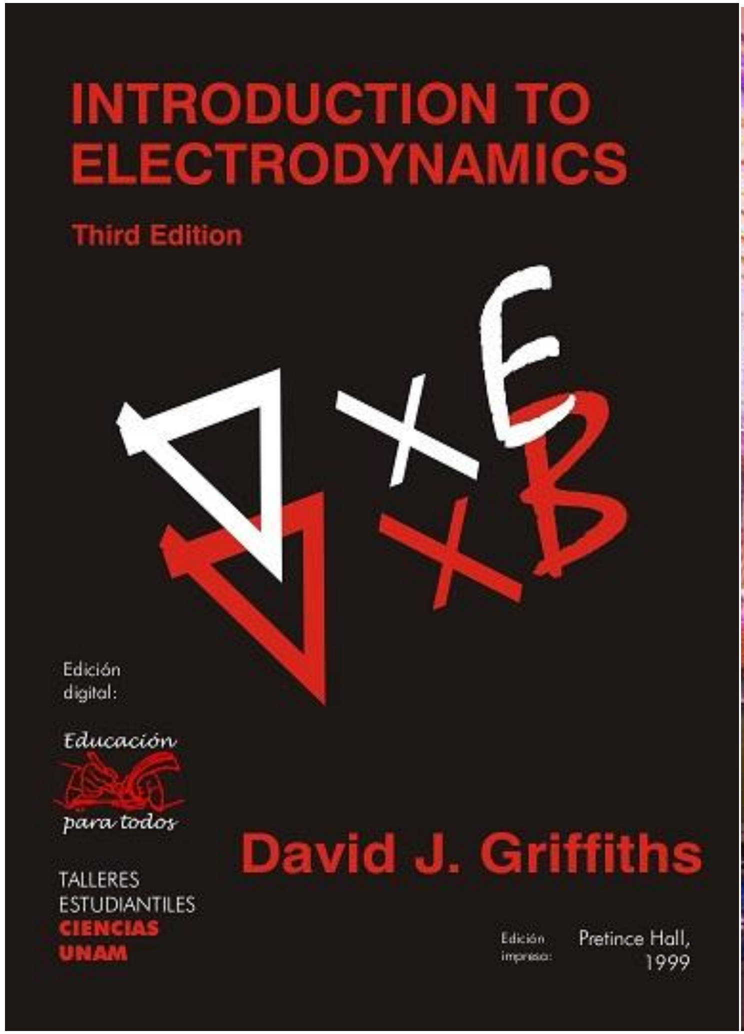
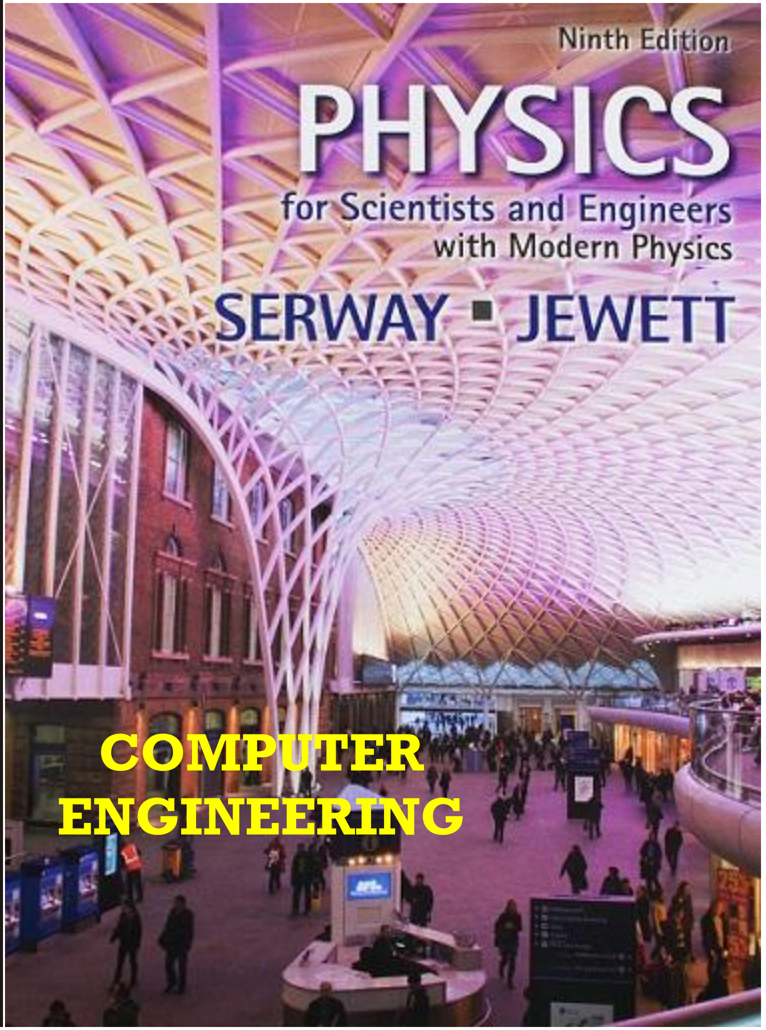
PHYSICS

