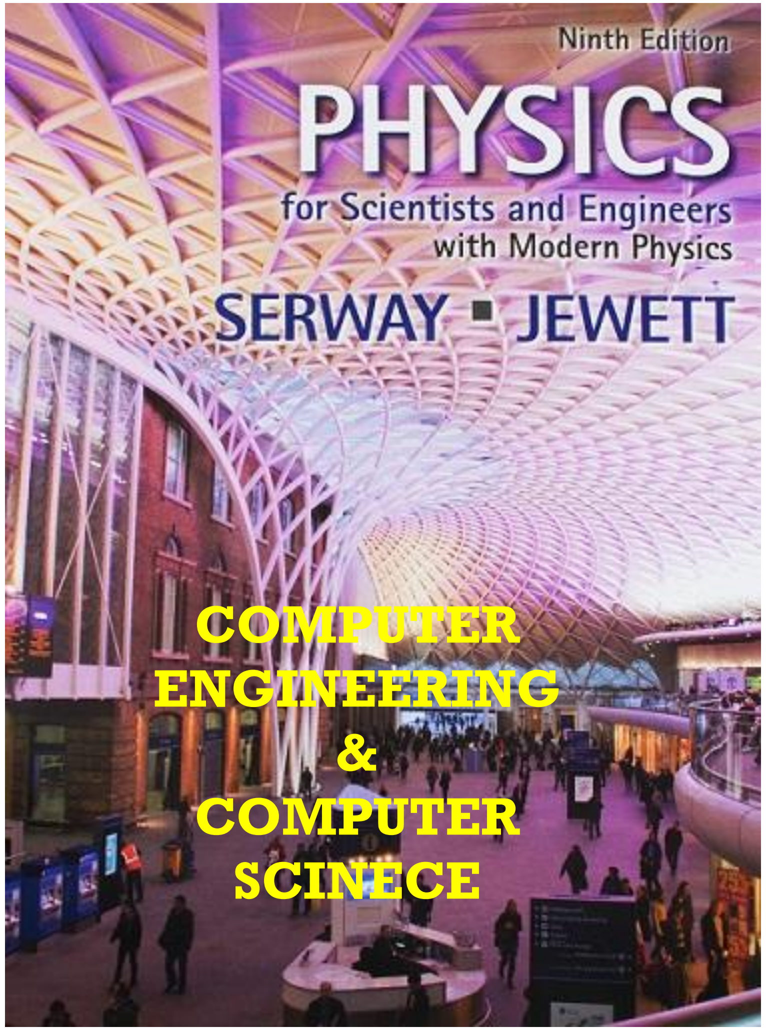
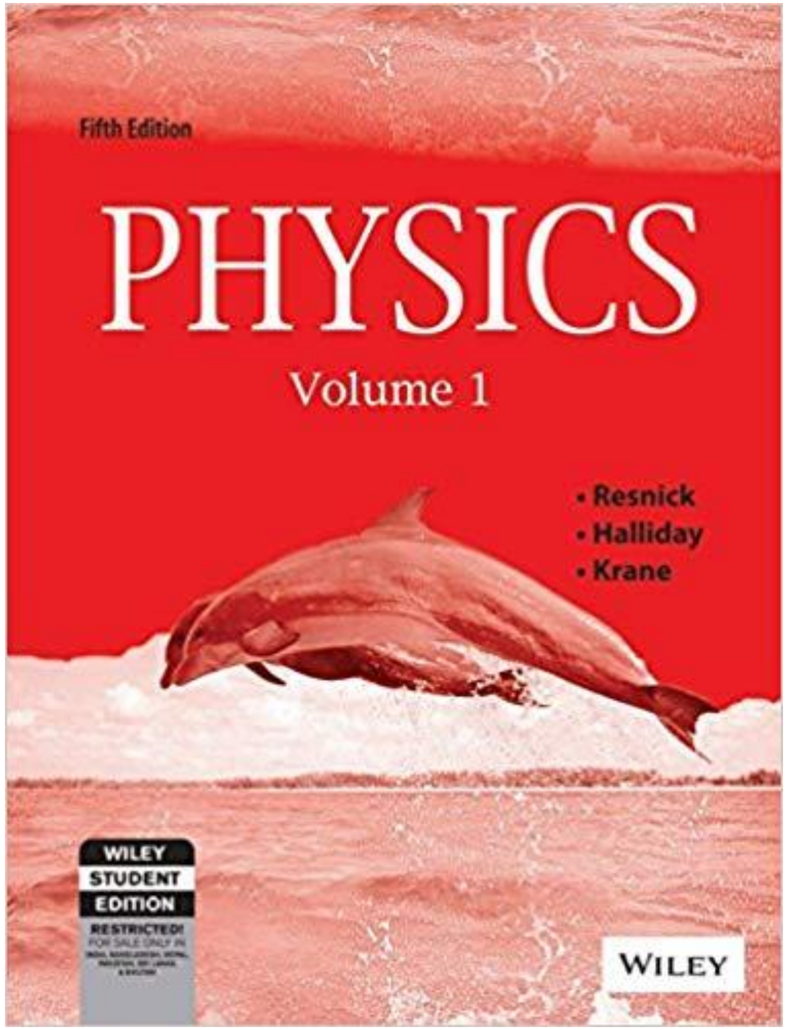
PHYSICS

General Physics I (PHYS 101)





Ganesh Kuwar Chhetri

Dept. of Physics, School of Science, Kathmandu University

Course Outline

- **System of Variable Mass**

- **Rocket**

- **Collision**
  - ➢ **Elastic Collision in One Dimension**
  - ➢ **Elastic Collision in Two Dimensions**
  - ➢ **Perfectly Inelastic Collision**
    - ✓ **Ballistic Pendulum**

System of Variable Mass

System of Variable Mass

- Figure R-1 shows a schematic view of a generalized system.

