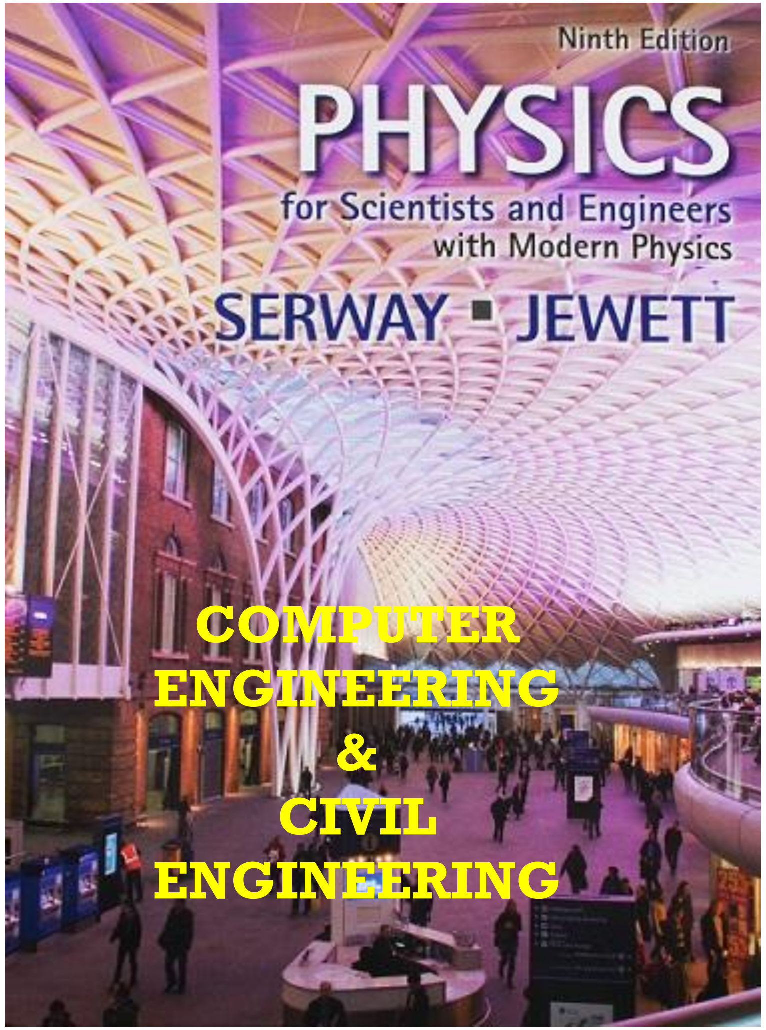
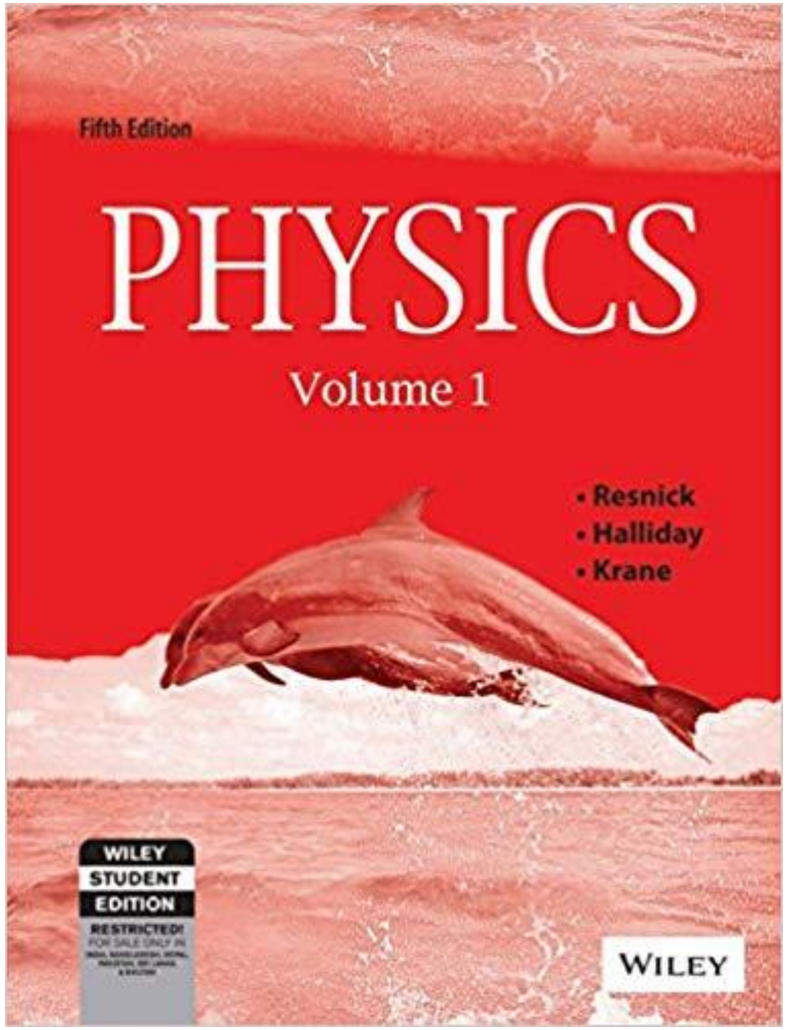
PHYSICS

General Physics I (PHYS 101)





Ganesh Kuwar Chhetri

Dept. of Physics, School of Science, Kathmandu University

April 08, 2021
Tuesday, 12.10 am - 1.50 am

Course Outline

- Work Energy Theorem
- Conservative and Non-Conservative Forces
- Force as Negative Gradient of Potential Energy
- Conservation of Linear Momentum
- Centre of Mass
- System of Variable Mass

Work –Energy Theorem

Kinetic Energy

- **Kinetic energy** *K* is energy associated with the *state of motion* of an object.
- For an object of mass *m* whose speed is well below the speed of light,

$$K = \frac{1}{2} m v^2 \qquad \text{(kinetic energy)}$$

- The faster the object moves, the greater is its kinetic energy.
- It is a scalar quantity.
- The SI unit of kinetic energy is the **joule** (J).
- Kinetic energy can never be negative.

Kinetic Energy : $\boxed{K = \frac{1}{2} m v^2 = \frac{1}{2} m \vec{v} \cdot \vec{v} = \frac{P^2}{2m}}$

Work-Energy Theorem

- The net work done by the forces acting on a body is equal to the change in the kinetic energy of the body:

$$W_{net} = \Delta K = K_f - K_i$$

Example

- Equal amount of work are performed on two bodies, A and B, initially at rest, and of masses M and 2M respectively. Find the relation between their speeds immediately after the work has been done on them.

