(OPTICS)

CHAPTER: 1: INTEFERENCE

Coherent Sources:

Two light sources are said to be coherent if they emit continuous light waves of same frequency, nearly equal or equal amplitude and same or constant phase difference.

To produce coherent source.

i) the sources must be monochromatic ie, same wavelength.

ii) the phase relation between the waves at the time of emission rapidly changes with time, not only in different sources but even in different parts of the same source.

Two virtual sources formed from a single source can act as coherent sources.

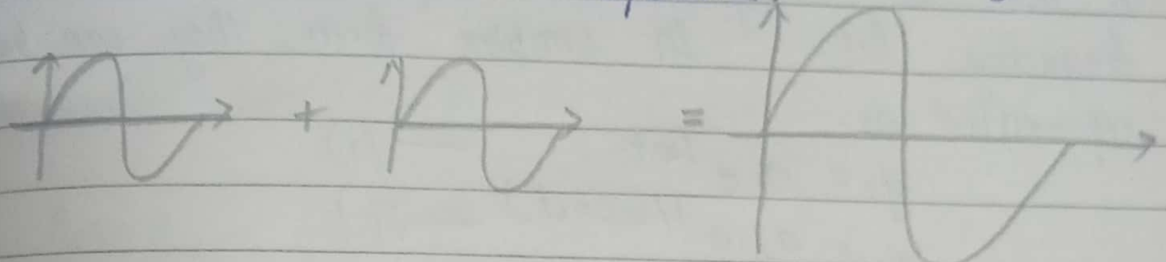
Inteference

The phenomenon of getting dark and bright fringes due to superposition of two coherent light sources is called interference.

Interference are of two types: constructive and destructive.

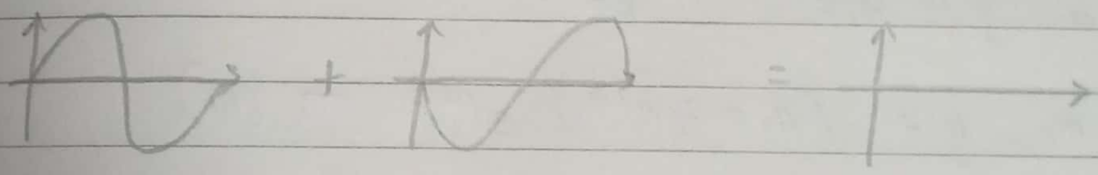
a) Constructive Interference

The phenomenon of getting bright fringes due to superposition of crest of one wave to crest of other wave is called constructive interference.



b): Destructive Interference:

The phenomenon of getting dark fringes due to superposition of trough of one wave to crest of another wave is called destructive interference.



Phase and Path Difference

For phase difference of 2π, path difference is λ.
Therefore, for phase difference of δ, path difference is $\frac{\lambda \times \delta}{2\pi}$

and

For path difference of λ, phase difference is 2π.
Therefore, for path difference of x, phase diffrence $= \frac{2\pi}{\lambda} x$

Superposition of Two Waves

Let us consider two waves with amplitude a_1 and a_2 having constant phase difference δ and frequency ω. In complex form, they can be represented as.

$$y_1 = a_1 e^{i\omega t} \quad \text{—— (i)}$$
$$y_2 = a_2 e^{i(\omega t+\delta)} \quad \text{—— (ii)}$$

Let y be the resultant wave after superposition.

So,

$$y = y_1 + y_2$$

or, $$y = a_1 e^{i\omega t} + a_2 e^{i(\omega t+\delta)}$$

or, $$y = (a_1 + a_2 e^{i\delta})\, e^{i\omega t} \quad \text{—— (iii)}$$

Let amplitude of resultant wave is R_0 and phase difference w.r.t first wave is ϕ.
This is written as.

$$y = R_0 e^{i(\omega t+\phi)} = R_0 e^{i\phi} e^{i\omega t} \quad \text{—— (iv)}$$

Comparing with eqn (iii), we get.