General Physics II (PHYS 102)





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Course Outline

- Electric Potential
- Poisson's Equation and Laplace's Equation
- Potential of a Uniformly charged spherical Shell
- Work Done to Move a Charge & Electric Potential Energy
- Problems
- Conductors & Insulators

Electric Potential

ELECTROSTATICS FIELD

Electric Potential:

- The potential energy per unit charge at a point in an electric field is called the **electric potential *V*** ***(or simply the potential)*** at that point.

$$\boxed{V = \frac{U}{q}}$$

- Electric potential is a scalar quantity.
- The SI unit of potential is the joules per coulomb which is defined as volt (V): $\boxed{1\ \text{V} = 1\ \text{J/C}}$
- The electric potential at an arbitrary point P in an electric field equals the work required per unit charge to bring a positive test charge from infinity to that point.

$$\boxed{\begin{aligned} V(\vec{r}) = V_{\text{P}} &= W_{\underset{\infty\ \to\ \text{P}}{(\text{unit})}} \\ &= -\int_{\infty}^{\text{P}} \vec{E}\cdot d\vec{l} \end{aligned}}$$

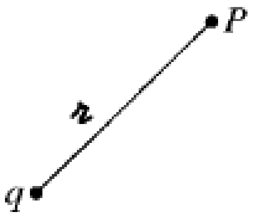
- Potential obeys the superposition principle: $\boxed{V = V_1 + V_2 + \ldots}$

  The potential at any given point is the sum of the potentials due to all the source charges separately.

Expression for Electric Potential :

The electric potential of a point charge at a point P:

$$V(\vec{\text{r}}) = \frac{1}{4\pi\varepsilon_0}\frac{q}{\mathscr{r}}$$



The potential of collection of charges:

$$V(\vec{\text{r}}) = \frac{1}{4\pi\varepsilon_0}\sum_{\text{i}=1}^{n}\frac{q_i}{\mathscr{r}_i}$$

The potential for a continuous distribution of charges

$$V(\vec{\text{r}}) = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{\mathscr{r}}$$

In particular, the potential for a surface charge is

$$V(\vec{\text{r}}) = \frac{1}{4\pi\varepsilon_0}\int\frac{\sigma(\text{r}')}{\mathscr{r}}\,da'$$

Potential Difference:

The potential difference between points a and b is equal to the work per unit charge required to carry a charged particle from a and b:

$$\boxed{\text{V(b)} - \text{V(a)} = W_{\underset{a\ \to\ b}{(\text{unit})}} = -\int_{a}^{b}\vec{E}\cdot d\vec{l}}$$

