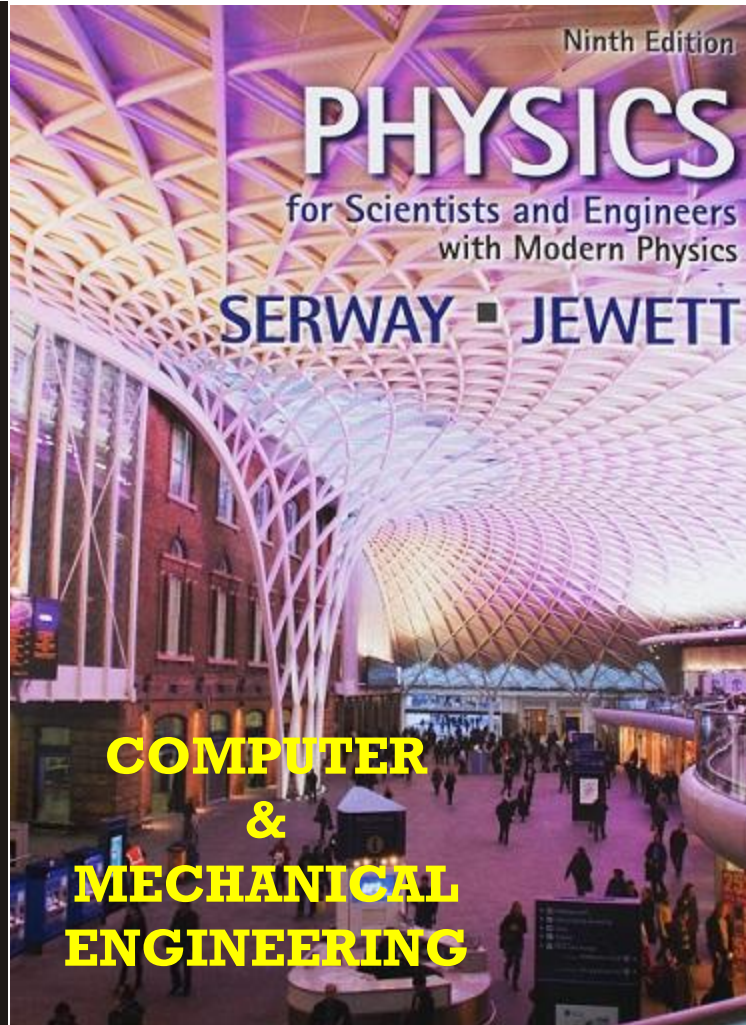
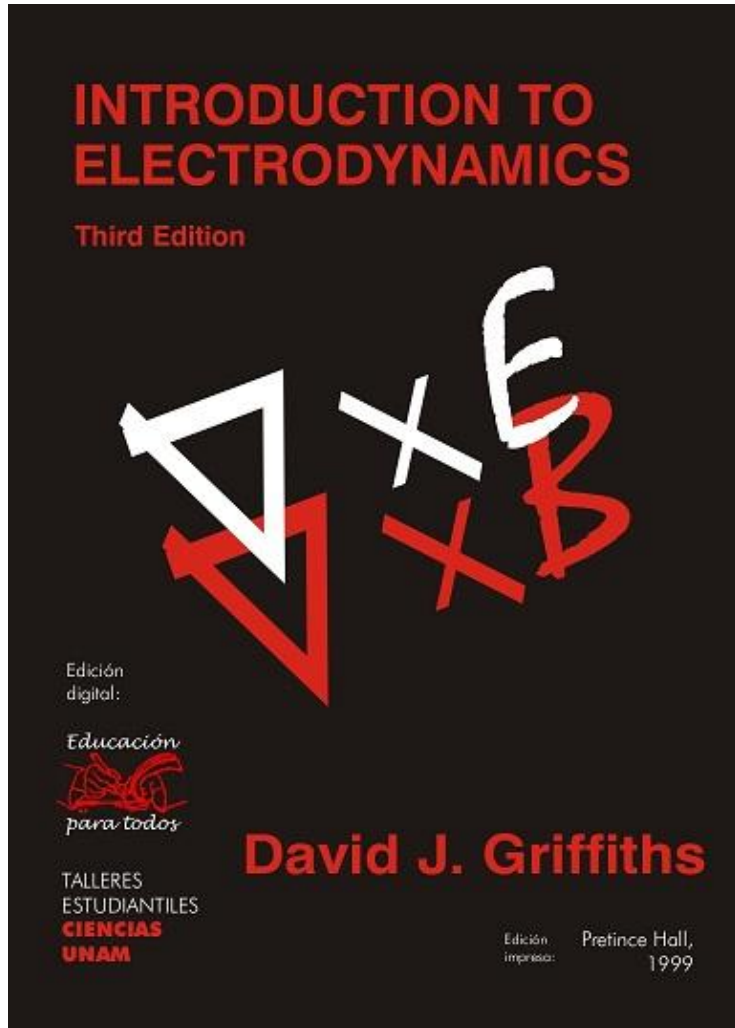


PHYSICS



General Physics II (PHYS 102)



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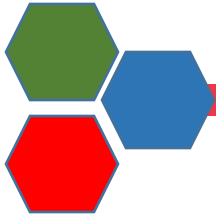
Course Outline



ELECTROSTATIC FIELD

- Electric Charge
- Coulomb's Law
- The Electric Field
- Continuous Charge Distributions
- Problems

Electric Charge



Electric Charge (q):

- Charge is a fundamental and characteristics property of the elementary particles which make up matter.
- It is a scalar quantity.
- SI unit of charge is coulomb (C).

