KATHMANDU UNIVERSITY

DHULIKHEL , KAVRE

Subject: MATH 104
Assignment: 1

Submitted by:

Ashraya Kadel
CE 1/2
Roll no: 25

Submitted to:

Ganga Ram Phaijoo
Department of Mathematics

Submission date: 07 / 09 / 2023

Q.17: Define polar, cylindrical and spherical coordinates of a point in space. Establish the relation among Cartesian, cylindrical and spherical coordinates in three-dimensional coordinates.

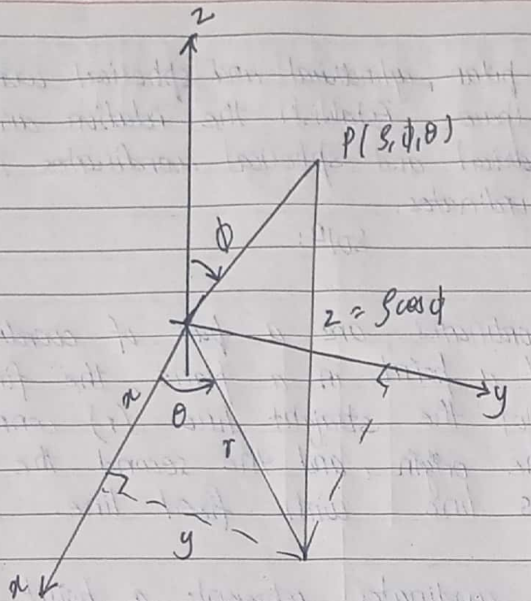
Soln:

Polar coordinates are a pair of coordinates locating the position of a point in a plane, the first being the length of the straight line (r) connecting the point to the origin, and the second the angle θ made by this line with fixed line.

Cylindrical coordinates represents a point in space by ordered triplets (r, θ, z) in which.

i) r and θ are polar coordinates for the vertical projection of P on xy-plane.
ii) z is the rectangular vertical coordinates.

Spherical coordinates represents a point P in space by ordered triplets (ρ, ϕ, θ) in which.
i) ρ is the distance from P to the origin.
ii) ϕ is the angle made by the vector $\vec{OP}$ in z-axis.
iii) θ is angle from cylinder coordinates.



(i): Cartesian and cylindrical coordinates:

$x = r\cos\theta$ $\quad$ $r = \sqrt{x^2+y^2}$

$y = r\sin\theta$ $\quad$ $\tan\theta = (y/x)$

$z = z$ $\quad$ $z = z$

(ii) Cartesian and spherical coordinates:

$x = \rho\sin\phi\cos\theta$ $\quad$ $\rho^2 = x^2+y^2+z^2$

$y = \rho\sin\phi\sin\theta$ $\quad$ $\cos\phi = z/\sqrt{x^2+y^2+z^2}$

$z = \rho\cos\phi$ $\quad$ $\tan\theta = (y/x)$

(iii) Cylindrical and spherical coordinates.

$r = \rho\sin\phi$ $\quad$ $\rho = \sqrt{r^2+z^2}$

$\theta = \theta$ $\quad$ $\cos\phi = (z/r)$

$z = \rho\cos\phi$ $\quad$ $\tan\theta = (y/x)$

(2): Replace the following Cartesian equations by equivalent polar equations.

(a): $y = -5.$

Soln,

Given,

$y = -5$

We know, $y = r\sin\theta$

so,

$r\sin\theta = -5.$

(b) $\frac{x^2}{4} + \frac{y^2}{4} = 1$

Soln:

Given,

$\frac{x^2}{4} + \frac{y^2}{4} = 1$

or, $x^2+y^2 = 4$

or $r^2\cos^2\theta + r^2\sin^2\theta = 4$

or $r^2(1) = 4$

$\therefore r = \pm 2$

(c): $(x-1)^2 + (y-2)^2 = 9$

Soln.

Given,

$(x-1)^2 + (y-2)^2 = 9$

or, $x^2 - 2x + 1 + y^2 - 4y - 4 = 9$

or, $x^2 + y^2 - 2x - 4y = 12$

Date
Page

or, $r^2\cos^2\theta + r^2\sin^2\theta - 2r\cos\theta - 4r\sin y = 12$

or, $r^2 - 2r\cos\theta - 4r\sin y = 12$

(d): $xy = 4$

Soln:

Given,

$xy = 4$

or, $r\cos\theta \, r\sin\theta = 4$

or, $r^2 \sin\theta\cos\theta = 4$

or, $r^2 \sin 2\theta = 8$

(3) (a): Find the rectangular coordinates for:

(i): $(2, \pi/2, \pi/3)$

Soln:

Given,

$\rho = 2$, $\phi = \pi/2$, $\theta = \pi/3$

Now,

$x = \rho \sin\phi \cos\theta$

$= 2 \times \sin \pi/2 \cdot \cos \pi/3 = 1$

$y = \rho \sin\phi \sin\theta$

$= 2 \times \sin \pi/2 \cdot \sin \pi/3 = \sqrt{3}$

$\therefore (2, \pi/2, \pi/3) \approx (1, \sqrt{3}, 0)$

$z = \rho \cos\phi$

$= \cancel{\rho \times \cos} \; 2 \times \cos \pi/2 = 0$

Date
Page

(ii): $(2, 2\pi/3, 1)$

Soln:

$r = 2$

$\theta = 2\pi/3$

$z = 1$

$x = r\cos\theta$

$= 2 \times \cos(2\pi/3) = -1$

$\therefore (2, 2\pi/3, 1) \approx (-1, \sqrt{3}, 1)$

$y = r\sin\theta$

$= 2 \times \sin(2\pi/3) = \sqrt{3}$

3 (b): i) Find spherical coordinates for $(0, 2\sqrt{3}, -2)$

Soln:

Given,

$x = 0$ $\quad y = 2\sqrt{3}$ $\quad z = -2$

$\rho = \sqrt{x^2 + y^2 + z^2}$

$= \sqrt{0^2 + (2\sqrt{3})^2 + (-2)^2} = 4$

$\phi = \dfrac{z}{\sqrt{x^2+y^2+z^2}} = \dfrac{-2}{\sqrt{0^2 + (2\sqrt{3})^2 + (2)^2}} = 2\pi/3$

$\theta = \tan^{-1}\left(\dfrac{y}{x}\right) = \tan^{-1}\left(\dfrac{2\sqrt{3}}{0}\right) = \dfrac{\pi}{2}$

$\therefore (0, 2\sqrt{3}, -2) \approx (4, 2\pi/3, \pi/2)$

(ii): Find cylindrical coordinates for $(3,-3,-7)$

Soln:

Given,

$x=3 \quad y=-3 \quad z=-7$

$r=\sqrt{x^2+y^2}$
$=\sqrt{(3)^2+(-3)^2} = 3\sqrt{2}.$

$\tan\theta = \frac{y}{x} = \frac{-3}{3} = -\pi/4$

$z=-7$

$\therefore (3,-3,-7) \approx (3\sqrt{2}, -\pi/4, -7)$

(Q.3):(c): Find the spherical coordinates for system eqn.

(i): $x^2+y^2+z^2=1.$

Soln.

Given,

$x^2+y^2+z^2=1$

or, $\rho^2=1$

$\therefore \rho = \pm 1$

(ii): $z=\sqrt{x^2+y^2}$

Soln:

~~Squaring both sides,~~

$\cancel{z^2 = x^2+y^2}$

$\cancel{\rho^2\cos^2\phi = \rho^2\sin^2\phi}$

Given,

$z=\sqrt{x^2+y^2}$

or, $\rho\cos\phi = r$

or, $\rho\cos\phi = \rho\sin\phi$

or, $\sin(\pi/2-\phi) = \sin\phi$

$\therefore \phi = \frac{\pi}{4}$

3(d): Find the equation in cylindrical polar coordinates for $x^2+(y-3)^2=9$

Soln:

Given,

$x^2+(y-3)^2=9$

or, $x^2+y^2-6y+9=9$

or, $x^2+y^2-6y=0$

or, $r^2\cos^2\theta + r^2\sin^2\theta - 6r\sin\theta = 0$

or, $r^2 - 6r\sin\theta = 0$

or, $r(r-6\sin\theta)=0$

3(e): Find the cartesian and cylindrical coordinate equations for the equation $\phi = 5\pi/6$, $0 \le \rho \le 2$ with proper ranges for z in Cartesian and r in cylindrical coordinates.

Soln:

Given,

$\phi = \frac{5\pi}{6} \qquad 0 \le \rho \le 2.$

Here,
if $\rho = 0$, $z = \rho\cos\phi = 0$

if $\rho = 2$, $z = 2\times\left(\frac{-\sqrt{3}}{2}\right) = -\sqrt{3}$

Again,

$\rho = 0$; $r = \rho\sin\phi \Rightarrow 0$
$\rho = 2$; $r = \rho\sin\phi \Rightarrow 1$

Also,

$\tan\phi = \tan 5\pi/6$

or, $\sin\phi = -1/\sqrt{3} \times \cos\phi$

or, $\sqrt{3}\,\rho\sin\phi = -\rho\cos\phi$

or, $\sqrt{3}\,r = -z$.

