1. Suppose a manufacturer claims that the mean lifetime of a light bulb is at least 10,000 hours. In a sample of 30 light bulbs, it was found that they only last 9,900 hours on average. Assuming the population standard deviation to be 120 hours, at 0.05 significance level, can we reject the claim by the manufacturer?

Step1 : State Null and Alternative hypothesis.

H_o: $\mu \geq 10000$, $H_1 : \mu < 10000$; This implies a lower tail test

Remember: The position of the tail is dependent on H_1.

*If H_1 includes a < sign, then the **lower tail** is used.*

*If H_1 includes a > sign, then the **upper tail** is used.*

Step2 : Choose Statistic:

Given: sample size n = 30, sample mean x´ = 9900, Population standard deviation σ = 120.

As the population standard deviation is given and sample size is large enough, we can choose z-statistic; Statistic = $(\bar{x} - \mu) / (\sigma/\sqrt{n})$

Step 3: Specify Significance level:

$\alpha = 0.05$

Step 4: Determine critical region; Compute critical value

The critical value @ 0.05 signif is = qnorm(0.05) = -1.64. (because it is lower tailed test)

Z-critical value = -1.64

Step 5: Determine the statistic value, and find its p-value;

Z-calculated = $(\bar{x} - \mu) / (\sigma/\sqrt{n}) = -4.56$

p-value in z-table for -4.56 does not exist in table, implies that its too small.

Step 6 : Does the calculated sample statistic value lie in the critical region?

- *Method 1: If Z-calculated is << Z-critical (in lower tail test) Or if If Z-calculated is >> Z-critical (in upper tail test), it implies the sample statistic we calculated is in critical region, which tells that we have enough evidence to reject the Null Hypothesis*

Method 2: If the p-value of Z-calculated is << p-value of Z-critical (in lower tail test) Or if the p-value Z-calculated is >> p-value of Z-critical (in upper tail test), it implies the sample statistic we calculated is in critical region, which tells that we have enough evidence to reject the Null Hypothesis

Now, as Z-calculated(-4.56) is << Z-critical(-1.64) (in lower tail test), the z-calculated is in critical region

Step 7: Make your decision:

We reject the Null hypothesis

2. Suppose a car manufacturer claims a model gets at least 25 mpg. A consumer group asks 10 owners of this model to calculate their mpg and the mean value was 22 with a standard deviation of 1.5. Is the manufacturer's claim supported at 95%confidence level.

   Ans. R-code

```
rm(list=ls(all=TRUE))
# Ho: mu >= 25, Ha : mu < 25.
n = 10
xBar = 22
mu = 25
s = 1.5
alpha = 0.05
# Since sample size is small, we will use Student's t distribution here.  This is lower tailed test.
se = s/sqrt(n)
test_Statistic = (xBar - mu)/ se
degrees_Of_Freedom = n-1
tValue= qt(alpha, degrees_Of_Freedom)
test_Statistic
tValue
# We reject the Null Hypothesis since the observation falls in rejection/critical region.
```

3. An outbreak of Salmonella-related illness was attributed to ice cream produced at a certain factory. Scientists measured the level of Salmonella in 9 randomly sampled batches of ice cream. The levels (in MPN/g) were: *0.593 0.142 0.329 0.691 0.231 0.7930.519 0.392 0.418.* Is there evidence that the mean level of Salmonella in the ice cream is greater than 0.3 MPN/g.

   Ans. R-code

```
rm(list=ls(all=TRUE))
# One-sample t-tests
# H0 <= 0.3 vs H1 > 0.3
Salmonella_level = c(0.593, 0.142, 0.329, 0.691, 0.231, 0.793, 0.519, 0.392, 0.418)

######### t-test ##########################
# General form
```

```
#t.test(x, y = NULL, alternative = c("two.sided", "less", "greater"), mu = 0, paired = FALSE, var.equal = FALSE, conf.level = 0.95)

t.test(Salmonella_level, alternative="greater", mu=0.3)
tValue= qt(0.95, 8)
tValue

# Since test statistic value t = 2.2051 >tValue, we reject the null hypothesis.
```

4. A study was performed to test whether cars get better mileage on premium gas than on regular gas. Each of 10 cars was first filled with either regular or premium gas, decided by a coin toss, and the mileage for that tank was recorded. The mileage was recorded again for the same cars using the other kind of gasoline. Test to determine whether cars get significantly better mileage with premium gas.

   Reg : 16, 20, 21, 22, 23, 22, 27, 25, 27, 28.

   Prem : 19, 22, 24, 24, 25, 25, 26, 26, 28, 32.

