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Unit 3:

APPLICATIONS OF DERIVATES

Diffrentials:

$$x\,dx + y\,dy = 0$$

Eg: $y = x^2 + 5x$

or, $\frac{dy}{dx} = 2x + 5$

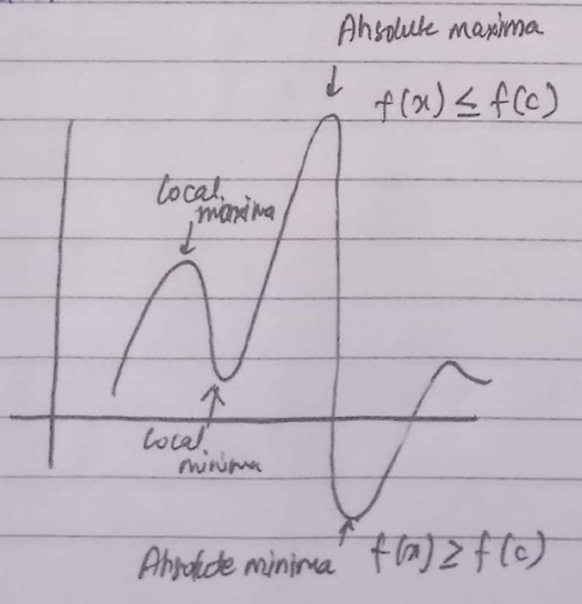
or, $dy - (2x+5)dx = 0$

$\therefore dy = (2x+5) \times dx$.

Hence, dy can be calculated if dx and x are known.

Extreme Values

Extreme values are of two types:
Absolute / Global extreme values and Local extreme values.
They are also local and global minimum and maximum.



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Extreme point:

The point ~~th~~ at which the nature of graph changes is called extreme point.

Definition:

Let 'f' be a function with domain D.
Then, f ~~is~~ has an absolute maximum value on D at point c if $f(x) \le f(c)$, $\forall x \in D$ and absolute minimum if $f(x) \ge f(c)$, $\forall x \in D$.

For local maximum value of f, if
$f(x) \le f(c)$, when x is nearer to c.
and local minimum value of f, if
$f(x) \ge f(c)$ when x nearer to c.

Critical point:

A critical point of a function 'f' is a point c in a domain of f such that $f'(c) = 0$ or $f'(c)$ is undefined.

Stationary point:

A ~~crit~~ stationary point of a function 'f' is a point c in a domain of f such that $f'(c) = 0$.

→ At critical point, the graph of function ~~is~~ has either vertical and horizontal tangent.
If $f'(c) = 0$ ie, horizontal tangent.
If $f'(c) =$ undefined ie, vertical tangent.

All stationary points are critical points, but not all critical points are stationary points.

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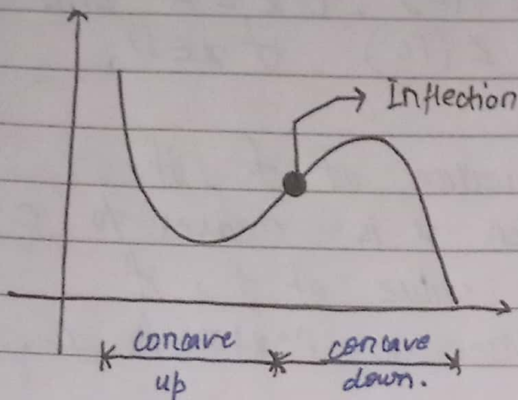
(*) Point of inflection:

The point of inflection is a point at which the concavity of the function changes.

ie, $f''(c) = 0$ at $x = c$.

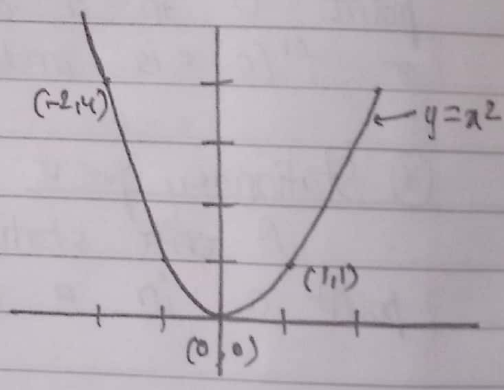
If $f''(x) > 0$ at $x = a$ then, concave up.

If $f''(x) < 0$ at $x = a$ then, concave down.



<Q>: Find global extreme value of $f(x) = x^2$ in $[-2, 1]$

Soln:



Given,

$f(x) = x^2$

or, $y = x^2$ —(i)

Differentiating (i) w.r.t x,

$\frac{dy}{dx} = \frac{dx^2}{dx}$

$\therefore f'(x) = 2x$

Putting $f'(x) = 0$,

$\therefore x = 0$

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So, $f(-2) = \cancel{x^2} \ (-2)^2$

$\therefore f(-2) = 4$

$f(1) = (1)^2$

$\therefore f(1) = 1$

$f(0) = (0)^2$

$\therefore f(0) = 0$

So, global maximum value = 4 at $x = -2$.
global minimum value = 0 at $x = 0$.

