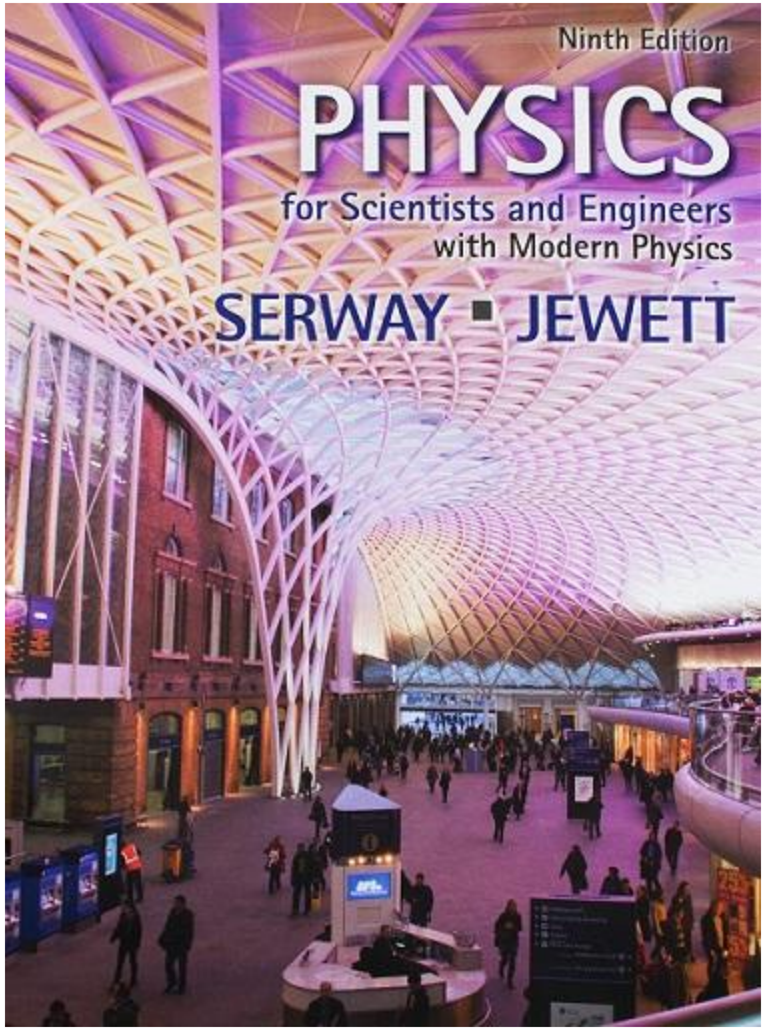
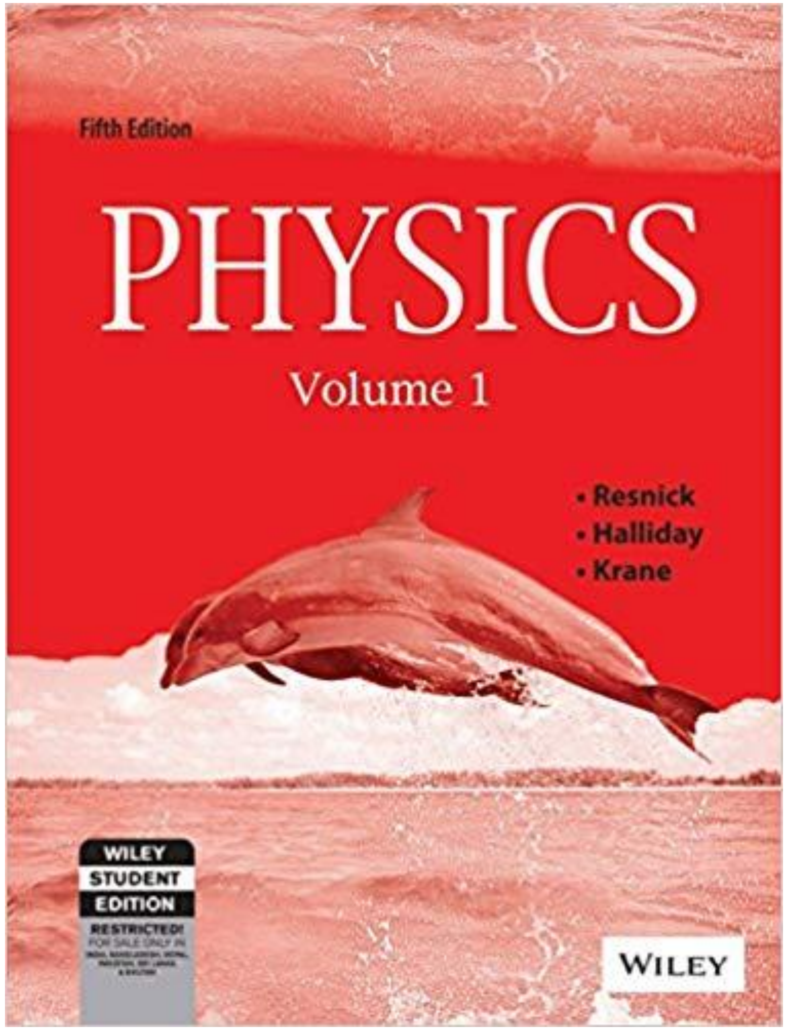
PHYSICS

General Physics I (PHYS 101)





Ganesh Kuwar Chhetri

Dept. of Physics, School of Science, Kathmandu University

VISCOSITY

Course Outline

- Fluids, Some General Characteristics of Fluid Flow
- Streamline, Laminar Flow and Turbulent Flow
- Viscosity , Newton’s law of Viscosity, Coefficient of Viscosity
- Turbulence, Reynolds's Number
- Steady Laminar Flow of Fluid in Pipe (Poiseuille’s Formula)
- Equation of Continuity
- Bernoulli’s Equation

General Characteristics of Fluid Flow

Fluids

- **Fluids** play a vital role in many aspects of everyday life. We drink them, breathe them, and swim in them. They circulate through our bodies and control our weather. Airplanes fly through them; ships float in them .
- A fluid is any substance that can flow; we use the term for both liquids and gases.

