Lab Assignment Questions

March 13, 2025

1. Continuity Equation

   Verify the continuity equation:

   $$\nabla \cdot \vec{v} = 0$$

   for an incompressible 2D velocity field:

   $$u(x, y) = 2x + y, \quad v(x, y) = -x - 3y.$$

   Compute:

   $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$$

   at:

   $$(x, y) = (1, 2).$$

2. Streamline Coordinates

   Given the velocity components:

   $$u = y^2, \quad v = 2xy,$$

   compute the streamline equation:

   $$\frac{dx}{u} = \frac{dy}{v}$$

   numerically starting at the point:

   $$(x_0, y_0) = (0, 1).$$

3. Vorticity Calculation

   For a velocity field:

   $$u = -y, \quad v = x,$$

   calculate the vorticity:

   $$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

   at:

   $$(x, y) = (2, -1).$$

4. Pathline Simulation

   For a flow field:

$$u = y, \quad v = -x,$$

   solve the system of ordinary differential equations (ODEs) numerically:

$$\frac{dx}{dt} = u(x, y), \quad \frac{dy}{dt} = v(x, y),$$

   with the initial position:

$$(x_0, y_0) = (1, 0).$$

   Compute the pathline for:

$$t \in [0, 5].$$

5. For the velocity field:

$$u(x, y) = 2x, \quad v(x, y) = -2y,$$

   compute the divergence:

$$\nabla \cdot \vec{v} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$$

   numerically over a grid.

Simulation Goal

Confirm that:

$$\nabla \cdot \vec{v} = 0$$

for incompressible flow.

Tools

Use Python libraries:

- `NumPy`: For computing numerical partial derivatives.
- `Matplotlib`: For visualizing the divergence results.

6. Non-Uniform Grid Continuity Check

Task

Simulate and validate:

$$\nabla \cdot \vec{v}$$

for a flow field:

$$u(x, y) = y, \quad v(x, y) = x^2,$$

on a non-uniform grid.

Goal

Explore how the divergence behaves on irregularly spaced grids.

Simulation Idea

Generate grids with varying densities along x and y, and check for continuity numerically.

7. Divergence-Free Perturbation Flow

Task

Define a perturbation flow:

$$u(x, y) = -\sin(y), \quad v(x, y) = \cos(x),$$

and validate its divergence.

Goal

Visualize the flow using streamlines and confirm numerically that:

$$\nabla \cdot \vec{v} = 0.$$

8. Rotational vs. Irrotational Flow Divergence

Task

Simulate and compare the divergence of:

- A rotational flow: $u = -y$, $v = x$,
- An irrotational flow (e.g., potential flow).

Goal

Illustrate the mathematical distinction between rotational and irrotational flows using numerical methods and visualization.

9. Continuity in Compressible Flow

Task

Extend the continuity equation to compressible flows:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0.$$

Simulate:

$$\rho(x, y, t) = 1 + 0.1t, \quad u(x, t) = x, \quad v(x, t) = 0.$$

Goal

Track density variation over time and verify continuity.