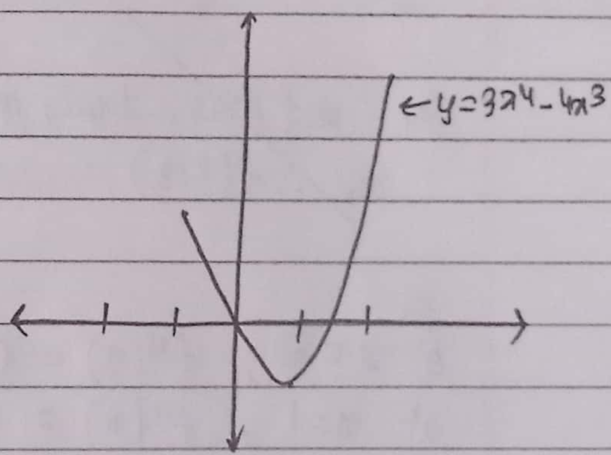
<Q>: If $f(x) = 3x^4 - 4x^3$ in $[-1, 2]$, find extreme values.

Sol^n:

Given,

$f(x) = 3x^4 - 4x^3$

$\therefore \cancel{f'(x)}$ or, $y = 3x^4 - 4x^3 \quad \text{--- (i)}$



Differentiating (i) w.r.t x,

$$\frac{dy}{dx} = \frac{d(3x^4)}{dx} - \frac{d(4x^3)}{dx}$$

$\therefore f'(x) = 12x^3 - 12x^2$

$= 12x^2(x-1)$

Putting $f'(x) = 0$,

$x = 0$ or, $x = 1$

$\therefore x = 0, 1$

At $x = 0$, $f(0) = 0$

At $x = 1$, $f(1) = -1$

At $x = -1$, $f(-1) = 7$

At $x = 2$, $f(2) = 16$

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Global maximum $= 16$ at $x=2$.

Global minimum $= -1$ at $x=1$

Now,

$$f''(x) = \frac{df'(x)}{dx}$$

$$\therefore f''(x) = 36x^2 - 24x.$$

At $f''(x) = 0$,

$x = 0$ $\quad$ $x = 2/3$

$\therefore x = 0, 2/3$

So,

at $x=0$, $f''(0) = 0$

at $x = 2/3$ $f''(2/3) = \frac{-16}{27} < 0$

So, f has local maximum at $x = 2/3$

So, $\therefore f(2/3) =$

At $f''(x) = 0$,

$0 = 36x^2 - 24x$

or $0 = 12x(3x-2)$

$\therefore x = 0, 2/3$

The points of inflection are $(0,0)$ and

and $\left(\frac{2}{3}, \frac{-16}{27}\right)$

So

at $x=0$, $f''(x) = 0$

at $x=1$, $f''(x) = 12 > 0$

At $x=1$, $f(x)$ has local minimum value.

$f_{min} = -1$

Here, there is no local maximum value

⟨Q⟩: $f(x) = x^{2/3}$ at $[-2, 3]$

Sol^n:

Given,

$f(x) = x^{2/3}$

$\therefore f'(x) = \frac{2}{3x^{1/3}}$ $\therefore f''(x) = \frac{-2}{9x^{4/3}}$

Here, $f'(0)$ = undefined.

and also, $f''(0)$ is undefined.

At $x = -2$, $f(-2) = 1.587$

At $x = -1$ $f(-1) = 1$

At $x = 0$ $f(0) = 0$

At $x = 1$ $f(1) = 1$

At $x = 2$ $f(2) = 1.587$

At $x = 3$ $f(3) = 2.0800$

Here, at $x = 0$, global minimum value. $= 0$

At $x = 3$, global maximum value $= 3$

⟨Q⟩: $f(x) = 8x - x^4$ at $[-2, 1]$, find global and local extremes.

Sol^n:

Given,

$f(x) = 8x - x^4$

$\therefore f'(x) = 8 - 4x^3$ $\therefore f''(x) = -12x^2$

Putting $f'(x) = 0$.

or, $0 = 8 - 4x^3$ $\therefore x = \sqrt[3]{2}$

At $x = -2$, $f(-2) = -32$
At $x = 0$, $f(0) = 0$
At $x = 1$, $f(1) = 7$

Here,
global maximum $= 7$ at $x = 1$
global minimum $= -32$ at $x = -2$.

In $f''(x)$, putting $x = \sqrt[3]{2}$,
$f''(\sqrt[3]{2}) = -19.04 < 0$
So, $f(x)$ has local maximum at $x = \sqrt[3]{2}$.

Local ~~minimum~~ maximum $= 7.56$ at $x = \sqrt[3]{2}$

n^{th} derivative test for Extreme Value

Let 'c' be a point in the interval in which the function $f(x)$ is defined and if $f'(c) = f''(c) = f'''(c) \ldots\ldots = f^{n-1}(c) = 0$ but $f^n(c) \neq 0$ then,

(i): $f(x)$ has neither max or min at $x = c$ if n is odd.
(ii) $f(x)$ has max at $x = c$ if n is even and $f^n(c) < 0$.
(iii) $f(x)$ has min at $x = c$, if n is even and $f^n(c) > 0$.

<Q>: $f(x) = x^6$.

Soln.

Given,
$f(x) = x^6$
$\therefore f'(x) = 6x^5$
$\therefore f''(x) = 30x^4$

For critical point, $6x^5 = 0$

$\therefore x = 0$

$\therefore f''(0) = 0.$

Now,

$f'''(x) = 120x^3$

$\therefore f'''(0) = 0$

Again,

$f''''(x) = 360x^2$

$\therefore f''''(0) = 0$

Also,

$f'''''(x) = 720x$

$\therefore f^{5}(0) = 0$

Again,

$f^{6}(x) = 720 \neq 0.$

Since $n = 6$ is even and $720 > 0$.