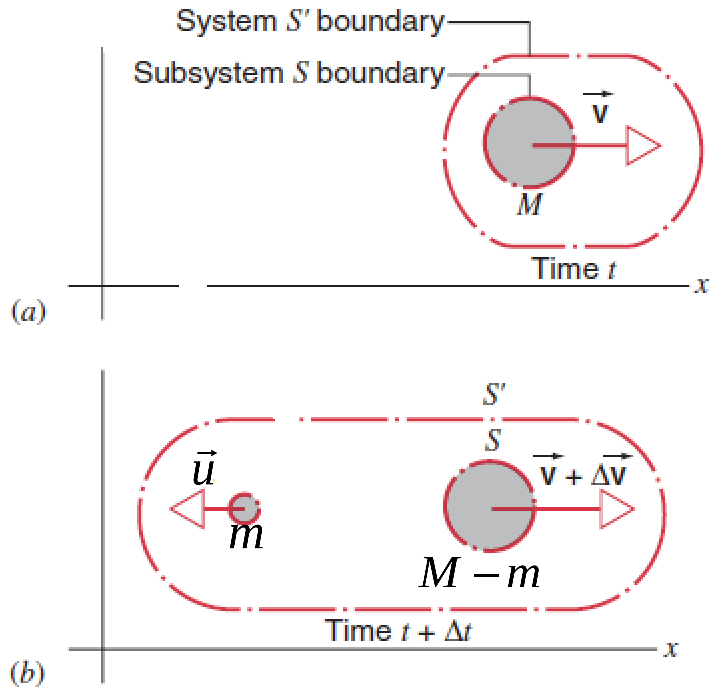


Figure R-1

At a time t, a system consists of a mass M whose centre of mass is moving with velocity $\vec{v}$ in the particular inertial frame of reference from which we are observing.

At a time $t+\Delta t$ the original mass M has ejected some mass m. The centre of mass of the remaining mass $M-m$, which we call the subsystem S, is now moves with velocity $\vec{v}+\Delta\vec{v}$, and the centre of mass of the ejected matter moves with velocity $\vec{u}$, both measured from our frame of reference.

An external force $\sum \vec{F}_{ext}$ acts on the entire system.

In the time interval Δt, the change in momentum $\Delta\vec{P}$ is

$$\Delta\vec{P} = \vec{P}_f - \vec{P}_i$$

where $\vec{P}_f$ is the final momentum of the system S' at time $t+\Delta t$

$\vec{P}_i$ is the initial momentum of the system S' at time t

$$= \left[(M-m)(\vec{v}+\Delta\vec{v}) + m\,\vec{u}\right] - \left[M\,\vec{v}\right]$$

$$= M\Delta\vec{v} + (\vec{u}-\vec{v})m - m\Delta\vec{v} \qquad \text{..........} \quad (1)$$

System of Variable Mass

In the time interval Δt, the change in mass of the subsystem S is

$$\Delta M = (M - m) - M = -m \Rightarrow m = -\Delta M$$

Hence the change in momentum is

$$\boxed{\Delta\vec{P} = M\Delta\vec{v} - (\vec{u} - \vec{v})\Delta M + \Delta\vec{v}\,\Delta M}$$

According to Newton's second law,

$$\sum \vec{F}_{ext} = \frac{d\vec{P}}{dt} = \lim_{\Delta t \to 0} \frac{\Delta\vec{P}}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{M\Delta\vec{v} - (\vec{u} - \vec{v})\Delta M + \Delta\vec{v}\Delta M}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \left[M\frac{\Delta\vec{v}}{\Delta t} - (\vec{u} - \vec{v})\frac{\Delta M}{\Delta t} + \Delta\vec{v}\frac{\Delta M}{\Delta t} \right]$$

$$= M\frac{d\vec{v}}{dt} - (\vec{u} - \vec{v})\frac{dM}{dt} \qquad \text{..........} \quad (2)$$

Equation (2) can be written in more instructive form as

$$M\frac{d\vec{v}}{dt} = \sum \vec{F}_{ext} + (\vec{u} - \vec{v})\frac{dM}{dt}$$

$$\therefore \boxed{M\frac{d\vec{v}}{dt} = \sum \vec{F}_{ext} + \vec{v}_{rel}\frac{dM}{dt}}$$

where $\vec{v}_{rel} = \vec{u} - \vec{v}$ is the velocity of the gained or lost matter relative to the subsystem S

This is the Newton's Second Law for a system of variable mass.

$$M\frac{d\vec{v}}{dt} = \sum \vec{F}_{ext} + \vec{F}_{reaction}$$

where $\vec{F}_{reaction} = \vec{v}_{rel}\dfrac{dM}{dt}$

$\downarrow$

reaction force exerted on the subsystem S by the mass that leaves it.

Rocket

Rocket

- A rocket is a space vehicle or projectile that is forced through space or the atmosphere by jet propulsion and that carries its own propellants (fuel and oxidizer).

- The rocket is the most interesting example of a system of variable mass.

- Its motion can be explained on the basis of **Newton's third law of motion** and the **momentum principle**.

  Let us choose a fixed-mass system (rocket + gas) and attach a reference frame to its centre of mass. The rocket forces a jet of hot gases from its exhaust; this is the action force. The jet of hot gases exerts a force on the rocket, propelling it forward. This is the reaction force. These forces are internal forces in the system (rocket + gas) .

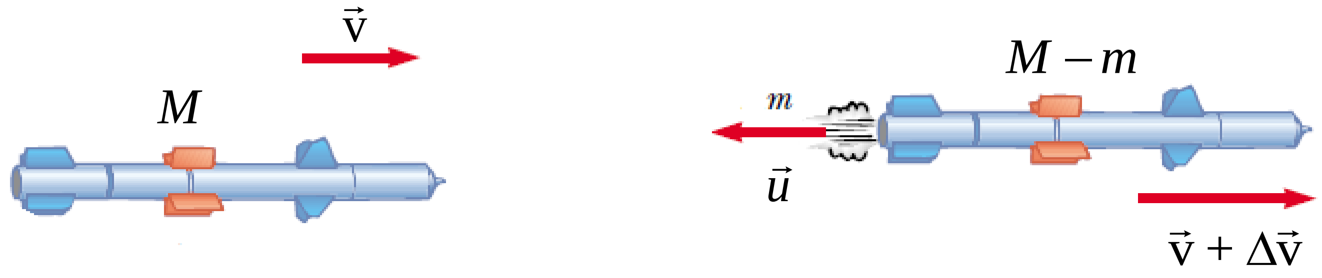
  In the absence of external forces, the total momentum of the system is constant (the centre of mass, initially at rest, remains at rest). The individual parts of the system (rocket and gases) may change their momentum, however; with respect to centre of mass frame, the hot gases acquire momentum in the backward direction and the rocket acquires an equal amount of momentum in the forward direction.