- Kinds of Charges

1. Positive Charge
2. Negative Charge

Elementary Charge :

The magnitude of charge on a proton or an electron.

$$e = 1.6 \times 10^{-19} \text{ C}$$

Properties of Charges:

1. Like charges repel each other and unlike charges attract each other.
2. Electric charge is quantized. $q = \pm ne$
3. Electric charge is conserved.
4. The electric charge is additive in nature.
5. The charge on a body is not affected by the speed of the body.

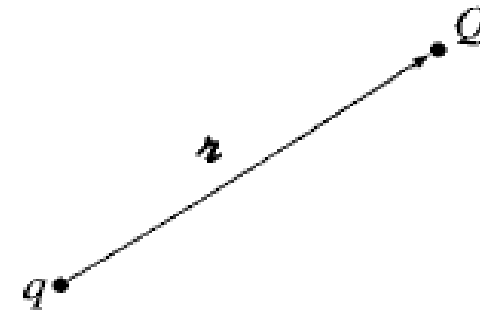
Coulomb's Law



Coulomb's Law:

- The force on a test charge Q due to a single point charge q , which is at rest a distance r away is given by **Coulomb's law**:

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{qQ}{r^2} \hat{r}$$



- The magnitude of the electric force between two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them.
- The constant ϵ_0 is called the **permittivity of free space**.

$$\epsilon_0 = 8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2}$$



Electric Field

ELECTROSTATIC FIELD

Coulomb's Law:

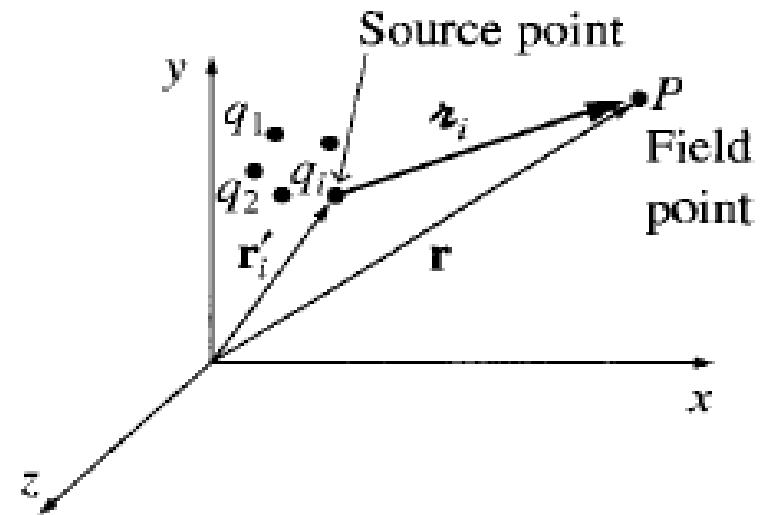
- If we have several point charges q_1, q_2, \dots, q_n at distances r_1, r_2, \dots, r_n from Q , the total force on Q is given by

$$\begin{aligned}\vec{F} &= \vec{F}_1 + \vec{F}_2 + \dots \\ &= \frac{1}{4\pi\epsilon_0} \left(\frac{q_1 Q}{r_1^2} \hat{r}_1 + \frac{q_2 Q}{r_2^2} \hat{r}_2 + \dots \right) \\ &= \frac{Q}{4\pi\epsilon_0} \left(\frac{q_1}{r_1^2} \hat{r}_1 + \frac{q_2}{r_2^2} \hat{r}_2 + \dots \right)\end{aligned}$$

$$\therefore \boxed{\vec{F} = Q\vec{E}}$$

where $\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \frac{q_i}{r_i^2} \hat{r}_i$

→ electric field of the source charges



- Principle of Superposition:**

$$\boxed{\vec{E} = \vec{E}_1 + \vec{E}_2 + \dots}$$

Electric Field



Electric Field:

- The electric field \vec{E} at a point in space is defined as the electric force \vec{F} acting on a positive test charge Q placed at that point divided by the magnitude of the test charge:

$$\vec{E} = \frac{\vec{F}}{Q}$$

- The **electric field** is a vector quantity that varies from point to point and is determined by the configuration of source charges.
- The SI unit of **electric field** \vec{E} is newton per coulomb (N C^{-1}).