$f(x)$ has minimum at $x = 0$.

<Q>: $f(x) = -2x^5$

Soln:

Given,

$f(x) = -2x^5$

$\therefore f'(x) = -10x^4$

$\therefore f''(x) = -40x^3$

Putting $f''(x) = 0$ $\therefore x = 0$. $\therefore f''(0) = 0$

So

$$f''''(x) = -120x^2$$
$$\therefore f''''(0) = 0$$

Now,

$$f'''''(x) = -240x$$
$$\therefore f'''''(0) = 0$$

Again,

$$f''''''(x) = -240 \neq 0$$

Here, $n = 5$ ie, odd. So, $f(x)$ has neither minimum and maximum values.

Rolle's theorem:

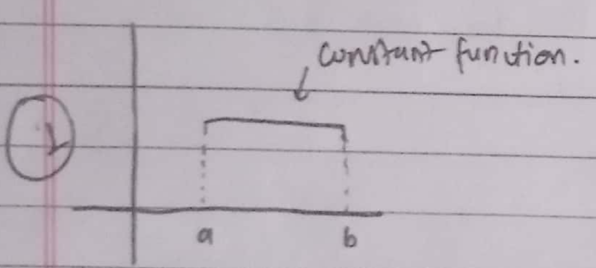
If a function $f(x)$ is

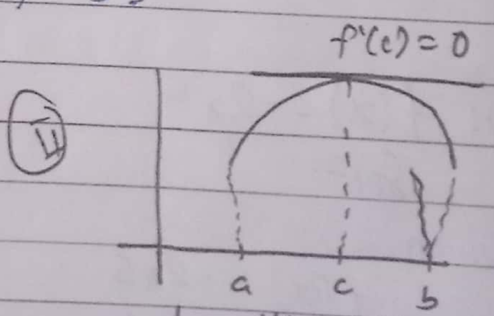
i) continuous on $[a, b]$

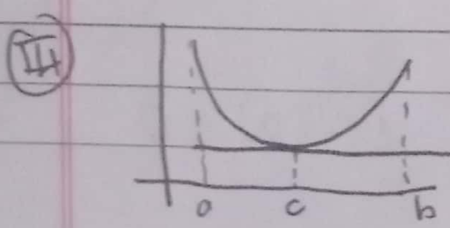
ii) differentiable on (a, b)

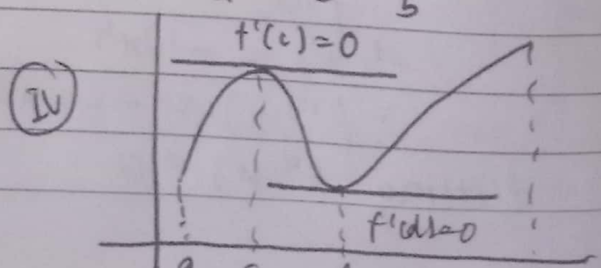
iii) $f(a) = f(b)$

then, there exists ~~atleast point~~ atleast one point $c \in (a, b)$ such that $f'(c) = 0$.









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⊛ Geometrical interpretation:

If a function satisfies all the condition of Rolle's theorem, then there exists a point $(c, f(c))$ on curve $y = f(x)$ where, f has horizontal tangent.

Eg: (i): $f(x) = 1/x^2$ at $[-1, 1]$.
→ not applicable as $x = 0$ undefined.

(ii): ~~log~~ $f(x) = \log x$ $[-1, 2]$
→ not applicable as $x = 0$ discontinuous.

(iii) $f(x) = \log x$ $[1, 2]$
→ ~~ap~~ not applicable as $f(1) \neq f(2)$

(iv): $f(x) = x^3 - 2x + 2$ $(1, 2)$
→ applicable.

(v): $f(x) = x^{2/3}$ in $[0, 1]$.
→ not applicable as $f(0) \neq f(1)$.

<Q>: Does Rolle's theorem satisfy $f(x) = \sin x / e^x$? If yes, find the point of horizontal tangent.

Soln:

Given,

$$f(x) = \frac{\sin x}{e^x} \quad \text{at } [0, \pi]$$

(i): Here, $f(x)$ is continuous at $[0, \pi]$ as $f(x)$ exists for all $x \in [0, \pi]$

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(ii): $f'(x) = \frac{d}{dx}\left(\frac{\sin x}{e^x}\right)$

$$= \frac{e^x \cdot \frac{d\sin x}{dx} - \sin x \frac{de^x}{dx}}{(e^x)^2}$$

$\therefore f'(x) = \frac{\cos x - \sin x}{e^x}$

Here, $(\cos x - \sin x)$ has solution for $([0, \pi)$

(iii) $f(0) = 0$

$f(\pi) = 0$

$\therefore f(0) = f(\pi)$

Hence, all the conditions for Rolle's theorem are satisfied.

~~Let 'c' be the point such that $f'(c) = 0$.~~

Putting $f'(x) = 0$.

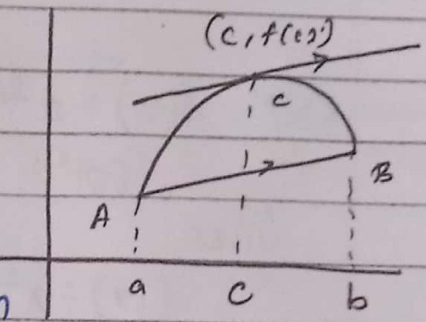
$0 = \cos x - \sin x$

$\therefore x = \frac{\pi}{4} \in \cancel{[0, \pi]}\ (0, \pi)$

Hence, horizontal tangent exists at $x = \pi/4$.

