1. Let us suppose, you have tossed two two-sided fair coins.
   a. Compute the PMF for heads in this experiment

   Ans: Outcome space={HH,HT,TH,TT}. Number of heads= {0,1,2}

   PMF= PX(k)= P(X=k)

   P(Heads=0)=1/4

   P(Heads=1)=2/4= 1/2

   P(Heads=2)= 1/4

   PX(k)= 1/4 for k=0

   1/2 for k=1

   1/4 for k=2

   0 otherwise

   PMF for the experiment

X	0	1	2
P(X)	1/4	1/2	1/4

   b. Compute Expectation of heads

   Ans: E(X)= ∑xP(x)

   = (0*1/4)+(1*1/2) +(2*1/4)

   = 0+1/2 +1/2=1

2. For a given probability density function, calculate

$$f(x) = \begin{cases} 3x^{-4}, & x > 1 \\ 0, & elsewhere \end{cases}$$

i) P(X = 2)
ii) P(X <= 4)
iii) P(X < 1)
iv) P(2 <= X <= 3)

Ans:

i) P(X = 2) By definition of PDF, its 0

ii) P(X<= 4)

$$P(X \leq 4) = \int_1^4 3x^{-4}\ \mathrm{dx}$$

$$P(X \leq 4) = \left[-x^{-3}\right]_1^4$$

$$P(X \leq 4) = -(4)^{-3} - -(1)^{-3}$$

$$P(X \leq 4) = -\frac{1}{64} + 1$$

$$P(X \leq 4) = \frac{63}{64}$$

iii) P(X<1) = 0

iv) P(2<=X<=3) = $\int_2^3 3x^{-4}$ dx

$$= \left[-x^{-3}\right]_2^3$$

$$= -\frac{1}{27} - -\frac{1}{8}$$

$$= \frac{19}{216}$$

ii) and iii) using R

```
f = function(x){3*x^(-4)}
integrate(f, lower = 1, upper = 4)
integrate(f, lower = 2, upper = 3)
```

3. The joint probability distribution of two random variables X and Y is given by:

   $P(X = 0, Y = 1) = \frac{1}{3}$, $P(X = 1, Y = -1) = \frac{1}{3}$, $P(X = 1, Y = 1) = \frac{1}{3}$.

   Find i) Marginal distribution of X and Y.
   ii) Conditional probability distribution of X given Y=1.

   Ans. The range of X is {0,1} and the range of Y is {-1,1}. The Joint distribution table is

X / Y	0	1	Marginal Y
-1	0	1/3	1/3
1	1/3	1/3	2/3
Marginal X	1/3	2/3	

The probability of X = 0 is
P(X=0)
= P(X=0,Y=-1) + P(X=0,Y=1)
= 0+1/3 = 1/3.

The probability of X = 1 is
P(X=1)
= P(X=1,Y=-1) + P(X=1,Y=1)
= 1/3+1/3 = 2/3.

Therefore, the marginal distribution of X is

x	0	1
p_x	1/3	2/3

Similarly, the marginal distribution of Y is

y	-1	1
P_y	1/3	2/3

ii) Now

P(X=0|Y=1) = $\frac{P(X=0,Y=1)}{P(Y=1)} = \frac{1/3}{2/3} = \frac{1}{2}$ and P(X=1|Y=1) = $\frac{P(X=1,Y=1)}{P(Y=1)} = \frac{1/3}{2/3} = \frac{1}{2}$

Thus the conditional distribution of X given Y = 1 is

x	0	1
P(X=x|Y=1)	1/2	1/2

4. Twelve volunteers were chosen for a blind-fold test to taste 2 soft-drinks A & B. What is the probability that 3 of them were able to correctly identify the drink that they had?

Ans: Binomial distribution with n = 12 and p = 0.5 and q = (1-p) = 0.5 $P(X=r) = {}^{n}C_{r} \times p^{r} \times q^{(n-r)}$

Hence $P(X = 3) = {}^{12}C_{3} \times (0.5)^{3} \times (0.5)^{9}$ * =0.05371

R: dbinom (3, 12, 0.5) = 0.05371

5. Customers arrive at a bus station at the rate of 5 per minute following Poisson distribution. What is the probability of 3 arrivals in a one-minute interval?