   Ans. R-code

```{r}
rm(list=ls(all=TRUE))
#H0: Mileage on premium gas <= Mileage on regular gas
#H1: Mileage on premium gas > Mileage on regular gas
# A study was performed to test whether cars
# Below is the relevant R-code:
prem = c(19, 22, 24, 24, 25, 25, 26, 26, 28, 32)
reg = c(16, 20, 21, 22, 23, 22, 27, 25, 27, 28)
t.test(prem,reg,alternative="greater", paired=TRUE)
tValue= qt(0.95, 9)
tValue
# Since test statistic value t >tValue, we reject the null hypothesis.
```

5. Do people have a preference for movie type?

   There are four categories of movies: comedy, horror, drama and science fiction. Let us say, we assumed a simple uniform distribution, where each category is liked by 25% of the people. Now we need to test how good our guess is:

	Comedy	Horror	Drama	Science fiction	
Observed	35	30	20	15	n=100

Ans. R-code

```
#Ho: the observed distribution fits the expected
#Ha : the observed  distribution does not fit that expected
rm(list = ls(all=T))
observed <- matrix(c(35, 30,20,15), ncol = 1)
observed
expected <- rep(sum(observed)/4,4)
expected
test_stat<-  sum((observed - expected)^2 / expected)
test_stat
crit<- qchisq(0.05, 3, lower.tail = F)
crit
pchisq(test_stat , 3, lower.tail = F)
chisq.test(observed)
#Hence, we reject the Null Hypothesis.
```

6. A survey is conducted by a gaming company that makes three video games. It wants to know if the preference of game depends on the gender of the player. Total number of participants is 1000. Here is the survey result

	Game A	Game B	Game C	Total
Male	200	150	50	400
Female	250	300	50	600
Total	450	450	100	1000

a. State the null hypothesis and alternate hypothesis.

b. Calculate the degrees of freedom.

c. Does men's preference is different from women's preference? Check with 0.05 level of significance.

Ans. R-code

```
rm(list = ls(all=T))

#Null hypothesis H0: the preference of game is independent of gender

#Alternate hypothesis Ha: the preference of game is dependent on gender

# If a contingency table of observed values is given, chisq. test does an independence test

observed <- matrix(c(200,150,50,250,300,50), byrow = TRUE, ncol = 3)

test_stat<- chisq.test(observed)

crit<- qchisq(p = 0.05, df = (nrow(observed)-1)*(ncol(observed)-1) ,lower.tail = FALSE)

#critical region lies at p(x2 > 5.991)

#test_stat$statistic>crit

# reject NULL Hypothesis
```

7. Laptop computer maker uses battery packs supplied by two companies, A and B. While both brands have the same average battery life between charges (LBC), the computer maker seems to receive more complaints about shorter LBC than expected for battery packs supplied by company B. The computer maker suspects that this could be caused by higher variance in LBC for Brand B. To check that, ten new battery packs from each brand are selected, installed on the same models of laptops, and the laptops are allowed to run until the battery packs are completely discharged. The following are the observed LBCs in hours.

   Brand A = 3.2, 3.4, 2.8, 3, 3, 3, 2.8, 2.9, 3, 3

   Brand B = 3, 3.5, 2.9, 3.1, 2.3, 2, 3, 2.9, 3, 4.1

Test, at the 10% level of significance, whether the variance of both the brands are similar.

Ans. R-code

```
rm(list = ls(all=T))

# H0: σ²(A) = σ²(B)

# H1: σ²(A) ≠ σ²(B)

obsA = c(3.2,3.4,2.8,3,3,3,2.8,2.9,3,3)

obsB = c(3,3.5,2.9,3.1,2.3,2,3,2.9,3,4.1)

sdA = sd(obsA)

sdB = sd(obsB)

Fstat = ((sdA)^2)/((sdB)^2)

Fstat

qf(p = 0.05,df1 = 9, df2 = 9)

qf(p = 0.95,df1 = 9, df2 = 9)

# They are not similar.
```

8. A car crash research team wants to examine the safety of compact cars, intermediate and full size cars. Given below are the hypothetical values of the mean pressure applied to the drivershead during the crash test for each of the car types. Check whether means are equal for each type of these cars at 5% significance level.

Compact	643	655	702
Intermediate	469	427	525
Full size	484	456	402

Ans. R-code

```
# ANOVA

##########################################################

#H0: The mean pressure applied to the driver heads for each of the groups are the same.
```

```
#Ha: Atleast one of the mean pressure applied to the driver heads for the groups are different

x1 <- c(643, 655, 702)

x2 <- c(469, 427, 525)

x3 <- c(484, 456, 402)

x <- c(x1,x2,x3)

sst<- sum((x- mean(x))^2)

sst

m <- 3

n <- length(x1)

ssw<- sum((x1- mean(x1))^2) + sum((x2- mean(x2))^2) + sum((x3- mean(x3))^2)

ssw

ssb<- sst - ssw

ssb

df_ssw<- m * (n-1)

df_ssb<- m-1

f_stat<- (ssb / df_ssb) / (ssw / df_ssw)

f_stat

F_crit<- qf(0.05, df_ssb, df_ssw, lower.tail = F)

F_crit

#F-statistic >> F-critical; Hence the statistic lies in the critical region.

#We fail to accept Ho; Reject Ho
```

Additional Practice Questions:

1. The life in hours of a 75- watt light bulb is known to be normally distributed with σ = 25 hours. A random sample of 100 bulbs has a mean life of x' = 1014 hours. Construct a 95 % two-sided confidence interval on the meanlife. Test the Hypothesis whether population mean is 1000 hours.