- It is used to place a satellite into an orbit.

Rocket

Rocket

- Consider a rocket fired in outer space. Figure R-2shows a rocket at (a) time t and (b) time $t + \Delta t$.



Time t

(a)

Time $t + \Delta t$

(b)

Figure R-2 Rocket Propulsion

a) The initial mass of the rocket plus all its fuel is M at a time t, and its speed is $\vec{v}$ in an inertial frame of reference (Earth).

b) At a time t + Δt , the rocket's mass has been reduced to M –m and an amount of fuel m has been ejected with a speed $\vec{u}$. The rocket's speed increases by an amount $\Delta\vec{v}$.

According to Newton's second law for a system of variable mass,

exhaust velocity $\vec{v}_{rel} = \vec{u} - \vec{v}$

$$M\frac{d\vec{v}}{dt} = \sum \vec{F}_{ext} + \vec{v}_{rel}\frac{dM}{dt} \quad \text{..........} \quad (1)$$

$\sum \vec{F}_{ext} \downarrow$ the total external force acting on the entire system

$$\boxed{\vec{v}_{rel}\frac{dM}{dt} \rightarrow \text{thrust exerted on the rocket by the ejecting gas-jet.}}$$

Rocket

Rocket

- If all the external forces, including gravity and air resistance, is zero i.e. $\sum \vec{F}_{ext} = 0$, then Eq. (1) can be written as

$$M \frac{d\vec{v}}{dt} = \vec{v}_{rel} \frac{dM}{dt}$$

or, $$d\vec{v} = \vec{v}_{rel} \frac{dM}{M} \quad \text{..........} \quad (2)$$

- Integrating Eq. (2) from the instant the velocity is $\vec{v}_0$, and the mass of the rocket plus its fuel is M_0 to the instant when the velocity is $\vec{v}$, and the mass of the rocket plus its fuel is M , we obtain

$$\int_{\vec{v}_0}^{\vec{v}} d\vec{v} = \vec{v}_{rel} \int_{M_0}^{M} \frac{dM}{M} \qquad \left[\vec{v}_{rel} = \text{constant } (\text{assumption})\right]$$

$$\therefore \boxed{\vec{v} - \vec{v}_0 = \vec{v}_{rel} \log_e \left(\frac{M}{M_0}\right)} \quad \text{..........} \quad (3)$$

Hence the change in speed of the rocket in any interval of time depends only on the exhaust velocity and on the fraction of mass exhausted during that interval.

Eq. (3) can be written as

$$\vec{v} = \vec{v}_0 + \vec{v}_{rel} \log_e \left(\frac{M}{M_0}\right)$$

$$\therefore \boxed{v = v_0 - v_{rel} \log_e \left(\frac{M}{M_0}\right)} \quad \text{..........} \quad (4)$$

If the rocket starts from rest $(v_0 = 0)$ with an initial mass M_0 and reaches a final velocity v_f at burnout when its mass is M_f , then Eq. (4) can be written as

$$v_f = -v_{rel} \log_e \left(\frac{M_f}{M_0}\right)$$

or, $$-\frac{v_f}{v_{rel}} = \log_e \left(\frac{M_f}{M_0}\right)$$

$$\therefore M_f = M_0 e^{-\frac{v_f}{v_{rel}}}$$

Rocket

- In the absence of external forces a **rocket** accelerates at an instantaneous rate given by

$$\vec{v}_{rel}\frac{dM}{dt} = M\frac{d\vec{v}}{dt}$$

$$\boxed{R\,\vec{v}_{rel} = M\,\vec{a} \quad (\text{First Rocket Equation})}$$

where M is the rocket's instantaneous mass (including unexpended fuel)

$R = \frac{dM}{dt} \rightarrow$ fuel consumption rate

$\vec{v}_{rel} \rightarrow$ exhaust velocity

$R\,\vec{v}_{rel} \rightarrow$ thrust of the rocket engine

- For a rocket with constant R and $\vec{v}_{rel}$, whose speed changes from v_i to v_f when its mass changes from M_i to M_f:

$$v_f = v_i - v_{rel}\log_e\left(\frac{M_f}{M_i}\right)$$

$$\boxed{\therefore v_f - v_i = v_{rel}\log_e\left(\frac{M_i}{M_f}\right) \quad (\text{Second rocket equation})}$$

- We have,

$$v_f = v_{rel}\log_e\left(\frac{M_0}{M_f}\right)$$

When $\frac{M_0}{M_f} = e^2$, then $v_f = v_{rel}\log_e\left(e^2\right)$

$$\therefore \boxed{v_f = 2v_{rel}}$$

The rocket speed is twice the exhaust speed .

Sample Problem

Rocket

- A rocket moving in space, far from all other objects, has a speed of $3.0\times10^{3}\ \mathrm{m\,s^{-1}}$ relative to the Earth.

  Its engines are turned on, and fuel is ejected in a direction opposite the rocket's motion at a speed of relative $5.0\times10^{3}\ \mathrm{m\,s^{-1}}$ to the rocket.

a. What is the speed of the rocket relative to the Earth once the rocket's mass is reduced to half its mass before ignition?

b. What is the thrust on the rocket if it burns fuel at the rate 50 kg/s ?

Hint:

$$\text{(a) } \mathrm{v_f} = \mathrm{v_i} + \mathrm{v_{rel}} \log_e\left(\frac{M_\mathrm{i}}{M_\mathrm{f}}\right) = 3.0\times10^{3} + \left(5.0\times10^{3}\right)\ln\left(\frac{M_\mathrm{i}}{0.5M_\mathrm{i}}\right) = 6.5\times10^{3}\ \mathrm{m\,s^{-1}}$$

$$\text{(b) Thrust} = \left|\mathrm{v_{rel}}\frac{dM}{dt}\right| = \left(5.0\times10^{3}\right)(50) = 2.5\times10^{5}\,N$$

Collision

Collision

- A collision is an isolated event in which two or more bodies [the colliding bodies] exert relatively strong forces on each other for a relatively short time.
- In collision of all kinds, momentum is always conserved.
- Collision processes are particularly important in nuclear and particle physics, where they are the major sources of experimental information.