Hence,
cartesian form: $z = -\sqrt{3(x^2+y^2)}$ with $-\sqrt{3} \le z \le 0$

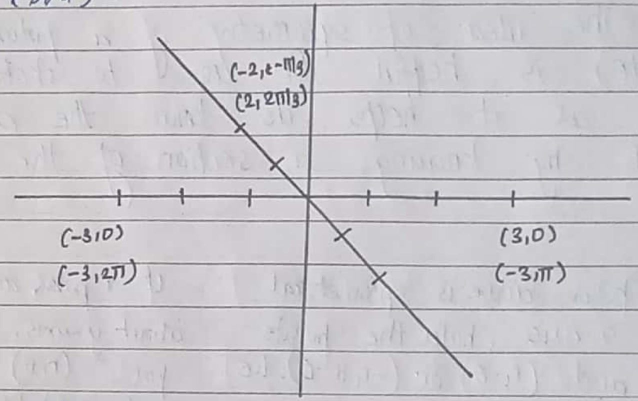
cylindrical form: $r = -\frac{1}{\sqrt{3}} z$ $\quad 0 \le r \le 1$, $-\sqrt{3} \le z \le 0$.

<Q.4>: Find the pair of polar coordinates that label the same point.

(a): $(3, 0)$ (b): $(-3, 0)$ (c): $(2, 2\pi/3)$
(d) $(-3, \pi)$ (e) $(-3, 2\pi)$ (f): $(-2, -\pi/3)$.

Sol^n:

For $(3, 0)$.



The pairs are:
(i) $(3,0)$ & $(-3,\pi)$
(ii) $(-3,0)$ & $(-3,2\pi)$
(iii) $(2,2\pi/3)$ & $(-2,-\pi/3)$

Date
Page

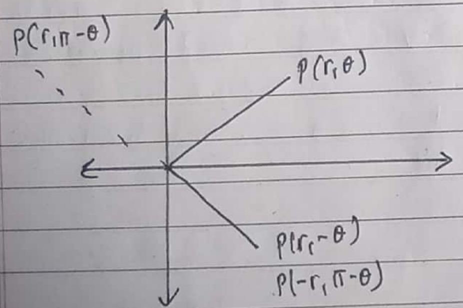
<Q.5>: How is the idea of symmetry of a polar curve $r = f(\theta)$ helpful to sketch the graph of the curve? Write the conditions when the curve $r = f(\theta)$ is symmetric about x-, y-axis. Check the symmetry, sketch the graph of curve $r = \sin 2\theta$ and find area covered by the curve.

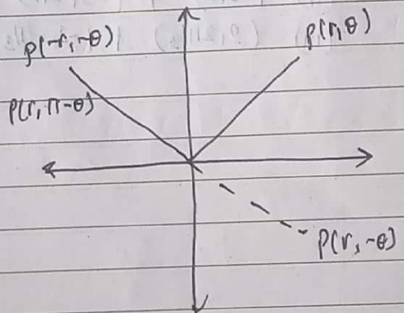
Soln:

The idea of symmetry of a polar curve $r = f(\theta)$ is helpful for us to sketch a graph as it helps us know the complete graph by knowing a section of the graph.

If a polar curve is symmetrical about x-axis, both the points (r, θ) and $(r, -\theta)$ or $(-r, \pi-\theta)$ lie on the curve.

If a polar curve is symmetrical about y-axis, both the points (r, θ) and $(-r, -\theta)$ or $(r, \pi-\theta)$ lie on the curve.





Date
Page

Given,
$$r = f(\theta) = \sin 2\theta$$

(x) Check Symmetry:

a) About x-axis.

~~About x-axis,~~ At $(r, -\theta)$

$r = \sin 2(-\theta)$

or, $r = -\sin 2\theta$ (F)

At $(-r, \pi-\theta)$

$-r = \sin(2\pi - 2\theta)$

or, $-r = -\sin 2\theta$ or, $r = \sin 2\theta$ (T)

Since $(-r, \pi-\theta)$ lies on the curve, it is symmetrical on x-axis.

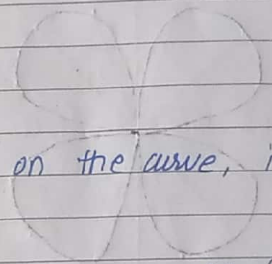
b) About y-axis.

At $(-r, -\theta)$

$-r = \sin 2(-\theta)$

or, $r = \sin 2\theta$ (T)

Since $(-r, -\theta)$ lies on the curve, it is symmetrical on y-axis.



c) About origin,

At $(-r, \theta)$

$-r = \sin 2\theta$ (F)

At $(r, \pi+\theta)$

$r = \sin(2\pi + 2\theta)$

or $r = \sin 2\theta$ (T)

Since $(r, \pi+\theta)$ lies on the curve, it is symmetrical about origin.

(x) Tangent at Pole:

At pole, $r = 0$

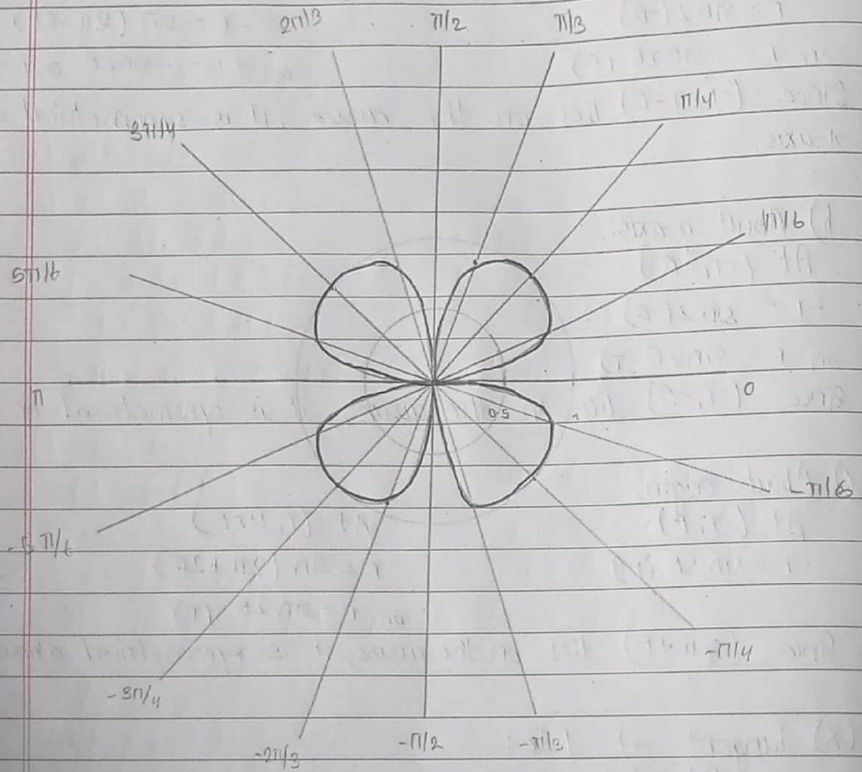
$0 = \sin 2\theta$

or, $2\theta = 0, \pi$

$\therefore \theta = 0, \pm\pi/2$

(x): r-θ table:

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	$5\pi/6$	π
$r=\sin 2\theta$	0	0.866	1	0.866	0	-0.866	-1	-0.866	0



Here,

$$\text{Area of the curve} = \frac{1}{2}\int_0^{2\pi} (\sin 2\theta)^2\, d\theta$$

Since the curve is symmetrical about x-axis, y-axis and origin,

$$= 4\times\frac{1}{2}\int_0^{\pi/2} (\sin 2\theta)^2\, d\theta$$

$$= 2\int_0^{\pi/2} \sin^2 2\theta\, d\theta$$

$$= \int_0^{\pi/2} (1-\cos 4\theta)\, d\theta$$

$$= \int_0^{\pi/2} 1.d\theta - \int_0^{\pi/2} \cos 4\theta\, d\theta$$

$$= \theta\Big|_0^{\pi/2} - \frac{\sin 4\theta}{4}\Big|_0^{\pi/2}$$

$$= (\pi/2 - 0) - \frac{1}{4}(\sin 4\times\pi/2 - \sin 4\times 0)$$

$$= \pi/2 \text{ sq. units.}$$

<Q.6>: Sketch the graph of the following equations.

(a): $r = -2\sin\theta$

Soln:

Given,

$r = -2\sin\theta$

~~= -2 × -1~~

(x) Symmetry:

(a): About x-axis:

At $(r, -\theta)$

$r = -2\sin(-\theta)$

or, $r = 2\sin\theta$ (F)

At $(-r, \pi-\theta)$

$-r = -2\sin(\pi-\theta)$

or, $-r = -2\sin\theta$ or, $r = 2\sin\theta$ (F)

The curve is not symmetrical on x-axis.

(b): At origin:

At $(-r, \theta)$

$-r = -2\sin\theta$

or, $r = 2\sin\theta$ (F)

At $(r, \pi+\theta)$

$r = -2\sin(\pi+\theta)$

or, $r = 2\sin\theta$ (F)

The curve is not symmetrical on origin

(c): At y-axis:

At $(-r, -\theta)$

$-r = -2\sin(-\theta)$

or, $-r = 2\sin\theta$

or, $r = -2\sin\theta$ (T)

Since $(-r, -\theta)$ passes through the ~~point~~ curve, it is symmetrical about y-axis.

(x) Tangent:

At pole, $r = 0$.