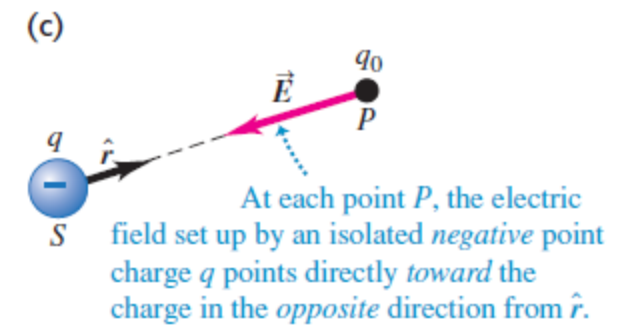
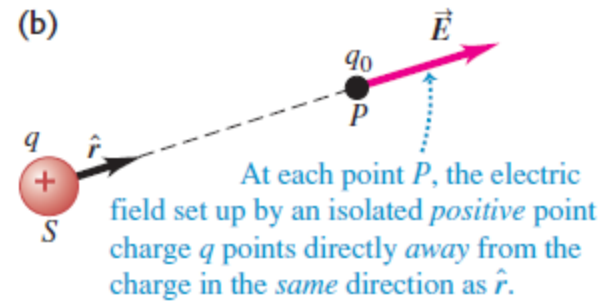
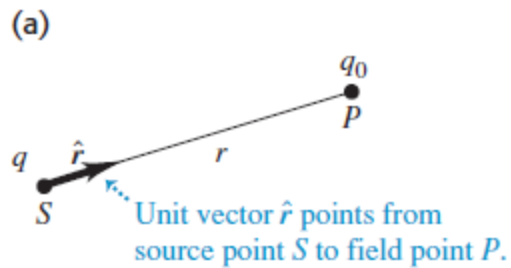
Electric Field

ELECTROSTATIC FIELD

Electric Field of a Point Charge:

- The electric field \vec{E} produced at point P by an isolated point charge q at the origin S is given by

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^3} \vec{r}$$



\vec{E} is produced by q but acts q_0 on the charge at point P .

- The electric field \vec{E} produced at field point P by an isolated point charge q at the source point S is given by

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^3} \vec{r}$$





Line Integral of Electric Field

Electric Field:

- The electric field at a point \vec{r} due to a point charge q located at the origin is given by

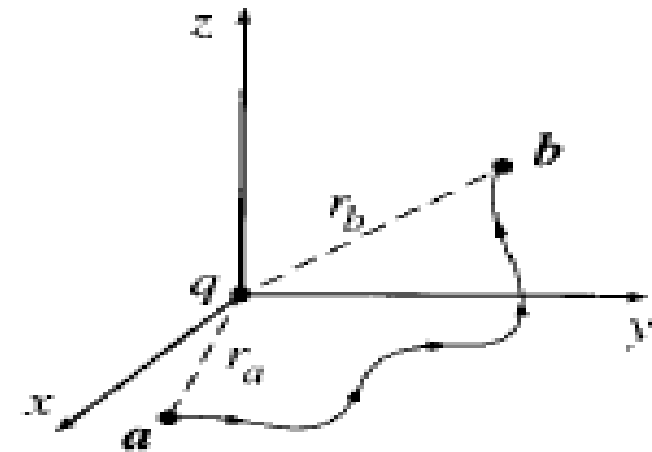
$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$$

- The line integral of electric field:

$$\begin{aligned} \int_a^b \vec{E} \cdot d\vec{l} &= \int_a^b \left(\frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r} \right) \cdot (dr \hat{r} + r d\theta \hat{\theta} + r \sin\theta d\phi \hat{\phi}) \\ &= \frac{q}{4\pi\epsilon_0} \int_a^b \frac{1}{r^2} dr = \frac{q}{4\pi\epsilon_0} \left[-\frac{1}{r} \right]_{r_a}^{r_b} \end{aligned}$$

$$\boxed{= \frac{q}{4\pi\epsilon_0} \left[\frac{1}{r_a} - \frac{1}{r_b} \right]}$$

- The electric field due to stationary charges is **conservative field**



The amount of work done by the electric field \vec{E} when a unit positive charge moves from point a to point b

$$\boxed{W_E = \int_a^b \vec{E} \cdot d\vec{l} = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{r_a} - \frac{1}{r_b} \right]}$$



The curl of Electric Field

The curl of Electric Field:

- The line integral of electric field around a closed path is zero.

$$\text{i.e. } \oint_c \vec{E} \cdot d\vec{l} = 0$$

$$\text{or } \int_s (\nabla \times \vec{E}) \cdot d\vec{a} = 0 \quad [\text{Using Stoke's Theorem}]$$

$$\therefore \boxed{\nabla \times \vec{E} = 0}$$

$$\nabla \times \vec{E} = 0$$

The electric field at point \vec{r} due to a point charge q located at the origin is

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{q}{r^3} \vec{r}$$

$$\begin{aligned} \text{So, } \nabla \times \vec{E} &= \nabla \times \left[\frac{1}{4\pi\epsilon_0} \frac{q}{r^3} \vec{r} \right] = \frac{q}{4\pi\epsilon_0} \left[\nabla \times \left(\frac{\vec{r}}{r^3} \right) \right] = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{r^3} (\nabla \times \vec{r}) + \nabla \left(\frac{1}{r^3} \right) \times \vec{r} \right] \\ &= \frac{q}{4\pi\epsilon_0} [0 + (-3r^{-3-2}) \vec{r} \times \vec{r}] \\ &= 0 \end{aligned}$$

Continuous Charge Distribution



Linear Charge Density

$$\lambda = \frac{dq}{dl'}$$



charge-per-unit-length

$[dl' \rightarrow \text{an element of length along the line}]$

Surface Charge Density

$$\sigma = \frac{dq}{da'}$$



charge-per-unit-area

$[da' \rightarrow \text{an element of area on the surface}]$

Volume Charge Density

$$\rho = \frac{dq}{d\tau'}$$



charge-per-unit-volume

$[d\tau' \rightarrow \text{an element of volume}]$

Small Charge Distribution

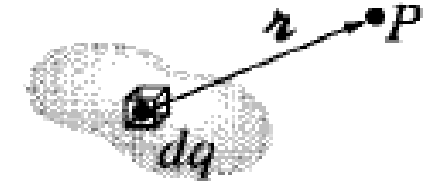
$$dq \rightarrow \lambda dl' \sim \sigma da' \sim \rho d\tau'$$

