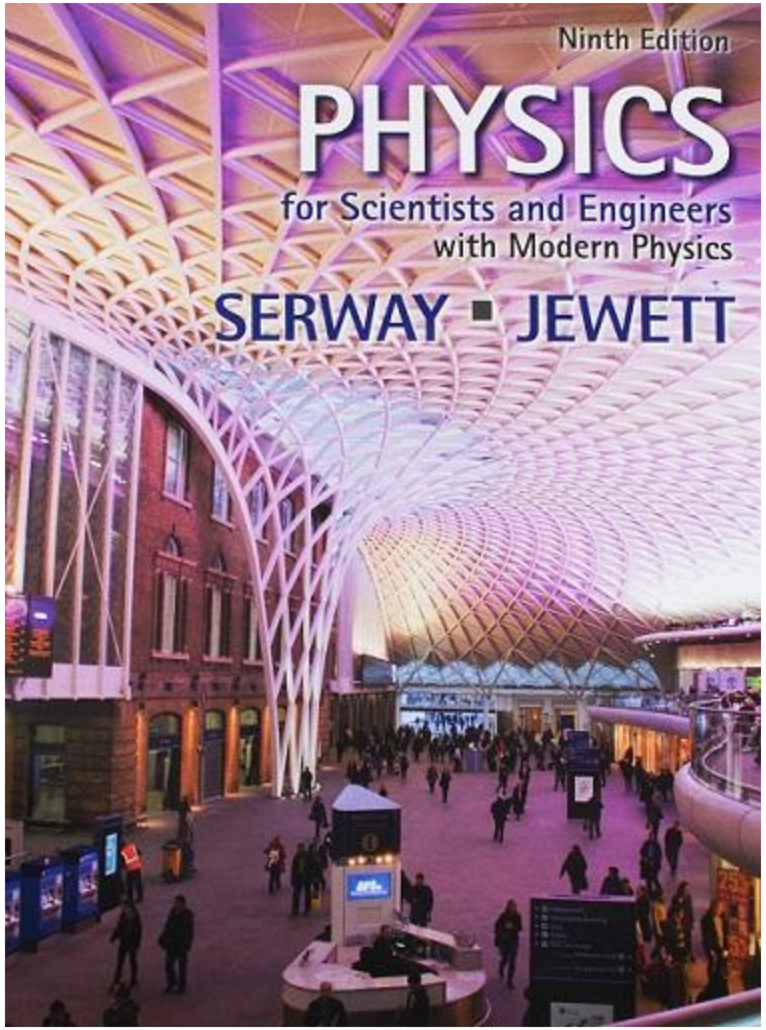
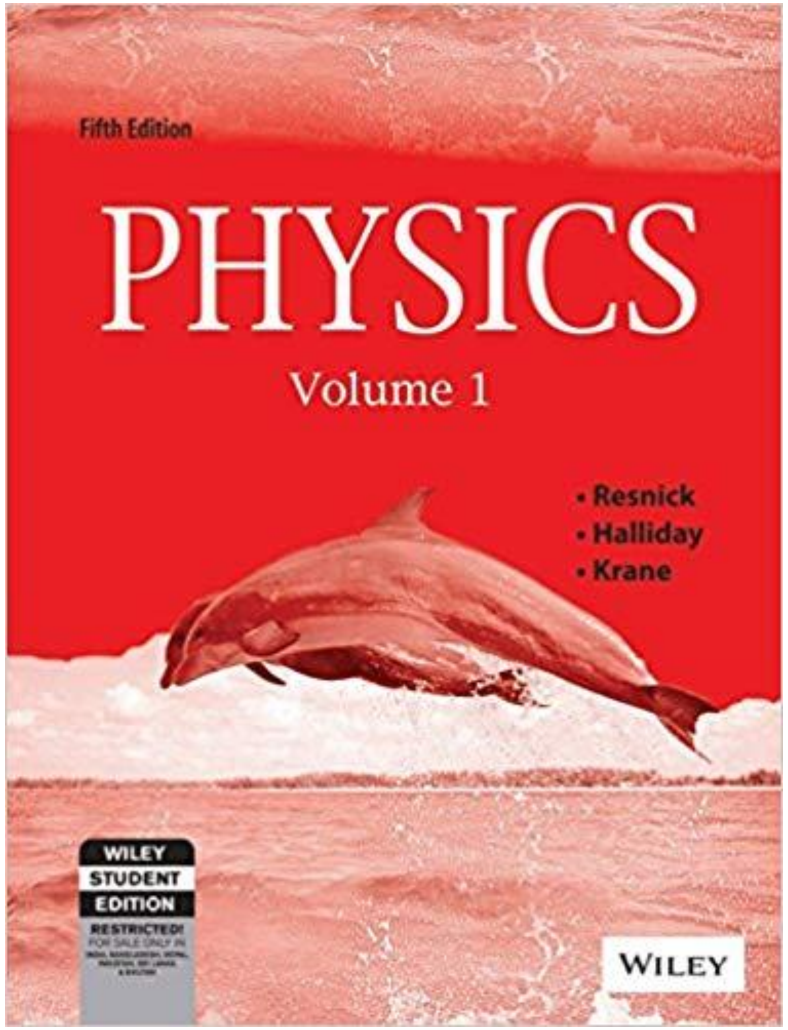
PHYSICS