Hint:

$$W_{net}^{A} = W_{net}^{B}$$

or $$\frac{1}{2} M_A v_A{}^2 - 0 = \frac{1}{2} M_B v_B{}^2 - 0$$

or $$M v_A{}^2 = 2M v_B{}^2$$

$$\Rightarrow \boxed{v_A = \sqrt{2}\, v_B}$$

Work –Energy Theorem

Work-Energy Theorem with a Constant Force

- Let a constant net force $\vec{F}_{net}$ acts on a body of mass *m*. As the body moves through displacement $\vec{s}$, this net force causes its velocity to change from $\vec{v}_i$ to $\vec{v}_f$.

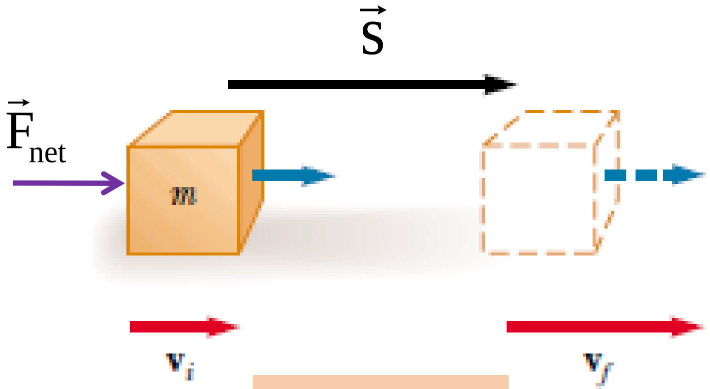


Figure WE-I

- According to Newton's second law,

$$\vec{F}_{net} = m\vec{a}$$

$$= m\left[\frac{\vec{v}_f - \vec{v}_i}{\Delta t}\right] \qquad .... \quad (1)$$

where Δt is the time interval for the body to move through the displacement $\vec{s}$.

- The net work done by the constant net force is

$$W_{net} = \vec{F}_{net} \cdot \vec{s}$$

$$= m\left[\frac{\vec{v}_f - \vec{v}_i}{\Delta t}\right] \cdot \left[\frac{\vec{v}_f + \vec{v}_i}{2}\right]\Delta t$$

$$= \frac{1}{2} m\left[\left(\vec{v}_f - \vec{v}_i\right) \cdot \left(\vec{v}_f + \vec{v}_i\right)\right]$$

$$= \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

$$= K_f - K_i$$

$$\therefore \boxed{W_{net} = \Delta K}$$

$$\because \vec{v}_{av} = \frac{\vec{s}}{\Delta t} = \frac{\vec{v}_f + \vec{v}_i}{2}$$

$$\Rightarrow \vec{s} = \left(\frac{\vec{v}_f + \vec{v}_i}{2}\right)\Delta t$$

Work –Energy Theorem

Work-Energy Theorem with a Variable Force

- Consider a body of mass m, moving along an axis and acted on by a net force $F_{net,x}$ that is directed along that axis.
- As the body moves through displacement $\vec{s}$, this net force causes its speed to change from $\vec{v}_i$ to $\vec{v}_f$.

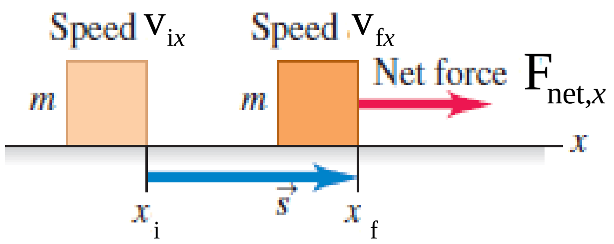


Figure WE-2

- According to Newton's second law,

$$F_{net,x} = ma_x = m\frac{dv_x}{dt} = m\frac{dv_x}{dx}\frac{dx}{dt}$$

$$= m\frac{dv_x}{dx}v_x$$

$$\therefore \boxed{F_{net,x} = mv_x\frac{dv_x}{dx}} \quad .... \quad (1)$$

- The net work done by the net variable force $F_{net,x}$ is

$$\boxed{\begin{aligned} W_{net} &= \int F_{net,x}\,dx \\ &= \int mv_x \frac{dv_x}{dx}\,dx \\ &= \int mv_x\,dv_x \end{aligned}}$$

- The variable of integration is now the velocity v_x. Let us integrate from initial velocity v_{ix} to final velocity v_{fx} :

$$W_{net} = \int_{v_{ix}}^{v_{fx}} mv_x\,dv_x = m\int_{v_{ix}}^{v_{fx}} v_x\,dv_x$$

$$= m\left[\frac{1}{2}\left(v_{fx}{}^2 - v_{fx}{}^2\right)\right]$$

$$= \frac{1}{2}mv_{fx}{}^2 - \frac{1}{2}mv_{ix}{}^2 = K_f - K_i$$

$$\therefore W_{net} = \Delta K$$

Sample Problem

- **An ideal spring S can be compressed 1.0 meter by a force of 100 N. This same spring is placed at the bottom of a frictionless inclined plane which makes an angle of θ = 30° with horizontal as shown in Figure SP-1. A 10-kg mass M is released from rest at the top of the incline and is brought to rest momentarily after compressing the spring 2.0 meters.**

(a) Through what distance does the mass slide before coming to rest?
(b) What is the speed of the mass just before it reaches the spring?

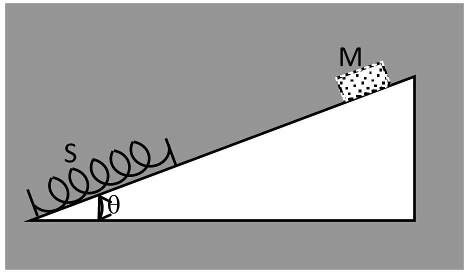


Figure SP-1

Hint:
Spring constant of the spring, $k = \frac{F}{x} = \frac{100\text{ N}}{1\text{ m}} = 100\text{ N/m}$

(a) Let 'd' be the required distance through which the mass slides before coming to rest.
According to the work-energy theorem,

$$\boxed{\begin{aligned} W_{net} &= \Delta K \\ \text{or} \quad W_g + W_s + W_N &= \Delta K \end{aligned}}$$

or $\quad mg \sin\theta\, d - \frac{1}{2}kx^2 + 0 = 0$

$$\therefore \quad d = \frac{\frac{1}{2}kx^2}{mg \sin\theta} = \frac{\frac{1}{2}\times 100\times(2)^2}{10\times 9.8\times \sin 30^0} = 4\text{ m}$$

(b) Let 'V_f' be the speed of the mass just before it reaches the spring.
According to the work-energy theorem,

$$\boxed{\begin{aligned} W_{net} &= K_f - K_i \\ \text{or} \quad W_g + W_N &= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \end{aligned}}$$

or $\quad mg\sin\theta(d-x) + 0 = \frac{1}{2}mv_f^2 - 0$

$$\therefore \quad v_f = \sqrt{2\times 9.8\times \sin 30^0\times(4-2)}$$
$$= 4.5\text{ m/s}$$

Potential Energy

Potential Energy

- **The potential energy U** is energy associated with the configuration of a system in which a conservative force acts.

