Lotka-Volterra Predator-Prey Model: Simulation Problems

Problem 1: Classic Lotka-Volterra Model
Given the equations

$$\frac{dN}{dt} = \alpha N - \beta NP, \quad \frac{dP}{dt} = \delta NP - \gamma P$$

with $\alpha = 0.1, \beta = 0.02, \delta = 0.01, \gamma = 0.1$, and initial conditions $N(0) = 40, P(0) = 9$, simulate the system for $t \in [0, 100]$. Plot the populations and discuss the oscillations in predator-prey dynamics.

Problem 2: Lotka-Volterra with Time-Dependent Parameters
For the system

$$\frac{dN}{dt} = \alpha(t) N - \beta NP, \quad \frac{dP}{dt} = \delta NP - \gamma(t) P$$

where $\alpha(t) = 0.1 + 0.05 \sin(0.1t)$, $\gamma(t) = 0.1 + 0.05 \cos(0.1t)$, simulate for $t \in [0, 100]$ with $N(0) = 40, P(0) = 9$. Analyze the impact of time-varying rates.

Problem 3: Lotka-Volterra with Harvesting
For the equations

$$\frac{dN}{dt} = \alpha N - \beta NP - hN, \quad \frac{dP}{dt} = \delta NP - \gamma P$$

with $h = 0.1$ and $N(0) = 40, P(0) = 9$, simulate for $t \in [0, 100]$. Discuss the effects of harvesting the prey population.

Problem 4: Lotka-Volterra with Refuge for Prey
Using the system

$$\frac{dN}{dt} = \alpha N - \beta NP(1 - r), \quad \frac{dP}{dt} = \delta NP - \gamma P$$

where $r = 0.3$, simulate for $t \in [0, 100]$ with $N(0) = 40, P(0) = 9$. Investigate how refuge affects population dynamics.

Problem 5: Lotka-Volterra with Allee Effect on Prey
Consider the system

$$\frac{dN}{dt} = \alpha N \left(1 - \frac{N}{K}\right) - \beta NP, \quad \frac{dP}{dt} = \delta NP - \gamma P$$

with $K = 50$ and $N(0) = 10, P(0) = 5$. Simulate for $t \in [0, 100]$ and explore the effect of the Allee effect on the prey population's dynamics.