Mean Value Theorem

If a function $f(x)$ is
i) continuous on $[a, b]$
ii) differentiable on (a, b)
then, there exists atleast one point $c \in (a, b)$ such that
$$f'(c) = \frac{f(b) - f(a)}{b - a} \quad \text{ie, } b \neq a$$



Geometrical Interpretation:

If a function $f(x)$ satisfied all conditions for mean value theorem, then there exists atleast a point $(c, f(c))$ on curve $y = f(x)$, from where tangent is drawn which is parallel to the chord joining A and B.

Eg:<Q>: $f(x) = Ax^2 + Bx + C$ in $[a, b]$, verify MVT.

Soln.

Here,
$f(x) = Ax^2 + Bx + C$.

(i) Since, $f(x)$ is a polynomial function, $f(x)$ is continuous on $[a, b]$.

(ii) $f'(x) = 2Ax + B$
This is differentiable for all $x \in (a, b)$

Here, all the conditions for Mean value theorem are satisfied. So, point 'c' exists in (a, b).
$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

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or, $2Ac + B = \dfrac{Ab^2 + Bb + C - Aa^2 - Ba - C}{b-a}$

or, $2Ac = \dfrac{Ab^2 - Aa^2}{b-a}$

or, $2c = \dfrac{(b+a)\cancel{(b-a)}}{\cancel{(b-a)}}$

$\therefore c = \dfrac{a+b}{2} \in (a,b)$.

<Q>: $f(x) = x^{2/3}$. $[0,1]$

Sol^n:

Given,

$f(x) = x^{2/3}$

(i): $f(x)$ is continuous at $[0,1]$ as $f(x)$ exists for all $x \in [0,1]$

(ii): $f'(x) = \dfrac{2}{3x^{1/3}}$

$f(x)$ is differentiable at $(0,1)$.

Hence, all the conditions for MVT are satisfied. Let c be the point on (a,b).

$f'(c) = \dfrac{f(b) - f(a)}{b-a}$

$\dfrac{2}{3c^{1/3}} = \dfrac{1^{2/3} - 0^{2/3}}{1-0}$

or, $\dfrac{2}{3} = c^{1/3}$ $\therefore c = \dfrac{8}{27} \in (0,1)$.

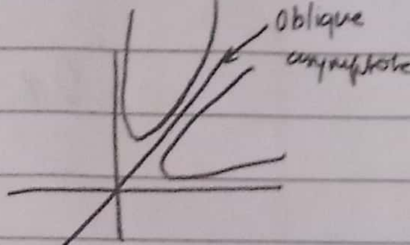
Asymptote of Graph

(i) $\lim_{x \to \pm a} f(x) = \pm\infty$

This gives vertical asymptote.

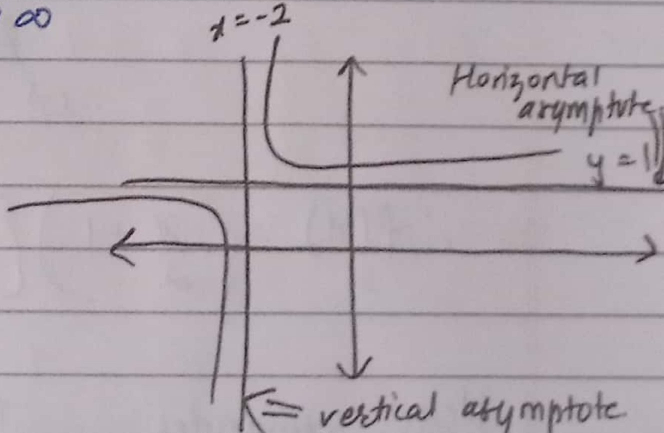
(ii) $\lim_{x \to \pm\infty} f(x) = b$

This gives horizontal asymptote.



(iii): Oblique asymptote:

The straight line meeting curve at infinity. If $n^\circ > d^\circ$, then oblique asymptote is possibility. If $\lim_{x \to \infty} f(x) = \infty$ then, ob. as. exists.



(Q): $y = \dfrac{x+3}{x+2}$

Soln:

(i) $\lim_{x \to -2} \dfrac{-2+3}{-2+2} = \infty$

ie, $x = -2$ is a vertical asymptote.

(ii): $\lim_{x \to \infty} \dfrac{x+3}{x+2} = \dfrac{x(1+3/x)}{x(1+2/x)} = 1$

$\therefore y = 1$ is horizontal asymptote.

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<Q>: $y = \dfrac{x^3 - 1}{2x - 4}$

Sol^n:

(i) $\lim\limits_{x \to 2} \dfrac{2^3 - 1}{2 \times 2 - 4} = \infty$

$\therefore x = 2$ is vertical asymptote.