Electric Field due to Continuous Charge Distribution



Electric Field Due to a Continuous Charge Distribution:

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r^2} \hat{r}$$



The electric field of a line charge

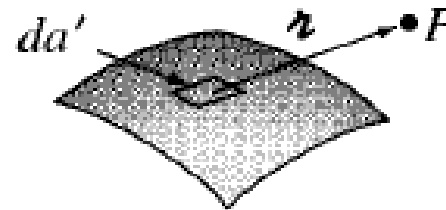
$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda(r') dl'}{r^2} \hat{r}$$



Line charge, λ

The electric field for a surface charge

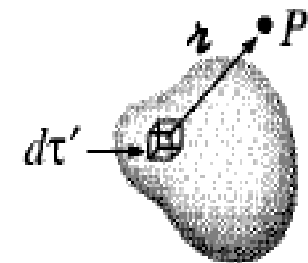
$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\sigma(r') da'}{r^2} \hat{r}$$



Surface charge, σ

The electric field for a volume charge

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(r') d\tau'}{r^2} \hat{r}$$



Volume charge, ρ

Problem



I. Electric Field a distance z above the midpoint between two equal charges, q , a distance d apart

From Figure

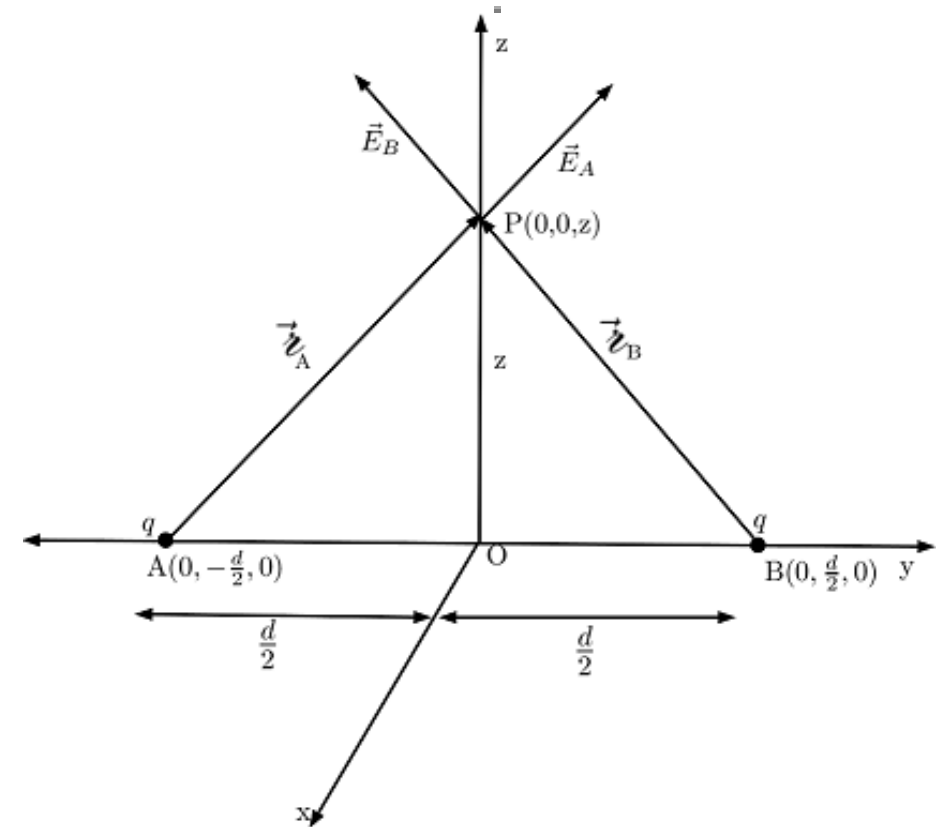
$$\vec{r}_A = (0-0)\hat{i} + \left(0 + \frac{d}{2}\right)\hat{j} + (z-0)\hat{k} = \frac{d}{2}\hat{j} + z\hat{k}$$

$$\Rightarrow r_A = \left(\frac{d^2}{4} + z^2\right)^{\frac{1}{2}}$$

and

$$\vec{r}_B = (0-0)\hat{i} + \left(0 - \frac{d}{2}\right)\hat{j} + (z-0)\hat{k} = -\frac{d}{2}\hat{j} + z\hat{k}$$

$$\Rightarrow r_B = \left(\frac{d^2}{4} + z^2\right)^{\frac{1}{2}}$$



Problem



I. Electric Field a distance z above the midpoint between two equal charges, q , a distance d apart

- **Electric Field** at P due to the charge at A is

$$\vec{E}_A = \frac{1}{4\pi\epsilon_0} \frac{q}{r_A^3} \vec{r}_A = \frac{1}{4\pi\epsilon_0} \frac{q}{\left(\frac{d^2}{4} + z^2\right)^{\frac{3}{2}}} \left(\frac{d}{2} \hat{j} + z\hat{k}\right)$$