Some General Characteristics of Fluid Flow

1. **Fluid flow can be steady or nonsteady.**

Steady Flow

- Flow speed - low
- The velocity of the moving fluid at any fixed point remains constant in time. Each particle of the fluid follows a smooth path, such that the paths of different particles never cross each other.
- The gentle flow of water near the center of a quiet stream is steady.

Nonsteady Flow

- Flow speed – high
- The velocities of the moving fluid vary erratically from point to point as well as from time to time (The velocities are functions of time).
- Nonsteady flow – a waterfall

2. **Fluid flow can be compressible or incompressible.**

- If the density of a fluid is a constant, independent of x, y. z and t, its flow is called incompressible flow.
- Liquids can usually be considered as flowing incompressibly. Even for a highly compressible gas, the variation in density may be insignificant, and for practical purpose, we can consider its flow to be incompressible.

For example, in flight at speeds much lower than the speed of sound in air, the flow of the air over the wings is nearly incompressible.

General Characteristics of Fluid Flow

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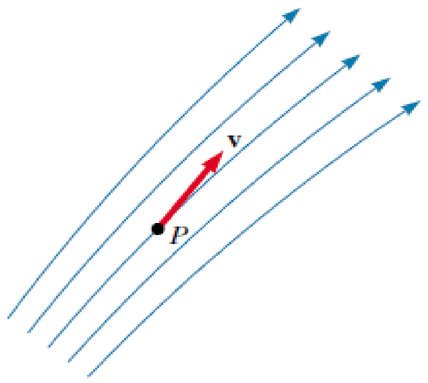
Some General Characteristics of Fluid Flow

3. **Fluid flow can be viscous or nonviscous.**
   - Viscosity in fluid flow is similar to friction in the motion of solid bodies.
   - The greater the viscosity, the greater the external force or pressure that must be applied to maintain the flow; under similar conditions, honey and motor oil are more viscous than water and air.
   - Although viscosity is present in all fluid flow, in some cases its effects may be negligible, in which case we can regard the flow as being nonviscous.

4. **Fluid flow can be rotational or irrotational.**
   - Imagine a tiny bit of matter, such as a small insect, that is carried along by a flowing stream. If the particle, as it moves with the stream, does not rotate about an axis through its centre of mass, the flow is irrotational; otherwise, it is rotational.

Streamline

- The path taken by a fluid particle under steady flow is called a streamline.
- The velocity of the particle is tangent to the streamline.
- A set of streamlines is called a ***tube of flow***.



Laminar Flow and Turbulent Flow

Laminar Flow

- In laminar flow,
  - Flow speed- low
  - Adjacent layers of fluid sliding smoothly over one another.
- The velocity of the moving fluid at any fixed point remains constant in time.
- Each particle of the fluid follows a smooth path, such that the paths of the different particles never cross each other.

Example: The gentle flow of water near the center of a quiet stream.

Turbulent Flow

- In turbulent flow,
  - Flow speed- sufficiently large
  - There is great disorder and a constantly changing flow pattern.
- The velocities vary erratically from point to point as well as from time to time.
- Turbulent flow occurs when the particles go above some critical speed.

Example: a waterfall

Flow of water from a faucet

Laminar at low speed

Turbulent at sufficiently high speed

Figure LT-1 shows the contrast between laminar and turbulent flow for smoke rising in air.

- Hot gases from a cigarette made visible by smoke particles.

  The flow of smoke rising from a cigarette is laminar up to a certain point, and then becomes turbulent.

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Viscosity

Viscosity

- All fluids offer resistance to any force tending to cause one layer to move another.

  Viscosity is the fluid property responsible for this resistance.
- Viscosity is internal friction in a fluid.
- Viscous forces oppose the motion of one portion of a fluid relative to another.
- The greater the viscosity, the greater the external force or pressure that must be applied to maintain the flow; under similar conditions, honey and motor oil are more viscous than water and air.
- Viscous effects are important in the flow of fluids in pipes, the flow of blood, the lubrication of engine parts, and many other situations.
- Viscosities of all fluids are strongly temperature dependent, increasing for gases and decreasing for liquids as the temperature increases (Figure V-1).

Figure V-1
Lava is an example of a viscous fluid. The viscosity decreases with increasing temperature: The hotter the lava, the more easily it can flow.

Newton's Law of Viscosity

Newton's Law of Viscosity

- Newton's law of viscosity states that " shear stress is directly proportional to velocity gradient".

For the straight and parallel motion of a given fluid, the shear stress between the two adjoining layers of fluid is directly proportional to the negative value of the velocity gradient between the same two adjacent fluid layers.