Here, since $n^\circ > d^\circ$, oblique asymptote exists.

$$2x - 4 \overline{\big) \; x^2 - 3 \quad} \big(x/2 + 1$$

$$x^2 - 2x$$

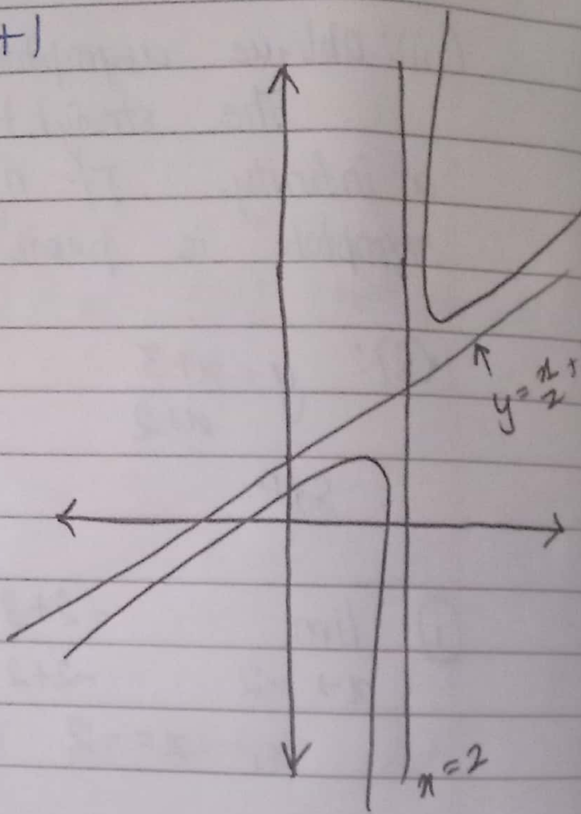
$$(-) \quad (+)$$

$$2x - 3$$

$$2x - 4$$

$$(-) \quad (+)$$

$$1$$



$\therefore f(x) = \left(\dfrac{x}{2} + 1 \right) + \left(\dfrac{1}{2x - 4} \right)$

Here, remainder $= \dfrac{1}{2x - 4}$

~~Horizontal asympto~~

So, here, $y = \left(\dfrac{x}{2} + 1 \right)$ is oblique asymptote.

Relative Rate of Growth

Let $f(x)$ and $g(x)$ are two functions for all positive x which is sufficiently large.

(i) $\lim\limits_{x \to \infty} \dfrac{f(x)}{g(x)} = \infty$ or 0

then, $f(x)$ grows faster than $g(x)$.

(ii) $\lim\limits_{x \to \infty} \dfrac{f(x)}{g(x)} =$ finite value $\neq 0$

then, $f(x)$ and $g(x)$ grows at the same rate.

<Q>: Show that e^x is faster than x^2.

Soln.

Let $f(x) = e^x$ and $g(x) = x^2$

So,

$$\lim_{x \to \infty} \frac{e^x}{x^2} = \lim_{x \to \infty} \frac{e^x}{2x} = \lim_{x \to \infty} \frac{e^x}{2} = \infty$$

Thus, e^x is faster than x^2.

<Q>: Show that 3^x faster than 2^x.

Soln.

Let $f(x) = 3^x$ and $g(x) = 2^x$

So,

$$\lim_{x \to \infty} \frac{3^x}{2^x} = \lim_{x \to \infty} \frac{3^x}{2^x} \frac{\ln 3}{\ln 2} = \lim_{x \to \infty} \frac{3^x}{2^x} \cdot \ln 1$$

$= 0$

Thus. ~~f(x)~~ 3^x grows faster than 2^x.

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L-Hopital Rule:

Suppose that $f(a) = g(a) = 0$ and functions f & g are differentiable in open interval I containing a and $g'(x) \neq 0$ on I if $x \neq a$ then,

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)} \text{ until } g'(x) \neq 0.$$

<Q>: $\lim_{x \to \infty} x^{1/x}$

Soln:

Here,

$$y = x^{1/x}$$

Taking log on both sides,

$$\log y = \frac{1}{x} \log x.$$

or, $$\lim_{x \to \infty} \log y = \lim_{x \to \infty} \frac{1}{x} \log x$$

Using L-Hopital rule,

$$= \lim_{x \to \infty} \frac{1}{x}$$

$$\lim_{x \to \infty} \log y = 0$$

or $$\lim_{x \to \infty} y = e^0$$

or, $$\lim_{x \to \infty} y = 1$$

$$\therefore \lim_{x \to \infty} x^{1/x} = 1$$

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<Q>: $\lim\limits_{x \to 0} \dfrac{x - \sin x}{x^3}$

Soln.

$= \lim\limits_{x \to 0} \dfrac{x - \sin x}{x^3} \quad \left[\dfrac{0}{0} \text{ form}\right]$

Using L-Hopital rule,

$= \lim\limits_{x \to 0} \dfrac{1 - \cos x}{3x^2}$

$= \lim\limits_{x \to 0} \dfrac{\sin x}{6x}$

$= \lim\limits_{x \to 0} \dfrac{\cos x}{6} = \dfrac{1}{6}$

<Q>: $\lim\limits_{x \to 0} \dfrac{1}{x^2} - \dfrac{1}{\sin^2 x}$

Soln.