General Physics I (PHYS 101)





Ganesh Kuwar Chhetri

Dept. of Physics, School of Science, Kathmandu University

ELASTICITY

Course Outline

- Stress, Strain, Elastic Limit
- Types of Elasticity
- Energy Stored in Stretched Wire
- Poisson's Ratio
- Numerical

Elasticity

Elasticity

- In reality, all objects are deformable and internal forces in the object resist the deformation.
- The property of matter by virtue of which it regains its original shape and size, when the deforming forces have been removed is called *elasticity*.

Stress

- The external force acting on an object per unit cross-sectional area.
- It characterizes the strength of the forces causing the deformation, on a "force per unit area" basis.
- $\text{Stress} = \dfrac{\text{Deforming force}}{\text{Cross-sectional area}} = \dfrac{F}{A}$
- SI unit of stress is the pascal. $\left[1\text{ Pa} = 1\text{N m}^{-2}\right]$
- Normal stress: Tensile stress

  Compressive stress

  Volume stress
- Shear stress

Strain:

- A measure of degree of deformation.
- It is the result of stress.
- It has no unit.

Hooke's Law

Hooke's Law: Robert Hooke (1635–1703), an English Physicist

- Within the proportional limit, $\text{stress} \propto \text{strain}$

$$\boxed{\text{Elastic Modulus} = \frac{\text{stress}}{\text{strain}}}$$

OR

- The extension is proportional to the load or tension in the wire when the proportional limit is not exceeded.

  It means applied force is proportional to the elongation produced. i.e., $F \propto e$

- Elastic modulus depends on the material being deformed and on the nature of deformation.
- Elastic modulus, in general, relates what is done to a solid object (a force is applied) and how that object responds (it deforms to some extent).
- It is not really a general law but an experimental finding that is valid only over a limited range.

$$\frac{F}{A} \propto \left(\frac{l}{A}\right)\left(\frac{e}{l}\right)$$

$$\downarrow$$

approximately constant

$$\boxed{\text{stress} \propto \text{strain}}$$

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Tensile and Compressive Stress and Strain

