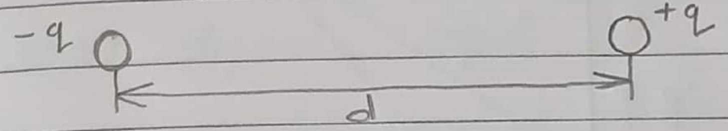
# CHAPTER : 3 : ELECTRIC FIELD IN MATTER

## # Electric dipole:

A pair of equal and opposite charges separated by a small distance.

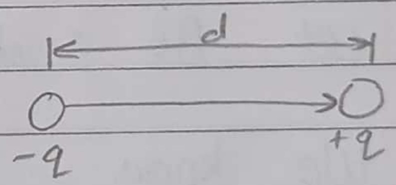



## # Electric Dipole Moment

The product of magnitude of either charge and the vector distance separating the two charges.

Mathematically,

$$\vec{p} = q\vec{d}$$



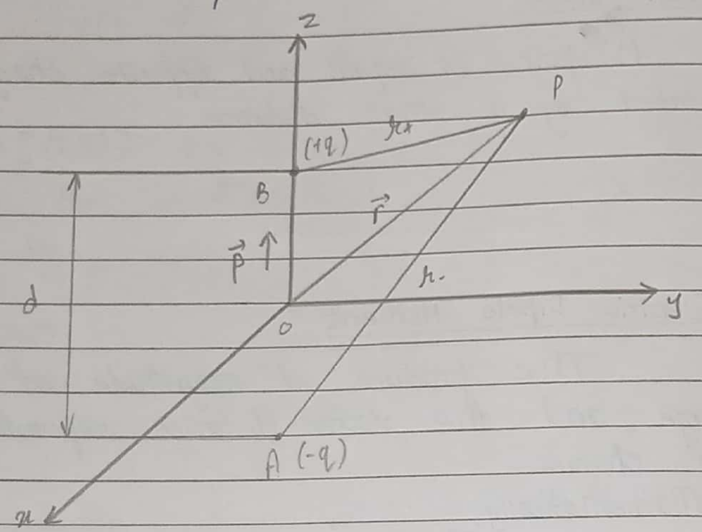

The direction of electric dipole moment vector is along the line joining two charges pointing from negative charge to positive charge.

It is vector quantity.

The SI unit is Coulomb meter (Cm).

# Electric Potential of Electric Dipole

Consider an electric dipole lying along the z-axis with its midpoint at the origin of the coordinate system.




Let $AB = d$. Then, $AO = OB = d/2$

We know,
Electric dipole moment $(\vec{p}) = q\vec{d}$

Now,
Electric potential of the dipole at point P is,

$$V_{dip} = \frac{1}{4\pi\varepsilon_0}\left(\frac{+q}{r_+}\right) + \frac{1}{4\pi\varepsilon_0}\left(\frac{-q}{r_-}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_+} - \frac{1}{r_-}\right) \quad \text{—(i)}$$

From the law of cosines,

$$r_+ = \sqrt{r^2 + \frac{d^2}{4} - 2 \cdot r \cdot \frac{d}{2}\cos\theta}$$

$$= \sqrt{r^2\left(1 + \frac{d^2}{4r^2} - \frac{d\cos\theta}{r}\right)}$$

For short dipoles, $r \gg d$.

$$r_+ \approx r\sqrt{1 - \frac{d\cos\theta}{r}}$$

or, $$\frac{1}{r_+} = \frac{1}{r}\left(1 - \frac{d\cos\theta}{r}\right)^{-1/2}$$

$$\therefore \frac{1}{r_+} = \frac{1}{r}\left(1 + \frac{d\cos\theta}{2r}\right)$$

Similarly, $$\frac{1}{r_-} = \frac{1}{r}\left(1 - \frac{d\cos\theta}{2r}\right)$$

