An introduction to R

R markdown basics

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About today

Sampling

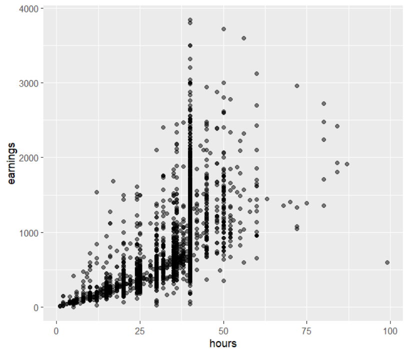
- Understand the notion of sampling, and dealing with **real world data**
- How it links with random variable (RV) and why the following elements are RV:
  - **OLS** estimators
  - Basic statistics
- Use our recent knowledge of plotting to highlight it

First regressions

- Perform several regressions
- Interpret results

Data

- Download the data and load it in R
- First, we are interested in the relationship between `hours worked` and `earnings`



Law of large numbers

Theory

Theorem

For a series of random variables $X_1, \cdots, X_n$, of the same distribution. Then,

$$\mu_n = \frac{1}{n} \sum_{i=1}^{n} X_i \xrightarrow[n \to \infty]{} \mathbb{E}[X]$$

Let's show it

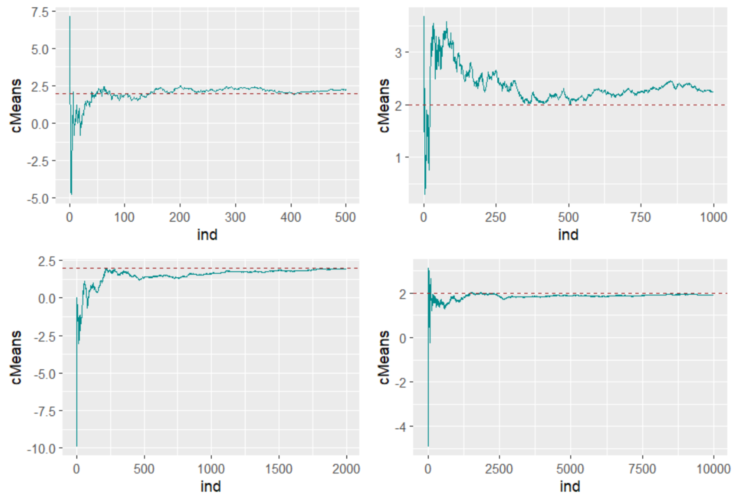
Monte Carlo Method : idea

- Generate a sample $X_1, \cdots, X_n$ of the same law
- Compute the mean of sample (X_1), then the mean of sample $(X_1,\ X_2)$, then sample $(X_1, \cdots,\ X_i)$ until the full sample.
- The series of means should converge to $\mathbb{E}[X]$

Bootstrapping

- Generate a sample $X_1, \cdots, X_n$ of the same law
- Compute the mean of sample (X_1), then the mean of sample $(X_i,\ X_j)$ randomly drawn, then sample $(X_{i_1}, \cdots,\ X_{i_p})$ until the full sample.
- You can also do it with p random sample f a fixed size k
- The series of means should converge to $\mathbb{E}[X]$

Law of large numbers



Regression analysis

Theory and empirics

- We want to estimate the relationship between two (for now) variables, X and Y
- We suppose that they are linked with a **theoretical** linear relationship: $Y = \alpha + \beta X + \epsilon$, this relationship is **always true** as ϵ can vary a lot
- Our goal is to estimate $\hat{\alpha}$ and $\hat{\beta}$, that are the ***estimators*** of our line.

Regression analysis

- $\hat{\alpha}$ and $\hat{\beta}$ **draw** our regression line
- Said otherwise, the regression line pass through all $\hat{y}_i = \hat{\alpha} + \hat{\beta} x_i$, that are predicted ordinates
- There is an **distance** $\hat{\epsilon}_i$ between the predicted value $\hat{y}_i$ and the true data value y_i: $\hat{\epsilon}_i = y_i - \hat{y}_i$

Remember

- The **OLS** estimator is defined by:

$$(\hat{\alpha},\ \hat{\beta}) = argmin_{a,b} \sum_{i=0}^{N} Y_i - (a + bX_i)$$

- The resolution of this problem gives:

$$\hat{\beta} = \frac{c\hat{o}v(X,Y)}{\hat{V}(X)}$$
$$\hat{\alpha} = \overline{Y} - \hat{\beta}\overline{X}$$

