ENGINEERING MECHANICS

Mechanics:

The science that describes and predicts the conditions of rest or motion of bodies under the action of forces is called mechanics.

It consists of:
- Rigid body Mechanics
- Deformable body mechanics
- Fluid mechanics.

Rigid Body:

The body in which deformation is zero or so small that it can be neglected is called rigid body.

The distance between any two given points on a rigid body remains constant in time regardless of external forces / moments exerted on it.

Date :
Page:

Engineering Mechanics

- Mechanical of Solid Bodies
  - Mechanics of Rigid body
    - Statics
    - Dynamics
      - Kinetics
      - Kinematics.
  - Mechanics of Deformable Body
    - Strength of Materials.
    - Theory of Elasticity.
- Mechanical of fluids.
  - Ideal fluids
  - Viscous fluids
  - Compressible fluids

Definitions:

i) Statics:

The study of distribution and effect of forces on rigid bodies which are at rest and remain at rest. is called statics.

ii) Dynamics:

The study of motion of rigid bodies and their correlation with the forces causing them is called dynamics.

Date: ....
Page: ....

(iii) Kinematics:

The study of motion of bodies without any reference to the forces causing motion or forces produced as a result of the motion is called kinematics.

(iv) Kinetics:

The study of the relationship between the forces and the resulting motion is called kinetics.

(v) Mechanics of deformable bodies

It is known as strength of materials.

The study dealing with internal force distribution and the deformation developed in actual engineering structures or machine components is called mechanics of deformable bodies.

(vi) Fluid mechanics:

The study of the liquids and gases (fluid) at rest or in motion is called fluid mechanics.

Force:

Any action that changes or tends to change the state of rest or of uniform motion of body is called force.

It is vector quantity.

Force

- Coplanar
  - Collinear
  - Concurrent
  - Parallel
  - Non-concurrent / Non-parallel
- Non-coplanar
  - Concurrent
  - Parallel
  - Non-concurrent, non-parallel

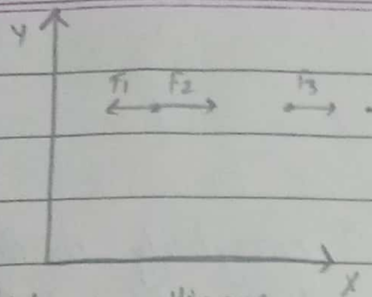
(i) Coplanar forces:
Line of action of all forces lies on same plane.

(ii) Non-coplanar forces:
Line of action of all forces do not lie on the same plane.

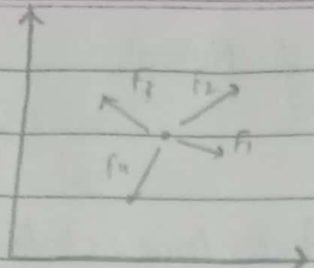
(iii) Collinear forces:
Line of action of all forces act along the same line.

(iv) Concurrent forces:
Line of action of all the forces passes through a single point.

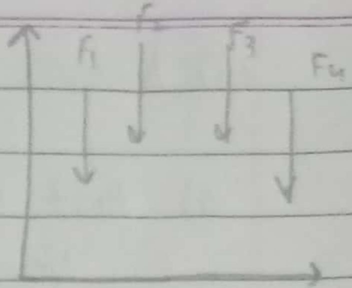
(v) Parallel forces:
Line of action of all forces parallel to each other.



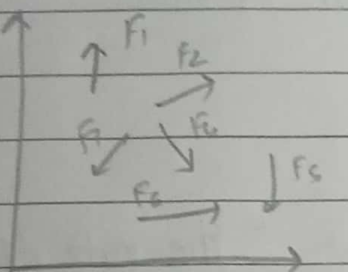
Coplanar collinear



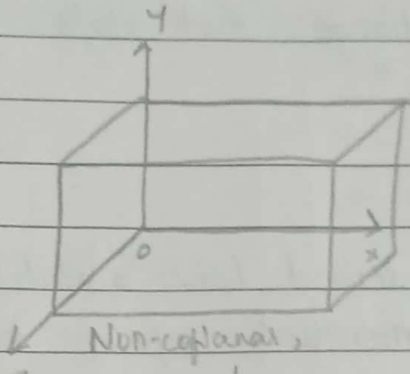
Coplanar-concurrent



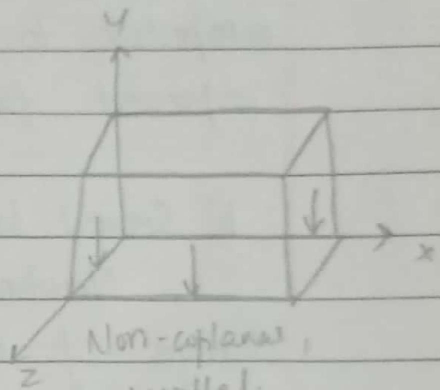
Coplanar parallel



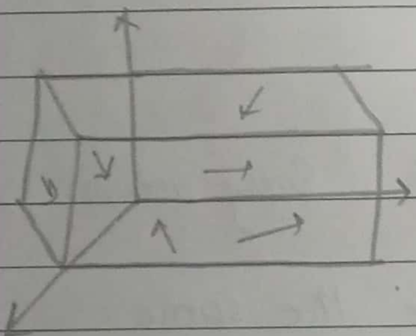
Coplanar, non concurrent, non-planar



Non-coplanar, concurrent



Non-coplanar, parallel.



non-coplanar, non-concurrent, non-parallel

Fundamental principle of Mechanics

The fundamental principle of mechanics are:

(i): Newton's three laws of motion

(ii) Newton's law of gravitation.