Mathematically the law can be stated as:

$$\tau \propto \frac{du}{dy}$$

or, $$\tau = -\eta \frac{du}{dy}$$

or, $$\frac{F}{A} = -\eta \frac{du}{dy}$$

$$\therefore \quad \boxed{F = -\eta A \frac{du}{dy}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

- Newton's law of viscosity for one-dimensional flow is given by

$$F_{\text{vis}} = -\eta A \frac{du}{dy}$$

This is the basic law of viscous resistance described by Newton in 1687.

- All gases and most simple liquids (water, oil) including liquid metals obey Newton's law of viscosity and are accordingly called Newtonian fluids.

where η is a constant for a particular fluid at a particular temperature.

This constant of proportionality η is called the coefficient of viscosity (absolute viscosity or dynamic viscosity) of the fluid.

Equation (1) is called the **Newton's law of viscosity** for one-dimensional flow.

Coefficient of Viscosity

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Coefficient of Viscosity (η)

- The magnitude of η is given by $\boxed{\eta = \dfrac{F_{\text{vis}}}{A\dfrac{du}{dy}}}$.

The **coefficient of viscosity** (dynamic viscosity) of fluid is defined as the tangential viscous force, which maintains a unit velocity gradient between its two adjoining parallel layers, each of unit area.

Dynamic viscosity (also known as **absolute viscosity**) is the measurement of the fluid's internal resistance to flow.

- The SI unit of viscosity is $\mathrm{N\cdot s/m^2}$ or Pa S.
- The equivalent cgs unit is $\mathrm{dyne\cdot s\,/cm^2}$, which is called the ***poise***.

The unit is named for the French Physician Jean-Louis Poiseuille (1799-1869), who investigated the flow of viscous fluids through tube.

$$\boxed{1\text{ poise} = 0.1\ \mathrm{N\cdot s/m^2}}$$

- The viscosity is large for fluids that offer a large resistance to flow and small for fluids that flow easily.

$$\boxed{\text{The viscosity of water at } \left(20^0 C\right) \text{ is } 1.0\times10^{-3}\ \mathrm{N\cdot s/m^2} \text{ or } 0.01\ \textit{poise}}$$

Table V-I: Viscosities of Selected Fluids

Fluid	$\eta\ \left(\mathrm{N\cdot s/m^2}\right)$
Glycerin $(20^0\mathrm{C})$	1.5
Motor oil $(0^0\mathrm{C})$	0.11
Motor oil $(20^0\mathrm{C})$	0.03
Blood $(37^0\mathrm{C})$	4.0×10^{-3}
Water $(20^0\mathrm{C})$	1.0×10^{-3}
Water $(90^0\mathrm{C})$	0.32×10^{-3}

Turbulence

Turbulence

- When the speed of a flowing fluid exceeds a certain critical value, the flow is no longer laminar. Instead, the flow pattern becomes extremely irregular and complex, and it changes continuously with time; there is no steady-state pattern. This irregular, chaotic flow is called **turbulence.**
- Bernoulli's equation is *not* applicable to regions where there is turbulence because the flow is not steady.
- **Application:**

  Listening for Turbulent Flow:

  Normal blood flow in the human aorta (The **aorta** is the main artery that carries blood away from your heart to the rest of your body.) is laminar, but a small disturbance such as a heart pathology can cause the flow to become turbulent. Turbulence makes noise, which is why listening to blood flow with a stethoscope is a useful diagnostic technique.

Reynolds Number

- Osborne Reynolds proposes the Reynolds Number on March 15,1883
- The Reynolds number is a dimensionless number used to determine whether a fluid is in laminar or turbulent flow.
- The Reynolds number for any flow speed v :

$$\boxed{R = \frac{\rho D v}{\eta}}$$

where $\rho \rightarrow$ density of the fluid

$D \rightarrow$ diameter of the pipe

$\eta \rightarrow$ viscosity of the fluid

Flow-through pipes are classified into three main flow regimes.

1. Laminar flow – R ≤ 2000
2. Transitional flow – R > 2000 and R < 4000
3. Turbulent flow – R ≥ 4000

- For cylindrical pipes, the Reynolds number corresponding to the critical speed is about 2000.

Thus for water flowing through a pipe of diameter 2 cm, the critical speed is

$$v_c = R\frac{\eta}{\rho D} = 2000\frac{1\times 10^{-3}\ \mathrm{N\cdot s/m^2}}{\left(10^3\ \mathrm{kg/m^3}\right)\left(0.02\ \mathrm{m}\right)} = 0.1\ \mathrm{m/s} = 10\ \mathrm{cm/s}$$

This is quite a low speed, which suggests that the flow of water is turbulent in ordinary household plumbing.

(The flow speed from a typical household tap is about 1 m/s.)

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Steady Laminar Flow in Pipe [Poiseuille's Formula]

Steady Laminar Flow in Pipe

- Let us consider a steady laminar flow of fluid of viscosity η through a horizontal pipe of radius R, and length L as shown in Figure P_F-1. The pressure P_1 at the left end is maintained greater than the pressure P_2 at the right end so that the fluid flows from left to right. The flow is axisymmetric.