Graphical example

Using previous classes

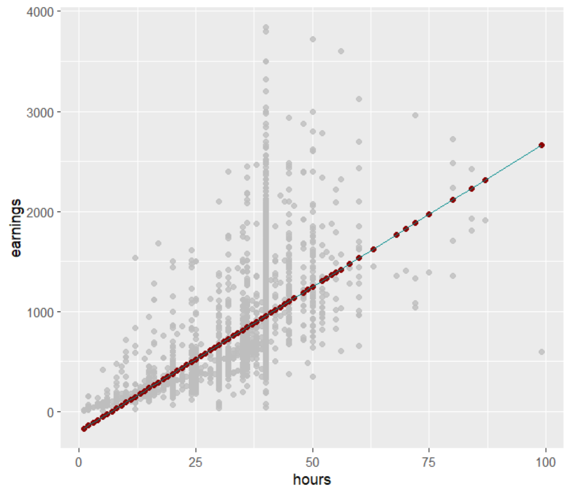
- Let's plot earnings as a function of hours worked
- Compute $\hat{\alpha}$, $\hat{\beta}$ and add a column hatEarnings ($\hat{y}_i$) to the data
- Add the hatEarning dots to the plot, change the color to highlight them
- Add a line
- Plot the same graph, and replace the `geom_line` with `geom_abline` (automatic regression plotting)

Graphical example

Solution

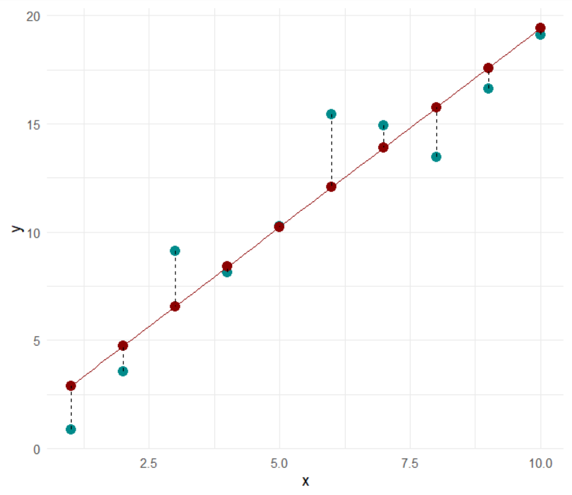
```
beta <- cov(data$earnings, data$hours)/var(data$hours)
alpha <- mean(data$earnings) - beta*mean(data$hours)

data %>%
  mutate(
    hatEarnings = alpha + beta*hours
  )%>%
  ggplot() +
  geom_point(aes(x = hours, y = earnings), alpha = .8, color = 'grey') +
  geom_point(aes(x=hours, y = hatEarnings), color = "darkred")+
  geom_line(aes(x=hours, y = hatEarnings), color = "darkcyan")
```

