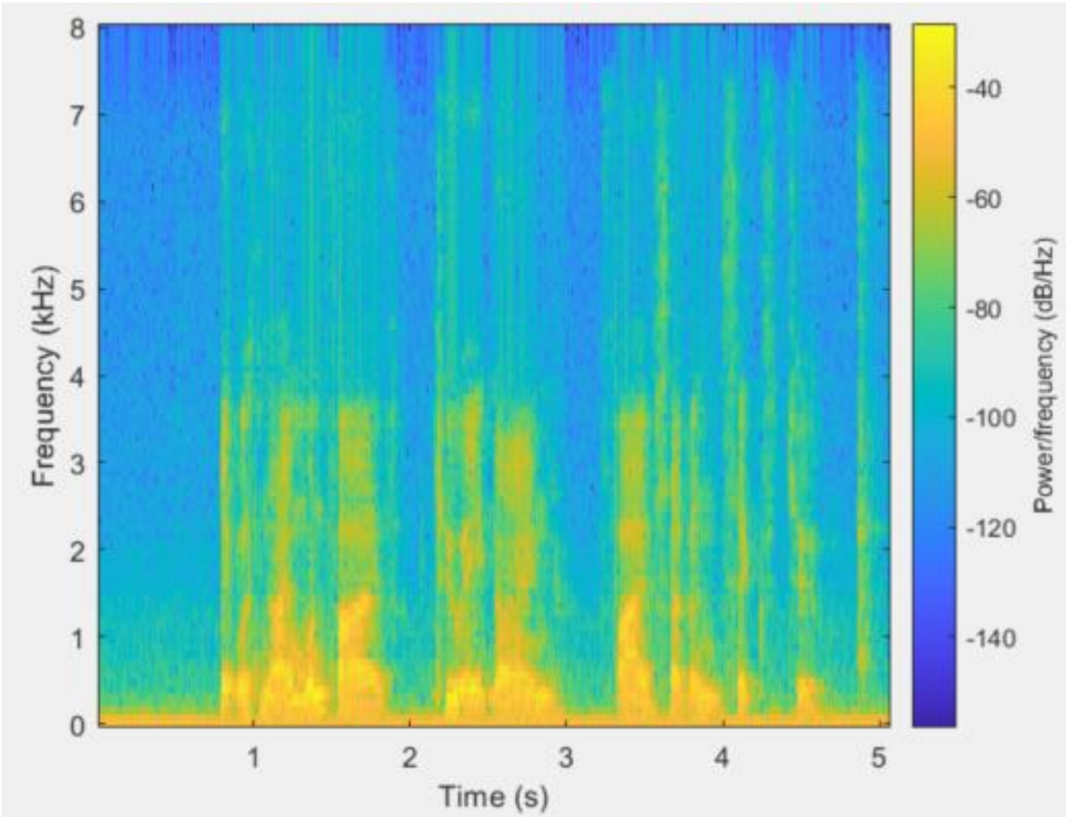
Problem 2

(a)