(iii) Parallelogram law

(iv) Principle of transmissibility.

Date:
Page:

(i): Newton's law of Motion:

a) First law:

Newton's first law states that, "everybody continues in its state of rest or of uniform motion, unless it is compelled to change that state by an external impressed force."

b) Second law:

Newton's second law states that, "the rate of change of momentum of a body is directly proportional to the force acting on it and takes place in the direction of force applied."

c) Third law:

Newton's third law states that, "Every action has equal and opposite reaction."

The force and reaction have the same magnitude and line of action but opposite in direction."

(ii) Newton's law of Gravitation:

Newton's law of ~~fra~~ gravitation states that, "the force of attraction between two bodies is directly proportional to the product of the masses and inversely proportional to the square of the distance between them"

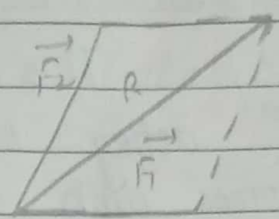
Then,

$$F \propto \frac{M_1 M_2}{d^2}$$

Date: ....
Page: ....

(iii) Parallelogram law:

Parallelogram law states,
" when two forces acting on the particle represented by two adjacent sides of parallelogram, the diagonal connecting the two sides represents the resultant force 'R' in magnitude and direction "



(iv) Principle of transmissibility:

Principle of transmissibility states that, "the conditions of equilibrium or the motion of a rigid body remains unchanged if a force acting at a given point of the rigid body is replaced by a force of same magnitude and direction, but acting at a different point provided that the forces have the same line of action."

$$R = \sqrt{F_1^2 + 2F_1F_2\cos\theta + F_2^2}$$

$$\tan\alpha = \frac{F_2 \sin\theta}{F_1 + F_2\cos\theta}$$

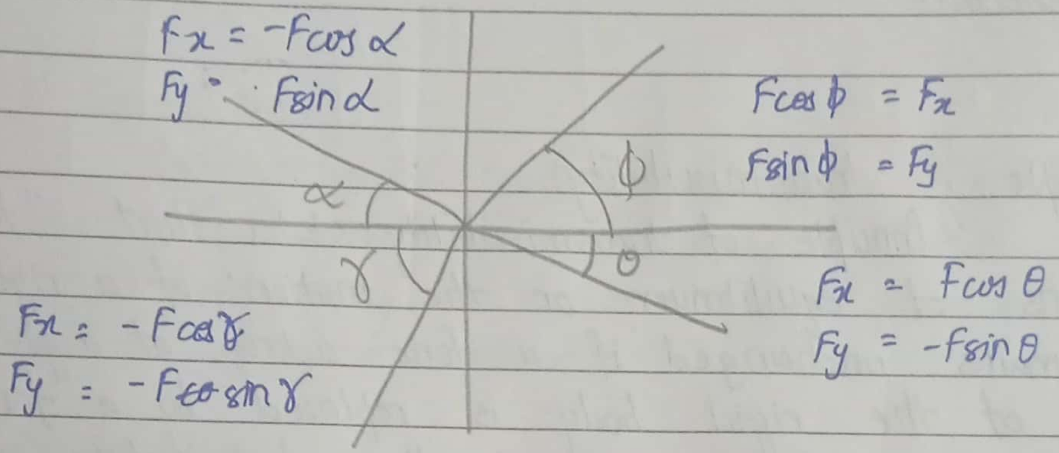
If $\theta = 90^\circ$, $R = \sqrt{F_1^2 + F_2^2}$ $\tan\alpha = F_2/F_1$

If $\theta = 0^\circ$ $R = F_1 + F_2$ $\tan\alpha = 0$, $\alpha = 0$

If $\theta = 180^\circ$ $R = F_1 - F_2$ $\tan\alpha = 0$, $\alpha = 0$.

Resolution of forces

The process of replacing a single force 'F' acting on a particle by two or more forces with together have the same effect as that the of a single force is called resolution of force into component.



If there are 'i' number of forces in system,

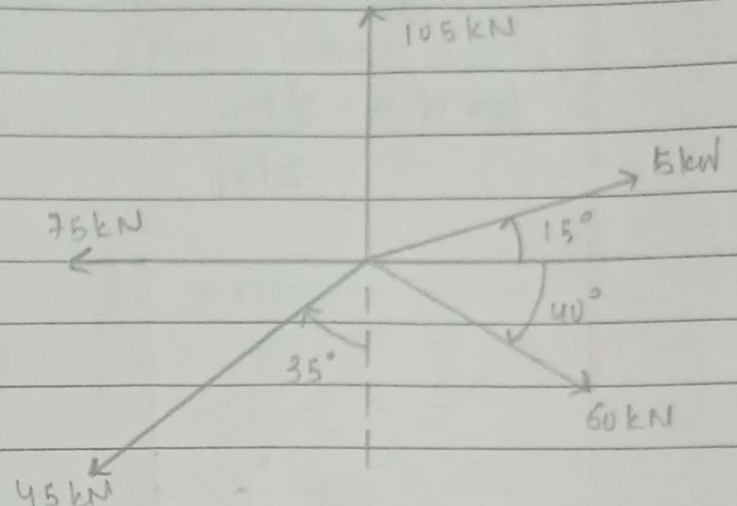
$$R_x = \sum F_{x_i} = \sum (F_i \cos\theta_i)$$
$$R_y = \sum F_{y_i} = \sum (F_i \sin\theta_i)$$