Result



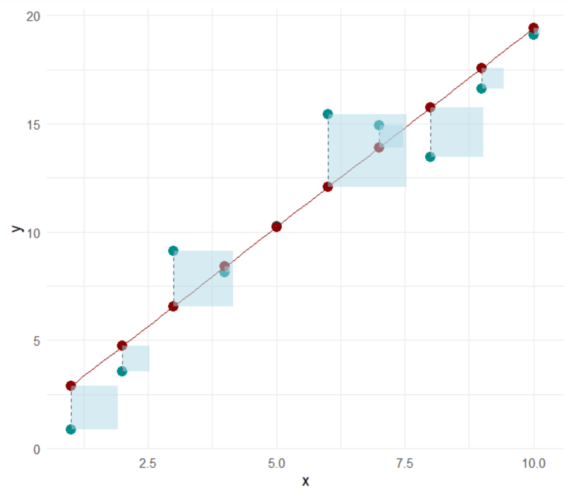
Errors

The goal is to minimize squared errors



Errors

The goal is to minimize squared errors



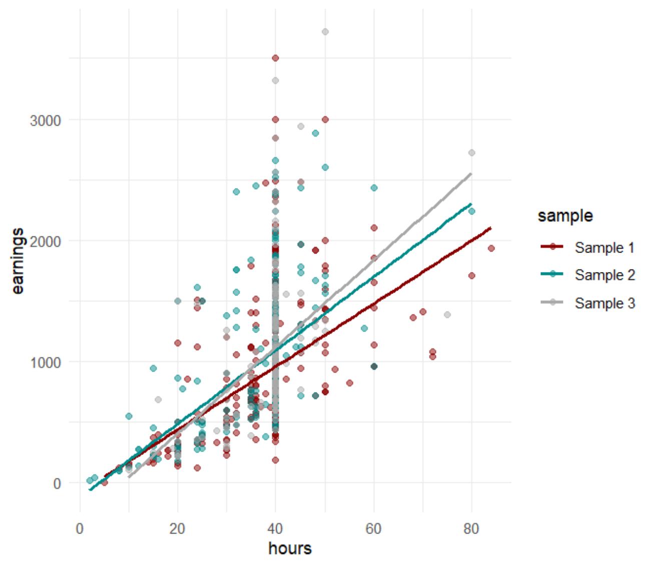
About sampling

About sampling

- From three different (not so) random sample, we can get three different regression lines

```
d1 <- sample_n(data, size = 500, weight = hours)
d2 <- sample_n(data, size = 300, weight = hourrt)
d3 <- sample_n(data, size = 200, weight = earnings)

d1$sample <- "Sample 1"
d2$sample <- "Sample 2"
d3$sample <- "Sample 3"

d <- bind_rows(d1, d2, d3) #Join data together
```

About sampling



A small exercise

Compute coefficients

- Create a function that takes in entry:
  - A dataframe
  - Two column names (the ones you want to perform the regression on)
- And gives both coefficients $\hat{\alpha}$ and $\hat{\beta}$ for the regression $Y = \alpha + \beta X + \epsilon$

Solution

Example

```r
computeCoefficients <- function(data, Y, X){
        #In: The data you will use the Y and X of your regression
        b <- cov(data[Y], data[X])/var(data[X])
        a <- mean(data[[Y]]) - b*mean(data[[X]])
        res <- t(data.frame(c(a, b)))
        colnames(res) <- c('alpha', 'beta')
        return(res)
}

computeCoefficients(d1, "earnings", "hours")
```

Sample	$\hat{\alpha}$	$\hat{\beta}$
d1	-82.9	26.01
d2	-121.3	30.34
d3	-314.02	35.8

About sampling

- **All** discrete statistics, as $\overline{X},\ \hat{\beta},\ \hat{V}(X)$ vary with the sample $X = (X_1, \ldots,\ X_n)$.
- As we can draw a sample, we indirectly draw estimators
- This is why we are able to compute their Variance, Expectation, etc.
- **For example:** $\mathbb{E}[\overline{X}] = \mathbb{E}[\mathbb{X}]$

About sampling

- That being said, we can **draw conclusions** about our estimators $\hat{\alpha}$ et $\hat{\beta}$, considering that we have our base hypothesis:
  1. $\forall n = 1, \ldots, N, \mathbb{E}[\epsilon_n] = 0$
  2. $\forall n = 1, \ldots, N, \mathbb{V}[\epsilon_n] = \sigma^2$
  3. $\forall n \neq m, cov(\epsilon_n, \epsilon_m) = 0$
- For example, β is **unbiased**: $\mathbb{E}[\hat{\beta}] = \beta$
- In the case of the **Gaussian Model** $\epsilon_n \sim \mathcal{N}(0, \sigma^2)$, we can have a *distribution* for $\hat{\beta} \sim \mathcal{N}(\beta,\ \sigma^2_{\hat{\beta}})$ and $\hat{\alpha} \sim \mathcal{N}(\alpha,\ \sigma^2_{\hat{\alpha}})$, with known values for the variances.
- **Increasing the sample size** means getting closer to the true laws, and that defines convergences as the **Central Limit Theorem**, the **Law of Large Numbers**, etc.

Regressions with R

Regressions with R

In practice

- There are built functions that can perform the regression for you
- For a simple regression, we use `lm` (linear model)
- A regression is an object you can store, it has attributes
- `summary()` gives you main infirmations

Example with the work data

```
reg1 <- lm(fml = earning ~ hours, data)
summary(reg1)
```

Regressions with R

```
Call:
lm(formula = y ~ x, data = data)

Residuals:
    Min      1Q  Median      3Q     Max 
-2.2695 -1.1248 -0.2785  0.7707  3.3628 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)   1.0509     1.3346   0.787    0.454    
x             1.8361     0.2151   8.537 2.73e-05 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.954 on 8 degrees of freedom
Multiple R-squared:  0.9011,	Adjusted R-squared:  0.8887 
F-statistic: 72.87 on 1 and 8 DF,  p-value: 2.729e-05
```