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Tensile Stress and Strain

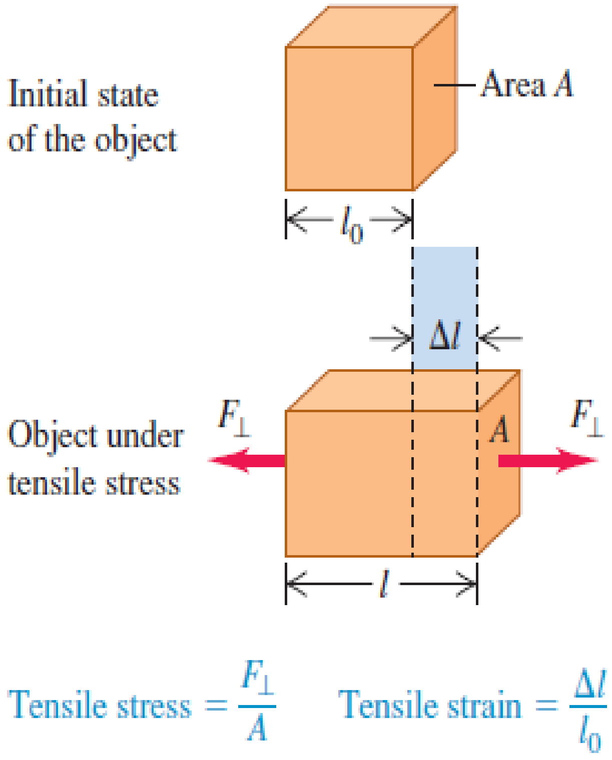


Figure E-1:

- **An object in tension. The net force on the object is zero, but the object deforms. The tensile stress (the ratio of the force to the cross-sectional area) produces a tensile strain (the elongation divided by the initial length).**
- **The elongation is exaggerated for clarity.**

Compressive Stress and Strain

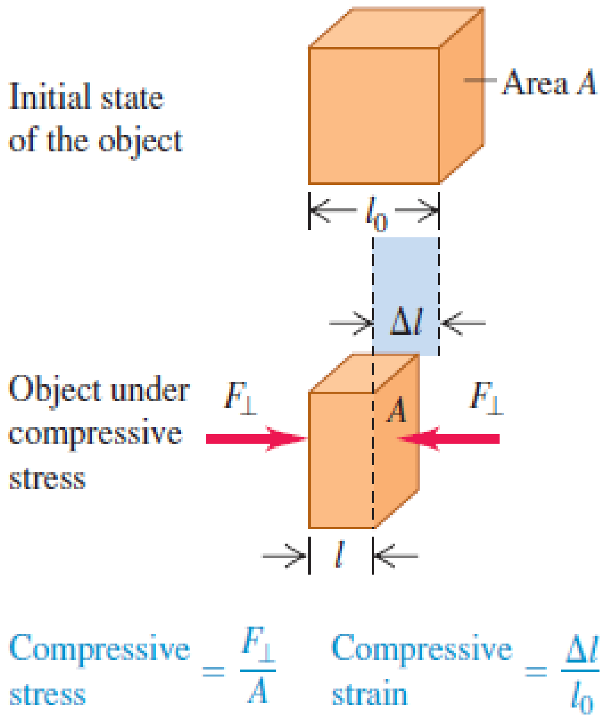


Figure E-2:

- **An object in compression. The net force on the object is zero, but the object deforms. The compressive stress (the ratio of the force to the cross-sectional area) produces a compressive strain (the contraction divided by the initial length).**
- **The contraction is exaggerated for clarity.**