.....(1)

- **Electric Field** at P due to the charge at B is

$$\vec{E}_B = \frac{1}{4\pi\epsilon_0} \frac{q}{r_B^3} \vec{r}_B = \frac{1}{4\pi\epsilon_0} \frac{q}{\left(\frac{d^2}{4} + z^2\right)^{\frac{3}{2}}} \left(-\frac{d}{2} \hat{j} + z\hat{k}\right)$$

.....(2)

- Therefore, **the total electric field at P:**

$$\begin{aligned} \vec{E} &= \vec{E}_A + \vec{E}_B \\ &= \frac{1}{4\pi\epsilon_0} \frac{q}{\left(\frac{d^2}{4} + z^2\right)^{\frac{3}{2}}} \left(\frac{d}{2} \hat{j} + z\hat{k}\right) + \frac{1}{4\pi\epsilon_0} \frac{q}{\left(\frac{d^2}{4} + z^2\right)^{\frac{3}{2}}} \left(-\frac{d}{2} \hat{j} + z\hat{k}\right) \end{aligned}$$

$$\therefore \boxed{\vec{E} = \frac{1}{4\pi\epsilon_0} q \frac{2z}{\left(\frac{d^2}{4} + z^2\right)^{\frac{3}{2}}} \hat{k}}$$

when $z \gg d$

$$\text{Electric field, } \vec{E} = \frac{1}{4\pi\epsilon_0} q \frac{2z}{(z^2)^{\frac{3}{2}}} \hat{k}$$

$$\therefore \boxed{\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{2q}{z^2} \hat{k}}$$

Problem



2. Electric Field a distance z above the midpoint of a straight line segment of length $2L$, which carries a uniform charge λ

From Figure

$$\vec{r} = (0-0)\hat{i} + (0-y')\hat{j} + (z-0)\hat{k} = -y'\hat{j} + z\hat{k}$$

$$\Rightarrow r = (y'^2 + z^2)^{\frac{1}{2}}$$

The charge on an element of length dy' at C along the line is

$$dq = \lambda dy'$$

The electric field at P due to the charge $dq (= \lambda dy')$ is given by

$$d\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{dq}{r^3} \vec{r} = \frac{1}{4\pi\epsilon_0} \frac{\lambda dy'}{(y'^2 + z^2)^{\frac{3}{2}}} (-y'\hat{j} + z\hat{k})$$

.....(1)

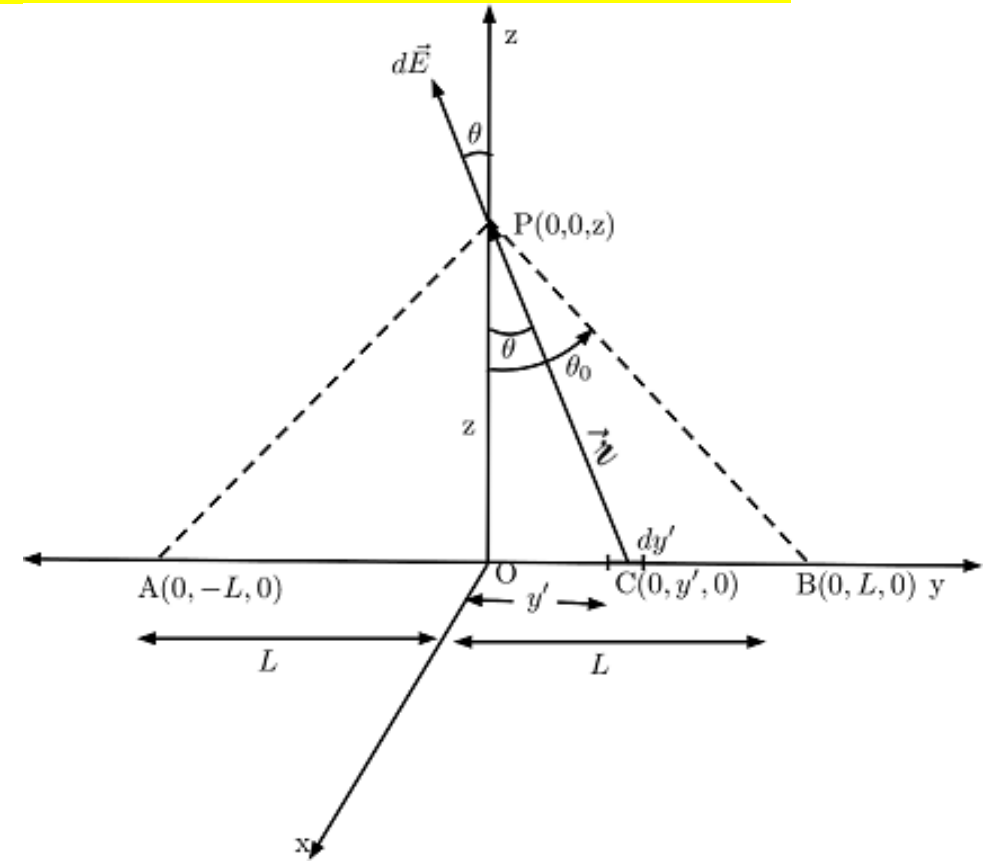


Figure E-2 illustrates the geometry and the coordinates to be used



Problem

2. Electric Field a distance z above the midpoint of a straight line segment of length $2L$, which carries a uniform charge λ

