Linearly Independent and Dependent:

The set of vectors $\{\vec{v_1}, \vec{v_2}, \dots, \vec{v_p}\} \in \mathbb{R}^n$ is said to be linearly independent if the vector equation $c_1\vec{v_1} + c_2\vec{v_2} + \dots + c_p\vec{v_p} = 0$ has a trivial solution.
ie, all cp's is equal to zero $(c_1 = c_2 = \dots = c_p = 0)$

The set of vectors $\{\vec{v_1}, \vec{v_2}, \dots, \vec{v_p}\} \in \mathbb{R}^n$ is said to be linearly dependent if for the vector equation $c_1\vec{v_1} + c_2\vec{v_2} + \dots + c_p\vec{v_p} = 0$
~~not all cp's is~~ atleast one cp's is not equal to zero.

<Q): Check linear dependence and independence for

$$\vec{v_1} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad \vec{v_2} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \quad \vec{v_3} = \begin{bmatrix} 2 \\ 1 \\ 6 \end{bmatrix}$$

Soln:

To check for dependency;

$$c_1\vec{v_1} + c_2\vec{v_2} + c_3\vec{v_3} = 0$$

or
$$c_1\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + c_2\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} + c_3\begin{bmatrix} 2 \\ 1 \\ 6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

which implies that,

$$c_1 + 4c_2 + 2c_3 = 0 \quad \text{--- (i)}$$
$$2c_1 + 5c_2 + c_3 = 0 \quad \text{--- (ii)}$$
$$3c_1 + 6c_2 + 6c_3 = 0 \quad \text{--- (iii)}$$

The augmented matrix of given equations

$$\left[\begin{array}{ccc|c} 1 & 4 & 2 & 0 \\ 2 & 5 & 1 & 0 \\ 3 & 6 & 6 & 0 \end{array}\right]$$

Applying $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - 3R_1$

$$\sim \left[\begin{array}{ccc|c} 1 & 4 & 2 & 0 \\ 0 & -3 & -3 & 0 \\ 0 & -6 & -6 & 0 \end{array}\right]$$

Applying, $R_3 \to R_3 - 2R_2$

$$\sim \left[\begin{array}{ccc|c} 1 & 4 & 2 & 0 \\ 0 & -3 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

from R_3, $0 \times c_3 = 0$
Thus, c_3 is a free variable.

Since c_3 can be other than zero, the given vectors are linearly dependent.

Show that the columns of matrix A are linearly independent.

$$A = \begin{bmatrix} 0 & 1 & 4 \\ 1 & 2 & -1 \\ 5 & 8 & 0 \end{bmatrix}$$

Soln

Let $\vec{v_1} = \begin{bmatrix} 0 \\ 1 \\ 5 \end{bmatrix}$, $\vec{v_2} = \begin{bmatrix} 1 \\ 2 \\ 8 \end{bmatrix}$, $\vec{v_3} = \begin{bmatrix} 4 \\ -1 \\ 0 \end{bmatrix}$

To check for dependency,

$$c_1\vec{v_1} + c_2\vec{v_2} + c_3\vec{v_3} = 0$$

or $c_1 \begin{bmatrix} 0 \\ 1 \\ 5 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 2 \\ 8 \end{bmatrix} + c_3 \begin{bmatrix} 4 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

which implies that,

$c_2 + 4c_3 = 0$ — (i)

$c_1 + 2c_2 - c_3 = 0$ — (ii)

$5c_1 + 8c_2 = 0$ — (iii)

The augmented matrix form is.

$$\left[\begin{array}{ccc:c} 0 & 1 & 4 & 0 \\ 1 & 2 & -1 & 0 \\ 5 & 8 & 0 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc:c} 1 & 4 & 0 & 0 \\ 2 & -1 & 1 & 0 \\ 8 & 0 & 5 & 0 \end{array}\right]$$

Applying $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - 8R_1$

$$\sim \left[\begin{array}{ccc:c} 1 & 4 & 0 & 0 \\ 0 & -9 & 1 & 0 \\ 0 & -32 & 5 & 0 \end{array}\right]$$

Applying $R_3 \to R_3 - \frac{32}{9} R_2$,

$$\sim \left[\begin{array}{ccc:c} 1 & 4 & 0 & 0 \\ 0 & -9 & 1 & 0 \\ 0 & 0 & -95/9 & 0 \end{array}\right]$$

From, R_3, $-95/9\, c_3 = 0$

$\therefore c_3 = 0$

From R_2, $-9c_2 + c_3 = 0$

$\therefore c_2 = 0$

From R_1, $c_1 + 4c_2 = 0$

$\therefore c_1 = 0$

Since, $c_1 = c_2 = c_3 = 0$, ie, trivial solution.