Electric Potential

ELECTROSTATICS FIELD

The Electric Field is the Gradient of a Scalar Potential

The potential difference between two points a and b:

$$V(\mathrm{b})-V(\mathrm{a})=-\int_a^b \vec{E}\cdot d\vec{l} \qquad \text{..........(1)}$$

The fundamental theorem for gradients states that:

$$V(\mathrm{b})-V(\mathrm{a})=\int_a^b (\nabla V)\cdot d\vec{l} \qquad \text{..........(2)}$$

So, $\quad -\int_a^b \vec{E}\cdot d\vec{l}=\int_a^b (\nabla V)\cdot d\vec{l}$

Since this is true for any points a and b, the integrands must be equal:

$$\therefore \boxed{\vec{E}=-\nabla V}$$

The expression for electric field in a region where potential : $V=-kxy$

$$\vec{E}=-\nabla V=-\left[\hat{i}\frac{\partial V}{\partial x}+\hat{j}\frac{\partial V}{\partial y}+\hat{k}\frac{\partial V}{\partial z}\right]$$

$$=-\left[\hat{i}\frac{\partial(-kxy)}{\partial x}+\hat{j}\frac{\partial(-kxy)}{\partial y}+\hat{k}\frac{\partial(-kxy)}{\partial z}\right]$$

$$=ky\,\hat{i}+kx\,\hat{j}$$

Poisson's Equation and Laplace's Equation:

Gauss's law in differential form:

$$\nabla\cdot\vec{E}=\frac{1}{\varepsilon_0}\rho$$

The electric field can be written as the gradient of a scalar potential

i.e. $\vec{E}=-\nabla V$

$$\therefore \qquad \nabla\cdot\vec{E}=\frac{1}{\varepsilon_0}\rho$$

$$\Rightarrow \nabla\cdot(-\nabla V)=\frac{1}{\varepsilon_0}\rho$$

$$\therefore \boxed{\nabla^2 V=-\frac{\rho}{\varepsilon_0}}$$

This is known as **Poisson's Equation.**

In regions where there is no charge, so that $\rho=0$, Poisson's equation reduces to **Laplace's Equation**

$$\boxed{\nabla^2 V=0}$$

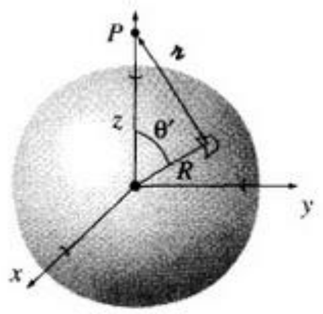
Electric Potential

1. Find the potential of a uniformly charged spherical shell of radius .

Solution:

- The potential for a surface charge is $V(\vec{r}) = \dfrac{1}{4\pi\varepsilon_0}\int \dfrac{\sigma}{\mathscr{r}}\, da'$.

- From the law of cosines, $\boxed{\mathscr{r}^2 = R^2 + z^2 - 2Rz\cos\theta'}$



- An element of surface area on this sphere is $R^2 \sin\theta' d\theta' d\phi'$.

- So,

$$V(z) = \frac{1}{4\pi\varepsilon_0}\left[\int \frac{\sigma}{\sqrt{R^2 + z^2 - 2Rz\cos\theta'}}\left(R^2 \sin\theta' d\theta' d\phi'\right)\right]$$

$$= \frac{\sigma R^2}{4\pi\varepsilon_0}\left[\left\{\int_0^{\pi} \frac{\sin\theta' d\theta'}{\sqrt{R^2 + z^2 - 2Rz\cos\theta'}}\right\}\left\{\int_0^{2\pi} d\phi'\right\}\right]$$

$$= \frac{\sigma R^2}{4\pi\varepsilon_0}\left[\left\{\frac{1}{Rz}\left(\sqrt{(R+z)^2} - \sqrt{(R-z)^2}\right)\right\}\{2\pi\}\right]$$

$$= \frac{\sigma R^2}{2\varepsilon_0}\left[\frac{1}{Rz}\left(\sqrt{(R+z)^2} - \sqrt{(R-z)^2}\right)\right]$$

$$\therefore \boxed{V(z) = \frac{\sigma R}{2\varepsilon_0 z}\left[\sqrt{(R+z)^2} - \sqrt{(R-z)^2}\right]}$$