$$R_0 e^{i\phi} = a_1 + a_2 e^{i\delta}$$

or.

$$R_0 \cos\phi + i R_0 \sin\phi = a_1 + a_2 \cos\delta + i a_2 \sin\delta \quad \text{—— (v)}$$

Equating real and imaginary part in (v), we get-

$$R_0 \cos\phi = a_1 + a_2 \cos\delta \quad -(vi)$$

$$R_0 \sin\phi = a_2 \sin\delta \quad -(vii)$$

Dividing (vi) from (vii)

$$\frac{R_0 \sin\phi}{R_0 \cos\phi} = \frac{a_2 \sin\delta}{a_1 + a_2 \cos\delta}$$

$$\tan\phi = \frac{a_2 \sin\delta}{a_1 + a_2 \cos\delta} \qquad \therefore \phi = \tan^{-1}\left(\frac{a_2 \sin\delta}{a_1 + a_2 \cos\delta}\right) \quad (viii)$$

Squaring and adding eq^n (vi) and (vii), we get

$$R_0^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos\delta \quad -(ix)$$

We know, $I \propto a^2$.

so,

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\delta \quad -(x)$$

*) For maximum intensity:

$\cos\delta = 1.$

ie, $\delta = 2n\pi$ for $n \in I$ ~~+ve~~ non-negative

Intensity is maximum when phase difference ~~is~~ betn superposing waves is equal to even integral multiplication of π.

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

$$I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2 \quad \text{— (xi)}$$

*) For minimum intensity:

$\cos\delta = -1$

ie, $\delta = (2n+1)\pi$ for $n \in I$ non-negative.

Intensity is minimum when phase difference betn superposing waves is equal to odd integral multiple of π.

$$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$

$$\therefore I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2 \quad \text{— (xii)}$$

Dividing eqn (xii) from (xi), we get.

$$\frac{I_{max}}{I_{min}} = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2} = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2$$

Analytical Treatment of Interference

Let us consider two interfering waves

$y_1 = a \sin wt$ — (i)

and

$y_2 = a \sin(wt + \delta)$. — (ii)

Then,

resultant wave after superposition. i.e. adding (i) and (ii).

$y = y_1 + y_2$

or, $y = a \sin wt + a \sin(wt + \delta)$

$= a\ [\sin wt + \sin(wt+\delta)]$

$= a\left[2 \sin \frac{1}{2}(2wt+\delta) \cos \frac{1}{2}(-\delta)\right]$

$= 2a \sin(wt + \phi) \cos\left(\frac{\delta}{2}\right)$ Here, $\phi = \frac{\delta}{2}$

$y = R \sin(wt + \phi)$

Here, $R = 2a \cos\left(\frac{\delta}{2}\right)$ is amplitude of resultant wave.

We know,

$I \propto R^2$

$\therefore I = 4a^2 \cos^2 \phi = 4a^2 \cos^2\left(\frac{\delta}{2}\right)$

(*) Bright fringes

Maximum intensity of resultant wave $I = 4I_0$

For phase ~~path~~ difference : $\delta = 0, 2\pi, 4\pi, \ldots, 2n\pi$

or,

For path ~~phase~~ difference : $x = 0, \lambda, 2\lambda, 3\lambda, \ldots, n\lambda$

ie, ~~phas~~ for bright fringe, the path difference between the waves should be equal to integral multiple of wavelength $n\lambda$.

(*) Dark fringes

Minimum intensity of resultant wave : $I = 0$

For phase difference : $\delta = \pi, 3\pi, 5\pi, \ldots, (2n+1)\pi$

or,

For path difference: $\delta = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots\ldots (2n+1)\frac{\lambda}{2}$

ie, for dark fringe, the path difference between the waves should be equal to half odd integral multiple of wavelength $(2n+1)\frac{\lambda}{2}$

