Optimization Technique Lab
Assignment 3: Solution of LPP using BFS

Consider the following linear programming:

$$
\begin{aligned}
&(P):\\
&\max \quad c_1x_1 + c_2x_2 + \ldots + c_nx_n\\
&a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1\\
&a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2 \qquad (1)\\
&\ldots\ldots\ldots\ldots\ldots\ldots\\
&\ldots\ldots\ldots\ldots\ldots\ldots\\
&a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n = b_m, \quad m \le n\\
&\text{All } x_j \ge 0
\end{aligned}
$$

Denote $f(x) = c_1x_1 + c_2x_2 + \ldots + c_nx_n$. If $X_1, X_2, \ldots X_k$ are the BFS (Basic feasible solution of the linear system (1)), then the optimal value of this LPP is

$$\max\left\{f\left(X_1\right), f\left(X_2\right), \ldots, f\left(X_k\right)\right\}$$

If the maximum occurs at X_i then X_i is the optimal solution of the LPP. Note that the optimal solution is not necessarily unique, whereas the optimal value is unique.
Similar logic should be used for the minimization problem.
Example: $Maximize$ $2x_1 + 3x_2 - x_3 + x_4$ subject to

$$
\begin{aligned}
&x_1 + x_2 - 2x_3 + x_4 = 1\\
&2x_1 - x_2 + x_3 + x_4 = 4\\
&x_1, x_2, x_3, x_4 \ge 0
\end{aligned}
$$

Basic solutions are: $A_1 = (0, -9, -5, 0)$

$$
\begin{aligned}
A_2 &= (9/5, 0, 4/10, 0)\\
A_3 &= (5/3, -2/3, 0, 0)\\
A_4 &= (0, -3/2, 0, 5/2)\\
A_5 &= (3, 0, 0, -2)\\
A_6 &= (0, 0, 1, 3)
\end{aligned}
$$

Basic feasible solutions are : A_2, A_6
Optimal value is : $maximum\ \{f\left(A_2\right), f\left(A_6\right)\} = maximum\ \{16/5, 2\} = 16/5$
Optimal solution is $(x_1, x_2, x_3, x_4) = (9/5, 0, 4/10, 0)$

ASSIGNMENT QUESTIONS:

1. Find the optimal value and optimal solution of the following problem manually.

Minimize $x_1 - 3x_2 + 2x_3 - x_4 + x_5$ subject to
$x_1 + x_2 - 2x_3 + x_4 + x_5 = 5$
$2x_1 - x_2 + x_3 + x_4 - x_5 = 4$
$x_1, x_2, x_3, x_4, x_5 \geq 0$

2. Write a program in C/C++ to find the optimal value and optimal solution of the following linear programming problem P. Modify your Assignment 2 code to get all basic feasible solutions. Upload the code in Moodle. Check the answers to the above two examples using your code. You may construct some new problems and verify your code. The output should be

- List of basic feasible solutions.
- Objective value at each basic feasible solution.
- Optimal value of the LPP.
- Optimal solution of the LPP.

(P):

$$
\begin{aligned}
&\text{max or min} \quad c_1x_1 + c_2x_2 + \ldots + c_nx_n \\
&a_{11}x_1 + a_{12}x_2 + \ldots a_{1n}x_n = b_1 \\
&a_{21}x_1 + a_{22}x_2 + \ldots a_{2n}x_n = b_2 \qquad (1) \\
&\ldots\ldots\ldots\ldots\ldots\ldots \\
&\ldots\ldots\ldots\ldots\ldots\ldots \\
&a_{m1}x_1 + a_{m2}x_2 + \ldots a_{mn}x_n = b_m, \quad m \leq n \\
&\text{All } x_j \geq 0
\end{aligned}
$$

Note that it is not always possible to find all basic feasible solutions by Assignment 2 code as the GS method is applicable under some conditions. To get rid of this difficulty we will develop code for SIMPLEX method in the next LAB. Therefore, you should read the simplex method before next week's LAB class.