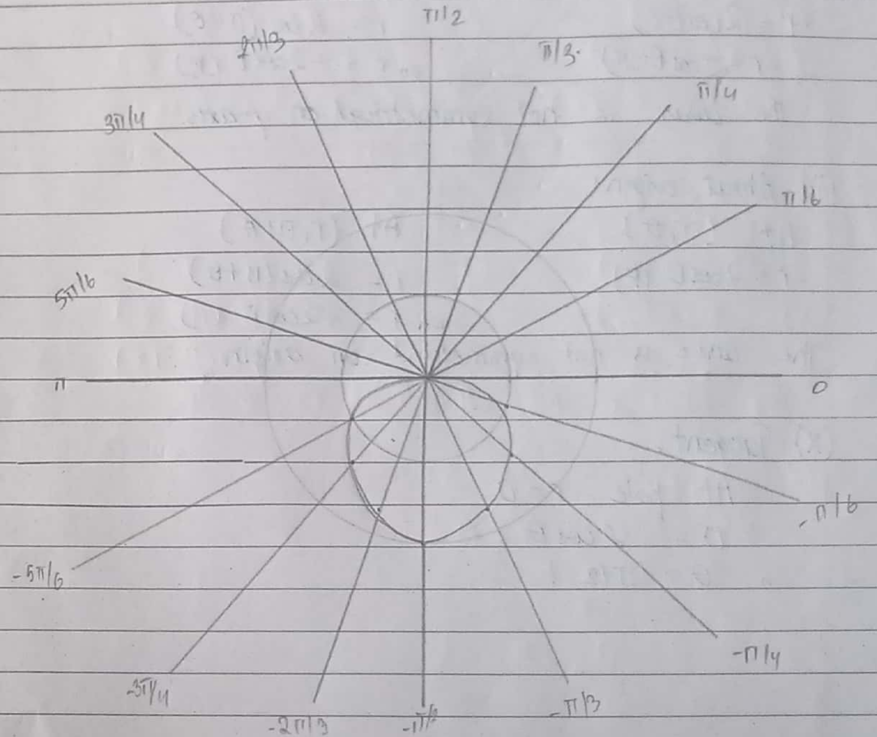
$0 = -2\sin\theta$

or, $\sin\theta = 0$

$\therefore \theta = 0, \pi$.

(x): $r-\theta$ table:

θ	$-\pi/2$	$-\pi/3$	$-\pi/4$	$-\pi/6$	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$r = -2\sin\theta$	2	1.73	1.414	1	0	-1	~~-1.73~~ -1.414	-1.73	-2



(b): $r = 2\cos\theta$

Soln:

Given, $r = 2\cos\theta$

(x) Symmetry:

(i): About x-axis:

At $(r, -\theta)$

$r = 2\cos(-\theta)$ or $r = 2\cos\theta$ (T)

Since $(r, -\theta)$ lies on the curve, it is symmetrical on x-axis.

(ii) About y-axis:

At $(-r, -\theta)$

$-r = 2\cos(-\theta)$

or $-r = 2\cos\theta$ (F)

At $(r, \pi - \theta)$

$r = 2\cos(\pi - \theta)$

or $r = -2\cos\theta$ (F)

The curve is not symmetrical on y-axis.

(iii) About origin:

At $(-r, \theta)$

$-r = 2\cos\theta$ (F)

At $(r, \pi + \theta)$

$r = 2\cos(\pi + \theta)$

or $r = -2\cos\theta$ (F)

The curve is not symmetrical on origin.

(x) Tangent.

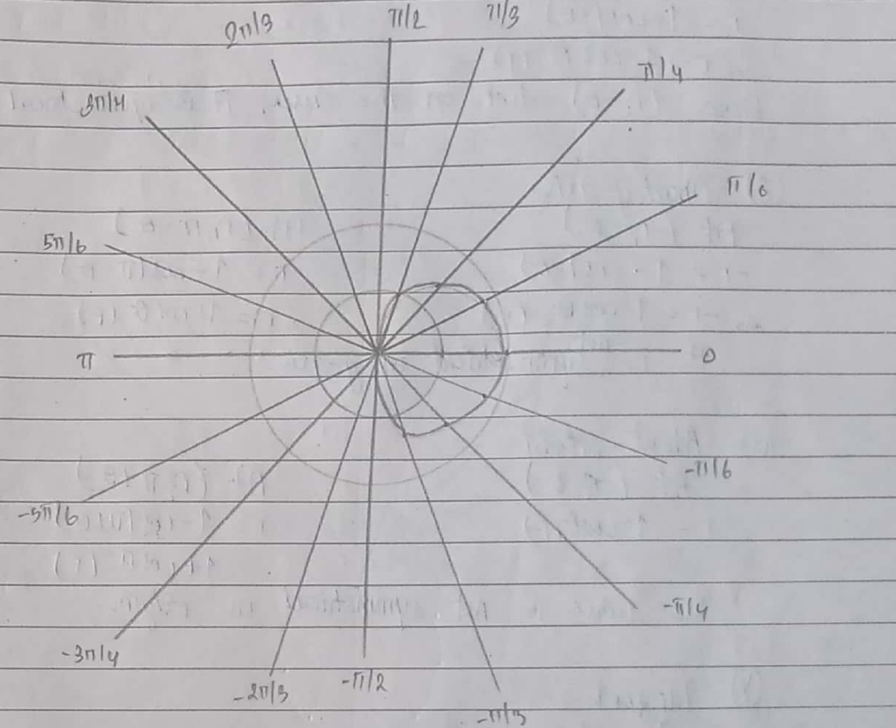
At pole, $r = 0$

$0 = 2\cos\theta$

or $\theta = \pi/2$.

(x): $r-\theta$ table:

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	$5\pi/6$	π
$r = 2\cos\theta$	2	1.732	1.414	1	0	−1	−1.414	−1.732	−2



(c): $r = 1-\cos\theta$.

Soln:

Given, $r = 1-\cos\theta$

x) Symmetry:

(i): About x-axis:

At $(r, -\theta)$

$r = 1-\cos(-\theta)$

or $r = 1-\cos\theta$ (T)

Since $(r, -\theta)$ lies on the curve, it is symmetrical on x-axis.

(ii): About y-axis:

At $(-r, -\theta)$

$-r = 1-\cos(-\theta)$

or $-r = 1-\cos\theta$ (F)

At $(r, \pi-\theta)$

$r = 1-\cos(\pi-\theta)$

or $r = 1+\cos\theta$ (F)

It is not symmetrical on y-axis.

(iii) About origin:

At $(-r, \theta)$

$-r = 1-\cos\theta$ (F)

At $(r, \pi+\theta)$

$r = 1-\cos(\pi+\theta)$

$= 1+\cos\pi$ (F)

The curve is not symmetrical on origin.

(x) Tangent:

At pole, $r=0$

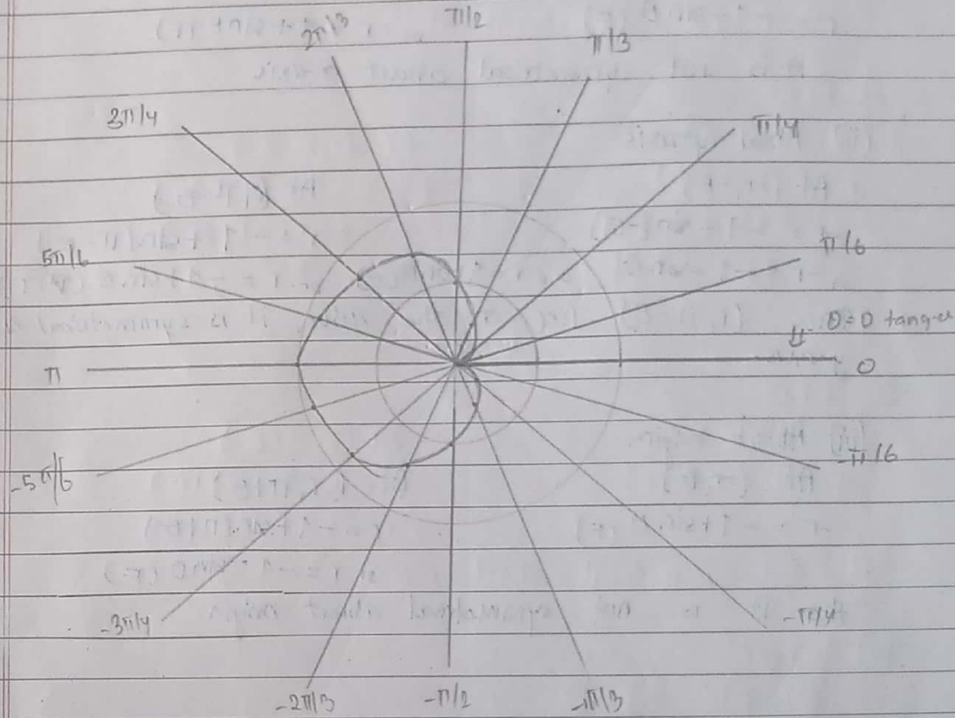
$0 = 1-\cos\theta$

or $\cos\theta = 1$

$\theta = 0, 2\pi$.

(x): $r-\theta$ table

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	$5\pi/6$	π
$r = 1-\cos\theta$	0	0.133	0.29	0.5	1	1.5	1.707	1.866	2



(d): $r = -1 + \sin\theta$

soln.

Given,

$r = -1 + \sin\theta$

(x) Symmetry:

(i): About x-axis:

At $(r, -\theta)$

$r = -1 + \sin(-\theta)$

$r = -1 - \sin\theta$ (F)

At $(-r, \pi - \theta)$

$-r = -1 + \sin(\pi - \theta)$

or $-r = -1 + \sin\theta$ (F)

It is not symmetrical about x-axis.

(ii) About y-axis:

At $(-r, -\theta)$

$-r = -1 + \sin(-\theta)$

or $-r = -1 - \sin\theta$ or, $r = 1 + \sin\theta$ (F)

At $(r, \pi - \theta)$

$r = -1 + \sin(\pi - \theta)$

or $r = -1 + \sin\theta$ (T)

Since $(r, \pi - \theta)$ lies on the curve, it is symmetrical about y-axis.

