MECHANICS

CHAPTER 5: VISCOCITY

Fluids:

A fluid is any substance that can flow. This term is used for both liquids and gases.

General Characteristics of Fluid Flow.

1: Fluid flow can be steady or non-steady.

Steady flow	Non-steady flow
In steady flow, flow speed is low.	In non-steady flow, flow speed is high.
The velocity of moving fluid at any fixed point remains constant in time. Each particle follows a smooth path such that path of different particles never cross each other.	The velocity of the moving fluid vary erratically from point-point and time-time. ie, velocity is the function of time.
Eg: gentle flow of water near the center of a quiet stream.	Eg: a waterfall.

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2) Fluid flow may be compressible or Incompressible.

If the density of a fluid is a constant, independent of x, y, z and t, its flow is called incompressible flow.

Liquids can be usually considered as flowing incompressibly.

*) For a highly compressible gas, the variation in its density is considered insignificant. Thus, its flow is considered in compressible.

Eg: flow of air over wings is nearly incompressible.

3) Fluid flow may be viscous or non-viscous:

Viscosity is also called resistive force in liquids.

If viscosity is greater, greater force must be applied to maintain the flow.

ie, honey and motor oil are more viscous than water and air.

Although all fluid flow has presence of viscosity, in some cases, its effect is negligible.

In such case, the flow is regarded as non-viscous.

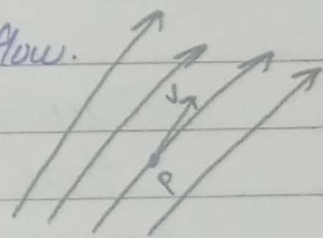
4: Fluid flow can be rotational or irrotational.
Let us consider a flowing stream.

If any particle moving with the stream doesn't rotate about an axis through its centre of mass, this flow is called irrotational.
Else, the flow is called rotational.

Types of Flow

Flow are of three types: Streamlined, Laminar and Turbulent.

a) Streamlined flow:
The path taken by a fluid particle under steady flow is called streamlined flow.



A set of streamlines is called a tube of flow.

Here, the velocity of the particle is tangent to the streamline.

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b) Laminar Flow:

In laminar flow, the fluid speed is low and the adjacent layers of fluid are sliding smoothly over one another.

Here, the velocity of the moving fluid at any fixed point remains constant in time.

Since each particles follows smooth path, their paths never cross each other.

Eg: Gentle water flow near center of quiet stream.

c) Turbulent flow:

In turbulent flow, the fluid speed is sufficiently large and there is great disorder and a constantly changing flow pattern.

Velocities vary erratically from point-point and time-time.

Turbulent flow occurs when particle goes above some critical speed.

Eg: waterfall.

Viscocity:

Viscocity is the property by virtue of which it opposes the relative motion between the different layers of liquid.

Viscous force opposes the motion of one portion of a fluid relative to another portion.

Viscocity is the internal friction of the fluid.

+) Importance:

i) Flow of fluids in pipes
ii) flow of blood
iii) Lubrication of engine parts.

- Viscocity is dependent on temperature.

For liquid, $V_f \propto \frac{1}{T}$

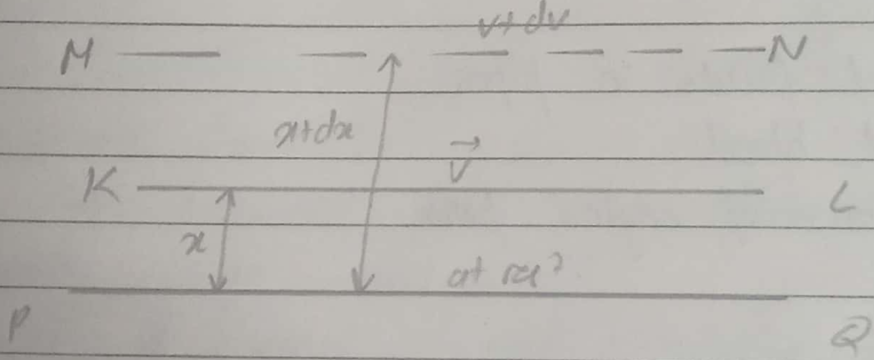
For gas, $V_f \propto T$

Newton's law of Viscocity

Newton's law of Viscocity states that, "Shear stress is directly proportional to velocity gradient."

or,

"For the straight and parallel motion of a given fluid, the shear stress between the two adjoining layers of the fluid is directly proportional to the negative value of the velocity gradient between the same two adjacent fluid layers."



