

1. Player A scores an average of 70 runs with SD of 20 runs. Player B scores an average of 40 runs with SD of 10 runs. In a particular game, player A scored 75 runs and player B scored 55 runs. Which of these two players have done better when compared to their own personal track records?

Ans:

$$\mu_A = 70, \sigma_A = 20;$$

$$\mu_B = 40, \sigma_B = 10;$$

$$Z = (x - \mu) / \sigma$$

$$Z_A = (75 - 70) / 20 = 0.25 \text{ and } Z_B = (55 - 40) / 10 = 1.5$$

The one with higher Z value has done better against their personal track records. Therefore player B has done better compared to his personal track record.

2. A college basketball team has a shortage of one team member and the coach wants to recruit a player. To be selected for training the minimum height for recruitment is 72 inches. The average height of the students is 67.2 inches with a variance of 29.34. What is the probability that the coach finds a player from that college?

Ans:

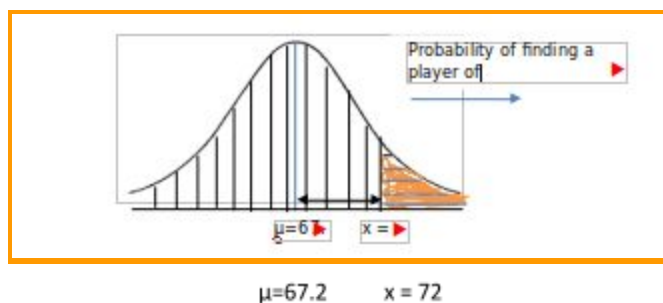
$$\mu = 67.2; \sigma^2 = 29.34, \sigma = 5.41, x = 72$$

$$Z = (72 - 67.2) / 5.41 = 0.8872$$

Using Z-table

$$P(X < 72) = P(Z = 0.88) = 0.811$$

$$P(X \geq 72) = 1 - 0.811 = 0.189 = 18.9\% \text{ probability}$$



$$R: 1 - \text{pnorm}(72, 67.2, 5.41) \text{ OR } 1 - \text{pnorm}(z\text{-score}) \text{ i.e. } 1 - \text{pnorm}(0.8872)$$

3. The engines made by Ford for speedboats had an average power of 220 horsepower (HP) and standard deviation of 15 HP. A potential buyer intends to take a sample of forty engines and will not place an order if the sample mean is less than 215 HP. What is the probability that the buyer will not place an order?

Sol:

Assign Variables

mu= 220

sigma= 15

Value= 215

n=40

se = sigma/sqrt(n)

zScore = (Value-mu)/se

zScore

pnorm(zScore)

pnorm(q = Value, mean = mu,sd = se)

4. A company manufactures rice in 10 kg bags with a standard deviation of 1.25 kg per bag. What is the probability that a random sample of 15 bags will have a mean between 9 and 9.5 kgs?

Ans. Let X_i denotes the amount of rice in i -th ($i = 1, 2, \dots, 15$) bag.

$E(X_i) = 10$ and $SD(X_i) = 1.25$.

Using CLT, we have \bar{X} , the sample mean, follows $N\left(10, \frac{1.25}{\sqrt{15}}\right)$.

$$P\left(9 < \bar{X} < 9.5\right)$$

$$= P\left(\frac{9-10}{\frac{1.25}{\sqrt{15}}} < \frac{\bar{X}-10}{\frac{1.25}{\sqrt{15}}} < \frac{9.5-10}{\frac{1.25}{\sqrt{15}}}\right)$$

$$= P(-3.09 < Z < -1.55)$$

[Z is a standard normal variable]

$$= \text{pnorm}(-1.55) - \text{pnorm}(-3.09)$$

= 0.059.

Therefore, the probability that a random sample of 15 bags will have a mean between 9 and 9.5 kgs is 0.059.

5. A random sample of 100 items is taken, producing a sample mean of 49. The population std. deviation is: 4.49. Construct a 90% confidence interval for the population mean.

Sol: $n = 100$, sample $\bar{x} = 49$, population standard deviation $\sigma = 4.49$;

90% confidence interval, $z = -1.64, +1.64$; R: `qnorm(0.05)` and `qnorm(0.95)`

$$C.I. = \bar{x} \pm z * \sigma / \sqrt{n}$$

$$C.I. = 49 - 1.64 \times 4.49 / \sqrt{100}; 49 + 1.64 \times 4.49 / \sqrt{100}$$

$$C.I. = (48.2, 49.7)$$

Rcode:

`n = 100`

`xbar = 49`

`cat("\n Sample Mean = ",xbar)`

`sigma = 4.49`

`se = sigma/sqrt(n)`

`cat("\n Standard Error= ",se)`

`alpha = 0.1`

`critical_value = qnorm(alpha/2,lower.tail = F)`

`cat("\n Critical Value = ",critical_value)`

*`margin_Of_Error = critical_value * se`*

`cat("\n Margin_Of_Error = ",margin_Of_Error)`

90% confidence interval, $z = -1.64$ to $+1.64$; R: `qnorm(0.05)` and `qnorm(0.95)`

```
upper_interval = xbar + margin_Of_Error
lower_interval = xbar - margin_Of_Error
cat("\n 90% confidence interval")
cat("\n lower_interval = ",lower_interval)
cat("\n upper_interval = ",upper_interval)
```

6. The average zinc concentration recovered from a sample of zinc measurements in 36 different locations is found to be 2.6 grams per milliliter. Find the 95% and 99% confidence intervals for the mean zinc concentration in the river. Assume that the population standard deviation is 0.3. Does the increase in the confidence level increases the confidence interval?

Sol: The point estimate $\bar{x} = 2.6$.

95% confidence interval, $z = -1.96$ to $+1.96$; R: `qnorm(0.025)` and `qnorm(0.975)`

99% confidence interval, $z = -2.57$ to $+2.57$; R: `qnorm(0.005)` and `qnorm(0.995)`

*C.I (95%) = $2.6 \pm 1.96 * 0.3 / \sqrt{36}$ C.I (99%) = $2.6 \pm 2.57 * 0.3 / \sqrt{36}$*