Magnitude of resultant $R = \sqrt{(\sum F_{x_i})^2 + (\sum F_{y_i})^2}$

Inclination with H-axis : $\tan\alpha = \dfrac{\sum F_{y_i}}{\sum F_{x_i}}$

Date: ....
Page: ....

<Q.1>: If five forces act on the particle, as shown, determine the resultant and its angle to the horizontal.



Sol^n:

For $F_1 = 15\,kN$,

$F_{x_1} = F_1 \cos\theta$

$= 15 \times \cos 15°$

$\therefore F_{x_1} = 14.489\ kN$

$F_{y_1} = F_1 \sin\theta$

$= 15 \times \sin 15°$

$\therefore F_{y_1} = 3.882\ kN$

For $F_2 = 105\,kN$

$F_{x_2} = 0$ $\quad$ $F_{y_2} = 105\,kN$

For $F_3 = 75\,kN$

$F_{x_3} = -75\,kN$ $\quad$ F_y

For $F_4 = 45\,kN$

$F_{x_4} = -45 \times \sin 35°$ $\quad$ $F_{y_4} = -45 \times \cos 35°$

$\therefore F_{x_4} = -25.811\ kN$ $\quad$ $\therefore F_{y_4} = -36.862\ kN$

For $F_5 = 60\,kN$

$F_{x_5} = 60 \times \cos 40°$ $\quad$ $F_{y_5} = -60 \times \sin 40°$

$\therefore F_{x_5} = 45.693\ kN$ $\quad$ $\therefore F_{y_5} = -38.567.$

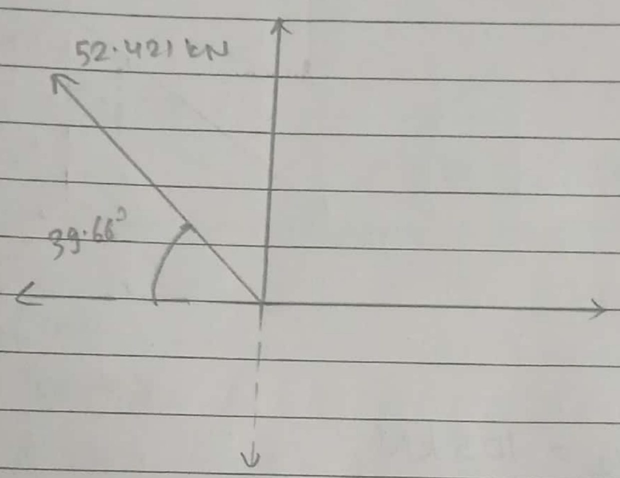
$\Sigma F_{x_i} = -40.359$ $\quad$ $\Sigma F_{y_i} = 33.453.$

Now,

$$R = \sqrt{(\Sigma F_{xi})^2 + (\Sigma F_{yi})^2}$$

$\therefore R = \cancel{40.539}$ 52.421 kN

$\tan\alpha = \dfrac{\Sigma F_{yi}}{\Sigma F_{xi}}$ $\therefore \alpha = 39.66^{\circ}$



Equilibrium of Particle

A body is said to be in equilibrium when the resultant of the force acting on it is zero.

If a body is in equilibrium, it will continue to remain in its original state.

$$\vec{R} = 0.$$

$\Sigma F_x = 0$ $\Sigma F_y = 0$

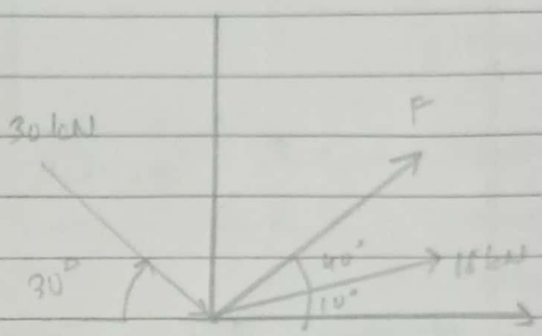
Date: ....
Page: ....

(Q.27) Three forces act on particle 'O' as figure. Determine the value of F such that the resultant of this force is horizontal. Find the magnitude and direction of the fourth force which when acting along the given three forces will keep O in equilibrium.

Soln.

Here,

angle betn F and O = 40+10°
= 50°



30 kN force acts towards particle.

Since the resultant force is horizontal, $\alpha = 0$.