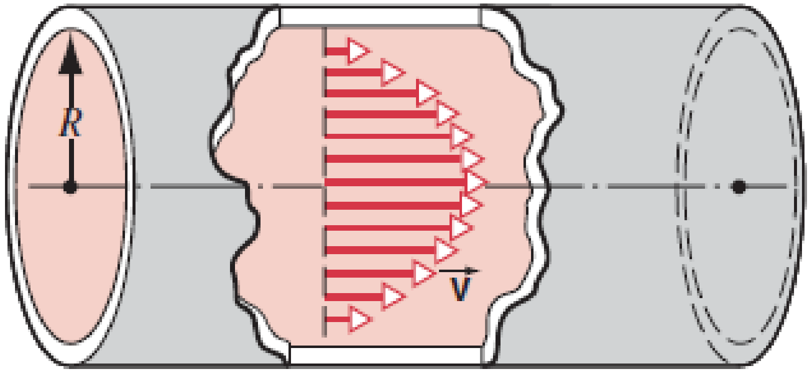

Figure P_F-1 Fluid flows through a cylindrical pipe

When the flow is fully developed the velocity profile is constant along the pipe axis. In laminar flow, the paths of individual particles of fluid do not cross, and so the pattern of flow may be imagined as a number of thin, concentric cylinders of varying radii, which slide over one another. The flow velocity varies with the radius; its maximum value occurs on the axis and its minimum value, which we assume to be zero, at the walls.

- Consider an arbitrary cylinder of fluid of radius r, length Land coaxial with the pipe.

The force on the left end of the cylinder of fluid is $P_1 \pi r^2$.

The force on the right end of the cylinder of fluid is $P_2 \pi r^2$

Steady Laminar Flow in Pipe [Poiseuille's Formula]

Steady Laminar Flow in Pipe

- The net force pushing that cylinder of fluid through the pipe is thus

$$F_{ext} = (P_1 - P_2)\pi r^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

- According to the Newton's law of viscosity for one-dimensional flow, the viscous force on that cylinder of fluid due to the neighboring layer is

$$F_{vis} = -\eta(2\pi r L)\frac{dv}{dr} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

where $\frac{dv}{dr}$ is the velocity gradient

For steady laminar flow of fluid,

$$F_{vis} = F_{ext}$$

or, $-\eta(2\pi r L)\frac{dv}{dr} = (P_1 - P_2)\pi r^2$

or, $\frac{dv}{dr} = -\frac{(P_1 - P_2) r}{2\eta L}$

$$\therefore \quad \boxed{dv = -\frac{(P_1 - P_2)}{2\eta L} r dr} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

Integrating equation (3), we get

$$\int dv = -\frac{(P_1 - P_2)}{2\eta L}\int r dr$$

or, $v = -\frac{(P_1 - P_2)}{2\eta L}\frac{r^2}{2} + C$

where C is the constant of integration

$$\therefore \quad \boxed{v = -\frac{(P_1 - P_2)}{4\eta L} r^2 + C} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

Steady Laminar Flow in Pipe

- The constant of integration, C , is evaluated by using the available boundary condition at the pipe wall: at r = R, v = 0. Consequently,

$$0 = -\frac{(P_1 - P_2)}{4\eta L} R^2 + C$$

This gives $$C = \frac{(P_1 - P_2)}{4\eta L} R^2$$

- Substituting the value of C in equation (4), we get

$$v = -\frac{(P_1 - P_2)}{4\eta L} r^2 + \frac{(P_1 - P_2)}{4\eta L} R^2$$

$$\therefore \quad \boxed{v = \frac{(P_1 - P_2)}{4\eta L}\left(R^2 - r^2\right)} \quad .... \quad (5)$$

- Equation (5) shows that the velocity decreases from maximum value $\frac{(P_1 - P_2)}{4\eta L} R^2$ at the centre to zero at the wall of the pipe.

Figure P_F2 shows the velocity profile for laminar flow of a viscous fluid in a long cylindrical pipe.

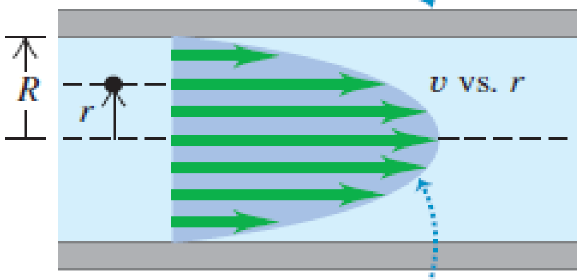

Figure P_F-2 velocity profile for laminar flow

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Steady Laminar Flow in Pipe [Poiseuille's Formula]

Steady Laminar Flow in Pipe

- Let us consider an elemental cylindrical tube of fluid of radius and thickness dr .

The cross section area of this elemental cylindrical tube of fluid is $dA = 2\pi r dr$.

The volume of fluid flowing through this elemental cylindrical tube of fluid per unit time is

$$dV = dA\, v$$

$$= 2\pi r dr \frac{(P_1 - P_2)}{4\eta L}(R^2 - r^2)$$

$$\therefore \boxed{dV = \frac{\pi(P_1 - P_2)}{2\eta L}(R^2 - r^2) r\, dr} \quad \ldots\ldots\ldots\ldots \quad (6)$$

The volume of fluid flowing through the pipe per unit time is therefore

$$V = \frac{\pi(P_1 - P_2)}{2\eta L}\int (R^2 - r^2) r\, dr$$

$$= \frac{\pi(P_1 - P_2)}{2\eta L}\left[R^2 \frac{r^2}{2} - \frac{r^4}{4}\right]_0^R$$

$$= \frac{\pi(P_1 - P_2)}{2\eta L}\left[\frac{R^4}{2} - \frac{R^4}{4}\right]$$

$$\therefore \boxed{V = \frac{\pi R^4 (P_1 - P_2)}{8\eta L}} \quad \ldots\ldots\ldots\ldots \quad (7)$$

This equation is known as **Poiseuille's formula.** Equation (7) is applicable only to the fully developed laminar flow of constant-density fluids.