Substituting value of $1/r_+$ and $1/r_-$ in eq$^n$(i),

$$V_{dip} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}\left(1 + \frac{d\cos\theta}{2r}\right) - \frac{1}{r}\left(1 - \frac{d\cos\theta}{2r}\right)\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{2d\cos\theta}{2r^2}\right]$$

$$\therefore V_{dip} = \frac{1}{4\pi\varepsilon_0}\frac{qd\cos\theta}{r^2}$$

Sol

$$V_{dip} = \frac{1}{4\pi\varepsilon_0} \frac{p\cos\theta}{r^2} \quad \text{— (ii)}$$

or

$$V_{dip} = \frac{1}{4\pi\varepsilon_0} \frac{\vec{p}\cdot\vec{r}}{r^3} \quad \text{— (iii)}$$

(x) Electric field of a Dipole:

$$\vec{E}_{dip} = -\nabla V_{dip}$$

$$= -\nabla\left[\frac{1}{4\pi\varepsilon_0}\left(\frac{\vec{p}\cdot\vec{r}}{r^3}\right)\right]$$

$$= \frac{-1}{4\pi\varepsilon_0} \nabla\left(\frac{\vec{p}\cdot\vec{r}}{r^3}\right)$$

$$= \frac{-1}{4\pi\varepsilon_0}\left[(\vec{p}\cdot\vec{r})\left(\nabla\left(\frac{1}{r^3}\right)\right) + \left(\frac{1}{r^3}\right)\left(\nabla(\vec{p}\cdot\vec{r})\right)\right]$$

$$= \frac{-1}{4\pi\varepsilon_0}\left[\frac{\vec{p}}{r^3} + \{(-3)r^{-3-2}\,\vec{r}\}(\vec{p}\cdot\vec{r})\right]$$

$$= \frac{-1}{4\pi\varepsilon_0}\left[\frac{\vec{p}}{r^3} - \frac{3\vec{r}\,(\vec{p}\cdot\vec{r})}{r^5}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{3(\vec{p}\cdot\hat{r})\hat{r}}{r^3} - \frac{\vec{p}}{r^3}\right]$$

$$= \frac{1}{4\pi\varepsilon_0 r^3}\left[3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}\right]$$

$$= \frac{1}{4\pi\varepsilon_0 r^3}\left(3p\cos\theta\,\hat{r} - p\hat{k}\right)$$

$$\therefore \vec{E}_{dip} = \frac{p}{4\pi\varepsilon_0 r^3}\left(3\cos\theta\,\hat{r} - \hat{k}\right)$$

Now,

$$E_{dip} = \sqrt{\vec{E}_{dip}\cdot\vec{E}_{dip}}$$

$$= \frac{p}{4\pi\varepsilon_0 r^3}\sqrt{(3\cos\theta\,\hat{r} - \hat{k})(3\cos\theta\,\hat{r} - \hat{k})}$$

$$= \frac{p}{4\pi\varepsilon_0 r^3}\sqrt{9\cos^2\theta - 3\cos^2\theta - 3\cos^2\theta + 1}$$

$$\boxed{\therefore E_{dip} = \frac{p}{4\pi\varepsilon_0 r^3}\sqrt{3\cos^2\theta + 1}}$$

x) Cases:

i) Case I :. At point on the axis of dipole. $\theta = 0$.

$$V_{dip} = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}, \quad E_{dip} = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}$$

(ii) Case II :

At a point on the perpendicular bisector of dipole,

$$V_{dip} = 0 \quad , \quad E_{dip} = \frac{1}{4\pi\varepsilon_0} \frac{p}{r^3}$$

(x) Electric ~~Dipole~~ field of a Dipole

In spherical polar coordinate system,

$$\nabla = \hat{r} \frac{\partial}{\partial r} + \hat{\theta} \frac{1}{r} \frac{\partial}{\partial \theta} + \hat{\phi} \frac{1}{r\sin\theta} \frac{\partial}{\partial \phi}$$

