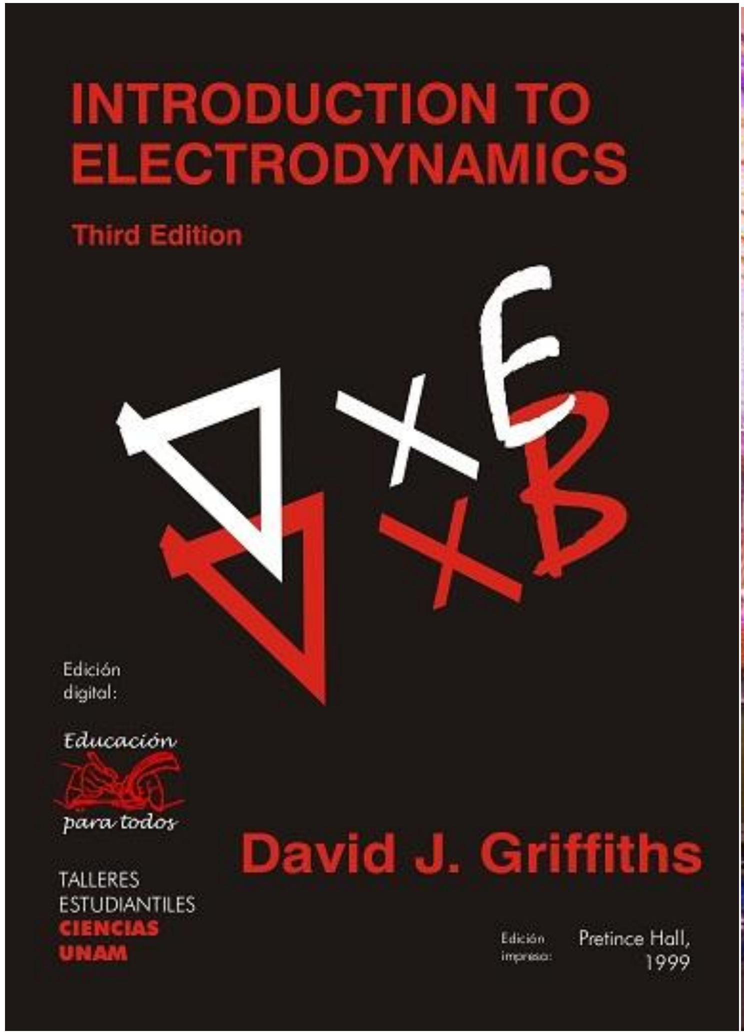
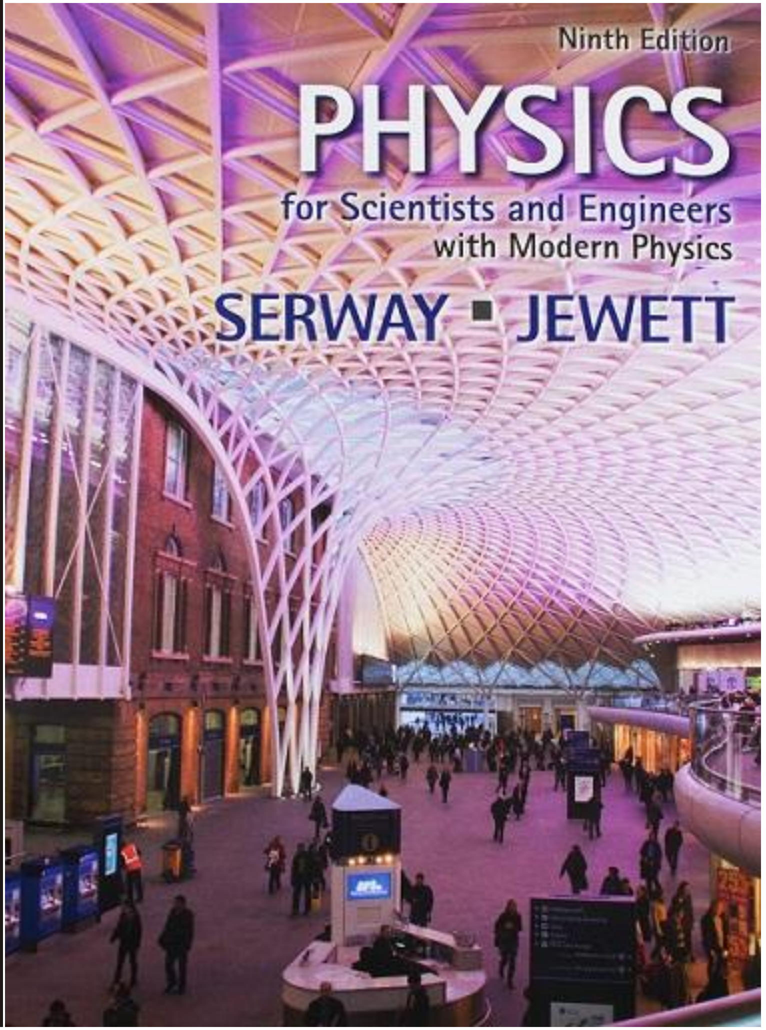
PHYSICS

