ENGINEERING MECHANICS

Course Instructor
Nischal Pokharel
Department of Mechanical Engineering,
Kathmandu University

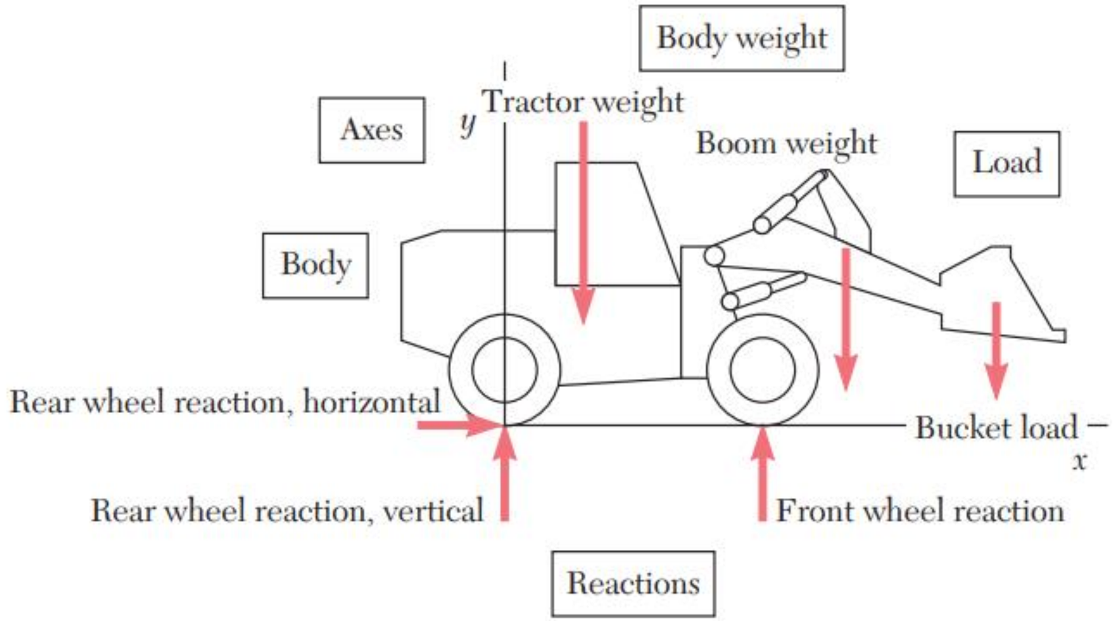
ENGINEERING MECHANICS

- Mechanics is defined as the science that describes and predicts the conditions of rest or motion of bodies under the action of forces.

- It consists:
  - Rigid body Mechanics
  - Deformable-body mechanics
  - Fluid mechanics

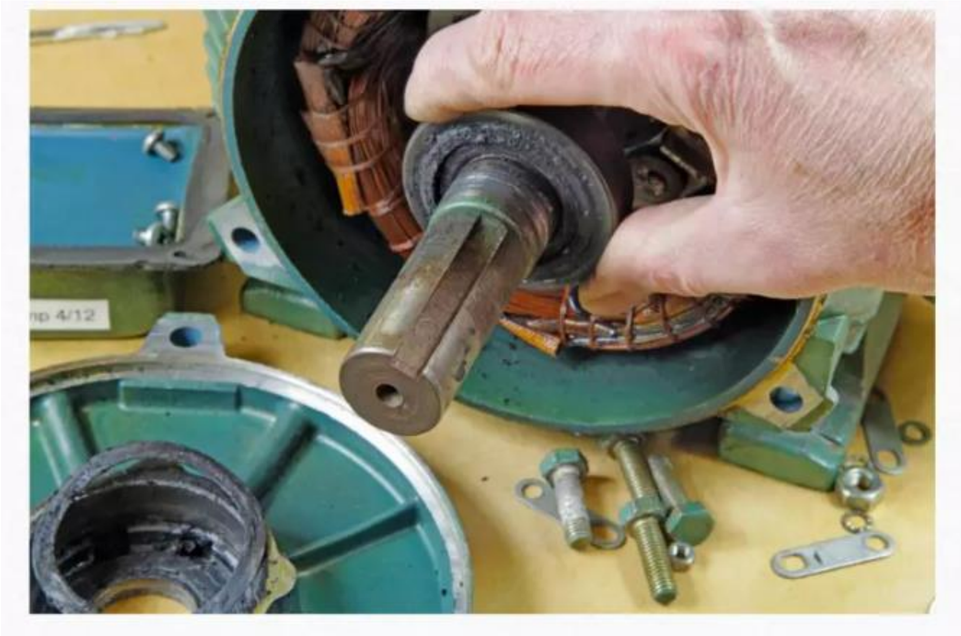
- A rigid body also known as a rigid object is a solid body in which deformation is zero or so small it can be neglected. The distance between any two given points on a rigid body remains constant in time regardless of external forces or moments exerted on it. A rigid body is usually considered as a continuous distribution of mass.