Configuration:	Configuration means how the parts of a system are located or arranged with respect to one another. For example: The compression or stretching of the spring in the block- spring system The height of the ball in the ball-Earth system

- The potential energy is a function of position whose negative derivative gives the force:

$$\boxed{F_x(x) = -\frac{dU(x)}{dx}}$$

- When the conservative force does work W on a particle within the system, the change in the potential energy of the system is

$$\boxed{\Delta U = U_f - U_i = -W}$$

- The difference in potential energy between any two locations x_i and x_f for a particle on which force $F_x(x)$ acts is calculated by

$$\boxed{U(x_f) - U(x_i) = -\int_{x_i}^{x_f} F_x(x)\,dx}$$

- The difference in potential energy between any two location for a particle on which force acts is calculated by

$$\boxed{U(x) - U(x_0) = -\int_{x_0}^{x} F_x(x)\,dx}$$

Example

A particle of mass m moves in one dimensional motion in a region where its potential energy is given by $U(x) = \frac{A}{x^3} - \frac{B}{x}$.

The force on the particle

$$F = -\frac{dU(x)}{dx} = -\frac{d}{dx}\left[\frac{A}{x^3} - \frac{B}{x}\right] = \frac{3A}{x^4} - \frac{B}{x^2}$$

Sample Problem

~~The potential energy function~~ for the force between two atoms in a diatomic molecule can be expressed approximately as follows: $\boxed{U(x)=\frac{a}{x^{12}}-\frac{b}{x^{6}}}$,

where a and b are positive constants and x is the distance between atoms.

(a) At what values of x is $U(x)$ equal to zero? At what values of x is a $U(x)$ minimum?

(b) Determine the force between the atoms.

(c) What is the dissociation energy of the molecule?

Hint:

(a) $$\frac{a}{x^{12}}-\frac{b}{x^{6}}=0$$

or $$\frac{a}{x^{12}}=\frac{b}{x^{6}}$$

$$\therefore \quad x=\left(\frac{a}{b}\right)^{\frac{1}{6}}$$

$U(x)$ also becomes zero as $x \rightarrow \infty$

The values of x at which $U(x)$ is a minimum is found from $$\left(\frac{dU(x)}{dx}\right)_{x=x_m}=0$$

That is,

$$-\frac{12a}{x_m^{\;13}}+\frac{6b}{x_m^{\;7}}=0$$

or $$x_m^{\;6}=\frac{2a}{b}$$

$$\therefore \quad x_m=\left(\frac{2a}{b}\right)^{\frac{1}{6}}$$

(b) $$F_x(x)=-\frac{dU(x)}{dx}=-\frac{d}{dx}\left[\frac{a}{x^{12}}-\frac{b}{x^{6}}\right]$$
$$=\frac{12a}{x^{13}}-\frac{6b}{x^{7}}$$

(c) Dissociation Energy

$$E_d=U(x=\infty)-U(x=x_m)=-U(x=x_m)$$
$$=-\frac{a}{x_m^{\;12}}+\frac{b}{x_m^{\;6}}=-\frac{a}{\left(\frac{2a}{b}\right)^2}+\frac{b}{\left(\frac{2a}{b}\right)}$$

$$\therefore \quad E_d=\frac{b^2}{4a}$$

DYNAMICS OF SYSTEM OF PARTICLES

Gravitational and Elastic Potential Energy

Gravitational Potential Energy

- The potential energy associated with a system consisting of Earth and a nearby particle is gravitational potential energy.
- If the particle moves from height y_i to height y_j the change in the gravitational potential energy of the particle-Earth system is

$$\Delta U = U(y_f) - U(y_i) = -\int_{y_i}^{y_f} F_y(y)dy = -\int_{y_i}^{y_f} (-mg)dy = mg(y_f - y_i)$$

- If the reference point of the particle is set as $y_0 = 0$ [at the surface of the earth] and the corresponding gravitational potential of the system is set as $U(y_0) = 0$ then the gravitational potential energy when the particle is at a height is

$$U(y) = -\int_0^y F_y(y)dy = -\int_0^y (-mg)dy$$

$$\therefore U(y) = mgy$$

Elastic Potential Energy

- **Elastic potential energy** is the energy associated with the state of compression or extension of an elastic object.
- For a spring that exerts a spring force $F = -k\,x$ when its free end has displacement x, the elastic potential energy is

$$U(x) = \frac{1}{2} k\, x^2$$

- The **reference configuration** has the spring at its relaxed length, at which $x = 0$ and U = 0 .

Path independence of conservative forces

- Suppose a particle, acted on by a conservative force, moves from a to b along the path 1 and back from b to a along path 2 as shown in Figure C-1.

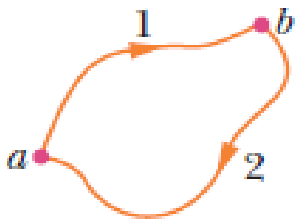


Figure C-1

The net work done by a conservative force on a particle moving around any closed path is zero.

That is,

$$W_{ab,1} + W_{ba,2} = 0$$

where $W_{ab,1} \rightarrow$ the work done by the force when the particle moves from a to b along path 1

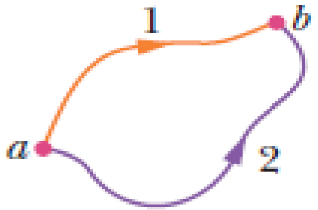
$W_{ba,2} \rightarrow$ the work done by the force when the particle moves from b to a along path 2

or, $$\int_{a\ (\text{path 1})}^{b} \vec{F}\cdot d\vec{s} + \int_{b\ (\text{path 2})}^{a} \vec{F}\cdot d\vec{s} = 0$$

or, $$\int_{a\ (\text{path 1})}^{b} \vec{F}\cdot d\vec{s} = -\int_{b\ (\text{path 2})}^{a} \vec{F}\cdot d\vec{s}$$

or, $$\int_{a\ (\text{path 1})}^{b} \vec{F}\cdot d\vec{s} = \int_{a\ (\text{path 2})}^{b} \vec{F}\cdot d\vec{s}$$

$$\therefore \quad \boxed{W_{ab,1} = W_{ab,2}}$$



The work done by a conservative force on a particle moving between two points does not depend on the path taken by the particle.

Conservative Forces

The Force of Gravity

- Figure C-2 shows an example of a system consisting of a ball acted on by the Earth's gravity.