General Physics II (PHYS 102)





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Course Outline

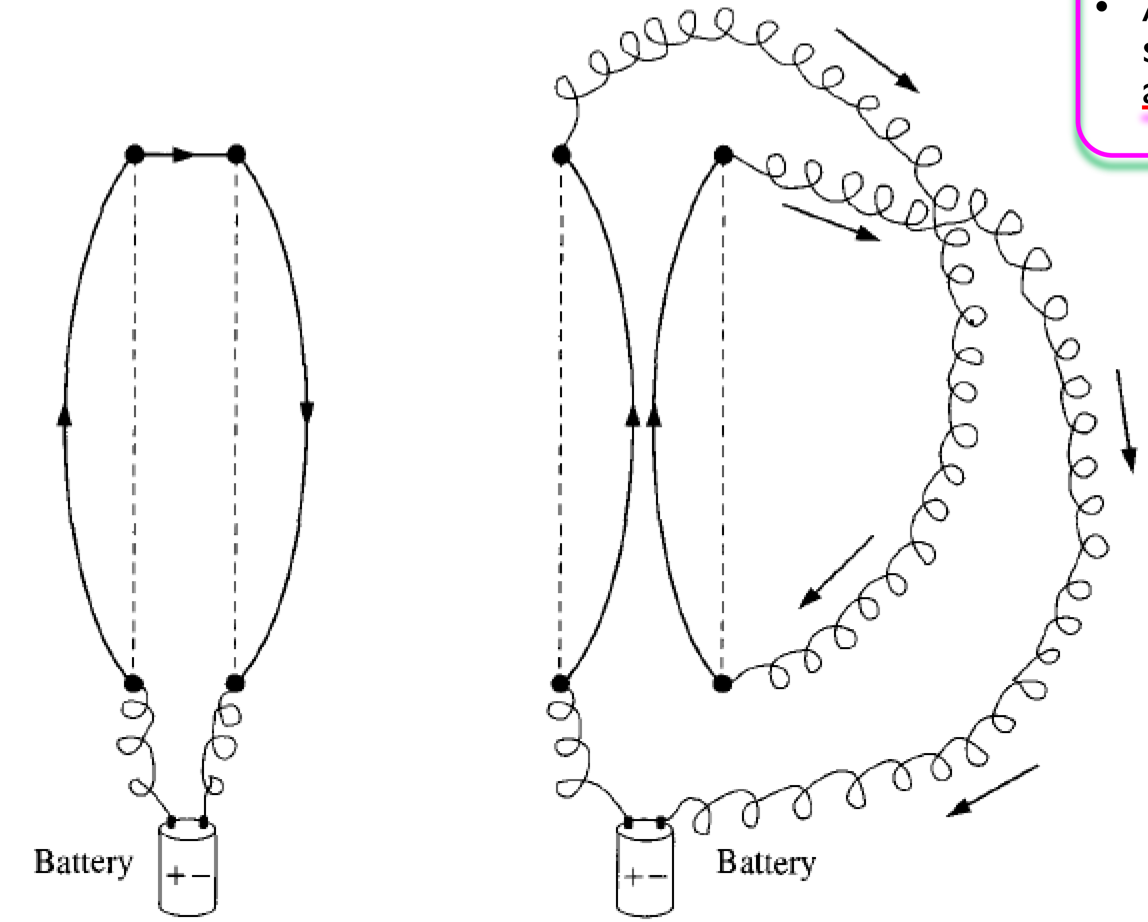
- Magnetic Fields, Magnetic Force, Lorentz Force, Magnetic Flux
- Cyclotron Motion, Cycloid Motion
- Magnetic Force on a System of Moving Charges
- Continuity Equation
- Sample Problems

Magnetic Fields

Magnetic Fields:

ELECTRIC FIELDS IN MATTER

- A stationary charge produces only an electric field $\vec{E}$ in the space around it, a moving charge generates, in addition, a magnetic field $\vec{B}$.