Now,

$$\vec{E}_{dip} = -\nabla V_{dip}$$

$$= -\nabla \left( \frac{1}{4\pi\varepsilon_0} \left( \frac{p\cos\theta}{r^2} \right) \right)$$

$$= -\frac{1}{4\pi\varepsilon_0} \left[ \nabla \left( \frac{p\cos\theta}{r^2} \right) \right]$$

$$= -\frac{\partial}{\partial r} \left( \frac{1}{4\pi\varepsilon_0} \frac{p\cos\theta}{r^2} \right) \hat{r} - \frac{1}{r} \frac{\partial}{\partial \theta} \left( \frac{1}{4\pi\varepsilon_0} \frac{p\cos\theta}{r^2} \right) \hat{\theta}$$

$$- \frac{1}{r\sin\theta} \frac{\partial}{\partial \phi} \left( \frac{1}{4\pi\varepsilon_0} \frac{p\cos\theta}{r^2} \right) \hat{\phi}$$

Now $\phi = 0$.

so,

$$\vec{E}_{dip} = -\frac{\partial}{\partial r} \left( \frac{1}{4\pi\varepsilon_0} \frac{p\cos\theta}{r^2} \right) \hat{r} - \frac{1}{r} \frac{\partial}{\partial \theta} \left( \frac{1}{4\pi\varepsilon_0} \frac{p\cos\theta}{r^2} \right) \hat{\theta}$$

$$= -\frac{p\cos\theta}{4\pi\varepsilon_0} \left[ \frac{\partial}{\partial r} (r^{-2}) \right] \hat{r} - \frac{p}{4\pi\varepsilon_0 r^3} \left[ \frac{\partial \cos\theta}{\partial \theta} \right] \hat{\theta}$$

$$= -\frac{p\cos\theta}{4\pi\varepsilon_0} \left( -\frac{2}{r^3} \right) \hat{r} - \frac{p}{4\pi\varepsilon_0 r^3} (-\sin\theta) \hat{\theta}$$

$$\therefore \vec{E}_{dip} = \frac{p}{4\pi\varepsilon_0 r^3} \left( 2\cos\theta\, \hat{r} + \sin\theta\, \hat{\theta} \right)$$

Now,

$$\vec{E}_{dip} = \sqrt{\vec{E}_{dip} \cdot \vec{E}_{dip}}$$

$$= \frac{p}{4\pi\varepsilon_0 r^3} \sqrt{(2\cos\theta\, \hat{r} + \sin\theta\, \hat{\theta})(2\cos\theta\, \hat{r} + \sin\theta\, \hat{\theta})}$$

$$= \frac{p}{4\pi\varepsilon_0 r^3} \sqrt{4\cos^2\theta + \sin^2\theta}$$

$$\boxed{\therefore E_{dip} = \frac{p}{4\pi\varepsilon_0 r^3} \sqrt{3\cos^2\theta + 1}}$$

# Dielectrics:

- A dielectric is a non-conducting material.
- They don't have practically free charges.
- An applied field causes a displacement of charges but no flow of charges in dielectric.

# Molecules of Dielectric

| Polar molecules | Non-polar molecules. |
|---|---|
| A polar molecules have a permanent dipole moment even in absence of polarizing field. | A non-polar molecule doesn't have a permanent dipole moment. |
| In polar molecules, the centre of gravity of the positive and negative charge distribution don't coincide. | In non-polar molecules, the center of gravity of positive and negative charge distribution coincide. |
| Eg: $H_2O$, $NH_3$, etc. | Eg: $H_2$, $N_2$, etc. |

# Induced Dipole Moment and Atomic Polarizability

In non-polar molecules, the center of +ve and -ve charge coincide and has no dipole moment. But if we place in an external electric field, the field distorts electric orbits and separate the centers of +ve and -ve charge.

The induced field shifts the electrons in a direction opposite the field due to which dipole moment is set up.

The dipole moment is said to be induced by the field and the atom or molecule is said to be polarized by the field.

Atomic polarizability is the electric dipole moment induced in the atom by an electric field of unit strength.

$$\alpha = \frac{p}{E}$$

where,
$\alpha$ = atomic polarizability
$p$ = induced dipole moment
$E$ = electric field.

Unit of Atomic Polarizability: $Fm^2$ ~~(Farad~~ (Farad meter square.)

