PHYS101 General Physics I

Mechanics

Lecture 1: Dynamics of system of particles

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Outline

Workdone by constant force

Work is said to be done if a force acting on a body moves it through a certain distance. If the force $\vec{F}$ and displacement $\vec{d}$ are in the same direction

i.e. $W = \vec{F}.\vec{d}$

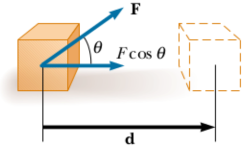

Figure 1: If an object under- goes a displacement $\vec{d}$ under the action of a constant force $\vec{F}$, the work done by the force is $(Fcos\theta)d$.

Workdone by constant force (contd.)

If the force and displacement are not in the same direction then

W = product of component of the force in the direction of displacement and displacement.

i.e. $W = Fcos\theta d$

$\rightarrow$ When $\theta < 90^0$, W is positive.

i.e. body gains kinetic energy.

$\rightarrow$ When $\theta = 90^0$, W is zero.

i.e. there is no transfer of energy from one object to the another. (Surface has no component in the direction of motion and the force does no work on the particle)

For example: Pushing the wall, Body moving in a circular motion.

$\rightarrow$ When $\theta > 90^0$, W is negative.

i.e. there is loss of kinetic energy of the body.

In SI system the unit of work done is joule.

We consider case where the numbers of forces acts on a body. Let $\vec{F}_1, \vec{F}_2, ...\vec{F}_n$ be the forces acting on the body and let the body move through a distance $\vec{d}$.

In general, $W = Fcos\theta d$ represents the scalar product of two vectors $\vec{F}$ and $\vec{d}$.

Work done by force $\vec{F}_1 = W_1 = \vec{F}_1.\vec{d}$

Work done by force $\vec{F}_2 = W_2 = \vec{F}_2.\vec{d}$

....

....

Work done by force $\vec{F}_n = W_n = \vec{F}_n.\vec{d}$

Now the total work done =

$W_1 + W_2 + ... + W_n = \vec{F}_1.\vec{d} + \vec{F}_2.\vec{d} + ... + \vec{F}_n.\vec{d}$

$= \vec{F}.\vec{d}$ = resultant force . $\vec{d}$

This is the work done by the resultant force so when a number of forces act on a body. Sum of the work done by all the individual forces = work done by the resultant forces.

Problem: A block of mass $10kg$ is to be raised from the bottom to the top of an inclined plane ($5m$ long and $3m$ up the ground). Assuming frictionless surface, how much work must be done by a force parallel to the incline pushing the block up to the constant speed?

Work done by variable force

A force which changes the position of the body. It may change the magnitude or direction or both . i.e. the force required to keep a rocket moving away from the earth, force exerted by a stretched spring etc.

One Dimensional Case:

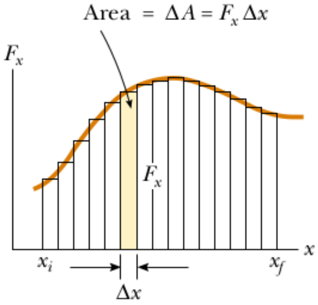

Figure 2: The work done by the force component F_x for the small displacement Δx is $F_x \Delta x$, which equals the area of the shaded rectangle. The total work done for the displacement from x_i to x_f is approximately equal to the sum of the areas of all the rectangles.

One Dimensional Case: (contd.)

Consider a particle being displaced along the x axis under the action of a varying force. The particle is displaced in the direction of increasing x from $x = x_i$ to $x = x_f$. In such a situation, we cannot use $W = (F cos\theta)d$ to calculate the work done by the force because this relationship applies only when $\vec{F}$ is constant in magnitude and direction. However, if we imagine that the particle undergoes a very small displacement Δx, as shown in Figure, then the x component of

One Dimensional Case: (contd.)

the force F_x is approximately constant over this interval; for this small displacement, we can express the work done by the force as

$$\Delta W = F_x \Delta x$$

This is just the area of the shaded rectangle in the Figure. If we imagine that the F_x versus x curve is divided into a large number of such intervals, then the total work done for the displacement from x_i to x_f is approximately equal to the sum of a large number of such terms:

$$W \approx \sum_{x_i}^{x_f} F_x \Delta x$$

One Dimensional Case: (contd.)

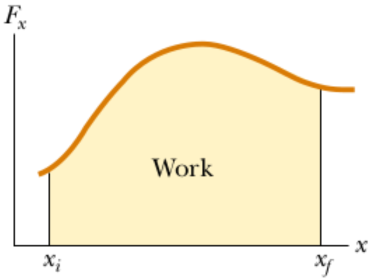

Figure 3: The work done by the component F_x of the varying force as the particle moves from x_i to x_f is exactly equal to the area under this curve.

One Dimensional Case: (contd.)

If the displacements are allowed to approach zero, then the number of terms in the sum increases without limit but the value of the sum approaches a definite value equal to the area bounded by the F_x curve and the x axis:

$$\lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F_x \Delta x = \int_{x_i}^{x_f} F_x dx$$

This definite integral is numerically equal to the area under the F_x versus x curve between x_i and x_f . Therefore, we can express the work done by F_x as the particle moves from x_i to x_f as

$$W = \int_{x_i}^{x_f} F_x dx$$

One Dimensional Case: (contd.)

This equation reduces to Equation $W = Fdcos\theta$ when the component $F_x = Fcos\theta$ is constant.