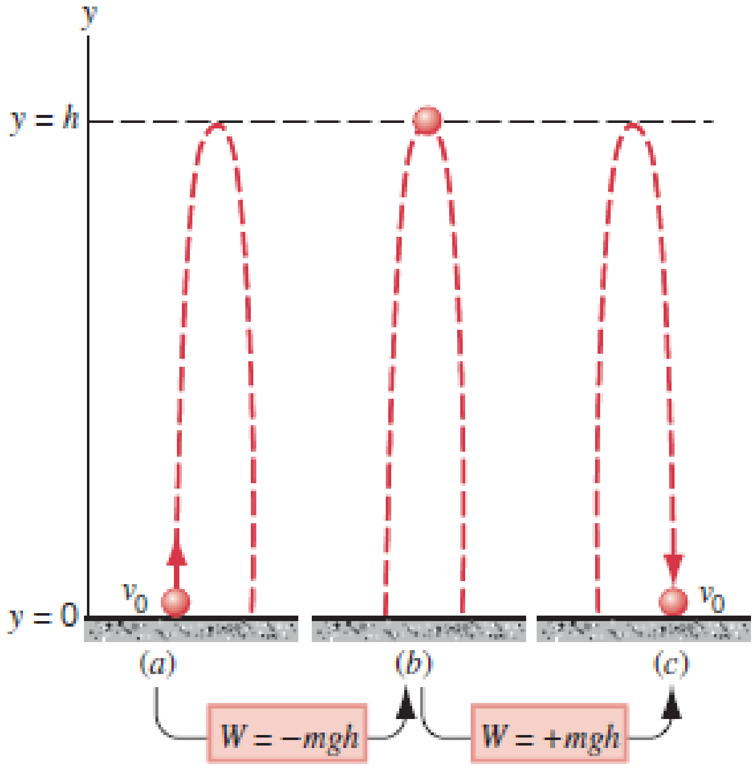


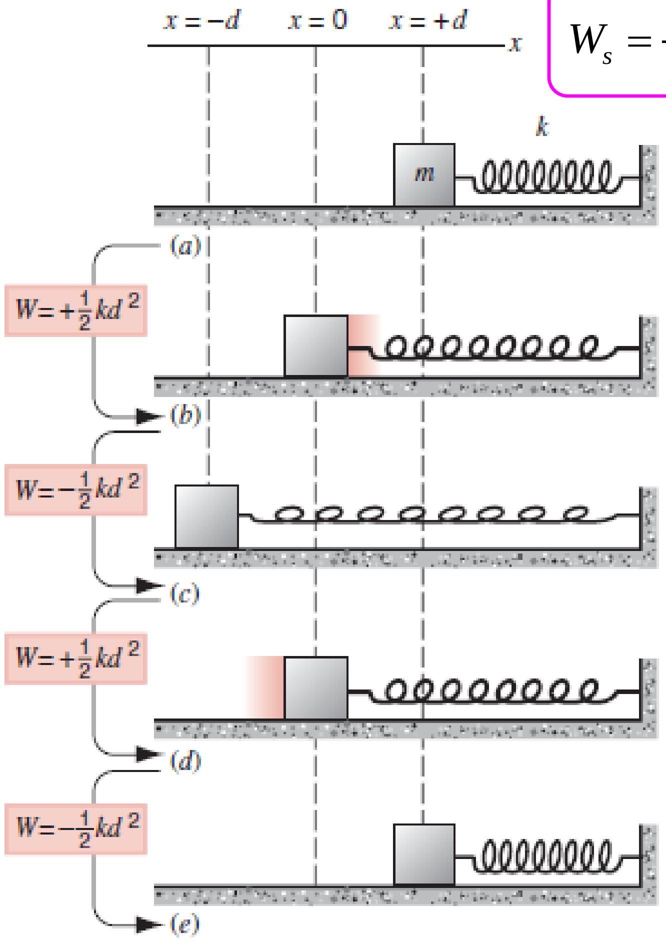
Figure C-2 A ball is thrown upward against the Earth's gravity. In *(a) it is just leaving its starting point, in (b) it has* reached the top of its trajectory, and in *(c) it has returned to its* original height. The work done by the Earth's gravity between the pairs of successive positions is shown in the boxes at the bottom.

The total work done on the ball by the force of gravity and by the spring force on the block for the round trip is zero

The Spring Force

Figure 12-1 shows a block of mass *m attached to a spring of force constant k; the block slides* without friction across a horizontal surface.

$$W_s = -\frac{1}{2}k\left(x_f^{\,2} - x_i^{\,2}\right)$$



Non- Conservative Force

The Frictional Force

- Figure C-3 shows a disk of mass m on the end of a thin but rigid rod of length R.

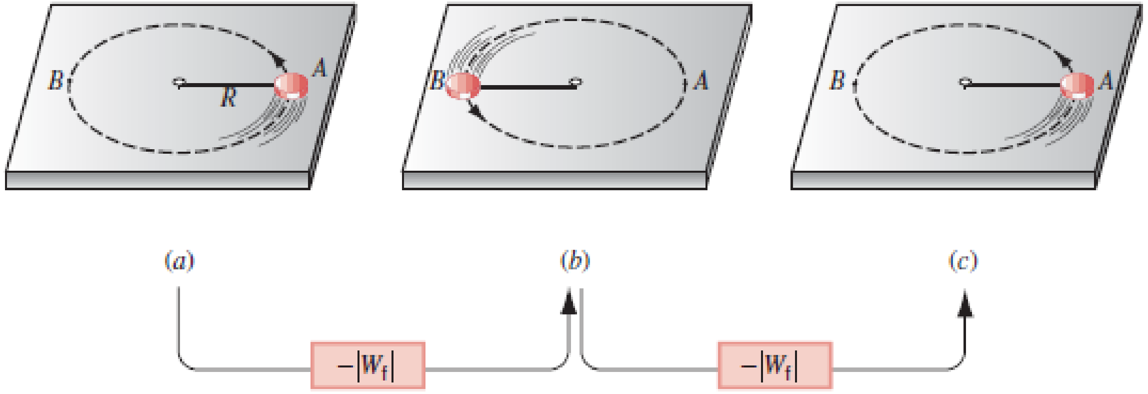


Figure C-3 A disk moves with friction in a circle on a horizontal surface. The positions shown represent (*a*) an arbitrary starting point *A*, *(b) one half* revolution later (at *B*), *and (c) another* half revolution later (back at *A*).

The work done by friction between successive positions is indicated in the boxes at the bottom.

The total work done by the frictional force on the disk is not zero for the round trip , but instead has the negative value $-2|W_f|$.