<Q>: Calculate the induced dipole moment per unit volume of He gas if placed in a field of 6000 volts/cm. The atomic polarizability of He = $0.18 \times 10^{-40}$ $Fm^2$ and density of He is $2.6 \times 10^{25}$ atoms/$m^3$.

Soln:

Given,

$\alpha = 0.18 \times 10^{-40}$ $Fm^2$

density ($\rho$) = $2.6 \times 10^{25}$ atoms/$m^3$

Electric field = 6000 volts/cm = $6 \times 10^5$ volts/m

Now,

(i): Dipole moment of He atom ($p$) = $\alpha E$

$= 0.18 \times 10^{-40} \times 6 \times 10^5$

$\therefore p = 1.08 \times 10^{-35}$ Cm

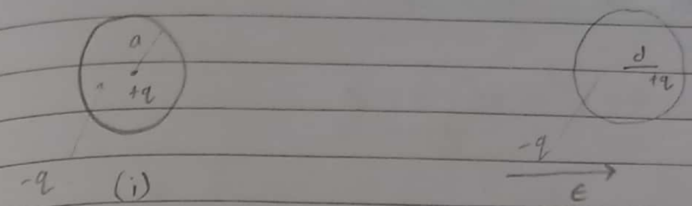
(ii) Induced dipole moment per unit volume ($P$) = $Np$

$= (2.6 \times 10^{25}) \times (0.18 \times 10^{-40})$

$\therefore P = 2.81 \times 10^{-10}$ $Cm^{-2}$

<Q>: A primitive model for an atom consists of point nucleus (+q) surrounded by a uniformly charged spherical cloud (-q) of radius a. Calculate the atomic polarizability of such atom.

Soln:




In the presence of electric field $\vec{E}$, the nucleus will be shifted slightly to the right and electron cloud to the left.

Say that equilibrium occurs when nucleus is displaced a distance 'd' from the sphere's centre.

At equilibrium,

the external field pushing the nucleus to the right ($E$) = the internal field pulling nucleus to the left ($E_e$)

or, $$E = \frac{1}{4\pi\varepsilon_0}\frac{qd}{a^3}$$

or, $$p = (4\pi\varepsilon_0 a^3) E$$

Here,

$$\alpha = 4\pi\varepsilon_0 a^3$$

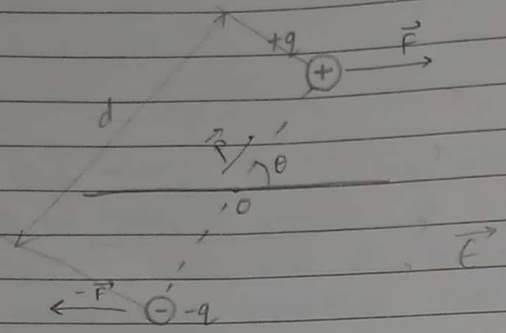
Now,

$$p = (4\pi\varepsilon_0 a^3) E$$

$$= 3\varepsilon_0 \left(\frac{4}{3}\pi a^3\right) E$$

$$\therefore p = 3\varepsilon_0 V E \qquad \therefore \alpha = 3\varepsilon_0 V$$

# Dipole in an Electric field




$\vec{p}$ at angle $\theta$ with direction of electric field $\vec{E}$ as shown in figure.

x) Net force on the Dipole:

Force acting on charge $+q$ $(\vec{F_+}) = +q\vec{E}$
Force acting on charge $-q$ $(\vec{F_-}) = -q\vec{E}$

$\therefore$ Net force $(\vec{F}) = \vec{F_+} + \vec{F_-} = +q\vec{E} - q\vec{E}$

$\therefore \vec{F} = 0$.

As two forces, acting at two ends of dipole are unlike parallel forces, they form a couple which rotates a dipole in clockwise direction, changing it in the direction of field.

