Homework1 · EE519.

Problem 1.

(a) sol:

$\Rightarrow |y[n]| = \left|\sum_{k=-\infty}^{\infty} h_k[n]\, x[k]\right|$ ... eq(1)

Now, assume $|x[n]| \le B_x < \infty, \forall n$

i.e., $x[n]$ is a bounded input.

$\Rightarrow$ eq(1) can be represented as follows

$$|y[n]| = \left|\sum_{k=-\infty}^{\infty} h_k[n]\, x[k]\right|$$
$$\le \sum_{k=-\infty}^{\infty} |h_k[n]|\,|x[k]|$$
$$\le B_x \cdot \underbrace{\sum_{k=-\infty}^{\infty} |h_k[n]|}_{<\infty} \quad \text{... eq(2)}$$

① If $\sum_{k=-\infty}^{\infty} |h_k[n]| < \infty$

Then $|y[n]| < \infty$

② If BIBO stable $\Rightarrow \sum |h_k[n]| < \infty\ \forall n$?

$\Leftrightarrow$ If $\sum |h_k[n]| = \infty\ \exists n$ then non BIBO stable

If $h_k[n]=1 \Rightarrow$ non BIBO stable

such n exists.

$\Rightarrow$ by ① & ② statements

The system is BIBO stable if and only if $\sum_{k=-\infty}^{\infty} |h_k[n]| < \infty$ $\forall n$

$\therefore$ Q.E.D

(b) It is worth reminding of meaning of causality

$\Rightarrow$ Causality: For $\forall k > n$, $y[n]$ is independent on $x[n]$

$\Rightarrow$ for $k>n$, $h[n-k] = 0 = h_k[n]$

Let's see if each case ① & ② below satisfy definition of causality.

① If $h_k[n] =$ for $n<k$

$\Rightarrow y[n]$ is determined regardless $x[n]$ for $n<k$

$\Rightarrow$ meaning it is causal

② If causal

$\Rightarrow y[n]$ is independent on $x[n]$ for $n<k$

$\Rightarrow$ meaning $h_k[n]$ for $n<k$ has to be zero.

$\Rightarrow$ By ① & ② case, the system is causal if and only if $h_k[n]=0$ for $n<k$

Q.E.D

Problem 2.

(a) $y[n] = \sin(2\pi x[n])$

sol)

(i) linear?

Let $x_1[n] \xrightarrow{H} y_1[n]$

$x_2[n] \xrightarrow{H} y_2[n]$

& $x_1[n]+x_2[n] \xrightarrow{H} y_3[n]$

For a system to be a linear system $y_3[n]$ should be equal to $y_1[n]+y_2[n]$

$\Rightarrow y_1[n] = \sin(2\pi x_1[n])$

$y_2[n] = \sin(2\pi x_2[n])$

$\Rightarrow y_3[n] = \sin(2\pi(x_1[n]+x_2[n])) \ne y_1[n]+y_2[n]$

$\therefore$ Non-linear system.

(ii) Time invariant ?

Let $x[n] \xrightarrow{H} y[n]$

$x_1[n] \xrightarrow{H} y_1[n]$

where $x_1[n] = x[n-n_0]$

$\Rightarrow$ see if $y_1[n] = y[n-n_0]$

① $y[n-n_0] = \sin(2\pi x[n-n_0])$

② $y_1[n] = \sin(2\pi x_1[n])$

$= \sin(2\pi x[n-n_0])$

$\Rightarrow y[n-n_0] = y_1[n]$

$\therefore$ Time Invariant system

(iii) Causal ?

$y[n] = \sin(2\pi x[n])$ is not depending on future value of $x[n]$

$\Rightarrow$ causal system

(iv) BIBO stable ?

For all possible real values for $x[n]$

$\Rightarrow$ $y[n]$ is within -1 & 1

$\Rightarrow$ BIBO stable system

(b) $y[n] = \sum_{k=n-n_0}^{n+n_0} x[k]$

(i) Linear ?