Put $R^2 + z^2 - 2Rz\cos\theta' = t^2$

$$\Rightarrow \sin\theta' d\theta' = \frac{1}{Rz}(t\, dt)$$

when $\theta'=0$, then t $=\sqrt{R^2 - z^2}$

when $\theta'=\pi$, then t $=\sqrt{R^2 + z^2}$

$$\Rightarrow \int_0^{\pi} \frac{\sin\theta' d\theta'}{\sqrt{R^2 + z^2 - 2Rz\cos\theta'}} = \frac{1}{Rz}\int_{\sqrt{R^2-z^2}}^{\sqrt{R^2+z^2}} \frac{(t\, dt)}{t}$$

$$= \frac{1}{Rz}\int_{\sqrt{R^2-z^2}}^{\sqrt{R^2+z^2}} dt$$

$$= \frac{1}{Rz}\left(\sqrt{(R+z)^2} - \sqrt{(R-z)^2}\right)$$

For points outsidethe sphere, $z > R$,

and hence $\sqrt{(R-z)^2} = z - R$

$$\therefore V_{out}(z) = \frac{R\sigma}{2\varepsilon_0 z}\left[(R+z) - (z-R)\right] = \frac{\sigma R^2}{\varepsilon_0 z}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\sigma(4\pi R^2)}{z} = \frac{1}{4\pi\varepsilon_0}\frac{q}{z}$$

For points inside the sphere, $z < R$,

and hence $\sqrt{(R-Z)^2} = R - z$

$$\therefore V_{in}(z) = \frac{R\sigma}{2\varepsilon_0 z}\left[(R+z) - (R-z)\right] = \frac{\sigma R}{\varepsilon_0}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\sigma(4\pi R^2)}{R} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

For points on the sphere, $z = R$,

$$\therefore V_{on}(R) = \frac{\sigma R}{\varepsilon_0} = \frac{1}{4\pi\varepsilon_0}\frac{\sigma(4\pi R^2)}{R}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{R} = V_{in}(z)$$

Work and Energy in Electrostatics

ELECTROSTATICS FIELD

The Work Done to Move a Charge

- Suppose we have a stationary configuration of source charges, and we want to move a test charge from a point a to point b [Figure W_W-1].
- At any point along the path, the electric force on Q is $\vec{F} = Q\vec{E}$ the force we exert, in opposition to this electrical force is $-Q\vec{E}$.
- The work done to move a test charge Q from a point a to point b is

$$W = \int_a^b \vec{F} \cdot d\vec{l} = \int_a^b \left(-Q\vec{E}\right) \cdot d\vec{l} = Q\left[-\int_a^b \vec{E} \cdot d\vec{l}\right]$$

$$= Q\left[V(b) - V(a)\right]$$

$$\therefore \boxed{V(b) - V(a) = V(\vec{r}_b) - V(\vec{r}_a) = \frac{W}{Q}}$$

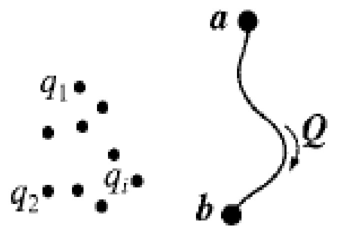


Figure Ww-3

- The *potential difference* between points a and b is equal to the work per unit charge required to carry a charged particle from a and b.
- The work done to bring the charge Q from infinity to the point $\vec{r}$ is

$$W = Q\left[V(\vec{r}) - V(\infty)\right]$$

$$\therefore \boxed{W = QV\left(\vec{r}\right)}$$

The potential energy per unit charge at a point in an electric field is called the *Electric potential* at that point.

