Optimization Technique Lab
Assignment 2: Basic Feasible Solution

Consider the following linear system

(P):

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m,\ m \leq n \end{aligned}$$

- Solution: Any vector $x = (x_1, x_2, \ldots, x_n)$ which satisfies this system is known as a solution of this system.
- Feasible solution: Any vector $x = (x_1, x_2, \ldots, x_n)$, with $x_j \geq 0$ for all $j = 1, 2, \ldots, n$ which satisfies this system is known as a feasible solution of this system.
- Basic solution: Any vector $x = (x_1, x_2, \ldots, x_n)$ which satisfies this system after substituting $n - m$ number of variables as 0 is known as the basic solution. These m variables are known as basic variables and $n - m$ variables are known as nonbasic variables. The maximum number of basic solutions is $C(n, m)$
- Basic feasible solution(BFS): If all components of a basic solution are nonnegative then it is known as a basic feasible solution.
- Degenerate solution: A basic solution in which one or more basic variables are zero-valued is known as a Degenerate solution.
- Non-Degenerate solution: A basic solution in which exactly m number of components are zero-valued is known as a Non-Degenerate solution.

Example:
$2x_1 + 6x_2 + 2x_3 + x_4 = 3$
$6x_1 + 4x_2 + 4x_3 + 6x_4 = 2$
Here $n - m = 2$. So put any two variables as zero and then solve for the rest. Maximum 6 possibilities exist.

Put $x_1 = 0, x_2 = 0$ and solve, then $x_3 = 2, x_4 = -1$. Solution is $(x_1, x_2, x_3, x_4) = (0, 0, 2, -1)$. This is a basic solution but not feasible.

Put $x_2 = 0, x_3 = 0$ and solve, then $x_1 = 8/3, x_4 = -7/3$. Solution is $(x_1, x_2, x_3, x_4) = (8/3, 0, 0, -7/3)$. This is a basic solution but not feasible.

Other solutions are
$(x_1, x_2, x_3, x_4) = (0, 1/2, 0, 0)$. This is a degenerate basic feasible solution.

$(x_1, x_2, x_3, x_4) = (-2, 0, 7/2, 0)$. This is a nondegenerate basic solution but not feasible.

* *

Assignment:

1. Manually solve the following system for Basic, Basic feasible, degenerate, and nondegenerate solutions. Basic and nonbasic variables of each solution

$3x_1 + 2x_2 + 4x_3 - 2x_4 + 5x_5 = 10$
$2x_1 + 4x_2 + 3x_3 + 2x_4 + x_5 = 12$

2. Using the Program of the previous LAB class on the Gauss-Seidel Method, classify the solutions of the following system

(I)

$$\begin{aligned} 3x_1 + 2x_2 + 4x_3 - 2x_4 + 5x_5 &= 10 \\ 2x_1 + 4x_2 + 3x_3 + 2x_4 + x_5 &= 12 \\ 4x_1 + 2x_2 + 3x_3 + 5x_4 + 3x_5 &= 15 \end{aligned}$$

(II)

$$\begin{aligned} 4x_1 + 2x_2 + 3x_3 &\leq 20 \\ x_1 - 3x_2 + 2x_3 &\leq 15 \end{aligned}$$

In II use slack variables and convert the system to linear eqations.

3. Write a Program in C or C + + to classify the solution of the general linear system (P). Modify your program on Gauss-Seidel Method Lab Assignment -1 of the previous class. In some cases, you may get a solution that does not exist by GS method. Incorporate that inside your program.

   Input: $a_{ij}, b_i, m, n, i = 1, 2, \ldots m,\ j = 1, 2, \ldots, n$.

   Output: Classify every solution, basic and nonbasic variables of each solution.

   Using this program, test Q1 and Q2.

   Construct your own system in more than 5 variables and solve using your program.

NOTE: SAVE YOUR PROGRAM ON YOUR PC. THIS PROGRAM WILL BE MODIFIED IN THE NEXT LAB CLASS. UPLOAD IN MOODLE.