Let $x_1[n] \xrightarrow{H} y_1[n]$

$x_2[n] \xrightarrow{H} y_2[n]$

$x_3[n] = x_1[n] + x_2[n] \xrightarrow{H} y_3[n]$

See if $y_3[n] = y_1[n] + y_2[n]$

$\Rightarrow y_1[n] = \sum_{k=n-n_0}^{n+n_0} x_1[k]$

$y_2[n] = \sum_{k=n-n_0}^{n+n_0} x_2[k]$

$y_3[n] = \sum_{k=n-n_0}^{n+n_0} x_3[k]$

$= \sum_{k=n-n_0}^{n+n_0} (x_1[k] + x_2[k])$

$= y_1[n] + y_2[n]$

$\Rightarrow$ linear system

(ii) Time invariant

Let $x[n] \xrightarrow{H} y[n]$

$x_1[n] = x[n-n_1] \rightarrow y_1[n]$

see if $y_1[n] = y[n-n_1]$

$\Rightarrow y[n] = \sum_{k=n-n_0}^{n+n_0} x[k] \Rightarrow y[n-n_1] = \sum_{k=n-n_1-n_0}^{n-n_1+n_0} x[k]$ ...①

② $y_1[n] = \sum_{k=n-n_0}^{n+n_0} x_1[k]$

$= \sum_{k=n-n_0}^{n+n_0} x[k-n_1]$

① & ② are eventually same

Therefore

$y_1[n] = y[n-n_1]$

$\Rightarrow$ Time-Invariant system

(iii) Causal

$y[n]$ is dependent on future value of $x[n]$

e.g. $y[n]$ determined by $x[n+n_0]$ where $n_0 > 0$

$\Rightarrow$ Non causal

(IV) BIBO Stable?

If $x_1[k] \le B_k < \infty$

$\Rightarrow y_1[n] = \sum_{k=n-n_0}^{n+n_0} x_1[k]$

$\le \sum_{k=n-n_0}^{n+n_0} B_k < \infty$

because sum of finite terms.

$\Rightarrow$ BIBO stable system

(c) $y[n] = n\,x[n]$

(I) linear?

Let $x_1[n] \xrightarrow{H} y_1[n]$

$x_2[n] \xrightarrow{H} y_2[n]$

$x_3[n] = x_1[n] + x_2[n] \xrightarrow{H} y_3[n]$

see if $y_3[n] = y_1[n] + y_2[n]$

$\Rightarrow y_3[n] = n\,x_3[n] = n(x_1[n] + x_2[n])$ ...①

$y_1[n] + y_2[n] = n\,x_1[n] + n\,x_2[n]$ ...②

$\Rightarrow$ ① = ② $\Rightarrow y_3[n] = y_1[n] + y_2[n]$

$\Rightarrow$ linear system

(II) Time Invariant?

Let $x[n] \xrightarrow{H} y[n]$

$x_1[n] = x[n-n_0] \xrightarrow{H} y_1[n]$

see if $y_1[n] = y[n-n_0]$

$y[n-n_0] = (n-n_0)\,x[n-n_0]$ ...①

$y_1[n] = n\,x_1[n]$

$= n\,x[n-n_0]$ ...②

① ≠ ② $\Rightarrow$ Non time invariant

(III) Causal?

$y[n]$ only depends on current value of $x[n]$

$\Rightarrow$ causal

(IV) BIBO stable?

Counter example

$n \to \infty \Rightarrow y[n] \to \infty$ even though $x[n] < \infty$

$\Rightarrow$ Non BIBO stable.

(d) $y[n] = 0.5^{|n-1|}\,x[n-1]$

sol:

(I) linear?

let $x_1[n] \to y_1[n] = 0.5^{|n-1|}\,x_1[n-1]$

$x_2[n] \to y_2[n] = 0.5^{|n-1|}\,x_2[n-1]$

$x_3[n] = x_1[n] + x_2[n] \to y_3[n] = 0.5^{|n-1|}(x_3[n-1])$

$= 0.5^{|n-1|}(x_1[n-1] + x_2[n-1])$ ...①

what is $y_1[n] + y_2[n]$

$\Rightarrow y_1[n] + y_2[n] = 0.5^{|n-1|}\,x_1[n-1]$

$+ 0.5^{|n-1|}\,x_2[n-1]$

$= 0.5^{|n-1|}(x_1[n-1] + x_2[n-1])$

= ①

$\Rightarrow y_3[n] = y_1[n] + y_2[n]$

$\Rightarrow$ linear system.

(II) Time invariant?

