(MECHANICS)

# CHAPTER: 4: ELASTICITY

## # Elasticity:

The property of a body by virtue of which it regains its original shape and size when the deforming force is removed is called elasticity of the body.

## # Perfectly Plastic:

If the body remains deformed and shows no tendency to recover its original condition on the removable of deforming force, it is called perfectly plastic body.

## # Stress:

The external force (ie, deforming force) acting on an object per unit cross-sectional area is called stress.

It characterizes the strength of the forces causing the deformation.

Mathematically,

$$\text{Stress} = \frac{\text{Deforming force}}{\text{Cross-sectional Area}} = \frac{F}{A}$$

SI unit = Pascal.

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# Strain:

The ratio of change in dimension to the original dimension of a body is called strain of the body.

It can also be defined as the ratio of change in size of elastic body under the action of stress to the original size.

Mathematically,

$$\text{Strain} = \frac{\text{Change in dimension}}{\text{Original dimension}}$$

It measures of degree of deformation.

SI unit = No unit (Ratio)

Strain is the result of stress.

# Normal Stress and Strain:

In this stress, the force acting on the body is normal to the surface.

It is of three types:

a) Tensile ~~strength.~~
b) Compressive
c) Bulk (Volume)

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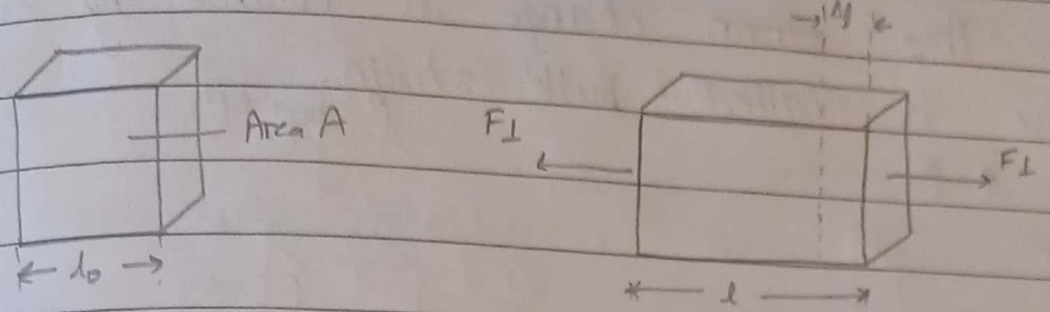
## a) Tensile stress and strain:

The net force acting on the object is zero, but the object deforms.



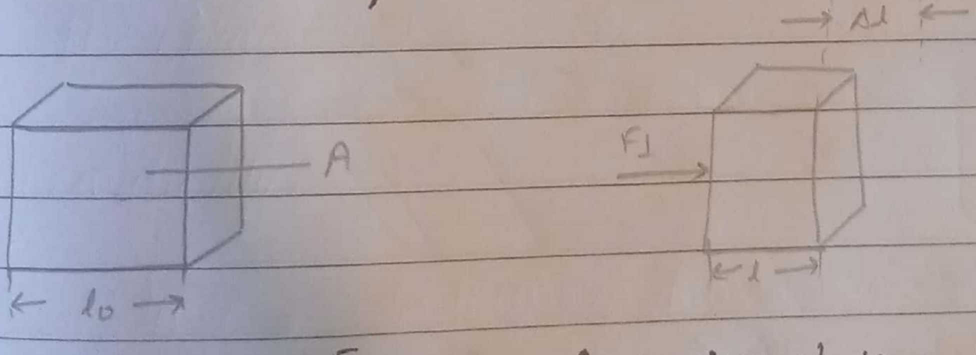

Tensile stress $= \dfrac{F_\perp}{A}$ Tension strain $= \dfrac{\Delta l}{l_0}$

Tensile stress produces tensile / longitudinal strain.

## b) Compressive Stress and strain:

The forces acting on the bar is pushing rather than pulling, the bar is compressed. Hence, stress is compressive stress.

Here, net force is zero but object deforms. Compressive stress produces compressive strain.