(x) Net Torque of the Dipole:

Net torque $(\vec{\tau}) = \vec{\tau_+} + \vec{\tau_-}$

$$= \left(\frac{\vec{d}}{2} \times \vec{F_+}\right) + \left\{\left(\frac{-\vec{d}}{2}\right) \times \vec{F_-}\right\}$$

$$= \frac{\vec{d}}{2} \times q\vec{E} + \left(\frac{-\vec{d}}{2}\right) \times (-q\vec{E})$$

or, $\vec{\tau} = \vec{d} \times q\vec{E}$

$\therefore \vec{\tau} = \vec{p} \times \vec{E}$

# Potential Energy of Dipole in an Uniform Electric Field

Let us consider a dipole of dipole moment '$\vec{p}$' oriented at an angle '$\theta$' with direction of an uniform electric field '$\vec{E}$' as shown in figure.

We know,

Torque on dipole $(\tau) = \vec{p} \times \vec{E}$ — (i).

The workdone by external field in turning dipole from initial angle '$\theta_0$' to final angle '$\theta$'.

$$W = \int_{\theta_0}^{\theta} dw$$

$$= \int_{\theta_0}^{\theta} -\tau \, d\theta = \int_{\theta_0}^{\theta} -(pE\sin\theta)\, d\theta$$

$$\therefore W = pE(\cos\theta - \cos\theta_0) \quad \text{— (ii)}$$

∴ The change in potential energy of system is negative of workdone.

$\therefore \Delta U = U(\theta) - U(\theta_0) = -W$.

We arbitary define reference angle $(\theta_0 = 90°)$ and PE $U(\theta_0) = 0$ at that angle.

$\therefore PE = U(\theta) = pE\cos\theta$

$\therefore U(\theta) = -\vec{p}\cdot\vec{E}$

At $\theta = 180°$, $U(\theta) = pE$

(*) Summary:

When an electric dipole is placed in a uniform electric field,

(i) Net force on the dipole $(\vec{F_{ext}} = 0)$

(ii) Net torque on dipole $(\vec{\tau} = \vec{p}\times\vec{E})$

(iii) Potential energy on dipole $(U = -\vec{p}\cdot\vec{E})$

Note:

(i) When dipole rotates ~~on~~ from an initial orientation $\theta_i$ to orientation $\theta_f$, the workdone $(W)$ done on the dipole by electric field is

$W = -\Delta U_e$

$= -(U_f - U_i)$

(ii) If the change in orientation is caused by an applied torque, the workdone $(W_\tau)$ on the dipole by the applied torque

$W_\tau = -W = U_f - U_i$

<Q7: A neutral water molecule $(H_2O)$ in its vapour state has an electric dipole moment of magnitude $6.2\times10^{-30}$ C.m.

(a) How far apart are the molecules's center of positive and negative charge?

(b) If the molecule is placed in an electric field of $1.5\times10^4$ N/C, what maximum torque can the field exert ~~of~~ on it?

(c) How much work must an external agent do to rotate this molecule by 180° in this field, starting from its fully aligned position, $\theta = 0$?

Soln:

(a): Soln:

We have 10 electrons in neutral water molecule.

Now,

$$d = \frac{\vec{p}}{q} = \frac{6.2\times10^{-30}}{10\times1.6\times10^{-19}} = 3.9\times10^{-12}\text{ m}$$

$\therefore d = 3.9$ pm.

distance is smaller than radius of hydrogen atom.

(b): Sol$^n$:

We know,

$$\tau = \vec{p} \times \vec{E}$$
$$= pE \sin\theta$$
$$= (6.2 \times 10^{-30}) \times (1.5 \times 10^{4}) \times \sin 90^\circ$$
$$\therefore \tau = 9.3 \times 10^{-26} \text{ N.m}$$

(c): Sol$^n$:

We know,

$$W = U_{180^\circ} - U_{0^\circ}$$
$$= (-pE\cos 180^\circ) - (pE\cos 0^\circ)$$
$$= 2pE$$
$$= 2 \times (6.2 \times 10^{-30}) \times (1.5 \times 10^{4})$$
$$\therefore W = 1.9 \times 10^{-25} \text{ J}$$

# Polarization:

If a piece of dielectric material is placed in an electric field, a lot of little dipoles pointing along the direction of the field and the material become polarized.