Examples:

- A bat strikes a base ball.
- A cat walks delicately through the grass.
- The deflection suffered by an alpha particle in passing close to a nucleus
- Neutrons hitting atomic nuclei a nuclear reactor.
- A meteorite collides with the Earth.

Elastic Collision:

A collision in which total kinetic energy (as well as total momentum) is the same before and after the collision.

Examples:

- The collisions that occur in Rutherford scattering are elastic.
- Billiard-ball collisions and the collision of air molecules with the walls of a container at ordinary temperatures are approximately elastic.

Inelastic Collision:

A collision in which the total kinetic energy after the collision is less than before the collision (even though momentum is constant) is called an ***inelastic collision***.

Examples:

- The collision between two automobiles on a road is inelastic.
- The collision of a rubber ball with a hard surface

Collision

Completely Inelastic Collision:

An inelastic collision in which the colliding bodies stick together (and move as one body) after the collision is often called a ***completely inelastic collision*.**

Example:

- When a meteorite collides with the Earth, the collision is called perfectly inelastic.

Remember this rule:

In any collision in which external forces can be neglected, momentum is conserved and the total momentum before equals the total momentum after; in elastic collisions *only,* the total kinetic energy before equals the total kinetic energy after.

Classifying collision:

- Elastic: A collision in which kinetic energy is conserved.
- Inelastic: A collision in which the total kinetic energy decreases.
- Completely Inelastic: An inelastic collision in which the colliding bodies have a common final velocity.

Elastic Collision in One Dimension

Elastic Collision in One Dimension

Consider two perfectly elastic bodies 1 and 2 of masses m_1 and m_2 and moving along the same straight line with velocities v_{1i} and v_{2i} respectively $(v_{1i} > v_{2i})$. The two bodies undergo a head-on collision and continue moving along the same straight line with velocities v_{1f} and v_{2f} in the same direction [Figure C-1].

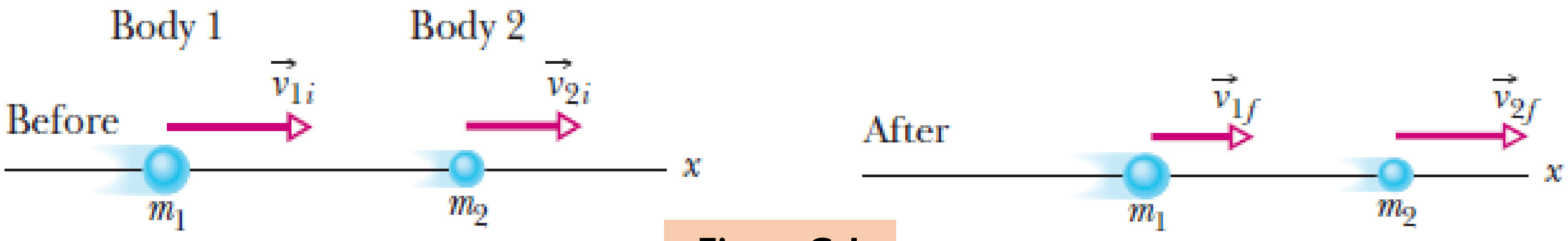


Figure C-1

By the conservation of momentum, we have

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \qquad \text{..........} \quad (A)$$

$$\Rightarrow \quad m_1 \left(v_{1i} - v_{1f}\right) = m_2 \left(v_{2f} - v_{2i}\right) \qquad \text{..........} \quad (1)$$

By the conservation of kinetic energy, we have

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

$$\Rightarrow \quad m_1 \left(v_{1i}^2 - v_{1f}^2\right) = m_2 \left(v_{2f}^2 - v_{2i}^2\right) \qquad \text{..........} \quad (2)$$

Dividing Eq. (2) by Eq. (1), we get

$$\frac{v_{1i}^2 - v_{1f}^2}{v_{1i} - v_{1f}} = \frac{v_{2f}^2 - v_{2i}^2}{v_{2f} - v_{2i}}$$

$$\text{or,} \quad v_{1i} + v_{1f} = v_{2f} + v_{2i}$$

$$\therefore \quad \boxed{v_{1i} - v_{2i} = v_{2f} - v_{1f}} \qquad \text{..........} \quad (3)$$

Thus, in one dimensional elastic collision, relative velocity of approach before the collision is equal to the relative velocity of separation after the collision.

Elastic Collision in One Dimension

For velocities after collision:

From Eq. (3), we have

$$v_{2f} = v_{1i} - v_{2i} + v_{1f}$$

Substituting v_{2f} in equation (A), we get

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 \left(v_{1i} - v_{2i} + v_{1f} \right)$$

$$\therefore \quad \boxed{v_{1f} = \left[\frac{m_1 - m_2}{m_1 + m_2}\right] v_{1i} + \left[\frac{2m_2}{m_1 + m_2}\right] v_{2i}} \quad \text{..........(4)}$$

Similarly,

$$\boxed{v_{2f} = \left[\frac{m_2 - m_1}{m_1 + m_2}\right] v_{2i} + \left[\frac{2m_1}{m_1 + m_2}\right] v_{1i}} \quad \text{..........(5)}$$

Special Cases

(i) For $m_1 = m_2$, $v_{1f} = v_{2i}$ and $v_{2f} = v_{1i}$

In one dimensional elastic collision of two bodies of equal masses, the two bodies simply exchange velocities as a result of collision.

(ii) For stationary target $v_{2i} = 0$

$$\boxed{v_{1f} = \left[\frac{m_1 - m_2}{m_1 + m_2}\right] v_{1i} \quad \& \quad v_{2f} = \left[\frac{2m_1}{m_1 + m_2}\right] v_{1i}}$$

Special Situations

(a) Equal masses: If $m_1 = m_2$, then $v_{1f} = 0$ and $v_{2f} = v_{1i}$ **In head-on collisions, bodies of equal masses simply exchange velocities.**

(b) A massive Target: If $m_2 \gg m_1$, then $v_{1f} \approx -v_{1i}$ and $v_{2f} \approx v_{2i} = 0$

When a very light particle collides head-on with a very heavy particle that is initially at rest, the light particle has its velocity reversed and the heavy one remains approximately at rest.