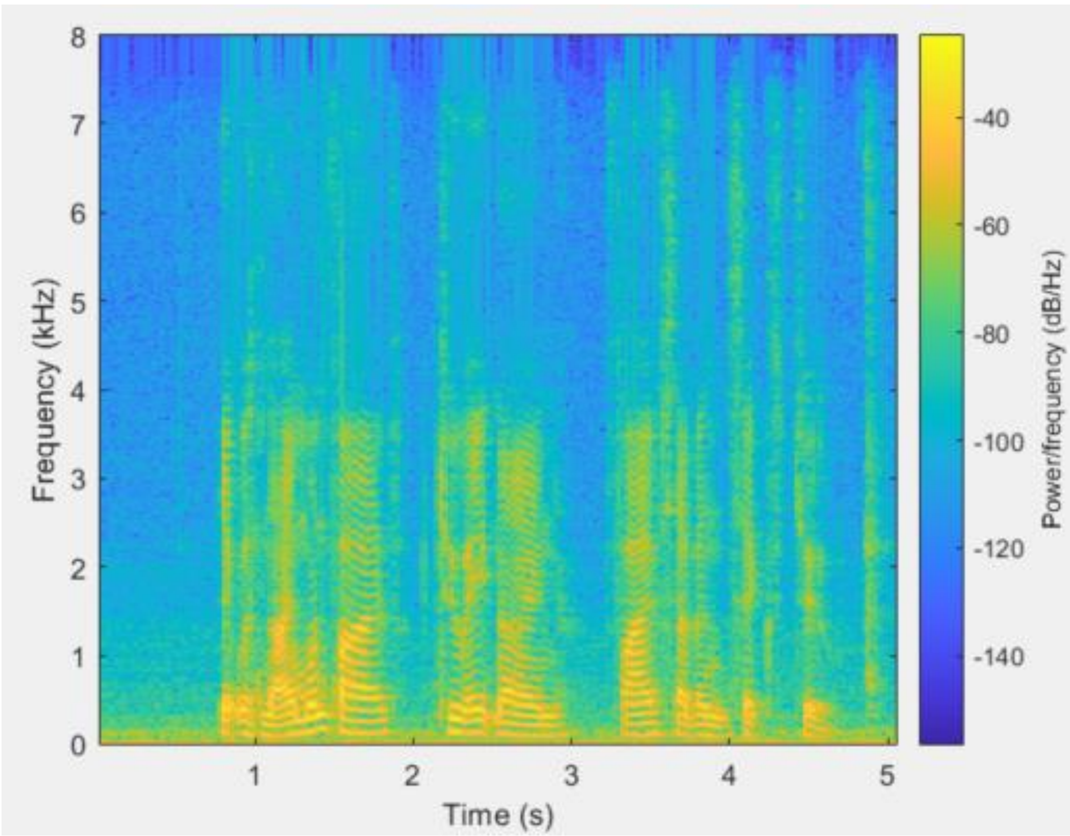
(a-1) Wideband Spectrogram




| | X | Window Length (Hamming) | Length of Overlap | f | fs |
|---|---|---|---|---|---|
| Chosen parameters | Provided audio signal | 200 | 100(i.e. 50%) | 200 | 16000 |

It has a good temporal resolution but poor frequency resolution.

(a-2) Narrowband Spectrogram

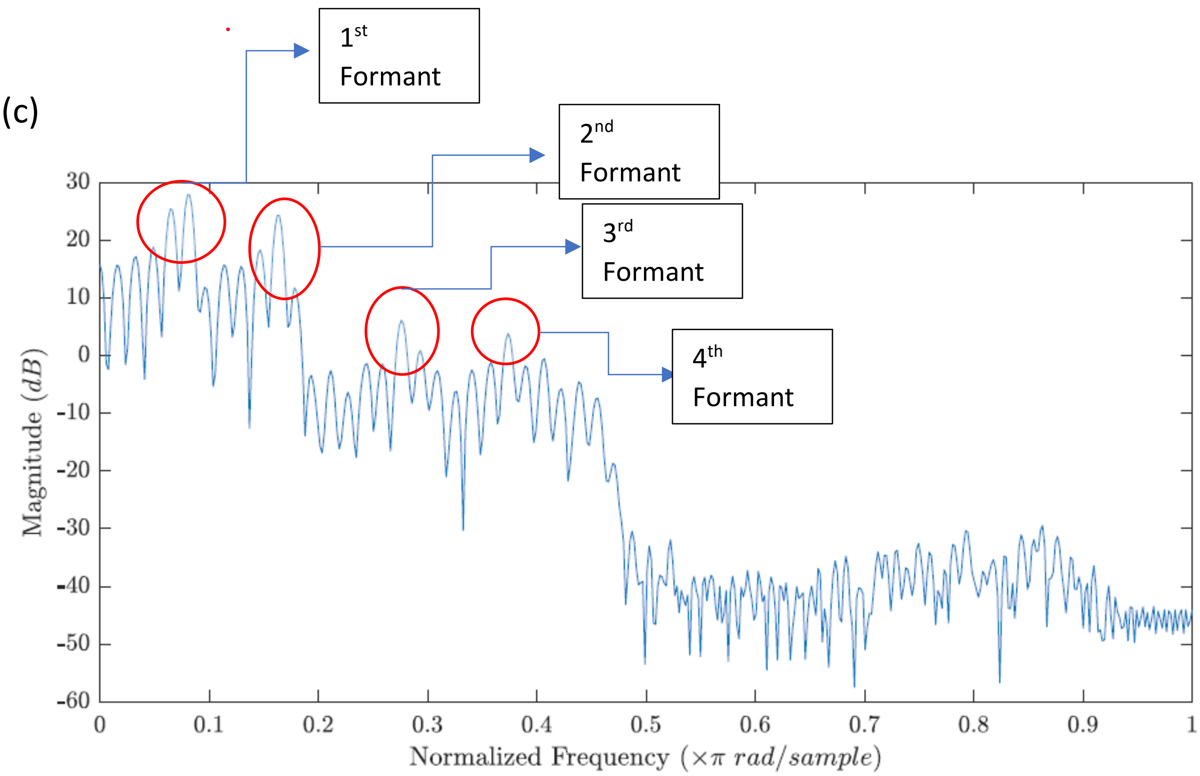



| | X | Window Length (Hamming) | Length of Overlap | f | fs |
|---|---|---|---|---|---|
| Chosen parameters | Provided audio signal | 800 | 400(i.e. 50%) | 800 | 16000 |