Conservative Force as Negative Gradient of Potential Energy

Conservative Force as Negative Gradient of Potential Energy $\vec{F} = -\nabla U$

- If a particle acted upon by a conservative force $\vec{F}$ moves from space point (x_0, y_0, z_0) described by the position vector $\vec{r}_0$, to another space point (x, y, z) described by the position vector $\vec{r}_0$ is

$$U(\vec{r}) = -\int_{\vec{r}_0}^{\vec{r}} \vec{F}\cdot d\vec{r} \qquad \text{..........} \quad (1)$$

where $\vec{r}_0$ refers to the position of zero potential energy.

- Expressing $\vec{F}$ and $d\vec{r}$ in rectangular co-ordinates, we have

$$\vec{F}\cdot d\vec{r} = \left(F_x\hat{i} + F_y\hat{j} + F_z\hat{k}\right)\cdot\left(dx\hat{i} + dy\hat{j} + dz\hat{k}\right)$$
$$= F_x dx + F_y dy + F_z dz$$

$$\therefore \qquad U(\vec{r}) = -\int_{\vec{r}_0}^{\vec{r}} \vec{F}\cdot d\vec{r}$$
$$= -\int_{\vec{r}_0}^{\vec{r}} \left(F_x dx + F_y dy + F_z dz\right)$$
$$= -\int_{x_0}^{x} F_x dx - \int_{y_0}^{y} F_y dy - \int_{z_0}^{z} F_z dz \qquad \text{..........} \quad (2)$$

Differentiating the equation (2) partially with respect to x, y, z, we get

$$F_x = -\frac{\partial U}{\partial x}, \qquad F_y = -\frac{\partial U}{\partial y} \qquad \text{and} \qquad F_z = -\frac{\partial U}{\partial z}$$

Now,

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k} = \hat{i}F_x + \hat{j}F_y + \hat{k}F_z$$
$$= \hat{i}\left(-\frac{\partial U}{\partial x}\right) + \hat{j}\left(-\frac{\partial U}{\partial y}\right) + \hat{k}\left(-\frac{\partial U}{\partial z}\right)$$
$$= -\left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)U = -\nabla U$$

$$\therefore \qquad \boxed{\vec{F} = -\nabla U = -\text{grad } U}$$

Sample Problem

- **A force acting on a particle moving in the plane is given by $\vec{F}=(2y\hat{i}+x^2\hat{j})$ N, where x and y are in meters. The particle moves from the origin to a final position having coordinates $x=5.00\ m$ and $y=5.00\ m$ and as shown in Figure C_c. Calculate the work done by on the particle as it moves along**

(a)OAC, (b) OBC, and (c) OC. (d) Is conservative or nonconservative? Explain

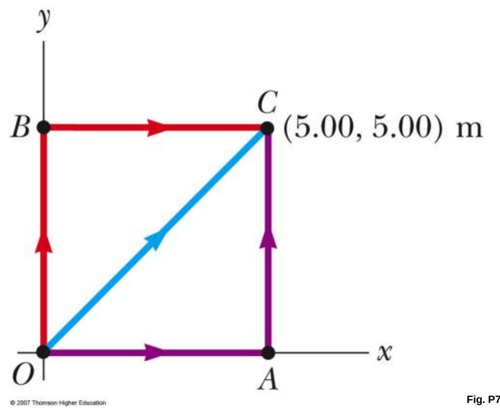


Fig. P7-33

Hint:

(a)

$$W_{OAC}=W_{OA}+W_{AC}=\int_0^{5.00}\left(2y\hat{i}+x^2\hat{j}\right)\cdot dx\hat{i}+\int_0^{5.00}\left(2y\hat{i}+x^2\hat{j}\right)\cdot dy\hat{j}=\int_0^{5.00}2y\ dx+\int_0^{5.00}x^2dy$$

$$=0+x^2\int_0^{5.00}dy \qquad [\because \text{ Along the path OA, y = 0, So } W_{OA}=0]$$

$$=(5)^2\left[(y)\Big|_0^{5.00}\right] = 125\text{ J}$$

(b)

$$W_{OBC}=W_{OB}+W_{BC}=\int_0^{5.00}\left(2y\hat{i}+x^2\hat{j}\right)\cdot dy\ \hat{j}+\int_0^{5.00}\left(2y\hat{i}+x^2\hat{j}\right)\cdot dx\ \hat{i}=\int_0^{5.00}x^2dy+\int_0^{5.00}2y dx$$

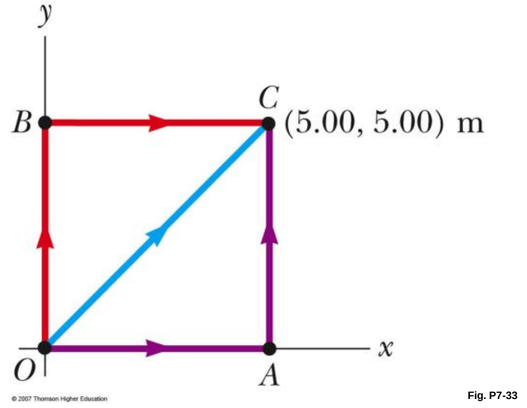
$$=0+2y\left[\int_0^{5.00}dx\right] \qquad [\because \text{ Along the path OB, x = 0, So } W_{OB}=0]$$

$$=10\left[(x)\Big|_0^{5.00}\right] = 50\text{J}$$

Sample Problem

- **A force acting on a particle moving in the plane is given by $\vec{F} = (2y\hat{i} + x^2\hat{j})$ N , where x and y are in meters. The particle moves from the origin to a final position having coordinates $x = 5.00\ m$ and $y = 5.00\ m$ and as shown in Figure C_c. Calculate the work done by on the particle as it moves along**

(a)OAC, (b) OBC, and (c) OC. (d) Is conservative or nonconservative? Explain



Hint:

(c)

$$W_{OC} = \int \vec{F}\cdot d\vec{S} = \int \left(2y\hat{i} + x^2\hat{j}\right)\cdot\left(dx\,\hat{i} + dy\,\hat{j}\right) = \int \left(2y\,dx + x^2 dy\right)$$

$$= \int \left(2x\,dx + x^2 dx\right) \qquad [\because \text{ Along the path OC, x = y, So dx=dy}]$$

$$= \int_{o}^{5.00} \left(2x + x^2\right) dx$$

$$= \left(x^2 + \frac{x^3}{3}\right)\Bigg|_0^{5.00}$$

$$= 25\text{ J} + \frac{125}{3}\text{J} = 66.7\text{J}$$

(d)

$\because W_{OAC} \neq W_{OBC} \neq W_{OC}$

The work done is path dependent.

So $\vec{F}$ is non-conservative.

