Optimization Technique LAB

ASSIGNMENT 4(Computation of initial simplex table)

Consider the following linear programming, which can be solved by SIMPLEX method:

Conditions are: All the constraints should be $\leq$ type. All $b_i \geq 0, i = 1, 2, ..., m$. All $x_j \geq 0, j = 1, 2, ..., n$.The objective function should be maximization type.

(P):
max $\;c_1x_1 + c_2x_2 + ... + c_nx_n$
subject to
$a_{11}x_1 + a_{12}x_2 + \ldots a_{1n}x_n \leq b_1$
$a_{21}x_1 + a_{22}x_2 + \ldots a_{2n}x_n \leq \; b_2 \qquad (1)$
....
....
$a_{m1}x_1 + a_{m2}x_2 + \ldots a_{mn}x_n \leq b_m, \quad m \leq n$
All $x_j \geq 0$.

The standard form is:

max $\;c_1x_1 + c_2x_2 + ... + c_nx_n + 0.x_{n+1} + 0.x_{n+2} + ... + 0.x_{n+m}$
$a_{21}x_1 + a_{22}x_2 + \ldots a_{2n}x_n \qquad + x_{n+2} \qquad\qquad = \; b_2$
....
....
$a_{m1}x_1 + a_{m2}x_2 + \ldots a_{mn}x_n \qquad\qquad\qquad + x_{n+m} = b_m$
All $x_j \geq 0, j = 1, 2, ..., x_{n+m}$.

Notation:

- $X_j = (p_{1j}, p_{2j}, ..., p_{mj})^T$ is the j^{th} column of this system. For all $i = 1, 2, ..., m$, $p_{ij} = a_{ij}$ if $j \leq n$, $p_{ij} = 1$ if $j > n, j = n + i$ and $p_{ij} = 0$ if $j > n, j \neq n + i$
- $b = (b_1, b_2, ..., b_m)^T$ column vector.
- Basis B. This is a row vector of basic variables. In this standard form it is $(x_{n+1}, x_{n+2}, ..., x_{n+m})$. The basis will change in every iteration of the simplex table, which will be used later.

- $X_B = (.....)^T$ is a column vector, whose components are the value of basic variables, which you will get after substituting nonbasic variables as zero in every iteration. In the standard form, $X_B = (b_1, b_2, ..., b_m)$. This will change in other iterations later.
- NB is the vector of nonbasic variables. This will change in other iterations. In the standard form $NB = (x_1, x_2, ..., x_n)$. The value of every nonbasic variable is zero.
- $C = (C_1, C_2, ..., C_n, C_{n+1}, C_{n+2}, ..., C_{n+m})$, is a row for coefficients of basic and non-basic variables in the objective function.
- C_B is the column vector, whose components are the value of c_j corresponding to basic variable x_j, i.e, $x_j \in B$ and C_{NB} is the vector of the value of c_j corresponding to non-basic variable x_j i.e $x_j \in NB$.
- $\Delta_j = z_j - c_j = C_B^T X_j - c_j$, $j = 1, 2, ..., n+m$

		C_1	C_2	$\dots$	C_n	C_{n+1}	C_{n+2}	$\dots$	C_{n+m}	
X_B	C_B	X_1	X_2	$\dots$	X_n	X_{n+1}	X_{n+2}	$\dots$	X_{n+m}	b
x_{n+1}	c_{n+1}	a_{11}	a_{12}	$\dots$	a_{1n}	1	0	$\dots$	0	b_1
x_{n+2}	c_{n+2}	a_{21}	a_{22}	$\dots$	a_{2n}	0	1	$\dots$	0	b_2
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
x_{n+m}	c_{n+m}	a_{m1}	a_{m2}	$\dots$	a_{mn}	0	0	$\dots$	1	b_n
$X_B^T C_B$		Δ_1	Δ_2	$\dots$	Δ_n	Δ_{n+1}	Δ_{n+2}	$\dots$	Δ_{n+m}	

Table 1: Simplex Table

$$\Delta_j = C_B^T X_j - C_j$$

Initial SIMPLEX table: ASSIGNMENT:

Q 1. Develop code in C/C++ to write the given LPP, which is suitable for the simplex method, to standard form and print the initial table, $b,\ B,\ C_B,\ X_B,\ \Delta_j$ for every $j = 1, 2, ..., n+m$.

Q2. Consider the following LPP and verify your code for the following output:

(a)$b,\ B,\ C_B,\ X_B,\ \Delta_j$ for every j

(b)Initial Table

$Minimize\ \ 3x_1 + 2x_2 - 4x_3 - x_4$
subject to
$3x_1 - x_2 + 2x_3 - 5x_4 \leq 10$
$3x_1 + 2x_2 - x_3 + x_4 \leq 4$
$3x_1 + 2x_2 - 3x_3 + 5x_4 \leq 5$
$x_1, x_2, x_3, x_4 \geq 0$

Standard form;
$Maximize\ \ -3x_1 - 2x_2 - -4x_3 + x_4$
subject to
$3x_1 - x_2 + 2x_3 - 5x_4 + x_5 \qquad = 10$
$3x_1 + 2x_2 - x_3 + x_4 \quad + x_6 = 4$
$3x_1 + 2x_2 - 3x_3 + 5x_4 \qquad x_7 \leq 5$
$x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0$

$b = (10, 4, 5)^T$
$B = (x_5, x_6, x_7)^T$
$X_B = (10, 4, 5)^T$
$C_B = (0, 0, 0)^T$
$\Delta_1 = 3, \Delta_2 = 2, \Delta_3 = -4, \Delta_4 = -1, \Delta_5 = 0, \Delta_6 = 0, \Delta_7 = 0$

		-3	-2	4	1	0	0	0	
X_B	C_B	X_1	X_2	X_3	X_4	X_5	X_6	X_7	b
x_5	0	3	-1	2	-5	1	0	0	10
x_6	0	3	2	-1	1	0	1	0	4
x_7	0	3	2	-3	5	0	0	1	5
$X_B^T C_B = 0$		3	2	-4	-1	0	0	0	

Table 2: Simplex Table