

Synthetic turbulence generation using statistical methods

Samy Braik

Supervisor: Aurélien Larcher¹, Jonathan Viquerat¹, Fabien Duval² and Aubin Brunel²

¹CEMEF, Mines Paris–PSL
²ASNR

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What is turbulence?

- Turbulence: complex, aperiodic fluid motion with strong vortical structures.
- We study *Homogeneous and Isotropic Turbulence (HIT)*: statistics invariant under space translations and rotations.

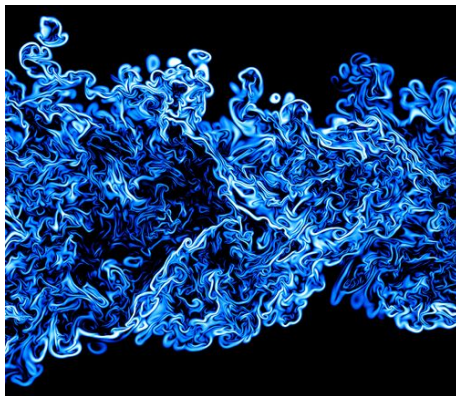


Figure: Slice through scalar dissipation (CNRS UMR 6614 CORIA and JSC).

Random Fourier model

- Build a synthetic velocity field by summing random Fourier modes.
- Frozen-turbulence assumption (no time dependence):

$$u^s(x) = 2 \sum_{n=1}^N \hat{u}_n \cos(\kappa^n \cdot x + \psi_n) \sigma^n \quad (1)$$

κ^n wave vector (random on a half-sphere to preserve isotropy)

σ^n direction (divergence-free: $\kappa^n \cdot \sigma^n = 0$)

ψ_n random phase (uniform $\mathcal{U}[0, 2\pi]$)

\hat{u}_n amplitude linked to prescribed energy spectrum $E(\kappa_n)$

Coefficient sampling

- Wave vector components (spherical coordinates):

$$\kappa_1 = \sin(\theta) \cos(\varphi) \quad (2)$$

$$\kappa_2 = \sin(\theta) \sin(\varphi) \quad (3)$$

$$\kappa_3 = \cos(\theta) \quad (4)$$

- Sampling densities: $f_\theta(\theta) = \frac{\sin \theta}{2}$, $f_\varphi(\varphi) = \frac{1}{2\pi}$.
- Direction vector σ obtained by fixing an angle $\alpha \sim \mathcal{U}[0, 2\pi]$ and ensuring $\kappa \cdot \sigma = 0$.

$$\sigma_1 = \cos(\varphi) \cos(\theta) \cos(\alpha) - \sin(\varphi) \sin(\alpha) \quad (5)$$

$$\sigma_2 = \sin(\varphi) \cos(\theta) \cos(\alpha) + \cos(\varphi) \sin(\alpha) \quad (6)$$

$$\sigma_3 = -\sin(\theta) \cos(\alpha) \quad (7)$$

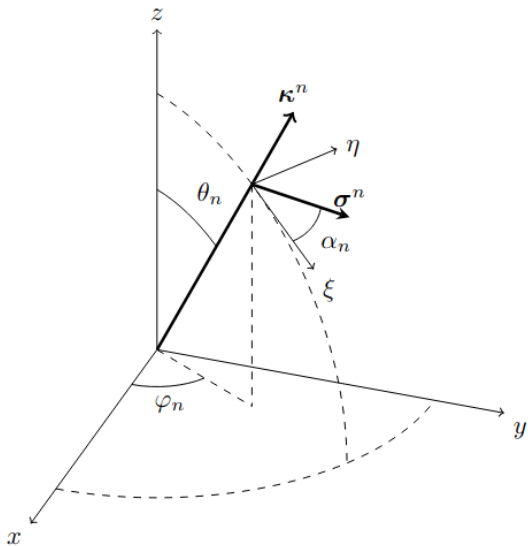


Figure: Wave vector geometry

Energy spectrum

- von Kármán–Pao (VKP) energy spectrum:

$$E_{\text{VKP}}(\kappa) = \frac{2}{3} \alpha_e \kappa L_e \frac{(\kappa L_e)^4}{[(\kappa L_e)^2 + 1]^{17/6}} \exp(-2(\kappa L_\eta)^2)$$