(c) A massive Projectile: If $m_1 \gg m_2$, then $v_{1f} \approx v_{1i}$ and $v_{2f} \approx 2v_{1i}$

when a very heavy particle collides head-on with a very light one that is initially at rest, the heavy particle continues its motion unaltered after the collision, and the light particle rebounds with a speed equal to about twice the initial speed of the heavy particle.

Elastic Collision in Two Dimensions

Figure C-2 shows a *glancing collision* (it is not head-on) between a projectile body and a target body initially at rest. The impulses between the bodies have sent the bodies off at angles θ_1 and θ_2 to the *x* axis, along which the projectile initially traveled and with velocities v_{1f} and v_{2f}.

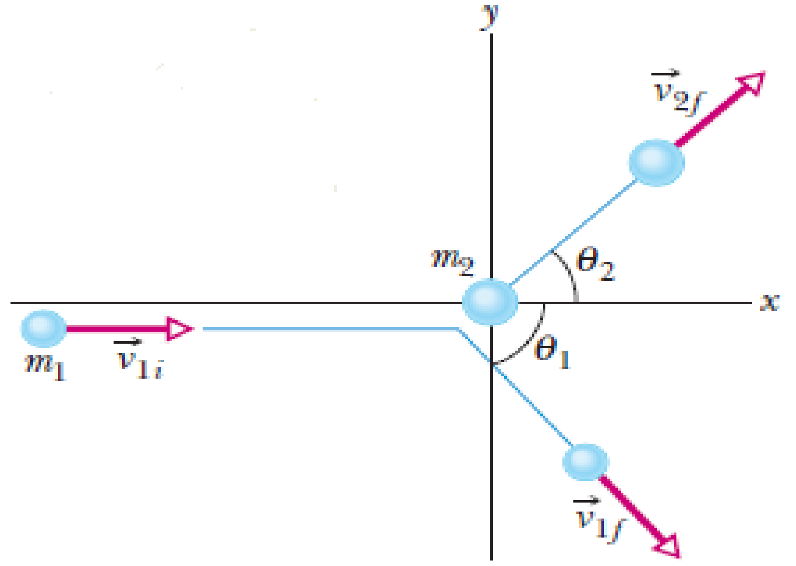


Figure C-2

By the conservation of momentum, we get

For components along x-axis:

$$m_1 v_{1i} = m_1 v_{1f}\cos\theta_1 + m_2 v_{2f}\cos\theta_2 \qquad \text{..........(1)}$$

For components along y-axis:

$$0 = -\, m_1 v_{1f}\sin\theta_1 + m_2 v_{2f}\sin\theta_2 \qquad \text{..........(2)}$$

By the conservation of kinetic energy, we get

$$\frac{1}{2} m_1 v_{1i}{}^2 = \frac{1}{2} m_1 v_{1f}{}^2 + \frac{1}{2} m_2 v_{2f}{}^2 \qquad \text{..........(3)}$$

For Equal Masses

$$v_{1f} = v_{1i}\cos\theta_1$$

$$v_{2f} = v_{1i}\sin\theta_1$$

$$\theta_1 + \theta_2 = 90^0$$

Equations 1 to 3 contain seven variables: two masses m_1 and m_2; three speeds, v_{1i}, v_{1f} and v_{2f}; and two angles θ_1 and θ_2. If we know any four of these quantities, we can solve the three equations for the remaining three quantities.

Sample Problem

- **A gas molecule having a speed of 300 meters/sec collides elastically with another molecule of the same mass which is initially at rest. After the collision the first molecule moves at an angle of 30^0 to its initial direction. Find the speed of each molecule after collision and the angle made with the incident direction by the recoiling target molecule.**

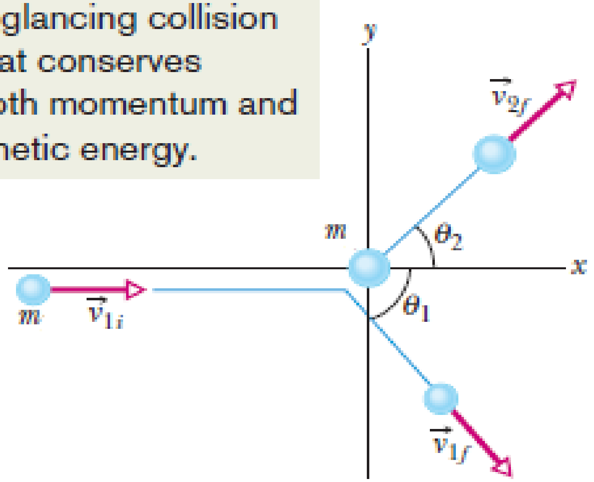
Hint:

Let 'm' be the mass of a gas molecule.

$$v_{1i} = 300 \text{ m s}^{-1},\ v_{2i} = 0 \text{ and } \theta_1 = 30^0$$

$$v_{1f} = ?, v_{2f} = ?,\ \theta_2 = ?$$

A glancing collision that conserves both momentum and kinetic energy.