Total Work Done by Conservative and Non-Conservative Forces

Total Work Done by Conservative Force

- Consider an isolated system in which only conservative forces act.

According to work-energy theorem:

The net work done by the forces acting on a body is equal to the change in the kinetic energy of the body.

$$W_{net} = \Delta K$$

$$\text{or, } W_1 + W_2 + \ldots + W_n = \Delta K$$

$$\text{or, } \sum W_c = \Delta K$$

where $\sum W_c$ is the total work done on the particle by conservative forces

$$\text{or, } \sum W_c = -\Delta U \quad [\because \Delta K + \Delta U = 0]$$

$$\therefore \boxed{\sum W_c = -\Delta U = \text{Negative Change in potential energy}}$$

In an isolated system in which only conservative forces act, the total mechanical energy remains constant.

$$K + U = \text{constant}$$

Total Work Done by Non-Conservative Force

- Consider an isolated system in which conservative forces and non-conservative forces act.

According to work-energy theorem:

The net work done by the forces acting on a body is equal to the change in the kinetic energy of the body.

$$W_{net} = \Delta K$$

$$\text{or, } W_1 + W_2 + \ldots + W_n = \Delta K$$

$$\text{or, } \sum W_c + \sum W_{nc} = \Delta K$$

where $\sum W_c$ is the total work done on the particle by conservative forces

$\sum W_{nc}$ is the total work done by non-conservative forces

$$\text{or, } -\Delta U + \sum W_{nc} = \Delta K \quad \left[\because \sum W_c = -\Delta U\right]$$

$$\therefore \boxed{\begin{aligned}\sum W_{nc} &= \Delta K + \Delta U \\ &= \text{change in total mechanical energy}\end{aligned}}$$

Principle of Conservation of Energy

- Consider an isolated system in which conservative forces and non-conservative forces act.

According to work-energy theorem:

The net work done by the forces acting on a body is equal to the change in the kinetic energy of the body.

$$W_{net} = \Delta K$$

or, $W_1 + W_2 + \ldots + W_n = \Delta K$

or, $\sum W_c + \sum W_{nc} = \Delta K$ where $\sum W_c$ is the total work done on the particle by conservative forces

$\sum W_{nc}$ is the total work done by non-conservative forces

$$\left[\because \sum W_c = -\Delta U, -\sum W_{nc} = \Delta Q\right.$$

or, $-\Delta U - \Delta Q = \Delta K$ where Q is the other forms of energy (heat, sound, light etc.) except mechanical energy$\left.\right]$

or, $\Delta K + \Delta U + \Delta Q = 0$

$\therefore$ $K + U + Q =$ constatnt

The total energy – kinetic plus potential plus other forms does not change.

Energy may be transformed from one kind to another, but it cannot be created or destroyed; the total energy is constant.

This is the principle of conservation of energy.

Collision

Collision

- A collision is an isolated event in which two or more bodies [the colliding bodies] exert relatively strong forces on each other for a relatively short time.
- In collision of all kinds, momentum is always conserved.

Examples:

- A bat strikes a base ball.
- A cat walks delicately through the grass.
- The deflection suffered by an alpha particle in passing close to a nucleus.
- Neutrons hitting atomic nuclei in a nuclear reactor.

Classifying collision:

- Elastic: A collision in which kinetic energy is conserved.
- Inelastic: A collision in which the total kinetic energy decreases.
- Completely Inelastic: An inelastic collision in which the colliding bodies have a common final velocity.

Linear Momentum

- The linear momentum $\vec{P}$ of a body is defined as the product of its mass and its velocity: $\vec{P} = m\vec{v}$
- It is a vector quantity.
- The SI unit of linear momentum is $\mathrm{kg\ m\ s^{-1}}$.
- Newton's second law in terms of momentum:

The rate of change of momentum of a body is equal to the resultant force acting on the body and is in the direction of that force.

$$\boxed{\sum \vec{F} = \frac{d\vec{P}}{dt}}$$

Law of Conservation of Linear Momentum

When the net external force acting on a system is zero, the total linear momentum of the system remains constant.

$$\boxed{\vec{P} = \text{ constant } \left(\text{closed, isolated system}\right)}$$

For a closed, isolated system,

$$\vec{P}_i = \vec{P}_f$$

$$\begin{pmatrix}\text{total linear momentum} \\ \text{at some initial time } t_i\end{pmatrix} = \begin{pmatrix}\text{total linear momentum} \\ \text{at some later time } t_f\end{pmatrix}$$

Impulse and Impulse Momentum Theorem

Impulse

- The **impulse** of the net force, denoted by $\vec{J}_{net}$ is defined to be the product of the net force and the time interval:

$$\boxed{\vec{J}_{net} = \sum\vec{F}\,(t_2 - t_1) = \sum\vec{F}\,\Delta t}$$

where is $\sum\vec{F}$ the constant net force acting on a particle during a time interval Δt from t_1 to t_2

- If the net force $\sum\vec{F}$ varies in magnitude and acts on a particle from a time t_i to t_f, then the impulse of the net force is

$$\boxed{\vec{J}_{net} = \int_{t_i}^{t_f} \sum\vec{F}\,dt}$$

- SI unit of impulse - newton-second

Impulse Momentum Theorem

- Newton's second law in terms of momentum: $\sum\vec{F} = \frac{d\vec{P}}{dt}$ ....(1)

Integrating both sides of Eq. (1) over time between the limits t_i to t_f, we get

$$\int_{t_i}^{t_f} \sum\vec{F}\,dt = \int_{t_i}^{t_f} \frac{d\vec{P}}{dt}\,dt = \int_{t_i}^{t_f} d\vec{P} = \vec{P}_f - \vec{P}_i$$

$$\therefore \quad \vec{J}_{net} = \Delta\vec{P}$$

$$\boxed{F\,\Delta t = mv_f - mv_i}$$

The impulse of the net force acting on a particle during a given time interval is equal to the change in momentum of the particle during that interval

Sample Questions

- If you know the impulse that has acted on a body of mass *m* you can calculate

  [a] its initial velocity.

  [b] its final velocity.

  [c] its final momentum.

  [d] the change in its velocity. $\Re$

- A 0.28 kg stone you throw rises 34.3 m in the air. The magnitude of the impulse the stone received from your hand while being thrown is

  [a] 0.27 N . s [b] 2.7 N . s [c] 7.3 N . S $\Re$ [d] 9.6 N . s

- A 3.00 kg stone is dropped from a 39.2 m high building. When the stone has fallen 19.6 m, the magnitude of the impulse it has received from the gravitational force is 58.8 N S.