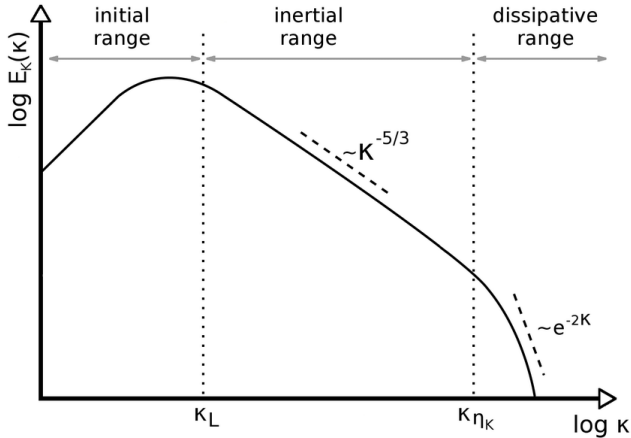


Figure: Energy spectrum of a turbulent flow

Results

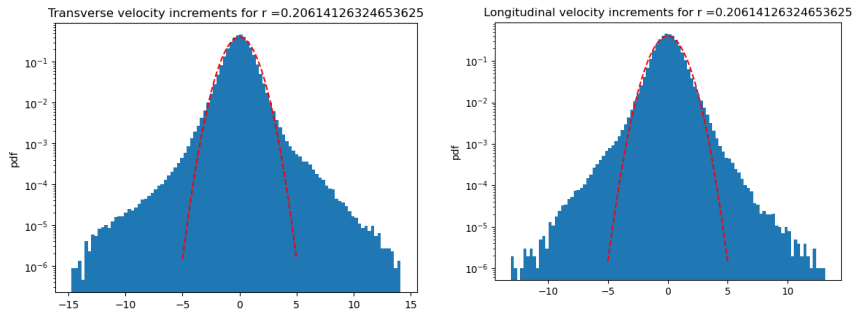


Figure: Velocity increments after learning on $(\varphi, \theta, \alpha)$

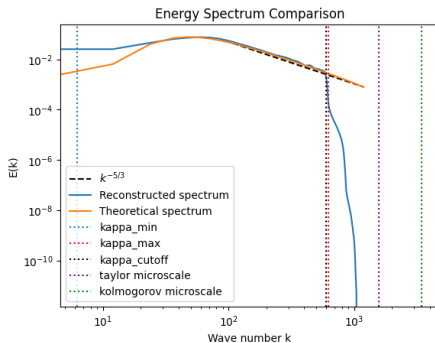


Figure: Energy spectrum reconstructed with the learned $(\varphi, \theta, \alpha)$

Direction	Mean
x	0.002564
y	-0.00652
z	-0.00093

Direction	RMS (expected: 0.222)
x	0.12904
y	0.31651
z	0.11406

Table: Velocity mean and RMS

Vorticity

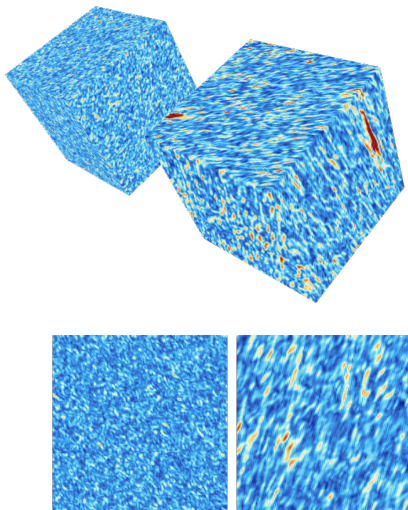


Figure: Vorticity field after learning on $(\varphi, \theta, \alpha)$