Let us consider a flow of liquid over a flat plate PQ as shown in figure.

We know, different liquid layers at different distance from PQ have different velocities.

Here, the layer with contact to PQ has zero velocity and velocity goes on increasing upwards.

According to Newton's law of viscous flow for streamline motion, the tangential viscous force F acting between two layers of area A at distance dx apart and moving with relative velocity dv is directly proportional to.

i) area of the layers. (A)
ii) velocity gradient (dv/dx).

Such that,

$$F \propto A \quad \text{— (i)}$$

and

$$F \propto \frac{dv}{dx} \quad \text{— (ii)}$$

Combining (i) and (ii), we get.

$$F \propto A\frac{dv}{dx}$$

or, $$F = -\eta A \frac{dv}{dx} \quad \text{— (iii)}$$

Here η = coefficient of viscosity of fluid. It is constant for particular fluid at particular temperature.

Here eq^n(iii) is called Newton's law of viscosity for one-dimensional flow.

If $A = 1\,m^2$, $dv/dx = 1$ then $F = -\eta$. $\therefore \eta = -F$

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Thus, coefficient of viscosity of fluid can be defined as the viscous force acting per unit area of contact between two layers having a unit velocity gradient between them.

SI unit = $N \cdot s/m^2$ or $Pa \cdot s$ or Decapoise.
CGS unit = $dyne \cdot s/cm^2$ or Poise.

$$1 \text{ poise} = 0.1 \ N \cdot s/m^2$$
$$= 0.1 \text{ Decapoise.}$$
$$\therefore 10 \text{ poise} = 1 \text{ Decapoise.}$$

At 20°C, water has viscocity 0.01 poise.

Reynolds Number

Reynolds number is a dimensionless number used to determine whether a fluid has laminar or turbulent flow.

Mathematically,
$$R = \frac{\rho D v}{\eta}$$

Here,
R = Reynolds number
ρ = density of fluid.
D = diameter of pipe.
v = velocity of flow.
η = viscocity of the flow.

According to Reynold's number, flow through pipes are of the following types.

1. Laminar flow : $R \le 2000$
2. Transitional flow: $R > 2000$ and $R < 4000$
3. Turbulent flow: $R \ge 4000$

For cylindrical pipes, the Reynolds number corresponding to critical speed is 2000.

Ideal Fluids

The fluids ~~whose~~ which is incompressible and non-viscous and whose flow is steady and irrotational is called ideal fluids.

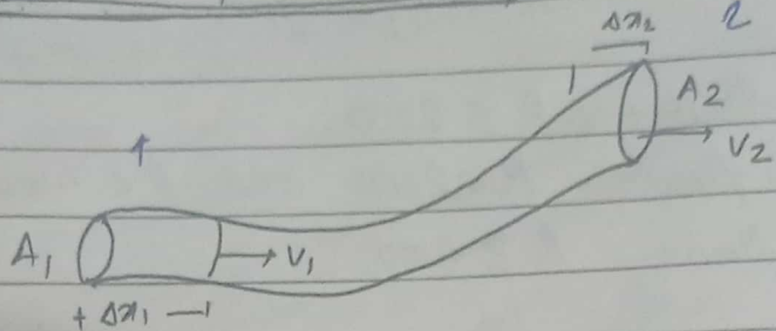
Volume flow rate / Volume flux of Fluid $(R_v) = A \times V$
[Eqⁿ of Continuity]

Since liquid is incompressible, the volume of fluid entering one end of the tube is equal to volume of leaving the other end of tube in same time interval.

