General Physics I (PHYS 101)

Lecture 05 Dynamics of system of particles

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Outline

1. Collision
   - Impact force and Impuls
   - Conservation of linear momentum during collision
   - Inelastic and Elastic collision
   - Examples of perfectly inelastic collision
   - One dimension elastic collision (head-on collision)

When two bodies are brought in contact for a short interval of time and they transfer the momentum to one another, then the situation is called collision. The force experienced by either body due to the other is called impact force or simply impact. The impact force starts to act on a body when collision starts, it increases to a maximum value and then decreases to zero as the collision ends. So the graph of impact force versus time during collision is a curve as shown in figure 1

Collision

Impact force and Impuls (contd.)

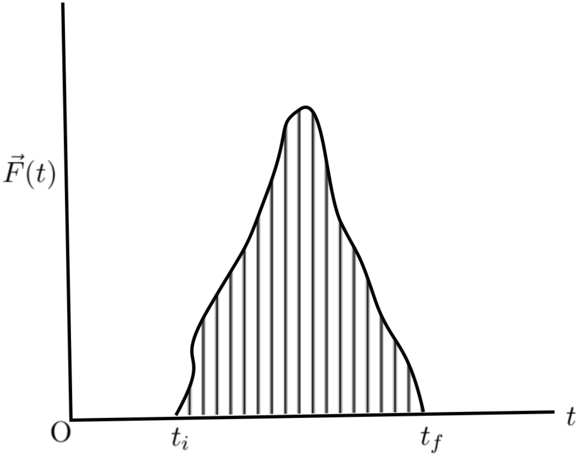

Figure 1: Impulsive force versus time

The impulse is defined as the time integral of impact force during the impact time. Let the collision starts at t_i and ends at t_f. Let the impact force $\vec{F}(t)$ is experienced by a body at any time t. The impulse is now defined as

$$\vec{J} = \int_{t_i}^{t_f} \vec{F}(t)dt \tag{1}$$

Graphically, it is also equal to the area under the impact force versus time graph. But from Newton's second law of motion, $\vec{F} = \dfrac{d\vec{P}}{dt}$.

Therefore

$$\vec{J} = \int_{t_i}^{t_f} \frac{d\vec{P}}{dt} dt = \int_{\vec{P}_i}^{\vec{P}_f} d\vec{P} = \vec{P}_f - \vec{P}_i$$

$$\text{i.e. } \vec{J} = \vec{P}_f - \vec{P}_i \tag{2}$$

where, $\vec{P}_i$ and $\vec{P}_f$ are the linear momenta of the body at time t_i and t_f respectively.

Hence the impulse on a body is also equal to the change in linear momentum of the body.

Collision

Conservation of linear momentum during collision

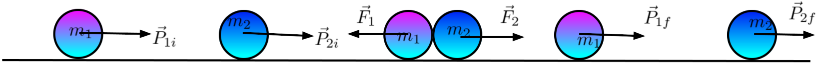

Figure 2: Conservation linear momentum

Consider two bodies of masses m_1 and m_2 moving with momentum $\vec{P}_{1i}$ and $\vec{P}_{2i}$, respectively collide for a short interval of time. The collision begins at t_i and ends at t_f. After collision the bodies move with momentum $\vec{P}_{1f}$ and $\vec{P}_{2f}$. During collision the first body experiences a time varying impact force $\vec{F}_1$ and the second body experiences the force $\vec{F}_2$. At any time during collision, these two

Conservation of linear momentum during collision (contd.)

forces are equal and opposite according to Newton's third law, i.e.

$\vec{F}_1 = -\vec{F}_2$.

From the definition of impulse

$$\vec{J}_1 = \int_{t_i}^{t_f} \vec{F}_1 dt = \vec{P}_{1f} - \vec{P}_{1i} \qquad \text{(for first body)} \tag{3}$$

$$\vec{J}_2 = \int_{t_i}^{t_f} \vec{F}_2 dt = \vec{P}_{2f} - \vec{P}_{2i} \qquad \text{(for second body)} \tag{4}$$

But,

$$\int_{t_i}^{t_f} \vec{F}_1 dt = -\int_{t_i}^{t_f} \vec{F}_2 dt$$

$$\implies \vec{P}_{1f} - \vec{P}_{1i} = -\left(\vec{P}_{2f} - \vec{P}_{2i}\right)$$

Collision

Conservation of linear momentum during collision (contd.)

$$\implies \vec{P}_{1f} + \vec{P}_{2f} = \vec{P}_{1i} + \vec{P}_{2i}$$

$$\implies \vec{P}_1 + \vec{P}_2 = \text{constant}$$

$$\implies m_1\vec{v}_1 + m_2\vec{v}_2 = \text{constant}$$

where, $\vec{v}_1$ and $\vec{v}_2$ are the velocities of the colliding bodies. So total momentum of two colliding bodies remains conserved during collision.