$\lim\limits_{x \to 0} \dfrac{1}{x^2} - \dfrac{1}{\sin^2 x} \quad \left[\dfrac{\infty}{\infty} \text{ form}\right]$

Using L-Hopital rule,

$\cancel{\lim\limits_{x \to 0} \dfrac{2}{x^3}} - \lim\limits_{x \to 0} \dfrac{\sin^2 x - x^2}{x^2 \sin^2 x}$

$= \lim\limits_{x \to 0} \dfrac{2 \sin x \cos x - 2x}{2 \sin x \cos x \cdot x^2 + \sin^2 x \cdot 2x}$

$= \lim\limits_{x \to 0} \dfrac{\sin x \cos x - x}{}$

<Q> : $\lim_{x \to 0} \frac{1}{x^2} - \frac{\cancel{\sin^2 x}\ 1}{\sin^2 x}$

Sol^n

$$\lim_{x \to 0} \frac{\sin^2 x - x^2}{x^2 \sin^2 x}$$

$$= \lim_{x \to 0} \frac{\sin^2 x - x^2}{x^4} \qquad \left[\because \lim_{x \to 0} \frac{\sin x}{x} = 1 \right]$$

$$= \lim_{x \to 0} \frac{\sin 2x - 2x}{4x^3} = \lim_{x \to 0} \frac{2\cos 2x - 2}{12x^2}$$

$$= \lim_{x \to 0} \frac{\cos 2x - 1}{6x^2} = \lim_{x \to 0} \frac{-\cancel{2}\sin 2x}{\cancel{6x}\, 2x}$$

$$= \lim_{x \to 0} \frac{-\cancel{2}\cos 2x}{\cancel{6}3} = \frac{-1}{3}.$$

Monotonic functions

The functions that are increasing or decreasing in the entire domain is called monotonic functions.

(*) First derivative test for Local Extrema:

Suppose 'c' be a critical point of a continuous function.

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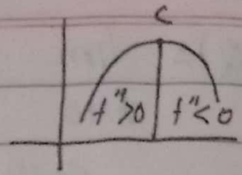
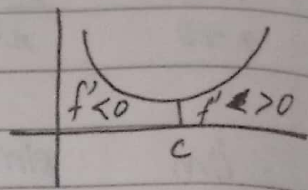
(i): If f' changes from +ve to −ve, then f has local maximum at c.



(ii): If f' changes from −ve to +ve, then f has local minimum at c.



(iii) If f' doesn't change its sign, then f has no extreme values at c.

⊛ Concavity:

The graph of a differentiable function $y = f(x)$ is concave up on an I where y' is increasing and concave down on I where y' is decreasing.

<Q>: Find local maxima and minimum value of $f(x) = x + 2\sin x$, $0 \le x \le 2\pi$.

Sol

Here,

$f(x) = x + 2\sin x$

$\therefore f'(x) = 1 + 2\cos x$.

For critical point, $f'(x) = 0$

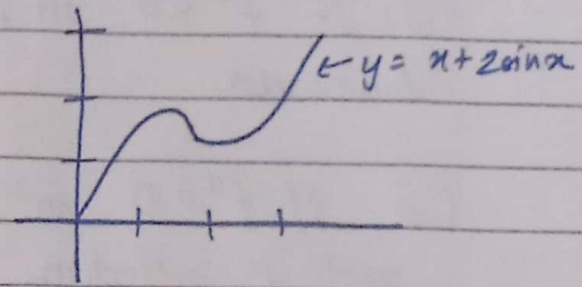
or $0 = 1 + 2\cos x$

$\therefore x = \frac{2\pi}{3}, \frac{4\pi}{3}$

Thus,

$f(0) = 0$ $\qquad\qquad$ $f(2\pi) \approx 6$

$f(2\pi/3) = \approx 3.83$ $\qquad\qquad$ $f(4\pi/3) \approx 2.46.$



To check increasing or decreasing:

$0 < x < 2\pi/3$	$2\pi/3 < x < 4\pi/3$	$4\pi/3 < x < 2\pi$
$f' > 0$	$f' < 0$	$f' > 0$
increasing	decreasing	increasing
+ve	-ve	+ve

From first derivative test ~~from~~ for local extreme value at $x = 2\pi/3$ and $x = 4\pi/3$,

At $x = 2\pi/3$, f' changes from +ve to -ve.
So, local maxima = 3.83 at $x = 2\pi/3$

At $x = 4\pi/3$, f' changes from -ve to +ve
So, local minima = 2.26 at $x = 4\pi/3$.

(*) <u>Second derivative test for Concavity</u>

Let $y = f(x)$ is differentiable on interval I.

(i): If $f'' > 0$ on I, then f over I is concave upward.

(ii): If $f'' < 0$ on I, then f over I is concave downward.

(iii) If $f'' = 0$ on I, then the point is called point of inflection.

⊛ Second derivative test for local extreme value.

Let c be the critical point of a continuous curve.

(i): If $f'(c) = 0$, $f''(c) < 0$, then f has local maximum value at $x = c$

(ii) If $f'(c) = 0$, $f''(c) > 0$, then f has local maximum value at $x = c$.

<Q>: Sketch the graph: $f(x) = x^4 - 4x^3 + 10$

Soln:

Given, $f(x) = x^4 - 4x^3 + 10$

(i): For symmetry;

Putting $x = -x$,

$f(-x) = (-x)^4 - 4x(-x)^3 + 10$

$= x^4 + 4x^3 + 10 \neq f(x)$

$\neq -f(x)$.

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This function has no symmetry.

(ii) Critical points:

$f(x) = x^4 - 4x^3 + 10$

$\therefore f'(x) = 4x^3 - 12x^2 = 4x^2(x-3)$

$\therefore f''(x) = 12x^2 - \cancel{24x}\ 24x = 12x(x-2)$

Putting $f'(x) = 0$,

$0 = \cancel{4x^3 - 12x^2}\ \cancel{4x}\ 4x^2(x-3)$

$\therefore x = 0, 3$

(iii) For increasing or decreasing:

Interval.	$(-\infty, 0)$	$(0, 3)$	$(3, \infty)$
$f'(x)$	-ve	-ve	+ve
	decreasing $f' < 0$	decreasing $f' < 0$	~~decreasing~~ increasing ~~$f' < 0$~~ $f' > 0$

The first derivative test for increasing and decreasing function shows f' changes from -ve to +ve at $x = 3$
. f is local minum at $x = 3$ $\therefore f_{min} = -17$
Here, there is no local ~~min~~ maximum.