(iii) About origin.

At $(-r, \theta)$

$-r = -1 + \sin\theta$ (F)

At $(r, \pi + \theta)$

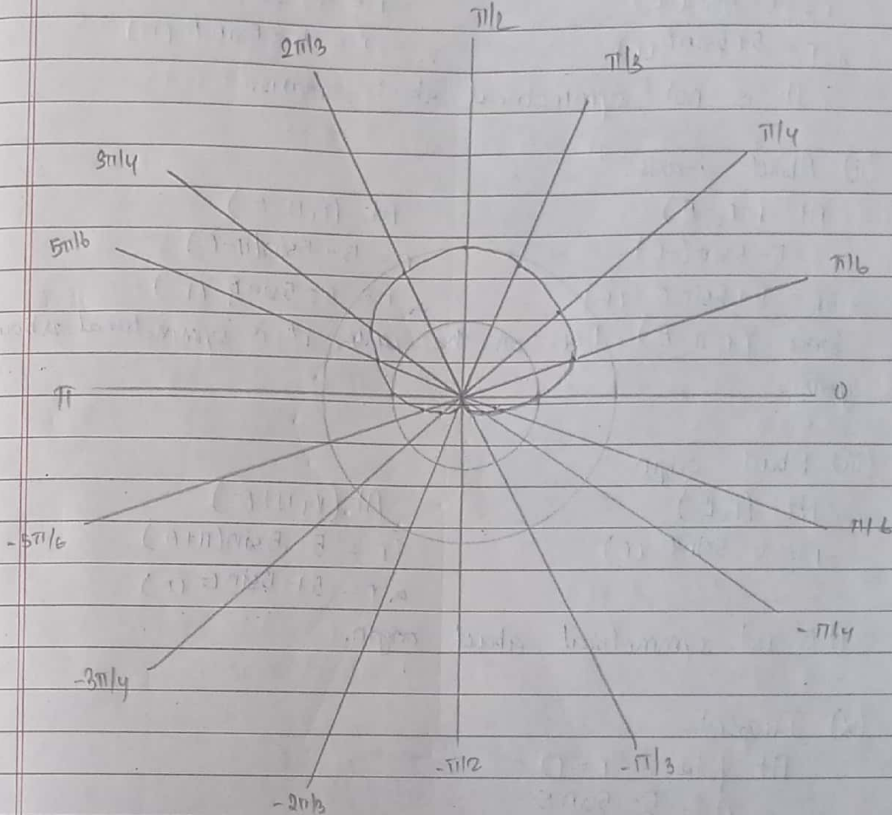
$r = -1 + \sin(\pi + \theta)$

or $r = -1 - \sin\theta$ (F)

~~The~~ It is not symmetrical about origin.

(x): r-θ table:

θ	$-\pi/2$	$-\pi/3$	$-\pi/4$	$-\pi/6$	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$r = -1 + \sin\theta$	-2	-1.86	-1.70	-1.5	-1	-0.5	-0.29	-0.13	0



Date ______
Page ______

(e): $r = 5 - 5\sin\theta$.

Soln:

Given, $r = 5 - 5\sin\theta$

(x): Symmetry:

(i): About x-axis:

At $(r, -\theta)$

$r = 5 - 5\sin(-\theta)$

or $r = 5 + 5\sin\theta$ (F)

At $(-r, \pi - \theta)$

$-r = 5 - 5\sin(\pi - \theta)$

or $-r = 5 - 5\sin\theta$ (F)

It is not symmetrical about x-axis.

(ii) About y-axis:

At $(-r, -\theta)$

$-r = 5 - 5\sin(-\theta)$

or $-r = 5 + 5\sin\theta$ (F)

At $(r, \pi - \theta)$

$r = 5 - 5\sin(\pi - \theta)$

or $r = 5 - 5\sin\theta$ (T)

Since $(r, \pi - \theta)$ lies on the curve, it is symmetrical about y-axis

(iii) About origin.

At (r, θ)

$-r = 5 - 5\sin\theta$ (F)

At $(r, \pi + \theta)$

$r = 5 - 5\sin(\pi + \theta)$

or $r = 5 + 5\sin\theta$ (F)

It is not symmetrical about origin.

(x) Tangent:

At pole, $r = 0$

$0 = 5 - 5\sin\theta$

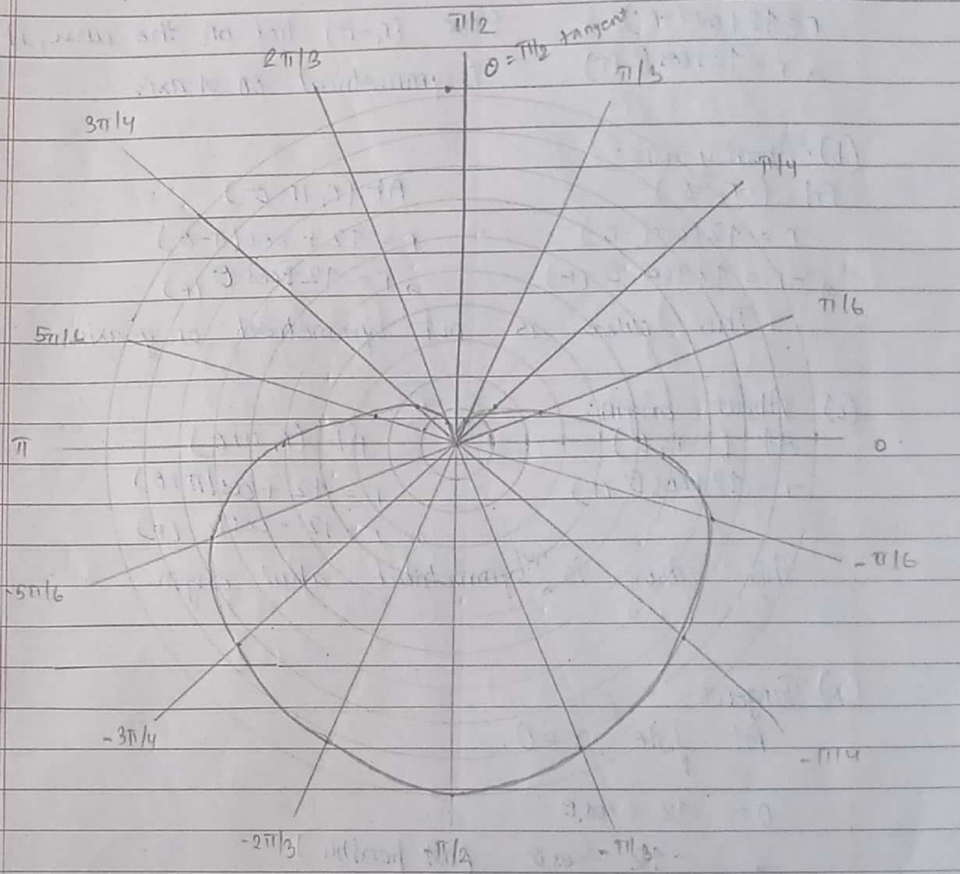
or $\sin\theta = 1$

$\therefore \theta = \pi/2$

Date ______
Page ______

(x) r - θ table:

θ	$-\pi/2$	$-\pi/3$	$-\pi/4$	$-\pi/6$	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$r = 5 - 5\sin\theta$	10	9.33	8.53	7.5	5	2.5	1.46	0.66	0



(f): $r = 12 + \cos\theta$.

Soln:

Given, $r = 12 + \cos\theta$

(x) Symmetry:

(a): About x-axis.

At $(r, -\theta)$

$r = 12 + \cos(-\theta)$

or $r = 12 + \cos\theta$ (T)

Since $(r, -\theta)$ lies on the curve, it is symmetrical on x-axis

(b): About y-axis:

At $(-r, -\theta)$

$-r = 12 + \cos(-\theta)$

or $-r = 12 + \cos\theta$ (F)

At $(r, \pi - \theta)$

$r = 12 + \cos(\pi - \theta)$

or $r = 12 - \cos\theta$ (F)

This curve is not symmetrical on y-axis.

(c) About origin:

At $(-r, \theta)$

$-r = 12 + \cos\theta$ (F)

At $(r, \pi + \theta)$

$r = 12 + \cos(\pi + \theta)$

or $r = 12 - \cos\theta$ (F)

This curve is not symmetrical about origin.

(x) Tangent:

At pole, $r = 0$.