Poiseuille's Formula

The volume flow rate (volume of fluid flowing through the pipe per unit time) is

$$\boxed{\frac{dV}{dt} = \frac{\pi R^4 (P_1 - P_2)}{8\eta L}}$$

Notes

Poiseuille's Formula

The volume flow rate (volume of fluid flowing through the pipe per unit time) is

$$\frac{dV}{dt} = \frac{\pi R^4 (P_1 - P_2)}{8\eta L}$$

Analogy between liquid-flow and current-flow

The volume flow rate (volume of fluid flowing through the pipe per unit time) is

$$Q = \frac{dV}{dt} = \frac{\pi R^4 (P_1 - P_2)}{8\eta L} = \frac{\pi R^4 P}{8\eta L} = \frac{P}{\left(\frac{8\eta L}{\pi R^4}\right)}$$

$$\Rightarrow \boxed{Q = \frac{P}{R}\left(\text{Poiseuille's formula}\right)} \leftrightarrow \boxed{I = \frac{V}{R} \quad \left(\text{Ohm's Law}\right)}$$

where $(P_1 - P_2) = P \rightarrow$ Pressure difference between two ends of the tube

$\frac{8\eta L}{\pi R^4} = R \rightarrow$ Effective viscous resistance

Sample Problem

Castor oil, which has a density of $0.96\times10^3 \mathrm{kg/m^3}$ at room temperature, is forced through a pipe of circular cross section by a pump that maintains a gauge pressure of 950 Pa. The pipe has a diameter of 2.6 cm and a length of 65 cm. The castor oil emerging from the free end of the pipe at atmospheric pressure is collected. After 90 s, a total of 1.23 kg has been collected. What is the coefficient of viscosity of castor oil at this temperature?

Hint:

Poiseuille's Law

The total mass flux: $\dfrac{dm}{dt} = \rho \dfrac{\pi R^4 \Delta P}{8\eta L}$

$$\Rightarrow \eta = \rho \frac{\pi R^4 \Delta P}{8\left(\frac{dm}{dt}\right)L}$$

$$= 1.15 \ \mathrm{N \cdot s/m^2}$$

Ideal Fluid

The motion of *real fluids* is very complicated and not yet fully understood. Instead, we will mostly consider the motion of *ideal fluids*. This greatly simplifies the mathematics of fluid dynamics and is often a good approximation to the behavior of real fluids.

Motion of ideal fluid can be considered as steady, incompressible, nonviscous and irrotational.

Flow of Ideal Fluids

- Volume flow rate (or volume flux) of the fluid :

$$\boxed{R_V = Av = \text{constant}}$$ [**Equation of Continuity**]

The volume of fluid that enters one end of a tube in a given time interval equals the volume leaving the other end of the tube in the same time interval if no leaks are present.

- Mass flow rate (or *mass flux*) of the fluid :

$$\boxed{R_m = \rho Av = \text{constant}}$$

This results expresses the ***law of conservation of mass*** in fluid dynamics.

Equation of Continuity

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Equation of Continuity

Consider ***an ideal fluid*** flowing with steady flow through a pipe of varying cross-sectional area, as illustrated in Figure VC-1.

Fluid can be considered as steady, incompressible, nonviscous and irrotational.

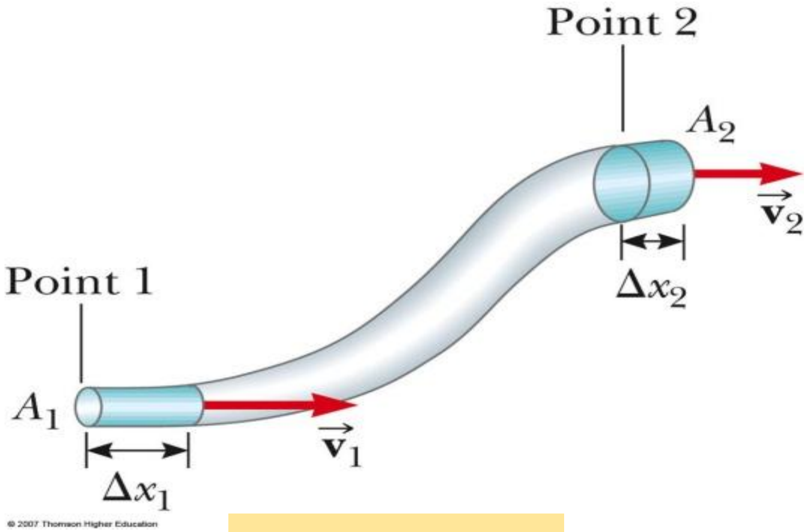

Figure VC-1

In a time Δt, the fluid at the bottom end of the pipe moves a distance $\Delta x_1 = v_1 \Delta t$.

If A_1 is the cross-sectional area in this region, then the mass of fluid contained in the left shaded region in Figure VC-1 is

$$m_1 = \rho A_1 \Delta x_1 = \rho A_1 v_1 \Delta t$$

where ρ is the (nonchanging) density of the ideal fluid.

