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Thus, $\delta = \min\left(1-\sqrt{1-\varepsilon}\ ,\ -1+\sqrt{1+\varepsilon}\right)$

which implies
$0 < |x - x_0| < \delta$ aties on $\left(\sqrt{1-\delta}\ ,\sqrt{1+\delta}\right)$

Thus, from the defn of limit,
we can say
$\lim_{x \to L} f(x) = L$ if $f(x) = \begin{cases} x^2 & x \neq 1 \\ 2 & x = 1 \end{cases}$ is proved.

One-Sided Limit

A function f has R.H.L. at x_0 if
$\lim_{x \to x_0^+} f(x) = L$,
for every $\varepsilon > 0$, there exists $\delta > 0$ such that
$0 < x < x_0 + \delta$ whenever $|f(x) - L| < \varepsilon$

A function f has L.H.L at x_0 if
$\lim_{x \to x_0^-} f(x) = L$
for every $\varepsilon > 0$, there exists $\delta > 0$ such that
$x_0 - \delta < x < 0$ whenever $|f(x) - L| < \varepsilon$

x) Theorem:
A function $f(x)$ has a limit L as x approaches to x_0 iff
$\lim_{x \to x_0^+} f(x) = \lim_{x \to x_0^-} f(x) = L$ i.e; $\lim_{x \to x_0} f(x) = L$

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Eg: $$\lim_{x \to -2^+} \frac{|x+2|}{x+2}(x+3)$$

$$= \lim_{x \to 2^+} \frac{\cancel{(x+2)}}{\cancel{(x+2)}} \times (x+3)$$

$$= 1$$

Eg: $$\lim_{x \to 1^-} \frac{\sqrt{2x} \cdot (x-1)}{|x-1|}$$

$$= \lim_{x \to 1^-} \frac{\sqrt{2x} \cdot \cancel{(x-1)}}{-\cancel{(x-1)}}$$

$$= -\sqrt{2}$$

Eg: Find LHL and RHL, for $f(x) = \begin{cases} 3-x & ; x < 2 \\ \frac{x}{2}+1 & , x > 2 \end{cases}$
and check if limit exists.

Soln:

For LHL,

$$\lim_{x \to 2^-} 3-x$$
$$= 3-2$$
$$= 1$$

For RHL,

$$\lim_{x \to 2^+} \frac{x}{2}+1$$
$$= 1+1 = 2$$

Since, $\lim_{x \to 2^-} f(x) \neq \lim_{x \to 2^+} f(x)$ or, LHL $\neq$ RHL,

So, for, $f(x)$ ~~no limit~~ limit doesn't exist

Limit of Logarithm and Exponential Function

i) $\lim_{x \to \infty} a^x = a^m$

ii) $\lim_{x \to \infty} a^x = \infty$

iii) $\lim_{x \to -\infty} a^x = 0$

iv) $\lim_{x \to 0} \frac{a^x - 1}{x} = \ln(a)$

v) $\lim_{x \to 0} \frac{a^{mx} - 1}{mx} = m \ln(a)$

vi) $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$

vii) $\lim_{x \to \infty} \log x = \infty$

viii) $\lim_{x \to 0} (1+x)^{\frac{1}{x}} = e$

ix) $\lim_{x \to 0} \log x = \infty$

x) $\lim_{x \to 0} \frac{\ln(1+x)}{x} = 1$

xi) $\lim_{x \to 0} \frac{\sin x}{x} = 1$

xii) $\lim_{x \to 0} \frac{\tan x}{x} = 1$

Limits at Infinity:

*) $\infty \times \infty$ is not an indeterminant form.