- Total Electric field** at P due to the charge on whole line segment AB:

$$\vec{E} = \int_{-L}^{+L} d\vec{E} = \frac{1}{4\pi\epsilon_0} \lambda \left[\int_{-L}^L \frac{(-y' \hat{j} + z \hat{k}) dy'}{(y'^2 + z^2)^{\frac{3}{2}}} \right] = \frac{1}{4\pi\epsilon_0} \lambda \left[\int_{-L}^L \frac{-y' \hat{j} dy'}{(y'^2 + z^2)^{\frac{3}{2}}} + \int_{-L}^L \frac{z \hat{k} dy'}{(y'^2 + z^2)^{\frac{3}{2}}} \right] = \frac{1}{4\pi\epsilon_0} \lambda \left[2 \int_0^L \frac{z dy'}{(y'^2 + z^2)^{\frac{3}{2}}} \right] \hat{k} \quad \dots\dots\dots(2)$$

put

$$y' = z \tan \theta$$

$$\Rightarrow dy' = z \sec^2 \theta d\theta$$

when $y' = 0$, then $\theta = 0$

$$\text{when } y' = L, \text{ then } \theta = \tan^{-1} \left(\frac{L}{z} \right) = \theta_0 \text{ (say)}$$

$$\begin{aligned} \therefore \int_{-\alpha}^{\alpha} f(x) dx &= 2 \int_0^{\alpha} f(x) dx \quad ; \text{ for even function } f(x) \\ &= 0 \quad ; \text{ for odd function } f(x) \end{aligned}$$



Problem

2. Electric Field a distance z above the midpoint of a straight line segment of length $2L$, which carries a uniform charge λ

$$\begin{aligned}
 \therefore \vec{E} &= \frac{1}{4\pi\epsilon_0} \lambda \left[2z \int_0^{\theta_0} \frac{z \sec^2 \theta d\theta}{z^3 \sec^3 \theta} \right] \hat{k} \\
 &= \frac{1}{4\pi\epsilon_0} \frac{2\lambda}{z} \left[\int_0^{\theta_0} \cos \theta d\theta \right] \hat{k} \\
 &= \frac{1}{4\pi\epsilon_0} \frac{2\lambda}{z} [\sin \theta_0] \hat{k} \\
 &= \frac{1}{4\pi\epsilon_0} \frac{2\lambda}{z} \left[\frac{L}{\sqrt{z^2 + L^2}} \right] \hat{k} \\
 \therefore \vec{E} &= \frac{1}{4\pi\epsilon_0} \frac{\lambda(2L)}{z\sqrt{z^2 + L^2}} \hat{k} \quad \dots\dots\dots(3)
 \end{aligned}$$

For points far from the line ($z \gg L$):

$$\begin{aligned}
 \vec{E} &\cong \frac{1}{4\pi\epsilon_0} \frac{\lambda(2L)}{z^2} \hat{k} \\
 \therefore \vec{E} &\cong \frac{1}{4\pi\epsilon_0} \frac{q}{z^2} \hat{k} \quad \dots\dots\dots(4)
 \end{aligned}$$

For far away the line "looks" like a point charge $q = \lambda(2L)$.

As $L \rightarrow \infty$:

$$\begin{aligned}
 \vec{E} &= \frac{1}{4\pi\epsilon_0} \frac{\lambda(2L)}{z\sqrt{z^2 + L^2}} \hat{k} = \frac{1}{4\pi\epsilon_0} \frac{2\lambda}{z\sqrt{\frac{z^2}{L^2} + 1}} \hat{k} \\
 \therefore \vec{E} &= \frac{1}{4\pi\epsilon_0} \frac{2\lambda}{z} \hat{k} \quad [\text{Field of an infinite straight wire}] \quad \dots\dots\dots(5)
 \end{aligned}$$

Problem



3. Electric Field a distance z above the centre of a circular loop of radius r , which carries a uniform line charge λ .

From Figure

$$\vec{r} = -x' \hat{i} - y' \hat{j} + z \hat{k} = -r \cos \phi \hat{i} - r \sin \phi \hat{j} + z \hat{k}$$

$$\text{and } r = \left(r^2 + z^2 \right)^{\frac{1}{2}}$$

The charge on an element of length dl' along a circular loop is

$$dq = \lambda dl' = \lambda (r d\phi)$$

An elemental length dl' on the ring with coordinates $(x', y', 0)$ subtends an elemental angle $d\phi$ at the centre.