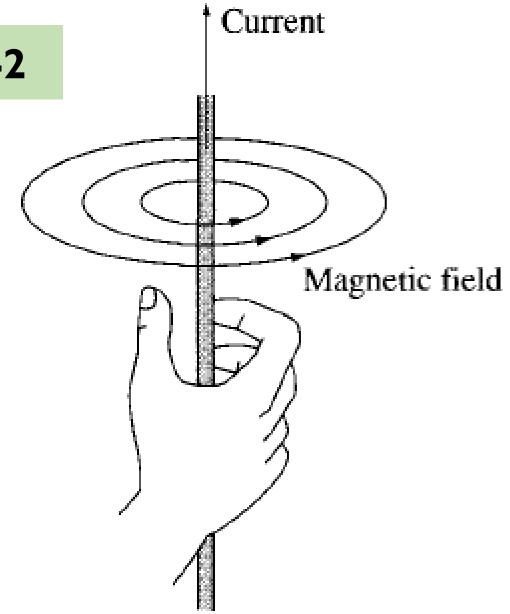


(a) Currents in opposite directions repel

(b) Currents in same directions attract

Figure M-1

Figure M-2



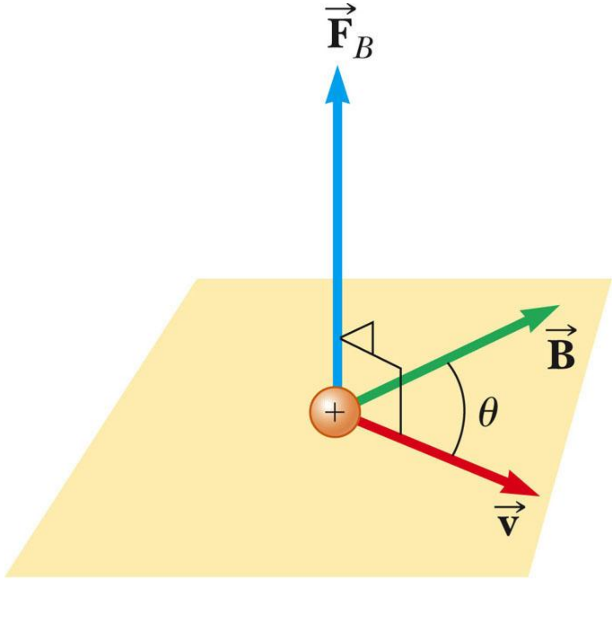
If you grab the wire with your right hand-thumb in the direction of the current - your fingers curl around in the direction of the magnetic field

Magnetic Force

Magnetic Force:

- The **magnetic force** in a charge Q, moving with velocity $\vec{v}$ in magnetic field $\vec{B}$ is given by

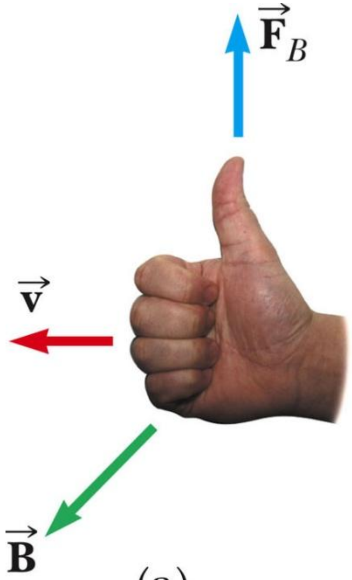
$$\vec{F}_B = Q(\vec{v} \times \vec{B})$$



(a)

Direction: Right-Hand Rule

- The fingers point in the direction of $\vec{V}$
- $\vec{B}$ comes out of your palm
  - Curl your fingers in the direction of $\vec{B}$
- The thumb points in the direction of $\vec{v} \times \vec{B}$ which is the direction of $\vec{F}_B$



(a)

Magnetic Force

Magnetic Force: $\vec{F}_B = Q(\vec{v} \times \vec{B})$

- The magnitude of the magnetic force on a charged particle moving in a magnetic field is

$$F_B = Q\,v\,B\sin\theta$$

where θ is the small angle between $\vec{v}$ and $\vec{B}$.