Work and Energy in Electrostatics

ELECTROSTATICS FIELD

Electric Potential Energy

- Consider that three point charges q_1, q_2 and q_3 are lying at locations $\vec{r}_1$, $\vec{r}_2$ and $\vec{r}_3$ respectively.
- First of all, let us remove all the three charges to infinite distance from each other.

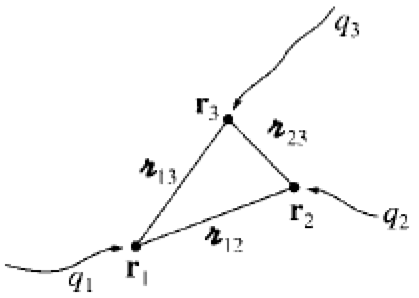


Figure Ww-3

(i) Let us move the charge q_1 from infinity to its location $\vec{r}_1$.

The work done to move the charge q_1 from infinity to its location $\vec{r}_1$ is $W_1=0$.

(ii) Let us move the charge q_2 from infinity to its location $\vec{r}_2$.

The work done to move the charge q_2 from infinity to its location $\vec{r}_2$ is

$$W_2 = q_2\left[V_1(\vec{r}_2)\right] \qquad \text{where } V_1(\vec{r}_2) \text{ is the potential due to } q_1.$$

$$= q_2\left[\frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{12}}\right] \qquad = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$$

(iii) Let us move the charge q_3 from infinity to its location $\vec{r}_3$.

The work done to move the charge q_3 from infinity to its location $\vec{r}_3$ is

$$W_3 = q_3\left[V_{1,2}(\vec{r}_3)\right] \quad \text{where } V_{1,2}(\vec{r}_3) \text{ is the potential due to carges } q_1 \text{ and } q_2.$$

$$= q_3\left[\frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{13}} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{23}}\right] \quad = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r_{13}} + \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_3}{r_{23}}$$

Electric Potential Energy

ELECTROSTATICS FIELD

Electric Potential Energy

- Therefore, the total work necessary to assemble the first three charges is $W = W_1 + W_2 + W_3$ and is equal to the potential energy *U*.

$$\therefore U = W = 0 + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{\mathscr{r}_{12}} + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{\mathscr{r}_{13}} + \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_3}{\mathscr{r}_{23}} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{\mathscr{r}_{12}} + \frac{q_1 q_3}{\mathscr{r}_{13}} + \frac{q_2 q_3}{\mathscr{r}_{23}}\right]$$

$$= \frac{1}{2}\times\frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{3}\sum_{\substack{j=1\\ j\neq i}}^{3}\frac{q_i q_j}{\mathscr{r}_{ij}}$$

- For a system of *n* - point charges, we have

$$U = \frac{1}{2}\times\frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\frac{q_i q_j}{\mathscr{r}_{ij}}$$

$$= \frac{1}{2}\sum_{i=1}^{n} q_i\left(\frac{1}{4\pi\varepsilon_0}\sum_{\substack{j=1\\ j\neq i}}^{n}\frac{q_j}{\mathscr{r}_{ij}}\right)$$

$$\therefore \boxed{U = \frac{1}{2}\sum_{i=1}^{n} q_i V(\vec{r}_i)}$$

where $V(\vec{r}_i) = \dfrac{1}{4\pi\varepsilon_0}\displaystyle\sum_{\substack{j=1\\ j\neq i}}^{n}\frac{q_j}{\mathscr{r}_{ij}}$ is the potential at point $\vec{r}_i$ (the postion of q_i) due to all other charges.

Electric Potential Energy

ELECTROSTATICS FIELD

The Energy of Continuous Charge Distribution

- The total work necessary to assemble the *n* - point charges is given by

$$W = \frac{1}{2}\sum_{i=1}^{n} q_i \; V(\vec{r}_i) \qquad \text{...........} \; (1)$$

where $V(\vec{r}_i)$ is the potential at point $\vec{r}_i$ (the postion of q_i)

due to all other charges.