*) <u>Intensity distribution</u>

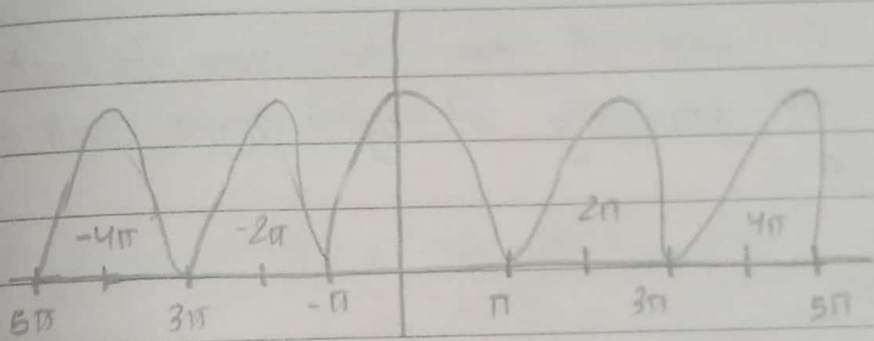
We know,

$$I = 4I_0 \cos^2\left(\frac{\delta}{2}\right) \quad \text{or,} \quad I = 4a^2 \cos^2\left(\frac{\delta}{2}\right)$$

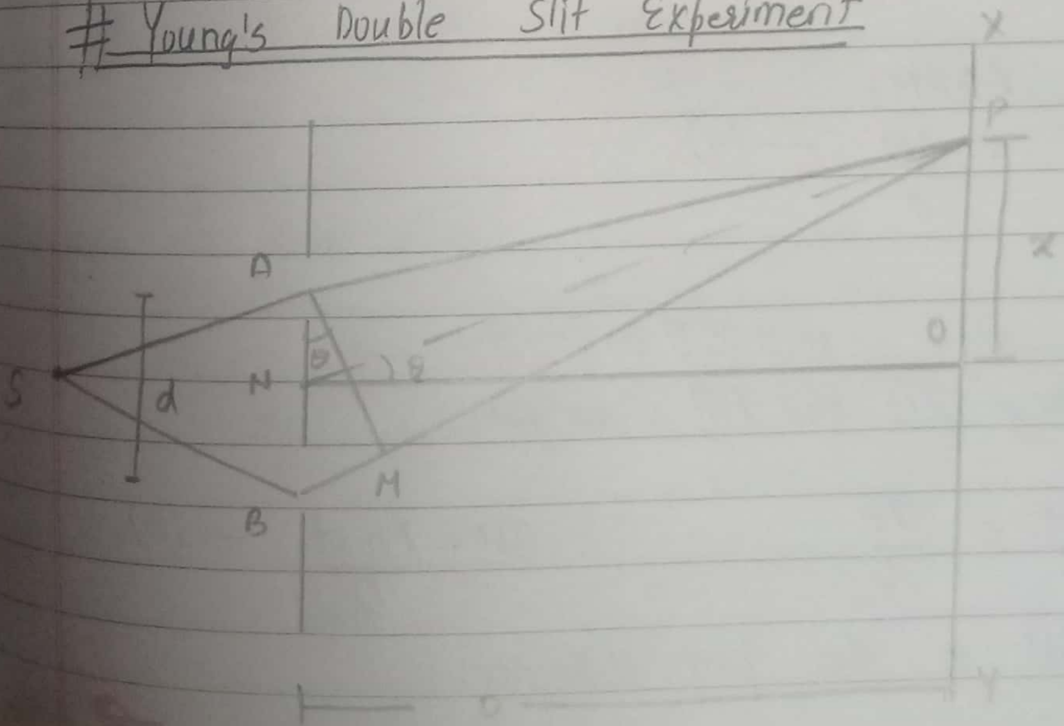
Here, $I_{max} = 4a^2 = 4I_0$ $\qquad I_{min} = 0$.

ie, intensity for each dark ~~to~~ and bright fringe is same.

Intensity distribution curve is as shown.



Young's Double Slit Experiment



Consider a source of monochromatic light S.

Let d be the width between slit A and B.
and D be the distance of screen from slit.

Let P be the position of n^{th} bright or dark fringe.

