Flow Matching

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April 30, 2025

We start by defining a probability density path $p : [0,1] \times \mathbb{R}^d \to \mathbb{R}^d$ meaning that for each time t, p_t is density function i.e. $\int p_t(x)dx = 1$.
A simple example of such a path is a path p interpolating two density p_0 and p_1 with $p_t = tp_1 + (1-t)p_0$

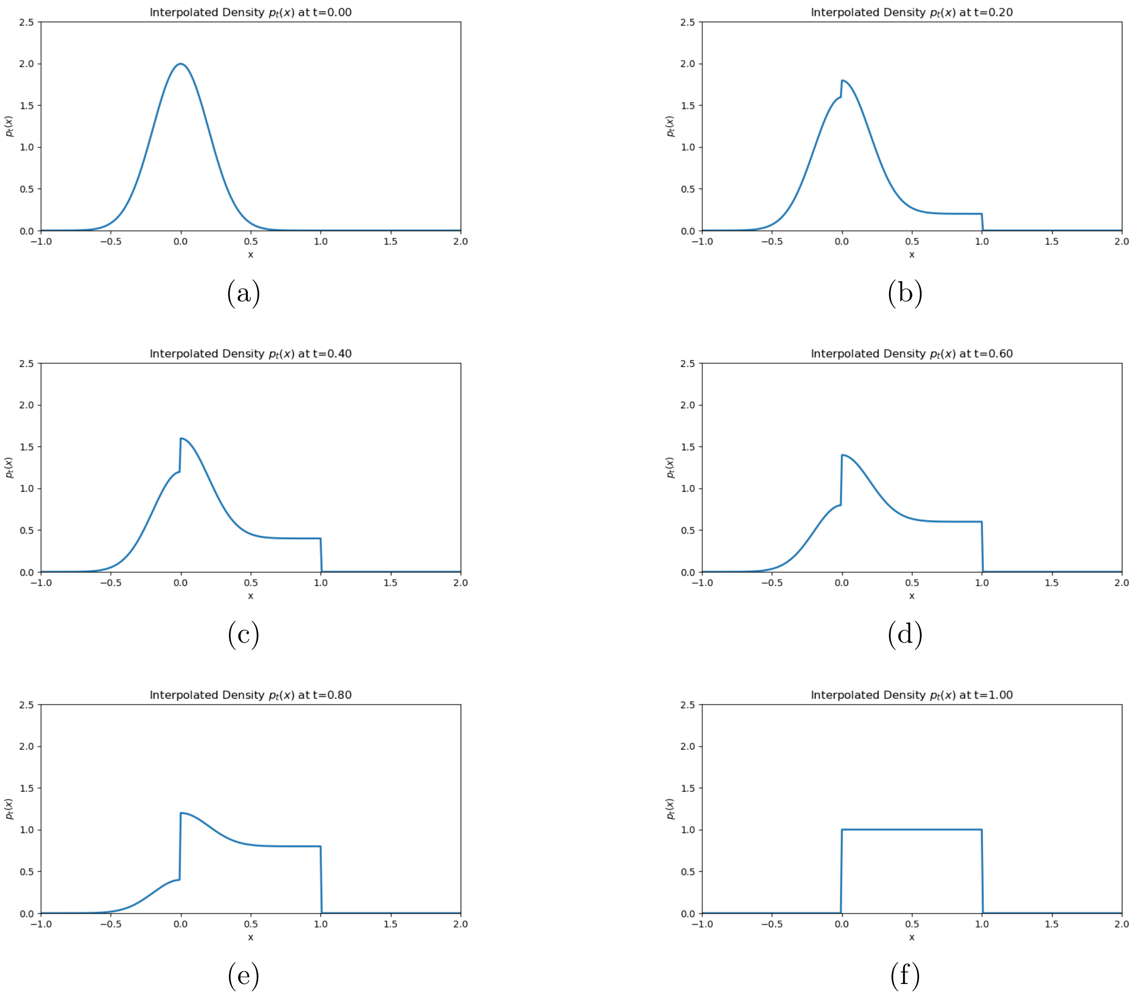

Figure 1: A probability path interpolating $\mathcal{N}(0, 0.2)$ and $\mathcal{U}([0,1])$

Next we introduce a core object, a time dependant vector field $v : [0, 1] \times \mathbb{R}^d \to \mathbb{R}^d$ which can be used to construct a map $\phi : [0, 1] \times \mathbb{R}^d \to \mathbb{R}^d$, called a flow, by the following ODE

$$\begin{aligned} \frac{d}{dt}\phi_t(x) &= v_t(\phi_t(x)) \\ \phi_0(x) &= x \end{aligned} \tag{1}$$

The link between the flow and the probability path is given by the change of variables formula

$$p_t(x) = q(\phi_t^{-1}(x)) \det \left[\frac{\partial \phi_t^{-1}}{\partial x}(x) \right] \tag{2}$$

This coincides with the normalizing framework.

Given a target probability path p_t and a corresponding v_t vector field, the naïve flow matching loss is

$$\mathcal{L}_{\mathrm{FM}}(\theta) = \mathbb{E}_{t,p_t(x)}[\|v_t^\theta(x) - v_t(x)\|^2] \tag{3}$$

But we don't have acces to v_t and p_t. To adress this problem and given a particular data sample x_1, we introduce conditional probability path $p_t(x|x_1)$ such that $p_0(x|x_1) = q(x)$ at time $t = 0$ and designed