- For a volume charge density ρ , Eq. (1) becomes

$$W = \frac{1}{2}\int \rho \; V d\tau$$

$$= \frac{1}{2}\int \left(\varepsilon_0 \nabla\cdot\vec{E}\right) \; V d\tau \qquad \left[\text{Using Gauss,s law:} \nabla\cdot\vec{E} = \frac{\rho}{\varepsilon_0}\right]$$

$$= \frac{\varepsilon_0}{2}\int V(\nabla\cdot\vec{E}) \; d\tau \qquad \left[\because \nabla\cdot(V\vec{E}) = V(\nabla\cdot\vec{E}) + (\nabla V)\cdot\vec{E}\right]$$

$$= \frac{\varepsilon_0}{2}\left[-\int (\nabla V)\cdot\vec{E} \; d\tau + \int \nabla\cdot(V\vec{E}) \; d\tau\right]$$

$$= \frac{\varepsilon_0}{2}\left[\int \vec{E}\cdot\vec{E} \; d\tau + \oint_S (V\vec{E}) \; \cdot d\vec{a}\right] \qquad \left[\text{Using } \vec{E} = -\nabla V \text{ and Divergece theorem}\right]$$

$$== \frac{\varepsilon_0}{2}\left[\int E^2 d\tau + \oint_S (V\vec{E}) \; \cdot d\vec{a}\right]$$

When the integration is taken over all space, the surface integral goes to zero.

$$\boxed{W = \frac{\varepsilon_0}{2}\int_{\text{all space}} E^2 d\tau = \int_{\text{all space}} u_E \; d\tau}$$

where $u_E = \dfrac{\varepsilon_0}{2} E^2$

$\downarrow$

Energy Density

Work and Energy in Electrostatics

Notes:

- **The work done to move a charge Q from point** a to point b: $\boxed{W = Q[V(b) - V(a)]}$

- **The work done to move a charge Q from** ∞ to point b **:** $\boxed{W = Q[V(a)]}$

- **The energy of a continuous charge distribution:**

$$\boxed{W = \frac{\varepsilon_0}{2}\int_{all\ \text{space}} E^2 d\tau = \int_{all\ \text{space}} u_E\, d\tau}$$

$$\boxed{\text{Energy density, } u_E = \frac{\varepsilon_0}{2}E^2 \rightarrow \text{energy per unit volume}\left[\text{Unit of } u_E \ \rightarrow Jm^{-3}\right]}$$

- **The electrostatic potential energy of configurations of three charges** q_1, q_2 and q_3 at locations $\vec{r}_1$, $\vec{r}_2$ and $\vec{r}_3$ respectively**:**

$$\boxed{U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right]}$$

Problem

Notes:

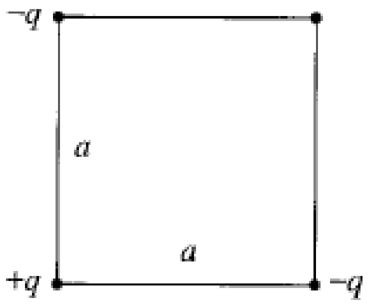
(a) Three charges are situated at the corners of a square (side), as shown in Figure P_P-1. How much work does it take to bring in another charge, $+q$, from far away and place it in the fourth corner?

(b) How much work does it take to assemble the whole configuration of four charges?

Hint:

(a)

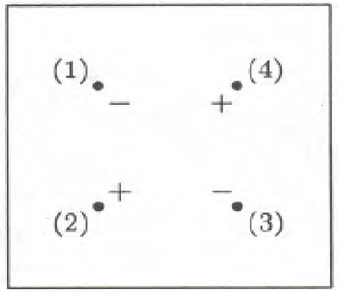
$$W_4 = qV$$
$$= (+q)\left[\frac{1}{4\pi\varepsilon_0}\left\{\frac{-q}{a} + \frac{q}{a\sqrt{2}} + \frac{-q}{a}\right\}\right]$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}\left[-2 + \frac{1}{\sqrt{2}}\right]$$



$$\because \qquad V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_{14}} + \frac{q_2}{r_{24}} + \frac{q_3}{r_{34}}\right)$$
$$W = U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_1q_4}{r_{14}} + \frac{q_2q_3}{r_{23}} + \frac{q_2q_4}{r_{24}} + \frac{q_3q_4}{r_{34}}\right]$$