Let θ be the angle made by NP with NO.

A perpendicular AM is drawn to BP.

Since A and B are too close to each other, AM meets NP practically at right angles ~~as~~ such that $\angle BAM = \theta$.

From ΔPNO,

$$\tan\theta = \frac{PO}{NO} = \frac{x_n}{D} \quad -(i)$$

and from ΔBAM,

$$\sin\theta = \frac{BM}{d} \quad -(ii)$$

For small angle, $\sin\theta \approx \theta \approx \tan\theta$
Equating (i) and (ii), we get.

$$\frac{BM}{d} = \frac{x_n}{D} \qquad \therefore BM = \frac{x_n d}{D} \quad -(iii)$$

Here, BM is the path difference.

Now, for bright fringe,

$$BM = n\lambda$$

or, $$\frac{x_n d}{D} = n\lambda$$

$$\therefore x_n = \frac{n\lambda D}{d} \quad \text{— (iv)}$$

Eq^n (iv) gives distance of n^{th} bright fringe from center of fringe system.

For $(n-1)^{th}$ bright fringe,

$$x_{n-1} = \frac{(n-1)\lambda D}{d} \quad \text{— (v)}$$

Hence, fringe width $(\beta) = x_n - x_{n-1}$

$$= \frac{n\lambda D}{d} - \frac{n\lambda D}{d} + \frac{\lambda D}{d}$$

$$\therefore \beta = \frac{\lambda D}{d} \quad \text{— (a)}$$

Now, for dark fringe.

$$BM = (2n+1)\frac{\lambda}{2}$$

or $$\frac{x_n}{D} = (2n+1)\frac{\lambda}{2}$$

$$\therefore x_n = \frac{(2n+1)\lambda D}{2d} \quad \text{— (vi)}$$

Eq^n (vi) gives distance of n^{th} dark fringe from the center of fringe ~~dis~~ system.

For $(n-1)^{th}$ dark fringe,

$$x_{n-1} = \frac{[2(n-1)+1]\lambda D}{2d} = \frac{(2n-1)\lambda D}{2d} \quad \text{—(vii)}$$

So, fringe width $(\beta) = x_n - x_{n-1}$

$$= \frac{(2n+1)\lambda D}{2d} - \frac{(2n-1)\lambda D}{2d}$$

$$\therefore \beta = \frac{\lambda D}{d} \quad \text{— (b)}$$

Here, $\beta = \frac{\lambda D}{d}$ for both dark and fringe width.

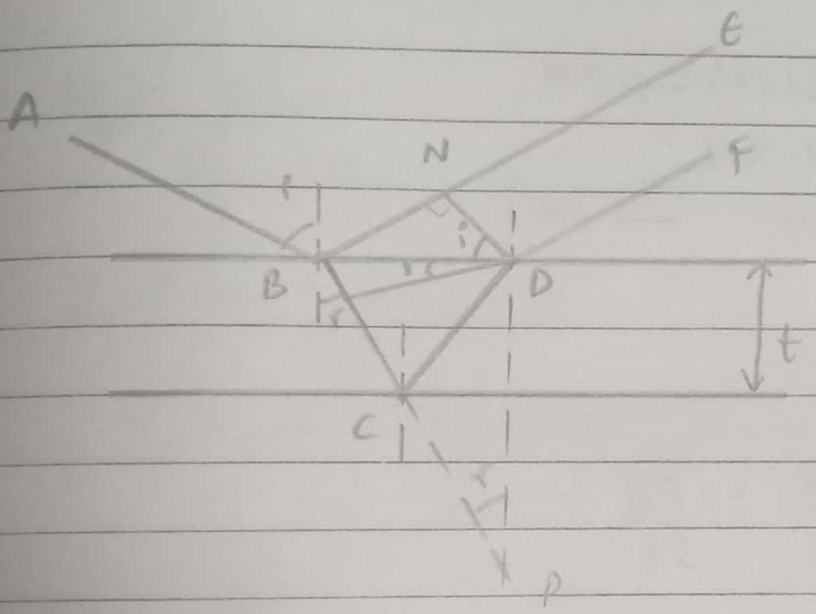
Hence, fringes in Young's double slit experiment are equally ~~width~~ space.
and
fringe width of both bright and dark fringe is equal.