Hint: Impulse-Momentum Theorem

$$J_{\text{net}} = mv - mu$$
$$= m\sqrt{2gh} - 0$$
$$= 3.00\sqrt{2 \times 9.8 \times 19.6}$$
$$= 58.8 \text{ N S}$$

Centre of Mass

Centre of Mass

- The ***center of mass*** of a system of particles is the point that moves as though (1) all of the system's mass were concentrated there and (2) all external forces were applied there.
- Suppose we have a system of particles $p_1, p_2, ..., p_N$ with masses $m_1, m_2, ..., m_N$, and position vectors $\vec{r}_1, \vec{r}_2, ..., \vec{r}_N$ respectively. Then, the ***centre of mass*** of S is the point of space whose position vector $\vec{r}_{cm}$ is defined by

$$\vec{r}_{cm} = \frac{\sum_{i=1}^{N} m_i \vec{r}_i}{\sum_{i=1}^{N} m_i} = \frac{\sum_{i=1}^{N} m_i \vec{r}_i}{M}$$

where M is the total mass of the system S.

For two particle system,

$$\vec{r}_{cm} = \frac{m_1 \vec{r}_1 + m_1 \vec{r}_2}{m_1 + m_2}$$

$$(x_{cm}, y_{cm}) = \left(\frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}, \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} \right)$$

The centre of mass of a system of particles is a mass-weighted average position of the particles.

Centre of mass of Solid Objects

$$\vec{r}_{cm} = \frac{1}{M} \int \vec{r}\ dm$$

First Moment of Mass for the system of particles

$$\sum_i m_i \vec{r}_i \ \& \int \vec{r}\ dm$$

- The centre of mass of the rigid body may or may not lie within the body.

Centre of Mass of a Homogeneous Semicircular Plate

- Let us consider a homogenous semicircular plate of radius R and mass M as shown in Figure S-1.

The homogeneous semicircular plate has rotational symmetry about the y-axis so that the centre of mass must lie on the y-axis.

Consider a thin strip of mass of this homogeneous semicircular plate.

Area of the thin strip, $da = 2\, x\, dy$

Mass of the thin strip, $dm = \dfrac{M}{\pi R^2/2}\, 2\, x\, dy = \dfrac{4M}{\pi R^2}\, x\, dy$

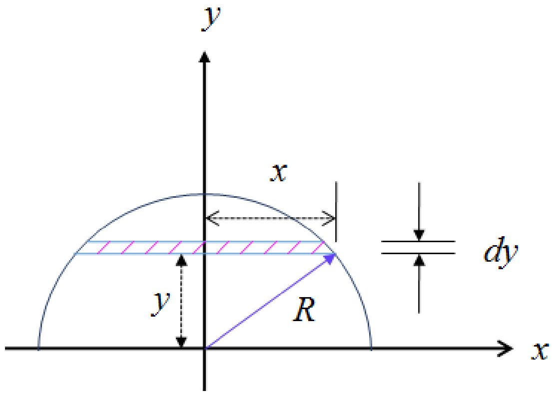


Figure S-1

- The centre of mass of homogeneous semicircular plate is given by

$$y_{cm} = \frac{1}{M}\int y\, dm = \frac{1}{M}\int y\left[\frac{4M}{\pi R^2}\, x\, dy\right] = \frac{4}{\pi R^2}\int y\, x\, dy = \frac{4}{\pi R^2}\int_0^R y\sqrt{R^2 - y^2}\; dy$$

Put,

$$R^2 - y^2 = t^2$$

$$\Rightarrow -2y\,dy = 2t\,dt$$

$$\therefore \boxed{y\,dy = -t\,dt}$$

when $y = 0$, then $t = R$

when $y = R$, then $t = 0$

$$\therefore \quad y_{cm} = \frac{4}{\pi R^2}\int_0^R \sqrt{R^2 - y^2}\; y\,dy$$

$$= \frac{4}{\pi R^2}\int_R^0 t\,(-t\,dt) = \frac{4}{\pi R^2}\int_0^R t^2\,dt$$

$$= \frac{4}{\pi R^2}\,\frac{R^3}{3} = \frac{4R}{3\pi}$$

Thus, the centre of mass of the homogeneous semicircular plate lies on the y-axis at a distance of $\dfrac{4R}{3\pi}$ from origin.

Motion of the Centre of Mass

Motion of the Centre of Mass

- Suppose we have a system S of particles $p_1, p_2, ..., p_N$ with masses $m_1, m_2, ..., m_N$, and position vectors $\vec{r}_1, \vec{r}_2, ..., \vec{r}_N$ respectively.

  Then, the position vector of **centre of mass** of the system S is given by

$$\vec{r}_{cm} = \frac{\sum_{i=1}^{N} m_i \vec{r}_i}{\sum_{i=1}^{N} m_i} = \frac{\sum_{i=1}^{N} m_i \vec{r}_i}{M}$$

where M is the total mass of the system.

$$\boxed{\therefore \vec{r}_{cm} = \frac{1}{M}\left[m_1\vec{r}_1 + m_1\vec{r}_2 + ... + m_N\vec{r}_N\right] \quad .... \quad (1)}$$

- Differentiating Eq.(1) with respect to time, we get the **velocity** of the centre of mass:

$$\vec{v}_{cm} = \frac{d\vec{r}_{cm}}{dt} = \frac{1}{M}\left[m_1\frac{d\vec{r}_1}{dt} + m_1\frac{d\vec{r}_2}{dt} + ... + m_N\frac{d\vec{r}_N}{dt}\right]$$

$$\Rightarrow \vec{v}_{cm} = \frac{1}{M}\left[m_1\vec{v}_1 + m_1\vec{v}_2 + ... + m_N\vec{v}_N\right] \quad .... \quad (2)$$

$$\therefore \boxed{M\vec{v}_{cm} = \sum_{i=1}^{N} m_i\vec{v}_i}$$

where $\sum_{i=1}^{N} m_i\vec{v}_i$ is the total momentum of the system.

In the absence of external forces, the total momentum is constant, so that $\vec{v}_{cm} =$ constant.

The velocity of the centre of mass is constant in the absence of external forces.

This is a remarkable property of the centre of mass.

Differentiating Eq. (2) with respect to time, we get the acceleration of the centre of mass:

$$a_{cm} = \frac{d\vec{v}_{cm}}{dt}$$

$$= \left[m_1\frac{d\vec{v}_1}{dt} + m_1\frac{d\vec{v}_2}{dt} + ... + m_N\frac{d\vec{v}_N}{dt}\right]$$

$$= \frac{1}{M}\left[\sum_{i=1}^{N} m_i\vec{a}_i\right] = \frac{1}{M}\left[\sum_{i=1}^{N}\vec{F}_i\right]$$

$$\therefore \boxed{M\vec{a}_{cm} = \sum \vec{F}_{ext}}$$

This states that the centre of mass of a system of particles moves as though all the mass of the system were concentrated at the centre of mass and all external forces were applied at that point.