(b)

$$W = \frac{1}{4\pi\varepsilon_0}\left[\frac{-q^2}{a} + \frac{q^2}{a\sqrt{2}} + \frac{-q^2}{a} + \frac{-q^2}{a} + \frac{q^2}{a\sqrt{2}} + \frac{-q^2}{a}\right]$$
$$= 2\frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}\left[-2 + \frac{1}{\sqrt{2}}\right]$$



Problem

Notes:

(a) Find the energy of a uniformly charged spherical shell of total charge q and radius R .

Solution:

For a uniformly charged spherical shell

Inside $E = 0$

Outside $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

Therefore

$$W_{tot} = \frac{\varepsilon_0}{2}\int_{\text{all space}} E^2 d\tau = \frac{\varepsilon_0}{2}\int_{outside}\left[\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\right]^2\left(r^2\sin\theta dr d\theta d\phi\right)$$

$$= \frac{\varepsilon_0}{2}\frac{1}{(4\pi\varepsilon_0)^2}q^2\left[\left\{\int_R^\infty \frac{1}{r^2}dr\right\}\left\{\int_0^\pi \sin\theta d\theta\right\}\left\{\int_0^{2\pi} d\phi\right\}\right]$$

$$= \frac{\varepsilon_0}{2}\frac{1}{(4\pi\varepsilon_0)^2}q^2(2)(2\pi)\left[\int_R^\infty \frac{1}{r^2}dr\right]$$

$$\therefore \boxed{W_{tot} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{2R}}$$

Problem

Notes:

(a) Find the energy stored in a uniformly charged solid sphere of radius *R* and charge *q* .

For a uniformly charged solid sphere of radius *R*:

Inside $E_{\text{in}} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{qr}{R^3}$

Outside $E_{\text{out}} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{\text{q}}{r^2}$

Therefore,

$$W_{tot} = \frac{\varepsilon_0}{2}\int\limits_{\text{all space}} E^2 d\tau \;=\; \frac{\varepsilon_0}{2}\int\limits_{\text{all space}} E^2\left(r^2\sin\theta dr d\theta d\phi\right)$$

$$= \frac{\varepsilon_0}{2}\left[\left\{\int_0^\infty E^2 r^2 dr\right\}\left\{\int_0^\pi \sin\theta d\theta\right\}\left\{\int_0^{2\pi} d\phi\right\}\right]$$

$$= \frac{\varepsilon_0}{2}(4\pi)\left[\int_0^R (E_{\text{in}})^2 r^2 dr + \int_R^\infty (E_{out})^2 r^2 dr\right]$$

$$= 2\pi\varepsilon_0\left[\int_0^R\left(\frac{1}{4\pi\varepsilon_0}\frac{qr}{R^3}\right)^2 r^2 dr + \int_R^\infty\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\right)^2 r^2 dr\right]$$

$$= 2\pi\varepsilon_0\left(\frac{1}{4\pi\varepsilon_0}q\right)^2\left[\frac{1}{R^6}\int_0^R r^4 dr + \int_R^\infty \frac{1}{r^2}dr\right] \;=\; \frac{1}{4\pi\varepsilon_0}\frac{q^2}{2}\left[\frac{1}{R^6}\frac{R^5}{5} + \frac{1}{R}\right]$$

$$\therefore \boxed{W_{tot} = \frac{6}{5}\left[\frac{1}{4\pi\varepsilon_0}\frac{q^2}{2R}\right]}$$

Conductors and Insulators

ELECTROSTATICS FIELD

Conductors

- **Conductors** are substances, which contain large numbers of essentially free charge carriers.
- The charge carriers are free to wander throughout the conducting material; they respond to almost infinitesimal electric fields, and they continue to move as long as they experience a field.