Inteference on Thin Films Due to Reflected light

Let us consider a thin film of thickness t and refractive index μ as in figure.

A ray of light AB strikes at point B with angle of incident (i) gets reflected along BE and also refracted along BC with angle of refraction (r).

At C, it again reflects along CD and finally emerges out along DF.



~~BM~~ DN $\perp$ BE and DH $\perp$ BC is drawn such that $\angle BDN = i$ and $\angle BDH = r$.

BC is produced to meet DP at P so that $DP = 2t$ and $\angle HPD = r$.

Let x be the path difference betn waves emerging from B and D respectively. Then,

$$x = \mu(BC + CD) - BN \quad \text{--- (i)}.$$

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From Snell's law,

$$\mu = \frac{\sin i}{\sin r} = \frac{BN/BD}{BM/BD} = \frac{BN}{BM}.$$

~~$\therefore BM =$~~ $\therefore BN = \mu BM$ — (ii)

And

$CD = CP$.

So eq^n (i) becomes,

$$x = \mu(BC + CP) - \mu BM.$$
$$= \mu(BP - BM)$$
$$\therefore x = \mu PM. \quad \text{— (iii)}$$

From ΔMPD,

$$\cos r = \frac{PM}{PD} = \frac{PM}{2t}$$

$$\therefore PM = 2t\cos r$$

Hence, path difference is

$$x = 2\mu t\cos r \quad \text{— (iv)}$$

According to stoke's law of reflection,

When a light wave is reflected ~~along~~ from the surface of an optically denser medium, it suffers a phase change of π or path difference $\lambda/2$. but suffers no change when reflected at the surface of ~~rar~~ optically rarer medium.

So,

corrected path difference becomes

$$x = 2\mu t \cos r + \frac{\lambda}{2} \quad \text{—(v)}$$

* For bright fringe:

$$x = n\lambda$$

$$\therefore 2\mu t \cos r + \frac{\lambda}{2} = n\lambda$$

or $$2\mu t \cos r = (2n-1)\frac{\lambda}{2} \quad \text{——— (a)}$$

*) For dark fringe:

$$x = (2n+1)\frac{\lambda}{2}$$

or, $$2\mu t \cos r + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$\therefore 2\mu t \cos r = n\lambda. \quad \text{— (b)}$$

Eq^n (a) and (b) are required conditions for bright and dark fringes for interference due to reflected light.

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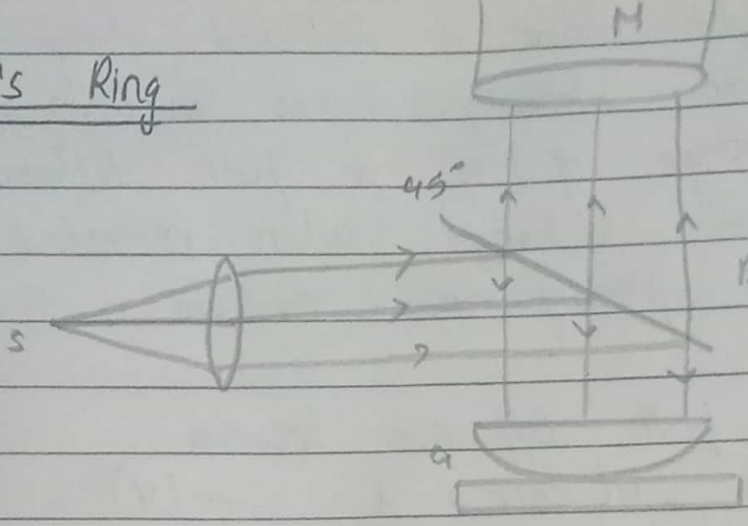
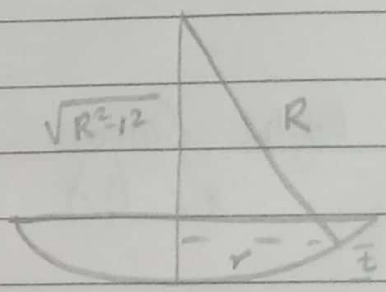
Newton's Ring





Suppose a monochromatic ~~light~~ source of light 'S' kept at focus of convex lens L_1 so that the light beam passing through it becomes parallel and strikes the plane glass plate at B. placed at angle 45° as shown in figure.