(iv): For point of inflection:
Putting $f''(x) = 0$.
or, $12x(x-2) = 0$ $\therefore x = 0, 2$.
~~Here,~~ So, $(0, 10)$ and $(2, -6)$ is the point of inflection.

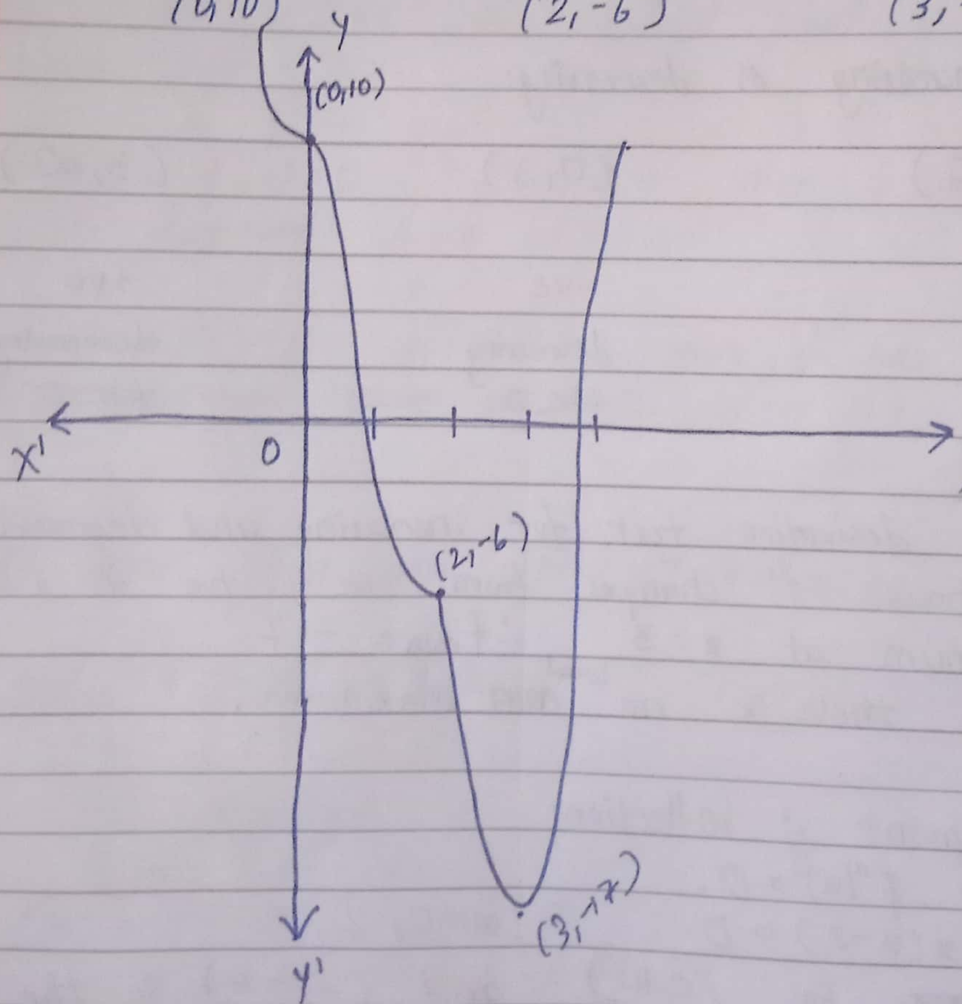
(v): For concavity:

Interval	$(-\infty, 0)$	$(0, 2)$	$(2, \infty)$
$f''(x)$	+ve	-ve	+ve
	increasing $f''>0$	decreasing $f''<0$	increasing $f''>0$

The second derivative test for concavity shows f is concave upward in interval $(-\infty, 0)$ and $(2, \infty)$ and concave downward in interval $(0, 2)$.

We have, ~~(0, 10)~~ three points,

(0, 10) (2, -6) (3, -17)



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<Q>: sketch the graph: $f(x) = x^3 - 3x + 3$.

Soln

Given, $f(x) = x^3 - 3x + 3$

(i): For symmetry:

Putting $x = -x$.

$f(-x) = (-x)^3 - 3x(-x) + 3$

$= -x^3 + 3x + 3 \neq f(x) \neq -f(x)$

This function has no symmetry.

(ii) For critical points:

$f(x) = x^3 - 3x + 3$

$\therefore f'(x) = 3x^2 - 3$

Putting $f'(x) = 0$

$3x^2 = 3$

$\therefore x = \pm 1$

(iii) For increasing and decreasing function.

Interval	$(-\infty, -1)$	$(-1, 1)$	$(1, \infty)$
$f'(x)$	+ve	-ve	+ve
	increasing	decreasing	increasing

From first derivative test for local extreme value at $x = 1$ and $x = -1$.

when f' changes from positive to negative at $x = -1$, it has local maximum value. $f_{max} = 5$

When f' changes from negative to positive at $x = 1$, it has local minimum value $f_{min} = 1$

So, two points are $(-1, 5)$ and $(1, 1)$

(iv): For point of inflection:

~~$f''(x) = 0$~~

$f'(x) = 3x^2 - 3$

$\therefore f''(x) = 6x$

Putting $f''(x) = 0$

or $6x = 0$ $\therefore x = 0$

Thus, point of inflection is $(0, 3)$.