Into details

	Dependent variable:
	earnings
hours	28.908***
	(0.591)
Constant	−198.809***
	(21.915)
Observations	4,609
R^2	0.342
Adjusted R^2	0.342
Residual Std. Error	393.284 (df = 4607)
F Statistic	2,391.645*** (df = 1; 4607)
Note:	*p<0.1; **p<0.05; ***p<0.01

Comparison

- With our previous function we found:

$$\hat{\alpha} = -198.8, \quad \hat{\beta} = 28.9$$

Into details

- **Coefficients:** the estimator values for our data sample
- **StdE, stars:** level of significativity given by a t-test
- $\mathbf{R^2}$**:** The part of the total variance that is explained by the model
- **Adjusted** $\mathbf{R^2}$**:** the same, but weighted by the number of independent variables
- **Residual Std. Error:** The standard error of the model (degrees of freedom: df)
- **F statistic:** a test statistic that is usually used to see if your model explains at least something

Regression, summary attributes

- The regression object `reg1` as well as the summary object `summary(reg1)` both have attributes.
- We can use it to plot residuals for example !

List of attributes of the regression object

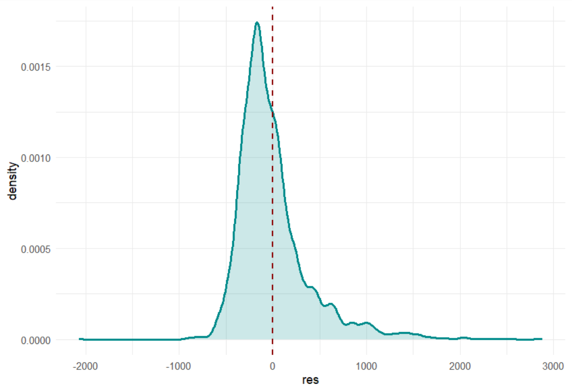
```
> str(reg1, give.attr = F)
List of 12
 $ coefficients : Named num [1:2] 1.05 1.84
 $ residuals    : Named num [1:10] -2.0079 -1.1834 2.5583 -0.2541 0.0274 ...
 $ effects      : Named num [1:10] -35.257 16.677 3.127 0.254 0.474 ...
 $ rank         : int 2
 $ fitted.values: Named num [1:10] 2.89 4.72 6.56 8.4 10.23 ...
 $ assign       : int [1:2] 0 1
 $ qr           :List of 5
  ..$ qr   : num [1:10, 1:2] -3.162 0.316 0.316 0.316 0.316 ...
  ..$ qraux: num [1:2] 1.32 1.27
  ..$ pivot: int [1:2] 1 2
  ..$ tol  : num 1e-07
  ..$ rank : int 2
 $ df.residual  : int 8
 $ xlevels      : Named list()
 $ call         : language lm(formula = y ~ x, data = data)
 $ terms        :Classes 'terms', 'formula'  language y ~ x
 $ model        :'data.frame':  10 obs. of  2 variables:
  ..$ y: num [1:10] 0.879 3.54 9.117 8.141 10.259 ...
  ..$ x: int [1:10] 1 2 3 4 5 6 7 8 9 10
> 
```

List of attributes of the summary object

```
> str(summary(reg1), give.attr = F)
List of 11
 $ call         : language lm(formula = y ~ x, data = data)
 $ terms        :Classes 'terms', 'formula'  language y ~ x
 $ residuals    : Named num [1:10] -2.0079 -1.1834 2.5583 -0.2541 0.0274 ...
 $ coefficients : num [1:2, 1:4] 1.051 1.836 1.335 0.215 0.787 ...
 $ aliased      : Named logi [1:2] FALSE FALSE
 $ sigma        : num 1.95
 $ df           : int [1:3] 2 8 2
 $ r.squared    : num 0.901
 $ adj.r.squared: num 0.889
 $ fstatistic   : Named num [1:3] 72.9 1 8
 $ cov.unscaled : num [1:2, 1:2] 0.4667 -0.0667 -0.0667 0.0121
> 
```

Residuals

```r
data.frame(res = reg1$residuals) %>% # Creates a dataframe with residuals as a single columns
  ggplot(aes(x = res)) + # We can now use ggplot on the dataframe using dplyr
  geom_density(color = 'darkcyan', fill = 'darkcyan', alpha = .2, linewidth = 1)+
  geom_vline(xintercept = 0, linetype = 'dashed', color = 'darkred', linewidth = .8) +
  theme_minimal()
```