Let $x[n] \to y[n] = 0.5^{|n-1|}\,x[n-1]$

$x_1[n] \to y_1[n] = 0.5^{|n-1|}\,x_1[n-1]$

$= x[n-n_0]$ $\quad = 0.5^{|n-1|}\,x[n-n_0-1]$

Is $y_1[n] = y[n-n_0]$?

$\Rightarrow y[n-n_0] = 0.5^{|n-n_0-1|}\, x[n-1-n_0]$

$\neq y_1[n]$

$\Rightarrow$ Non time invariant.

(iii) causal ?

$y[n]$ only depends on past value of $x[n]$.

$\Rightarrow$ causal system

(iv) BIBO stable ?

$0.5^{|n-1|} \leq B < \infty$ for $\forall n$

let $|x[n]| \leq B_x < \infty$ for $\forall n$ (Bounded input)

$\Rightarrow y[n] = 0.5^{|n-1|}\, x[n-1]$

$\leq 0.5^{|n-1|}\, |x[n-1]| \leq B \cdot B_x < \infty$

$\Rightarrow$ Bounded output

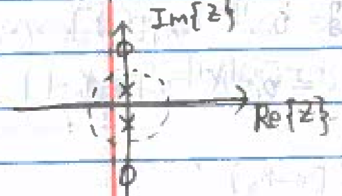
$\Rightarrow$ BIBO stable system.

Problem 3.

(a) (i) All-pass system

We can tell if a system is an all-pass system by checking if all poles & zeros are conjugate reciprocal to each other

ex)



all pass filter's zero pole plot

(ii) minimum phase system

We can call a system a minimum phase system if all zeros & poles of system are in the unit circle.

(b) causal LTI

$$H(z) = \frac{(1+0.2z^{-1})(1-9z^{-2})}{1+0.81z^{-2}}$$

$$\Rightarrow = \frac{(1+0.2z^{-1})(1-3z^{-1})(1+3z^{-1})}{(1-0.9iz^{-1})(1+0.9iz^{-1})}$$

can be decomposed as follows

$$= \frac{(1+0.2z^{-1})(1+\frac{1}{3}z^{-1})(1-\frac{1}{3}z^{-1})}{(1-0.9iz^{-1})(1+0.9iz^{-1})} \times \frac{(1+3z^{-1})(1-3z^{-1})}{(1+\frac{1}{3}z^{-1})(1-\frac{1}{3}z^{-1})}$$

$$= H_{min}(z) \times H_{ap}(z)$$

$$\Rightarrow H_{min}(z) = \frac{(1+0.2z^{-1})(1+\frac{1}{3}z^{-1})(1-\frac{1}{3}z^{-1})}{(1-0.9iz^{-1})(1+0.9iz^{-1})}$$

$$H_{ap}(z) = \frac{(1+3z^{-1})(1-3z^{-1})}{(1+\frac{1}{3}z^{-1})(1-\frac{1}{3}z^{-1})}$$

Problem 4

(a)

sol)

LTI system

$$x[n] \xrightarrow{T_1} \boxed{h_1[n]} \xrightarrow{w[n]} \xrightarrow{T_2} \boxed{h_2[n]} \longrightarrow y[n]$$

$\Rightarrow$ $y[n] = w[n] \circledast h_2[n] \quad \cdots ①$

$w[n] = x[n] \circledast h_1[n] \quad \cdots ②$

by ① & ②

$\Rightarrow$ $y[n] = x[n] \circledast h_1[n] \circledast h_2[n]$

$= x[n] \circledast (h_1[n] \circledast h_2[n])$

$= x[n] \circledast h[n]$

$\Rightarrow$ $h[n] = h_1[n] \circledast h_2[n]$

(b) Let's use definition of convolution

$\Rightarrow$ $h_1[n] \circledast h_2[n] = \sum_{k=-\infty}^{\infty} h_1[k] h_2[n-k]$