Eg: $\lim_{x \to \infty} (x^2 - x)$

$= \lim_{x \to \infty} x(x-1) = \infty$

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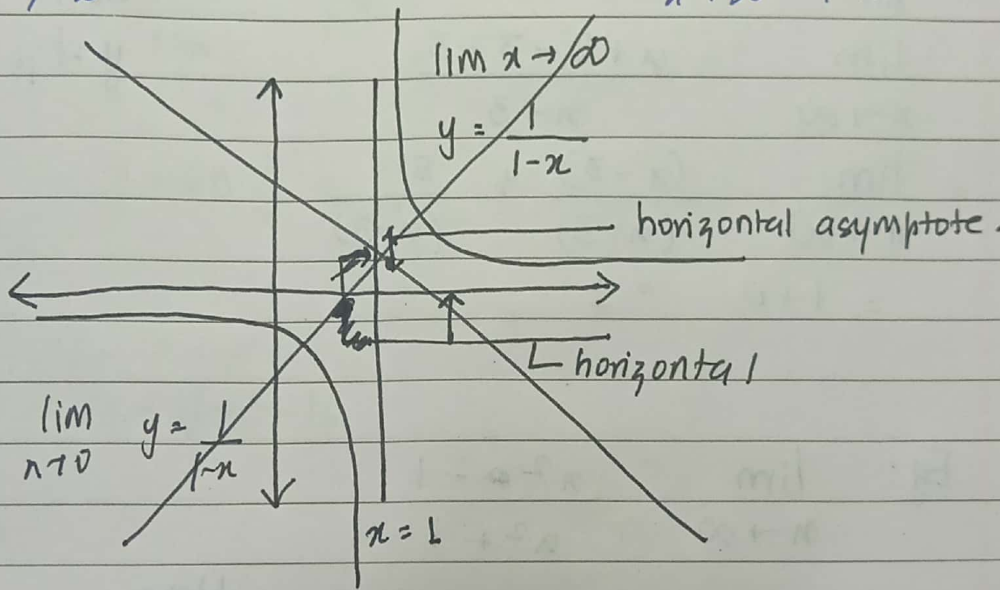
⊛Cases:

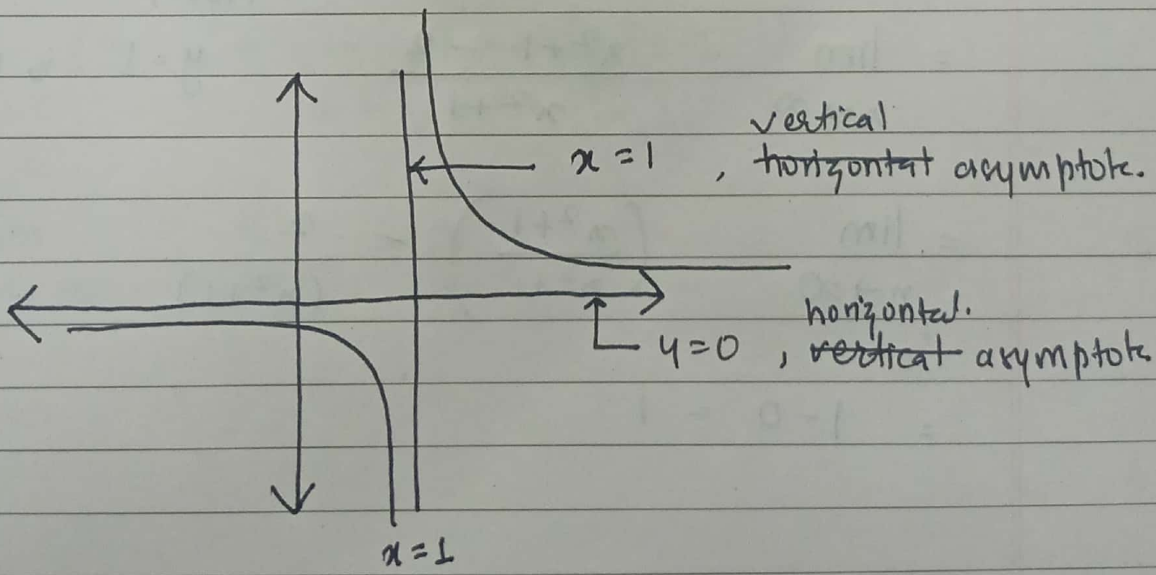
(i): $\lim_{x \to a} f(x) = \infty$ it gives vertical asymptote.

(ii) $\lim_{x \to \infty} f(x) = L$ it gives horizontal asymptote.

Now, for

$\lim_{x \to 1} \frac{1}{1-x} = \infty$ and $\lim_{x \to \infty} \frac{1}{1-x} = 0$





So,

if $\lim_{x \to \text{infinity}} y = \cancel{\text{in}}\text{finite}$, it gives $y = L$
ie, gives horizontal asymptote.

if $\lim_{x \to \text{finite}} y = \text{infinite}$, it gives $x = A$
ie, gives vertical asymptote.

Eg: $\lim_{x \to \infty} \left(\frac{x+2}{x-3}\right)$

Here, $y = 1$ is ~~vertical~~ horizontal asymptote.