So,

$$\tan\alpha = \frac{\Sigma F_y}{\Sigma F_x} = 0$$

ie, $(\uparrow +)\ \Sigma F_y = 0$

or $-30 \times \sin 30^\circ + 18 \times \sin 180^\circ + F \sin 50^\circ = 0$

or $F = \frac{30 \sin 30^\circ - 18 \sin 10^\circ}{\sin 50^\circ}$

$\therefore F = 15.5$ kN

Since R (resultant) is horizontal,

ΣF_x $R = 30 \cos 30^\circ + 18 \times \cos 10^\circ + 15.5 \times \cos 50^\circ$

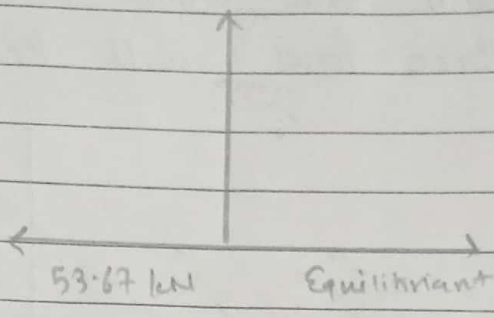
$\therefore R = 53.67$ kN.

Let E be the equilibriant $\Sigma F_x = 0$.

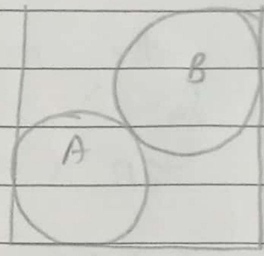
ie,

$$R + E = 0$$

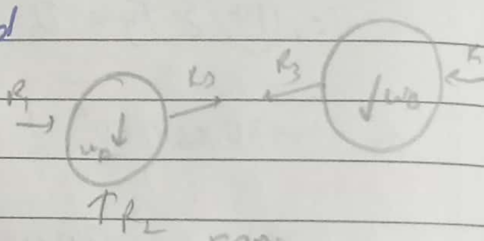
$$\therefore E = -53\cdot 67 \text{ kN.}$$



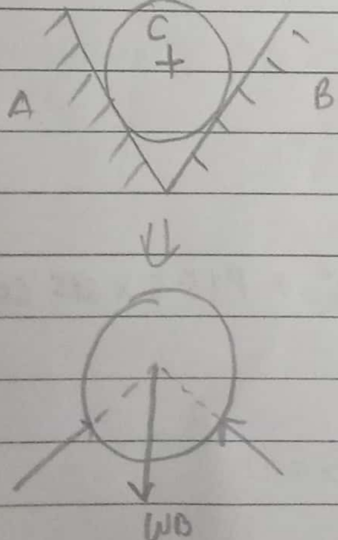
Free-Body Diagram

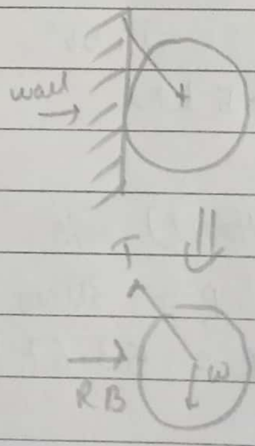


A diagram of a body (or part of it) which shows all the forces and couples applied on it, and which has all forces and couples labelled for use in solution of problem is free body diagram

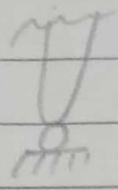
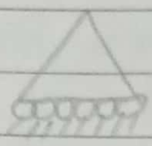
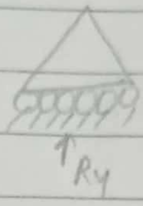
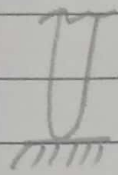
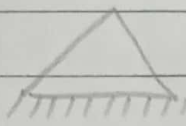
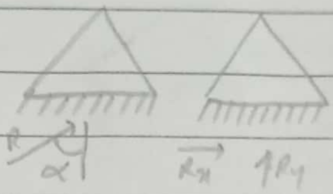
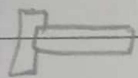
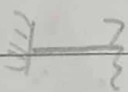
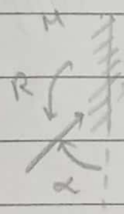


FBDiagram





Types of Support or Connection

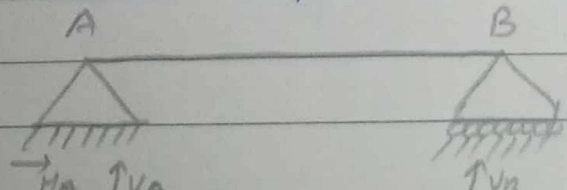
Name	Example	Representation	Reaction	Unknowns	Remarks
Smooth / Roller Support				One (R_y)	Displacement in vertical direction is prevented.
Hinge / Rough surface.				Two (R, α) or (R_x, R_y)	Displacement in both vertical and horizontal direction is prevented
Fixed support.				Three (R, α, M) or (R_x, R_y, M)	Displacement in both vertical & horizontal direction and rotation is prevented.

Beam

A member which bends when subjected to load applied transverse to the long dimensions (axis of member) is called beam.

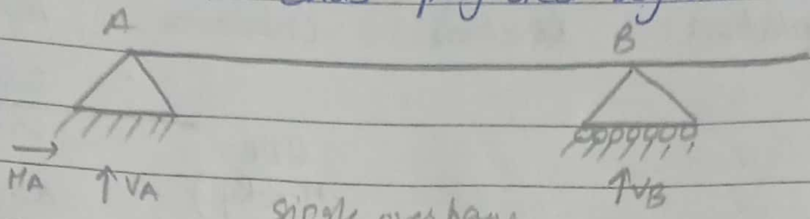
*) Types of Beam:

(i): Simple Supported Beam: Beam supported by hinge / roller on the smooth surface at the ends having one span.