Let $n-k = k'$

$\Rightarrow$ $\sum_{k'=-\infty}^{\infty} h_1[n-k'] h_2[k']$

Let $k' = k$

$\Rightarrow$ $\sum_{k=-\infty}^{\infty} h_1[n-k] h_2[k]$

$= h_2[n] \circledast h_1[n]$

$\Rightarrow$ $h_2[n] \circledast h_1[n] = h_1[n] \circledast h_2[n]$

(c)

$$H(z) = \left(\sum_{r=0}^{M} b_r z^{-r} \right) \left(\frac{1}{1 - \sum_{k=1}^{N} a_k z^{-k}} \right)$$

$$= H_1(z) H_2(z)$$

sol)

$$\Rightarrow \frac{Y(z)}{X(z)} = \left(\sum_{r=0}^{M} b_r z^{-r} \right) \left(\frac{1}{1 - \sum_{k=1}^{N} a_k z^{-k}} \right)$$

$$= \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_M z^{-M}}{1 - (a_1 z^{-1} + a_2 z^{-2} + \cdots + a_N z^{-N})}$$

$$\Rightarrow Y(z) \{ 1 - (a_1 z^{-1} + a_2 z^{-2} + \cdots + a_N z^{-N}) \}$$

$$= X(z) \{ b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_M z^{-M} \}$$

Inverse z-transform

$\Rightarrow$ Difference equation of the system

$$y[n] - a_1 y[n-1] - a_2 y[n-2] - \cdots - a_N y[n-N]$$

$$= b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] + \cdots + b_M x[n-M]$$

Problem 5.

(a)

sol) Let

$$\tilde{X}[k] = \sum_{m=-\infty}^{\infty} x[m] e^{-j\frac{2\pi}{N}km} \quad \cdots eq(1)$$

$$\tilde{x}[n] = \frac{1}{N} \sum_{k=0}^{N-1} \tilde{X}[k] e^{j\frac{2\pi}{N}km} \quad \cdots eq(2)$$

plug in eq(1) to (2)

$\Rightarrow$ continue on the next page.

$$\Rightarrow \tilde{x}[n] = \frac{1}{N}\sum_{k=0}^{N-1}\left(\sum_{m=-\infty}^{\infty} x[m] e^{-j\frac{2\pi}{N}km}\right)\cdot e^{j\frac{2\pi}{N}kn}$$

Assume all summations converge.

$$= \sum_{m=-\infty}^{\infty} x[m]\cdot \underbrace{\frac{1}{N}\sum_{k=0}^{N-1} e^{-j\frac{2\pi}{N}k(m-n)}}$$

This term has a spetial representation

$$\Rightarrow \frac{1}{N}\sum_{k=0}^{N-1} e^{+j\frac{2\pi}{N}k(m+n)} = \begin{cases} 1 & \text{for } m = n+rN \\ & r: \text{integer} \\ 0 & \text{, otherwise} \end{cases}$$

$\Rightarrow$ Let $m = n + rN$

$\Rightarrow rN = m - n$

$\Rightarrow r = \frac{m-n}{N} \Rightarrow$ r's range also $[-\infty, \infty]$

$$\Rightarrow \sum_{r=-\infty}^{\infty} x[n+rN]\cdot 1$$

$$\therefore \tilde{x}[n] = \sum_{r=-\infty}^{\infty} x[n+rN]$$

Q.E.D.

(b)

Sol)

$$Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} y[n] e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} x[nM] e^{-j\omega n}$$

Let $nM = k$

$$= \sum_{k=-\infty}^{\infty} x[k]\,\delta[\langle k\rangle_M]\, e^{-j\frac{\omega k}{M}}$$

Let $k = n$

$$= \sum_{n=-\infty}^{\infty} x[n]\,\delta[\langle n\rangle_M]\, e^{-j\frac{\omega n}{M}}$$

fact $\frac{1}{M}\sum_{k=0}^{M-1} e^{\frac{j2\pi kn}{M}} = \begin{cases} \frac{1}{M}\cdot M = 1, & \text{for } n = lM \\ 0, & n \neq lM \end{cases}$

use fact

$$= \sum_{n=-\infty}^{\infty} x[n]\cdot\frac{1}{M}\sum_{k=0}^{M-1} e^{\frac{j2\pi kn}{M}}, e^{-\frac{j\omega n}{M}}$$

$$= \frac{1}{M}\sum_{k=0}^{M-1}\sum_{n=-\infty}^{\infty} x[n] e^{\frac{-jn(\omega-2\pi k)}{M}}$$

$$= \frac{1}{M}\sum_{k=0}^{M-1} X\left(e^{j\left(\frac{\omega-2\pi k}{M}\right)}\right)$$

Q.E.D.