A convenient measure of this effect is $\vec{P}$ = dipole moment per unit volume which is called polarization.

The dipole moment per unit volume of the polarized material.

$$\text{Polarization } (\vec{P}) = \frac{d\vec{p}}{d\tau} = \frac{\text{net electric dipole moment}}{\text{an elemental volume}}$$

$$= \frac{1}{d\tau}\left(\sum_m \vec{p}_m\right)$$

SI unit $= C\,m^{-2}$

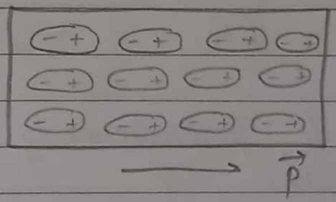
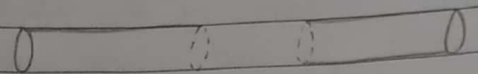


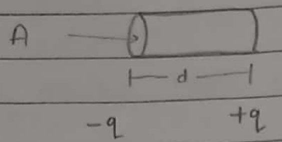
Fig: piece of polarized dielectric material

# Calculation of Actual Amount of Bound Charge Resulting from Polarization

(a) For uniform polarization:

Consider a tube of dielectric parallel to uniform polarization.






The dipole moment of the piece,

$$p = P(A\,d) \quad \text{--- (i)}$$

$$\therefore\ p = qd \quad \text{--- (ii)}$$

From eq$^n$ (i) and (ii), $q = PA$

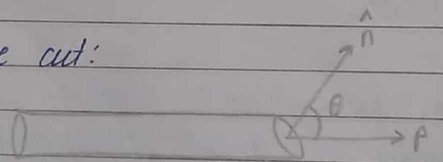
The bound charge that piles up at the right end of tube '$q$'.

$$q = PA.$$

For the ends sliced off perpendicularly, surface charge density $(\vec{\sigma_b}) = P$

$$(\sigma_b) = \frac{q}{A}.$$

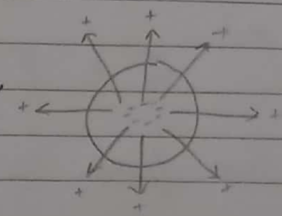
For an oblique cut:




Surface charge density $(\sigma_b) = \dfrac{q}{A_{end}} = \dfrac{q}{\frac{A}{\cos\theta}} = P\cos\theta = \vec{P}\cdot\hat{n}$

The effect of the polarization is to paint a bound charge $\sigma_b = \vec{P}\cdot\hat{n}$ over the surface of the material.

(b) For non-uniform polarization:



If the polarization is non-uniform, we get accumulations of bound charge within the material as well as the surface.

The net bound charge in a given volume is equal and opposite to the amount that has been pushed out through the surface

$$\int_V \rho_b\, d\tau = -\oint_S \sigma_b\, da$$

$$= -\oint_S (\vec{P}\cdot\hat{n})\, da$$

$$\therefore \int_V \rho_b\, d\tau = -\int_V (\nabla\cdot\vec{P})\, d\tau.$$

Since it is true for any volume, we have

$$\rho_b = -\nabla\cdot\vec{P}$$

The effect of polarization is to produce accumulation of bound charges $\rho_b = -\nabla\cdot\vec{P}$ with the dielectric and $\vec{\sigma_b} = \vec{P}\cdot\hat{n}$ on the surface.

# Gauss's Law of Presence of Dielectrics

We know that,
the effect of polarization is to produce accumulation of bound charge

$\rho_b = -\nabla \cdot \vec{P}$ within the dielectric

and

$\sigma_b = \vec{P} \cdot \hat{n}$ on the surface.