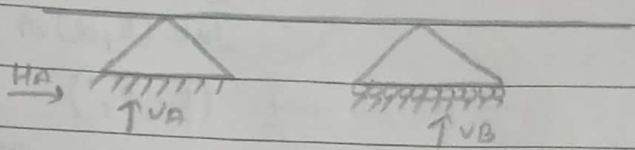


(ii) Overhanging Beam:

Both ends projected beyond the support.



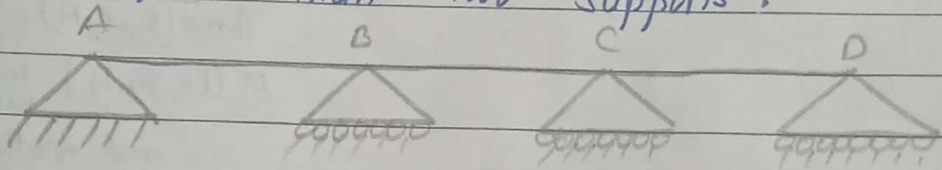
single overhang



Double over hang

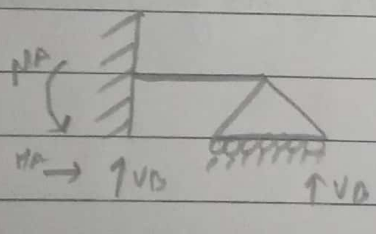
(iii) Continuous beam:

More than two supports.



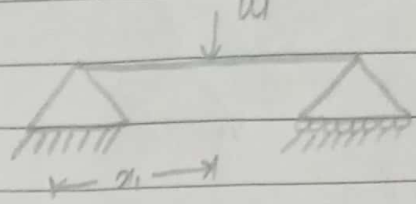
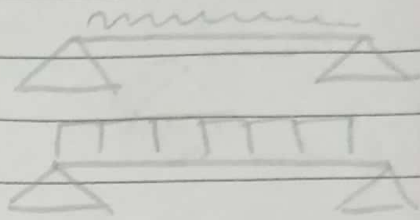
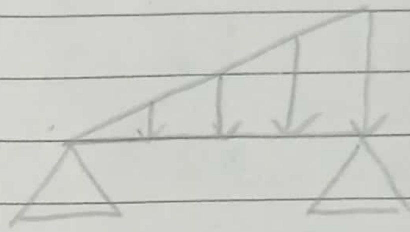
(iv) Cantilever beam:

One end is built into a wall so that it can't rotate or move transversly.



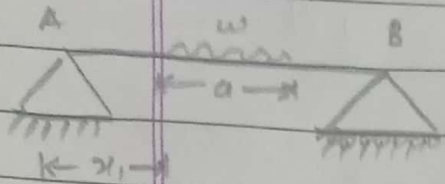
Date :
Page:

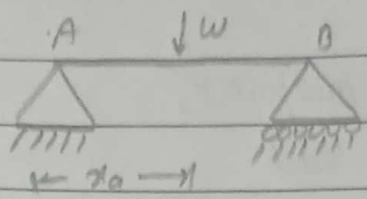
Types of Load

Types	Definitions	Example.
Concentrated load	Acting at midpoint, point load	
Uniformly Distributed load.	Load is constant.	
Uniformly varying load.	Varying linearly over considerable length.	

