{d}{dw}\frac{1}{n}\sum_{i}(\phi(x_i)^\top w - y_i)^2 = \frac{2}{n}\Phi^\top(\Phi w - y) \quad \text{where} \quad \Phi = \begin{bmatrix} \phi(x_1)^\top \\ \phi(x_2)^\top \\ \cdots \end{bmatrix}, y = \begin{bmatrix} y_1 \\ y_2 \\ \cdots \end{bmatrix}.$$

Hint:

$$\sum_{i}^{n} x_i^2 = \begin{bmatrix} x_1 & x_2 & x_3 & \ldots \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \cdots \end{bmatrix} = x^\top x.$$

$$\begin{bmatrix} \phi(x_1)^\top w - y_1 \\ \phi(x_2)^\top w - y_2 \\ \phi(x_3)^\top w - y_3 \\ \cdots \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} \phi(x_1)^\top \\ \phi(x_2)^\top \\ \phi(x_3)^\top \\ \cdots \end{bmatrix} w - \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \cdots \end{bmatrix} \end{bmatrix} = (\Phi w - y)$$

Note: Really try to do it yourself first, give it a good 10 min effort. If you are still stuck, study the following code `https://github.com/endsley/ml_examples/blob/master/gradient_descent/polyR_end_of_class.ipynb`
  ```