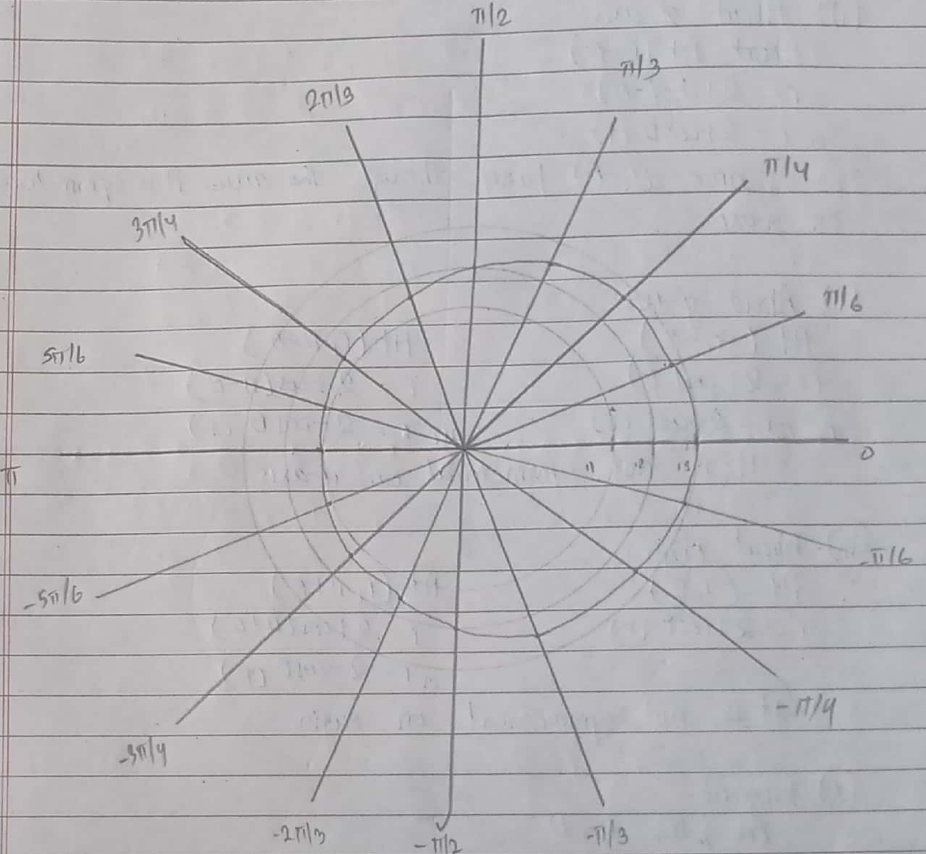
$0 = 12 + \cos\theta$

or $-12 = \cos\theta$ (not possible)

There's not tangent at pole

(x): r-θ table

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	$5\pi/6$	π
$r = 12 + \cos\theta$	13	12.86	12.7	12.5	~~11.5~~ 12	11.5	11.3	11.13	11



(g): $r = 2+\cos\theta$

Soln:

Given,

$r = 2+\cos\theta$

(x) Symmetry:

(i): About x-axis:

About At $(r, -\theta)$

$r = 2+\cos(-\theta)$

or $r = 2+\cos\theta$ (T)

Since $(r, -\theta)$ passes through the curve, it is symmetrical on x-axis.

(ii) About y-axis:

At $(-r, -\theta)$

$-r = 2+\cos(-\theta)$

or, $-r = 2+\cos\theta$ (F)

At $(r, \pi-\theta)$

$r = 2+\cos(\pi-\theta)$

or, $r = 2-\cos\theta$ (F)

It is not symmetrical on y-axis.

(iii) About origin:

At $(-r, \theta)$

$-r = 2+\cos\theta$ (F)

At $(r, \pi+\theta)$

$r = 2+\cos(\pi+\theta)$

or $r = 2-\cos\theta$ (F)

It is not symmetrical on origin.

(x) Tangent:

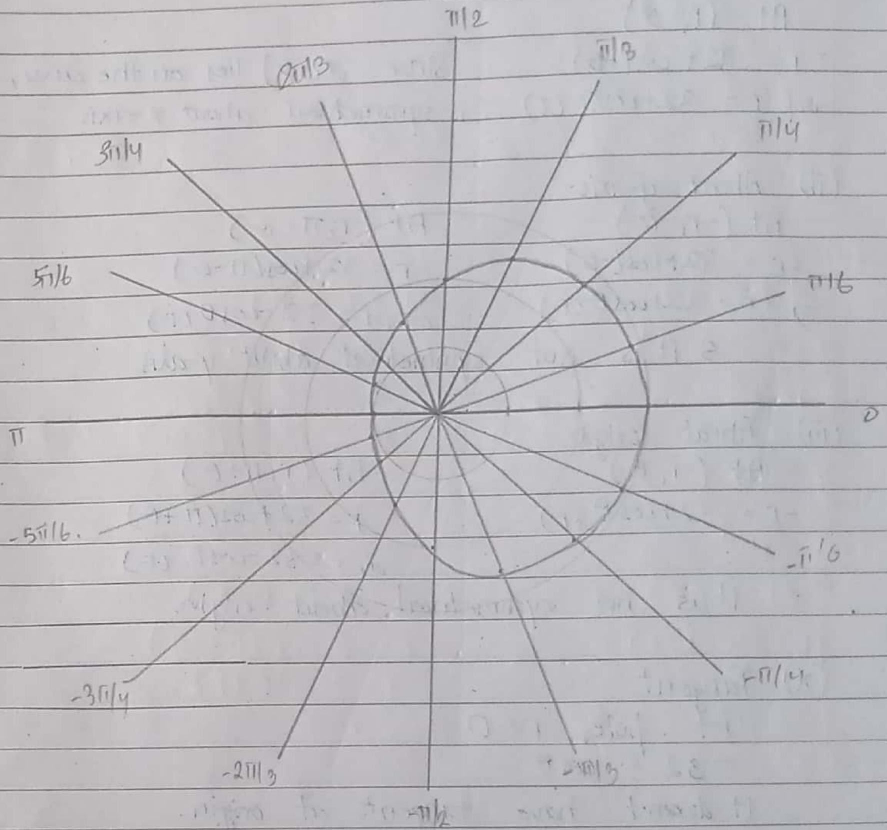
At pole, $r = 0$

$0 = 2+\cos\theta$

or $-2 = \cos\theta$ (Tangent doesn't exist at pole).

(x): r–θ table.

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	$5\pi/6$	π
$r = 2+\cos\theta$	3	2.86	2.7	2.5	2	1.5	1.29	1.13	1



(h): $r = 32 + \cos\theta$

Soln:

Given,

$r = 32 + \cos\theta$

(x) Symmetry:

(i): About x-axis:

At $(r, -\theta)$

$r = 32 + \cos(-\theta)$

or, $r = 32 + \cos\theta$ (T)

Since $(r, -\theta)$ lies on the curve, it is symmetrical about x-axis

(ii) About y-axis:

At $(-r, -\theta)$

$-r = 32 + \cos(-\theta)$

or $-r = 32 + \cos\theta$ (F)

At $(r, \pi - \theta)$

$r = 32 + \cos(\pi - \theta)$

or, $r = 32 - \cos\theta$ (F)

$\therefore$ It is not symmetrical about y-axis.

(iii) About origin.

At $(-r, \theta)$

$-r = 32 + \cos\theta$ (F)

At $(r, \pi + \theta)$

$r = 32 + \cos(\pi + \theta)$

or, $r = 32 - \cos\theta$ (F)

It is not symmetrical about origin.

(x) Tangent.

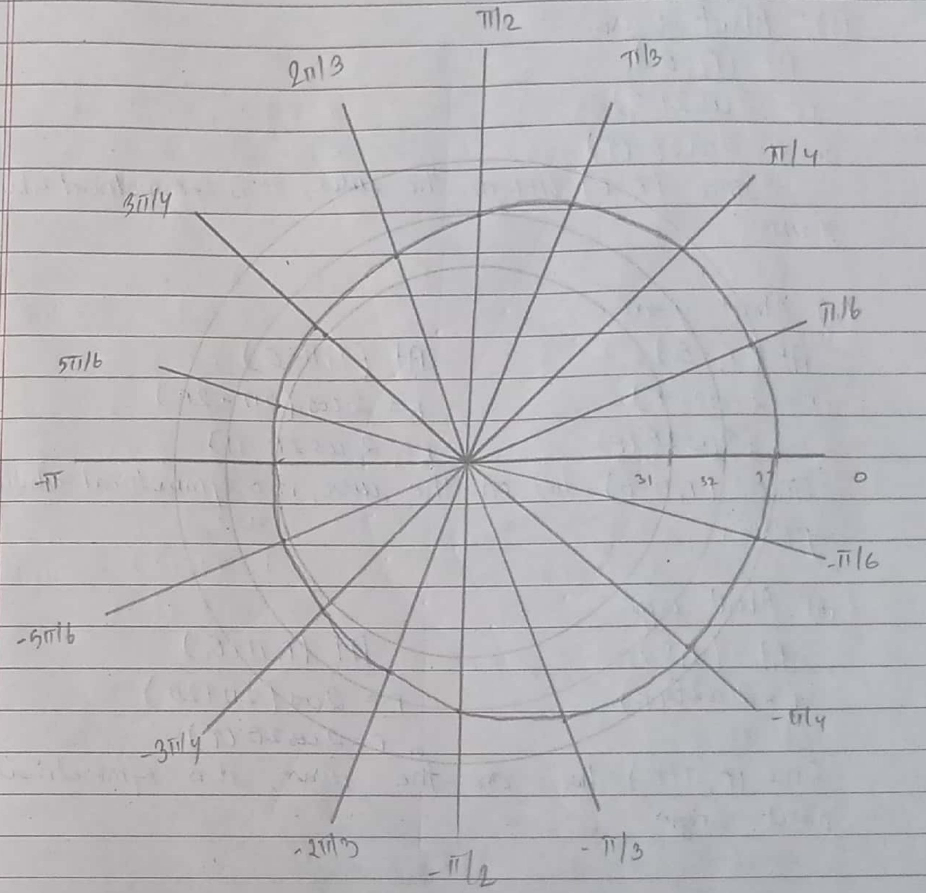
At pole, $r = 0$

$-32 = \cos\theta$

It doesn't have tangent at origin.

(x) r-θ table.

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	$5\pi/6$	π
$r = 32 + \cos\theta$	33	32.86	32.70	32.5	32	31.5	31.29	31.13	31



(i): $r = 2\cos 2\theta$

Soln:

Given,

$r = 2\cos 2\theta$

x) Symmetry

(i): About x-axis.

