Optimization Technique LAB

ASSIGNMENT 3B(Standard form with basis)

Consider the following linear programming:

(P):
$\max or \min \quad c_1x_1 + c_2x_2 + ... + c_nx_n$
$a_{11}x_1 + a_{12}x_2 + \ldots a_{1n}x_n \leq \ or \ = or \geq \ b_1$
$a_{21}x_1 + a_{22}x_2 + \ldots a_{2n}x_n \leq \ or \ = or \geq \ b_2 \qquad (1)$
.....
.....
$a_{m1}x_1 + a_{m2}x_2 + \ldots a_{mn}x_n \leq \ or \ = or \geq \ b_m, \quad m \leq n$
All $x_j \geq 0$

- Convert minimization to maximization form(min $f(x) = -max(-f(x))$)
- If any x_j is unrestricted then replace this by $x_j^1 - x_j^2$ in the entire system.
- All b_i should be non-negative. Change the constraints accordingly.
- Using slack, surplus, and artificial variables convert each constraint to an equality constraint so that the whole linear system will have a basis, which is an identity matrix of order m.

The transformed problem is in standard form.
Example:
Convert the following LPP to standard form:
$Minimize \ \ x_1 - 3x_2 + 2x_3 - x_4 + x_5$
subject to
$x_1 + x_2 - 2x_3 + x_4 + x_5 \leq 5$
$2x_1 - x_2 + x_3 + x_4 - x_5 \geq 4$
$3x_1 - 2x_2 + x_3 + 2x_4 - 3x_5 = 10$
$x_1, x_3, x_4, x_5 \geq 0$

Standard form:
$Maximize \ \ -x_1 + 3(a-b) - 2x_3 + x_4 - x_5$ subject to
$x_1 + (a-b) - 2x_3 + x_4 + x_5 + x_6 = 5$
$2x_1 - (a-b) + x_3 + x_4 - x_5 - x_7 + x_8 = 4$
$3x_1 - 2(a-b) + x_3 + 2x_4 - 3x_5 + x_9 = 10$
$x_j \geq 0 \forall j = 1, 3, 4, 5, 6, 7, 8, 9 \ \ and \ \ a, b \geq 0$

Here x_6 is a slack variable, x_7 is a surplus variable, and x_8, x_9 are artificial variables. Since x_2 unrestricted so it is replaced by $a - b$ with $a, b \geq 0$
The basis is (x_6, x_8, x_9), whose columns form the identity matrix.
ASSIGNMENT

Q. Write a program in C/C++ to convert a general linear programming problem (P) into standard form and declare the basis.
Using your code convert the following LPP to standard form. Print the list of slack, surplus, and artificial variables. Print the basis and the standard form.

1. $Maximize \;\; 3x_1 + 2x_2 - 4x_3 - x_4$
   subject to
   $3x_1 - x_2 + 2x_3 - 5x_4 \geq -10$
   $3x_1 + 2x_2 - x_3 + x_4 \leq 4$
   $3x_1 + 2x_2 - 3x_3 + 5x_4 = 5$
   $x_1, x_2, x_3, x_4 \geq 0$
   Ans:
   $Maximize \;\; 3x_1 + 2x_2 - 4x_3 - x_4$
   subject to
   $-3x_1 + x_2 - 2x_3 + 5x_4 + x_5 = 10$
   $3x_1 + 2x_2 - x_3 + x_4 + x_6 = 4$
   $3x_1 + 2x_2 - 3x_3 + 5x_4 + x_7 = 5$
   $x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0$
   x_5 and x_6 is a slack variable, x_7 artificial variable. Basis is (x_5, x_6, x_7).

2. $Minimize \;\; 2x_1 - 3x_2 - 3x_3 - x_4$
   subject to
   $2x_1 + x_2 - 3x_3 + x_4 \geq 5$
   $3x_1 - 2x_2 + x_3 + x_4 = -3$
   $x_1 + 3x_2 + 2x_3 + 2x_4 \leq 2$
   $x_1, x_2, x_3 \geq 0$, x_4 is unrestricted.
   Ans:
   $Maximize \;\; -2x_1 + 3x_2 + 3x_3 + x_4^{'} - x_4^{''}$
   subject to
   $2x_1 + x_2 - 3x_3 + x_4^{'} - x_4^{''} - x_5 + x_6 = 5$
   $-3x_1 + 2x_2 - x_3 - x_4^{'} + x_4^{''} + x_7 = 3$
   $x_1 + 3x_2 + 2x_3 + 2x_4^{'} - 2x_4^{''} + x_8 = 2$
   $x_1, x_2, x_3, x_4^{'}, x_4^{''}, x_5, x_6, x_7, x_8 \geq 0.$
   x_8 is a slack variable. x_5 is a surplus variable. x_7, x_6 are artificial variables.
   The basis is (x_6, x_7, x_8).