HEAT TRANSFER

⟨2⟩: Calculate the average energy $\bar{\varepsilon}$ of an oscillator of frequency $0.60 \times 10^{14}\ sec^{-1}$ at temperature $T = 1800\ K$ treating it as

i) Classical oscillator ii) Planck's eoscillators.

Soln:

Given,

frequency of oscillator $(f) = 0.60 \times 10^{14}\ sec^{-1}$

temperature $(T) = 1800\ K$

Boltzmann's constant $(k) = 1.38 \times 10^{-23}$ Joule/K

Planck's constant $(h) = 6.6 \times 10^{-34}$ Js

Average energy of classical oscillator $(\bar{\varepsilon}_c) = ?$

Average energy of planck's oscillator $(\bar{\varepsilon}_p) = ?$

We know,

For Planck's oscillators,

$$\bar{\varepsilon}_p = \frac{hf}{e^{\frac{hf}{kT}} - 1} = \frac{6.6 \times 10^{-34} \times 0.60 \times 10^{14}}{e^{\frac{6.6 \times 10^{-34} \times 0.60 \times 10^{14}}{1.38 \times 10^{-23} \times 1800}} - 1}$$

$$\therefore \bar{\varepsilon}_p = 1.01 \times 10^{-20}\ J$$

For classical oscillators,

$$\bar{\varepsilon}_p = kT$$

$$= (1.38 \times 10^{-23}) \times 1800$$

$$\therefore \bar{\varepsilon}_p = 2.484 \times 10^{-20}\ J$$

<Q.3>: When the temperature of a black body increases, it is observed that the wavelength corresponding to maximum energy changes from 0.26 μm to 0.13 μm. Calculate the ratio of emissive power of the body at respective temperature.

Soln:

Given,

Wavelength corresponding to max energy 1 (λ_1) = 0.26 μm

Wavelength corresponding to max energy 2 (λ_2) = 0.13 μm

From Wein's displacement law,

$$\lambda_1 T_1 = \lambda_2 T_2$$

$$\text{or } \frac{T_1}{T_2} = \frac{\lambda_2}{\lambda_1} = \frac{0.13}{0.26} \quad \therefore \frac{T_1}{T_2} = \frac{1}{2}$$

Now,

$$\text{ratio of emissive power} = \left(\frac{T_1}{T_2}\right)^4$$

$$= \left(\frac{1}{2}\right)^4 = \frac{1}{16}$$

$$= 1 : 16$$

<Q.4>: The radiation emitted by a star is 10000 times more than that of sun. If the surface temperature of sun and the star is 6000 K and 2000 K respectively. Calculate the ratio of the radii of the star and the sun.

Soln:

Given,

surface temperature of star $(T_1) = 2000$ K

Surface temperature of sun $(T_2) = 6000$ K

Let Q_1 = heat radiation emitted by star.

Q_2 = heat radiation by sun.

$Q_1 = 10000\, Q_2$ [According to question].

From stefan's law, for 1 sec of time interval.

$$Q_1 = \sigma A_1 T_1^4 = \sigma \pi r_1^2 T_1^4 \quad \text{— (i)}$$

$$Q_2 = \sigma A_2 T_2^4 = \sigma \pi r_2^2 T_2^4 \quad \text{— (ii)}$$

Dividing (i) by (ii); we get.

$$\frac{Q_1}{Q_2} = \frac{\sigma \pi r_1^2 T_1^4}{\sigma \pi r_2^2 T_2^4}$$

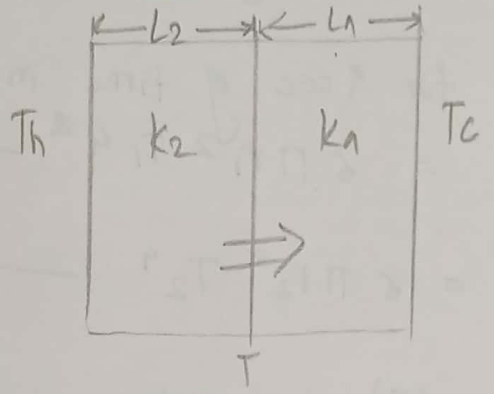
or, $$\frac{10000\, \cancel{Q_2}}{\cancel{Q_2}} = \frac{r_1^2}{r_2^2} \times \frac{(2000)^4}{(6000)^4}$$

or, $$\left(\frac{10000 \times 6000^4}{2000^4}\right)^{1/2} = \left(\frac{r_1^2}{r_2^2}\right)^{1/2}$$

$\therefore r_1 : r_2 = 900 : 1$

<Q.1>: The slab of thickness L_1 and L_2 and thermal conductivity k_1 and k_2 are in thermal contact with each other as shown in figure. The temperature of outer surfaces are T_H and T_C. such that $T_h > T_C$. Determine the temperature at the interface and the rate of energy transfer by conduction through slabs in steady condition.

Soln:



Soln.

Given,

Thickness of slab 1 = L_1

Thermal conductivity of 1st slab = K_1

Thickness of slab 2 = L_2

Thermal conductivity of 2nd slab = K_2

Let θ be the temperature of interface.

As $T_H > T_C$,

Rate of heat loss in 2nd slab $\left(\frac{dQ}{dt}\right)_2 = \frac{K_2 A (T_H - \theta)}{L_2}$ ... (i)

Rate of heat gain in 1st slab $\left(\frac{dQ}{dt}\right)_1 = \frac{K_1 A (\theta - T_C)}{L_1}$ ... (ii)

We know, rate of heat loss = rate of heat gain

Equating (i) and (ii), we get.

$$\frac{K_2 A (T_H - \theta)}{L_2} = \frac{K_1 A (\theta - T_c)}{L_1}$$

or, $K_2 L_1 T_H - K_2 \theta L_1 = K_1 \theta L_2 - K_1 L_2 T_c$

or, $K_2 L_1 T_H + K_1 L_2 T_c = \theta (K_2 L_1 + K_1 L_2)$

$$\therefore \theta = \frac{K_1 L_2 T_c + K_2 L_1 T_H}{K_2 L_1 + K_1 L_2}$$

Now,

Rate of heat transfer $\left(\frac{dQ}{dt}\right) = \frac{K_2 A}{L_2}\left(T_H - \frac{K_2 L_1 T_H + K_1 L_2 T_c}{K_2 L_1 + K_1 L_2}\right)$

$$= \frac{K_2 A}{L_2}\left(\frac{T_H K_2 L_1 + T_H K_1 L_2 - K_2 L_1 T_H + K_1 L_2 T_c}{K_2 L_1 + K_1 L_2}\right)$$

$$= \frac{K_2 A \cdot K_1 L_2 (T_H - T_c)}{L_2 \cancel{K_2} (K_2 L_1 + K_1 L_2)}$$

$$\therefore \frac{dQ}{dt} = \frac{A K_1 K_2 (T_H - T_c)}{K_1 L_2 + K_2 L_1}$$