From Gauss's law,

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

or, $\nabla \cdot \varepsilon_0 \vec{E} = \rho_b + \rho_f$ {within dielectric, $\rho = \rho_b + \rho_f$ }

or, $\nabla \cdot \varepsilon_0 \vec{E} = -\nabla \cdot \vec{P} + \rho_f$

or, $\nabla \cdot (\varepsilon_0 \vec{E} + \vec{P}) = \rho_f$

We know,

$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$ is the electric displacement

$$\therefore \nabla \cdot \vec{D} = \rho_f$$

So,

$$\int_V (\nabla \cdot \vec{D})\, d\tau = \int_V \rho_f \cdot d\tau$$

$$\therefore \oint \vec{D} \cdot d\vec{a} = Q_{f_{enc}}$$

{ total free charge enclosed in volume }.

## (x) Susceptibility, Permittivity and Dielectric Constant

For many substances, the polarization is proportional to the electric field $\vec{E}$ is provided it is not too strong,

$$\vec{P} = \varepsilon_0 \chi_e \vec{E} \quad \text{—— (i)}$$

$\chi_e$ = electric susceptibility of the medium
and materials obeying eq$^n$ (i) are linear dielectrics.

In linear media,

$$\vec{D} = \varepsilon_0 \vec{E} + \varepsilon_0 \chi_e \vec{E}$$
$$= \varepsilon_0 (1 + \chi_e) \vec{E} \quad \text{—— (ii)}$$

$$\therefore \vec{D} = \varepsilon \vec{E}$$

where, $\varepsilon = \varepsilon_0 (1 + \chi_e)$

$\varepsilon$ = permittivity of the material.

x) Relative Permittivity / Dielectric Constant: $K = \frac{\varepsilon}{\varepsilon_0} = (1 + \chi_e)$

In homogeneous linear dielectric,

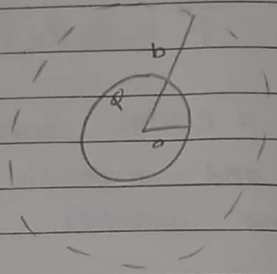
$$\rho_b = -\nabla \cdot \vec{P}$$

$$\therefore \rho_b = -\left(\frac{\chi_e}{1 + \chi_e}\right) \rho_f$$

<Q7: A metal sphere of radius a, carries a charge Q. It is surrounded out to radius b, linear dielectric of permittivity $\varepsilon_0$. Find the potential at the center (relative to infinity).

Soln:

We know,
Gauss's law in presence of dielectric,




$$\oint \vec{D}\cdot d\vec{a} = Q_{fenc} \quad \text{—(i)}$$

Drawing spherical Gaussian surface of radius r (r>a) and applying eqn (i).

$$D(4\pi r^2) = Q$$

$$\therefore \vec{D} = \frac{Q}{4\pi r^2}\hat{r} \quad \text{for all } r>a.$$

So, $$\vec{E} = \frac{1}{\varepsilon_0}\frac{Q}{4\pi r^2}\hat{r} \quad \text{for } r>b$$

$$\vec{E} = \frac{1}{\varepsilon}\frac{Q}{4\pi r^2}\hat{r} \quad \text{for } a<r<b$$

$$\vec{E} = 0 \quad \text{for } r<a.$$

The potential at center relative to infinity.

$$V = -\int_\infty^0 \vec{E}\cdot d\vec{l}$$

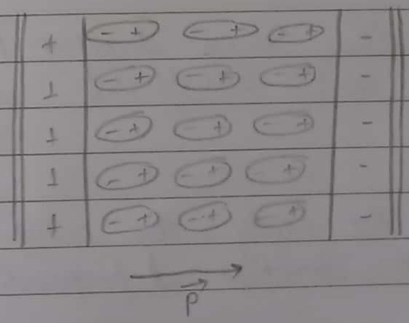
$$= -\int_0^\infty E\,dr$$

$$= -\int_\infty^b E\,dr - \int_b^a E\,dr - \int_a^0 (0)\,dr$$

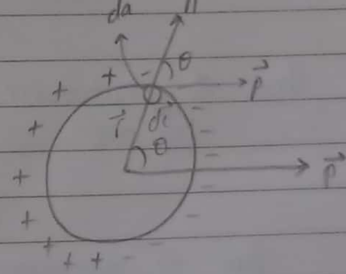
$$\therefore V = \frac{Q}{4\pi}\left(\frac{1}{\varepsilon_0 b} + \frac{1}{\varepsilon a} - \frac{1}{\varepsilon b}\right)$$

# Clausius- Mossoti Equation:

An expression for the electric field at the center of a spherical cavity inside a polarized dielectric due to the charges on the wall of the cavity.