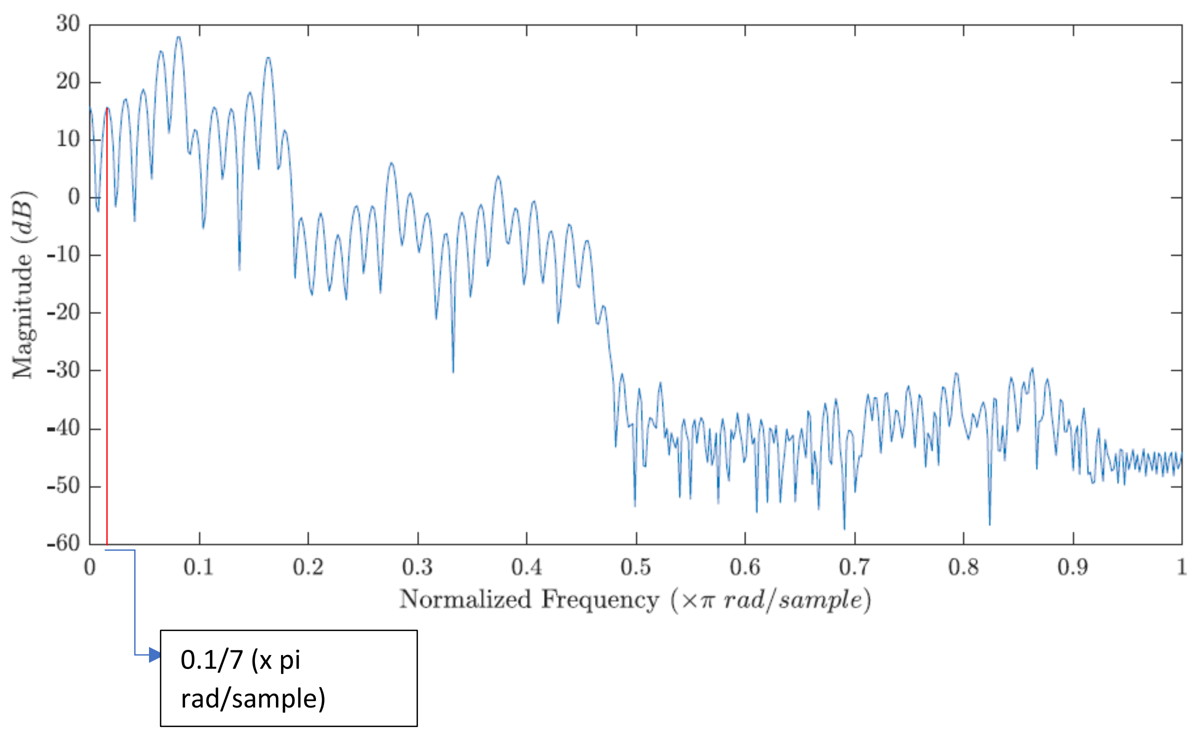
It has a good frequency resolution but poor temporal resolution.

(b) I think it is a part of a narrow band because it is showing a lot of dynamics in terms of frequency (i.e., good frequency resolution).

(c)