Similarly, the fluid that moves through the upper end of the pipe in the time t has a mass

$$m_2 = \rho A_2 \Delta x_2 = \rho A_2 v_2 \Delta t$$

Since ***mass is conserved*** and the ***flow is steady***, the mass that crosses in a time must equal the mass that crosses in the time t.

That is $$m_1 = m_2$$

$$\rho A_1 v_1 \Delta t = \rho A_2 v_2 \Delta t$$

$$\therefore \boxed{A_1 v_1 = A_2 v_2}$$

This expression is called the **equation of continuity**.

It states that, "The product of the area and the fluid speed at all points along the pipe is a constant for an incompressible fluid".

Continuity equation tells us that the speed is high where the tube is constricted (small A) and low where the tube is wide (large A).

Bernoulli's Equation

The Swiss physicist ***Daniel Bernoulli*** first derived the relationship between fluid speed, pressure, and elevation in 1738.

Consider the flow of an ideal fluid through a nonuniform pipe in a time Δt, as illustrated in Figure BE-1.

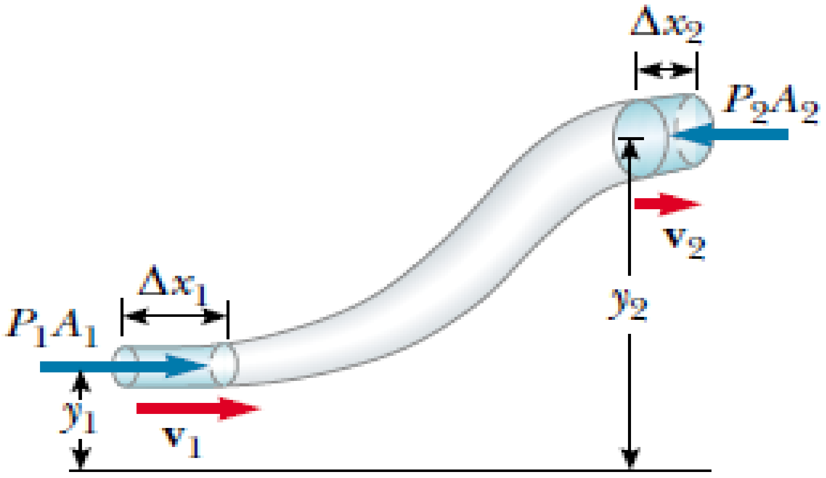

Figure BE-1
A fluid in laminar flow through a constricted pipe.

In Figure BE-1, we consider the element of fluid that at some initial time lies between the two cross sections *a* and *c*. The speeds at the lower and upper ends are $\vec{v}_1$ and $\vec{v}_2$. In a small time interval Δt, the fluid that is initially at *a* moves to *b*, a distance $\Delta x_1 = v_1\Delta t$, and the fluid that is initially at *c* moves to *d*, a distance $\Delta x_2 = v_2\Delta t$. The cross-sectional areas at the two ends are A_1 and A_2 as shown in Figure BE-1.

From the continuity equation,

$$A_1 v_1 = A_2 v_2$$

$$\text{or,} \quad A_1 v_1 \Delta t = A_2 v_2 \Delta t$$

$$\text{or,} \quad A_1 \Delta x_1 = A_2 \Delta x_2 = \Delta V$$

The volume of the fluid passing *any* cross section during time is the same.

The net work done on a fluid element by the pressure of the surrounding fluid is

$$W = P_1 A_1 \Delta x_1 - P_2 A_2 \Delta x_2$$

$$\therefore \quad W = (P_1 - P_2)\Delta V \qquad \text{....................} \quad (1)$$

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Bernoulli's Equation

The work W is due to forces other than the conservative force of gravity, so it equals the change in the total mechanical energy (kinetic energy plus gravitational potential energy) associated with the fluid element.

The net change in kinetic energy ΔK during time Δt is

$$\Delta K = \frac{1}{2}(\rho \Delta V)\ v_2^2 - \frac{1}{2}(\rho \Delta V)\ v_1^2$$

$$\therefore \quad \Delta K = \frac{1}{2}\rho \Delta V\left(v_2^2 - v_1^2\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

The net change in gravitational potential energy ΔU during time Δt is

$$\Delta U = (\rho \Delta V) g\, y_2 - (\rho \Delta V) g\, y_1$$

$$\therefore \quad \Delta U = \rho \Delta V\ g\left(y_2 - y_1\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

Combining Eqs. (1), (2), and (3) in the energy equation $W = \Delta K + \Delta U$, we obtain

$$\left(P_1 - P_2\right)\Delta V = \frac{1}{2}\rho \Delta V\left(v_2^2 - v_1^2\right) + \rho \Delta V\ g\left(y_2 - y_1\right)$$

or, $$\left(P_1 - P_2\right) = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) + \rho\, g\left(y_2 - y_1\right)$$

$$\therefore \quad P_1 + \frac{1}{2}\rho v_1^2 + \rho\, g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho\, g y_2$$

This is **Bernoulli's equation** as applied to an ideal fluid.

It is often expressed as

$$P + \frac{1}{2}\rho v^2 + \rho\, g y = \text{constant}$$

This expression specifies that, in laminar flow, the sum of the pressure (P), kinetic energy per unit volume $\left(\frac{1}{2}\rho v^2\right)$ and gravitational potential energy per unit volume $(\rho\, g y)$ has the same value at all points along a streamline.

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Application of Bernoulli's Equation

Aircraft Wing

The ***Bernoulli effect*** can explain the lift on an aircraft wing, in part.