By conservation of momentum:

$$v_{1i} - v_{1f}\cos\theta_1 = v_{2f}\cos\theta_2 \quad \text{........(1)}$$

$$v_{1f}\sin\theta_1 = v_{2f}\sin\theta_2 \quad \text{..........(2)}$$

By conservation of kinetic energy:

$$v_{1i}^2 - v_{1f}^2 = v_{2f}^2 \quad \text{..........(3)}$$

Squaring and adding Equations (1) and (2), we get

$$(v_{1i} - v_{1f}\cos\theta_1)^2 + (v_{1f}\sin\theta_1)^2 = (v_{2f}\cos\theta_2)^2 + (v_{2f}\sin\theta_2)^2$$

$$\Rightarrow \quad v_{1i}^2 - 2v_{1i}v_{1f}\cos\theta_1 + v_{1f}^2\cos^2\theta_1 + v_{1f}^2\sin^2\theta_1 = v_{2f}^2\cos^2\theta_2 + v_{2f}^2\sin^2\theta_2$$

$$\Rightarrow \quad v_{1i}^2 - 2v_{1i}v_{1f}\cos\theta_1 + v_{1f}^2 = v_{2f}^2$$

$$\Rightarrow \quad v_{1i}^2 - 2v_{1i}v_{1f}\cos\theta_1 + v_{1f}^2 = v_{1i}^2 - v_{1f}^2 \qquad \text{[using Equation (3)]}$$

$$\Rightarrow \quad 2v_{1f}^2 = 2v_{1i}v_{1f}\cos\theta_1$$

$$\Rightarrow \quad v_{1f} = v_{1i}\cos\theta_1$$

$$\therefore \quad \boxed{v_{1f} = v_{1i}\cos\theta_1 = 300\times\cos 30^0 = 260 \text{ m s}^{-1}}$$

From Eq.(3),

$$v_{2f}^2 = v_{1i}^2 - v_{1f}^2 = v_{1i}^2 - (v_{1i}\cos\theta_1)^2 = v_{1i}^2\sin^2\theta_1$$

$$\therefore \quad \boxed{v_{2f} = v_{1i}\sin\theta_1 = 300\times\sin 30^0 = 150 \text{ m s}^{-1}}$$

$$\boxed{\begin{aligned} v_{1f} &= v_{1i}\cos\theta_1 \\ v_{2f} &= v_{1i}\sin\theta_1 \\ \theta_1 + \theta_2 &= 90^0 \end{aligned}}$$

$$\boxed{\begin{aligned} \theta_2 &= 90^0 - \theta_1 \\ &= 90^0 - 30^0 \\ &= 60^0 \end{aligned}}$$

Perfectly Inelastic Collision in One Dimension

Perfectly Inelastic Collision in One Dimension

The collision, in which momentum is conserved with some loss in kinetic energy and the colliding bodies stick together after the collision is called perfectly inelastic collision.

Let us consider a perfectly inelastic one dimensional collision between two bodies. Let the masses of the bodies and initial and final velocities are as shown in Figure C-3.

Figure C-3

According to the conservation of momentum, we have

$$m_1 v_{1i} = (m_1 + m_2) V$$

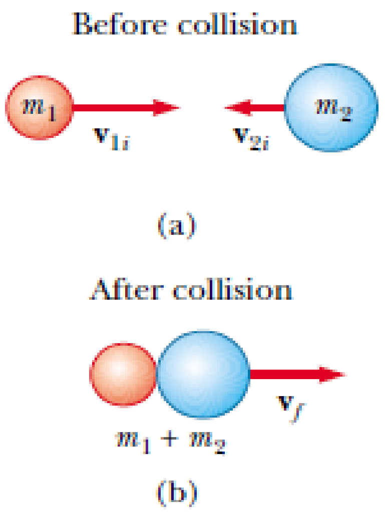
$$\therefore \quad V = \frac{m_1 v_{1i}}{m_1 + m_2} \quad .... \quad (1)$$

The loss in kinetic energy of the system during an inelastic collision is given by

$$\Delta T = \frac{1}{2} m_1 v_{1i}^2 - \frac{1}{2}(m_1 + m_2) V^2$$

$$= \frac{1}{2} m_1 v_{1i}^2 - \frac{1}{2}(m_1 + m_2)\left[\frac{m_1 v_{1i}}{m_1 + m_2}\right]^2 \qquad \text{[using Eq.(1)]}$$

$$\therefore \quad \boxed{\Delta T = \frac{1}{2}\left[\frac{m_1 m_2}{m_1 + m_2}\right] v_{1i}^2}$$



Loss in Kinetic Energy of the system during collision:

$$\boxed{\Delta T = \frac{1}{2}\left[\frac{m_1 m_2}{m_1 + m_2}\right](v_{1i} + v_{2i})^2}$$

A Ballistic Pendulum

A Ballistic Pendulum

- The ballistic pendulum was used to measure the speeds of bullets before electronic timing devices were available.
- The ballistic pendulum [Figure B-1) is a large wooden block of mass M hanging vertically by two cords.

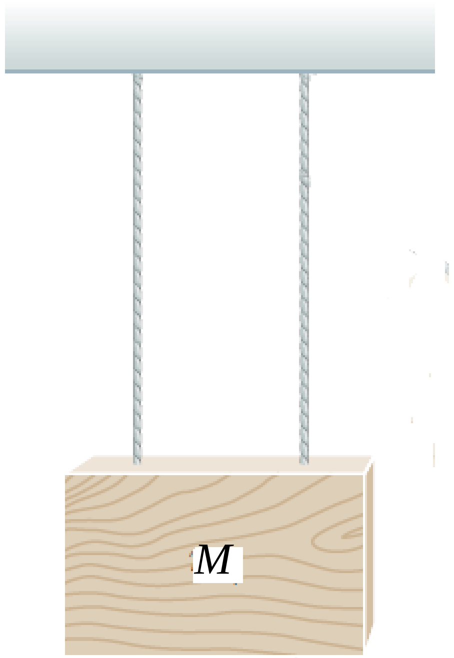


Figure B-1

A Ballistic Pendulum

The ballistic pendulum is an apparatus used to measure the speed of a fast-moving projectile such as a bullet. A projectile of mass m is fired into a large block of wood of mass *M* suspended from some light wires. The projectile embeds in the block, and the entire system swings through a height h. How can we determine the speed of the projectile from a measurement of h?

Figure B-2 shows a ballistic pendulum.

It consists of a large block of mass *M*, hanging from two long pairs of cords. The bullet of mass m is fired into a large block of wood suspended from some light wires. The bullet embeds in the block, and the entire system swings through a height *h*.