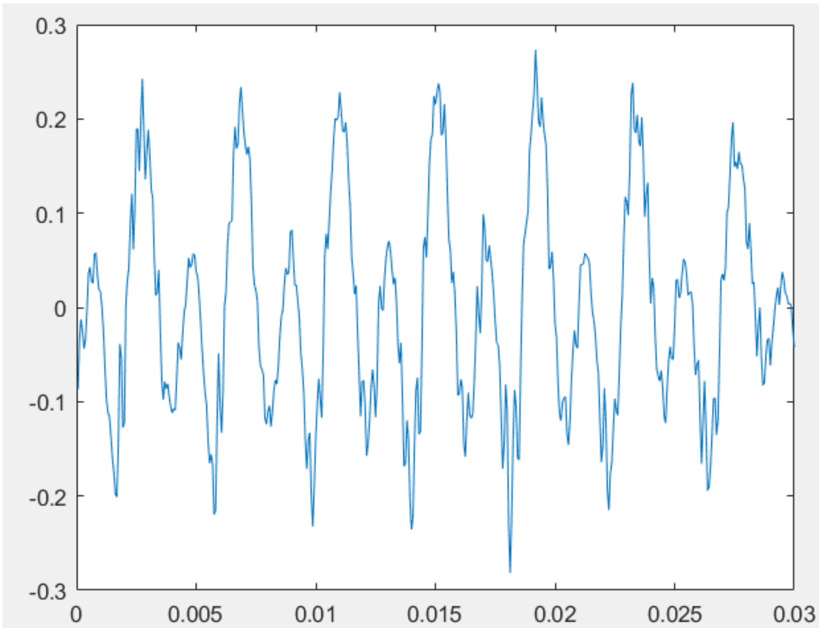

(d)



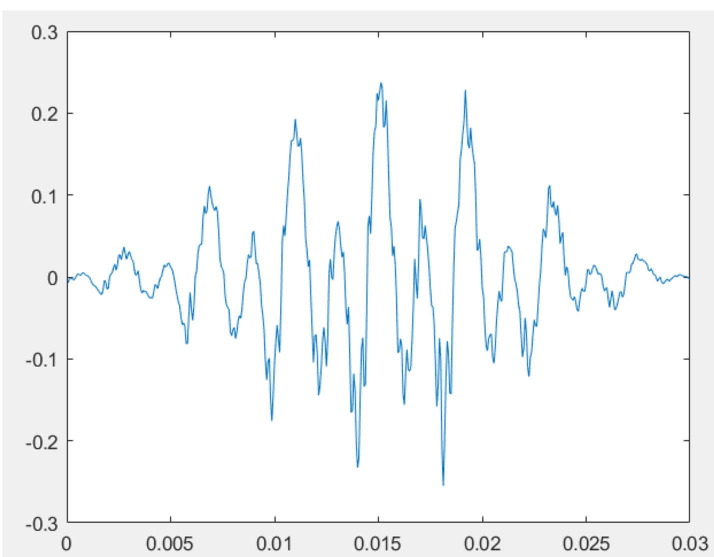

=>0.1*pi/7*16000/(2*pi) = 114.2857 Hz = fundamental frequency

Problem 4.

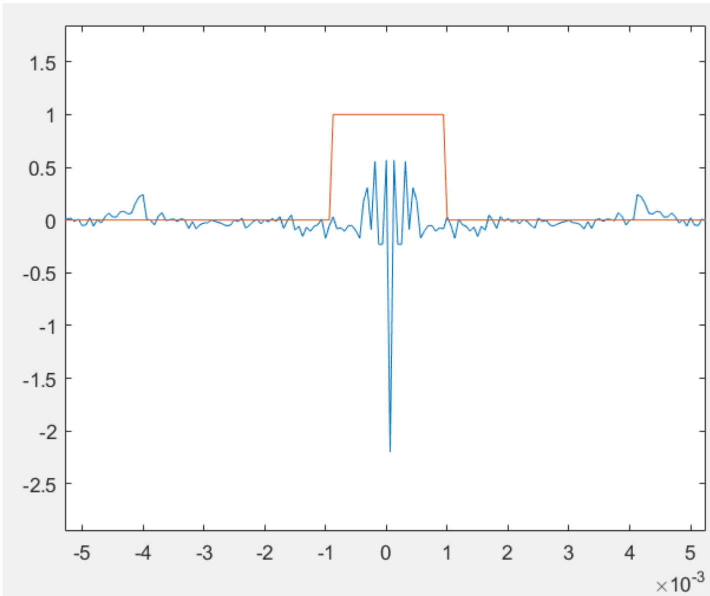
(i)Non-windowed signal s[n].




(ii)windowed signal x[n]