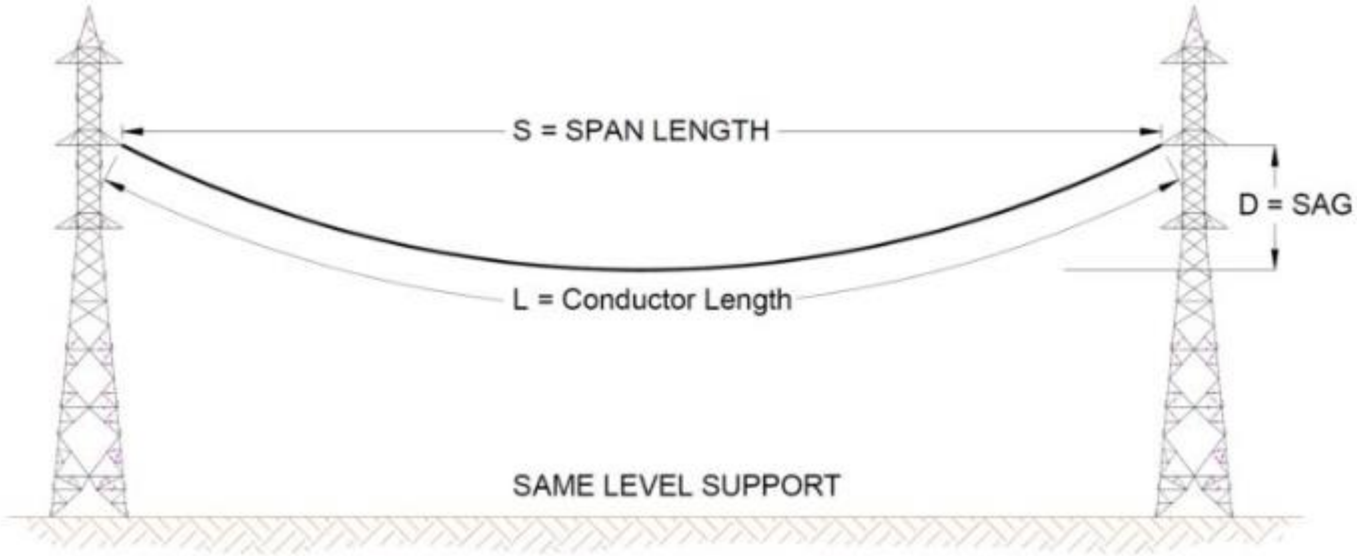
Mechanical Engineering



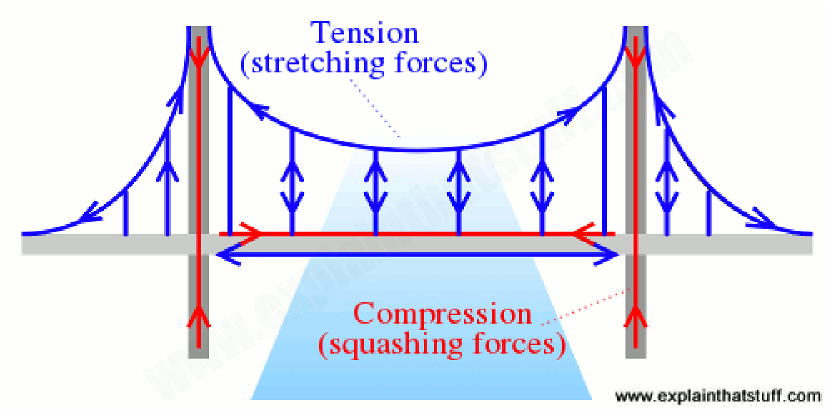
MECHANICS APPLICATIONS

Electrical Engineering

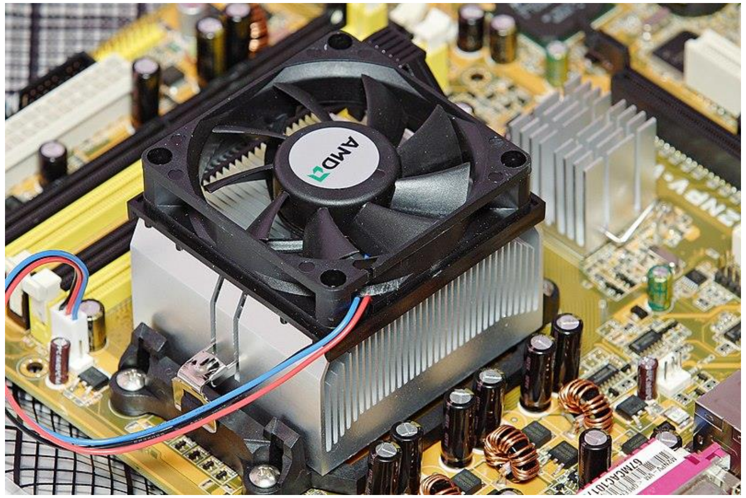


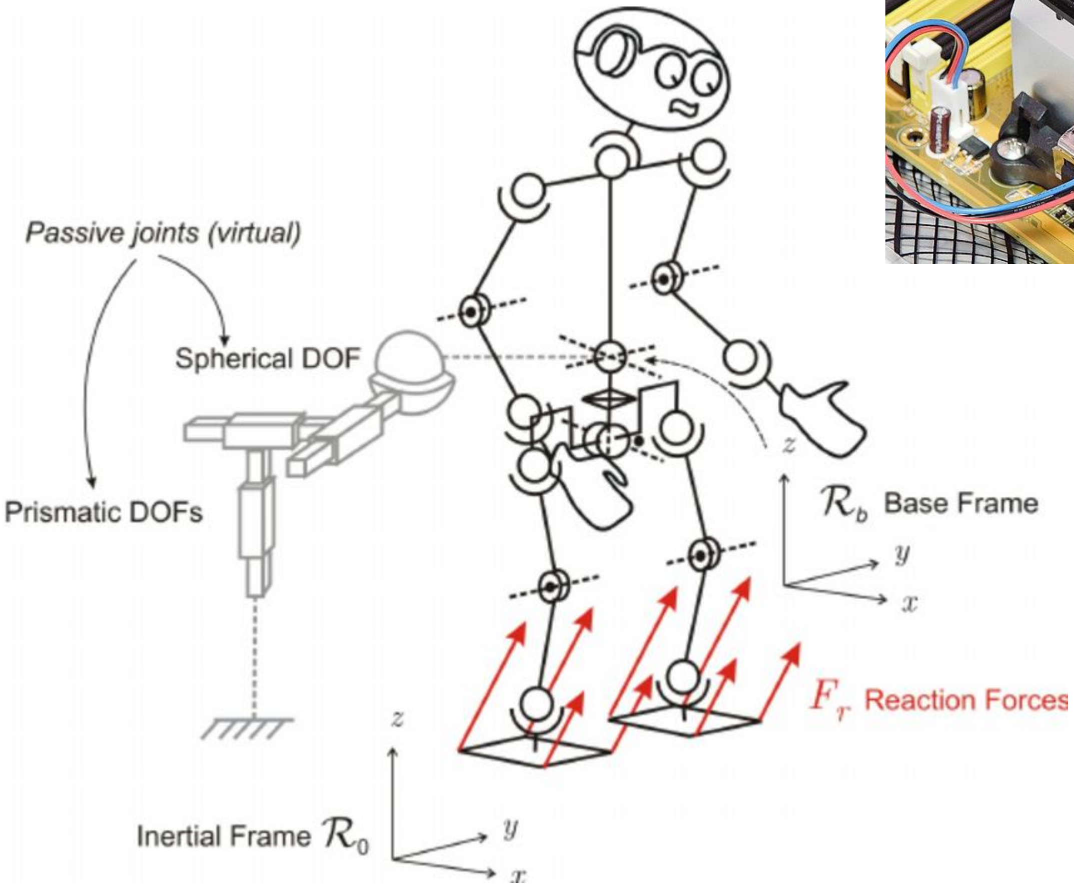


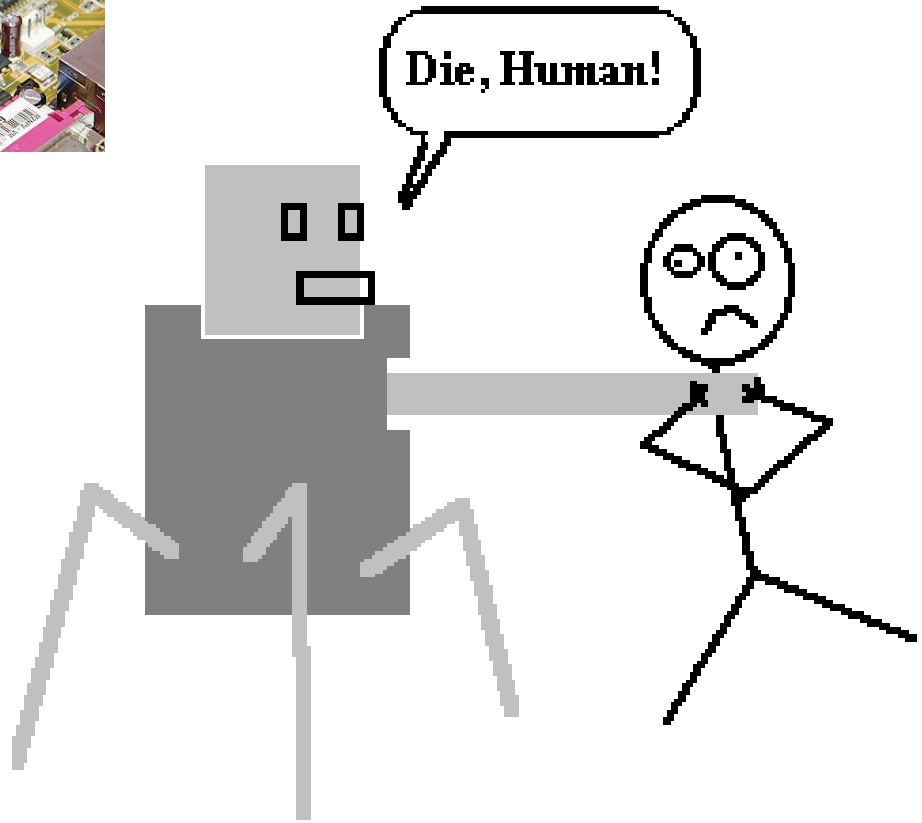
Civil Engineering



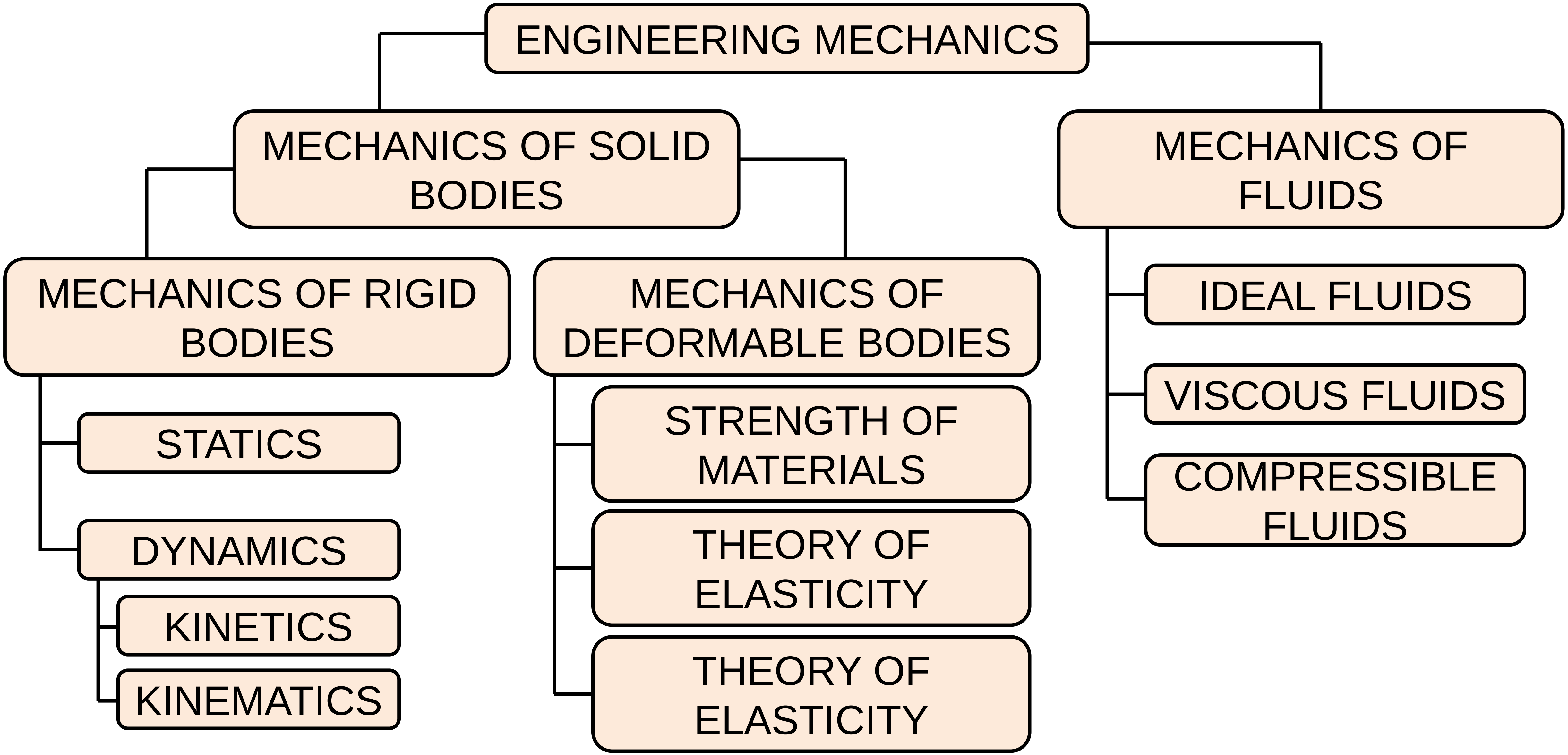
Computer Engineering







ENGINEERING MECHANICS

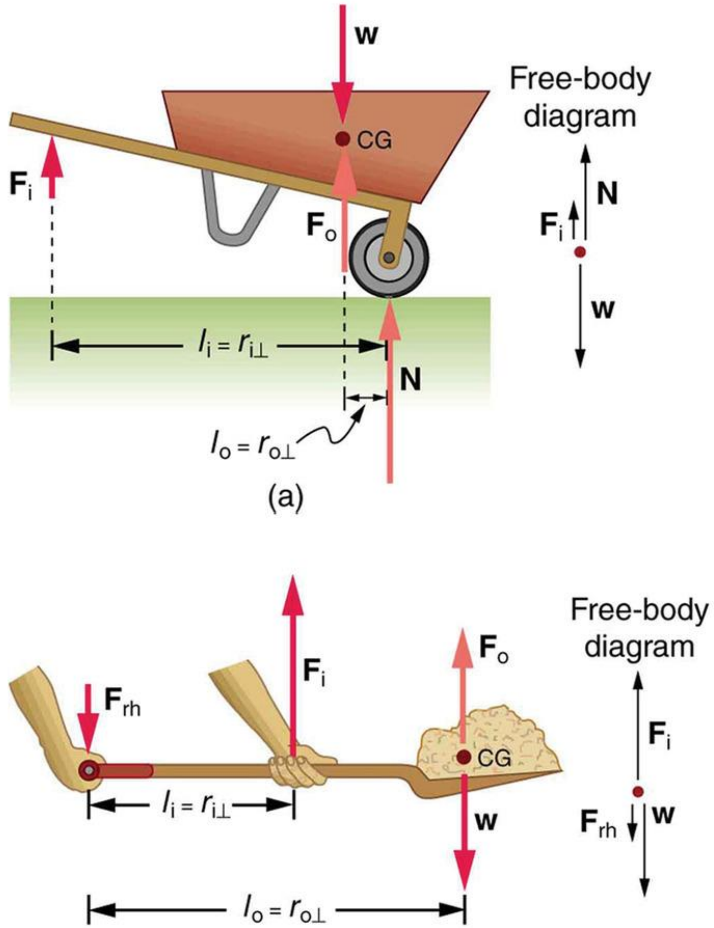


DEFINATIONS

- **Statics** is the study of distribution and effect of forces on rigid bodies which are at rest and remain at rest.
- **Dynamics** is the study of motion of rigid bodies and their correlation with the forces causing them
- **Kinematics** is the study of motion of bodies without any reference to the forces causing motion or forces produced as a result of the motion
- **Kinetics** is the study of the relationship between the forces and the resulting motion
- **Mechanics of deformable bodies** is the study dealing with internal force distribution and the deformation developed in actual engineering structures and machine components. It is also popularly known as strength of materials or mechanics of materials.
- **Fluid mechanics** is the study of liquids and gases(fluid) at rest or in motion.