At $(r, -\theta)$

$r = 2\cos 2(-\theta)$

or $r = 2\cos 2\theta$ (T)

Since $(r, -\theta)$ lies on the curve, it is symmetrical about x-axis

(ii) About y-axis:

At $(-r, -\theta)$

$-r = 2\cos 2(-\theta)$

or $-r = 2\cos 2\theta$ (F)

At $(r, \pi-\theta)$

$r = 2\cos(2\pi - 2\theta)$

$r = 2\cos 2\theta$ (T)

Since $(r, \pi-\theta)$ lies on the curve, it is symmetrical about y-axis.

(iii) About origin

At $(-r, \theta)$

$-r = 2\cos 2\theta$ (F)

At $(r, \pi+\theta)$

$r = 2\cos(2\pi + 2\theta)$

or $r = 2\cos 2\theta$ (T)

Since $(r, \pi+\theta)$ lies on the curve, it is symmetrical about origin.

(x) Tangent:

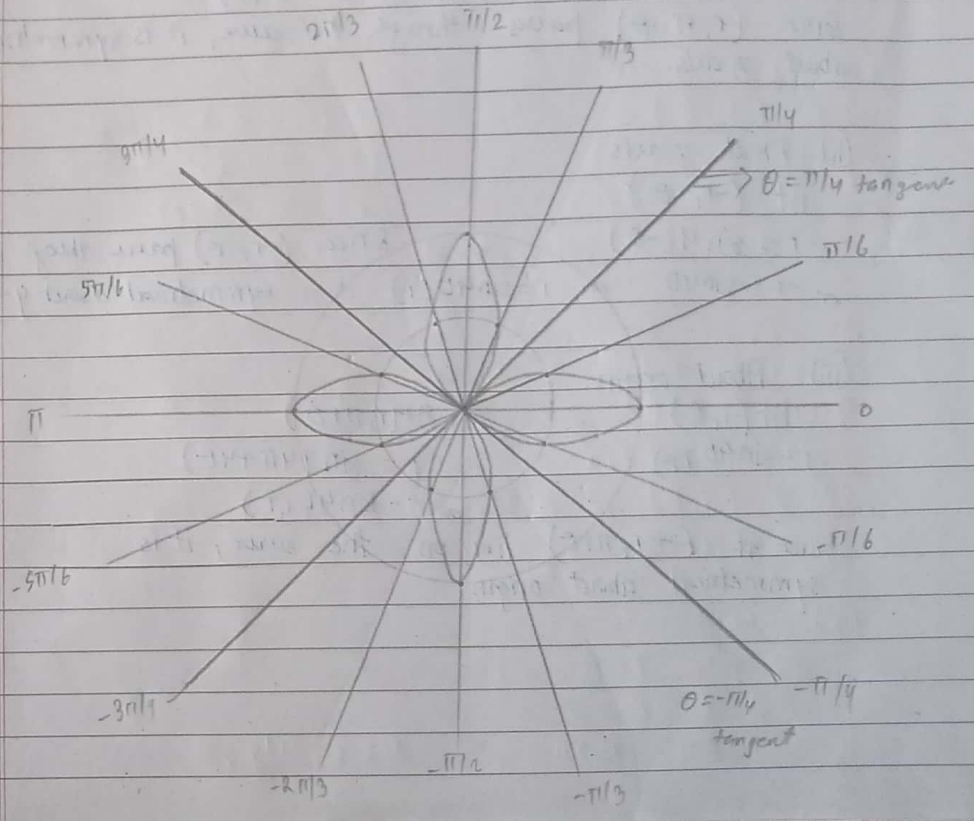
At pole, $r = 0$

$0 = 2\cos 2\theta$

or $\cos 2\theta = \pi/2$ $\therefore \theta = \pi/4, -\pi/4$

(x): $r-\theta$ table.

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	$5\pi/6$	π
$r = 2\cos 2\theta$	2	1	0	-1	-2	-1	0	1	2



(j): $r = \sin 4\theta$

Soln:

Given,

$r = \sin 4\theta$

(x) Symmetry:

(i): About x-axis:

At $(r, -\theta)$

$r = \sin 4(-\theta)$

or $r = -\sin 4\theta$ (F)

At $(-r, \pi - \theta)$

$-r = \sin(4\pi - 4\theta)$

$-r = -\sin 4\theta$

or $r = \sin 4\theta$ (T)

Since $(-r, \pi - \theta)$ passes through the curve, it is symmetrical about x-axis.

(ii) About y-axis.

At $(-r, -\theta)$

$-r = \sin 4(-\theta)$

or $-r = -\sin 4\theta$ or $r = \sin 4\theta$ (T)

Since $(-r, -\theta)$ pass through curve, it is symmetrical about y-axis.

(iii) About origin.

At $(-r, \theta)$

$-r = \sin 4\theta$ (F)

At $(r, \pi + \theta)$

$r = \sin(4\pi + 4\theta)$

or $r = \sin 4\theta$ (T)

Since it $(r, \pi + \theta)$ lies on the curve, it is symmetrical about origin.

(x): Tangent:

At pole, $r = 0$

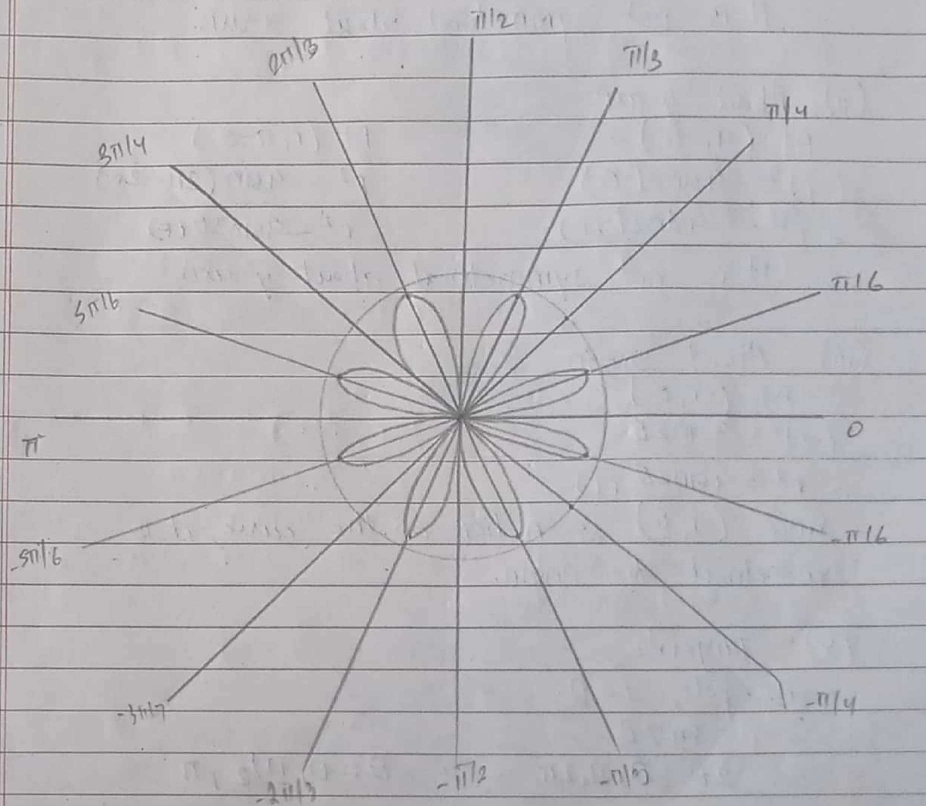
or, $0 = \sin 4\theta$

or $4\theta = 0, 2\pi, \pi$

$\therefore \theta = 0, \pi/2, \pi/4$

(x): r-θ table

θ	$-\pi/2$	$-\pi/3$	$-\pi/4$	$-\pi/6$	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$r = \sin 4\theta$	0	0.86	0	-0.86	0	0.86	0	-0.86	0



(k): $r^2 = 4\sin 2\theta$

Soln:

Given,

$r^2 = 4\sin 2\theta$

(x) Symmetry

(i): About x-axis.

At $(r, -\theta)$

$r^2 = 4\sin 2(-\theta)$

or, $r^2 = -4\sin 2\theta$ (F)

At $(-r, \pi-\theta)$

$(-r)^2 = 4\sin(2\pi - 2\theta)$

or $r^2 = -4\sin 2\theta$ (F)

It is not symmetrical about x-axis.

(ii) About y-axis

At $(-r, -\theta)$

$(-r)^2 = 4\sin 2(-\theta)$

or $r^2 = -4\sin 2\theta$ (F)

At $(r, \pi-\theta)$

$r^2 = 4\sin(2\pi - 2\theta)$

or $r^2 = -4\sin 2\theta$ (F)

It is not symmetrical about y-axis.

(iii) About origin.

At $(-r, \theta)$

$(-r)^2 = 4\sin 2\theta$

or $r^2 = 4\sin 2\theta$ (T)

Since $(-r, \theta)$ ~~is~~ a lies on the curve, it is symmetrical to origin.

(x): Tangent.