Compressive stress $= \dfrac{F_\perp}{A}$ Compressive strain $= \dfrac{\Delta l}{l_0}$

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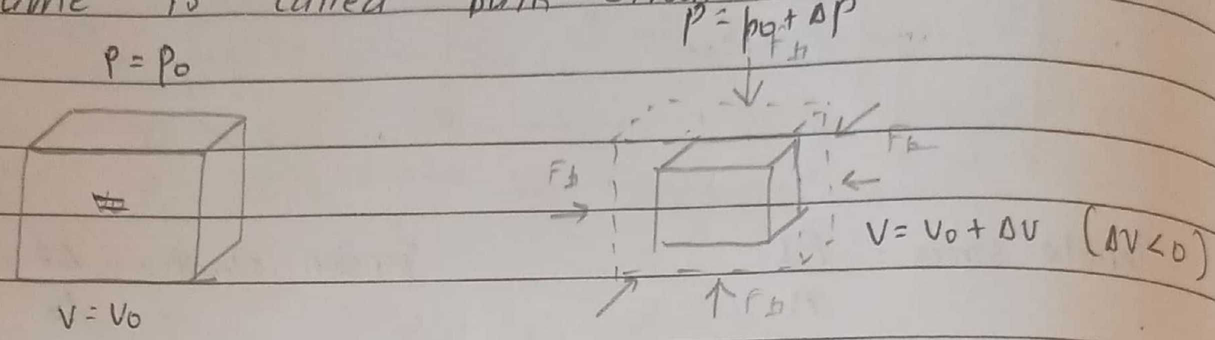
## c) Bulk (Volumetric) stress and strain

The stress is responsible for the change in volume is bulk stress.

The ~~strain~~ change in volume by original volume is called bulk strain.

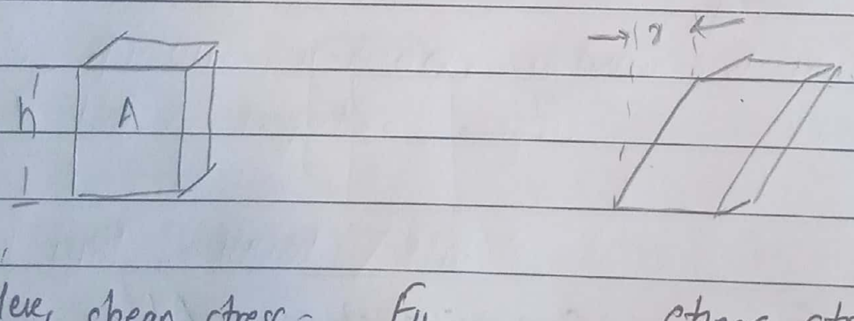



Bulk stress = $\Delta P$ Bulk strain = $\frac{\Delta V}{V_0}$

## d) Shear stress and strain:

Shear stress is responsible for changing the shape of the object. This force acts tangential to a surface opposite to rigid surface.

Shear strain is the angle in radian through which a side of a body originally $\perp^r$ to the fixed surface is turned is called shear strain.




Here, shear stress = $\frac{F_{\parallel}}{A}$ shear strain = $\frac{x}{h}$

Here, x = horizontal distance sheared face moves.

→ Only applied on solid.

# Hooke's law and Modulus of Elasticity

Hooke's law states that, "within elastic limit, stress is directly proportional to strain."

ie, stress $\alpha$ strain.

or, "the extension is proportional to the load or tension in the wire" when the proportional limit is not exceeded.

Mathematically,

$$\text{Elastic modulus} = \frac{\text{Stress}}{\text{strain}}$$

Elastic modulus depends upon
(i): nature of the material
(ii) nature of the deformation.

Theree are three types of modulus of elasticity. They are as follows:

a) Young's Modulus
b) Bulk Modulus
c) Shear Modulus.

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a) Young's Modulus: Elasticity in Length

- This measures the resistance of a solid to a change in its ~~F~~ length.