Ans: Poisson distribution:

$P(X = r) = (\lambda^{r} \times e^{-\lambda}) / r!$

$\lambda = 5$, $r = 3$

R: dpois (3, 5) = 0.1404

6. Player A scores an average of 70 runs with SD of 20 runs. Player B scores an average of 40 runs with SD of 10 runs. In a particular game, player A scored 75 runs and player B scored 55 runs. Which of these two players have done better when compared to their own personal track records?

Ans:

$\mu_A = 70$, $\sigma_A = 20$;

$\mu_B = 40$, $\sigma_B = 10$;

$Z = (x-\mu) / \sigma$

$Z_A = (75-70) / 20 = 0.25$ and $Z_B = (55-40) / 10 = 1.5$

The one with higher Z value has done better against their personal track records. Therefore player B has done better compared to his personal track record.

7. A college basketball team has a shortage of one team member and the coach wants to recruit a player. To be selected for training the minimum height for recruitment is 72 inches. The average height of the students is 67.2 inches with a variance of 29.34. What is the probability that the coach finds a player from that college?

Ans:

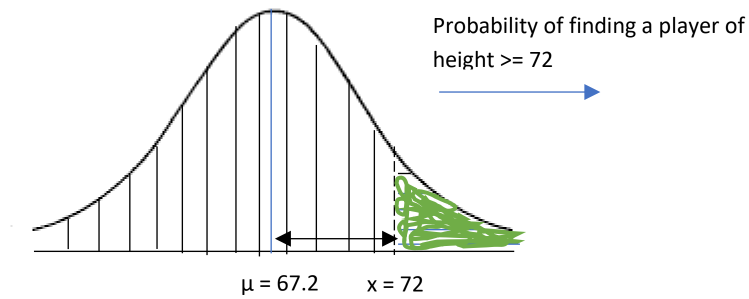
μ = 67.2; σ2 = 29.34, σ = 5.41, x = 72

Z = (72-67.2)/5.41 = 0.8872

Using Z-table

P(X<72) = P (Z = 0.88) = 0.811

P(X>=72) = 1-0.811 = 0.189 = 18.9% probability

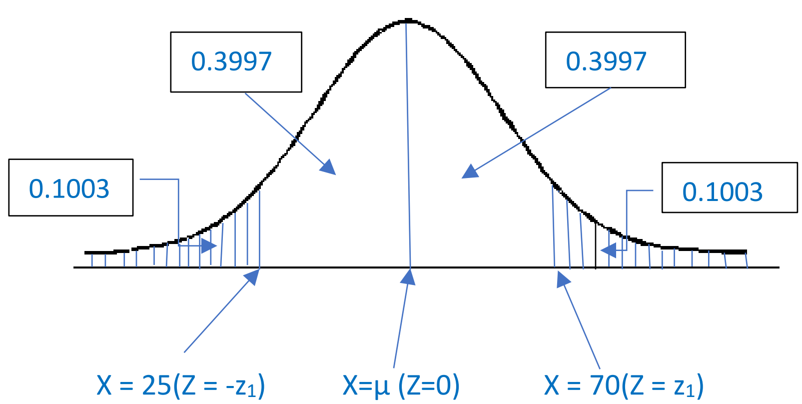


R: 1-pnorm(72,67.2, 5.41) OR 1-pnorm(z-score) i.e. 1-pnorm(0.8872)

8. In a distribution exactly normal, 10.03% of the items are under 25 kilogram weight and 89.97% of the items are under 70 kilogram weight. What are the mean and standard deviation of the distribution?

Ans. Let X denotes the weight of the items. If $X \sim N(\mu, \sigma^2)$, then we are given

P(X<25) = 0.1003 and P(X<70) = 0.8997. Therefore, P(X>70) = 0.1003.



We have seen from the above graph that X = 25 and X = 70, are symmetric points under the normal curve, therefore, if we standardize X, then

when X = 25, $Z = \frac{25-\mu}{\sigma} = -z_1$ (say) and when X = 70, $Z = \frac{70-\mu}{\sigma} = z_1$ (say).

Therefore, $P(Z < -z_1) = 0.1003$, implies $-z_1 = -1.28$

and hence, $z_1 = 1.28$ by symmetry.

Therefore, we have

$$\frac{25-\mu}{\sigma} = -1.28 \rightarrow 25 - \mu = -1.28\sigma,$$

and $$\frac{70-\mu}{\sigma} = 1.28 \rightarrow 70 - \mu = 1.28\sigma.$$

Solving, we get $\mu = 47.5$ and $\boldsymbol{\sigma = 17.58}$ **.**