The beam now travels downwards and strikes plano-convex lens ~~as~~ L as shown.

The plano-convex lens is placed on a plane glass plate G with curved surface touching glass plate. G.

Since there is thin air film in between the ~~plane~~ lens and plane glass plate, interference takes place between the light rays reflected from lower surface of plano-convex lens and upper surface of plane glass plate G.

The inteference fringes consists of dark and bright concentric rings called Newton's rings observed through travelling microscope.

Let R be the radius of curvature of lens L,
t = thickness of thin air film
r = radius of n th dark or bright fringe.

From figure,

$$t = R - \sqrt{R^2 - r^2}$$

$$\therefore t = R - R\left[1 - \left(\frac{r}{R}\right)^2\right]^{\frac{1}{2}} \quad \text{— (i).}$$

Expanding using binomial theorem and neglecting higher powers of (r/R), we get.

$$t = R - R\left[1 - \frac{r^2}{2R^2}\right]$$

$$\text{or, } t = \frac{\cancel{R} r^2}{2R}$$

$$\therefore 2t = \frac{r^2}{R} \quad \text{—— (ii)}$$

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For bright fringe,

$$2\mu t \cos\theta = (2n-1)\frac{\lambda}{2}$$

For air, $\mu = 1$ and for small θ, $\cos\theta \approx 1$

So,

$$2t = (2n-1)\frac{\lambda}{2} \quad \text{——— (iii)}$$

From eq^n (ii) and (iii),

$$\frac{r^2}{R} = (2n-1)\frac{\lambda}{2}$$

$$\therefore r = \sqrt{(2n-1)\frac{\lambda R}{2}} \quad \text{— (a)}$$

Eq^n(a) gives the radius of n^{th} bright fringe from centre.

For diameter of n^{th} bright ring,

$$D_n = 2r = 2\sqrt{(2n-1)\frac{\lambda R}{2}} \quad \text{— (I)}$$

For dark fringe,

$$2\mu t \cos\theta = n\lambda$$

For air, $\mu = 1$ and for small θ, $\cos\theta \approx 1$

$$2t = n\lambda \quad \text{—— (iv)}$$

from eq^n (ii) and (iv),

$$\frac{r^2}{R} = n\lambda$$

or, $$r = \sqrt{n\lambda R} \quad \text{—— (b)}$$

Eq^n (b) gives radius of n^{th} dark fringe from centre.

For diameter of n^{th} dark ring,

$$D_n = 2r = 2\sqrt{n\lambda R} \quad \text{—— (II)}$$

So, when $n = 0$, $D_n = 2\sqrt{0 \times \lambda R} = 0$ ie, corresponds to central dark ring.

Hence, centre of the ring system is dark in case of reflected light.

Now,

$D_4 - D_1 = 4\sqrt{\lambda R} - 2\sqrt{\lambda R} = 2\sqrt{\lambda R}$

$D_{16} - D_9 = \cancel{99\sqrt{\lambda R}}\ 8\sqrt{\lambda R} - 6\sqrt{\lambda R} = 2\sqrt{\lambda R}$.

Thus, this shows that as we move far from center, the rings are found to be closely packed.

With increasing numbers of fringes, fringe width decreases.