- $(F_B)_{max} = Q\,v\,B$ when $\vec{v}$ is perpependicular to $\vec{B}\left(\theta = 90^0\right)$
- $F_B = 0$ when $\vec{v}$ is parallel or antiparallel to $\vec{B}$ $\left(\theta = 0 \text{ or } 180^0\right)$

Magnetic Forces do no work.

$$W_{mag} = \int dW_{mag} = \int \vec{F}_{mag} \cdot d\vec{l}$$
$$= \int Q(\vec{v} \times \vec{B}) \cdot \vec{v}dt$$
$$= 0$$

- ❑ The magnetic force vector is perpendicular to the magnetic field.
- ❑ The magnetic force acts on a charged particle only when the particle is in motion.
- ❑ **The magnetic force associated with a steady magnetic field does no work when a particle is displaced because the force is perpendicular to the displacement of its point of application.**

From the last statement and on the basis of the work–kinetic energy theorem, we conclude that the kinetic energy of a charged particle moving through a magnetic field cannot be altered by the magnetic field alone. The field can alter the direction of the velocity vector, but it cannot change the speed or kinetic energy of the Particle.

Magnetic Field

ELECTRIC FIELDS IN MATTER

Magnetic Field:

$$B = \frac{F_B}{Q\,v\,\sin\theta}$$

- The SI unit of magnetic field is the newton per coulomb-meter per second, which is called the **tesla (T):**

$$\boxed{1\,\mathrm{T} = 1\frac{\mathrm{N}}{\mathrm{C \cdot m/s}} = 1\frac{\mathrm{N}}{\mathrm{A \cdot m}}}$$

- A non-SI magnetic-field unit in common use, called the **gauss (G)**.

$$\boxed{1\,\mathrm{T} = 10^4 G}$$

Lorentz Force:

- In the presence of both electric and magnetic fields, the **net force** in a charge Q, moving with velocity $\vec{v}$ is given by

$$\vec{F}_L = \left(Q\vec{E}\right) + Q\left(\vec{v} \times \vec{B}\right) = Q\left[\vec{E} + \vec{v} \times \vec{B}\right]$$

Magnetic Flux:

- Magnetic flux is a measurement of the total magnetic field which passes through a given area.
- Magnetic Flux through a Surface:

$$\phi_B = \int_s \vec{B} \bullet d\vec{a}$$

- The unit of magnetic flux is weber [Wb].

Motion of Charged Particle in a Uniform Magnetic Field

Cyclotron Motion

When a positively charged particle enters a magnetic field in a direction perpendicular to the field:

- The particle moves in a circle because the magnetic force $\vec{F}_B$ is perpendicular $\vec{v}$ and $\vec{B}$ and has a constant magnitude $q\,v\,B$.
- Newton's second law for the particle:

$$\sum F = F_B = ma$$

$$\text{or,}\quad q\,v\,B = \frac{mv^2}{r}$$

$$\text{or,}\quad qB = \frac{mv}{r} \longrightarrow \boxed{r = \frac{mv}{qB}}$$

Radius of Circular Path

$$\therefore \quad p = q\,B\,r \quad \ldots\ldots\ldots\ (M.1)$$

where m is the particle's mass

and $p = mv$ is its momentum.

Equation M.1 is known as the **cyclotron formula** because it describes the motion of a particle in a cyclotron – the first of the modern particle accelerators.
It also suggests a simple experimental technique for finding the momentum of a particle.

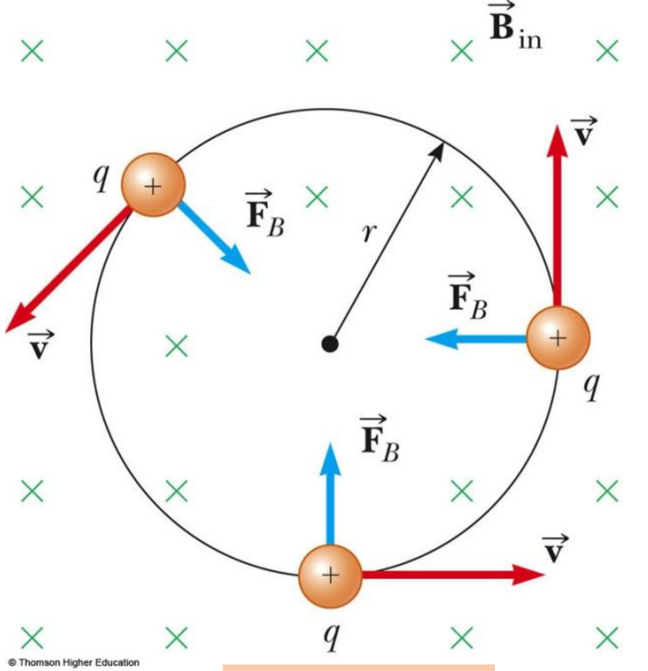


Figure B-1

Cyclotron Frequency

$$\omega = \frac{v}{r} = \frac{qB}{m}$$

Time Period

$$T = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$$

The angular speed of the particle and the period of the circular motion do not depend on the speed of the particle or on the radius of the orbit

