Vector space :

V is said to be vector space if the following properties are satisfied.

Let $u_1, u_2, u_3, u_4 \in V$ then,

(i) $\vec{u_1} + \vec{u_2} = \vec{u_3} + \vec{u_4}$

(ii) $\vec{u_1} + (\vec{u_2} + \vec{u_3}) = (\vec{u_1} + \vec{u_2}) + \vec{u_3}$

(iii) $\vec{u_1} + \vec{0} = \vec{u_1} = \vec{0} + \vec{u_1}$

(iv) $\vec{u_1} + (-\vec{u_1}) = 0 = (-\vec{u_1}) + \vec{u_1}$

*) Scalar multiplication:

If $c\vec{u_1} \in V$

i) $c_1(\vec{u_1} + \vec{u_2}) = c_1\vec{u_1} + c_2\vec{u_2}$

ii) $(c_1 + c_2)\vec{u_1} = c_1\vec{u_1} + c_1\vec{u_2}$

iii) $(c_1 c_2)\vec{u_1} = c_1(c_2\vec{u_1})$

iv) $1 \cdot \vec{u_1} = \vec{u_1}$

Vector subspace

A non-empty subset 'S' of $\mathbb{R}^n$ is called a subspace if the following properties are satisfied.

(i): Zero vector is in 'S'

(ii): if $\vec{u}$ and $\vec{v}$ are in 'S', then $\vec{u} + \vec{v} \in S$.

(iii) If $\vec{u}$ is in 'S' and c be any sular then,

$c\vec{u} \in S$.

<Q>: Prove that $(4t, -2t, t)$ is a subspace of $\mathbb{R}^3$.

Soln:

Let $S = \begin{bmatrix} 4t \\ -2t \\ t \end{bmatrix}$

P_1: Putting $t = 0$,

$$S = \begin{bmatrix} 4 \times 0 \\ -2 \times 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

The ~~set~~ zero vector $\vec{0} \in S$.

P_2: Let $\vec{u_1} = \begin{bmatrix} 4t_1 \\ -2t_1 \\ t_1 \end{bmatrix}$ and $\vec{u_2} = \begin{bmatrix} 4t_2 \\ -2t_2 \\ t_2 \end{bmatrix}$

then,

$$\vec{u} + \vec{v} = \begin{bmatrix} 4t_1 \\ -2t_1 \\ t_1 \end{bmatrix} + \begin{bmatrix} 4t_2 \\ -2t_2 \\ t_2 \end{bmatrix}$$

$$\therefore \vec{u} + \vec{v} = \begin{bmatrix} 4(t_1+t_2) \\ -2(t_1+t_2) \\ 1(t_1+t_2) \end{bmatrix} \in S$$

$\therefore \vec{u} + \vec{v} \in S$

P_3: Let $\vec{u_1} = \begin{bmatrix} 4t_1 \\ -2t_1 \\ t_1 \end{bmatrix}$ and $\vec{u_2} = \begin{bmatrix} 4t_2 \\ -2t_2 \\ t_2 \end{bmatrix}$

Let c be any scalar.

$$c\vec{u_1} = \begin{bmatrix} 4(ct_1) \\ -2(ct_1) \\ 1(ct_1) \end{bmatrix} \in S$$

Since all three conditions are satisfied, $(4t, -2t, t)$ is a subspace of $\mathbb{R}^3$.

(*) If $\{\vec{v_1}, \vec{v_2}, \dots, \vec{v_p}\}$ be a vector space V, then span $\{\vec{v_1}, \vec{v_2}, \dots, \vec{v_p}\}$ is subspace of V.

<Q>: Show that $S = \{u = (x, y) \in \mathbb{R}^2 : x + 2y = 0$ ~~$x+2y=1$~~

is subspace of $\mathbb{R}^2$.

Soln:

Given eqn,

$x + 2y = 0$

or $x = -2y$

Here, y is a free variable.

Let $y = t$.

Then,

$x = -2t$

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Let $\vec{u} = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2t \\ t \end{bmatrix}$

P_1 : Putting $t=0$,

$\vec{u} = \begin{bmatrix} -2\times 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

P_2 : Let $\vec{u_1} = \begin{bmatrix} -2t_1 \\ t_1 \end{bmatrix}$ and $\vec{u_2} = \begin{bmatrix} -2t_2 \\ t_2 \end{bmatrix}$

~~So~~ $\vec{u_1}, \vec{u_2} \in S$.

So,

$\vec{u_1} + \vec{u_2} = \begin{bmatrix} -2t_1 \\ t_1 \end{bmatrix} + \begin{bmatrix} -2t_2 \\ t_2 \end{bmatrix}$

$\vec{u_1} + \vec{u_2} = \begin{bmatrix} -2(t_1+t_2) \\ (t_1+t_2) \end{bmatrix} \in S$

P_3 : Let c be an scalars

$c\vec{u} = \begin{bmatrix} -2(ct) \\ (ct) \end{bmatrix} \in S$

Since all three conditions are satisfied, S is a subspace of $\mathbb{R}^2$.

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<Q>: Show that $S = \{\vec{u} = (x, y, z) \in \mathbb{R}^3,\ x - 2y + 3z = 0\}$ is a subspace of $\mathbb{R}^3$.

Soln.

Given eqn,

$x - 2y + 3z = 0$

$\therefore x = 2y - 3z$.

Here, y and z are free variables.

Let $y = s$ and $z = t$.

So,

$x = 2s - 3t$.

We know,

$\vec{u} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2s-3t \\ s \\ t \end{bmatrix}$

$= s\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$

Let $\vec{u_1} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$ and $\vec{u_2} = \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$

So, $\vec{u} = s\vec{u_1} + t\vec{u_2} \in S$ —— P_1

P_2 : Putting $s=0$ and $t=0$,

$\vec{u} = \begin{bmatrix} 2\times 0 - 3\times 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

P_3: Let c be any scalar,

$$c\vec{u} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} c(2s-3t) \\ cs \\ ct \end{bmatrix} \in S$$

$$\cancel{= s\begin{bmatrix} 2c \\ c \\ 0 \end{bmatrix} + t\begin{bmatrix} -3c \\ 0 \\ c \end{bmatrix}}$$

Since, all three conditions are satisfied,
thus, S is a subspace of $\mathbb{R}^3$.