Presentation Title

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April 25, 2025

Outline

We denote by p the target distribution and q an easy-to-sample distribution, for example a centered Gaussian.

Diffusion

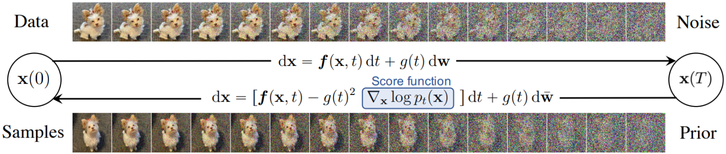
Let $X_0 \sim p$. We want to add noise until we reach pure noise, and denoise it afterward. We choose an horizon of time $T \in \mathbb{N}^*$ and a noise schedule $\beta : [0, T] \to \mathbb{R}^*$, continuous and non decreasing.

Forward process

$$d\overrightarrow{X}_t = \frac{-\beta(t)}{2\sigma^2}\overrightarrow{X}_t dt + \sqrt{\beta(t)} dB_t, \quad \overrightarrow{X}_0 \sim p$$

Backward process

$$\begin{aligned} d\overrightarrow{X}_t = &\left(\frac{\beta(T-t)}{2\sigma^2}\overleftarrow{X}_t + \beta(T-t)\nabla \log p_{T-t}\left(\overleftarrow{X}_t\right)\right) dt \\ &+ \sqrt{\beta(T-t)} dB_t, \quad \overleftarrow{X}_0 \sim p_T \end{aligned}$$



We learn the score by using score-matching techniques

Score matching

$$\mathcal{L}_{\text{score}}(\theta) = \mathbb{E}\left[\left\| s_\theta\left(\tau, \vec{X}_\tau\right) - \log p_\tau\left(\vec{X}_\tau | X_0\right)\right\|^2\right]$$

Plug it in the backward process and generate by discretizing the dynamics.

Normalizing flow

Let $X_0 \sim q$ and $X_1 \sim p$. We want to learn f_θ such that $X_1 \simeq f_\theta^{-1}(X_0) = Z \sim p_Z$. To do that, we set a structure on f_θ, with $f_1, \ldots, f_k$ simpler function (all parametrized by θ) such that

$$f_\theta = f_1 \circ f_2 \circ \ldots \circ f_k$$

We determine f_θ by minimizing

$$\mathcal{L}_{\mathrm{NF}}(\theta) = \mathbb{E}\left[-\log p_Z(f_\theta(x)) - \log \left| \det \frac{\partial f_\theta}{\partial x}(x) \right| \right]$$

Flow matching