(v) Concavity:

Int.	$(-\infty, 0)$	$(0, \infty)$
$f''(x)$	-ve decreasing	+ve increasing

from second derivative test for concavity, it shows that

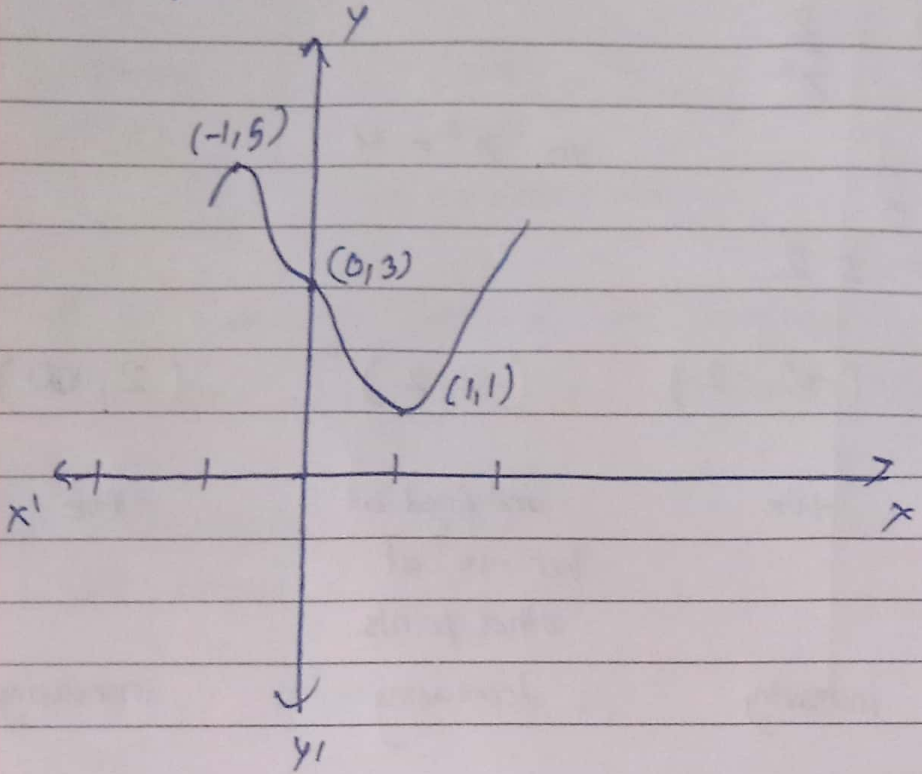
f is concave upwards in $(0, \infty)$

and f is concave downwards in $(-\infty, 0)$.

We have points;

$(0,3)$, $(-1,5)$, $(1,1)$

(iv): Graph.



<Q>: ~~ff~~ Sketch the graph: $f(x) = \frac{x^2+4}{2x}$

Sol^n.

(I): For symmetry:

Putting $x = -x$.

$$f(-x) = \frac{(-x)^2+4}{2\times(-x)} = -\left(\frac{x^2+4}{2x}\right) = -f(x)$$

This function is symmetry about origin.

(ii) For increasing or decreasing.

$$f(x) = \frac{x^2+4}{2x}$$

$$\therefore f'(x) = \frac{1}{2} - \frac{2}{x^2}$$

Putting $f'(x) = 0$.

$$0 = \frac{1}{2} - \frac{2}{x^2}$$

or $\frac{2}{x^2} = \frac{1}{2}$ or $x^2 = 4$

$\therefore x = \pm 2$

Interval.	$(-\infty, -2)$	$(-2, 2)$	$(2, \infty)$
$f'(x)$	+ve	undefined at 0 but -ve at other points.	+ve
	increasing	decreasing	increasing

From the first derivative test to find local maxima and local minima, we observe that $f'(x)$ is undefined at $x = 0$.

Hence, the curve of $f(x)$ increases, decreases and then increases but doesn't intersect at $x = 0$.

(iv): ~~For~~ Point of inflection.

$$f''(x) = \frac{4}{x^3}$$

Putting $f''(x) = 0$, ~~it is undefined.~~

$$0 = \frac{4}{x^3}$$

$\therefore x = \infty$

Here, point of inflection doesn't exist.
i.e. condition for asymptote

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(v): Concavity:

Interval	$(-\infty, 0)$	$(0, \infty)$
$f''(x)$	-ve	+ve
	concave downward	concave upwards

f is concave upward for interval $(0, \infty)$ and concave downward for interval $(-\infty, 0)$.

(vi) Asymptote:
We know,

$$\lim_{x \to 0} \frac{x^2+4}{2x} = \infty.$$

So, $x = 0$ is vertical asymptote.

$$\lim_{x \to \infty} \frac{x^2+4}{2x} = \infty.$$

So, horizontal asymptote doesn't exist

but $n^\circ > d^\circ$. So, oblique asymptote may exist.

$$2x \,)\, x^2 + 4 \,(\, x/2 + 2/x$$
$$\underline{-\,x^2}$$
$$+4$$
$$\underline{-\,4}$$
$$0$$

So, $y = \frac{x}{2}$ is oblique asymptote.

We have, $x = 0 \Rightarrow VA$ $\quad y = x/2 \Rightarrow O.A.$
points $(2, 2)$ and $(-2, -2)$

Graph.