Bulk Stress and Strain

Bulk Stress and Strain

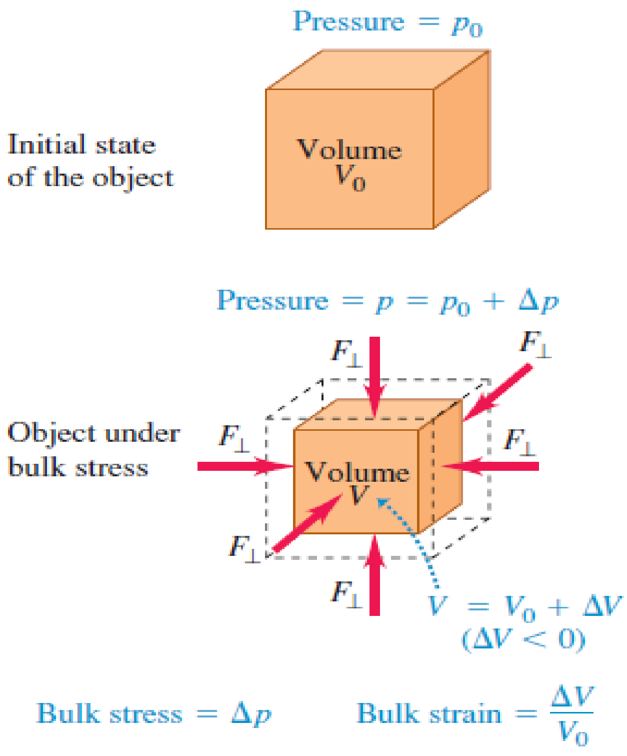


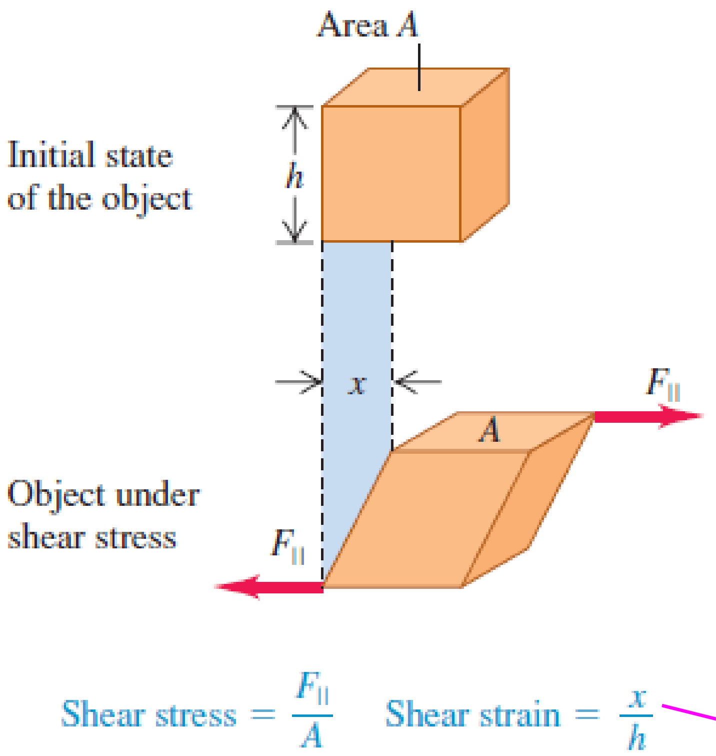
Figure E-3:

An object under bulk stress. Without the stress, the cube has volume when the stress is applied; the cube has a smaller volume *V*.

The volume change is exaggerated for clarity.

[Pressure plays the role of stress in a volume deformation]

Shear Stress and Strain



horizontal distance that the shared face moves

height of the object

Figure E-4:

An object under shear stress. Forces are applied tangent to opposite surfaces of the object.

The deformation *x* is exaggerated for clarity.

Figure E-5:
A book under shear stress.

Types of Moduli of Elasticity

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Young's Modulus: Elasticity in Length

- It measures the resistance of a solid to a change in its length.

$$\text{Young's Modulus (Y)} = \frac{\text{Tensile stress}}{\text{Tensile strain}} = \frac{F_\perp / A}{\Delta l / l_0}$$

- SI unit of Young's Modulus is the pascal. $\left[1\text{Pa} = 1 \text{ N m}^{-2}\right]$
- Young's modulus is typically used to characterize a rod or wire stressed under either tension or compression.

Bulk Modulus : Volume Elasticity

- It measures the resistance of solids or fluids to changes in their volume.

$$\text{Bulk Modulus (B)} = \frac{\text{Bulk stress}}{\text{Bulk strain}} = -\frac{\Delta p}{\Delta V / V_0}$$

- SI unit of Bulk Modulus is the pascal.
- The reciprocal of bulk modulus is **compressibility (k)**.

$$\text{k} = \frac{1}{\text{B}}$$

Shear Modulus: Elasticity of Shape

- It measures the resistance of motion of the planes within the solid parallel to each other.

$$\text{Shear Modulus (S)} = \frac{\text{Shear stress}}{\text{Shear strain}} = \frac{\text{F}_\parallel / \text{A}}{x / \text{h}}$$

- SI unit of Shear Modulus is N m^{-2}.