Ans. R-code

σ= 25

n = 100

x´ = 1014

Confidence level = 95%

critical_Value = qnorm(0.05/2, lower.tail = F) or qnorm(0.975,0,1) standard_error = σ/sqrt(n)

*margin_of_error = critical_Value * standard_error*

 # Confidence interval (for 95% confidence level)

*x´ ± margin_of_error 1014 ± 1.96*25/sqrt(100)*

2. 6 subjects were given a drug (treatment group) and an additional 6 subjects a placebo (control group). Their reaction time to a stimulus was measured (in *ms*). The outcome of both the group as follows

Control = 91, 87, 99, 77, 88, 91

Treat = 101, 110, 103, 93, 99, 104

Test whether the drug has an effect or not (Assume higher reaction time is better)?

Ans. R-code

```
rm(list=ls(all=TRUE))
#H0: Treatment reaction time <= Control reaction time
#H1: Treatment reaction time > Control reaction time

# Below is the relevant R-code when assuming equal standard deviation:
Treat = c(101, 110, 103, 93, 99, 104)
Control = c(91, 87, 99, 77, 88, 91)

t.test(Treat, Control, alternative="greater", var.equal=TRUE)

#Below is the relevant R-code when not assuming equal standard deviation:
# Welch Two Sample t-test
```

```
t.test(Treat, Control, alternative="greater")

tValue= qt(0.95, 10)
tValue

# Since test statistic value t >tValue, we reject the null hypothesis.
```

3. A bank manager observed that the standard deviation in waiting in line for service during the Christmas holidays season is about 10 minutes per customer. Hoping to implement a new policy of single line service, the standard deviation of waiting in line for 25 customers were observed with the new policy by a pilot study and was calculated to be 5 minutes. Should the manager adopt the new single line policy based on this pilot study?

Ans. R-code

```
rm(list = ls(all=T))
#Ho : σ2 ≥ 100
#H1 : σ2< 100.
```

If you want be 99 % certain that the test is true, then = 0.01 = α.
df = n-1=25-1=24
Observed sample variance $S^2 = 5^2 = 25$.

Test statistic = $\frac{(n-1)S^2}{\sigma_0^2} = \frac{24*25}{100} = 7.25$

Crit_val = *qchisq(0.01, 24) = 10.86*

Since 7.25 < 10.86, then we reject the null hypothesis that s=5 belongs to a population whose standard deviation is 10.
So indeed 5 is truly smaller than 10.
Hence, we conclude that a policy of single waiting line improves the variance of waiting time from 10 to 5 minutes. So, adopting the single line policy since it improves variability.

4. A national survey agency conducts a nationwide survey on consumer satisfaction and finds out the response distribution as follows:

Excellent:	8%
Good:	47%
Fair:	34%
Poor:	11%

A store manager wants to find if these results of customer survey apply to the customers of super market in her city. So, she interviews 207 randomly selected customers and asked them to rate their responses. The results of this local survey are given below. Determine if the local responses from this survey are the same as expected frequencies of the national survey, at 95% confidence level.

Response	Frequency
Excellent	21
Good	109
Fair	62
Poor	15

Ans. R-code

```
rm(list = ls(all=T))
#Null hypothesis H0: the observed distribution fits the expected
#Alternate hypothesis Ha: the observed distribution does not fit the expected

observed <- c(21, 109, 62, 15)
expected <- (c(8, 47,34, 11)/100)* sum(observed)
test_stat<- sum((observed - expected)^2 / expected)
test_stat
crit<- qchisq(0.05, 3, lower.tail = F)
crit
chisq.test(observed, p = c(0.08,0.47, 0.34,0.11 ))
# We fail to reject Null Hypothesis/accept the Null Hypothesis.
```

5. Suppose you want to check whether a coin is biased or unbiased with the following hypothesis.

   Ho: p = 0.5 vs H1 = 0.8
   where p is the probability of head in a single toss of a coin. You have decided that if you find more than 7 heads in 10 tosses, you will reject the null hypothesis.
   i) What is Type-I error of your test?
   ii) What is the Type-II error?

iii) What is the power of your test?

Ans. R-code

```
rm(list=ls(all=TRUE))
#TT = X1 + ... + X10 ~Binomial (10,p)
#i)
Alpha = dbinom(8,10,0.5)+dbinom(9,10,0.5)+dbinom(10,10,0.5)
#or
Alpha = sum(dbinom(8:10,10,0.5))
#or
Alpha = 1-pbinom(7,10,0.5,lower.tail = T)

#ii)
Beta = pbinom(7,10,0.8)

#iii)
Power = 1 - Beta
```