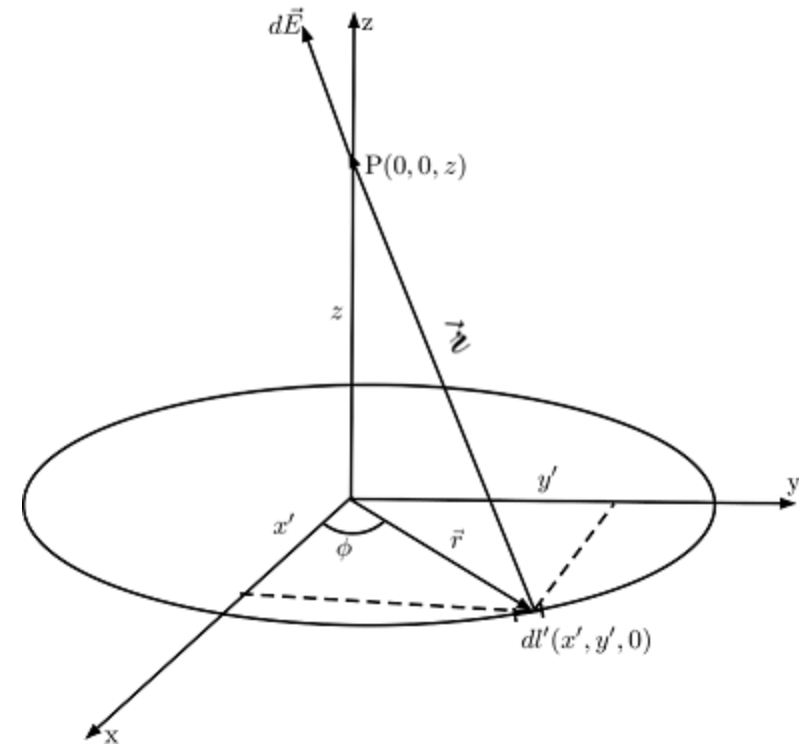


Figure E-3 illustrates the geometry and the coordinates to be used

Problem



3. Electric Field a distance z above the centre of a circular loop of radius r , which carries a uniform line charge λ .

The electric field at P

due to the charge $dq (= \lambda dl')$ is given by

$$\begin{aligned} d\vec{E} &= \frac{1}{4\pi\epsilon_0} \frac{dq}{r^3} \vec{r} \\ &= \frac{1}{4\pi\epsilon_0} \frac{\lambda dl'}{(r^2 + z^2)^{\frac{3}{2}}} (-r \cos \phi \hat{i} - r \sin \phi \hat{j} + z \hat{k}) \end{aligned}$$

$$\therefore \boxed{d\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{\lambda (rd\phi)}{(r^2 + z^2)^{\frac{3}{2}}} (-r \cos \phi \hat{i} - r \sin \phi \hat{j} + z \hat{k})}$$

.....(1)

\therefore The net electric field at P due to the charge on whole circular loop is

$$\begin{aligned} \vec{E} &= \int d\vec{E} \\ &= \frac{1}{4\pi\epsilon_0} \frac{r\lambda}{(r^2 + z^2)^{\frac{3}{2}}} \left[-\left(r \int_0^{2\pi} \cos \phi d\phi\right) \hat{i} - \left(r \int_0^{2\pi} \sin \phi d\phi\right) \hat{j} + z \left(\int_0^{2\pi} d\phi\right) \hat{k} \right] \\ &= \frac{1}{4\pi\epsilon_0} \frac{r\lambda}{(r^2 + z^2)^{\frac{3}{2}}} [0 - 0 + z(2\pi) \hat{k}] \\ &= \frac{1}{4\pi\epsilon_0} \lambda (2\pi r) \frac{z}{(r^2 + z^2)^{\frac{3}{2}}} \hat{k} \end{aligned}$$

$$\therefore \boxed{\vec{E} = \frac{1}{4\pi\epsilon_0} q \frac{z}{(r^2 + z^2)^{\frac{3}{2}}} \hat{k}} \quad \text{.....(2)}$$

where $q = \lambda [2\pi r]$ is the total charge on the circular loop.



Problem

4. **Electric Field** a distance z above the centre of a flat circular disc of radius R , which carries a uniform surface charge σ .

- The disk can be considered as the combination of an infinite number of infinitesimally thin rings.

Consider a ring of radius r and thickness dr of this disk.

The charge on this ring is $dq = \sigma(2\pi r dr)$

..... (1)

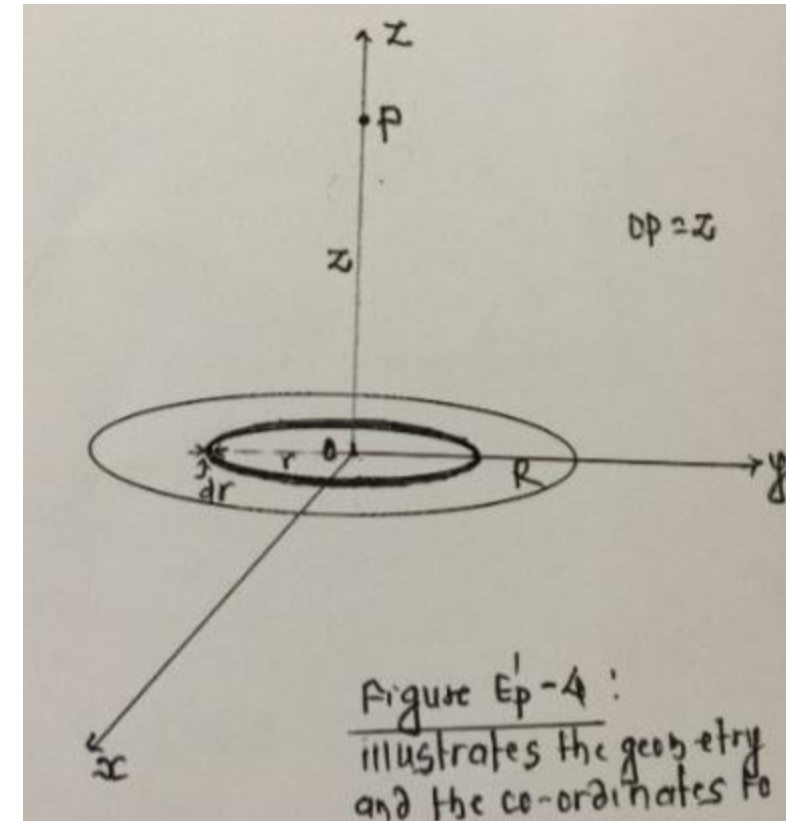
The electric field at P due to the charge $dq (= \sigma 2\pi r dr)$ on the ring is given by