Airplane wings are designed so that the air speed above the wing is greater than that below the wing. As a result, the air pressure above the wing is less than the pressure below, and a net upward force on the wing, called *lift*, results .

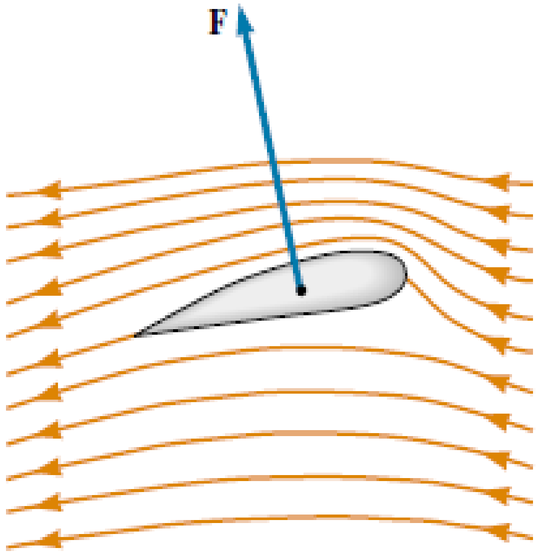


Figure B_A-1

Streamline flow around an airplane wing. The pressure above the wing is less than the pressure below, and a dynamic lift upward results.

Atomizer

-Used in perfume bottles and paint sprayers

A stream of air passing over one end of an open tube, the other end of which is immersed in a liquid, reduces the pressure above the tube, as illustrated in Figure B_A-2 . This reduction in pressure causes the liquid to rise into the air stream. The liquid is then dispersed into a fine spray of droplets. You might recognize that this so-called atomizer is used in perfume bottles and paint sprayers.

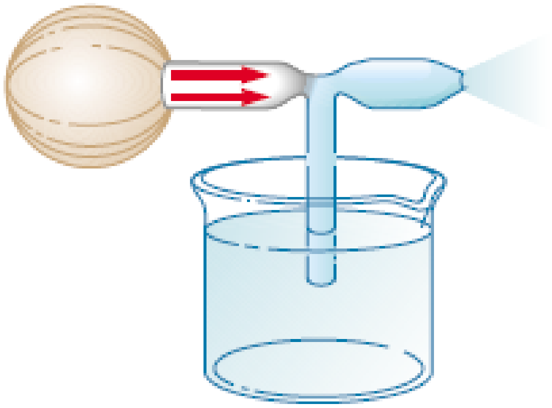


Figure B_A-2

A stream of air passing over a tube dipped into a liquid causes the liquid to rise in the tube.

Rate of Liquid Flow Through Capillaries

Rate of Liquid - Flow Through:

(A) Capillaries in series

Let two capillaries A and B, of lengths l_1 and l_2 and radii r_1 and r_2 respectively connected in series , as shown in Figure B-1 and let a liquid of coefficient of viscosity η flow through them in steady or streamline motion.

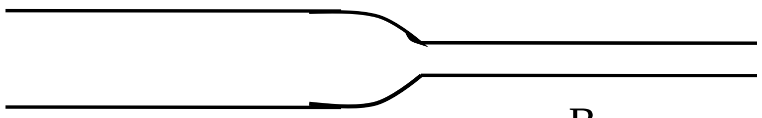


Figure B-1

Rate of Liquid-Flow Through the combination is

$$Q = \frac{P}{R} = \frac{P}{R_1 + R_2} = \frac{P}{\frac{8\eta}{\pi}\left(\frac{l_1}{r_1^4} + \frac{l_2}{r_2^4}\right)}$$

$$\therefore \boxed{Q = \frac{\pi P}{8\eta}\left(\frac{l_1}{r_1^4} + \frac{l_2}{r_2^4}\right)^{-1}}$$

(B) Capillaries in Parallel

Let two capillaries A and B, of lengths l_1 and l_2 , and radii r_1 and r_2 respectively lying in the same horizontal plane and parallel with each other , as shown in Figure B-2 and let a liquid of coefficient of viscosity η flow through them in steady or streamline motion.

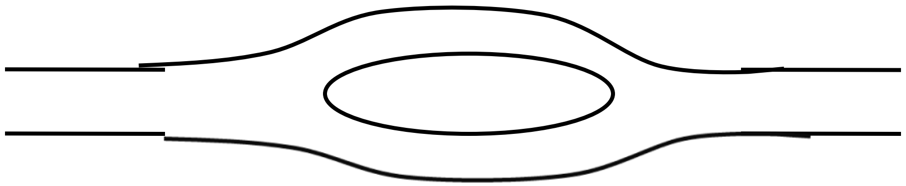


Figure B-2

Rate of Liquid-Flow Through the combination is

$$Q = \frac{P}{R} = P\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = P\left[\frac{\pi}{8\eta}\left(\frac{r_1^4}{l_1} + \frac{r_2^4}{l_2}\right)\right]$$

$$\therefore \boxed{Q = \frac{\pi P}{8\eta}\left(\frac{r_1^4}{l_1} + \frac{r_2^4}{l_2}\right)} = Q_1 + Q_2$$

Two Important Concepts Regarding Ideal Fluid Flow Through a Pipe of Nonuniform Size

(A) Equation of Continuity

The flow rate (volume flux) through the pipe is constant; this is equivalent to stating that the product of the cross-sectional area *A* and the speed *v* at any point is a constant. This result is expressed in the **equation of continuity:**

$$\boxed{Av = \text{constant}}$$

We can use this expression to calculate how the velocity of a fluid changes as the fluid is constricted or as it flows into a more open area.