The collision can be divided into two types depending under the situation whether the total kinetic energy of the colliding bodies remain conserved before and after collision or not. If the total kinetic energy of the colliding bodies remains conserved before and after collision, then the collision is said to be elastic collision. Otherwise, the collision is said to be inelastic collision. If the total kinetic energy after collision is always less than that of initial, then the collision is said to be perfectly inelastic collision.

Example (I) Sticking collision:- Consider a body of mass m moving with speed u collides with another body of mass M at rest as shown in figure 3.

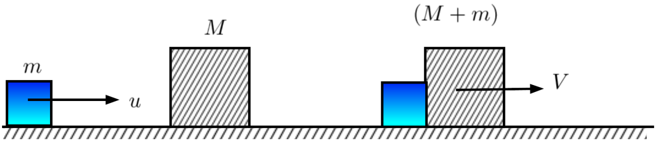

Figure 3: An example of sticking collision

After collision both the bodies move together (with sticking) with a common velocity along the same initial direction of first body. Now from the conservation of linear momentum, we have

$$mu = (M+m)V \implies V = \frac{m}{M+m}u \tag{5}$$

The initial total kinetic energy of the system $(K_i) = \dfrac{1}{2}mu^2$

Collision

Examples of perfectly inelastic collision (contd.)

The final total kinetic energy of the system $(K_f) = \frac{1}{2}(M+m)V^2 = \frac{1}{2}(M+m)\left(\frac{m}{M+m}u\right)^2 = \frac{m}{M+m}\frac{1}{2}mu^2 = \frac{m}{M+m}K_i$

i.e. $K_f = \frac{m}{M+m}K_i$. Since, $(M+m)$ is always greater than m, K_f must always be less than K_i. So that the collision of this type is inelastic collision. The decrease in kinetic energy before and after collision is $\Delta K = K_i - K_f = K_i - \frac{m}{M+m}K_i = \frac{M}{M+m}K_i$. The percentage decrease in kinetic energy

$$= \frac{\Delta K}{K_i} \times 100\% = \frac{M}{M+m} \times 100\%$$

Example (II) Ballistic pendulum:- A heavy wooden block vertically suspended by a light string as shown in figure 4 is called ballistic pendulum. It can be used to measure the muzzle velocity of the bullet as fired by the gun. Consider the mass of the block and bullet are M and m, respectively. Let u be the muzzle velocity of the bullet as fired from the gun to be determined.

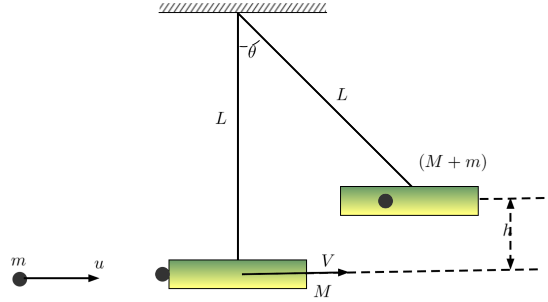

Figure 4: A ballistic pendulum

As the bullet fired very closed the block, it embeds into the block and the system (bullet plus block) acquires the common velocity V (say) which causes the system to attain the maximum height h as shown in figure 4. From the conservation of linear momentum, we have

$$mu = (M+m)V \implies u = \frac{M+m}{m}V \tag{6}$$

Only the force of gravity contribute to the work done on the system, the total mechanical energy of the system remain conserved. Hence,

$$\frac{1}{2}(M+m)V^2 = (M+m)gh \implies V = \sqrt{2gh} \tag{7}$$

Therefore, equation (6) becomes

$$u = \frac{M+m}{m}\sqrt{2gh} \tag{8}$$

Measuring the mass of bullet and block as well as the height that is attained by the system after collision, one can calculate the velocity of the bullet using equation (8)

The total kinetic energy just before collision

$$(K_i) = \frac{1}{2}mu^2 = \frac{1}{2}\frac{(M+m)^2}{m}(2gh)$$

The total kinetic energy just after collision

$$(K_f) = \frac{1}{2}(M+m)V^2 = \frac{1}{2}(M+m)(2gh)$$

Decrease in kinetic energy

$$= K_i - K_f = \frac{1}{2}\frac{M(M+m)}{m}(2gh)$$

The decrease percentage

$$= \frac{\text{Decreased kinetic energy}}{K_i} \times 100\% = \frac{M}{M+m} \times 100\%.$$

This shows the collision is perfectly inelastic.

