

Presentation Title

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Outline

Introduction

We denote by p the target distribution and q an easy-to-sample distribution, for example a centered Gaussian.

Diffusion

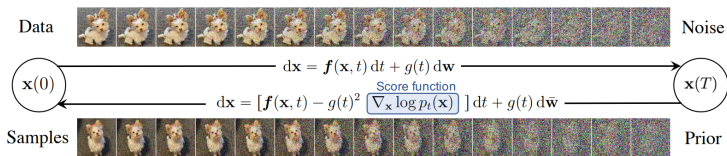
Let $X_0 \sim p$. We want to add noise until we reach pure noise, and denoise it afterward. We choose an horizon of time $T \in \mathbb{N}^*$ and a noise schedule $\beta : [0, T] \rightarrow \mathbb{R}^*$, continuous and non decreasing.

Forward process

$$d\vec{X}_t = \frac{-\beta(t)}{2\sigma^2} \vec{X}_t dt + \sqrt{\beta(t)} dB_t, \quad \vec{X}_0 \sim p$$

Backward process

$$d\overleftarrow{X}_t = \left(\frac{\beta(T-t)}{2\sigma^2} \overleftarrow{X}_t + \beta(T-t) \nabla \log p_{T-t}(\overleftarrow{X}_t) \right) dt + \sqrt{\beta(T-t)} dB_t, \quad \overleftarrow{X}_0 \sim p_T$$



We learn the score by using score-matching techniques

Score matching

$$\mathcal{L}_{\text{score}}(\theta) = \mathbb{E} \left[\left\| s_{\theta} \left(\tau, \vec{X}_{\tau} \right) - \log p_{\tau} \left(\vec{X}_{\tau} | X_0 \right) \right\|^2 \right]$$

Plug it in the backward process and generate by discretizing the dynamics.

Normalizing flow

Let $X_0 \sim q$ and $X_1 \sim p$. We want to learn f_θ such that $X_1 \simeq f_\theta^{-1}(X_0) = Z \sim p_Z$. To do that, we set a structure on f_θ , with f_1, \dots, f_k simpler function (all parametrized by θ) such that

$$f_\theta = f_1 \circ f_2 \circ \dots \circ f_k$$

We determine f_θ by minimizing

$$\mathcal{L}_{\text{NF}}(\theta) = \mathbb{E} \left[-\log p_Z(f_\theta(x)) - \log \left| \det \frac{\partial f_\theta}{\partial x}(x) \right| \right]$$

Flow matching