Equivalent Road

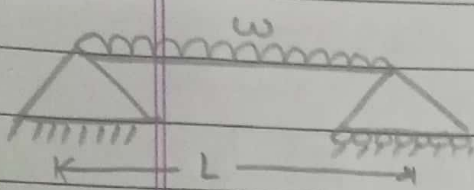
Load | Equivalent | Remarks

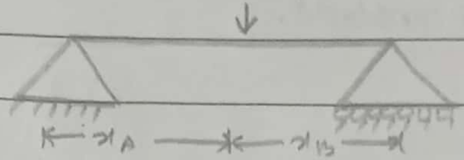




$W = wa$

$\vec{x_a} = x_1 + \frac{a}{2}$

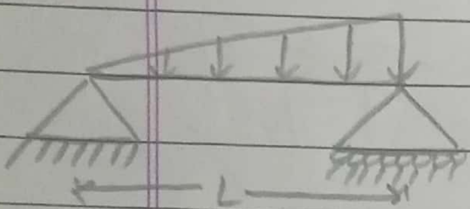


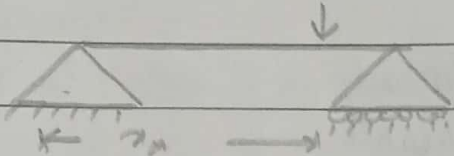


$W = wL$

$x_A = L/2$

$x_B = L/2$

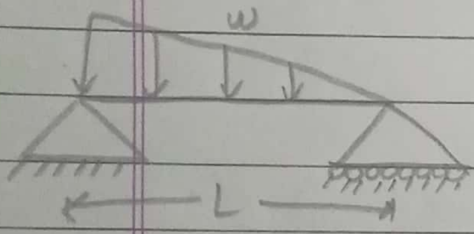


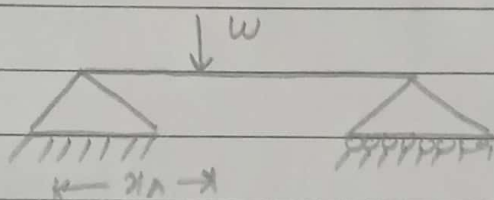


$W = wL/2$

$x_A = 2L/3$

$x_B = L/3$

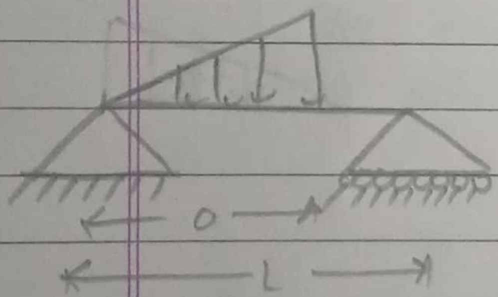


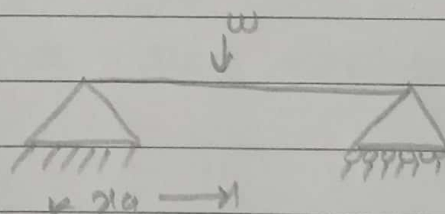


$W = wL/2$

$x_A = L/3$

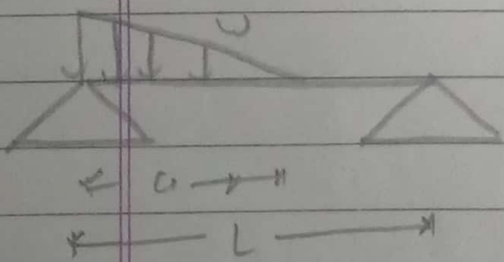
$x_B = 2L/3$

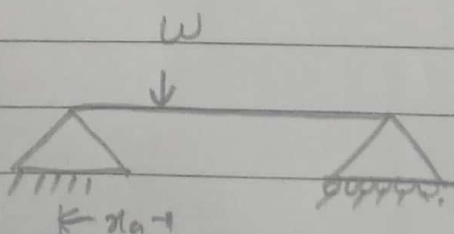




$W = wa/2$

$x_A = \frac{2a}{3}$

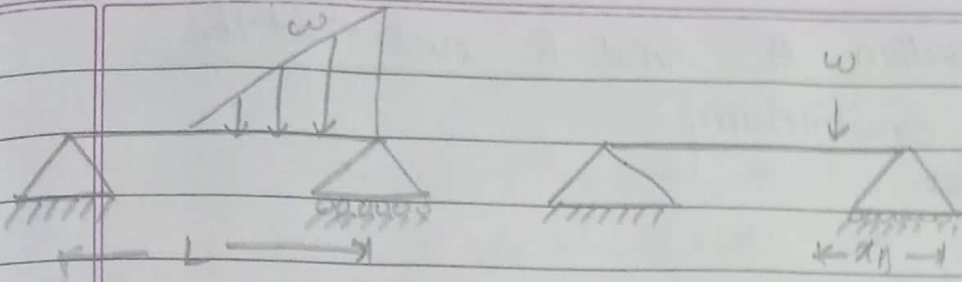




$W = wa/2$

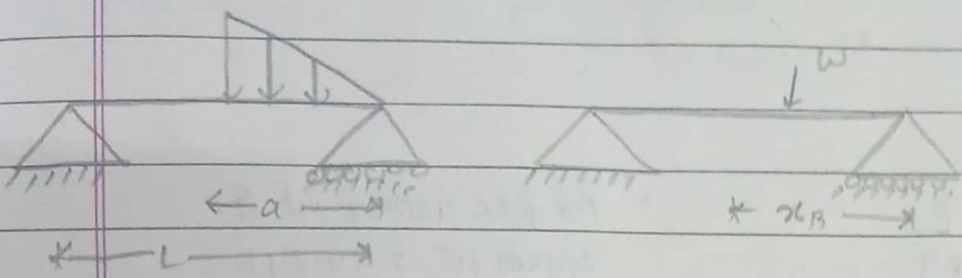
$x_a = \frac{a}{3}$

Date :
Page:



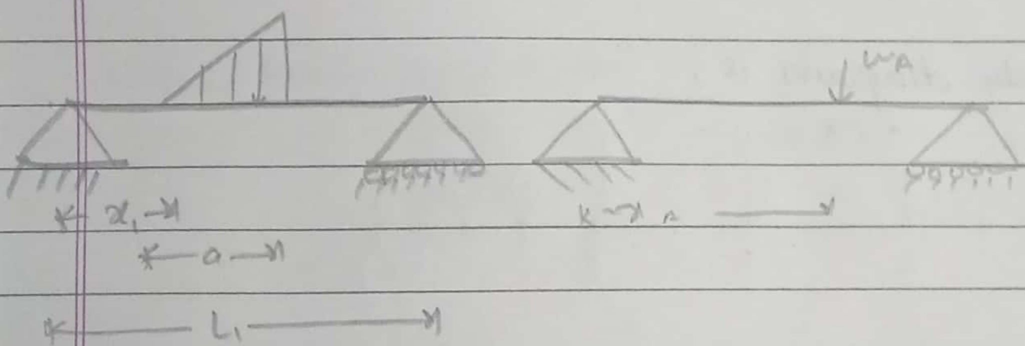
$W = wa/2$

$\vec{x}_B = a/3$



$W = wa/2$

$\vec{x}_B = \frac{2a}{3}$

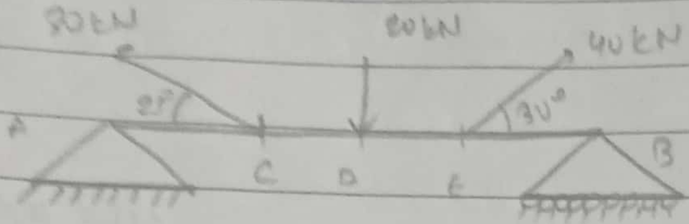


$W = wa/2$

$\vec{x}_a = (x_1 + 2a/3)$.