Motion of Charged Particle in a Uniform Magnetic Field

Helical Path

When a positively charged particle enters a uniform magnetic field obliquely:

- If the velocity of a charged particle has a component parallel to the (uniform) magnetic field, the particle will move in a helical path about the direction of the field vector.

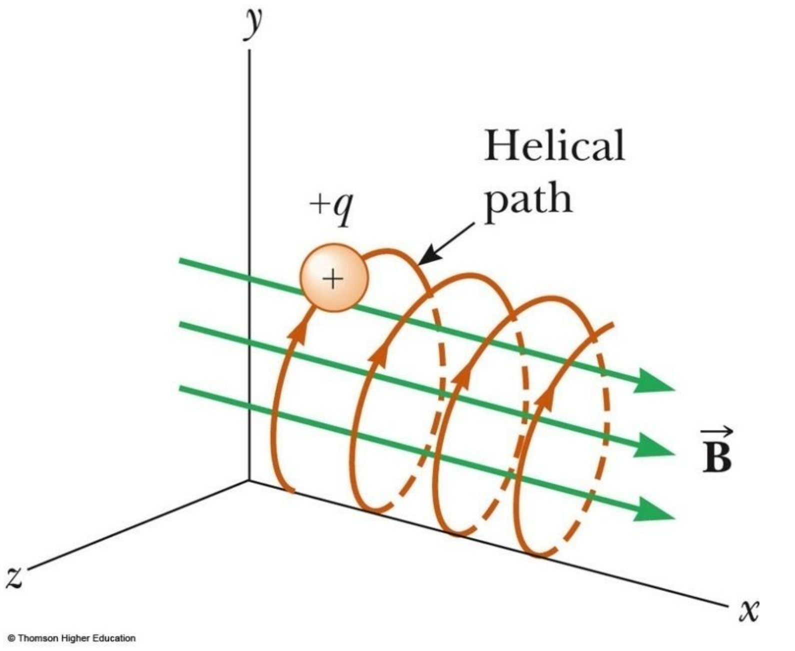


Figure B-2

The velocity component perpendicular to the magnetic field creates circular motion, whereas the component of the velocity parallel to the field moves the particle along a straight line. The resulting motion is helical.

Motion of Charged Particle in a Uniform Electric Field at right angles to the Magnetic Field

Cycloid Motion

Suppose that $\vec{B}$ points in the x-direction, and $\vec{E}$ in the z-direction, as shown in Figure B-3. A particle at rest is released from the origin; what path will if follow?

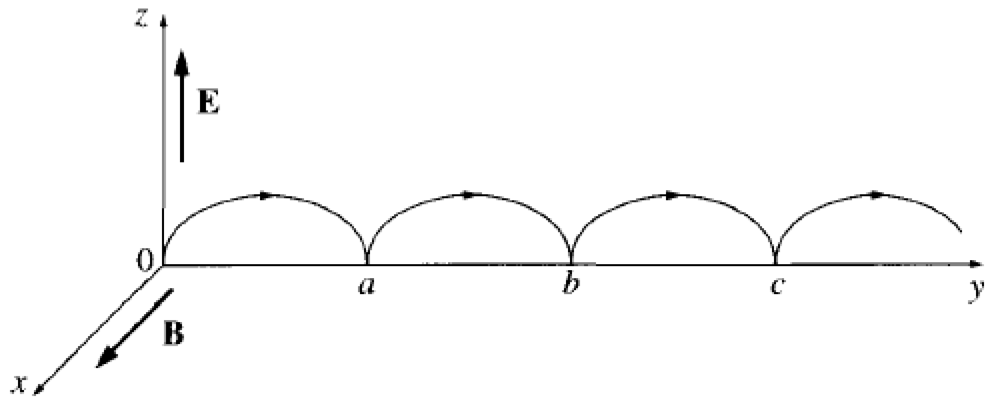


Figure B-3

Initially, the particle is at rest, so the magnetic force is zero, and the electric field accelerates the charge in the z-direction.