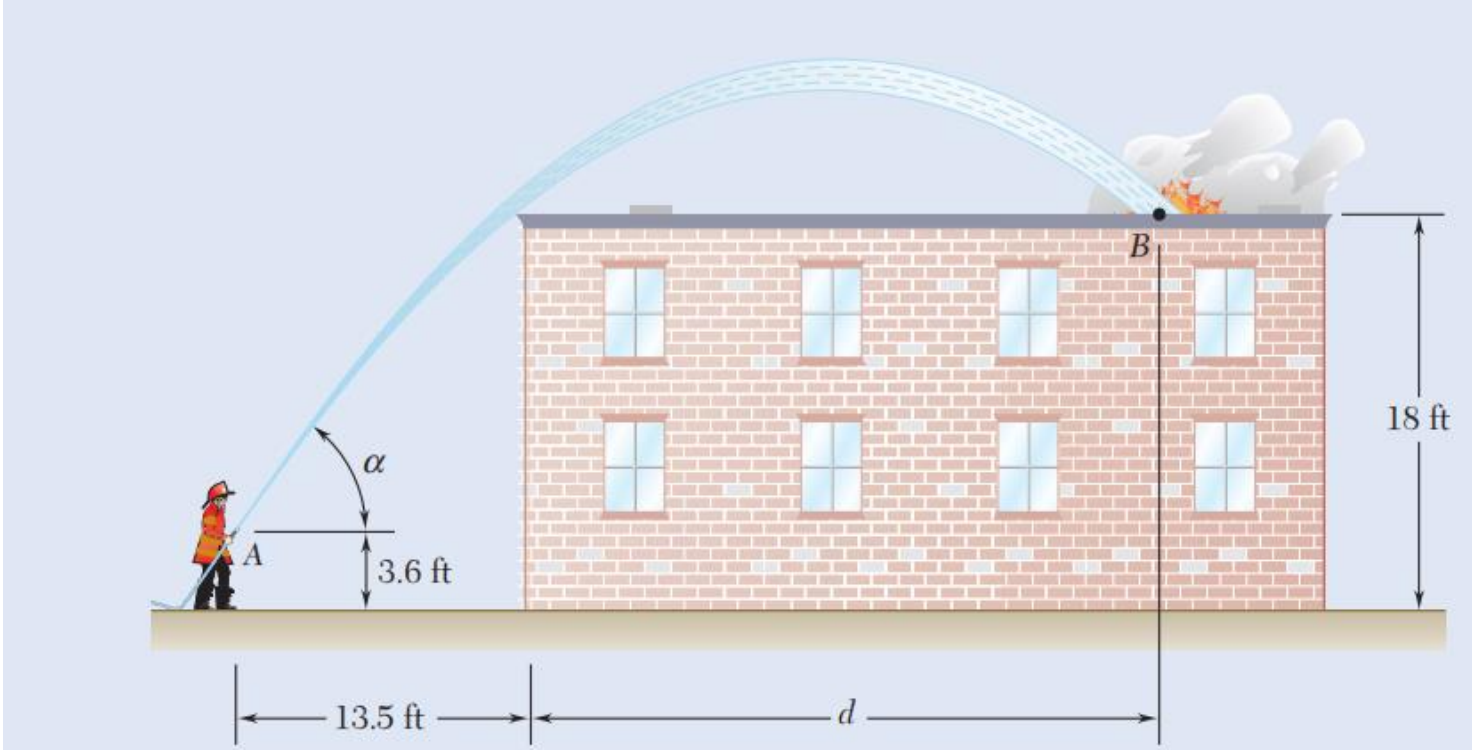
MECHANICS TYPES

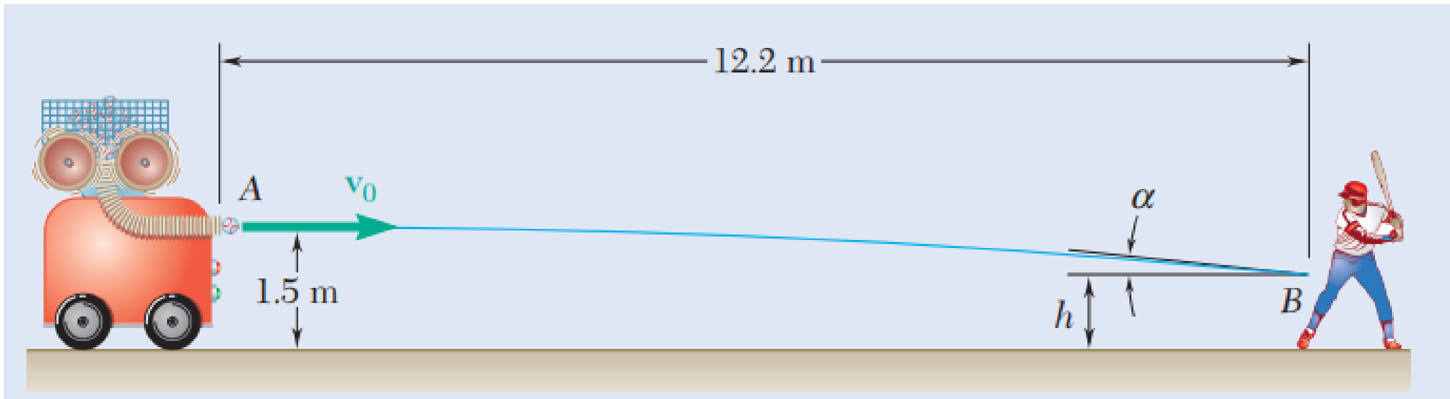
- Statics



MECHANICS TYPES

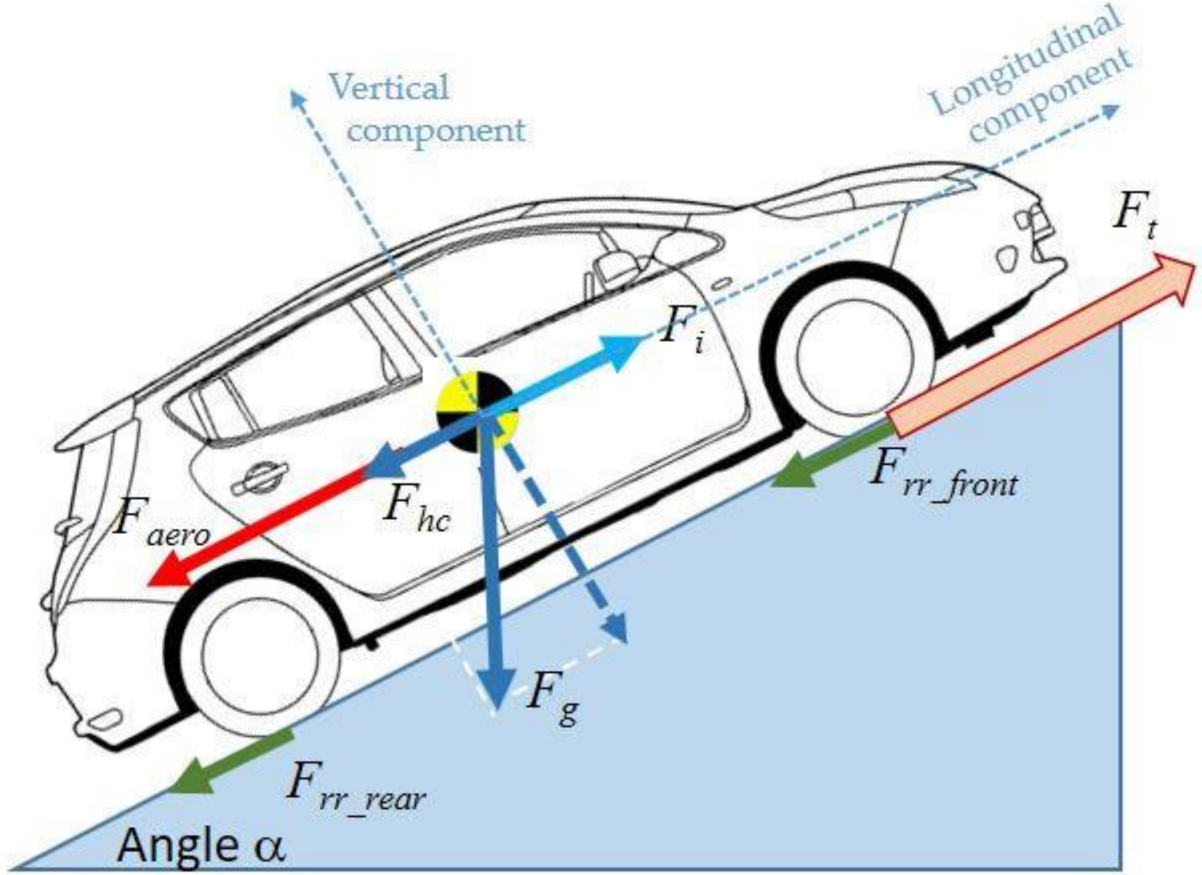
- Kinamatics





MECHANICS TYPES

- Kinetics



MECHANICS TYPES

- Strength of Materials

FORCE

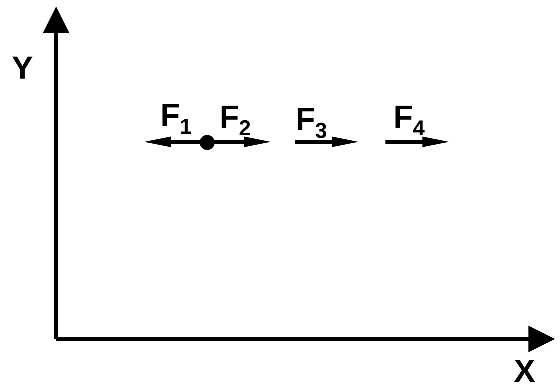
- Any action that changes or tends to change the state of rest or of uniform motion of body. The characteristics of force are magnitude, direction and point of application (line of action). It is a vector quantity

FORCE

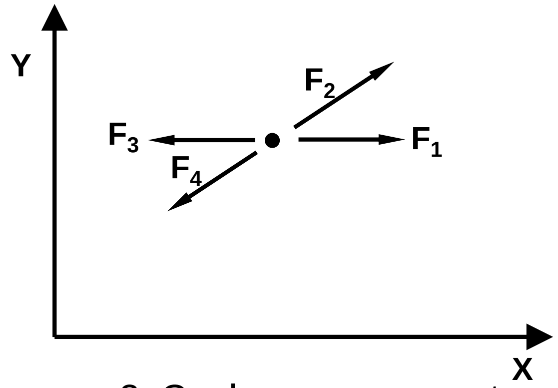
- FORCE SYSTEM
  - Coplanar (Plane)
    - Collinear
    - Concurrent
    - Parallel
    - Non concurrent, Non parallel
  - Non-Coplanar (Space)
    - Concurrent
    - Parallel
    - Non concurrent, Non parallel

S.N	Force system	Characteristics
1.	Coplanar forces	Lines of action of all forces lie on the same plane
2.	Non-coplanar forces	Lines of action of all forces do not lie on the same plane
3.	Collinear forces	Line of action of all forces act along the same line
4.	Concurrent forces	Line of action of all forces pass through a single point
5.	Parallel forces	Line of action of all forces that are parallel to each other

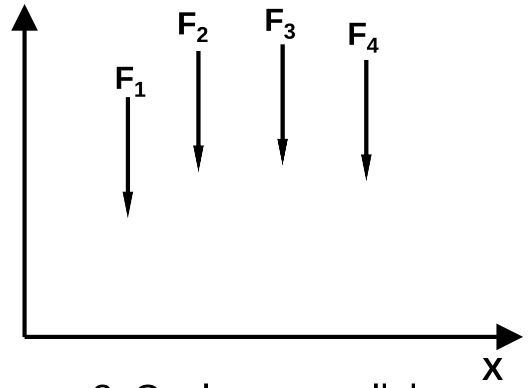
FORCE



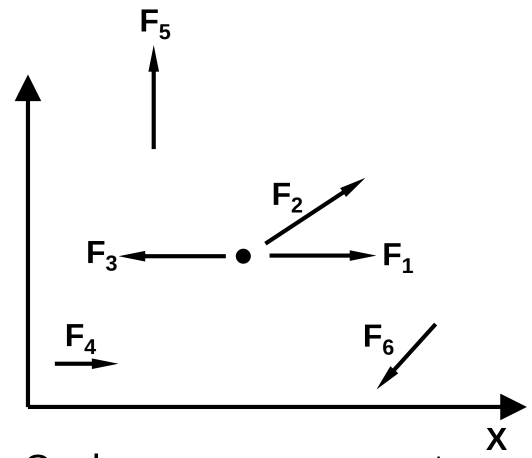
1. Coplanar-collinear



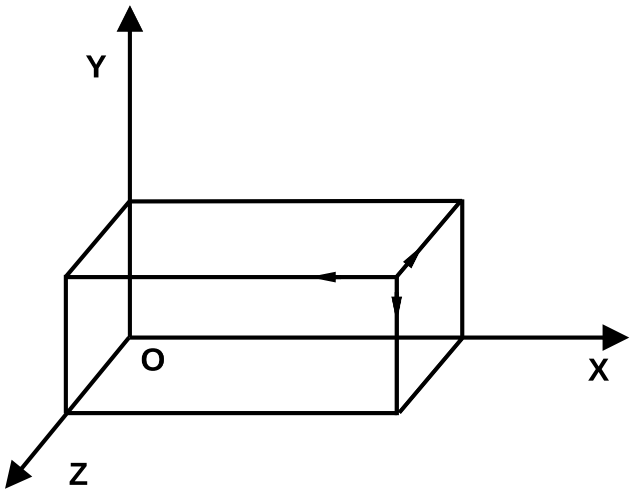
2. Coplanar-concurrent



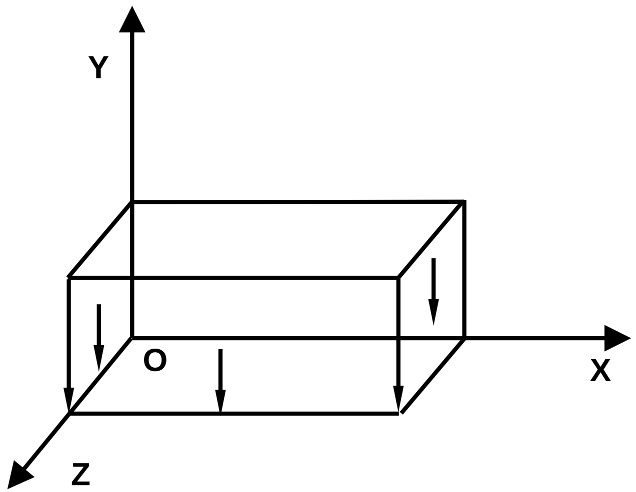
3. Coplanar-parallel



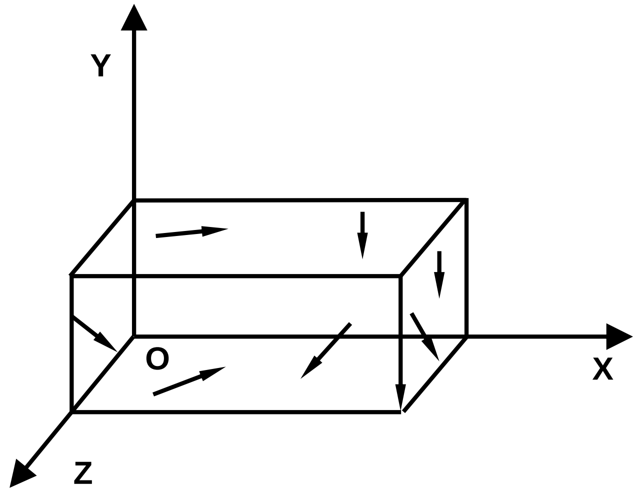
4. Coplanar, non-concurrent non parallel



5. Non-coplanar, concurrent

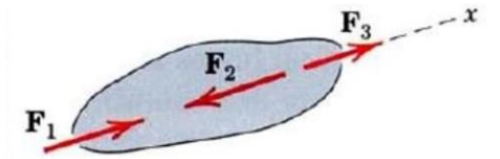
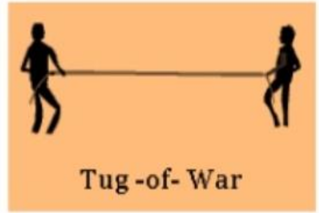