$$d\vec{E} = \frac{1}{4\pi\epsilon_0} dq \frac{z}{(r^2 + z^2)^{\frac{3}{2}}} \hat{k}$$

$$= \frac{1}{4\pi\epsilon_0} (\sigma 2\pi r dr) \frac{z}{(r^2 + z^2)^{\frac{3}{2}}} \hat{k}$$

using Eq.(1)

$$\therefore d\vec{E} = \frac{\sigma z}{2\epsilon_0} \frac{r dr}{(r^2 + z^2)^{\frac{3}{2}}} \hat{k} \quad \text{..... (2)}$$





Problem

4. Electric Field a distance z above the centre of a flat circular disc of radius R , which carries a uniform surface charge σ .

- Hence the electric field at P due to the charge on the whole flat circular disk is given by

$$\vec{E}_{disk} = \int d\vec{E} = \frac{\sigma z}{2\epsilon_0} \left[\int_0^R \frac{r dr}{(r^2 + z^2)^{\frac{3}{2}}} \right] \hat{k} \quad \dots\dots\dots (3)$$

put $r^2 + z^2 = t^2$

$\Rightarrow r dr = t dt$

when $r = 0$, then $t = z$

when $r = R$, then $t = \sqrt{R^2 + z^2}$

$$\begin{aligned} \therefore \vec{E} &= \frac{\sigma z}{2\epsilon_0} \left[\int_z^{\sqrt{R^2 + z^2}} \frac{t dt}{t^3} \right] \hat{k} = \frac{\sigma z}{2\epsilon_0} \left[\int_z^{\sqrt{R^2 + z^2}} \frac{1}{t^2} dt \right] \hat{k} \\ &= \frac{\sigma z}{2\epsilon_0} \left[-\frac{1}{t} \right]_z^{\sqrt{R^2 + z^2}} \hat{k} = \frac{\sigma z}{2\epsilon_0} \left[\left(-\frac{1}{\sqrt{R^2 + z^2}} \right) - \left(-\frac{1}{z} \right) \right] \hat{k} \\ \therefore \vec{E} &= \frac{\sigma}{2\epsilon_0} \left[1 - \frac{z}{\sqrt{R^2 + z^2}} \right] \hat{k} \end{aligned}$$



Problem

4. **Electric Field** a distance z above the centre of a flat circular disc of radius R , which carries a uniform surface charge σ .

ELECTROSTATIC FIELD

As $R \rightarrow \infty$:

$$\vec{E} = \frac{\sigma}{2\epsilon_0} [1-0] \hat{k}$$
$$= \frac{\sigma}{2\epsilon_0} \hat{k}$$

↑

Electric field due to
infinite sheet of charge

For points far from the disk ($z \gg R$):

$$\vec{E} = \frac{\sigma}{2\epsilon_0} \left[1 - \frac{z}{\sqrt{R^2 + z^2}} \right] \hat{k} = \frac{\sigma}{2\epsilon_0} \left[1 - \frac{1}{\left(\frac{R^2 + z^2}{z^2} \right)^{\frac{1}{2}}} \right] \hat{k} = \frac{\sigma}{2\epsilon_0} \left[1 - \left(1 + \frac{R^2}{z^2} \right)^{-\frac{1}{2}} \right] \hat{k}$$

$$\cong \frac{\sigma}{2\epsilon_0} \left[1 - \left\{ 1 - \frac{1}{2} \frac{R^2}{z^2} + \dots \right\} \right] \hat{k} \quad \text{using binomial expansion}$$

$$\cong \frac{\sigma}{2\epsilon_0} \frac{1}{2} \frac{R^2}{z^2} \hat{k} = \frac{1}{4\pi\epsilon_0} \frac{\sigma(\pi R^2)}{z^2} \hat{k}$$

$$\therefore \vec{E} \cong \frac{1}{4\pi\epsilon_0} \frac{q}{z^2} \hat{k}$$

where $q = \sigma(\pi R^2)$ is the total charge on the disk

For far away the disk "looks" like a point charge $q = \sigma(4\pi R^2)$.

Text Books & References



1. **David J. Griffith**, **Introduction to Electrodynamics**
2. **R.A. Serway and J.W. Jewett**, **Physics for Scientist and Engineers with Modern Physics**
3. **Halliday and Resnick**, **Fundamental of Physics**
4. **D. Halliday, R. Resnick, and K. Krane** , **Physics, Volume 2, Fourth Edition**

A decorative graphic in the top left corner consisting of three hexagons (green, blue, and red) and a horizontal line that is red on the left and green on the right.

*Thank
you*