As it picks up speed, a magnetic force develops which, according to Eq. $\vec{F}_B = Q(\vec{v}\times\vec{B})$, pulls the charge around to the right. The faster it goes, the stronger F_B becomes; eventually, it curves the particle back around towards the y axis.

At this point the charge is moving against the electrical force. so it begins to slow down-the magnetic force then decreases, and the electrical force takes over, bringing the charge to rest at point a, in Figure B-3. There the entire process commences anew, carrying the particle over to point b, and so on.

So it will follow **cycloid** path.

Magnetic Force on a System of Moving Charges

Magnetic Force on a System of Moving Charges

- Consider a number of point charges $q_1, q_2, \ldots, q_n$ are moving with velocities $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n$ respectively in magnetic field.

The net magnetic force is

$$\vec{F}_m = q_1\left(\vec{v}_1 \times \vec{B}\right) + q_2\left(\vec{v}_2 \times \vec{B}\right) + \ldots + q_n\left(\vec{v}_n \times \vec{B}\right)$$

$$= \sum_{n=i}^{n} q_i\left(\vec{v}_i \times \vec{B}\right) \qquad \ldots\ldots\ldots\ldots \quad (1)$$

- For the continuous system of moving charges , Eq. (1) becomes

$$\vec{F}_m = \int dq\left(\vec{v} \times \vec{B}\right)$$

where $\vec{v}$ is the velocity of elemental charge dq in magnetic filed $\vec{B}$

Line Current

The magnetic force on the line current

$$\vec{F}_{mag} = \int \lambda dl(\vec{v} \times \vec{B}) = \int dl($$

$$= \int I(d\vec{l} \times \vec{B})$$

$$\boxed{\therefore \vec{F}_{mag} = I\int (d\vec{l} \times \vec{B})}$$

Figure B.F - I

$$\because \quad dq = \lambda dl$$

$$\Rightarrow \frac{dq}{dt} = \frac{\lambda dl}{dt}$$

$$\therefore I = \lambda v$$

$$\left[\begin{array}{l}\text{A line charge } \lambda \text{ travelling down a wire} \\ \text{at speed v constitutes a current } \boxed{I = \lambda v}\end{array}\right]$$

Magnetic Force on a System of Moving Charges

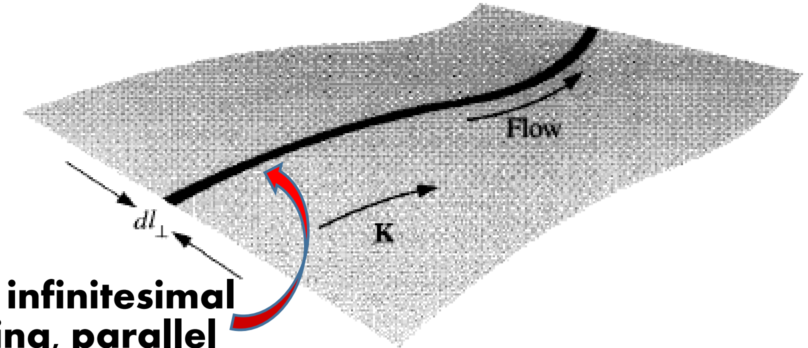
Surface Current

The magnetic force on the surface current

$$\vec{F}_{mag} = \int dq(\vec{v}\times\vec{B}) = \int \sigma da(\vec{v}\times\vec{B})$$

$$= \int (\sigma\,\vec{v}\times\vec{B})\, da$$

$$\boxed{\therefore \vec{F}_{mag} = \int (\vec{K}\times\vec{B})\, da}$$



a ribbon of infinitesimal width running, parallel to the flow

$$\because \quad dq = \sigma da$$

$$\Rightarrow \frac{dq}{dt} = \sigma dl_\perp \frac{dl_\parallel}{dt}$$

$$\Rightarrow \frac{dI}{dl_\perp} = \sigma \frac{dl_\parallel}{dt}$$

$$\therefore \quad K = \sigma\vec{v}$$

Surface Current Density

$$\vec{K} \equiv \frac{d\vec{I}}{dl_\perp}$$

Current per unit width-perpendicular-to-flow

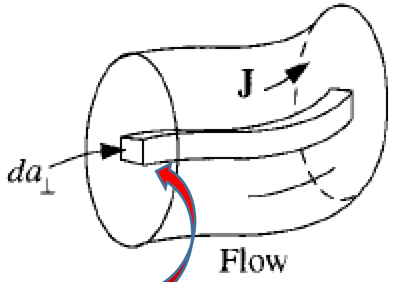
Volume Current

The magnetic force on a volume current

$$\vec{F}_{mag} = \int dq(\vec{v}\times\vec{B}) = \int \rho d\tau(\vec{v}\times\vec{B})$$

