Activities

1. Tickets numbered 1 to 20 are mixed up and then a ticket is drawn at random. What is the probability that the ticket drawn has a number which is a multiple of 3 or 5?

   Ans: Here, S = {1, 2, 3, 4, ..., 19, 20}.

   Let E = event of getting a multiple of 3 or 5 = {3, 6 , 9, 12, 15, 18, 5, 10, 20}.

   $\therefore$ $P(E) = \frac{n(E)}{n(S)} = \frac{9}{20}$.

2. What is the probability of a randomly selected leap year will contain 53 Sundays?

   Ans: A leap year contains 366 days, i.e, 52 weeks and 2 days. These two days could be any one of the following:

   i) Sunday & Monday
   ii) Monday & Tuesday
   iii) Tuesday & Wednesday
   iv) Wednesday & Thursday
   v) Thursday & Friday
   vi) Friday & Saturday
   vii) Saturday & Sunday.

   Therefore, the required probability is 2/7.

3. A committee of three is chosen from five councilors - Adams, Burke, Cobb, Dalby and Evans.

   What is the probability that Burke is in the committee?

   Ans. Abbreviate the names of the five councilors with the letters A, B, C, D and E.

   There are 10 possible committees: (A, B, C), (A, B, D), (A, B, E), (A, C, D), (A, C, E), (A, D, E), (B, C, D), (B, C, E), (B, D, E) and (C, D, E)

   Of these, Burke is included in 6: (A, B, C), (A, B, D), (A, B, E), (B, C, D), (B, C, E) and (B, D, E)

   So:

   The Number of ways it can happen = 6

   The Total number of outcomes = 10

$$\text{Probability of an event happening} = \frac{\text{Number of ways it can happen}}{\text{Total number of outcomes}}$$

$$\text{Therefore, the probability Burke is on the committee} = \frac{6}{10} = \frac{3}{5}$$

4. The probability that a Ramesh passes a Math test is 2/3 and the probability that he passes both Math and English test is 14/45. The probability that he passes at least one test is 4/5. What is the probability that he passes the English test?

   Ans: Let A: is the event that Ramesh passes the Math test and B: is the event that he passes the English test.
   Given, P(A) = 2/3, P(A∩B) = 14/45 and P(AUB) = 4/5, and we want P(B).
   We know,
   P(AUB) = P(A) + P(B) - P(A∩B)
   → P(B) = P(AUB) - P(A) + P(A∩B)
   → P(B) = 4/5 - 2/3 + 14/45
   → P(B) = 4/9.

5. Let three fair coins be tossed. Let
   A = {all heads or all tails},
   B = {at least two heads},
   C = {at most two tails}.
   Of the pairs of events, (A, B), (A, C), and (B, C), which are independent and which are dependent? (Justify).

   Ans: If A and B are independent, then P(A ∩ B) = P(A) * P(B). If this condition is not satisfied, then they are dependent.
   We write the event space for each of A, B and C.
   A = {HHH, TTT},
   B = {HHH, HHT, HTH, THH},
   C = {HHH, HHT, HTH, THH, HTT, THT, TTH}.
   P(A ∩ B) = 1/8 and P(A) · P(B)= (2/8)(4/8) =1/8 so A and B are independent.
   P(A ∩ C) = 1/8 and P(A) · P(C)= (2/8)(7/8), so A and C are dependent.
   P(B ∩ C) =4/8 and P(B) · P(C)= (4/8)(7/8), so B and C are dependent.

6. In a region during a 1-year period, there were 1000 deaths. It was observed that 321 people died of a renal failure and 460 people had atleast one parent with renal failure. Of these 460 people, 115 died of renal failure.
   (i) Calculate the Probability that a person dies of Renal Failure in the population if you pick him at random
   (ii) If you pick a person at random from the population, calculate the Probability that a person dies of Renal Failure and at least one of his parents died due to a Renal Failure
   (iii) Calculate the probability that a patient dies of renal failure if neither of his parents had a renal failure

   Ans:

   (i) Marginal Probability

   P (death due to renal failure) = 321/1000 = 0.321

   (ii) Joint Probability

   P (death due to RF and parent died of RF) = 115/1000 = 0.115

   (iii) Conditional Probability

   Let H=the event that atleast one of parents of the randomly selected man die of cause related to renal failure.

   D=event that the randomly selected man died of renal failure.

D/H	Parent died of RF	Parent !died of RF	Total
People died of RF	115	206	321
People !died of RF	345	334	679
Total	460	540	1000

   206/540=0.381

7. The probability that you park in a no-parking zone and get a parking ticket is 0.06. The probability that you must park in a no-parking zone (as you cannot find a legal parking

space) is 0.20. Today, you arrive at INSOFE and must park in a no-parking zone. What is the probability that you will get a parking ticket?

N = You park in a no-parking zone, T = You get a parking ticket
P (N and T) = 0.06, $P(N)$ = 0.20

$$\mathrm{P(T|N)} = \frac{P(T\ and\ N)}{P(N)} = \frac{0.06}{0.02} = 0.30.$$

8. Approximately 1% of women aged 40-50 have breast cancer. A woman with breast cancer has a 90% chance of a positive test from a mammogram, while a woman without has a 10% chance of a false positive result. What is the probability a woman aged 40 - 50 has breast cancer given that she just had a positive test?

Ans: Let the two events B and A be, B = "the woman has breast cancer" and A = "a positive test". We wish to calculate P(B|A).

$$P(B|A) = \frac{P(B\cap A)}{P(A)} = \frac{P(B\cap A)}{P(B\cap A)+P(B^c\cap A)}$$

$$\begin{aligned} P(B\cap A) &= P(B)P(A|B) = 0.01\cdot 0.9 = 0.009 \\ P(B^c\cap A) &= P(B^c)P(A|B^c) = 0.99\cdot 0.1 = 0.099 \end{aligned}$$

$$P(B|A) = \frac{0.009}{0.009+0.099} = \frac{9}{108}$$

Assignment:

9. A bag contains 2 red, 3 green and 2 blue balls. Two balls are drawn at random. What is the probability that none of the balls drawn is blue?

Ans. Total number of balls = (2 + 3 + 2) = 7.

Let S be the sample space.

Then, $n(S)$ = Number of ways of drawing 2 balls out of 7

$= {}^7C_2$

$= \frac{(7 \times 6)}{(2 \times 1)}$

$= 21.$

Let E = Event of drawing 2 balls, none of which is blue.

$\therefore$ $n(E)$ = Number of ways of drawing 2 balls out of (2 + 3) balls.

$= {}^5C_2$

$= \frac{(5 \times 4)}{(2 \times 1)}$

$= 10.$

$\therefore$ $P(E) = \frac{n(E)}{n(S)} = \frac{10}{21}.$

10. Below is a table of graduates and post graduates

	Graduate	Post Graduate	Total
Male	19	41	60
Female	12	28	40
Total	31	69	**100**

a) What is the probability that a randomly selected individual is a male and a graduate? What kind of probability is it (Marginal/ Joint/Conditional)
b) What is the probability that a randomly selected individual is a male
c) What is the probability of a randomly selected individual being a graduate? What kind of probability is this?
d) What is the probability that a randomly selected person is a female given that the selected person is a post graduate? What kind of probability is this?

Ans. a) Joint Probability: P(Male and Graduate)= 19/100.
b) Marginal Probability: P(Male) = 60/100.
c) Marginal Probability: P(Graduate)=31/100.
d) Conditional Probability: P(Female|Post Graduate)=28/69.