Optimization Technique LAB

ASSIGNMENT 5

First, follow the steps of the simplex method of this example and solve one problem manually. Then modify Assignment 4 code for the general simplex method and upload it in Moodle.

Example. Max. $z = 3x_1 + 2x_2 + 5x_3$, subject to the constraints:

$$x_1 + 2x_2 + x_3 \leq 430, 3x_1 + 2x_3 \leq 460, x_1 + 4x_2 \leq 420, \text{ and } x_1, x_2, x_3 \geq 0.$$

This problem is suitable for the Simplex method since all variables are positive, constraints are less than equal to type and all b_j are positive.
Convert this to standard form:
Standard form;
$Maximize\ \ 3x_1 + 2x_2 + 5x_3$
subject to
$x_1 + 2x_2 + x_3 + x_4 \qquad\qquad = 430$
$3x_1 \quad + 2x_3 \quad + x_5 = 460$
$x_1 + 4x_2 \qquad\quad x_6 = 420$
$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$ Write the initial table as in Assignment 4. Calculate all Δ_j and objective value $z = C_B^T X_B$
Initial table:

$\Delta_1 = -3, \Delta_2 = -2, \Delta_3 = -5, \Delta_4 = 0, \Delta_5 = 0, \Delta_6 = 0$

The basic feasible solution at this stage is $(x_1, x_2, x_3, x_4, x_5, x_6) = (0, 0, 0, 430, 460, 420)$
The objective value is:$z = X_B^T C_B = 0$
Basis=(x_4, x_5, x_6)
All Δ_j are not positive.
Hence optimal solution is not reached. We have to move to the next iteration as follows.
Select the column corresponding to the most negative Δ_j. Let the most negative value occur corresponding to X_k column. Here, most negative $\Delta_j = -5$

			3	2	5	0	0	0	
$Basic\ Var$	C_B	X_B	X_1	X_2	X_3	X_4	X_5	X_6	X_B/X_k
x_4	0	430	1	2	1	1	0	0	430/1
x_5	0	460	3	0	2	0	1	0	460/2
x_6	0	420	1	4	0	0	0	1	$xxxx$
$x_1 = x_2 = x_3 = 0$		$z = X_B^T C_B = 0$	-3	-2	-5	0	0	0	

Table 1: Simplex Table

occurs corresponding to X_3 column. This column is known as the pivot column.
Divide the column X_B by the pivot column componentwise corresponding to those components of the pivot column which has a strictly positive quantity. Then consider it's minimum. This is known as the minimum ratio rule. Here we have:
$\min\{430/1, 460/2\} = 460/2$. The minimum ratio occurs at 2nd row. This row is known as the pivot row.
Pivot row corresponds to x_5 variable and Pivot column corresponds to x_3 variable.
Decision: x_5 will be removed from the basis and x_3 will enter into the basis. Their intersecting element is known as the pivot element. Here the pivot element is 2.
Using the pivot element and pivot row, modify the table using row operation so that new basis can be accommodated. In general, this can be done as follows:
Consider all the columns $X_B, X_1, X_2, ...X_{n+m}$

$$X_B = \begin{pmatrix} x_{B1} \\ x_{B2} \\ .. \\ .. \\ x_{Bm} \end{pmatrix} \text{ and } X_j = \begin{pmatrix} p_{1j} \\ p_{2j} \\ .. \\ .. \\ p_{mj} \end{pmatrix}, j = 1, 2, ..., m+n.$$

If s^{th} row is the pivot row and k^{th} column is the pivot column then p_{sk} is the pivot element. In the new table,

$$p_{ij} \to \begin{pmatrix} p_{ij}/p_{sk} & for \quad i = s, j = 1, 2, ..., n+m \\ p_{ij} - (p_{sj}/p_{sk})p_{ik} & for \quad i \neq s, j = 1, 2, ..., n+m \end{pmatrix}$$

and

$$x_{B_i} \to \begin{pmatrix} x_{Bi}/p_{sk} & for \quad i = s \\ x_{Bi} - (x_{Bs}/p_{sk})p_{ik} & for \quad i \neq s \end{pmatrix}$$

As a result of this transformation, x_s will be removed from the basis and x_k will enter into the basis.

In our example, $X_1 = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix} \to \begin{pmatrix} 1 - (3/2).1 \\ 3/2 \\ 1 - (3/2).0 \end{pmatrix} = \begin{pmatrix} -1/2 \\ 3/2 \\ 1 \end{pmatrix}$

$$X_2 = \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix} \to \begin{pmatrix} 2 - (0/2).1 \\ 0/2 \\ 4 - (0/2).0 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix}$$

$$X_3 = \begin{pmatrix}1\\2\\0\end{pmatrix} \rightarrow \begin{pmatrix}1-(2/2).1\\2/2\\0-(2/2).0\end{pmatrix} = \begin{pmatrix}0\\1\\0\end{pmatrix} (\ \textit{this is the new basis vector})$$

$$X_4 = \begin{pmatrix}1\\0\\0\end{pmatrix} \rightarrow \begin{pmatrix}1-(0/2).1\\0/2\\0-(0/2).0\end{pmatrix} = \begin{pmatrix}1\\0\\0\end{pmatrix}$$

$$X_5 = \begin{pmatrix}0\\1\\0\end{pmatrix} \rightarrow \begin{pmatrix}0-(1/2).1\\1/2\\0-(1/2).0\end{pmatrix} = \begin{pmatrix}-1/2\\1/2\\0\end{pmatrix} (\textit{this is no more a basis vector})$$

$$X_6 = \begin{pmatrix}0\\0\\1\end{pmatrix} \rightarrow \begin{pmatrix}0-(0/2).0\\0/2\\1-(0/2).1\end{pmatrix} = \begin{pmatrix}0\\0\\1\end{pmatrix}$$

$$X_B = \begin{pmatrix}430\\460\\420\end{pmatrix} \rightarrow \begin{pmatrix}430-(460/2).1\\460/2\\420-(0/2).0\end{pmatrix} = \begin{pmatrix}200\\230\\420\end{pmatrix}$$