$$= \int (\rho\,\vec{v}\times\vec{B})\, d\tau$$

$$\boxed{\therefore \vec{F}_{mag} = \int (\vec{J}\times\vec{B})\, d\tau}$$



a tube of infinitesimal cross section, running parallel to the flow

Volume Current Density

$$\vec{J} \equiv \frac{d\vec{I}}{da_\perp}$$

Current per unit area-perpendicular-to-flow

$$\because \quad dq = \rho d\tau$$

$$\Rightarrow \frac{dq}{dt} = \rho da_\perp \frac{dl_\parallel}{dt}$$

$$\Rightarrow \frac{dI}{da_\perp} = \rho \frac{dl_\parallel}{dt}$$

$$\therefore \quad \mathrm{J} = \rho\vec{v}$$

Continuity Equation

Continuity Equation

- When the flow of charge is distributed throughout a three-dimensional region, we describe it by the volume current density, $\vec{J}$.

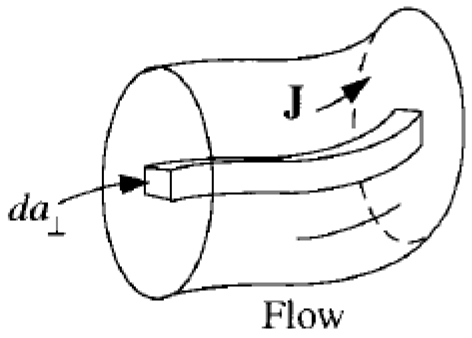


Figure C.E-1

- Consider a "tube" of infinitesimal cross section $da_\perp$, running to the flow (Figure C.E-1). If the current in this tube is dI , the volume current density is

$$\vec{J} \equiv \frac{d\vec{I}}{da_\perp} \quad \text{.........} \quad (1)$$

According to Eq. (1), the current crossing a surface S can be written as

$$\boxed{I = \int_S J \, da_\perp = \int_S \vec{J} \cdot d\vec{a}} \quad \text{.........} \quad (2)$$

In particular, the total charge per unit time leaving a volume V is

$$\boxed{I = \oint_s \vec{J} \cdot d\vec{a} = \int_V \left(\nabla \cdot \vec{J}\right) d\tau} \quad \text{........} \quad (3)$$

Because charge is conserved, whatever flows out through the surface must come at the expense of that remaining inside:

$$\boxed{\int_V \left(\nabla \cdot \vec{J}\right) d\tau = -\frac{d}{dt}\int_V \rho \, d\tau = -\int_V \left(\frac{\partial \rho}{\partial t}\right) d\tau} \quad \text{........}(4)$$

The minus sign reflects the fact that an outward flow decreases the charge left in V

Since this applies to any volume, we conclude that

$$\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

Continuity Equation

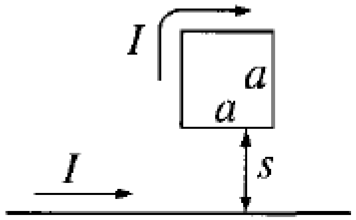
This is the precise mathematical statement of local charge conservation.

Questions

Sample Problems

- **A square loop is placed near an infinite straight wire as shown in Figure Q. Both the wires carry same current along the direction as indicated in figure.**

  The square loop **tends to move away from the straight wire.**



- **A charge of 3 C is moving with velocity** $\vec{v} = (4\hat{i}+3\hat{j})\ \mathrm{m\,s^{-1}}$ **in a magnetic field** $\vec{B} = (4\hat{i}+3\hat{j})\ \mathrm{Wb\,m^{-2}}$**. The force acting on the test charge is** **Zero** **.**

- **A proton is moving in a circular orbit of radius 14 cm in a uniform 0.35-T magnetic field perpendicular to the velocity of the proton. Find the speed of the proton.**

  What if an electron, rather than a proton, moves in a direction perpendicular to the same magnetic field with this same speed? Will the radius of its orbit be different?