Coefficient interpretation

Super important

- Depends on the framework you ar in (log - log, etc.)
- Being wrong changes completely your results

Choosing the appropriate transformation

- Most of the time, $\log$ transformations are used
- They compress high values, and can be understood in terms of **utility**. For instance, $\log$(salary) can mean that workers have a smaller gain in utility going from 5000 to 5500 per month that from 1000 to 1500 (*decreasing marginal utility*)
- They have another interpretation
- You can find keys for interpretation at the end of the slide ▸ HERE

Our results

$$\hat{\alpha} = -198.8, \quad \hat{\beta} = 28.9$$

Coeff.	Value	Interpretation
α	-198.8	
β	28.9	

Our results

$$\hat{\alpha} = -198.8, \quad \hat{\beta} = 28.9$$

Coeff.	Value	Interpretation
α	-198.8	Working 0 hours imply no money (negative salary in fact)
β	28.9	An additional hour worked implies more 28.9\$

- **Beware** about interpretation of α. sometimes, it is **difficult to link it with reality** (would someone really work 0 hour? What about social minimas, etc.)

Binary Variable

Set-up

Let's assume that we want to explain `earnings` by the variable `sex`, 1 for man and 0 for woman.

- A first look at `data$sex` gives us the values `"Male"` or `"Female"`
- Even though R is capable of performing regressions on categorical variables, to link it with theory, let's recode the variable `sex`

```
data <- data %>% mutate(sexNum = ifelse(sex=="Male", 1, 0))
```

Binary variables

Let's assume that we want to explain `earnings` by the variable `sex`, 1 for man and 0 for woman.

$$earnings = a + bsex + epsilon$$

A bit of theory

$$\begin{aligned} & E[earnings \mid Sex = 1] - E[earnings \mid Sex = 0] \\ = & \, a + b - (a + b \times 0) \\ = & \, b \end{aligned}$$

- **Interpretation:** In expectation, being a male (sex=1) implies b more $ of salary than for being a woman (sex = 0), all other things equal.

Binary variables

Let's assume that we want to explain `earnings` by the variable `sex`, `1` for man and `0` for woman.

$$earnings = a + bsex + epsilon$$

A bit of theory

$$E[earnings \mid Sex = 0] \;=\; a$$

- **Interpretation:** a is the mean of the reference group
- **Therefore:** b is the difference in means

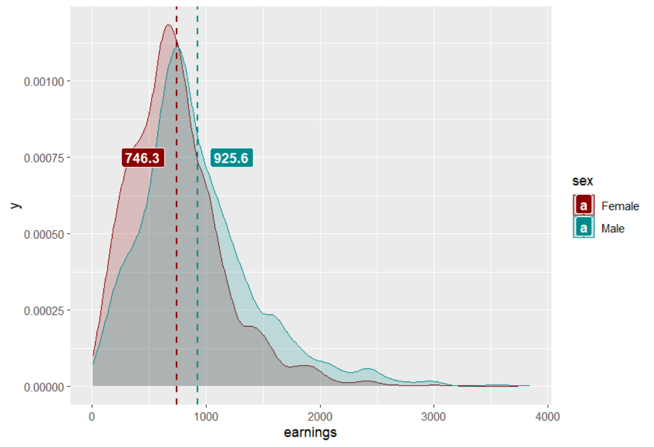
Graphical interpretation

Try to plot the earnings distribution as a function of the sex.

```
sumData <- data %>%
group_by(sex) %>%
summarize(m.grp = mean(earnings, na.rm = T))

data %>%
    ggplot(aes(x = earnings, fill = sex, color = sex)) +
    geom_density(alpha = .2) +
    scale_color_manual(values = c('darkred', 'darkcyan')) +
    scale_fill_manual(values = c('darkred', 'darkcyan')) +
    geom_vline(
        data = sumData, aes(xintercept = m.grp, color = sex),
        linewidth = .6, linetype = 'dashed') +
    geom_label(
        data = sumData,
        aes(x = m.grp + (-1)^(sex=="Female")*300,
        y = 0.00075,
        label = as.character(round(m.grp, 1))),
        color = 'white', fontface = 'bold')
```

See that I used the categorical value for `sex`

Graphical interpretations



Regression

Let's regress earnings on sex, with both the categorical character variable and the binary one `sexnum`

Regress earnings on sex

```
reg2 <- lm(earnings ~ sex, data)
summary(reg2)

reg3 <- lm(earnings ~ sexNum, data)
summary(reg3)
```