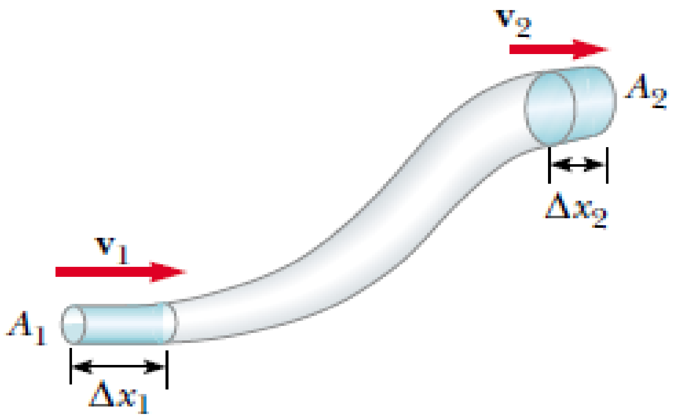
(B) Bernoulli's Equation

The sum of the pressure, kinetic energy per unit volume, and gravitational potential energy per unit volume has the same value at all points along a streamline. This result is summarized in **Bernoulli's equation**:

$$\boxed{P + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}}$$

When the fluid is at rest,

$$P + \rho\, g y = \text{constant}$$
$$\Rightarrow (P_1 - P_2) = \rho\, g\,(y_2 - y_1)$$



Hints for Problems

Poiseuille's Law

The total mass flux, $\dfrac{dm}{dt} = \rho \dfrac{\pi R^4 \Delta P}{8\eta L} \quad \Rightarrow \quad \boxed{\eta = \rho \dfrac{\pi R^4 \Delta P}{8\left(\dfrac{dm}{dt}\right) L}}$

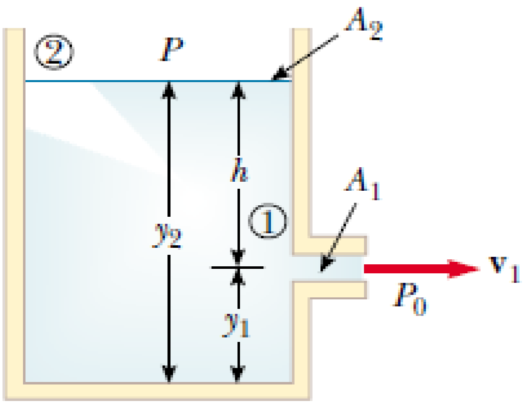
Equation of Continuity and Bernoulli's Equation

A large storage tank, open at the top and filled with water, develops a small hole it its side at a point m below the water level.

$A_2 >> A_1$

$\downarrow$

the liquid is approximately at rest at the top of the tank



Applying **Bernoulli's equation** between points 1 and 2:

$$P_0 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_0 + 0 + \rho g y_2$$

$$\Rightarrow v_1 = \sqrt{2g\left(y_2 - y_1\right)} = \sqrt{2gh}$$

Flow Rate: $A_1 v_1 = \left(\dfrac{\pi d^2}{4}\right) v_1$

P_0

$\searrow$

the atmospheric pressure

Equation of Continuity:

$$A_1 v_1 = A_2 v_2$$

Bernoulli's Equation :

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

$$\Rightarrow P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2 \quad \left[\text{when the tubes are at the same elevation}\right]$$

$$\Rightarrow P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2$$

$$\Rightarrow \Delta P = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)$$

Problems

(Q)

A large storage tank, open at the top and filled with water, develops a small hole it its side at a point 16.0 m below the water level. The rate of flow from the leak is $2.50\times10^{-3}\,\mathrm{m^3/min}$.
Determine (a) the speed at which the water leaves the hole, and (b) the diameter of the hole.

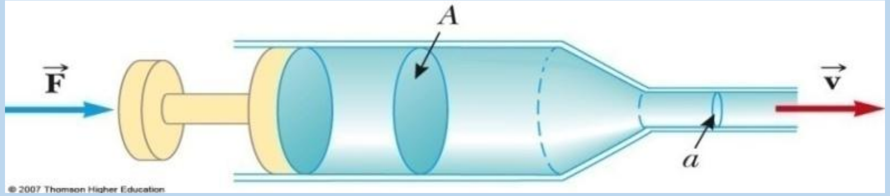
(Q)

In ideal flow, a liquid of density 850 kg/m^3 moves from a horizontal tube of radius 1.00 cm into a second horizontal tube of radius 0.500 cm. A pressure difference ΔP exists between the tubes.

a) Find the volume flow rate as a function of ΔP .
b) Evaluate the volume flow rate for ΔP =6.00 kPa.
c) State how the volume flow rate depends on ΔP .

(Q)

A hypodermic syringe contains a medicine having density of water [Figure F-1]. The barrel of the syringe has a cross-sectional area $A = 2.50\times10^{-5}\,\mathrm{m^2}$, and the needle has a cross-sectional area $a = 1.00\times10^{-8}\,\mathrm{m^2}$. In the absence of a force on the plunger, the pressure everywhere is 1 atm. A force of magnitude 2.00 N acts on the plunger, making medicine squirt horizontally form the needle. Determine the speed of the medicine as it leaves the needle's tip.



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Thank you