Conservation of Momentum in a System of Particles

- Suppose we have a system S of particles $p_1, p_2, ..., p_N$ with masses $m_1, m_2, ..., m_N$, and move with velocities $\vec{v}_1, \vec{v}_2, ..., \vec{v}_N$ and momenta $\vec{p}_1, \vec{p}_2, ..., \vec{p}_N$ respectively.

The total momentum of the system S is given by

$$\vec{P} = \sum_{i=1}^{N} \vec{p}_n = \vec{p}_1 + \vec{p}_2 + ... + \vec{p}_N = m_1\vec{v}_1 + m_1\vec{v}_2 + ... + m_N\vec{v}_N = \sum_{i=1}^{N} m_i\vec{v}_i$$

$$\boxed{\vec{P} = M\vec{V}_{cm}}$$ where M is the total mass of the system. .... (1)

The total momentum of a system of particles is equal to the product of the total mass of a system and the velocity of its center of mass.

- Differentiating Eq. (1) with respect to time, we get

$$\frac{d\vec{P}}{dt} = M\frac{d\vec{V}_{cm}}{dt} = M\vec{a}_{cm} = \sum \vec{F}_{ext}$$

- Newton's second law for a system of particles: $\sum \vec{F}_{ext} = \frac{d\vec{P}}{dt}$ .... (2)

Equation (2) states that, in a system of particles, the net external force equals the rate of change of the linear momentum of the system.

If the net external force acting on a system is zero, then $\frac{d\vec{P}}{dt} = 0$ and so the total linear momentum of the system $\vec{P}$ remains constant.

Sample Problems

- **Three particles are placed in the xy plane. A 30-g particle is located at (3, 4) m, and 40-g particle is located at (-2,-2) m. Where a 20-g particle must be placed so that the centre of mass of the three-particle system is at the origin?**

Ans: $(x, y) = (-0.5, -2)m$

- **Two bodies are moving along same direction with acceleration** a_1 and a_2 **respectively. Both of them are acted by equal forces. Find acceleration of the centre of mass of the system?**

Hint:

$$a_{cm} = \frac{m_1 a_1 + m_2 a_2}{m_1 + m_2} = \frac{2a_1 a_2}{a_1 + a_2} \qquad \because F_1 = F_2 \Rightarrow m_1 = \frac{a_2}{a_1} m_2$$

- **Three forces of magnitudes 50N, 40N, 20N are directed along + ve x-axis, +ve y-axis and –ve x-axis respectively. These forces are acted on the bodies of masses 2 kg, 3 kg, and 5 kg respectively. Find the magnitude of acceleration of centre of mass of the system.**

Hint:

$$a_{cm} = \frac{\left|\sum \vec{F}_{ext}\right|}{M} = \frac{\sqrt{\left(\sum F_{ext,x}\right)^2 + \left(\sum F_{ext,y}\right)^2}}{M} = \frac{\sqrt{\left(F_{1x} + F_{2x} + F_{3x}\right)^2 + \left(F_{1y} + F_{2y} + F_{3y}\right)^2}}{M}$$

$$= \frac{\sqrt{(50 + 0 - 20)^2 + (0 + 40 + 0)^2}}{2 + 3 + 5} = 5 \text{ m s}^{-2}$$

Sample Problem

- **A projectile is fired from a gun at an angle of 45^0 with the horizontal and with a muzzle speed of $457.2\ \mathrm{m\,s^{-1}}$. At the highest point in its flight the projectile explodes into two fragments of equal mass. One fragment, whose initial speed is zero, falls vertically. How far from the gun does the other fragment land, assuming a level terrain?**

Hint:

Let the mass of the projectile be 2m.

Since there is no external force, centre of mass follows the true trajectory.

We have,

$$x_{cm} = \frac{m_A x_A + m_B x_B}{m_A + m_B}$$

$$\Rightarrow 2x_{cm} = x_A + x_B \qquad [\because m_A = m_B = \mathrm{m}]$$

$$\therefore \quad x_B = 2x_{cm} - x_A$$

$$= 2\times\frac{u^2 \sin 2\theta}{g} - u\cos\theta\left(\frac{u\sin\theta}{g}\right) \qquad \left[\begin{array}{l}\because x_{cm} = \text{Range} = \dfrac{u^2\sin 2\theta}{g} \\ x_A = u_x t = u\cos\theta\left(\dfrac{u\sin\theta}{g}\right)\end{array}\right]$$

$$= 2\times\frac{(457.2)^2 \sin 90^0}{9.8} - 457.2\times\cos 45^0\left(\frac{457.2\sin 45^0}{9.8}\right)$$

$$= 4.27\times10^4 m - 1.11\times10^4$$

$$= 3.16\times10^4 m$$

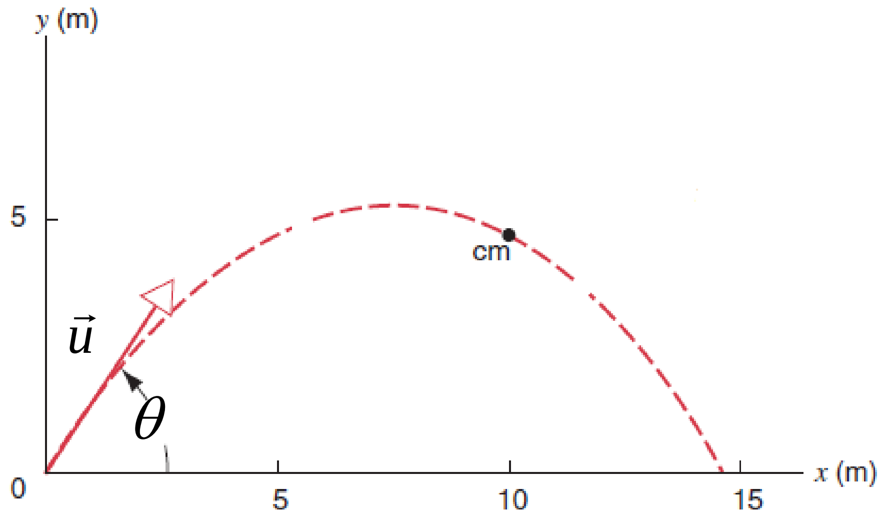


Figure C-I
The dashed line shows the parabolic trajectory of the center of mass of the two fragments.

References

1. **R.A. Serway and J.W. Jewett**, Physics for Scientist and Engineers with Modern Physics

2. **Halliday and Resnick**, Fundamental of Physics

3. **D. Halliday, R. Resnick, and K. Krane** , Physics, Volume 2, Fourth Edition

4. **Hugh D.Young, Roger A. Freedman,** University Physics with Modern Physics, 13TH Edition


Thank you