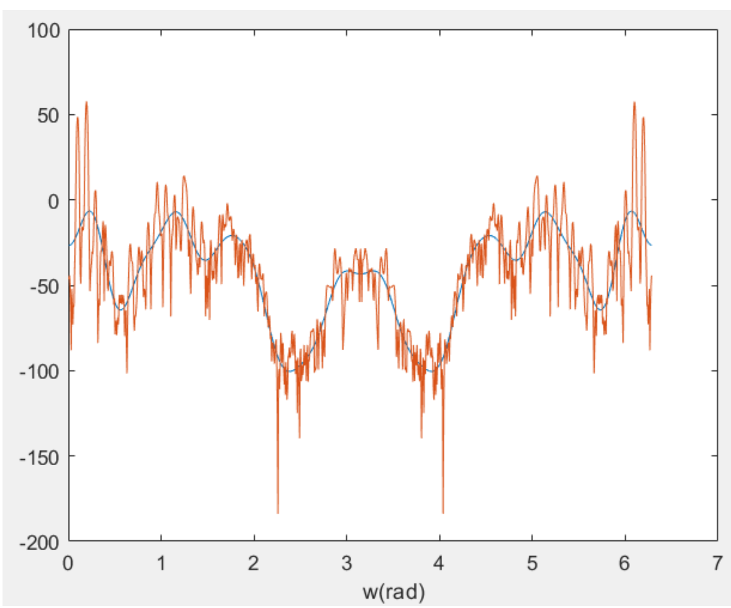
(iii)cepstrum c[n] with the cepstrum window $w_c$




$N' = 15$

Blue: c[n], Orange: window $w_c$

(iv)Spectrum of y[n] and spectrum of x[n] in dB




Blue: spectrum of y[n] in dB, Orange: spectrum of x[n] in dB

$N' = 15$

As $N'$ gets bigger but still less than L, envelope patterns become sharpener. Otherwise, envelope shapes become smoother.

In (i), we can find a fundamental period by picking 2 nearest peaks, subtract their index location and convert the result to proper time.

In (iii), we can pick location of the very first envelope and convert it to proper scale.

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Problem2

```
fileName = 'hw3_SpectralAnalysis.wav';
[audioSignal,Fs ]= audioread(fileName);
audioSignal = audioSignal' ; % make it 1xlength form

lengthOfHamming4Narrowband = 800;
lengthOfHamming4Wideband = 200;

%50 percent overlap
lengthOfOverlap4Narrowband = 400;
lengthOfOverlap4Wideband = 100;

%Wideband spectrogram
figure(1)
spectrogram(audioSignal,lengthOfHamming4Wideband,lengthOfOv
erlap4Wideband,lengthOfHamming4Wideband,Fs,'yaxis')

%Narrowband spectrogram
figure(2)
spectrogram(audioSignal, lengthOfHamming4Narrowband,
lengthOfOverlap4Narrowband,
lengthOfHamming4Narrowband,Fs,'yaxis')
```

Problem 4

```
fileName = 'hw2_TIMIT_LDC93S1.wav';
[audioSignal,Fs ]= audioread(fileName);
audioSignal = audioSignal' ; % make it 1xlength form

%Find abs maximum in signal and normalize
absMaxOfSignal = max(abs(audioSignal));

normalizedAudioSignal = audioSignal./absMaxOfSignal;

%Pick voiced sound segment based on results from HW2
lengthOfSegment = 0.03*Fs;
```

```
normalizedAudioSignalSegment =
normalizedAudioSignal(5200:5200+lengthOfSegment-1);

%Create continuous time for display purpose
Ts = 1/Fs;
continuousTimeArray = [1:lengthOfSegment].*Ts;

%Windowing signal segment
%Generate Hamming window
hammingWindow = hamming(lengthOfSegment);
hammingWindow = hammingWindow'; % transpose

%Window s[n]
windowed_normalizedAudioSignalSegment =
normalizedAudioSignalSegment .* hammingWindow;


%% First section of Homomorphic signal analysis for voiced
speech in Figure 2
numberOfDFT = 1024;
numberOfIDFT = 1024;
%Find DFT
dft_windowed_normalizedAudioSignalSegment =
fft(windowed_normalizedAudioSignalSegment,numberOfDFT);

%Calculate natural logarithm
LogSpectrum_absOfX =
log(abs(dft_windowed_normalizedAudioSignalSegment));

%Take inverse DFT to find cepstrum c[n]
c_n = ifft(LogSpectrum_absOfX,numberOfIDFT);

%Shift c_n properly then c_n can have negative time range.
shifted_c_n = ifftshift(c_n);

%Generate continuous time array for c_n
amountOfShift = 512;
continuousTimeArray_cepstrum = Ts.*[1:numberOfIDFT] -
(Ts*amountOfShift);


%% Second section of Homomorphic signal analysis for voiced
speech in Figure 2
```

```
lengthOfLifter = 30 ; %any number less than L = 66 in this
case.
halfOfLengthOfLifter = lengthOfLifter/2;

%initialize
cepstrumLifter = zeros(1,numberOfDFT);

cepstrumLifter(amountOfShift-
halfOfLengthOfLifter+1:amountOfShift+halfOfLengthOfLifter)
= rectwin(lengthOfLifter);

lifted_c_n = cepstrumLifter .* shifted_c_n; %

%Calculate dft of windowed c[n]
dft_lifted_c_n =
fft(fftshift(lifted_c_n),numberOfDFT); %need to be shifted
because lifted_c_n was shifted before.

%Convert back from log spectrum by taking exponential
originalSpectrum_lifted_x_n = exp(dft_lifted_c_n);

%Inverse DFT
y_n =
ifft(originalSpectrum_lifted_x_n,numberOfIDFT); %lifted x_n

%Calculate dft of y_n
dft_y_n = fft(y_n,numberOfDFT);

%frequency array for plotting
frequencyArray = [1:numberOfDFT] .*(2*pi/numberOfDFT);


%% plotting

figure(1)
plot(continuousTimeArray,normalizedAudioSignalSegment)

figure(2)
plot(continuousTimeArray,windowed_normalizedAudioSignalSegm
ent)

figure(3)
plot(continuousTimeArray_cepstrum,shifted_c_n)
hold on
plot(continuousTimeArray_cepstrum,cepstrumLifter)
```

```
hold off

figure(4)
plot(frequencyArray,20*log(abs(dft_y_n)))
xlabel('w(rad)')
hold on
plot(frequencyArray,20*log(abs(dft_windowed_normalizedAudio
SignalSegment)))
hold off
```