Mass flow rate / Mass flux of fluid $(R_m) = \rho A v =$ constant.

This demonstrates law of conservation of mass in fluid dynamics.

Equation of Continuity



Let us consider an ideal fluid flowing with steady flow through a pipe of varying cross-sectional area.

Let v_1 and v_2 be the velocities of fluid at cross-section A_1 and A_2 of tube respectively.

In time Δt, the fluid at bottom of pipe moves distance Δx_1. Here, $\Delta x_1 = \Delta t \times v_1$

Mass of fluid at region 1 $(m_1) = \rho A_1 \Delta x_1$
$= \rho A_1 v_1 \Delta t$ — (i)

Here, ρ = density of ideal fluid.

Similarly,
Mass of fluid at region 2 $(m_2) = \rho A_2 \Delta x_2$
$= \rho A_2 \cdot v_2 \Delta t$ — (ii)

Since the mass is conserved and flow is steady,

mass entering region 1 = mass ~~entering~~ exiting from region 2.
~~So, from $eq^n(i)$~~ So, equating eq^n(i) and (ii),

$$m_1 = m_2$$

or, $\rho A_1 v_1 \Delta t = \rho A_2 v_2 \Delta t$

so, $A_1 v_1 = A_2 v_2$ —— (iii)

i.e, $A \times v =$ constant. —— (iv)

Here eq^n (iii) and eq^n (iv) is called equation of continuity.

Equation of continuity states that, "the product of the area and the fluid speed at all points along the pipe is constant for an incompressible fluid."

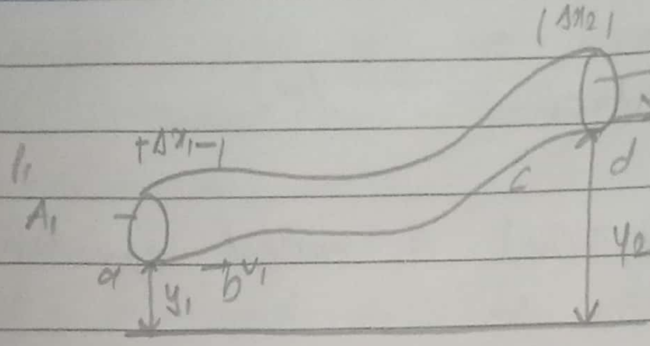
From eq^n (iv).

$$A \propto \frac{1}{v}.$$

This shows that velocity is inversely proportional to the area of the pipe.

Bernoulli's Equation

Bernoulli's theorem states that, "For steady flow, the sum of pressure energy, potential energy and kinetic energy ie, total energy of a fluid remains constant."



Consider the flow of an ideal fluid through a non-uniform pipe in time Δt.

The lower end of the pipe lies at height y_1 and upper end lies at height y_2 from ground.

The speeds at the upper end and the lower ends are v_1 and v_2.

In small time interval Δt, the fluid moves by Δx_1 from a to b. $\Delta x_1 = V_1 \cdot \Delta t_1$

and ~~from~~ by Δx_2 from c to d $\Delta x_2 = V_2 \cdot \Delta t_2$.

The cross-sectional area of the two ends are A_1 and A_2.

From continuity eq^n;

$A_1 v_1 = A_2 v_2$

Multiplying by Δt on both sides,

$A_1 v_1 \Delta t = A_2 v_2 \Delta t$

or, $A_1 \Delta x_1 = A_2 \Delta x_2 = \Delta V$

The volume of fluid ΔV passing any cross-section at time Δt is same.

Hence,
the networkdone on fluid element by pressure of surrounding fluid is

$W = P_1 A_1 \Delta x_1 - P_2 A_2 \Delta x_2$

$\therefore W = (P_1 - P_2) \Delta V$ —— (i)

Here, the work (W) is due to the forces other than conservative force of gravity.
So, it equals to the change in the total mechanical energy associated with fluid element.