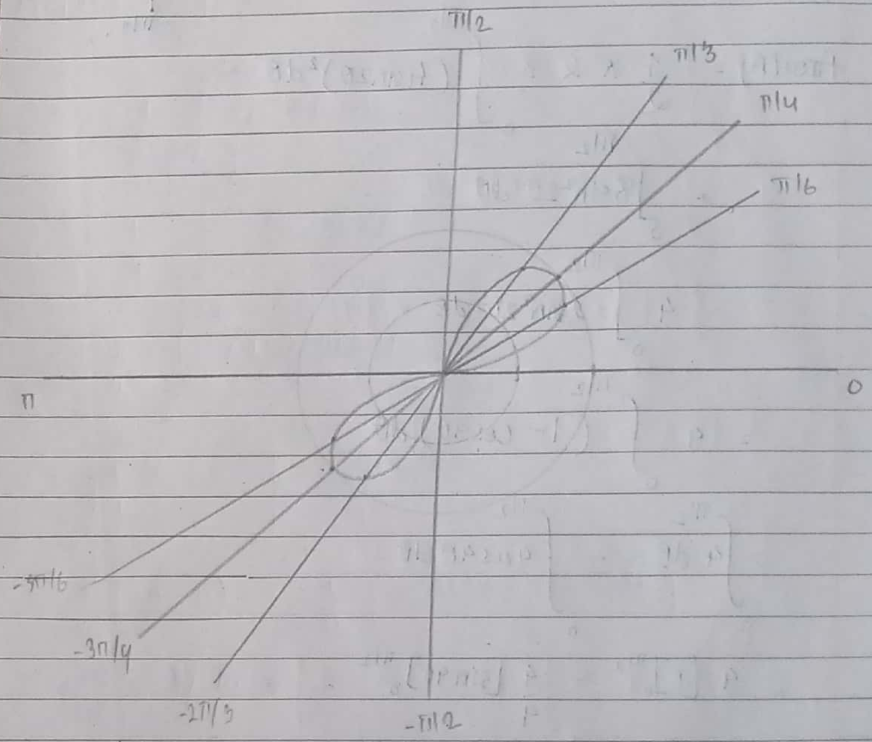
At pole, $r = 0$.

$0 = \sin 2\theta$

$2\theta = 0, \pi, 2\pi$ $\therefore$ $\theta = 0, \pi/2, \pi$

(x): r-θ table

θ	π	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$-\pi/2$	$-2\pi/3$	$-3\pi/4$	$-5\pi/4$
$r = \sqrt{4\sin 2\theta}$	0	0	1.86	2	1.86	0	0	1.86	2	1.86



<Q.7>: Find the area of the following regions

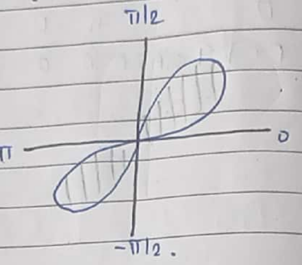
(a) Inside the lemniscate $r^2 = 4\sin 2\theta$ (Not from file)

Soln:

At $r=0$,

$0 = 4\sin 2\theta$

$\therefore \theta = 0, \pi/2$



Now,

$$\text{Area}(A) = \frac{1}{2} \times 2 \times \int_0^{\pi/2} (4\sin 2\theta)^2 \, d\theta$$

$$= \int_0^{\pi/2} 8\sin^2 2\theta \, d\theta$$

$$= 4\int_0^{\pi/2} 2\sin^2 2\theta \, d\theta$$

$$= 4\int_0^{\pi/2} (1-\cos 4\theta)\, d\theta$$

$$= \int_0^{\pi/2} 4\, d\theta - \int_0^{\pi/2} 4\cos 4\theta \, d\theta$$

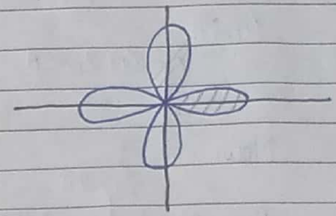
$$= 4[\theta]_0^{\pi/2} - \frac{4}{4}[\sin 4\theta]_0^{\pi/2}$$

$$= 4(\pi/2 - 0) - (\sin 4\times\pi/2 - \sin 0)$$

$$= 2\pi \text{ sq units.}$$

(b): Inside one leaf of four leafed rose $r = \cos 2\theta$.

Soln:



Given, $r = \cos 2\theta$

At $r=0$,

$0 = \cos 2\theta$

$2\theta = \pm\pi/2 \quad \therefore \theta = \pm\pi/4$

Now,

$$\text{Area }(A) = \frac{1}{2}\int_0^{2\pi} r^2 \, d\theta$$

$$= \frac{1}{2}\int_0^{2\pi} \cos^2 2\theta \, d\theta$$

$$= \frac{1}{4}\int_0^{2\pi} 2\cos^2 2\theta \, d\theta$$

$$= \frac{1}{4}\int_0^{2\pi} (1+\cos 4\theta)\, d\theta$$

$$= \int_0^{2\pi} \frac{1}{4}\, d\theta - \int_0^{2\pi} \frac{\cos 4\theta}{4}\, d\theta$$

$$= \frac{1}{4}(2\pi - 0) - \frac{1}{16}\left(\cos 4\times 2\pi - \cos 4\times 0\right)$$

$$= \frac{\pi}{2} \text{ sq. units}$$

Now,

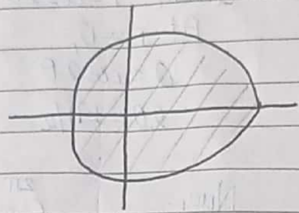
$$\text{Area of a leaf} = \frac{1}{4} \times \frac{\pi}{2} = \frac{\pi}{8} \text{ sq. units.}$$

(c): Inside the limacon: $r = 4+2\cos\theta$.

Sol^n:

Given,

$r = 4+2\cos\theta$



Now,

$$\text{Area}(A) = \frac{1}{2}\int_0^{2\pi} r^2\, d\theta$$

$$= \frac{1}{2}\int_0^{2\pi} (4+2\cos\theta)^2 d\theta$$

$$= \frac{1}{2}\int_0^{2\pi} [2(2+\cos\theta)]^2 d\theta$$

$$= 2\int_0^{2\pi} [4+4\cos\theta+\cos^2\theta]\, d\theta$$

$$= \int_0^{2\pi} [8+8\cos\theta+2\cos^2\theta]\, d\theta$$

$$= \int_0^{2\pi} [8+8\cos\theta+1+\cos 2\theta]\, d\theta$$

$$= \int_0^{2\pi} 9\cdot d\theta + 8\int_0^{2\pi} \cos\theta\, d\theta + \int_0^{2\pi} \cos 2\theta\, d\theta$$

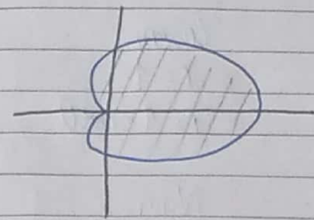
$$= 9(2\pi-0) + 8(\sin 2\pi - \sin 0) + \frac{1}{2}\sin(4\pi-0) = 18\pi \text{ sq.units.}$$

(d): inside the cardial $r = 2(1+\cos\theta)$

Sol^n:

Given,

$r = 2(1+\cos\theta)$



Now,

$$\text{Area}(A) = \frac{1}{2}\int_0^{2\pi} r^2\, d\theta$$

$$= \frac{1}{2}\int_0^{2\pi} [2(1+\cos\theta)]^2 d\theta$$

$$= 2\int_0^{2\pi} [1+2\cos\theta+\cos^2\theta]\, d\theta$$

$$= \int_0^{2\pi} [2+4\cos\theta+2\cos^2\theta]\, d\theta$$

$$= \int_0^{2\pi} [3+4\cos\theta+\cos 2\theta]\, d\theta$$

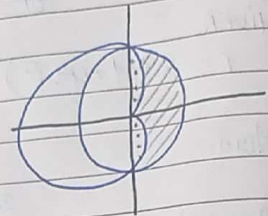
$$= \int_0^{2\pi} 3\, d\theta + \int_0^{2\pi} 4\cos\theta\, d\theta + \int_0^{2\pi} \cos 2\theta\, d\theta$$

$$= 3(2\pi-0) + 4[\sin(2\pi)-\sin(0)] + \frac{1}{2}[\sin(2\pi)-\sin(0)]$$

$$= 6\pi \text{ sq.units.}$$

(e): inside circle $r = 1$, outside cardiod $r = 1-\cos\theta$

Soln:



Given,

$r = 1$

$r = 1-\cos\theta$

Now,

Area of common semicircle $(A_1) = \frac{1}{2}\int_{-\pi/2}^{\pi/2} r^2 d\theta$

$$= \frac{1}{2}\int_{-\pi/2}^{\pi/2} 1 \cdot d\theta$$

$$= \frac{1}{2}[\theta]_{-\pi/2}^{\pi/2} = \frac{1}{2}\left[\frac{\pi}{2}+\frac{\pi}{2}\right] = \frac{\pi}{2} \text{ sq.units.}$$

Area of dotted region $(A_2) = \frac{1}{2} \times 2 \int_{0}^{\pi/2} r^2 d\theta$

$$= \int_0^{\pi/2} (1-\cos\theta)^2 d\theta$$

$$= \int_0^{\pi/2} (1 - 2\cos\theta + \cos^2\theta)\, d\theta$$

$$= \frac{1}{2}\int_0^{\pi/2} (2 - 4\cos\theta + 1 + \cos 2\theta)\, d\theta$$

$$= \frac{1}{2}\int_0^{\pi/2} (3 - 4\cos\theta + \cos 2\theta)\, d\theta$$

$$= \frac{1}{2}\left[\int_0^{\pi/2} 3 \cdot d\theta - \int_0^{\pi/2} 4\cos\theta + \int_0^{\pi/2} \cos 2\theta\right]$$

$$= \frac{1}{2}\left[3\left(\frac{\pi}{2}-0\right) - 4\left(\sin\frac{\pi}{2} - \sin 0\right) + \frac{1}{2}(\sin 2\pi - \sin 0)\right]$$

$$= \frac{1}{2}\left[\frac{3\pi}{2} - 4\right] = \frac{3\pi}{4} - 2 \text{ sq. units.}$$