6. Non-coplanar, parallel



7. Non-coplanar, non-concurrent, non-parallel

FORCE

<table>
<tr><th>Force System</th><th>Free body diagram</th><th>Example</th></tr>
<tr><td>Collinear</td><td>
</td><td>
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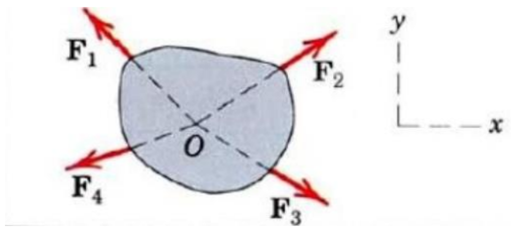
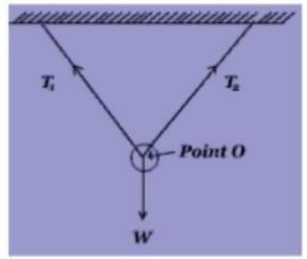
<tr><td>Concurrent</td><td>
</td><td>
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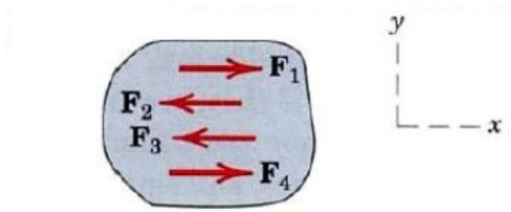
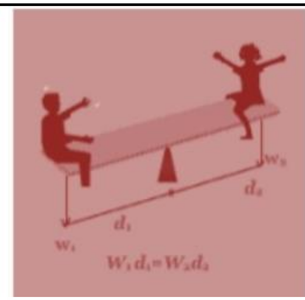
<tr><td>Parallel</td><td>
</td><td>
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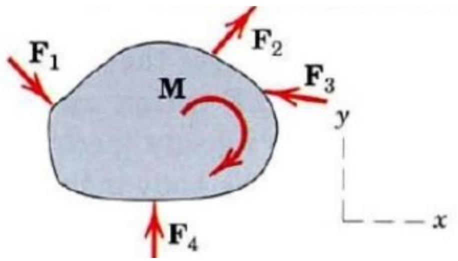
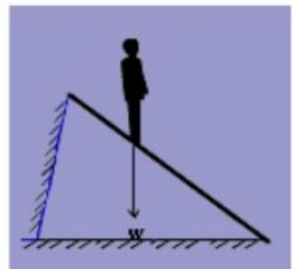
<tr><td>General</td><td>
</td><td>
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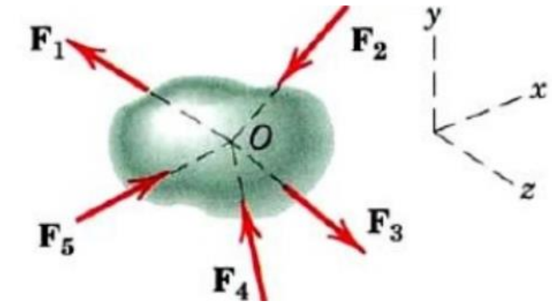
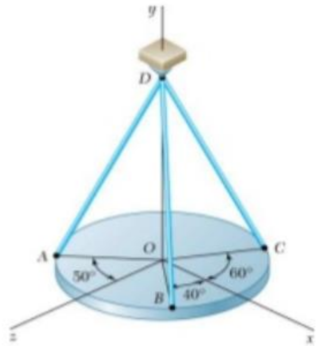
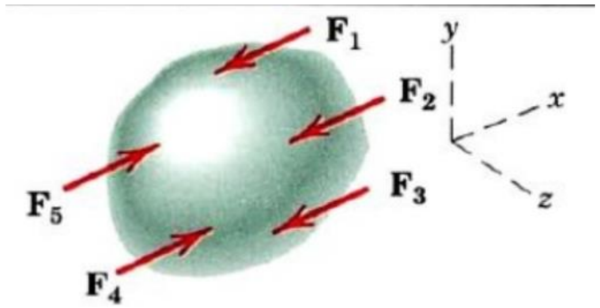
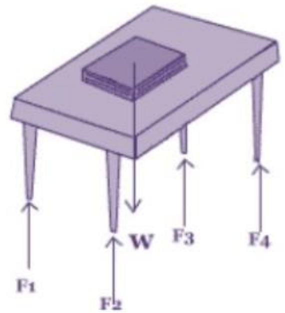
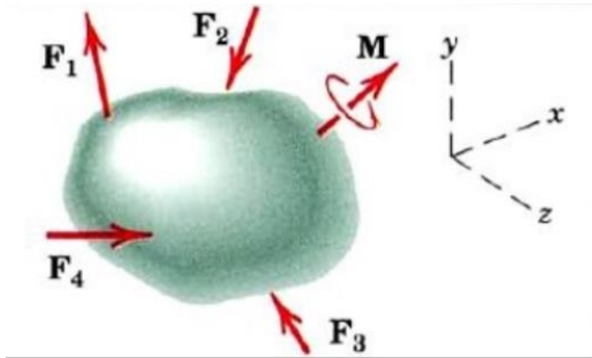
FORCE

Force System	Free body diagram	Example
Concurrent		
Parallel		
General		

FUNDAMENTAL PRINCIPLE OF MECHANICS

- Fundamental principle of mechanics are:

  1) Newton's three laws of motion
  2) Newton's law of gravitation
  3) Parallelogram law
  4) Principle of transmissibility

NEWTON'S LAWS OF MOTION

- Formulated by Sir Isaac Newton in the late seventeenth century which can be stated as:

1. First Law: Every body continues in its state of rest or of uniform motion in a straight line, unless it is compelled to change that state by an external impressed force.

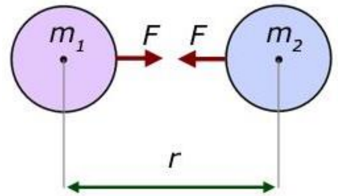
2. Second Law: The rate of change of momentum of a body is directly proportional to the force acting on it and takes place in the direction of force. If the resultant force acting on the particle is non-zero, the acceleration of the particle will be proportional to the magnitude and direction of the resultant force.

$$F=ma$$

3. Third Law: To every action there is an equal and opposite reaction. i.e. the forces of action and reaction between the bodies in contact have the same magnitude and line of action but opposite in direction.

NEWTON'S LAW OF GRAVITATION

- Two particles of mass m_1 and m_2 are attracted towards each other along the line connecting them with a force whose magnitude 'F' is proportional to the product of their masses and inversely proportional to the square of the distance (r) between them.



Hence,

$$F = G\frac{m_1 m_2}{r^2}$$

Where 'G' is the universal constant or constant of gravitation and its value is $(66.73\pm0.03)\times10^{-12}$ m^3 kgs^2. Thus, a particle of mass 'm' lying on the surface of earth of mass 'M' and radius 'R' is attracted towards the earth by a force, 'F' which is given by:

$$F = m\frac{GM}{R^2}$$

Thus, the weight of the particle is:

$$W = mg$$

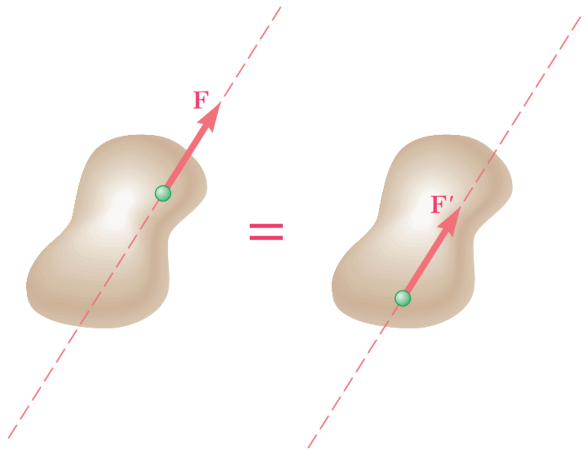
and

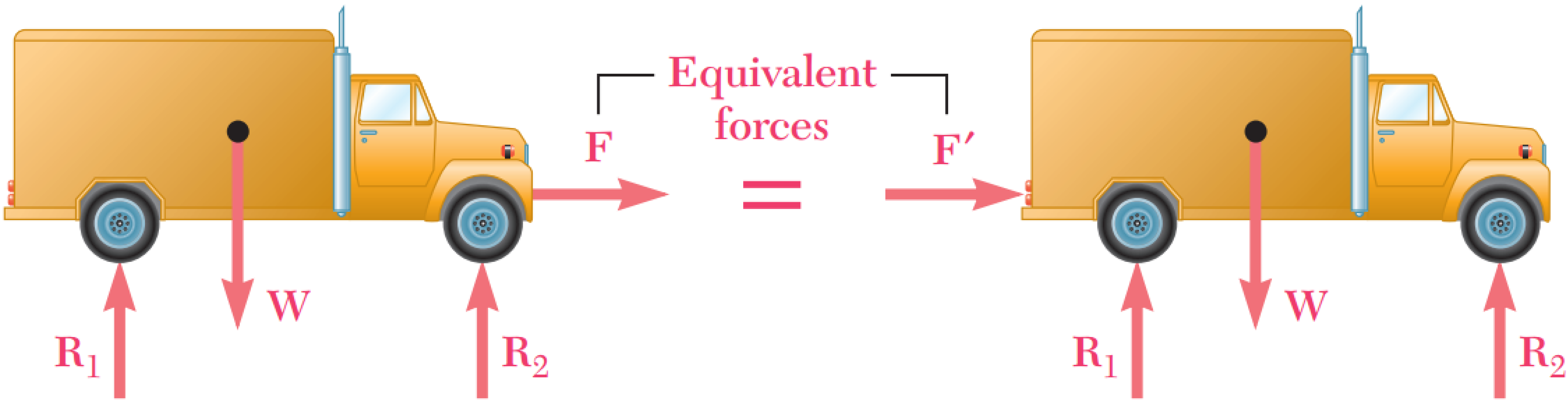
$$g = \frac{GM}{R^2}$$

Where R= 6.378×10^{6} m, M=5.98×10^{24} kg. The value of 'R' depends upon the elevation of point considered. It also depends upon its latitude, since the earth is not truly spherical. It is sufficiently accurate to assume in most engineering computations that g= 9.81 m/s^2

PRINCIPLE OF TRANSMISSIBILITY

- The conditions of equilibrium or the motion of a rigid body remains unchanged if a force acting at a given point of the rigid body is replaced by a force of same magnitude and direction, but acting at a different point provided that the two forces have the same line of action.