Mathematically,

$$\text{Young modulus } (Y) = \frac{\text{Tensile stress}}{\text{Tensile strain}} = \frac{F_\perp / A}{\Delta l / l_0}$$

$$\text{or } Y = \frac{F_\perp \, l_0}{A \times \Delta l}$$

SI unit = Pascal or $Nm^{-2}$

→ This is used to characterize a rod or wire under stressed.

b) Bulk modulus: Elasticity in Volume

- This measures the resistance ~~and~~ of solid and fluids to changes in their volume.

Mathematically,

$$\text{Bulk modulus } (B) = \frac{\text{Bulk stress}}{\text{Bulk strain}} = \frac{-\Delta P}{\Delta V / V_0}.$$

As pressure increases, volume decreases. Hence, negative sign.

SI unit: Pascal / $Nm^{-2}$

(*) Compressibility:

The reciprocal of bulk modulus is called compressibility.

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Mathematically,

$$\text{Compressibility } (k) = \frac{1}{B} = \frac{-\Delta V / V_0}{\Delta p} = -\frac{1}{V_0}\frac{\Delta V}{\Delta p}$$

SI unit = $\text{Pascal}^{-1}$

Compressibility can also be defined as the fractional decrease in volume $(-\Delta V/V_0)$ per unit increase $\Delta p$ in pressure.

c) Shear Modulus: Elasticity of Shape.

- This measures the resistance of motion of the planes within the solid parallel to each other.

Mathematically,

$$\text{Shear modulus } (S) = \frac{\text{Shear stress}}{\text{Shear strain}} = \frac{F_{\parallel}/A}{x/h}$$

It can also be defined as,

$$\text{Modulus of rigidity } (\eta) = \frac{\text{shear stress}}{\text{Shear strain}} = \frac{F_{\parallel}/A}{\theta}$$

Unit of shear Modulus = Pascal.

$E = \frac{9KC}{3K+C}$

$\frac{E}{9} = \frac{KC}{3K+C}$

$\frac{9}{E} = \frac{3K+C}{KC} = \frac{3}{C} + \frac{1}{K}$

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## ⊛ Relation between Elastic Constants :

$$\frac{9}{Y} = \frac{3}{\eta} + \frac{1}{B}$$

# Stress - strain curve - for Elastic Solid.

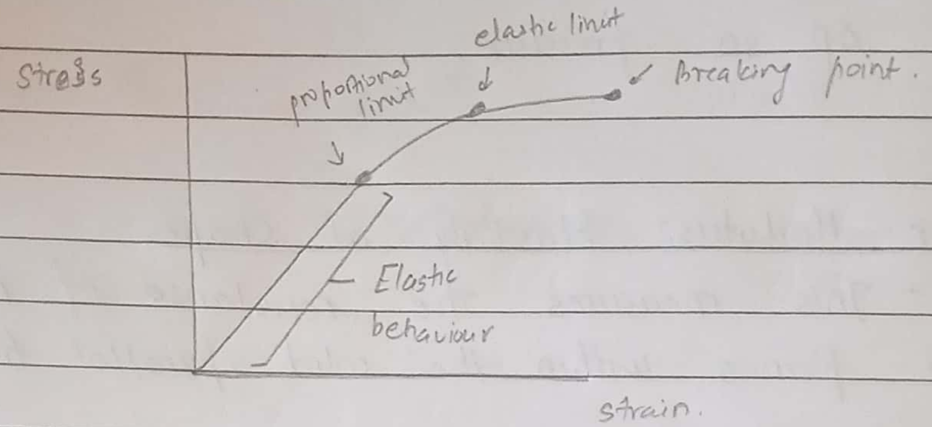



Fig: Stress - strain curve for elastic solid

Initially, until proportional limit, the stress - strain curve is a straight line. Here, Hooke's law is obeyed.

As stress further increases, strain increases exponentially than stress until the elastic limit.

After the stress is further increased, the beyond elastic limit, the object gets permanently distorted and doesn't return to the original shape.