X) Determination of wavelength of Sodium light:

For n^{th} dark ring,

$$D_n = 2\sqrt{n\lambda R}$$

Squaring, we get, $D_n^2 = 4n\lambda R$ — (i)

For m^{th} dark ring $(m>n)$

$$D_m^2 = 4m\lambda R \quad \text{— (ii)}$$

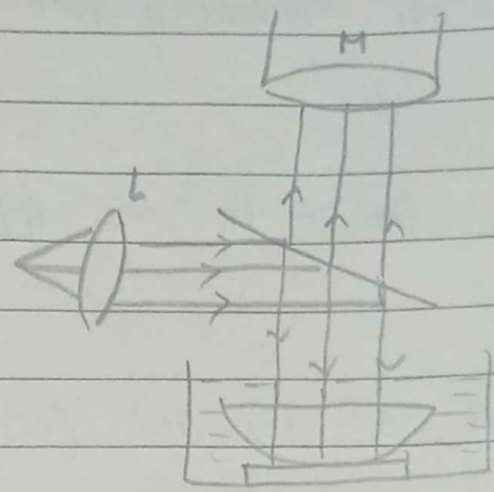
Subtracting eq^n (i) from (ii),

$$D_m^2 - D_n^2 = 4m\lambda R - 4n\lambda R$$

$$\therefore \lambda = \frac{D_m^2 - D_n^2}{4R(m-n)} \quad \text{— (A)}$$

Eq^n (A) measures wavelength of sodium light.

X) Determination of R.I. of transparent tigt liquid



The experimental setup is given below.

We know,

~~Dn~~ diameter of n^{th} fringe, without liquid.

$$D_n = 2\sqrt{n\lambda R}$$

So, for n^{th} and m^{th} dark ring without liquid.

$$D_m^2 - D_n^2 = 4(m-n)\lambda R \quad \text{— (i)}$$

Suppose liquid is poured then, n^{th} dark ring has.

$$2\mu t \cos\theta = n\lambda.$$

For small θ, $\cos\theta \approx 1$.

So,

$$2\mu t = n\lambda.$$

$$2t = \frac{r'^2}{R}$$

Thus,

$$\mu \frac{r'^2}{R} = n\lambda \qquad \therefore r' = \sqrt{\frac{n\lambda R}{\mu}} \quad \text{— (ii)}$$

for diameter $(D_n') = 2r' = 2\sqrt{\frac{n\lambda R}{\mu}}$ — (iii)

Now for diameter of n^{th} and m^{th} $(m>n)$ dark ring with liquid.

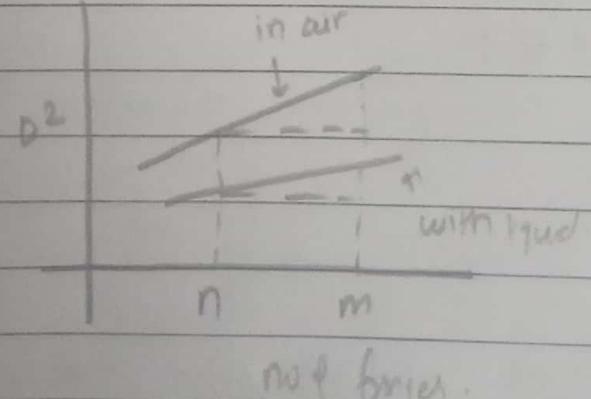
$$D_m'^2 - D_n'^2 = \frac{4(m-n)\lambda R}{\mu} \quad -(iv)$$

From eq^n (i) and (iv),

$$D_m'^2 - D_n'^2 = \frac{D_m^2 - D_n^2}{\mu}$$

$$\therefore \mu = \frac{D_m^2 - D_n^2}{D_m'^2 - D_n'^2} \quad -(B)$$

Eq^n (B) is the required relation for measurement of R.I. of given transparent liquid.



To determine R.I. graphically, we plot the graph betn square of diameter and no of fringes at y and x-axis respectively as shown in figure.

Since,

$$\mu = \frac{D_m^2 - D_n^2}{D_m'^2 - D_n'^2}$$

So,

$$\mu = \frac{\text{Slope of line AB}}{\text{slope of line CD}}$$