Date :
Page:

Q(8·1): Find the reaction A and B such that the system is in equilibrium.



Soln:

For force acting at C,

$30\cos 25° = 27.189$

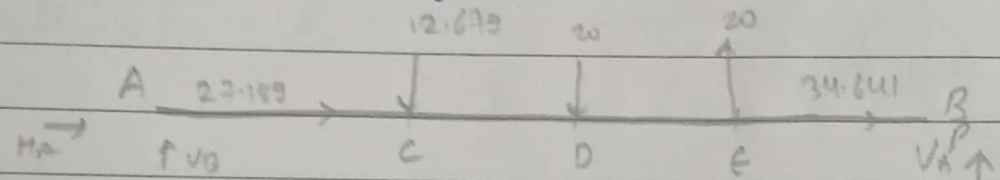
$30\sin 25° = 12.679$

For force acting at E.

$40\cos 30° = 34.641$

$40\sin 30° = 20$

The free body diagram is,



At equilibrium, $\sum F_x = 0$, $\sum F_y = 0$, $\sum M_a = 0$

So,

$\sum F_x = 0$ ($\overrightarrow{+}$)

or, $27.189 + H_A + 34.641 = 0$

$\therefore H_A = -61.84$ — (i)

This shows HA acts towards left.

Also,

$\sum F_y = 0$ ($\uparrow +$)

or $V_A - 12.679 - 20 + 20 + V_B = 0$

or $V_A + V_B = 12.679$ — (iii)

Date :
Page:

Again,

(↺+) $\sum M_a = 0$

or $-12.679 \times 2 - 20 \times 5 + 20 \times 8 + V_B \times 10 = 0$

$\therefore V_B = -3.464$ kN ——— (iii)

Here, V_B acts downward.

Putting value of eqn (ii),

$V_A = 16.143$ kN

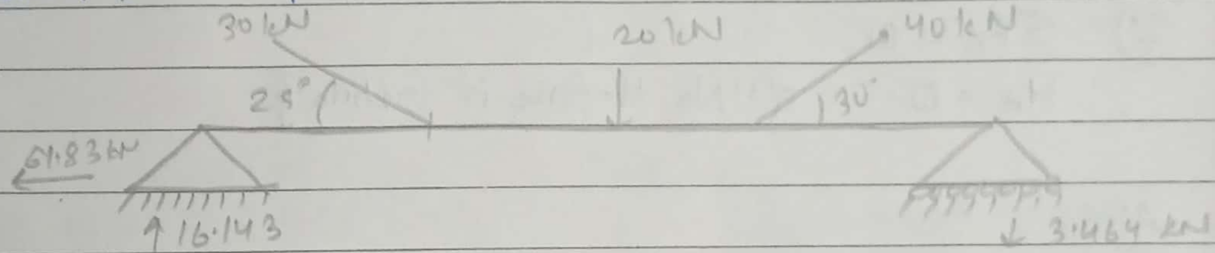
So,

$H_A = 61.83$ kN (←)

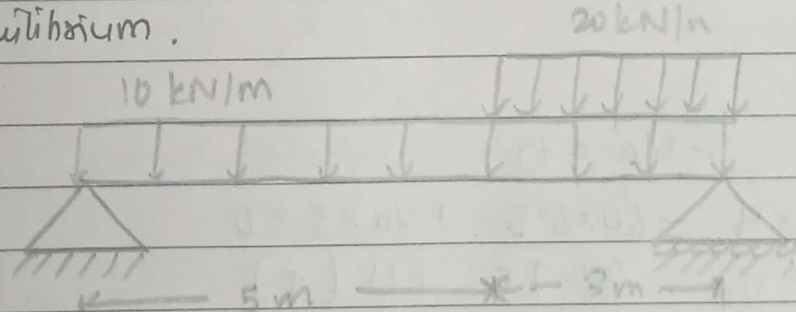
$V_A = 16.143$ kN (↑)

$V_B = 3.464$ kN (↓)

Hence, final equilibrium.



<Q.2>: Find the reaction at A and B so that system is at equilibrium.



Soln.

Date :
Page:

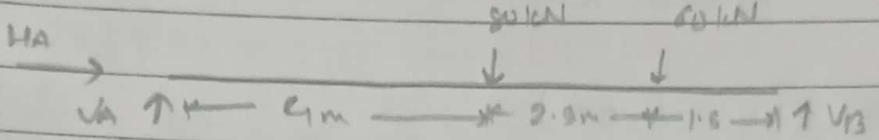
The equivalent loads acting are.

