Lab on 27.03.25

Complex Potential Problems

1. Flow Around a Cylinder

The complex potential for flow around a cylinder is given by:

$$F(z) = U\left(z + \frac{a^2}{z}\right)$$

Where:

- U: Uniform velocity.
- a: Radius of the cylinder.
- $z = x + iy$: Complex variable representing the position.

The velocity potential (ϕ) and stream function (ψ) are:

$$\phi = \mathrm{Re}(F(z)), \quad \psi = \mathrm{Im}(F(z))$$

1. Enter U = 5 (arbitrary unit for velocity, e.g., meters/second).
2. Enter a = 2 (arbitrary unit for radius, e.g., meters).
3. Enter x = 3 and y = 4 (coordinates of the point in the flow field, in the same units as the radius).

2. Uniform Flow

The complex potential for uniform flow is given by:

$$F(z) = Uz$$

Where:

- U: Velocity of the flow.
- $z = x + iy$: Complex variable.

The velocity potential and stream function are:

$$\phi = \mathrm{Re}(F(z)), \quad \psi = \mathrm{Im}(F(z))$$

1. Uniform Velocity ((U)): The velocity of the uniform flow. Example: (U = 5 , m/s).
2. Coordinates ((x, y)): The point in the flow field where you want to calculate (ϕ) and (ψ). Take (x = 3), (y = 2).

3. Source or Sink Flow

The complex potential for a source or sink flow is:

$$F(z) = \frac{m}{2\pi} \ln(z)$$

Where:

- m: Strength of the source (positive) or sink (negative).
- $z = x + iy$: Complex variable.

The velocity potential and stream function are:

$$\phi = \mathrm{Re}(F(z)), \quad \psi = \mathrm{Im}(F(z))$$

1. Strength , m = 10 units
2. x = 2, y = 1.

4. Doublet Flow

The complex potential for a doublet flow is:

$$F(z) = -\frac{m}{2\pi z}$$

Where:

- m: Strength of the doublet.
- $z = x + iy$: Complex variable.

The velocity potential and stream function are:

$$\phi = \mathrm{Re}(F(z)), \quad \psi = \mathrm{Im}(F(z))$$

1. The strength of the doublet, m = 15 units.
2. Coordinates (x, y): The point in the flow field where you want to compute (ϕ) and (ψ). Take (x = -1), (y = 4).

5. Flow Past a Circular Cylinder

Problem Statement

To analyze the 2D flow past a circular cylinder using complex potential theory, where the goal is to calculate and visualize the velocity potential (ϕ) and stream function (ψ) in the flow field.

Complex Potential

The complex potential is given by:

$$F(z) = U \left(z + \frac{a^2}{z} \right)$$

where:

- U: Uniform velocity far from the cylinder.
- a: Radius of the cylinder.
- $z = x + iy$: Complex variable representing the position.

The velocity potential (ϕ) and stream function (ψ) are:

$$\phi = \mathrm{Re}(F(z)), \quad \psi = \mathrm{Im}(F(z))$$

Flow Characteristics

The flow exhibits:

- Symmetry about the horizontal axis passing through the cylinder's center.
- Streamlines (ψ) showing the flow patterns around the cylinder.
- Equipotential lines (ϕ) depicting the variation of velocity potential in the flow field.

Visualization

The streamlines and equipotential lines can be visualized using a computational tool, such as Python, with the following details:

- Generate a grid of points (x, y) to represent the flow field.
- Compute ϕ and ψ at each point in the grid using the complex potential.
- Plot ψ as contour lines to represent streamlines.
- Plot ϕ as contour lines to represent equipotential lines.
- Highlight the cylinder boundary in the plots to show its interaction with the flow.

1. Uniform Velocity ((U)): U = 5
2. a = 1.
3. Grid Points ((x, y)): The code automatically creates a grid (e.g., ([-3, 3]) in both (x) and (y)) to cover the area around the cylinder.

Mathematical Formulas

1. Complex potential:

$$F(z) = U\left(z + \frac{a^2}{z}\right)$$

2. Velocity potential (ϕ):

$$\phi = \mathrm{Re}\left(U\left(z + \frac{a^2}{z}\right)\right)$$

3. Stream function (ψ):

$$\psi = \mathrm{Im}\left(U\left(z + \frac{a^2}{z}\right)\right)$$