If more than one force acts on a particle, the total work done is just the work done by the resultant force. If we express the resultant force in the x direction as $\sum F_x$, then the total work (or net work) done as the particle moves from x_i to x_f is

$$\sum W = W_{net} = \int_{x_i}^{x_f} (F_x dx)$$

Graphically, the work done by a variable force F_x from an initial point x_i to final point x_f is given by the area under the force - displacement curve as shown in the figure.

Three Dimensional Case:

Consider a body moved along a curved path by a force. When the magnitude and direction of a force vary in three dimensions, it can be expressed as a function of the position $\vec{F}(\vec{r})$, or in terms of the coordinate $\vec{F}(x, y, z)$. The work done by such a force in an infinitesimal displacement $d\vec{s}$ is

$$\Delta W = \vec{F}.d\vec{s}$$

The total work done in going from point A to point B as shown in the figure.

Three Dimensional Case: (contd.)

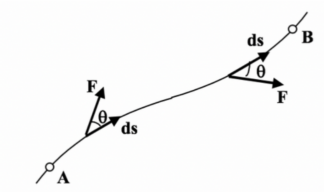

Figure 4: A particle moves along a curved path subject to a variable force $\vec{F}$.

The work done by the force in a displacement $d\vec{s}$ is $\Delta W = \vec{F}.d\vec{s}$

$$\text{i. e.} \quad W_{A\to B} = \int_A^B \vec{F}.d\vec{s} = \int_A^B (F cos\theta) ds$$

In terms of rectangular components,

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k} \quad \text{and} \quad d\vec{s} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

Three Dimensional Case: (contd.)

Therefore,

$$W_{A\to B} = \int_{x_A}^{x_B} F_x dx + \int_{y_A}^{y_B} F_y dy + \int_{z_A}^{z_B} F_z dz$$

Work Done by a Spring

If x be the displacement of the free end of the spring from its equilibrium position then, the magnitude of spring force is given by

$$F_x = -kx$$

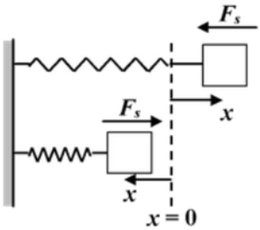

Figure 5: The force exerted by an ideal spring is given by Hooke's law: $F_s = -kx$, where x is the extension or compression of the spring.

The negative sign signifies that the force always opposes the extension $(x > 0)$ or the compression $(x < 0)$ of the spring. In other words, the force tends to restore the system to its equilibrium position.

The work done by the spring force for a displacement from x_i to x_f is given by

$$W_s = \int_{x_i}^{x_f} F_s dx = -\int_{x_i}^{x_f} kxdx$$

$$\text{or} \quad W_s = -\frac{1}{2}k\left({x_f}^2 - {x_i}^2\right)$$

Work Done by a Spring (contd.)

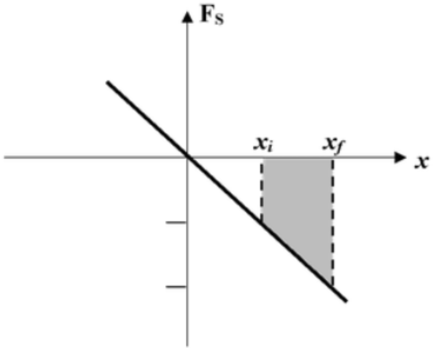

Figure 6: The work done by the spring when the displacement of its free end changes from x_i to x_f is the area of the trapezoid: $W_s = -\frac{1}{2}k\left({x_f}^2 - {x_i}^2\right)$

Work Done by a Spring (contd.)

Graphically, the work done by the spring force in a displacement from x_i to x_f is the shaded area (as shown in the figure above) which is the difference in the areas of two triangles.

Note:

The work done by a spring force is negative.

The work done by the spring force only depends on the initial and final points.

The net work done by the spring force is zero for any path that returns to the initial point.

Problem:

A small object of mass m is suspended from a string of length l. The object is pulled sideways by a force F that is also horizontal. Until the string finally makes an angle θ_m with the vertical. The displacement is accomplished of a small constant speed. Find the work done by all forces that act on the object.

Problem: (contd.)

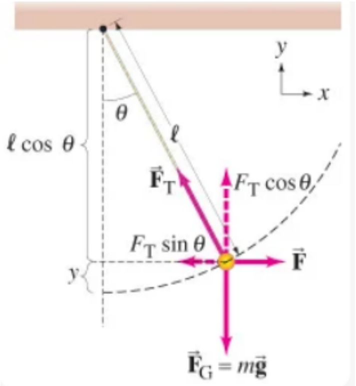

Figure 7: A particle is suspended from a string of length l and is pulled a side by a horizontal force $\vec{F}$. The maximum angle reached is θ_m

Problem: (contd.)

Hints: Applying Newton's second law

Along x-axis, $F = Tsin\theta$

Along y-axis, $mg = Tcos\theta$

$\implies F = mgtan\theta$

The work done by force F is $W_F = \int Fdx = \int_0^{\theta_m} mgtan\theta dx$

At an arbitrary position $x = lsin\theta$

$\implies dx = lcos\theta d\theta$

$\therefore W_F = \int_0^{\theta_m} mgtan\theta lcos\theta d\theta = mgl\left[1 - cos\theta_m\right]$

$\because \left[cos\theta_m = \frac{l-h}{l}\right] \implies h = l\left(1 - cos\theta_m\right)$

Hence, $W = mgh$