Problem 6.

(a) Please see my source code in my report / or you can see matlab files as well

(b)

$$r[n] = \begin{cases} 1, & 0 \le n \le L-1 \\ 0, & \text{otherwise} \end{cases}$$

$$= u[n] - u[n-L]$$

$$R(e^{j\omega}) = \sum_{n=-\infty}^{\infty} r[n] e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} u[n] - \sum_{n=-\infty}^{\infty} u[n-L]$$

$(\because)\ \sum_{n=-\infty}^{\infty} u[n] = \frac{1}{1-e^{-j\omega}}$

$$= \frac{1}{1-e^{-j\omega}} - e^{-jL\omega}\cdot\left(\frac{1}{1-e^{-j\omega}}\right)$$

(DTFT shift property)

$$= \frac{1}{(1-e^{-j\omega})}\cdot\left(1-e^{-jL\omega}\right)$$

$$= \frac{\left(e^{+j\frac{L}{2}\omega} - e^{-j\frac{L}{2}\omega}\right)\cdot e^{-j\frac{L}{2}\omega}}{\left(e^{j\frac{\omega}{2}} - e^{-j\frac{\omega}{2}}\right)\cdot e^{-j\frac{\omega}{2}}}$$

$$= \frac{\sin(\omega L/2)}{\sin(\omega/2)}\cdot e^{-j\omega(L-1)/2}$$

Q.E.D

($\because$ Euler's formula)

⊕ Plotting part is in my report.

(c)

When $L=31$, height of the main lobe increases & number of positions of the points where the magnitude is equal to 0 increases as well.

In theory,

maximum of $R(e^{j\omega})$ is L

As we observe, $\max|R(e^{j\omega})|=24$ with $L=24$

& $\max|R(e^{j\omega})|=31$ with $L=31$.

Also, as L increases term $\omega L/2$ increases meaning frequency of zero increases.

Therefore, pattern we could find in (c) & (b) agrees with theory.

problem 7.

(a) sol)

$$d(t) = \sin(\omega_1 t)+\sin(\omega_2 t)$$
$$= \sin(2\pi f_1 t)+\sin(2\pi f_1 t)$$
$$(\because \omega = 2\pi f \ [rad/sec]).$$

$$\Rightarrow d[n] = d(nT_s) \quad (\because t=nT_s)$$

where $T_s = \frac{1}{8000}$, $f_s = 8000Hz$

$$\Rightarrow d_0[n] = \sin\left(2\pi \cdot 941 \cdot \frac{1}{8000} \cdot n\right) + \sin\left(2\pi \cdot 1336 \cdot \frac{1}{8000} \cdot n\right)$$
$$= \sin(0.7391 n) + \sin(1.0493 n)$$

$$d_2[n] = \sin\left(2\pi \cdot 697 \cdot \frac{1}{8000} n\right) + \sin\left(2\pi \cdot 1336 \cdot \frac{1}{8000} \cdot n\right)$$
$$= \sin(0.5474 n) + \sin(1.0493 n)$$

(b) please check my answer in the report.

(c) ~ (e)

please check my report.