  Hint:
  - $$v_p = \frac{qBr}{m_p} = 4.7\times10^{6}\ \mathrm{m/s}$$
  - **Radius of orbit of electron will be smaller**

- **A charge particle is circling in a magnetic field with cyclotron frequency** $1.5\times10^{8}\ \mathrm{rad/s}$ **. If speed of charge particle is doubled, the new cyclotron frequency is** $1.5\times10^{8}\ \mathrm{rad/s}$ **.**

Questions

Sample Problem

1. In 1897 J. J. Thomson "discovered" the electron by measuring the charge-to-mass ratio of "cathode rays (actually, streams of electrons, with charge *q* and mass *m)* as follows:

(a) First he passed the beam through uniform crossed electric and magnetic fields (mutually perpendicular, and both of them perpendicular to the beam), and adjusted the electric field until he got zero deflection. What, then, was the speed of the particles (in terms of *E* and *B)?*

(b) Then he turned off the electric field, and measured the radius of curvature, *R*, of the beam as deflected by the magnetic field alone. In terms of *E*, *B*, and *R*, what is the charge-to-mass ratio *(q / m)* of the particles?

Hint:

- $qE = q\,\mathrm{v}\,B \Rightarrow \boxed{\mathrm{v} = \dfrac{E}{B}}$

- $m\mathrm{v} = q\,B\,\mathrm{R} \Rightarrow \dfrac{q}{m} = \dfrac{\mathrm{v}}{BR} \Rightarrow \boxed{\dfrac{q}{m} = \dfrac{E}{B^2 R}}$

2. You set out to reproduce Thomson's e/m experiment with an accelerating potential of 150 V and a deflecting electric field of magnitude 6.0×10^6 N/C .

(a) At what fraction of the speed of light do the electrons move?

(b) What magnetic-field magnitude will yield zero beam deflection?

Hint:

- $\dfrac{1}{2}\mathrm{m}\,\mathrm{v}^2 = eV \Rightarrow \mathrm{v} = \sqrt{2\left(\dfrac{e}{m}\right)V} = 0.024c$

- $B = \dfrac{E}{\mathrm{v}} = 0.83\ \mathrm{T}$

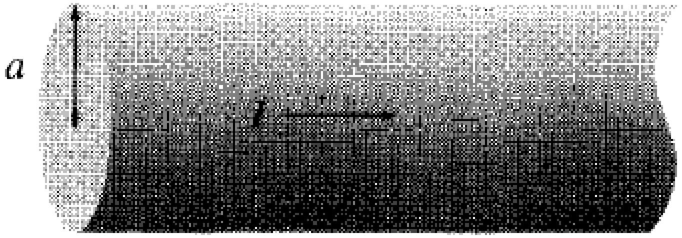
Questions

Sample Problem

1. A current I is uniformly distributed over a wire of circular cross section, with radius a (Figure Q.B-2).

(a) Find the volume current density. Suppose the current density in the wire is proportional to the distance from the axis, J = k s (for some constant k).

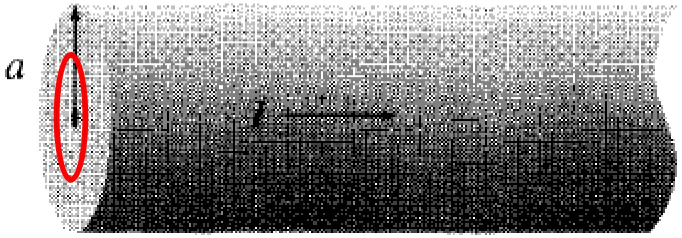
(b) Find the total current in the wire.

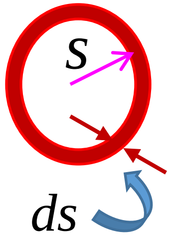


Hint:

- $J = \dfrac{I}{\pi a^2}$

- $I = \int_S J \, da_{\perp} = \int_0^a (ks)(2\pi s ds) = 2\pi k \int_0^a s^2 ds = \dfrac{2\pi k a^3}{3}$





Text Books & References

1. **David J. Griffith**, Introduction to Electrodynamics
2. **R.A. Serway and J.W. Jewett**, Physics for Scientist and Engineers with Modern Physics
3. **Halliday and Resnick**, Fundamental of Physics
4. **D. Halliday, R. Resnick, and K. Krane** , Physics, Volume 2, Fourth Edition


Thank you