# EE 519 Home work 3

Problem 1.

(a) Sol)

$$R[-k] = \sum_{m=-\infty}^{\infty} x[m]\,w[n-m]\,x[m-k] \cdot w[n+k-m]$$

Let $-m+k = -m'$

$$= \sum_{m'=-\infty}^{\infty} x[m'+k]\,w[n-m'-k]\,x[m']\,w[n-m']$$

Let $m'$ be $m$

$$= \sum_{m=-\infty}^{\infty} x[m+k]\,w[n-m-k]\,x[m]\,w[n-m]$$

$$= R[k]$$

$\therefore$ $R[k]$ is an even function of $k$

Q.E.D

(b) Sol)

$$S_n(e^{j\omega}) = \left(X_n(e^{j\omega})\right)^* \cdot X_n(e^{j\omega})$$

$$= \left(\sum_{m=-\infty}^{\infty} w[n-m]\cdot x[m]\cdot e^{-j\omega m}\right)^* \cdot \left(\sum_{l=-\infty}^{\infty} w[n-l]\,x[l]\,e^{-j\omega l}\right)$$

$$= \left(\sum_{m=-\infty}^{\infty} w[n-m]\,x[m]\,e^{+j\omega m}\right) \cdot \left(\sum_{l=-\infty}^{\infty} w[n-l]\,x[l]\,e^{-j\omega l}\right) \quad \text{... eq(1)}$$

($\because$ $w[n]$ & $x[n]$ are real)

On the other hand,

$$FT[R[k]] = \sum_{k=-\infty}^{\infty} R_n[k]\,e^{-j\omega k}$$

$$= \sum_{k=-\infty}^{\infty} \left(\sum_{m=-\infty}^{\infty} x[m]\,w[n-m]\,x[m+k]\,w[n-k-m]\right) \cdot e^{-j\omega k}$$

~~switch order of sums~~

$$= \sum_{m=-\infty}^{\infty} x[m]\,w[n-m] \cdot \sum_{k=-\infty}^{\infty} x[m+k]\,w[n-k-m] \cdot e^{-j\omega k}$$

Let $m+k = l$

then, $k = l-m$, $-m-k = -l$

$$\Rightarrow \sum_{m=-\infty}^{\infty} x[m]\,w[n-m] \sum_{l=-\infty}^{\infty} x[l]\,w[n-l]\,e^{-j\omega l} \cdot e^{j\omega m}$$

$$= \sum_{m=-\infty}^{\infty} x[m]\,w[n-m] \cdot e^{j\omega m}, \quad \sum_{l=-\infty}^{\infty} x[l]\,w[n-l] \cdot e^{-j\omega l} \quad \text{... eq(2)}$$

change $m \to l$

$\Rightarrow$ eq(1) = eq(1)

$$\therefore S_n(e^{j\omega}) = FT[R[k]]$$

Q.E.D

(C) Sol)

