Discontinuity of a Function

A function $f(x)$ is said to be discontinuous at a point $x=c$ if it fails to meet any or all conditions mentioned.

(i): $f(c)$ exists. (c lies in $f(x)$ domain)

(ii) $\lim_{x \to c} f(x)$ exists (f has limit at $x \to c$)

(iii) $\lim_{x \to c} f(x) = f(c)$.

(*) Types

They are: jump discontinuity, oscillating discontinuity, infinite discontinuity & removable discontinuity.

(i) Jump discontinuity:

In this case, one sided limits exists but they are unequal.

ie, LHL $\neq$ RHL

$$\lim_{x \to a^-} f(x) \neq \lim_{x \to a^+} f(x)$$

(ii) Oscillating discontinuity:

In this case, function $f(x)$ is oscillating discontinuity at $x=c$, if $f(x)$ is not defined at $x=c$.

(iii) Infinite discontinuity:

In this case, any one of the one side limit is infinite.

(iv) Removable discontinuity:

In this case, the one sided limits are ~~equal an~~ finite and equal but it is not equal to the functional value.