The net change in KE ΔK during time Δt is

$\Delta K = \frac{1}{2}(\rho \Delta V) v_2^2 - \frac{1}{2}(\rho \Delta V) v_1^2$

$\therefore \Delta K = \frac{1}{2} \rho \Delta V (v_2^2 - v_1^2)$ —— (ii)

The net change in gravitational potential energy ΔU during time Δt is

$$\Delta U = (\rho \Delta V) g y_2 - (\rho \Delta V) g y_1$$
$$\therefore \Delta U = \rho \Delta V g (y_2 - y_1) \quad \text{--- (iii)}$$

We know,

$$W = \Delta K + \Delta U \quad \text{--- (iv)}$$

~~Putting~~ Combining eq^n (i), (ii), (iii) in (iv),

$$(P_1 - P_2)\Delta V = \frac{1}{2}\rho \Delta V (v_2^2 - v_1^2) + \rho \Delta V g (y_2 - y_1)$$

or $$P_1 - P_2 = \frac{1}{2}\rho (v_2^2 - v_1^2) + \rho g (y_2 - y_1)$$

Arranging, we get.

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2 \quad \text{--- (v)}$$

Eq^n (v) is Bernoulli's equation to an ideal fluid.
So,

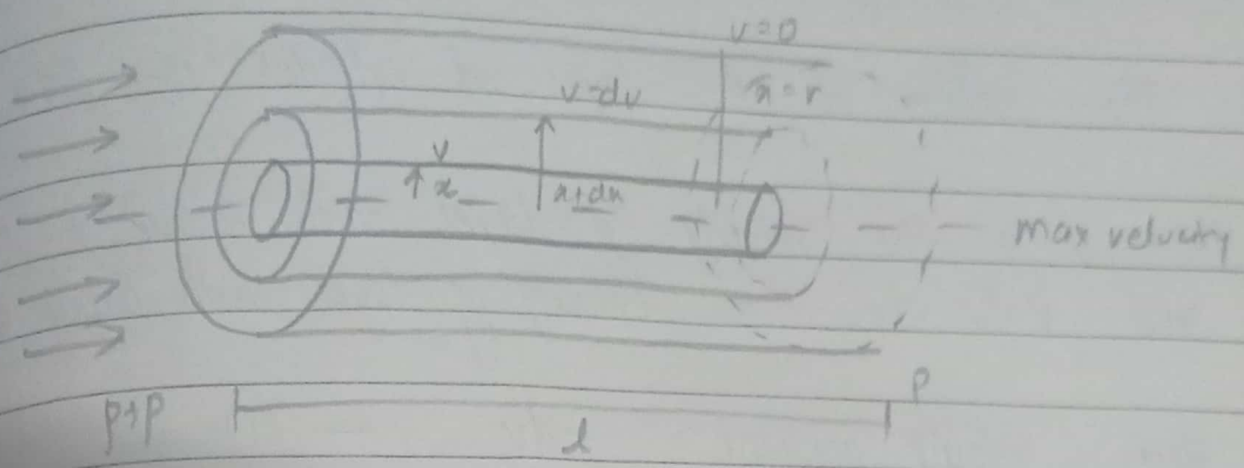
$$P + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$$

*) Significance

In laminar flow, the sum of pressure energy (P), kinetic energy per unit volume ($\frac{1}{2}\rho v^2$) and gravitational potential energy per unit volume ($\rho g y$) has same value at all points along streamline.

Steady Laminar Flow of Fluids in Pipe / Poiseuille's Formula

Liquid flow through a narrow tube.



Let us consider a steady laminar flow of fluid of viscosity η through a horizontal capillary tube of radius 'r' and length 'l' under constant pressure difference p as shown in figure.

The pressure at left side is maintained greater than at the right side such that liquid flows from left to right. The flow is axis symmetric.

When the flow is fully developed, the velocity profile is constant along pipe axis.

In laminar flow, the path of individual particles of fluid don't cross so, the pattern of flow can be imagined as a number of thin concentric cylinders of varying radii which slide over one another.

Here, the flow velocity varies with radius is, maximum value occurs at axis and is minimum at the walls.

Consider an arbitrary co-axial cylinder of fluid of radius x and thickness dx.