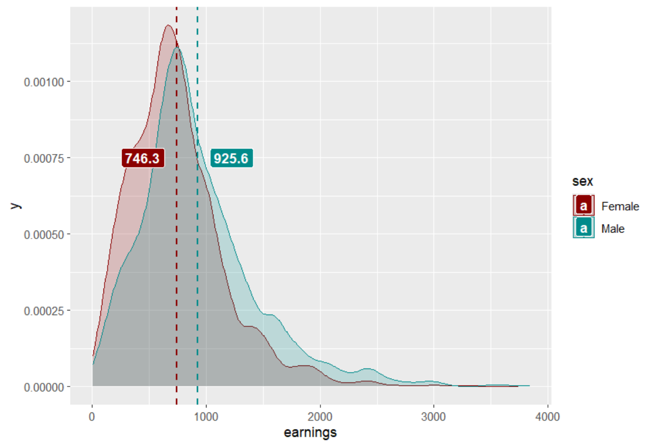
Regression

```
> reg2 <- lm(earnings ~ sex, data)
> summary(reg2)

Call:
lm(formula = earnings ~ sex, data = data)

Residuals:
   Min     1Q Median     3Q    Max 
-921.6 -314.3  -75.6  216.4 3053.7 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  746.256      9.862   75.67   <2e-16 ***
sexMale      179.346     14.035   12.78   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 476.4 on 4607 degrees of freedom
Multiple R-squared:  0.03423,	Adjusted R-squared:  0.03402 
F-statistic: 163.3 on 1 and 4607 DF,  p-value: < 2.2e-16

> reg3 <- lm(earnings ~ sexNum, data)
> summary(reg3)

Call:
lm(formula = earnings ~ sexNum, data = data)

Residuals:
   Min     1Q Median     3Q    Max 
-921.6 -314.3  -75.6  216.4 3053.7 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  746.256      9.862   75.67   <2e-16 ***
sexNum       179.346     14.035   12.78   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Regression

- **Notice** how `R` produces strictly the same regression
- It automatically defines a reference category.
- Above, the coefficient associated with `sexMale` tells us R defines `Female` as the reference category
- It represents the difference in means

Graphical interpretations



Our regression gives $\hat{\alpha} = 746.3$ and $\hat{\beta} = 179.3 = 925.6 - 746.3$

Broader categorical variables

More than two values

```
data$education
```

```
> unique(data$educ)
[1] "High school"        "Bachelor's degree" "No high school"     "Associate degree"
```

- A n levels category variable is equivalent to $n - 1$ dummy variables
- Commonly called öne hotëncoding

educ	Bachelor's degree	High school	No high school
High school	0	1	0
Bachelor's degree	1	0	0
No high school	0	0	1
Associate degree	0	0	0

Integration in R

- Of course, R does not need us to encode every time we want to perform a regression analysis
- It is possible to run regressions directly and the coefficient associated with `variableCategory` will describe the difference in means w.r.t the reference category (the only one not printed)

Let's pratice

```
reg4 <- lm(earnings ~ educ, data)
summary(reg4)
```

Integration in R

```
> reg4 <- lm(earnings ~ educ, data)
> summary(reg4)

Call:
lm(formula = earnings ~ educ, data = data)

Residuals:
     Min       1Q   Median       3Q      Max 
-1006.06  -304.01   -64.01   193.29  2829.94 

Coefficients:
                      Estimate Std. Error t value Pr(>|t|)    
(Intercept)             904.01      19.47  46.430  < 2e-16 ***
educBachelor's degree   106.06      24.65   4.302 1.72e-05 ***
educHigh school         -97.30      21.58  -4.509 6.68e-06 ***
educNo high school     -328.85      28.19 -11.665  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 468.9 on 4605 degrees of freedom
Multiple R-squared:  0.06466,	Adjusted R-squared:  0.06405 
F-statistic: 106.1 on 3 and 4605 DF,  p-value: < 2.2e-16
```

Integration in R

Reference category

- See how the only category no printed is `Associate degrees`
- **Key for reading:** In **expectation**, *all other things equal*, a Bachelor's degree implies 106.06$ more than an Associate degree.

Change the reference category

- Can be **useful** to gain ***interpretation* power**
- **Factors:** allow us to define implicit orders for variables (levels)
- Levels are **only implicit**, R knows they do not have a specific signification

Integration in R

Re-level all your data