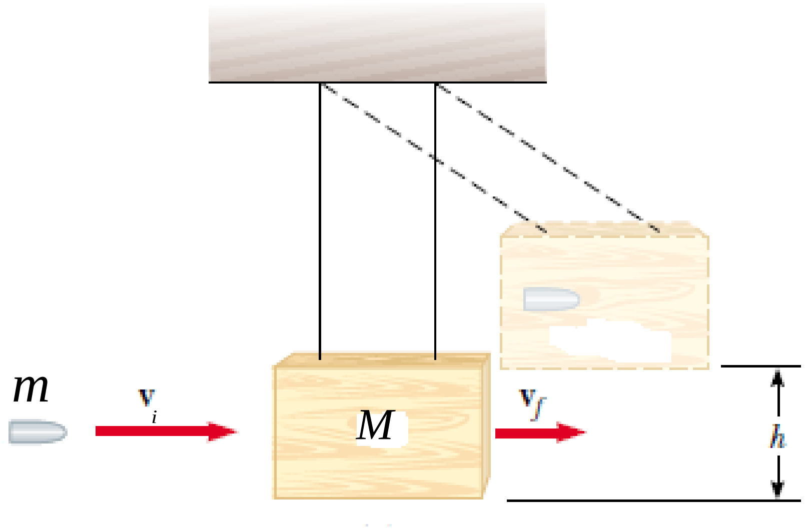


Figure B-2

$v_{2i} = 0$, the block is initially at rest

Let us split this complicated motion into two parts: (1) the bullet-block collision and (2) the bullet-block rise.

A Ballistic Pendulum

Part 1: The bullet-block collision

The bullet moving with speed enters the block and comes to rest relative to the block, after which the bullet + block combination moves with a common speed . We assume this happens very quickly.

This is an example of a completely inelastic collision.

Momentum is conserved in the collision:

$$\begin{pmatrix}\text{total momentum before} \\ \text{the collision}\end{pmatrix} = \begin{pmatrix}\text{total momentum after} \\ \text{the collision}\end{pmatrix}$$

$$m\mathrm{v}_i = (m+M)\,\mathrm{v}_f$$

$$\therefore \boxed{\mathrm{v}_f = \frac{m\mathrm{v}_i}{m+M}} \;\ldots\ldots\ldots\ldots\; (1)$$

Part 2: The bullet-block rise

The combination, now moving with speed v_f , swings upward until it comes to rest.

The mechanical energy of the bullet block Earth system is conserved:

$$\begin{pmatrix}\text{mechanical energy} \\ \text{at bottom}\end{pmatrix} = \begin{pmatrix}\text{mechanical energy} \\ \text{at top}\end{pmatrix}$$

$$\frac{1}{2}(m+M)\,\mathrm{v}_f^{\,2} + 0 = 0 + (m+M)\,gh$$

$$\therefore \boxed{\mathrm{v}_f = \sqrt{2gh}} \;\ldots\ldots\ldots\ldots\; (2)$$

Suppose the block's initial level as our reference level of zero gravitational potential energy.

From the equations (1) and (2), we get

$$\frac{m\mathrm{v}_i}{m+M} = \sqrt{2gh}$$

$$\therefore \mathrm{v}_i = \left(\frac{m+M}{m}\right)\sqrt{2gh}$$

This expression tells us that it is possible to obtain the initial speed of the bullet by measuring *h* and the two masses.

The ratio between initial and final kinetic energies

$$\frac{K_f}{K_i} = \frac{\frac{1}{2}(m+M)\,\mathrm{v}_f^{\,2}}{\frac{1}{2}m\mathrm{v}_i^{\,2}} = \frac{\frac{1}{2}(m+M)\left[\frac{m\mathrm{v}_i}{m+M}\right]^2}{\frac{1}{2}m\mathrm{v}_i^{\,2}} = \frac{m}{m+M}$$

Sample Problem

- **A ballistic pendulum consists of large block of wood of mass M=5.4 kg hanging vertically by two cords. When a bullet of mass m =9.5 g and v_i velocity is fired into the block, it gets embedded and the block-bullet combination swings upward, rising a maximum vertical distance h = 6.3 cm .**

(a) What is the initial speed the bullet?

(b) What fraction of the initial kinetic energy is lost in this collision?

Hint:

(a) $$v_i = \left(\frac{m+M}{m}\right)\sqrt{2gh} = \left(\frac{0.0095+5.4}{0.0095}\right)\sqrt{2\times 9.8\times 0.063} = 630 \text{ m s}^{-1}$$

(b) $$\frac{K_f}{K_i} = \frac{m}{m+M} = \frac{0.0095}{0.0095+5.4} = 0.0018.$$

Only 0.18% of the initial kinetic energy remains after the collision .

DYNAMICS OF SYSTEM OF PARTICLES

Central Force

Central Force

- A force which always acts towards or away from a fixed point, and whose magnitude depends only on the distance from that point is called ***a central force***.

  Mathematically, a central force acting on a particle may be represented by

  $\vec{F} = \pm F(\mathrm{r})\,\hat{\mathrm{r}}$ where $F(\mathrm{r})$ is a function of the distance r of the particle from a fixed point and $\hat{\mathrm{r}}$ is a unit vector along $\vec{\mathrm{r}}$.

- Central forces are (i) long range forces & (ii) conservative forces.

Examples:

a) Gravitational force of attraction between two masses.

$$\vec{F} = -\frac{Gm_1m_2}{\mathrm{r}^2}\hat{\mathrm{r}} = -F(\mathrm{r})\,\hat{\mathrm{r}} \qquad \text{where } F(\mathrm{r}) = -\frac{Gm_1m_2}{\mathrm{r}^2}$$

The Earth moves around the Sun under a central force which is always directed towards the Sun.

b) Electrostatic force of attraction or repulsion between two charges.

$$\vec{F} = \pm\left[\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{\mathrm{r}^2}\right]\hat{\mathrm{r}} = \pm F(\mathrm{r})\,\hat{\mathrm{r}} \qquad \text{where } F(\mathrm{r}) = \pm\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{\mathrm{r}^2}$$

The electron in Hydrogen atom moves under a central force which is always directed towards the nucleus.

Some important properties of central force:

- Central force motion is always motion in a plane.
- The angular momentum of the particle subjected to a central force is conserved.
- The areal velocity - the area swept out by the radius vector per unit time, is constant.