Stress – Versus – Strain Curve for Elastic Solid

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Stress Vs Strain Curve for Elastic Solid

- Figure E-5 shows the stress-versus-strain curve for elastic solid.

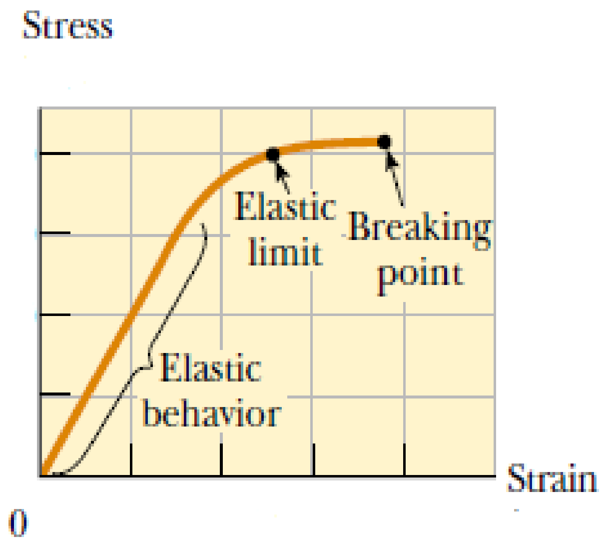


Figure E-5:

- Initially, a stress–strain curve is a straight line. As the stress increases, however, the curve is no longer straight. When the stress exceeds the elastic limit, the object is permanently distorted and does not return to its original shape after the stress is removed. Hence, the shape of the object is permanently changed. As the stress is increased even further, the material ultimately breaks.
- **The proportional limit** is the maximum stress for which stress and strain are proportional. Beyond the proportional limit, Hooke's law is not valid.
- **The elastic limit** is the stress beyond which irreversible deformation occurs.
- **The breaking stress, or ultimate strength**, is the stress at which the material breaks.

Ductile Material:

- A ductile material is one that can be stressed well beyond its elastic limit without breaking

  [a large amount of plastic deformation takes place between the elastic limit and the fracture point]
- A soft iron wire – Ductile material

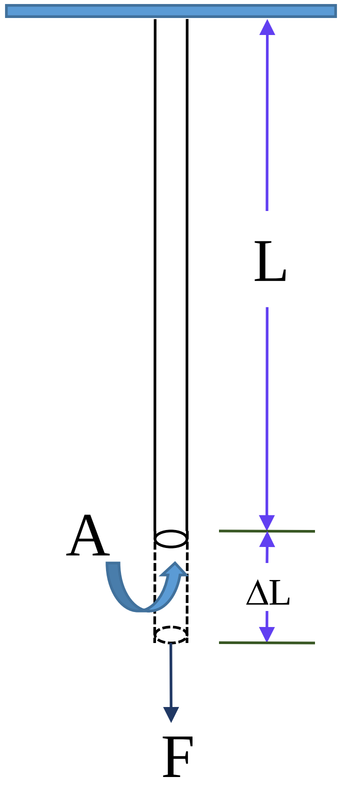
Brittle Material:

- A brittle material is one that breaks soon after the elastic limit is reached.
- A steel piano string – Brittle material

Energy Stored in a Stretched Wire

Energy Stored in a Stretched Wire

- Suppose a wire of original length L, Young's modulus Y, and cross-sectional area A, suspended vertically with upper end is attached to a rigid support and is stretched elastically by an amount when a normal force F is applied at lower end [Figure W-1].



$$\text{Tensile stress} = \frac{F}{A}$$

$$\text{Tensile strain} = \frac{\Delta L}{L}$$

$$\text{Young's Modulus (Y)} = \frac{\text{Tensile stress}}{\text{Tensile strain}} = \frac{F/A}{\Delta L/L} = \frac{FL}{A\Delta L}$$

$$\therefore \boxed{F = \frac{YA\Delta L}{L}} \quad \text{..............(1)}$$

The work done in stretching the wire by an amount is

$$W = \text{average force} \times \text{extension}$$

$$= \frac{1}{2}(0+F)\times \Delta L$$

$$= \frac{1}{2}F\Delta L$$

$$= \frac{1}{2}\left[\frac{YA\Delta L}{L}\right]\Delta L \qquad \text{[using Eq. (1)]}$$

$$\therefore \boxed{W = \frac{1}{2}YA\frac{(\Delta L)^2}{L}}$$

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Energy Stored in a Stretched Wire