The column of matrix A are linearly independent.

<Q>: Check linear dependency for

$$\begin{bmatrix}-2\\0\\1\end{bmatrix}, \begin{bmatrix}3\\2\\5\end{bmatrix}, \begin{bmatrix}6\\-1\\1\end{bmatrix}, \begin{bmatrix}7\\0\\-2\end{bmatrix}$$

Sol^n:

Let $\vec{v_1} = \begin{bmatrix}-2\\0\\1\end{bmatrix}$, $\vec{v_2} = \begin{bmatrix}3\\2\\5\end{bmatrix}$, $\vec{v_3} = \begin{bmatrix}6\\-1\\1\end{bmatrix}$, $\vec{v_4} = \begin{bmatrix}7\\0\\-2\end{bmatrix}$

To check for linear dependency,

$$c_1\vec{v_1} + c_2\vec{v_2} + c_3\vec{v_3} + c_4\vec{v_4} = 0$$

or $c_1\begin{bmatrix}-2\\0\\1\end{bmatrix} + c_2\begin{bmatrix}3\\2\\5\end{bmatrix} + c_3\begin{bmatrix}6\\-1\\1\end{bmatrix} + c_4\begin{bmatrix}7\\0\\2\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$

The augmented matrix is.

$$\left[\begin{array}{cccc:c}-2 & 3 & 6 & 7 & 0\\ 0 & 2 & -1 & 0 & 0\\ 1 & 5 & 1 & 2 & 0\end{array}\right]$$

Here, the equations are.

$$-2c_1 + 3c_2 + 6c_3 + 7c_4 = 0$$
$$2c_2 - c_3 = 0$$
$$c_1 + 5c_2 + c_3 + 2c_4 = 0$$

Here, we will have atleast one free variable. as there are 4 vectors in $\mathbb{R}^3$.

Thus, the given vectors are linearly dependent.

<Q>: For what value of 'h' is the vectors linearly dependent:

$v_1 = \begin{bmatrix}1\\-3\\2\end{bmatrix}$, $V_2 = \begin{bmatrix}-3\\9\\-6\end{bmatrix}$, $V_3 = \begin{bmatrix}5\\-7\\h\end{bmatrix}$

Sol^n:

Given,

$V_1 = \begin{bmatrix}1\\-3\\2\end{bmatrix}$, $V_2 = \begin{bmatrix}-3\\9\\-6\end{bmatrix}$, $V_3 = \begin{bmatrix}5\\-7\\h\end{bmatrix}$

To check for linear dependency;

$V_1 = \begin{bmatrix}1\\-3\\2\end{bmatrix}$

$V_2 = \begin{bmatrix}-3\\9\\-6\end{bmatrix} = -3\begin{bmatrix}1\\-3\\2\end{bmatrix} = -3V_1$

Since V_2 can be expressed in terms of V_1, the given set of vectors are linearly dependent.

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To check for linear dependency:

$$c_1\vec{v_1} + c_2\vec{v_2} + c_3\vec{v_3} = 0$$

or, $c_1\begin{bmatrix}1\\-3\\2\end{bmatrix} + c_2\begin{bmatrix}-3\\9\\-6\end{bmatrix} + c_3\begin{bmatrix}5\\-7\\h\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$

which implies that,

$c_1 - 3c_2 + 5c_3 = 0$ — (i)

$-3c_1 + 9c_2 - 7c_3 = 0$ — (ii)

$2c_1 - 6c_2 + hc_3 = 0$ — (iii)

Representing the eqn's in augmented matrix form,

$$\left[\begin{array}{ccc:c} 1 & -3 & 5 & 0 \\ -3 & 9 & -7 & 0 \\ 2 & -6 & h & 0 \end{array}\right]$$

Applying, $R_2 \to R_2 + 3R_1$ and $R_3 \to R_3 - 2R_1$

$$\sim \left[\begin{array}{ccc:c} 1 & -3 & 5 & 0 \\ 0 & 0 & 8 & 0 \\ 0 & 12 & h-10 & 0 \end{array}\right]$$

For the ~~eqn~~ vectors to be linearly independent, $c_3 \neq 0$.

so,

$$c_3 = h - 10 = 0$$

$$\therefore h = 10$$