After crossing the breaking point, the material ultimately breaks. The stress at which the material breaks is called breaking stress / ultimate stress.

## *) Ductile material:

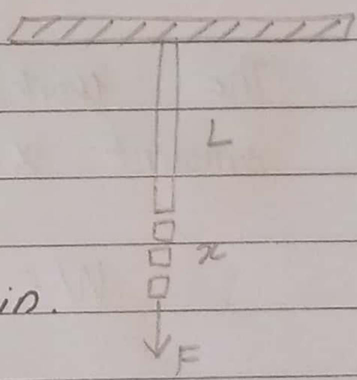
A ductile material is the material that can be stressed well beyond its elastic limit without breaking. Eg: soft iron wire.

Here, large amount of plastic deformation takes place between elastic limit and fracture point.

## *) Brittle material:

A brittle material is the material that breaks soon after the elastic limit is reached.
Eg: piano string.

# # Energy stored in a stretched wire




The applied force must do some work to deform a body. The energy so used is stored in the form of potential energy and called energy of strain.

Let $L$ = original length of wire
$Y$ = Young's modulus
$A$ = cross-sectional area
is suspended vertically with its upper end attached to a rigid support.
Let $F$ = normal force applied at lower end.
$x$ = amount stretched elastically by force F.

Here,

Tensile stress $= \frac{F}{A}$

Tensile strain $= \frac{x}{L}$

We know,

Young's modulus $(Y) = \frac{\text{Tensile stress}}{\text{Tensile strain}}$

$$= \frac{F/A}{x/L} = \frac{FL}{Ax}$$

$$\therefore F = \frac{YAx}{L}$$

The workdone in stretching the wire by an amount $x$ from Original position O is.

$$W = \int_0^x F \cdot dx$$

$$= \int_0^x \frac{YAx}{L} dx$$

$$= \frac{1}{2} \frac{YA x^2}{L} \quad \text{Or,} \quad W = \frac{1}{2}\left(\frac{YAx}{L}\right) x$$

$\therefore W = \frac{1}{2} \times F \times x = \frac{1}{2} \times$ force $\times$ extension

We know,
Energy stored in wire $(U)$ = workdone in stretching wire $(W)$

$$\therefore U = \frac{1}{2} YA \frac{x^2}{L}$$

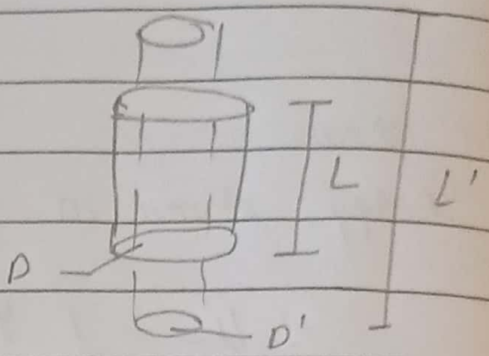
Now,
$$\text{Energy density} = \frac{U}{V} = \frac{\frac{1}{2}\frac{YAx^2}{L}}{AL} = \frac{1}{2} \times \frac{F \times x}{A \times l}$$

$$= \frac{1}{2} \times \text{stress} \times \text{strain}.$$

// Note:
Elasticity decreases as temperature increases.
Since steel is more elastic than rubber, for a given stress, the strain produced in steel is much smaller than in rubber.
Thus, this implies that Young's modulus for steel is greater than that of rubber.

# Poisson's Ratio

Let a wire of original length 'L' and diameter 'D' is subjected to equal and opposite force F along its length.




The length increases to L' and diameter decreases to D'. Then,

Longitudinal strain ($\alpha$) = $\dfrac{L-L'}{L}$ {in direction of force applied.}

and

Lateral strain ($\beta$) = $\dfrac{D-D'}{D}$ {in direction $\perp^r$ to force applied}.

So,

Poisson's ratio ($\sigma$) = $\dfrac{\text{Lateral strain } (\beta)}{\text{longitudinal strain } (\alpha)}$

Poisson's ratio is a pure number.

It is constant for a given material.