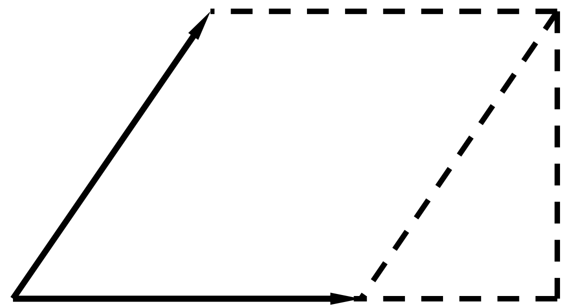


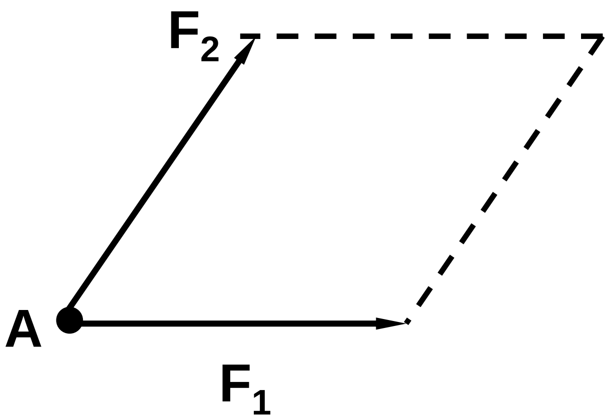


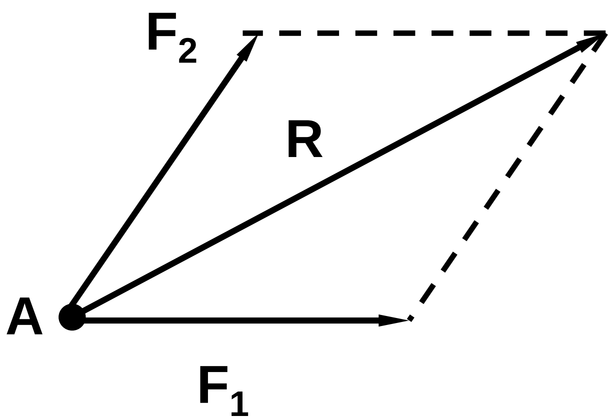
PARALLELOGRAM LAW

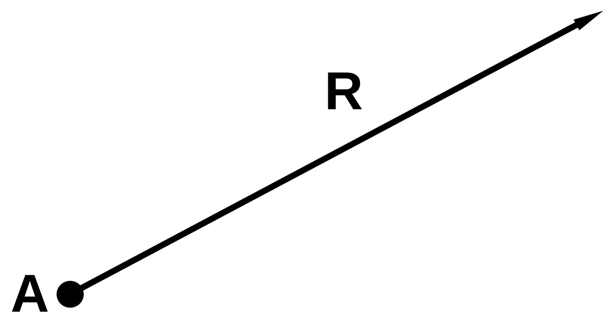
- When two forces (F1, F2) acting on a particle are represented by two adjacent sides of a parallelogram, the diagonal connecting the two sides represents the Resultant force 'R" in magnitude and direction.

- This law is based on the experimental evidence and hence it cannot be proved or derived mathematically.

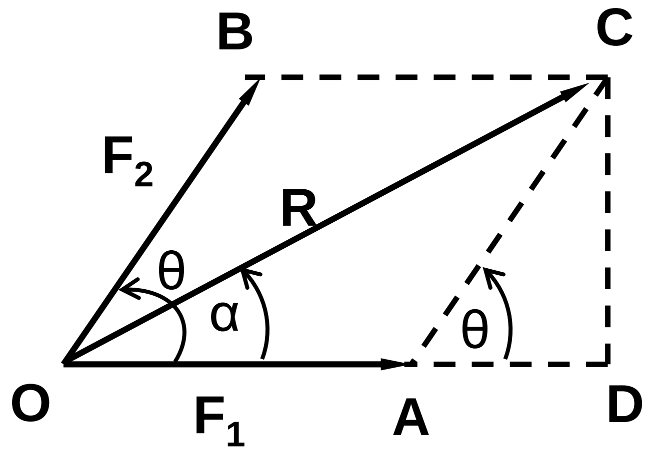








PARALLELOGRAM LAW



- To derive the relations between F1, F2 and R:
- Consider the parallelogram OACB. Let OA and OB represents the forces $\vec{F_1}$ and $\vec{F_2}$ acting at a point O. The diagonal OC represents the resultant $\vec{R}$ which can be expressed as:

$OC^2=(OA+AD)^2+CD^2$

$=OA^2+2OA.AD+AD^2+CD^2$

$=OA^2+2OA.AD+AC^2 \quad (AC^2=AD^2+CD^2)$

$$R^2 = F_1^2 + 2F_1.F_2\cos\theta + F_2^2$$

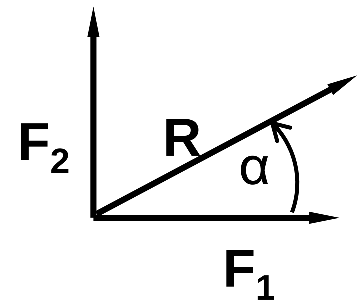
$$R = \sqrt{F_1^2 + 2F_1.F_2\cos\theta + F_2^2}$$

$$\tan\alpha = \frac{CD}{OA+AD} = \frac{F_2\sin\theta}{F_1 + F_2\cos\theta}$$

PARALLELOGRAM LAW

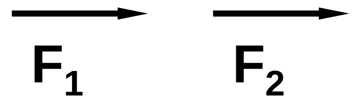
- Case I, If F_1 and F_2 are at right angle, then $\theta = 90^0$

$$R = \sqrt{F_1^{\,2} + F_2^{\,2}} \qquad \tan\alpha = \frac{F_2}{F_1}$$



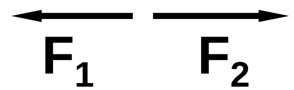
- Case II, If F_1 and F_2 are collinear and are in same direction then, $\theta = 0^0$

$$R = F_1 + F_2 \qquad \tan\alpha = 0; \alpha = 0$$



- Case III, If F_1 and F_2 are collinear and are in opposite direction, ($F_1 > F_2$) then $\theta = 180^0$

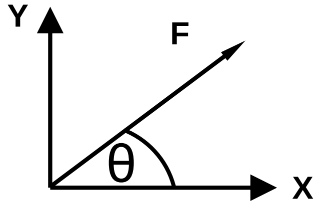
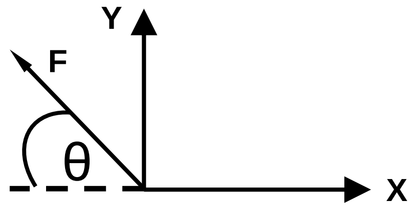
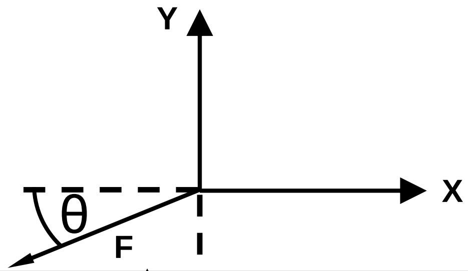
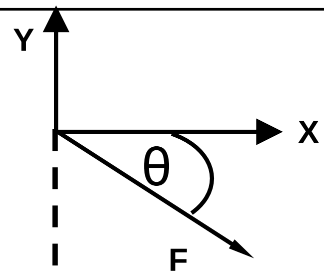
$$R = F_1 - F_2 \qquad \tan\alpha = 0; \alpha = 0$$



A quantity which has both, magnitude, direction and also obeys parallelogram law is called a vector. Hence force, displacement, velocity, etc. are vectors. Small rotation are also considered vectors. However large rotations are not vector quantities.

