Lab Test on 03.04.25

1. Solve for the population size $P(t)$ over time using the equation $\frac{dP}{dt} = rP - \frac{bP^2}{K}$, where $r = 0.05$, $b = 0.02$, $K = 1000$, and $P(0) = 100$.

2. Solve the delay equation $\frac{dP}{dt} = rP(1 - P(t-\tau)/K)$ using numerical methods for $r = 0.02$, $\tau = 5$, $K = 300$, and $P(0) = 150$.

3. Solve the predator-prey system $\frac{dx}{dt} = ax - bxy$, $\frac{dy}{dt} = -cy + dxy$ for $a = 0.2$, $b = 0.01$, $c = 0.1$, $d = 0.02$, $x(0) = 30$, and $y(0) = 10$.

4. Compute the eigenvalues of the Jacobian matrix at equilibrium points for $x(a - bx - cy) = 0$, $y(-c + dxy) = 0$, where $a = 0.5$, $b = 0.02$, $c = 0.4$, $d = 0.01$.

5. Using SIR model equations, solve numerically for $S(t)$, $I(t)$, and $R(t)$ when $\frac{dI}{dt} = \beta S - \gamma I$, with $S(0) = 990$, $I(0) = 10$, $R(0) = 0$, $\beta = 0.4$, $\gamma = 0.1$, and $N = S + I + R = 1000$.

6. Compute the time to peak infection for $\frac{dI}{dt} = \beta S - \gamma I$, with initial conditions $S(0) = 1000$, $I(0) = 5$, $\beta = 0.03$, and $\gamma = 0.01$.

7. A uniform flow with velocity $U = 2\,\mathrm{m/s}$ interacts with a cylinder of radius $R = 1\,\mathrm{m}$.
   (a) Write a Python program to calculate the velocity field V_r and V_θ in polar coordinates around the cylinder.
   (b) Plot the streamlines of the flow.

8. Consider a laminar flow between two parallel plates separated by $H = 0.01\,\mathrm{m}$, with a pressure gradient of $\frac{\partial P}{\partial x} = -100\,\mathrm{Pa/m}$. Write Python code to compute the velocity profile $u(y)$ across the gap.

9. The upper plate moves with velocity $U = 0.5\,\mathrm{m/s}$, while the lower plate is stationary. The fluid viscosity is $\mu = 0.001\,\mathrm{Pa{\cdot}s}$. Develop a Python code to calculate the velocity profile $u(y)$ using the Navier-Stokes equations.

10. A source of strength $Q = 10\,\mathrm{m^2/s}$ is located at $(-2, 0)$ and a sink of equal strength is located at $(2, 0)$.
    (a) Write Python code to calculate the stream function $\psi(x, y)$.
    (b) Visualize the flow pattern by plotting streamlines.

11. Given the velocity field $\vec{V} = (2x, -3y)$, verify whether it satisfies the continuity equation for incompressible flow.
    (a) Write Python code to calculate $\nabla \cdot \vec{V}$.
    (b) Use your program to check if the flow is divergence-free.

12. Simulate the spread of an epidemic using the SIR model with $\beta = 0.3$, $\gamma = 0.1$, and initial populations $S_0 = 999$, $I_0 = 1$, $R_0 = 0$.

(a) Write Python code to solve the system of differential equations using the Runge-Kutta method.

(b) Plot $S(t), I(t), R(t)$ over time.

13. The Lotka-Volterra equations describe the prey-predator interaction:

$$\frac{dx}{dt} = 2x - 0.01xy, \quad \frac{dy}{dt} = -y + 0.02xy.$$

(a) Implement Python code to solve these equations numerically.

(b) Plot the phase-plane diagram $x(t)$ vs $y(t)$.

14. The complex potential for a doublet and uniform flow is $\Phi(z) = Uz + \frac{\kappa}{z}$, where $U = 1\,\mathrm{m/s}$, $\kappa = 1$.

(a) Write Python code to calculate the streamlines of this flow.

(b) Visualize the flow using contour plots.

15. For a logistic growth model with harvesting, $\frac{dP}{dt} = rP(1 - \frac{P}{K}) - h$, determine the critical harvesting rate h that leads to population collapse, given $r = 0.1$, $K = 500$, and $P(0) = 50$.