Insulators

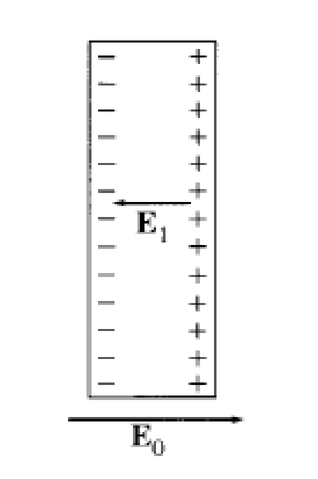
- **Insulators** (Dielectrics) are substances in which all charged particles are bound rather strongly to constituent molecules.
- The charged particles may shift their positions slightly in response to an electric field, but they do not leave the vicinity of their molecules.

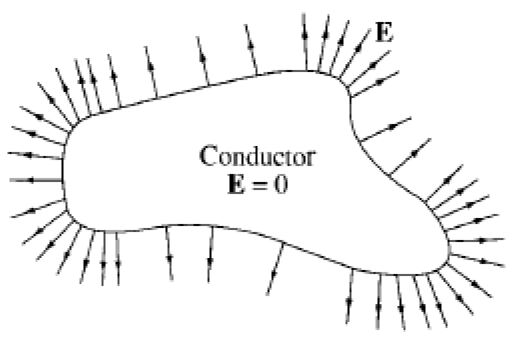
Perfect Conductor

- A ***Perfect*** conductor is a material containing an *unlimited* supply of completely free charges.
- In real life there are no perfect conductors, but many substances come amazingly close.

Basic Electrostatic Properties

- Electric field $E = 0$, inside a conductor





- Volume charge density $\rho = 0$ inside a conductor

  From Gauss's law: $\nabla \cdot \vec{E} = \dfrac{\rho}{\varepsilon_0}$

  $\vec{E} = 0$ inside a conductor $\Rightarrow \rho = 0$ inside a conductor.

- Any net charge resides on the surface.
- $\vec{E}$ is perpendicular to the surface, just outside a conductor.
- A conductor is an equipotential.

  For any two points within (or at the surface of) a given conductor, $V(a) - V(b) = -\int_a^b \vec{E} \cdot d\vec{l} = 0$

  $\Rightarrow V(a) = V(b)$

Questions

ELECTROSTATICS FIELD

Notes:

- If E and V are electric field and electric potential at the midpoint of two equal and opposite point charges, then $E \neq 0,\ V = 0$.

- A thin spherical conducting shell of radius R has a charge q. Another charge Q is placed at the centre of the shell. The electrostatic potential at a point p at a distance from the centre of the shell is

$$V = V_1 + V_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R/2} + \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

- The work done in displacing a charge 2C through 0.5m on an equipotential surface is $zero$.

- The electrostatic potential energy of configuration of four charges $+q, -2q, -q$ and $+2q$ placed at four corners A, B, C and D of a square of side 'a' is

$$U = -\frac{1}{4\pi\varepsilon_0}\left[\frac{5q^2}{a\sqrt{2}}\right].$$

- The electrostatic potential energy of configuration of three charges +2e,-e and -2e placed at three corners A, B and C of a equilateral triangle of side ' l ' is

$$\boxed{U = -\frac{e^2}{\pi\varepsilon_0 l}}.$$

Text Books & References

1. **David J. Griffith**, Introduction to Electrodynamics
2. **R.A. Serway and J.W. Jewett**, Physics for Scientist and Engineers with Modern Physics
3. **Halliday and Resnick**, Fundamental of Physics
4. **D. Halliday, R. Resnick, and K. Krane** , Physics, Volume 2, Fourth Edition


Thank you