```
data <- data %>%
    mutate(
        educf = factor(educ,
        levels = c("No high school",
        "High school",
        "Associate degree",
        "Bachelor's degree"))
        )
```

Only set the reference level

```
data <- data %>%
    mutate(
        educf = relevel(as.factor(educ), ref = "No high school")
        )
```

Your column does not change **at all**, but R understand there is an implicit order for some funtions (**regression**, **ggplot**, etc.)

Multivariate regressions

Set-up

Interest

- Performing ***one*** regression at a time is **not productive**
- Multivariate regressions allow us to **disentangle** every effect.

Frisch - Waught's theorem, idea :

- In the basic setup $Y = \alpha + \beta X + \epsilon$, the error term ϵ may **encapsulates** **other effects** than β
- Writing $Y = \alpha + \beta X_1 + \gamma X_2 + \mu$ allows to highlight effect γ and "*strip it* out of ϵ". Then, if X_1 and X_2 are correlated, there is a chance that the effect for X_1 changes, it would be **cleaned** by X_2

This theorem is one of the solution for the **omitted variable bias problem**

Model and Theorem

Consider a multiple regression model:

$$y = X_1\beta_1 + X_2\beta_2 + \varepsilon,$$

where:

- y is the vector of the dependent variable ($n \times 1$),
- X_1 ($n \times k_1$) and X_2 ($n \times k_2$) are matrices of explanatory variables,
- β_1 and β_2 are vectors of coefficients,
- ε is the error term.

Theorem: The coefficients $\hat{\beta}_1$ can be obtained in three steps:

1. Regress y on X_2 and obtain the residuals r_y.
2. Regress X_1 on X_2 and obtain the residuals R_{X_1}.
3. Regress r_y on R_{X_1}. The coefficients are $\hat{\beta}_1$.

Demonstration

The initial model is:

$$y = X_1\beta_1 + X_2\beta_2 + \varepsilon.$$

1. Project y onto X_2: $P_2 = X_2(X_2'X_2)^{-1}X_2'$.

$$r_y = (I - P_2)y.$$

2. Project X_1 onto X_2:

$$R_{X_1} = (I - P_2)X_1.$$

3. Regress r_y on R_{X_1}:

$$\hat{\beta}_1 = (R_{X_1}'R_{X_1})^{-1}R_{X_1}'r_y.$$

This approach is equivalent to directly estimating the full model!

Graphical Intuition

- The residuals r_y represent the part of y that is not explained by X_2.
- The residuals R_{X_1} represent the part of X_1 that is not explained by X_2.
- Regressing r_y on R_{X_1} measures the net effect of X_1 on y.

Advantage: This theorem allows for interpreting coefficients in complex contexts.

A Simple multivariate regression

An example

- Let's regress `earnings` on `sex`, `educ`, `hours`
- We **assume** the relationship for individual i is given by:

$$earnings_i = \beta_0 + \beta_1 sex_i + \beta_2 educ_i + \beta_3 hours_i + \epsilon_i$$

Let's run it

```
multiReg <- lm(earnings ~ sex + educ + hours, data)
summary(multiReg)

data <- data %>%
    mutate(
        logEarnings = log(earnings)
    )

multiRegLog <- lm(logEarnings ~ sex + educ + hours, data)
summary(multiRegLog)
```

A simple multivariate regression (level - level)

```
Call:
lm(formula = earnings ~ sex + educf + hours, data = data)

Residuals:
    Min      1Q  Median      3Q     Max 
-1976.4  -232.1   -75.3   140.6  2707.6 

Coefficients:
                        Estimate Std. Error t value Pr(>|t|)    
(Intercept)            -370.0241    24.6203 -15.029  < 2e-16 ***
sexMale                 118.6189    11.3734  10.430  < 2e-16 ***
educfAssociate degree   220.7786    22.7767   9.693  < 2e-16 ***
educfBachelor's degree  367.6253    20.5479  17.891  < 2e-16 ***
educfHigh school        114.6405    18.0911   6.337 2.57e-10 ***
hours                    27.3701     0.5789  47.280  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 374.4 on 4603 degrees of freedom
Multiple R-squared:  0.4041,	Adjusted R-squared:  0.4034 
F-statistic: 624.2 on 5 and 4603 DF,  p-value: < 2.2e-16
```

A simple multivariate regression (level - level)

- *All other things equal* **in our model**, men earn in average 118$ more than women (monthly)
- *All other things equal* **in our model**, an Associate degree allows 220$ more monthly than no High School
- *All other things equal* **in our model**, an Bachelor's degree allows 367$ more monthly than no High School

A simple multivariate regression (level - level)

A recall on Frisch Waugh's theorem

- In the simple regression framework $earnings = \alpha + \beta Male + \epsilon$, we had $\hat{\beta} = 179$
- In the multivariate regression framework $earnings = \beta_0 + \beta_1 Male + \beta_2 educf + \beta_3 hours + \epsilon$, we have $\hat{\beta}_1 = 118$, which is **lower**
- Somehow, we have cleaned the coefficient, maybe getting closer to **causality**
- It is the all other things equal that changes, meaning that the difference on all people between men and women is lower than the difference on all people that have the same degree and work the same hours

A simple multivariate regression (log - level)