Samwoo Seong
7953-6137-66
samwoose@usc.edu

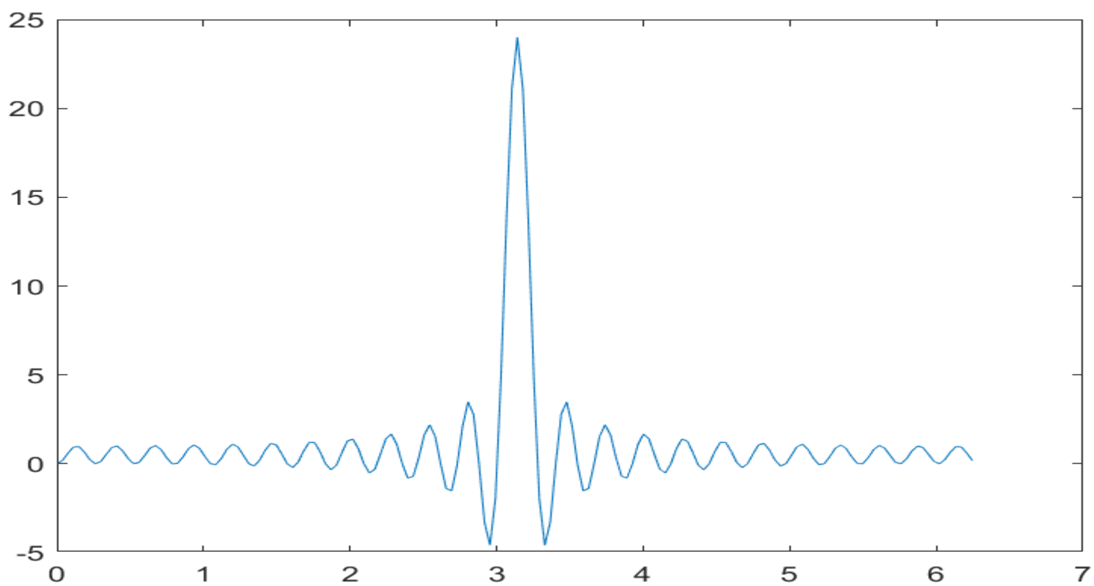
EE519 Homework #1
January 30, 2020

Problem6

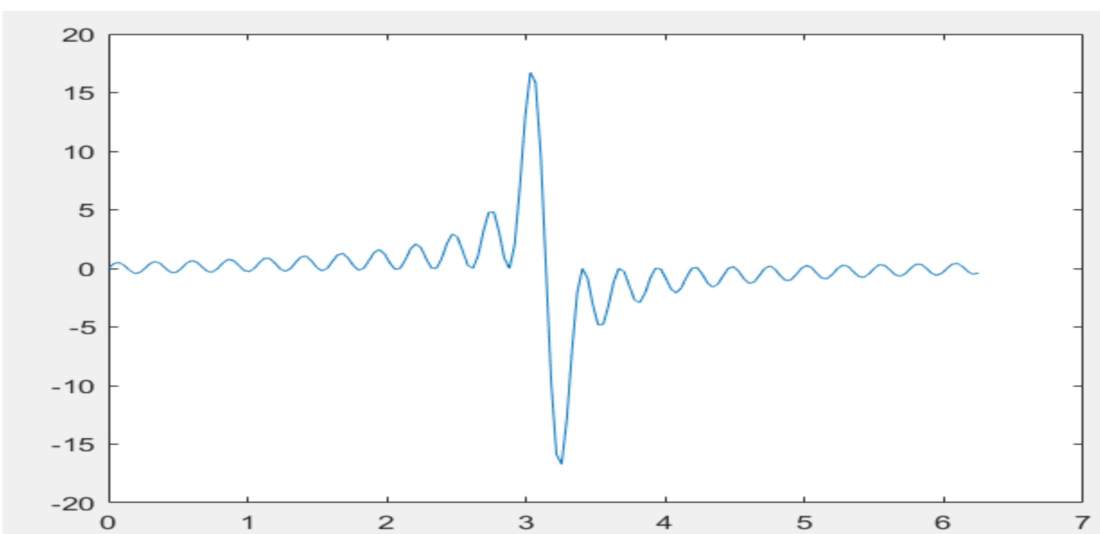
(b)