Central Force

Angular momentum of a particle under a central force is always conserved

- The torque acting on a particle subjected to central force is

$$\vec{\tau} = \vec{r}\times\vec{F} = \vec{r}\times F(r)\hat{r} = F(r)\,\vec{r}\times\frac{\vec{r}}{r} = 0$$

- The time rate of change of angular momentum is

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}\left[\vec{r}\times\vec{P}\right] \quad = \frac{d\vec{r}}{dt}\times\vec{P} + \vec{r}\times\frac{d\vec{P}}{dt} = \vec{v}\times m\vec{v} + \vec{r}\times\vec{F} = 0 + \ \vec{\tau} = \ \vec{\tau} = 0$$

$$\boxed{\therefore \quad \vec{L} = \text{ a constant}}$$

Hence, the angular momentum of a particle under a central force is conserved.

Two Body Problem and Reduced Mass

Two Body Problem

- Consider two bodies with masses m_1 and m_2 interacting through central force whose instantaneous position vectors with respect to an origin in an inertial reference frame are $\vec{r}_1$ and $\vec{r}_2$ (Figure T-1) .

The **relative position vector** $\vec{r}$ pointing from body 2 to body 1 is

$$\vec{r} = \vec{r}_1 - \vec{r}_2$$

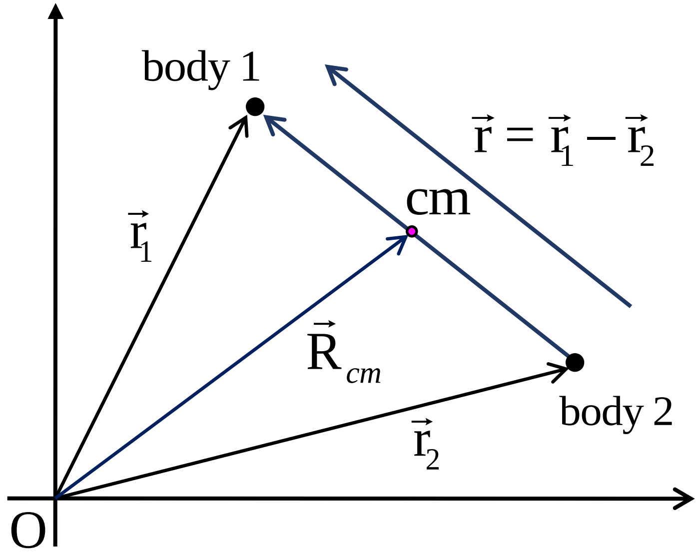


Figure T-1

- Let $\vec{F}_{12}$ and $\vec{F}_{21}$ be the forces on body 1 and body 2 (due to the interaction of the two bodies) respectively.

By Newton's third law,

$$\vec{F}_{12} = -\vec{F}_{21} = F(r)\hat{r} \qquad .... \quad (1)$$

- Applying Newton's second law for the body 1:

$$m_1 \frac{d^2\vec{r}_1}{dt^2} = \vec{F}_{12} \qquad \Rightarrow \frac{d^2\vec{r}_1}{dt^2} = \frac{F(r)\hat{r}}{m_1} \qquad .... \quad (2)$$

Applying Newton's second law for the body 2:

$$m_2 \frac{d^2\vec{r}_2}{dt^2} = \vec{F}_{21} \qquad \Rightarrow \frac{d^2\vec{r}_2}{dt^2} = -\frac{F(r)\hat{r}}{m_2} \qquad .... \quad (3)$$

Two Body Problem

- Subtracting Eq. (3) from Eq. (2), we get

$$\frac{d^2\vec{r}_1}{dt^2} - \frac{d^2\vec{r}_2}{dt^2} = \frac{F(r)\hat{r}}{m_1} + \frac{F(r)\hat{r}}{m_2}$$

or, $$\frac{d^2(\vec{r}_1 - \vec{r}_2)}{dt^2} = \left(\frac{1}{m_1} + \frac{1}{m_2}\right)F(r)\hat{r}$$

or, $$\frac{d^2\vec{r}}{dt^2} = \frac{1}{\mu}F(r)\hat{r} \qquad \text{where } \frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2}$$

$$\Rightarrow \mu = \frac{m_1 m_2}{m_1 + m_2}$$ is the reduced mass.

$\therefore$ $$\boxed{\mu\frac{d^2\vec{r}}{dt^2} = F(r)\hat{r}}$$

This is exactly the same as the equation of motion of a single body of mass μ at a vector distance $\vec{r}$ from a fixed centre under a central force $F(r)\hat{r}$.

Thus, the original two-body problem involving two vectors $\vec{r}_1$ and $\vec{r}_2$ and has been reduced to a one-body problem involving a single vector $\vec{r}$.

Reduced Mass

Two Body Problem

- The **reduced mass** μ of the two particles of masses m_1 and m_2 is given by $\mu = \dfrac{m_1 m_2}{m_1 + m_2}$.

Reduced Mass of the Hydrogen Atom

- Hydrogen atom consists of an electron of mass m_e (say) and a much heavier nucleus (proton) of mass m_p (say), which exert electrostatic attraction on each other. Both the electron and the proton revolve round their common centre of mass with same angular velocity. This two-particle system can be treated as a one-particle system; the electron with reduced mass revolving around the fixed nucleus.

- The reduced mass of the hydrogen atom is

$$\mu = \frac{m_e m_p}{m_e + m_p} = m_e \left[1 + \frac{m_e}{m_p}\right]^{-1} \approx m_e \left[1 - \frac{m_e}{m_p}\right]$$

But, $\dfrac{m_e}{m_p} = \dfrac{1}{1836}$

$$\therefore \boxed{\mu = m_e \left[1 - \frac{1}{1836}\right] \approx m_e} \Longrightarrow$$

Therefore, the reduced mass of Hydrogen atom is nearly equal to the mass of the electron.

Reduced Mass of the Positronium

- Positronium is a hydrogen-like atom made up of a positron and an electron, with no proton. A positron is a particle which has a mass equal to the electron mass but has positive charge e .

- The reduced mass of the positronium is

$$\boxed{\mu = \frac{m_e m_e}{m_e + m_e} = \frac{m_e}{2}}$$

Therefore, the reduced mass of positronium is about one-half that of atomic hydrogen.

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Thank you