$\therefore$ The area outside cardiod inside circle $= \frac{\pi}{2} - \left(\frac{3\pi}{4} - 2\right)$

$$= \frac{\pi}{2} - \frac{3\pi}{4} + 2$$

$$= \frac{2\pi - 3\pi}{4} + 2$$

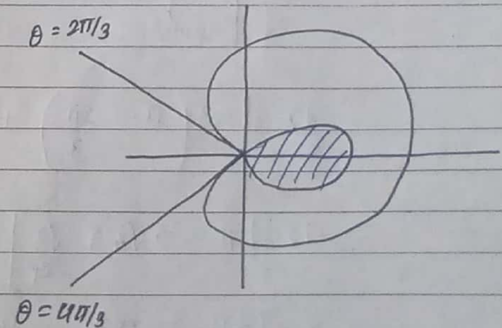
$$= 2 - \pi/4 \text{ sq. units.}$$

(f): Inner loop of limacon $r = 12 + 24\cos\theta$

Soln:

Given,

$r = 12 + 24\cos\theta$



At pole, $r = 0$

$12 + 24\cos\theta = 0$

or, $\cos\theta = \frac{-12}{24}$

or, $\cos\theta = \frac{-1}{2}$

or $\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

Date ______ Page ______

Now, the area of the limacon inner loop is between $\theta = 2\pi/3$ and $\theta = 4\pi/3$

$$\text{Area} = \frac{1}{2}\int_{2\pi/3}^{4\pi/3} r^2\, d\theta$$

$$= \frac{1}{2}\int_{2\pi/3}^{4\pi/3} (12+24\cos\theta)^2\, d\theta$$

$$= \frac{144}{2}\int_{2\pi/3}^{4\pi/3} (1+2\cos\theta)^2\, d\theta$$

Since the inner loop is symmetrical about x-axis

$$= 72\times 2\int_{2\pi/3}^{\pi} [1 + 2\cos\theta + \cos^2\theta]\, d\theta$$

$$= 72\int_{2\pi/3}^{\pi} [2+4\cos\theta + 1+\cos 2\theta]\, d\theta$$

$$= 72\left[\int_{2\pi/3}^{\pi} 3\, d\theta + \int_{2\pi/3}^{\pi} 4\cos\theta\, d\theta + \int_{2\pi/3}^{\pi} \cos 2\theta\, d\theta\right]$$

$$= 72\left[3\left(\pi - \frac{2\pi}{3}\right) + 4\left[\sin\pi - \sin\frac{2\pi}{3}\right] + \frac{1}{2}\left[\sin 2\pi - \sin 4\pi/3\right]\right]$$

$$= 72\left[\pi - 2\sqrt{3} + \frac{\sqrt{3}}{4}\right]$$

$$= 72\pi - \frac{7\sqrt{3}}{4} \text{ sq. units.}$$

Date ______ Page ______

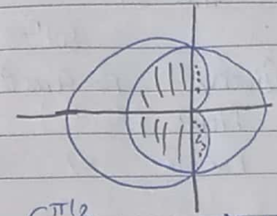
(g): shared by circle $r=2$ and cardiod $r = 2-2\cos\theta$.

Soln:

Given,

$r=2$

$r = 2-2\cos\theta$



Now,

$$\text{Area of dotted region } (A_1) = \frac{1}{2}\times 2\int_0^{\pi/2} (2[1-\cos\theta])^2\, d\theta$$

$$= 4\int_0^{\pi/2} (1-2\cos\theta+\cos^2\theta)\, d\theta$$

$$= 2\int_0^{\pi/2} (3-4\cos\theta+\cos 2\theta)\, d\theta$$

$$= 2\left[\int_0^{\pi/2} 3\cdot d\theta - \int_0^{\pi/2} 4\cos\theta\, d\theta + \int_0^{2\pi} \cos 2\theta\, d\theta\right]$$

$$= 6\theta\Big|_0^{\pi/2} - 8\sin\theta\Big|_0^{\pi/2} + \sin 2\theta\Big|_0^{\pi/2}$$

$$= 3\pi - 8 \text{ sq units.}$$

$$\text{Area of common semicircle } (A_2) = 1/2\int_{-\pi/2}^{\pi/2} r^2\, d\theta$$

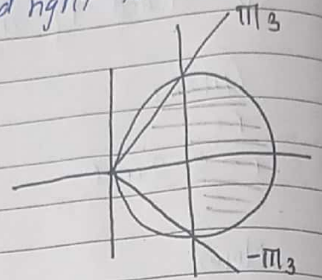
$$= 1/2\int_{-\pi/2}^{\pi/2} 4\, d\theta$$

$$= 2\,\theta\Big|_{-\pi/2}^{\pi/2} = 2\pi$$

$\therefore$ Total area $(A) = 5\pi - 8$ sq units.

Q7: Inside the circle $r = 4\cos\theta$ and right to vertical line $r = \sec\theta$.

Sol^n:



Given, $r = 4\cos\theta$

Now,

$r\sec\theta \Rightarrow x = 1.$

Now, solving eqns:

$4\cos\theta = \sec\theta$

or $\cos^2\theta = \frac{1}{4}$ or, $\cos\theta = \frac{1}{2}$

$\therefore \theta = \pm \pi/3.$

Now the area of shaded region (A)

$$= \frac{1}{2}\int_{-\pi/3}^{\pi/3} [(4\cos\theta)^2 - (\sec\theta)^2]\,d\theta$$

$$= \frac{1}{2}\int_{-\pi/3}^{\pi/3}\left(16\cos^2\theta - \frac{1}{\cos^2\theta}\right)d\theta$$

Since the curve is symmetrical about x-axis.

$$A = \frac{1}{2}\times 2\int_0^{\pi/3}(16\cos^2\theta - \sec^2\theta)\,d\theta$$

$$= \int_0^{\pi/3} 16\cos^2\theta\,d\theta - \int_0^{\pi/3}\sec^2\theta\,d\theta$$

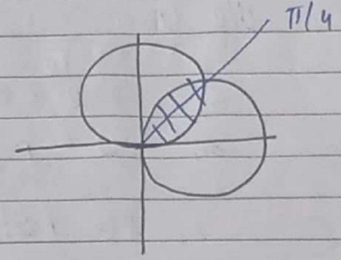
$$= 8\int_0^{\pi/3}(1+\cos 2\theta)\,d\theta - \int_0^{\pi/3}\sec^2\theta\,d\theta$$

$$= 8\,\theta\Big|_0^{\pi/3} + 4\sin 2\theta\Big|_0^{\pi/3} - \tan\theta\Big|_0^{\pi/3}$$

$$= \frac{8\pi}{3} + 2\sqrt{3} - \sqrt{3} \qquad = \frac{8\pi}{3} + \sqrt{3} \text{ sq units.}$$

Q (ii): shared by $r = 2a\cos\theta$ and $r = 2a\sin\theta$

Sol^n:



Given,

$r = 2a\cos\theta$ —(i)

$r = 2a\sin\theta$ —(ii)

Solving (i) & (ii), we get.

$\theta = \pi/4.$

The common area is symmetrical above and below $\pi/4$.

$$\text{Area}(A) = \frac{1}{2}\times 2\times\int_0^{\pi/4}(2a\sin\theta)^2\,d\theta$$

$$= 2a^2\int_0^{\pi/4} 2\sin^2\theta\,d\theta$$

$$= 2a^2\left[\int_0^{\pi/4} 1\,d\theta - \int_0^{\pi/4}\cos 2\theta\,d\theta\right]$$

$$= 2a^2\left[\frac{\pi}{4} - 0 - \left(\frac{\sin 2\times\pi}{4} - \frac{\sin 2\times 0}{3}\right)\right]$$

$$= 2a^2\left[\frac{\pi}{4} - 1\right] \text{ sq. units.}$$

Date
Page

<8>: (a): Find the polar equation of the line passing through $P_0(2, \pi/3)$.

Soln:

Given,

$$P_0(r_0, \theta_0) = P_0(2, \pi/3)$$

We know,

$$r_0 = r\cos(\theta - \cancel{\theta_0}\ \theta_0)$$

or, $$2 = r\cos(\theta - \pi/3)$$

or, $$2 = r\left[\cos\theta \cdot \cos\frac{\pi}{3} + \sin\theta \cdot \sin\frac{\pi}{3}\right]$$

or, $$2 = r\cos\theta \cdot \frac{1}{2} + r\sin\theta \cdot \frac{\sqrt{3}}{2}$$

or, $$r\cos\theta + \sqrt{3}\, r\sin\theta = 4$$

which is the reqd polar eqn of line.

<b>: Find the cartesian eqn of polar line $r\cos(\theta - 2\pi/3) = 1$

Soln:

Given, $$r\cos(\theta - 2\pi/3) = 1$$

or, $$r\left[\cos\theta \cdot \cos\frac{2\pi}{3} + \sin\theta \cdot \sin\frac{2\pi}{3}\right) = 1$$

or, $$r\cos\theta \cdot \left(\frac{-1}{2}\right) + r\sin\theta\left(\frac{\sqrt{3}}{2}\right) = 1$$

or, $$-\frac{x}{2} + \frac{\sqrt{3}\,y}{2} = 1$$

or $$\sqrt{3}y - x = 2$$

which is the reqd eqn of polar line