(i)




(ii)

Let $da$ = an elemental area on the surface of cavity surface.

The charge on an elemental area $da$ is.

$$dq = -\sigma_b \, da = -(\vec{P}\cdot\hat{n})\, da$$

$$= -P\cos\theta \, (r^2 \sin\theta \, d\theta \, d\phi)$$

The electric field at the center of the cavity due to charge $dq$.

$$d\vec{E_c} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^3}\vec{r}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\{-P\cos\theta\, r^2\sin\theta\, d\theta\, d\phi\}\vec{r}}{r^3}$$

$$\therefore d\vec{E_c} = \frac{P}{4\pi\varepsilon_0}(\cos\theta\sin\theta\, d\theta\, d\phi)\,\hat{n}$$

The component of $d\vec{E_c}$ along direction $\vec{P}$ is

$$d\vec{E_c}\cos\theta = \frac{P}{4\pi\varepsilon_0}\cos^2\theta\,\sin\theta\, d\theta\, d\phi$$

Due to symmetry of the cavity, the components of $d\vec{E_c}$ along the direction perpendicular to $\vec{P}$ is zero.

Therefore, the electric field at center $C$ of spherical cavity due to entire surface charge on the cavity surface.

$$E_c = \int \frac{1}{4\pi\varepsilon_0} P\cos^2\theta\sin\theta\, d\theta\, d\phi$$

$$= \frac{P}{4\pi\varepsilon_0}\left\{\int_0^{\pi}\cos^2\theta\,\sin\theta\right\}\left\{\int_0^{2\pi} d\phi = 2\pi\right\}$$

$$\vec{E_c} = \frac{\vec{P}}{3\varepsilon_0}$$

~~Clausius~~ Therefore, the net electric field experienced by this molecule is the sum of electric field due to bound charge on the cavity surface and the resultant of all other fields except due to the bound charges on the cavity surface.

ie, $\vec{E_m} = \vec{E_c} + \vec{E}$

The dipole moment of a molecule per unit molecular field is called its polarizability ($\alpha_m$).

$$\vec{P_m} = \alpha_m \vec{E_m} \quad \text{---- (ii)}$$

If there are $N$ molecules per unit volume, then the polarization, Date. No.

$$\vec{P} = N p_m$$
$$= N \alpha_m \vec{E}_m = N \alpha_m [\vec{E}_c + \vec{E}]$$

or, $$\vec{P} = N \alpha_m \left[ \frac{\vec{P}}{3\varepsilon_0} + \frac{\vec{P}}{\chi_e \varepsilon_0} \right]$$ $[\because \vec{P} = \varepsilon_0 \chi_e \vec{E}]$

or, $$1 = N \alpha_m \left[ \frac{1}{3\varepsilon_0} + \frac{1}{\chi_0 \varepsilon_0} \right]$$ or $1 = N \alpha_m \left[ \frac{\chi_e + 3}{3\varepsilon_0 \chi_e} \right]$

or $$\alpha_m = \frac{3\varepsilon_0}{N} \left[ \frac{\chi_e}{\chi_e + 3} \right]$$

We know, $1 + \chi_e = K$.

So,
$$\alpha_m = \frac{3\varepsilon_0}{N} \left[ \frac{K-1}{K-1+3} \right]$$

$$\therefore \alpha_m = \frac{3\varepsilon_0}{N} \left[ \frac{K-1}{K+2} \right]$$

$$\therefore \alpha_m = \frac{3\varepsilon_0}{N} \left[ \frac{\varepsilon_r - 1}{\varepsilon_r + 2} \right]$$

This is Clausius- Mossoti equation.