Sol^n:

$\lim_{x \to \infty} \frac{x+2-3+3}{x-3}$

$= \lim_{x \to \infty} \frac{(x-3)}{(x-3)} + \frac{5}{(x-3)}$

$= 1 + 0 = 1$

Eg: $\lim_{x \to \infty} \frac{x^2 \cancel{x} - 1}{x^2+1}$

Here, $y = 1$ is ~~vert~~ horizontal asymptote.

$= \lim_{x \to \infty} \frac{x^2+1-2}{x^2+1}$

$= \lim_{x \to \infty} \left(\frac{x^2+1}{x^2+1}\right) - \frac{2}{(x^2+1)}$

$= 1 - 0 = 1$

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Eg: $\lim_{x\to\infty} \left(\frac{x+2}{x-3}\right) = 1$ ie, $y=1$ is horizontal asymptote.

$\lim_{x\to 3} \left(\frac{x+2}{x-3}\right) = \infty$ is $x=3$ is vertical asymptote.

Eg: $\lim_{x\to\infty} x \times \sin \frac{1}{x}$

Soln.

$\lim_{1/x\to 0} \frac{\sin 1/x}{1/x} = 1$

$\therefore \lim_{x\to\infty} x \times \sin \frac{1}{x} = 1$

Eg: $\lim_{x\to\infty} x - \sqrt{x^2-16}$

Soln:

$= \lim_{x\to\infty} \frac{(x - \sqrt{x^2-16})(x+\sqrt{x^2-16})}{(x+\sqrt{x^2-16})}$

$= \lim_{x\to\infty} \frac{16}{\infty} = 0$

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Continuity of a Function:

For a function $f(x)$ to be continuous, then at $x = c$

(i): $f(c)$ is exists.

(ii) LHL = RHL i.e, $\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x)$

(iii) $\lim_{x \to c} f(x) = f(c)$.

ie; $f(c \pm h) = f(c)$.

*) Constant functions are ~~constant~~ continuous.

Find $f(x) = \sec(y\sec^2 y - \tan^2 y - 1)$ at $y = 1$.

Soln:

$= \sec(y\sec^2 y - \tan^2 y - 1)$

$= \sec(y\sec^2 y - (1 + \tan^2 y))$

$= \sec(y\sec^2 y - \sec^2 y)$

$= \sec(\sec^2 y(y - 1))$

$= \sec 0 \quad = 1.$

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Eg: For what value of a and b is

$$f(x) = \begin{cases} -2 & , x \le -1 \\ ax - b & -1 < x < 1 \\ 3 & ; x \ge 1 \end{cases}$$ is continuous ??

Soln:

At $x = -1$,

$a \times (-1) - b = -2$ ——— ~~(i)~~

or, $-a - b = -2$ ——— (i)

At $x = 1$

$a \times 1 - b = 3$ ——— (ii)

Adding (i) and (ii),

$$\begin{aligned} \cancel{a} - b &= 3 \\ -\cancel{a} - b &= -2 \\ \hline -2b &= 1 \end{aligned}$$

$\therefore b = \frac{-1}{2}$

Subtracting (i) from (ii);

$$\begin{aligned} a - \cancel{b} &= 3 \\ -a - \cancel{b} &= -2 \\ (+) \quad (\cancel{+}) \quad & (+) \\ \hline 2a &= 5 \end{aligned}$$

$\therefore a = 5/2$

Continuous Extension of Point

Defn: If $f(c)$ is not defined, but $\lim_{x \to c} f(x) = L$ exists

and we can define a new function $f(x)$

i.e.,

$$f(x) = \begin{cases} f(x) \text{ if } x \text{ is in a domain} \\ L \text{ if } x = c \end{cases}$$

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Thus, F is said to continuous extension of f at $x = c$.

Eg: $g(x) = \begin{cases} \dfrac{x^2-16}{x^2-3x-4} & \text{if } x \neq 4 \\ g\ ?? & \text{at } x = 4 \end{cases}$

Now,
Continuous extension of point at $x = 4$,

$$g(4) = \frac{4^2-16}{4^2-3\times4-4} = \frac{0}{0} \text{ (indeterminant form)}$$

So,

$$g(x) = \frac{x^2-16}{x^2-3x-4}$$
$$= \frac{(x)^2-(4)^2}{x^2-4x+x-4}$$
$$= \frac{(x+4)\cancel{(x-4)}}{\cancel{(x-4)}(x+1)}$$

$$\therefore g(4) = \frac{8}{5}$$

Thus, the continuous ~~eq~~ extension of $g(x)$ at $x = 4$ is $8/5$.