L = 24, N = 7 x L

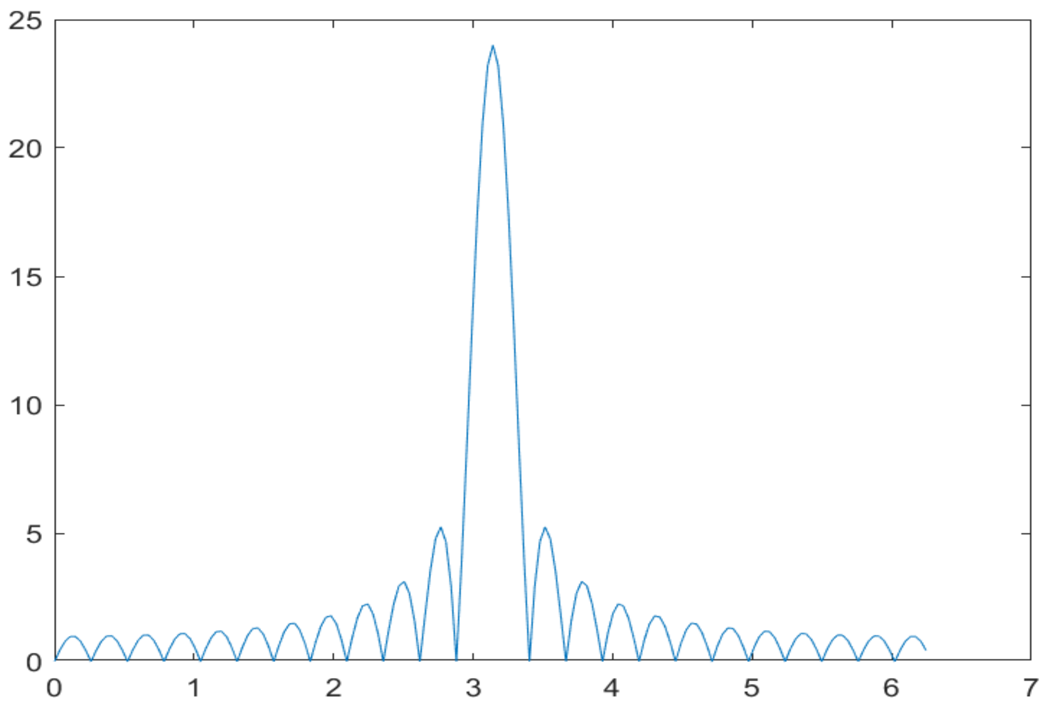
(1) Real part of R



(2) Imaginary Part

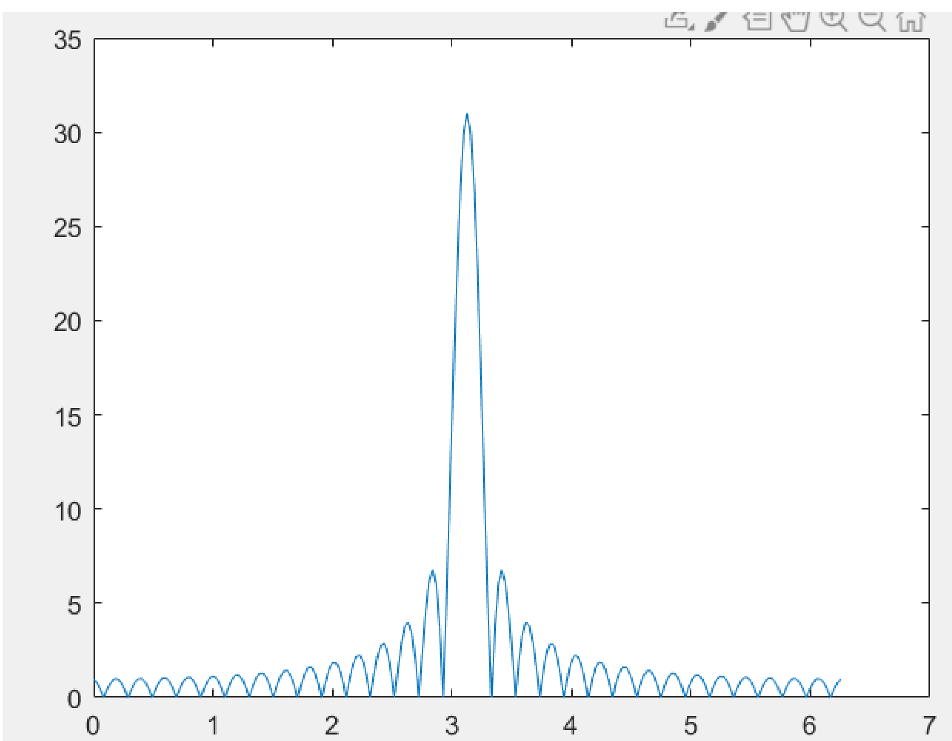


(3) Magnitude part



(c)

