General Physics I (PHYS 101)

Lecture 17

Diffraction

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Outline

Rectilinear propagation of light

According to wave theory of light, every point on primary wave behaves as source for secondary wave and the forward envelope of secondary wavelets for certain time gives the new position of wave at that time. Thus, after a long time or at a great distance from the source, light may be considered as a plane wave. Different experiments on light wave showed that it has straight line motion. And this can be verified by placing an obstacle of large size in between the path of light to the screen. The shadow would not be produced if the light wave would have curved type motion. The phenomenon by which light wave travels in straight line is called the rectilinear propagation of light. But an unexpected result would be

Rectilinear propagation of light (contd.)

obtained if we place an obstacle of small size; comparable to the wavelength of light. This peculiarity is explained by the term diffraction on the basis of wave theory.

The bending of light at small apertures or at the sharp edges to form a band of dark and bright fringes of varying intensities is called diffraction. Thus, for diffraction, the size of obstacle must be comparable to wavelength of light used. Theory of diffraction is explained on the basis of wave theory of light. There are two types of diffraction depending up on their nature.

1. Fresnel diffraction and
2. Fraunhofer diffraction

Fresnel diffraction: The diffraction phenomenon in which source and screen both are separated by a finite distance from slits is called Fresnel diffraction. In such diffraction spherical wave from a point source falls up on a slit and gets diffracted forming dark and bright fringes of varying intensities. Observations of Fresnel diffraction phenomena do not require any lenses.

Fraunhofer diffraction: The diffraction phenomenon in which source and screen both are separated by an infinite distance from slit is called Fraunhofer diffraction. In such diffraction, light from a source at infinity falls up on slit and gets diffracted resulting dark and bright fringes with different intensities. Fraunhofer diffraction pattern can be easily observed in practice. The incoming light is rendered parallel with a lens and the diffracted beam is focused on the screen with another lens.

Resultant amplitude of n waves:

Consider there are n number of waves with same amplitude and frequencies as well as successive phase difference of δ given by

$y_1 = ae^{i\theta}$,

$y_2 = ae^{i(\theta+\delta)}$,

$y_3 = ae^{i(\theta+2\delta)}$,

$y_4 = ae^{i(\theta+3\delta)}$,

$y_5 = ae^{i(\theta+4\delta)}$,

..............,

$y_n = ae^{i[\theta+(n-1)\delta]}$.

Resultant amplitude of n waves: (contd.)

Now their resultant y is given by

$$
\begin{aligned}
y &= y_1 + y_2 + y_3 + y_4 + y_5 + \cdots + y_n \\
&= ae^{i\theta} + ae^{i(\theta+\delta)} + ae^{i(\theta+2\delta)} + ae^{i(\theta+3\delta)} + ae^{i(\theta+4\delta)} + \cdots + ae^{i[\theta+(n-1)\delta]} \\
&= ae^{i\theta}[1 + e^{i\delta} + e^{2i\delta} + e^{3i\delta} + e^{4i\delta} + \cdots + e^{i(n-1)\delta}] \\
&= ae^{i\theta}\left[\frac{e^{in\delta}-1}{e^{i\delta}-1}\right] = ae^{i\theta}\frac{e^{in\delta/2}}{e^{i\delta/2}}\left[\frac{e^{in\delta/2}-e^{-in\delta/2}}{e^{i\delta/2}-e^{-i\delta/2}}\right] \\
&= ae^{i\left[\theta+\frac{(n-1)}{2}\delta\right]}\frac{2i\sin\left(\frac{n\delta}{2}\right)}{2i\sin\left(\frac{\delta}{2}\right)} = Re^{i[\theta+\frac{(n-1)}{2}\delta]}
\end{aligned}
$$

Where

$$R = a\frac{\sin\left(\frac{n\delta}{2}\right)}{\sin\left(\frac{\delta}{2}\right)} \tag{1}$$

is the resultant amplitude of n waves.

Let a collimating lens L_1 be placed at a distance equal to its focal length from a monochromatic source S. The collimated beam of light from L_1 falls on a narrow slit AB of width a. The converging lens L_2, on the other side of the slit, converge the undiffracted beam at center C and diffracted beam at P on a screen XY as shown. Draw $AK \perp BK$ and suppose θ be the angle of diffraction. Then from figure $\measuredangle BAK = \theta$ since AK meets OP at right angles.

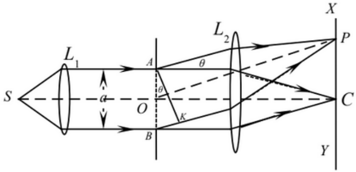

Figure 1

Fraunhofer diffraction at single slit: (contd.)

We now calculate the path difference between the waves originating from A and B. From figure BK is path difference.

i.e. path difference, $BK = AB\sin\theta = asin\theta$

Now phase difference $= \frac{2\pi}{\lambda} \times$ path difference,

$$\therefore \text{Phase difference} = \frac{2\pi}{\lambda} a\sin\theta$$

Let us divide AB in to large number n of equal parts so that each part behaves as a point source for secondary wave. Then phase difference between two consecutive waves is

$$\delta = \frac{1}{n}\frac{2\pi}{\lambda} a\sin\theta$$

Fraunhofer diffraction at single slit: (contd.)