Since we proved $R[k]$ is an even function of $k$

$$\Rightarrow R[k] = R[-k]$$

$$= \sum_{m=-\infty}^{\infty} x[m]\,w[n-m]\,x[m-k]\,w[n+k-m]$$

$$= \sum_{m=-\infty}^{\infty} x[m]\,x[m-k]\,\underbrace{w[n-m]\,w[n-m+k]}$$

This is $h_k[n-m]$ where

$$h_k[n] = w[n]\,w[n+k]$$

There fore

$$R_n[k] = \sum_{m=-\infty}^{\infty} x[m]x[m-k]h_k[n-m]$$

where $h_k[n] = w[n]w[n+k]$

(d) sol)

$$w[n] = \begin{cases} a^n & , \text{ if } n \ge 0 \\ 0 & , \text{ if } n < 0 \end{cases}$$

$$w[n+k] = \begin{cases} a^{n+k} & , \text{ if } n+k \ge 0 \\ 0 & , \text{ if } n+k < 0 \end{cases}$$

$$\Rightarrow h_k[n] = \begin{cases} a^n \cdot a^{n+k} & , \text{ if } n \ge 0 \\ 0 & , \text{ if } n < 0 \end{cases}$$

for $k \ge 0$

or

$$= \begin{cases} a^n \cdot a^{n+k} & , \text{ if } n \ge -k \\ 0 & , \text{ if } n < -k \end{cases}$$

for $k < 0$.

(e) sol) for $k \ge 0$

$$H_k(z) = \sum_{n=-\infty}^{\infty} h_k[n] \cdot z^{-n}$$

$$= \sum_{n=0}^{\infty} h_k[n] z^{-n} \quad \because h_k[n] = 0 \text{ for } n < 0$$

$$= \sum_{n=0}^{\infty} a^n \cdot a^{n+k} z^{-n}$$

$$= \sum_{n=0}^{\infty} a^k (a^2 z^{-1})^n$$

$$= \frac{a^k}{1 - a^2 z^{-1}} \quad \text{,,, (1)}$$

For $k \ge 0$

$$R_n[k] = \sum_{m=-\infty}^{\infty} x[m]\, x[m-k] h_k[n-m]$$

$$= \sum_{m=-\infty}^{\infty} s_k[m]\, h_k[n-m]$$

where $s_k[m] = x[m]\, x[m-k]$

$$= s_k[m] \circledast h_k[m]$$

$$\Rightarrow ZT[R_n[k]] = S_k(z) H_k(z)$$

$$\oplus \; ZT[R_{n-1}[k]] = S_k(z) H_k(z) z^{-1}$$

$$ZT[R_{n-2}[k]] = S_k(z) H_k(z) z^{-2}$$

$$\Rightarrow ZT[R_n[k]] = \frac{(ZT[R_{n-1}[k]])^2}{(ZT[R_{n-2}[k]])}$$

$\Rightarrow$ let

$$\therefore R_n[k] = ZT^{-1}\left( \frac{(ZT[R_{n-1}[k]])^2}{ZT[R_{n-2}[k]]} \right)$$

where

$$H_k(z) = \frac{a^k}{1 - a^2 z^{-1}}$$

(f)

$$w[n] = \begin{cases} n a^n & , \text{ if } n \ge 0 \\ 0 & , \text{ if } n < 0 \end{cases}$$

$$w[n+k] = \begin{cases} (n+k) a^{n+k} & , \text{ if } n+k \ge 0 \\ 0 & , \text{ if } n+k < 0 \end{cases}$$

$$\Rightarrow h_k[n] = \begin{cases} n(n+k) a^{n+k} & , \text{ if } n \ge 0 \\ 0 & , \text{ if } n < 0 \end{cases}$$

for $k \ge 0$

EE519 HW3

Problem 3. proof is closely followed from week 6 Lecture note.

sol)

$$\hat{h}[n] = ZT^{-1}[\log H(z)]$$

$$= ZT^{-1}\left[\log\left(\frac{G}{1-\sum_{k=1}^{p}\alpha_k z^{-k}}\right)\right]$$

$$= ZT^{-1}\left[\log G - \log\left(1-\sum_{k=1}^{p}\alpha_k z^{-k}\right)\right]$$

let $A(z) = 1-\sum_{k=1}^{p}\alpha_k z^{-k}$

$$= ZT^{-1}[\log G - \log(A(z))]$$

$$= \log G \cdot \delta[n] - \hat{a}[n]$$

Let's take a look at $\hat{a}[n]$

$A(z)$ is min phase

$\Rightarrow \hat{A}(z) = \log(A(z))$

$\Rightarrow$ Take derivatives.