These can be summarized in a table as follows: Modified table(Iteration 2):
All Δ_j are not positive.

			3	2	5	0	0	0	
Basic Var	C_B	X_B	X_1	X_2	X_3	X_4	X_5	X_6	X_B/X_k
x_4	0	200	$-1/2$	2	0	1	-1/2	0	
x_3	5	230	$3/2$	0	1	0	$1/2$	0	
x_6	0	420	1	4	0	0	0	1	
$x_1 = x_2 = x_5 = 0$		$z = X_B^T C_B = 1150$	$9/2$	-2	0	0	$5/2$	0	

Table 2: Iteration 2

Hence optimal solution is not reached. We have to move to the next iteration
The basic feasible solution at this stage is $(x_1, x_2, x_3, x_4, x_5, x_6) = (0, 0, 230, 200, 0, 420)$
The objective value is:$z = X_B^T C_B = 1150$
Basis=(x_4, x_3, x_6)
Similarly do other iterations. The complete simplex table is :

		$c_j \rightarrow$	3	2	5	0	0	0	
Basic Variables	C_B	X_B	X_1	X_2	X_3	X_4	X_5	X_6	Min Ratio (X_B/X_k)
x_4	0	430	1	2	1	1	0	0	$430/1$
$\leftarrow x_5$	0	460	3	0	$\boxed{2}$	0	1	0	$460/2 \leftarrow$
x_6	0	420	1	4	0	0	0	1	$-$
$x_1 = x_2 = x_3 = 0$	$z = 0$		-3	-2	-5^*	0	0	0	$\leftarrow \Delta_j$
					$\uparrow$		$\downarrow$		
$\leftarrow x_4$	0	200	$-1/2$	$\boxed{2}$	0	1	$-1/2$	0	$200/2 \leftarrow$
$\rightarrow x_3$	5	230	$3/2$	0	1	0	$1/2$	0	$-$
x_6	0	420	1	4	0	0	0	1	$420/4$
$x_1 = x_2 = x_5 = 0$	$z = 1150$		$9/2$	-2^*	0	0	$5/2$	0	$\leftarrow \Delta_j$
				$\uparrow$		$\downarrow$			
$\rightarrow x_2$	2	100	$-1/4$	1	0	$1/2$	$-1/4$	0	
x_3	5	230	$3/2$	0	1	0	$1/2$	0	
x_6	0	20	2	0	0	-2	1	1	
$x_1 = s_4 = x_5 = 0$	$z = 1350$		4	0	0	1	2	0	$\leftarrow \Delta_j \geq 0$

Since all $\Delta_j \geq 0$, the solution is: $x_1 = 0, x_2 = 100, x_3 = 230$, max z=1350.

ASSIGNMENT 5(SIMPLEX METHOD CONTINUED)

Q1. Solve the following LPP by the simplex method manually. Show the results in tabular form. At every iteration find the basic feasible solution, basic variables, nonbasic variables, objective value, pivot row, pivot column, and pivot element with its value.
Maximize $x_1 + 2x_2 + x_3$
subject to
$2x_1 + x_2 - x_3 \leq 2$,
$-2x_1 + x_2 - 5x_3 \geq -6$,
$4x_1 + x_2 + x_3 \leq 6$, $x_j \geq 0$
Ans: Maximum value:10
Optimal solution: $(x_1, x_2, x_3) = (0, 4, 2)$

Q2 . Modify your code for Assignment 4 and develop code for the general simplex method to solve a linear programming problem.

The following are the key points of the simplex method code.

- Check if the problem is suitable for the simplex method or not.(unrestricted variables should be taken care)
- Convert the problem to standard form.
- Write the initial table,C_B, x_B, c_j and Δ_j for all j.This is Iteration 1.

- Declare the basic feasible solution and objective value at Iteration 1.
- If all $\Delta_j \geq 0$ then this basic feasible solution is the optimal solution and the corresponding objective value is the optimal value. Otherwise, change the table as explained in the example.
- Continue the process until all Δ_j are positive.

OUTPUT:

- Total number of iterations=–
- at every iteration
  1. Basic feasible solution—
  2. Basic variables—
  3. Nonbasic variables—
  4. Δ_j for all j
  5. Pivot element and it's value—–
  6. Incoming variable—-
  7. outgoing variable—–
  8. Minimum ratio—–
  9. objective value—-
- Optimal solution—
- Optimal value—–
- Values of Δ_j

NOTE: Try to complete it by 5 PM today and upload it in Moodle. If you are not able to complete then you will upload it in the next lab class. Moodle will be closed at 5 PM and will be opened in the next class.