Now resultant amplitude of n waves reaching at P is given by

$$R = a\frac{\sin\left(\frac{n\delta}{2}\right)}{\sin\left(\frac{\delta}{2}\right)} = a\frac{\sin\left(\frac{\pi a \sin\theta}{\lambda}\right)}{\sin\left(\frac{\pi a \sin\theta}{n\lambda}\right)} = a\frac{\sin\alpha}{\sin\left(\frac{\alpha}{n}\right)} \tag{2}$$

Where

$$\alpha = \frac{\pi a \sin\theta}{\lambda} \tag{3}$$

For large n, $\sin\left(\frac{\alpha}{n}\right) \approx \frac{\alpha}{n}$

$$\therefore R = a\frac{\sin\alpha}{(\alpha/n)} = \frac{n\,a\sin\alpha}{\alpha} = \frac{A\sin\alpha}{\alpha} \tag{4}$$

Where $A = na$.

Since intensity at P is proportional to the square of the amplitude of resultant wave, we can write

$$I = R^2$$

$$\therefore I = \frac{A^2 \sin^2\alpha}{\alpha^2} \tag{5}$$

Maximum and minimum intensities at P of resultant wave can be found out by differentiating I with respect to α and equating to zero as

$$\frac{dI}{d\alpha} = 0 \;\Rightarrow\; \frac{d}{d\alpha}\left(\frac{A^2 \sin^2\alpha}{\alpha^2}\right) = 0$$

Fraunhofer diffraction at single slit: (contd.)

$$\Rightarrow 2A^2 \left(\frac{\sin\alpha}{\alpha}\right)\left(\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2}\right) = 0$$

$$\text{i.e. either} \frac{\sin\alpha}{\alpha} = 0 \tag{6}$$

$$\text{or, } \frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2} = 0$$

$$\implies \alpha = \tan\alpha \tag{7}$$

Position of minimum intensity:

When $\dfrac{\sin\alpha}{\alpha} = 0$ then from equation (5) $I = 0$ and this is minimum intensity. So for minimum intensity,

$$\frac{\sin\alpha}{\alpha} = 0 \implies sin\alpha = 0$$

$$\text{or, } \alpha = \pm m\pi\,;\, m = 1,\, 2,\, 3, \ldots$$

$$\text{or, } \frac{\pi a \sin\theta}{\lambda} = \pm m\pi \tag{8}$$

$$\therefore\ a\sin\theta = \pm m\lambda \tag{9}$$

This is the condition for minimum intensity and minimum intensity occurs at the points $\alpha = \pm\pi, \pm 2\pi, \pm 3\pi, \ldots$

Position of maximum intensity:

For maximum intensity we solve $\alpha = \tan\alpha$ graphically i.e. $y = \alpha$ and $y = \tan\alpha$. From graph, solution of these two curves is approximately, $\alpha = 0^0, \dfrac{3\pi}{2}, \dfrac{5\pi}{2}, \dfrac{7\pi}{2}, \ldots$

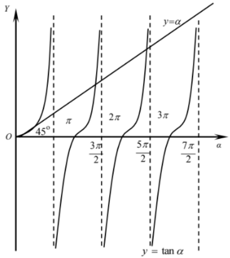

Figure 2

Fraunhofer diffraction at single slit: (contd.)

So points of maximum intensity are $\alpha = 0^0, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}, \ldots$

For central maximum,

$$\alpha = 0$$

$$\text{or, } \frac{\pi a \sin\theta}{\lambda} = 0 \quad \text{or, } \sin\theta = 0$$

$$\therefore\ \theta = 0^\circ$$

This is the condition for principal maxima. And directions of secondary maxima are approximately

$$\alpha = \pm(2m+1)\frac{\pi}{2} \implies \qquad \frac{\pi a \sin\theta}{\lambda} = \pm(2m+1)\frac{\pi}{2}$$

Fraunhofer diffraction at single slit: (contd.)

$$\therefore \; a\sin\theta = \pm(2m+1)\frac{\lambda}{2}$$

This is the condition for secondary maxima.

Intensity distribution.

The intensity of central principal maximum is

$$(\text{at } \alpha = 0^\circ)$$

$$I_0 = R^2 = \lim_{\alpha\to 0}\frac{A^2\sin^2\alpha}{\alpha^2} = A^2$$

Also intensity of 1^{st} secondary maxima,

$$I_1 = A^2\frac{[\sin(3\pi/2)]^2}{[3\pi/2]^2} \approx \frac{A^2}{22} = \frac{I_0}{22}$$

Fraunhofer diffraction at single slit: (contd.)

Again intensity of 2^{nd} secondary maxima,

$$I_2 = A^2 \frac{[\sin(5\pi/2)]^2}{[5\pi/2]^2} \approx \frac{A^2}{62} = \frac{I_0}{62}$$

And intensity of 3^{rd} secondary maxima,

$$I_3 = A^2 \frac{[\sin(7\pi/2)]^2}{[7\pi/2]^2} \approx \frac{A^2}{121} = \frac{I_0}{121}$$

Thus, it is seen that intensity of secondary maxima goes on decreasing when number of secondary maxima increases. The intensity distribution in Fraunhofer single slit diffraction is as shown.we get the positions of first, second, third … secondary maxima.

Fraunhofer diffraction at single slit: (contd.)

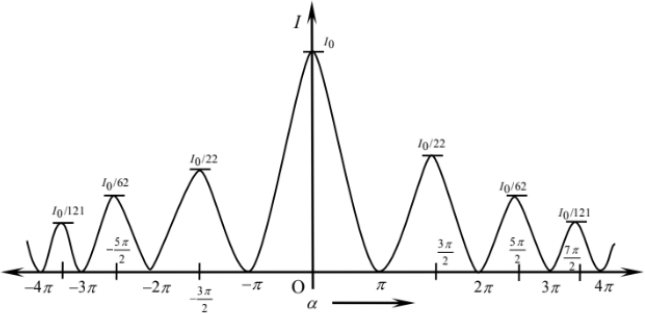

Figure 3