The surface of this layer of cylinder is $2\pi x l$.

Let v be the velocity of layer with radius x and $v-dv$ be the velocity of layer with radius $x+dx$.

Here, velocity gradient $= \frac{-dv}{dx}$ — (i)

The viscous force experienced by layer of radius x is

$$-\eta A \frac{dv}{dx} = -\eta 2\pi x l \frac{dv}{dx} \quad \text{— (ii)}$$

and the force experienced by this layer due to pressure difference $p = p \times \pi x^2$ — (iii) $[\because F = P \times A]$.

We know,

for steady laminar flow of fluid,

$$F_{vis} = F_{ext}$$

or, $-\eta (2\pi x l) \frac{dv}{dx} = p \times \pi x^2$ $[\because$ From eq^n(iii) & (iv)$]$

or, $dv = \frac{-p x \, dx}{2\eta l}$ ——— (v)

Integrating eq^n (v), we get

$$\int dv = \int \frac{-px}{2\eta l} dx$$

or, $$v = \frac{-p}{2\eta l} \frac{x^2}{2} + C$$

$$\therefore v = -\frac{p}{4\eta l} x^2 + C \quad \text{—— (vi).}$$

We know,
when radius of lamina is equal to radius of capillary tube, velocity is 0.
So,
Putting $x = r$ and $v = 0$ in eq^n (vi),

$$0 = -\frac{p}{4\eta l} r^2 + C \qquad \therefore C = \frac{p}{4\eta l} r^2 \quad \text{— (vii)}$$

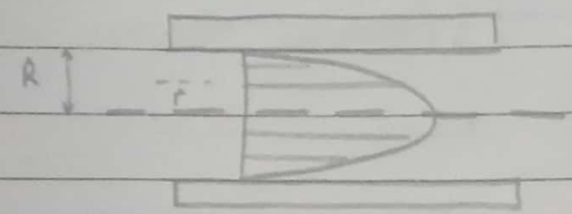
Putting value of C in eq^n (vi),

$$v = \frac{pr^2}{4\eta l} - \frac{px^2}{4\eta l}$$

$$\therefore v = \frac{p}{4\eta l}(r^2 - x^2) \quad \text{—— (viii)}$$

Here, eq^n (viii) shows that as liquid approaches the wall of pipe, velocity decreases from max value $v_0 = \frac{P}{4\eta l} r^2$ to zero.

This shows that the velocity distribution curve for laminar flow of a viscous fluid in long cylindrical pipe is parabolic.



The cross-sectional area of this elemental cylindrical layer of liquid flowing through layer of radius x and $x+dx$ is

$$dA = 2\pi x\, dx \quad \text{— (ix)}$$

The volume of liquid flowing through this elemental cylindrical tube of fluid per unit time is

$$dv = dA \cdot v$$
$$= 2\pi x\, dx \cdot \frac{P}{4\eta l}(r^2 - x^2)$$

$$\therefore dv = \frac{\pi P}{2\eta l}(r^2 - x^2)\, x\, dx. \quad \text{— (x)}$$

Integrating eq^n (x),

the volume of fluid flowing through the pipe per unit time is.

$$V = \frac{\pi p}{2\eta l}\int_0^r (r^2 - x^2)\, x\, dx$$

$$= \frac{\pi p}{2\eta l}\left[\frac{r^2 x^2}{2} - \frac{x^4}{4}\right]_0^r$$

$$= \frac{\pi p}{2\eta l}\left[\frac{r^4}{2} - \frac{r^4}{4}\right]$$

$$\therefore V = \frac{\pi p r^4}{8\eta l} \qquad \text{——— (xi)}$$

This equation is known as Poiseuille's formula.
This is applicable only to the fully developed laminar flow of constant-density fluids.

Therefore,
coefficient of viscosity $(\eta) = \frac{\pi p r^4}{8 L V}$

*) Limitations
i) The flow of liquid must be streamlined
ii) The velocity of layer in contact to walls must be zero.
iii) The acceleration of liquid should be zero.
iv) The formula is not applicable for gas.