Collision

One dimension elastic collision (head-on collision)

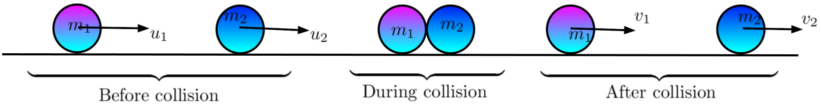

Figure 5: One dimentional elastic collision

Consider a body of mass m_1 moving with velocity u_1 elastically collides with another body of mass m_2 moving with velocity u_2 along same initial direction of the first body. If $u_1 > u_2$, the collision occurs and the bodies move with velocities v_1 and v_2 respectively after collision. If both of the bodies move along same of their initial direction, then the collision is said to be one dimension collision or head-on collision.

From the conservation of linear momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \tag{9}$$

$$\text{or, } m_1\left(u_1 - v_1\right) = m_2\left(v_2 - u_2\right) \tag{10}$$

Since the collision is elastic, the total kinetic energy remains conserved. Therefore,

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

Collision

One dimension elastic collision (head-on collision) (contd.)

$$\implies m_1u_1^2+m_2u_2^2=m_1v_1^2+m_2v_2^2 \tag{11}$$

$$\implies m_1\left(u_1^2-v_1^2\right)=m_2\left(v_2^2-u_2^2\right)$$

$$\implies m_1\left(u_1+v_1\right)\left(u_1-v_1\right)=m_2\left(v_2+u_2\right)\left(v_2-u_2\right) \tag{12}$$

Dividing equation (12) by equation (10), we get

$$u_1+v_1=v_2+u_2 \tag{13}$$

$$\text{Or, } v_1=v_2+u_2-u_1 \tag{14}$$

$$\text{Or, } u_1-u_2=-\left(v_1-v_2\right) \tag{15}$$

Here, $(u_1 - u_2)$ is the relative velocity of first body with respect to the second body before collision and $(v_1 - v_2)$ is that of after collision. According equation (15) these are equal and opposite. Hence, in one dimension elastic collision, the rate of approach before collision is equal to the rate of separation after collision.

Substituting the value of v_1 from equation (14) to equation (9), we can have

$$m_1u_1 + m_2u_2 = m_1\left(v_2 + u_2 - u_1\right) + m_2v_2$$

$$m_1u_1 + m_2u_2 = m_1v_2 + m_1u_2 - m_1u_1 + m_2v_2$$

Collision

One dimension elastic collision (head-on collision) (contd.)

$$(m_1 + m_2)\, v_2 = 2m_1 u_1 + (m_2 - m_1)\, u_2$$

$$v_2 = \frac{2m_1}{m_1 + m_2} u_1 + \frac{m_2 - m_1}{m_1 + m_2} u_2 \tag{16}$$

Substituting the value of v_2 in equation (14), we get

$$v_1 = \frac{2m_1}{m_1 + m_2} u_1 + \frac{m_2 - m_1}{m_1 + m_2} u_2 + u_2 - u_1$$

$$v_1 = \left(\frac{2m_1}{m_1 + m_2} - 1 \right) u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2} + 1 \right) u_2$$

$$\implies v_1 = \frac{m_1 - m_2}{m_1 + m_2} u_1 + \frac{2m_2}{m_1 + m_2} u_2 \tag{17}$$

Using equation (16) and (17) we can calculate the velocities of the colliding bodies after collision in term of initial information.

Case I:- For equal masses i.e. $m_1 = m_2$ If two colliding bodies are of equal masses, then equations (16) and (17) reduces to

$$v_1 = u_2$$

$$v_2 = u_1$$

That means, when two bodies of equal masses elastically collides leading one dimensional collision, then after collision they exchange their velocities.

Case II:- For second body at rest i.e. $u_2 = 0$ In this situation the equations (16) and (17) becomes

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2} u_1 \tag{18}$$

$$v_2 = \frac{2m_1}{m_1 + m_2} u_1 \tag{19}$$

Sub-case (a):- For equal masses i.e. $m_1 = m_2$ In this situation equation (18) and (19) becomes

$$v_1 = 0$$

$$v_2 = u_1$$

That means, when a body elastically collides another body of equal mass at rest leading one dimensional collision, then after collision the first body comes at rest and the second body moves with the initial velocity of the first body.

Sub-case (b):- For the second body is massive i.e. $m_2 \gg m_1$ In this situation equations (18) and (19) reduces to

$$v_1 \approx -u_1$$

$$v_2 \approx 0$$

One dimension elastic collision (head-on collision) (contd.)

That means, when a light body elastically collides with massive (heavy) body at rest providing the one dimensional collision, then after collision, the light body rebounds with approximately the same of its initial velocity and the heavy body approximately remains at rest.

Sub-case (c):- For the first body is massive i.e. $m_1 \gg m_2$ In this situation, the equations (18) and (19) reduces to

$$v_1 \approx u_1$$

$$v_2 \approx 2u_1$$

That means, when a massive body elastically collides with a light body at rest providing the one dimensional collision, then after collision, the massive body continues its motion with approximately the same of its initial velocity along same direction and the light body scatters with approximately the double of the initial velocity of the massive body along same direction.