RESOLUTION OF FORCES

- The process of replacing a single force F acting on a particle by two or more forces with together have the same effect as that of a single force is called resolution of the force into component,

S.N.	Force	F_x	F_y
		$F \cos \theta$	$F \sin \theta$
		$-F \cos \theta$	$F \sin \theta$
		$-F \cos \theta$	$-F \sin \theta$
		$F \cos \theta$	$-F \sin \theta$

RESOLUTION OF FORCES

- If there are i number of forces in the system then,

$$\sum F_{Xi} = \sum (F_i \cos\theta_i) = R_X$$

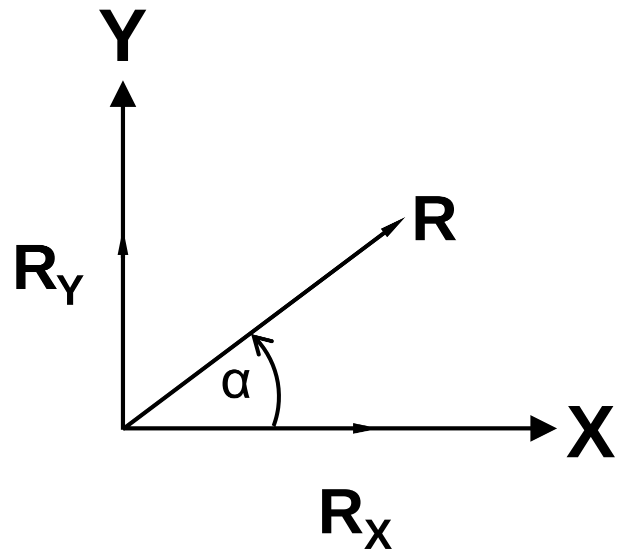
$$\sum F_{Yi} = \sum (F_i \sin\theta_i) = R_Y$$

- And all the forces act away from the particle. The magnitude of resultant 'R' is calculated as:

$$R = \sqrt{(\sum F_{Xi})^2 + \sum (F_{Yi}{}^2)}$$

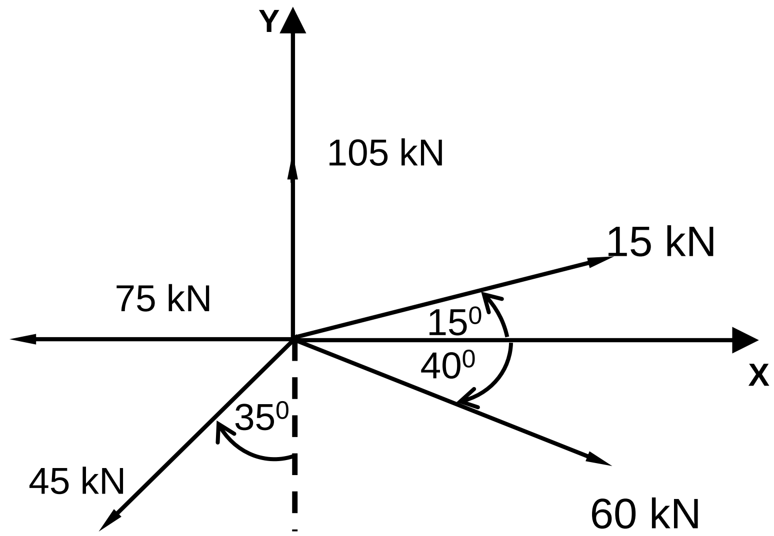
- The inclination with horizontal axis is given as:

$$\tan\alpha = \frac{(\sum F_{Yi})}{(\sum F_{Xi})}$$

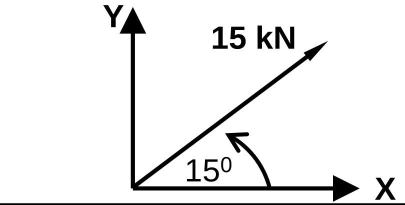
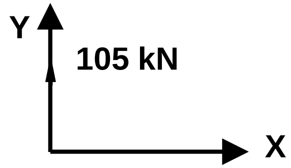
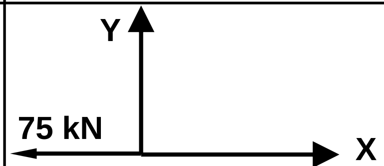
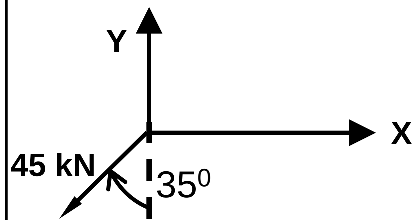
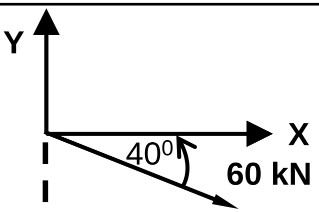


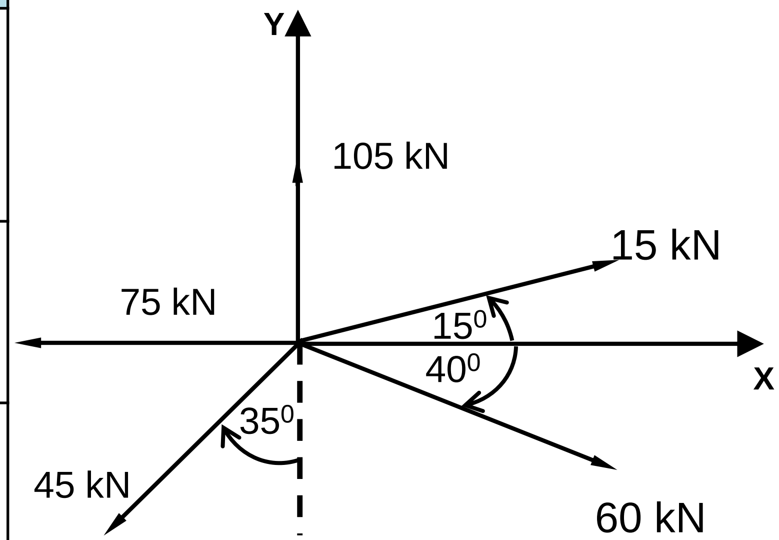
SAMPLE PROBLEM 1

- If five forces act on the particle as shown in figure determine the resultant and its angle to the horizontal



SAMPLE PROBLEM 1

S.N.	Force	F_{Xi}	F_{Yi}
1.		$15\cos 15^0$ $=14.489$	$15\sin 15^0$ $=3.882$
2.		0	105
3.		-75	0
4.		$-45\sin 35^0$ $=-25.811$	$-45\cos 35^0$ $=-36.862$
5.		$60\cos 40^0$ $=45.963$	$-60\sin 40^0$ $=-38.567$
		$\sum F_{Xi} = -40.359$	$\sum F_{Yi} = 33.453$



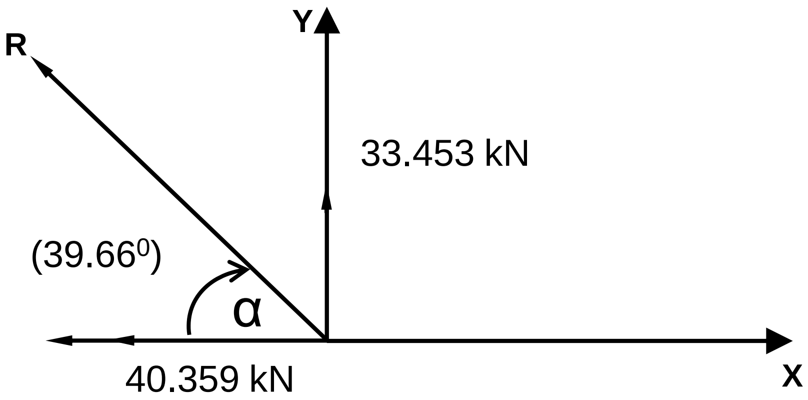
$$R = \sqrt{(-40.359)^2 + (33.453)^2}$$

$$= 52.421 kN$$

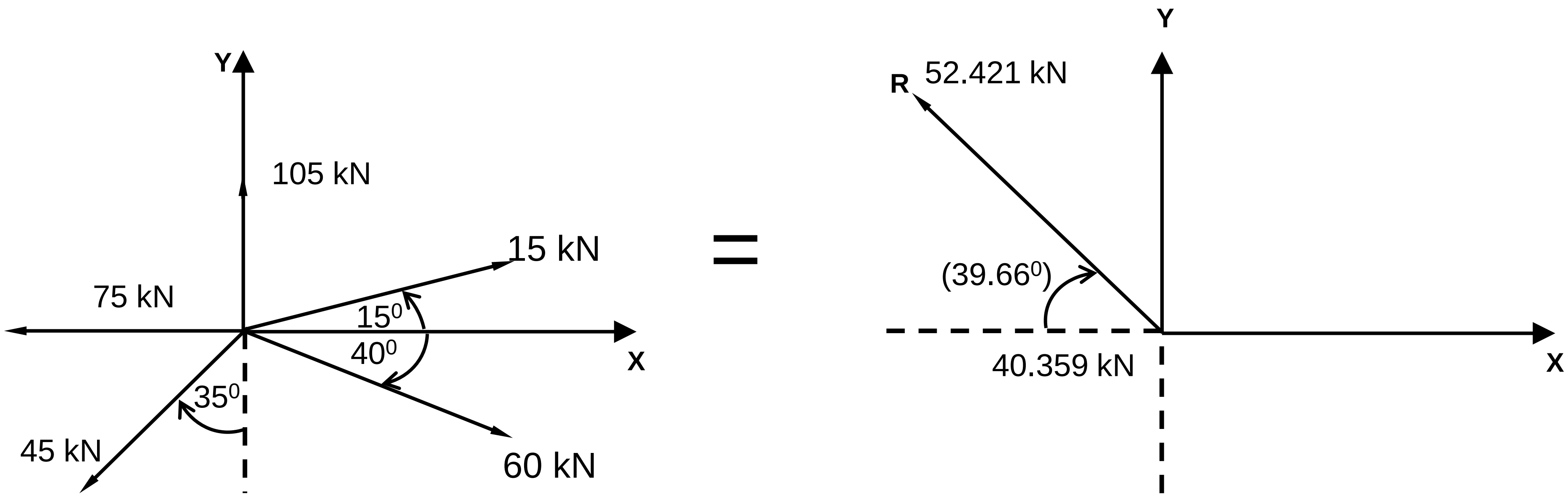
$$\tan\alpha = \frac{(\sum F_{Yi})}{(\sum F_{Xi})} = \frac{33.453}{-40.3589}$$

$$\alpha = (39.66)$$

SAMPLE PROBLEM 1



- From the sign of $\sum F_{Xi}$ and $\sum F_{Yi}$ the quadrant in which the resultant lies can be determined. From the magnitude of $\sum F_{Xi}$ and $\sum F_{Yi}$, the inclination of the resultant with horizontal α can be determined.