$$\frac{d}{dz}\hat{A}(z) = \frac{1}{A(z)} \cdot \frac{dA(z)}{dz}$$

$$\Rightarrow -z\frac{d\hat{A}(z)}{dz} \cdot A(z) = -z\frac{dA(z)}{dz}$$

Take $Z^{-1}$

$\Rightarrow n\hat{a}[n] \circledast a[n] = na[n]$

$$\Rightarrow \sum_{k=-\infty}^{\infty} k\hat{a}[k]\cdot a[n-k] = na[n]$$

if $n \neq 0$

$$a[n] = \sum_{k=-\infty}^{\infty}\frac{k}{n}\hat{a}[k]\cdot a[n-k]$$

since $a[n]$ is min phase, previous summation is valid for $n-k \geq 0$ & $k \geq 0$

$\Leftrightarrow n \geq k \geq 0$

$$\Rightarrow \hat{a}[n] = \frac{a[n]}{a[0]} - \sum_{k=0}^{n-1}\frac{k}{n}\cdot\frac{a[n-k]}{a[0]}\cdot\hat{a}[k], \; n>0$$

$$= -\alpha_n - \sum_{k=1}^{n-1}\frac{k}{n}\alpha_{n-k}\hat{h}[k], \; n>0$$

since LPC filter causal

$\hat{h}[n] = 0$ for $n<0$

Therefore

$$\Rightarrow \hat{h}[n] = \begin{cases} 0, & \text{for } n<0 \\ \log G, & \text{for } n=0 \\ \alpha_n + \sum_{k=1}^{n-1}\frac{k}{n}\alpha_{n-k}\hat{h}[k], & \text{for } n>0 \end{cases}$$

problem 1 (f) continue.

for $k<0$

$$h_k[n] = \begin{cases} n(n+k)\, a^{n+k}, & \text{if } n \ge -k \\ 0, & \text{if } n < -k \end{cases}$$

$$\Rightarrow H_k(z) = \sum_{n=-\infty}^{\infty} h_k[n] z^{-n}$$

$$= \sum_{n=0}^{\infty} h_k[n] z^{-n} \quad \because h_k[n]=0 \text{ for } n<0$$

$$= \sum_{n=0}^{\infty} n(n+k)\, a^{2n+k} \cdot z^{-n}$$

Also,

$\Rightarrow R_n[k]$ is the same as $R_n[k]$ from (e)

$$\Rightarrow R_n[k] = ZT^{-1}\left(\frac{(ZT[R_{n-1}[k]])^2}{ZT[R_{n-2}[k]]}\right)$$

where $H_k(z) = \sum_{n=0}^{\infty} n(n+k)\, a^{2n+k} \cdot z^{-n}$

ii) To be periodic

$R_n[k] = R_n[k + rN_p]$

left hand side

$$= \sum_{m=-\infty}^{\infty} x[m]\, w[n-m]\, x[m+k]\, w[n-k-m]$$

Right hand side

$$= \sum_{m=-\infty}^{\infty} x[m]\, w[n-m]\, x[m+k+rN_p]$$

$$, w[n-(k+rN_p)-m]$$

$\Rightarrow w[n-(k+rN_p)-m] \neq w[n-k-m]$

$\Rightarrow$ non periodic Q.E.D

iii) To be periodic

$\phi[k+rN_p] = \phi[k]$

Right hand side

$\Rightarrow \phi[k+rN_p]$

$$= \lim_{L\to\infty} \frac{1}{2L+1} \sum_{m=-L}^{L} x[m]\, x[m+k+rN_p]$$

$$= \lim_{L\to\infty} \frac{1}{2L+1} \sum_{m=-L}^{L} x[m]\, x[m+k]$$

$\because x[n] = x[n+rN_p]$

$\Rightarrow \phi[k+rN_p] = \phi[k] \Rightarrow$ periodic

Q.E.D

(g) Sol)

i) $$\phi[k] = \lim_{L\to\infty} \frac{1}{2L+1} \sum_{m=-L}^{L} x[m]\, x[m+k]$$

$$\phi[-k] = \lim_{L\to\infty} \frac{1}{2L+1} \sum_{m=-L}^{L} x[m]\, x[m-k]$$

Let $m-k$ be $m'$

$$\Rightarrow \lim_{L\to\infty} \frac{1}{2L+1} \sum_{m'=-L-k}^{L-k} x[m'+k]\, x[m']$$

$\Rightarrow \phi[k] = \phi[-k] \Rightarrow$ even. Q.E.D