```
Call:
lm(formula = logEarnings ~ sex + educf + hours, data = data)

Residuals:
    Min      1Q  Median      3Q     Max 
-3.8581 -0.2353 -0.0236  0.2299  1.8256 

Coefficients:
                        Estimate Std. Error t value Pr(>|t|)    
(Intercept)            4.4561736  0.0269149 165.565  < 2e-16 ***
sexMale                0.0848793  0.0124334   6.827 9.82e-12 ***
educfAssociate degree  0.2955869  0.0248995  11.871  < 2e-16 ***
educfBachelor's degree 0.4502549  0.0224630  20.044  < 2e-16 ***
educfHigh school       0.1888843  0.0197772   9.551  < 2e-16 ***
hours                  0.0506223  0.0006329  79.991  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.4093 on 4603 degrees of freedom
Multiple R-squared:  0.6252,	Adjusted R-squared:  0.6248 
F-statistic:  1535 on 5 and 4603 DF,  p-value: < 2.2e-16
```

Interactions and interpretations

The principle of interactions

Purpose

- Sometimes, you want to have access to some precise coefficients
- *Example:* What is the additional earning for men for an additional hour worked.

A look at different specifications

See in next class !

Coefficient interpretation

Coefficient interpretation

In the following, I will consider a multivariate regression:

$$Price_i = \beta_0 + \beta_1 Surface + \beta_2 Garden + \epsilon_i$$

Where :

- $Price_i$ is the price of house i
- $Surface_i$ is the Surface of house i
- $Garden_i$ equals 1 if house i has a garden, 0 otherwise

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Traditional framework

$$Price_i = \beta_0 + \beta_1 Surface_i + \beta_2 Garden_i + \epsilon_i$$

In this classic framework:

- $\frac{\partial Price_i}{\partial Surface_i} = \beta_1$, so β_1 is the marginal effect of Surface on the price.
- **We can say that** β_1 is the variation in **units** induced by an increase of one additional **unit** of *Surface*, ***all other things equal***

Log - log framework

$$\log Price_i = \beta_0 + \beta_1 \log Surface_i + \beta_2 Garden_i + \epsilon_i$$

In this framework:

$$\begin{aligned} Price_i =& e^{\beta_0 + \beta_1 \log(Surface_i) + \beta_2 Garden_i + \epsilon_i} \\ \implies \frac{\partial Price_i}{\partial Surface_i} =& \frac{\beta_1}{Surface_i} e^{\beta_0 + \beta_1 \log(Surface_i) + \beta_2 Garden_i + \epsilon_i} \\ =& \beta_1 \frac{Price_i}{Surface_i} \end{aligned}$$

So:

$$\beta_1 = \frac{Surface_i}{\partial Surface_i} \frac{\partial Price_i}{Price_i}$$

- We can say that that is is an elasticity. An change of 1% of $Surface_i$ implies a change of β_1% of the price

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Log - level framework

$$\log Price_i = \beta_0 + \beta_1 Surface_i + \beta_2 Garden_i + \epsilon_i$$

In this framework: (I don't show it, same method)

$$100 \times \beta_1 = \frac{100 \frac{\partial Price_i}{Price_i}}{Surface_i}$$

We can say that one additional unit of *Surface* implies a change of salary of $100 \times \beta_1$%

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Level - log framework

$$Price_i = \beta_0 + \beta_1 \log Surface_i + \beta_2 Garden_i + \epsilon_i$$

In this framework: (I don't show it, same method)

$$\frac{\beta_1}{100} = \frac{\partial Price_i}{100 \frac{Surface_i}{Surface_i}}$$

We can say that one additional percent of *Surface* implies a change of salary of $\frac{\beta_1}{100}$ unit

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