$W_1 = 10\,kN \times 8 = 80\ kN$ at 4m from A

$W_2 = 20\,kN \times 3 = 60\ kN$ at 1.5m from B.

[∵ uniform force acts at half distance]

The free body diagram,



At equilibrium, $\sum F_x = 0$, $\sum F_y = 0$, $\sum M_A = 0$.

So,

$(\overset{+}{\rightarrow})\ \sum F_x = 0$

$\therefore H_A = 0$ {∵ No H-force is acting}

Also,

$(+\uparrow)\ \sum F_y = 0$

or $V_A - 80 - 60 + V_B = 0$

$\therefore V_A + V_B = 140$ —— (i)

Now,

$(\overset{\curvearrowleft}{+})\ \sum M_A = 0$

or $-80\times 4 - 60\times 6.5 + V_B \times 8 = 0$

$\therefore V_B = 88.75\ kN\ (\uparrow)$

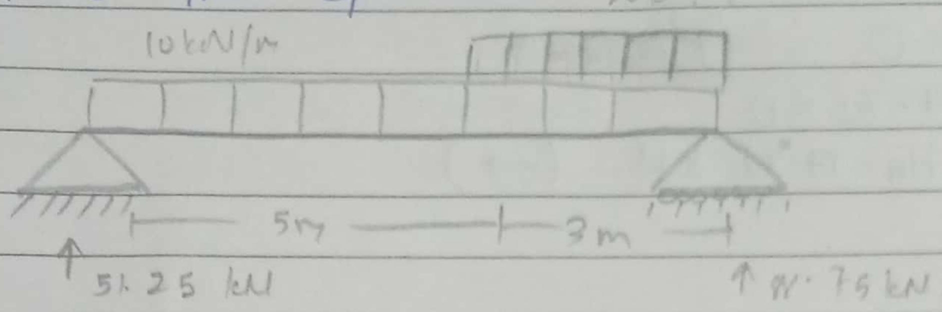
Putting V_B in eqn (i)

$V_A = 140 - 88.75$

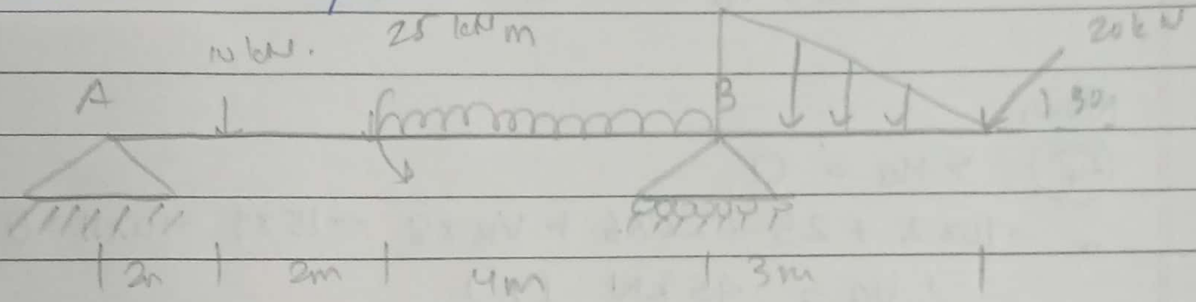
$\therefore V_A = 51.25\ kN\ (\uparrow)$

Date :
Page:

Hence, the final equilibrium



<Q.37>: Find the reaction at A and B so that system is in equilibrium.



Solⁿ:

The equivalent loads acting are

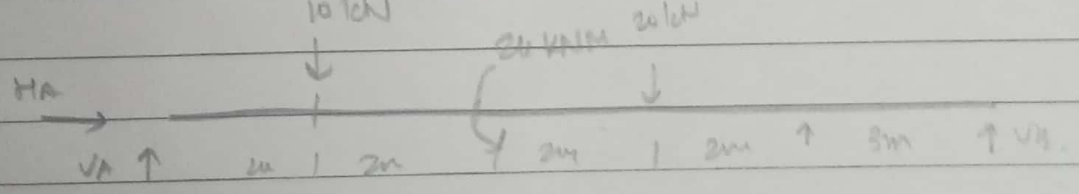
$20 \cos 30^{\circ} = 17.32$ kN $\qquad 20 \times \sin 30 = 10$ kN

The equivalent loads are

$w_1 =$ ~~15~~. $30/2 =$ 15 kN at 2m from E

$w_2 =$ $5 \times 4 = 20$ kN at 2m mark from B.

The free body diagram is.



At equilibrium, $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M_a = 0$.

So,

$\sum F_x = 0$ $\quad (\overrightarrow{+})$

$H_A - 17.32 = 0$

$\therefore H_A = 17.32$ kN. $(\rightarrow)$

Also,

$\sum F_y = 0.$

or $V_A - 10 - 20 + V_B - 15 - 10 = 0$

$\therefore V_A + V_B = 55$ kN $\quad$ — (i)

Now,

$(\circlearrowleft +)$ $\sum M_a = 0$

or $-10 \times 2 + 25 - 20 \times 6 + V_B \times 8 - 15 \times 9 - 10 \times 11 = 0$

$\therefore V_B = 45$ kN $(\uparrow)$

$V_A = 55 - 45 = 10$ kN $(\uparrow)$

Hence, the final equilibrium.