Energy Stored in a Stretched Wire

Energy stored in the wire (U) = Work done in stretching the wire [W]

$$\therefore \text{Energy stored in the wire } (U) = \frac{1}{2} YA \frac{(\Delta L)^2}{L}$$

$$\text{Energy Density} = \frac{U}{V} = \frac{\frac{1}{2} YA \frac{(\Delta L)^2}{L}}{AL} = \frac{1}{2}\frac{F}{A}\frac{\Delta L}{L} = \frac{1}{2}\text{stress} \times \text{strain}$$

Examples

A wire of length L and cross-sectional area A is made of a material of Young's modulus of elasticity Y. If the wire is stretch by an amount X, then work done on the wire is ……

$$W = \frac{YAX^2}{2L}$$

If S is the stress and Y is the Young's modulus of a material of a wire, then the energy stored in the wire per unit volume is ……

$$\frac{U}{V} = \frac{S^2}{2Y}$$

Poisson’s Ratio, Relation between Different Modulii

Poisson’s Ratio

$$\text{Poisson's Ration} = \frac{\text{Lateral strain}\,(\beta)}{\text{Longitudinal strain}\,(\alpha)} = \frac{\dfrac{\text{lateral contraction}}{\text{original diameter}}}{\dfrac{\text{longitudinal extension}}{\text{original length}}} = \frac{-\Delta d/d}{\Delta L/L}$$

- It is a pure number.
- It is constant for given material.

Relation between Different Moduli

Relation between Young’s modulus (Y), Bulk modulus (K) and Modulus of rigidity(η):

$$\frac{9}{Y} = \frac{3}{\eta} + \frac{1}{K}$$

Notes:

- Elasticity decreases as the temperature increases.
- **Steel is more elastic than rubber:**
  For a given stress, the strain produced in steel is much smaller than that produced in the rubber. This implies that Young’s modulus $\left(=\frac{\text{stress}}{\text{strain}}\right)$ for steel is greater than that for rubber.

Sample Problems

- **A 200 kg load is hung on a wire of length 4.00 m, cross-sectional area $0.200\times10^{-4}\ \text{m}^2$, and Young's Modulus $8.00\times10^{10}\ \text{N/m}^2$. What is its increase in length?**

Hint:

$$\text{Young's Modulus } (Y) = \frac{F/A}{\Delta L/L} = \frac{F\ L}{A\ \Delta L}$$

$$\Rightarrow \Delta L = \frac{F\ L}{A\ \text{Y}} = 4.90\ \text{mm}$$

- **Assume Young's modulus for bone is $1.50\times10^{10}\ \text{N/m}^2$. The bone breaks if stress greater than $1.50\times10^{8}\ \text{N/m}^2$ is imposed on it. (a) What is the maximum force that can be exerted on the femur bone in the leg if it has a minimum effective diameter of 2.50 cm? (b) If this much force is applied compressively, by how much does the 25.0-cm-long bone shorten?**

Hint:

(a) $\text{Stress} = F/A$

$$\Rightarrow F = \text{Stress}\left[\pi\left(\frac{d}{2}\right)^2\right] = \left(1.50\times10^{10}\ \text{N/m}^2\right)\left[\pi\left(\frac{2.50\times10^{-2}\ \text{m}^2}{2}\right)^2\right] = 73.6\ \text{kN}$$

(b) $\text{Stress} = \text{Y(strain)} = \text{Y}\left(\frac{\Delta L}{L_i}\right)$

$$\Rightarrow \Delta L = \text{Stress}\left(\frac{L_i}{Y}\right) = 2.50\ \text{mm}$$

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Thank you