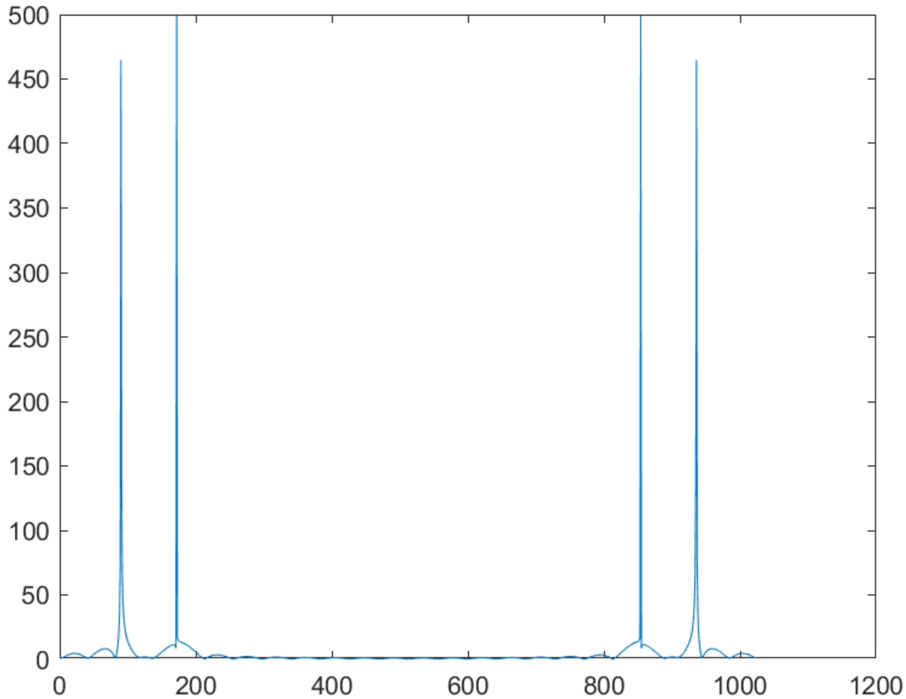
L = 31, N = 7 x L



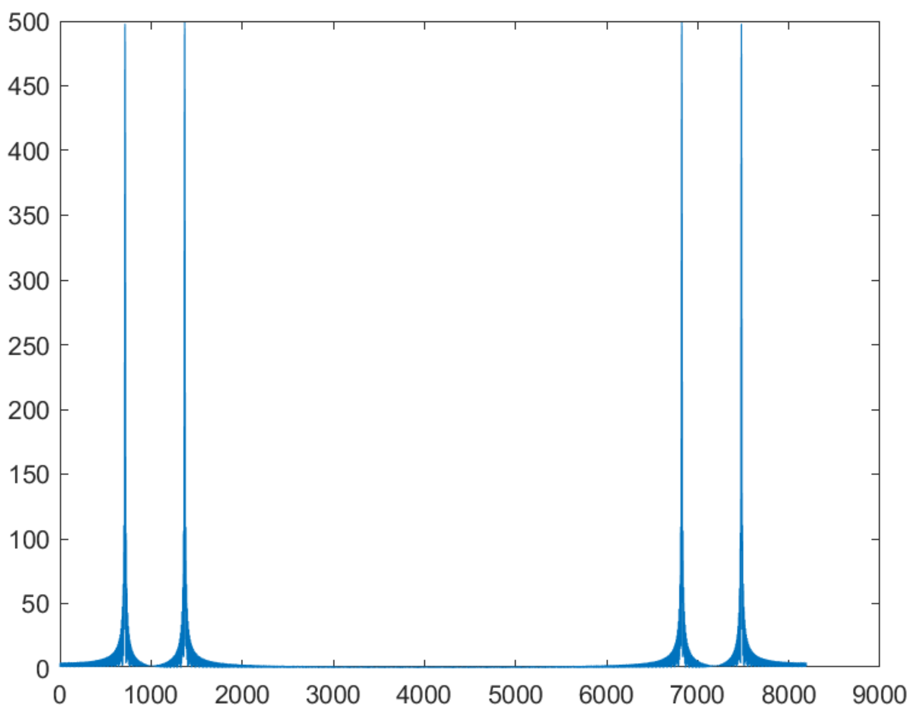
Problem7.

(b)

N = 2^10



N = 2^13



As we can observe, the case N = 2^13 has fine resolution. Two peaks become closer with N = 2^13. Also, the results seem 4 Dirac pulses for each N, but not exactly the same. This is because length of N is finite.

(d)

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Please check my audio file, USCID.wav

(e)

Decoded numbers (Sig1) = 584730

Decoded numbers (Sig2) = 66874129

Decoded numbers (Sig3) = 50037