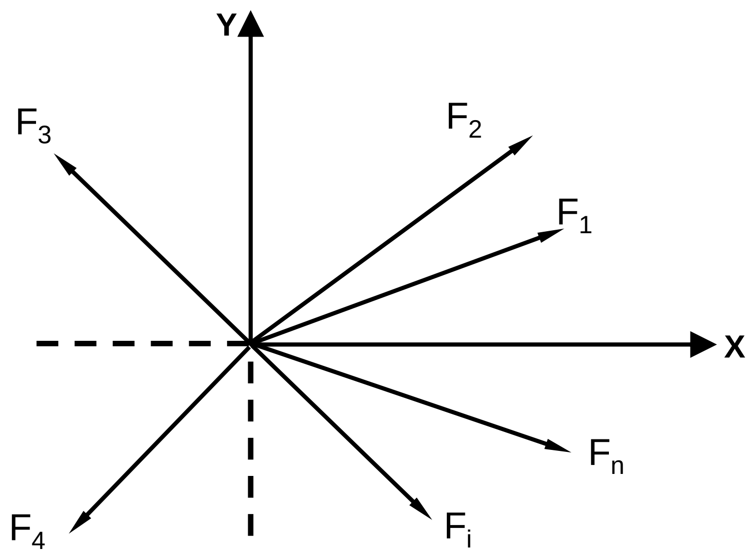
EQUILIBRIUM OF PARTICLE

- A body is said to be in equilibrium when the resultant of the force acting on it is zero. If a body is in equilibrium, it will continue to remain in a state of rest or uniform motion.

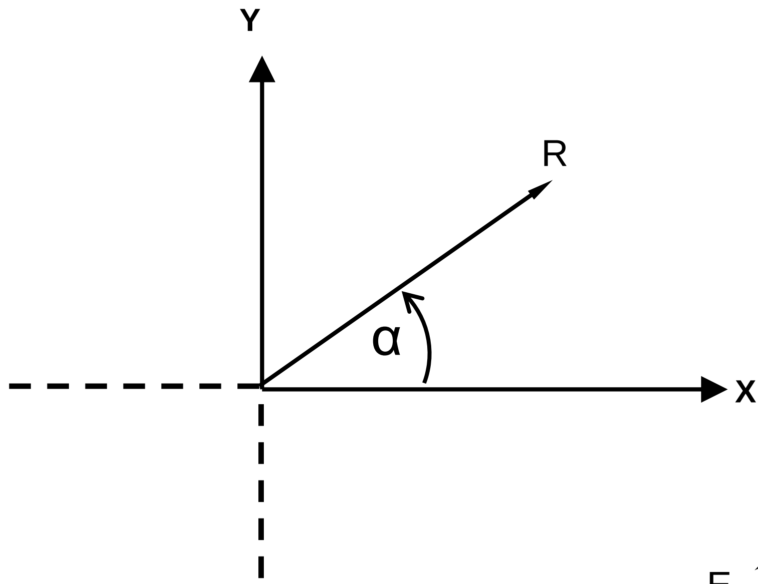
$$\vec{R} = 0$$

or

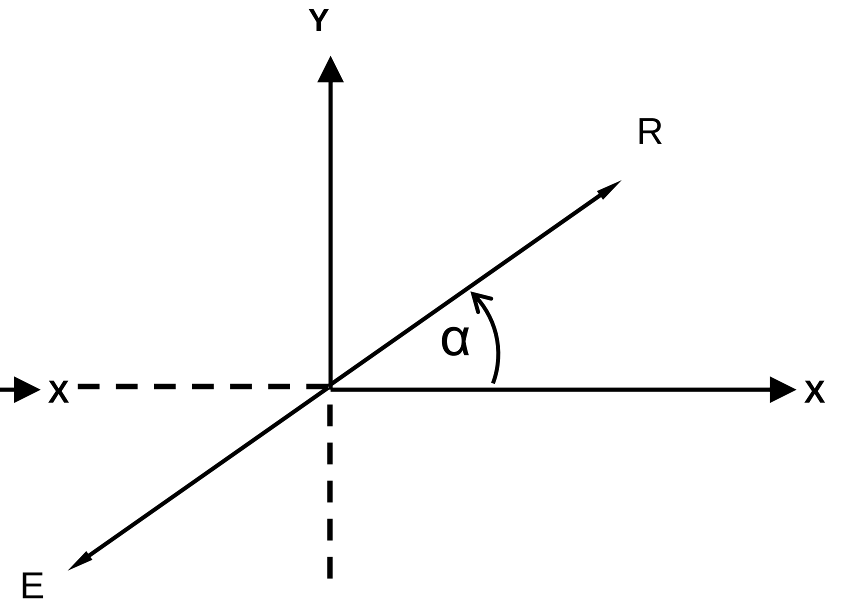
$$\sum F_X = 0 \quad \sum F_Y = 0$$



Given system of forces



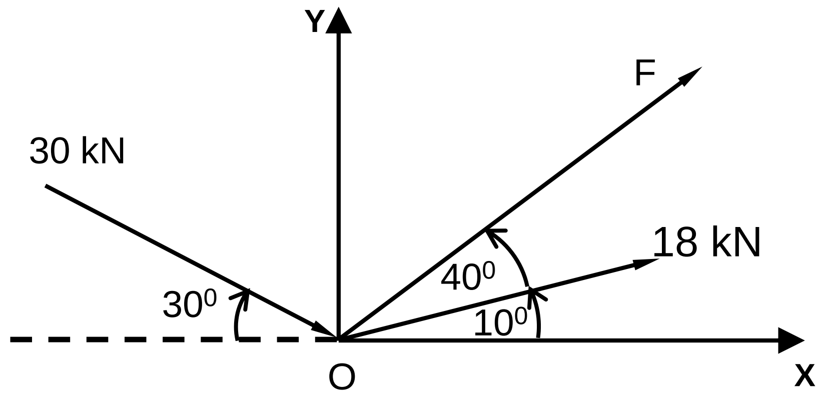
Resultant



Equilibrant

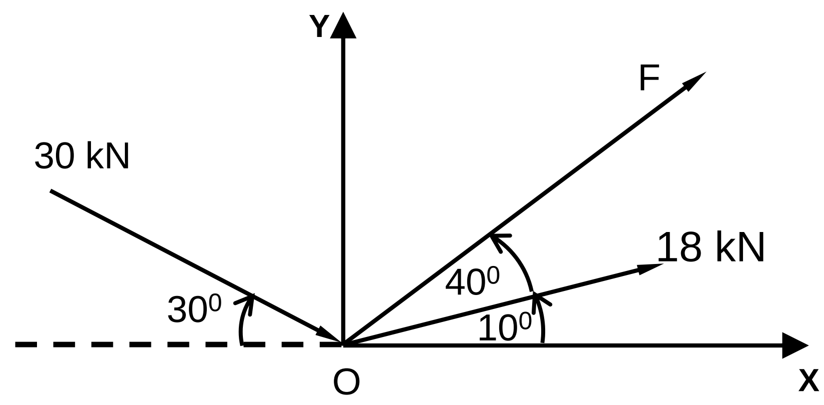
SAMPLE PROBLEM 2

- Three forces act on particle 'O' as shown in the figure. Determine the value of F such that the resultant of this forces is horizontal. Find the magnitude and direction of fourth force which when acting along with the given three forces will keep O in equilibrium.



SAMPLE PROBLEM 2

- Angle between F and O is 40+10=50^0 and 30 kN is acting towards the particle. Since the resultant is horizontal α=0



So, $\tan\alpha = \dfrac{(\sum F_Y)}{(\sum F_X)} = 0$

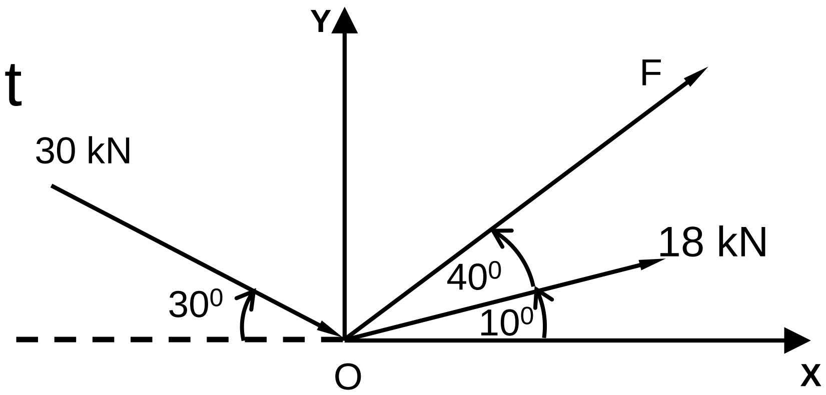
gives, $(\uparrow +) \sum F_Y = 0$

-30sin30^0+ 18sin10^0+Fsin50^0=0

$$F = \frac{30\sin 30^0 - 18\sin 10^0}{\sin 50^0} = 15.5kN$$

SAMPLE PROBLEM 2

- For the equilibrium condition both, $\sum F_X$ $\sum F_Y$ should be zero.
- Since first from condition it is given that the resultant is horizontal, Let R be that horizontal force so that,

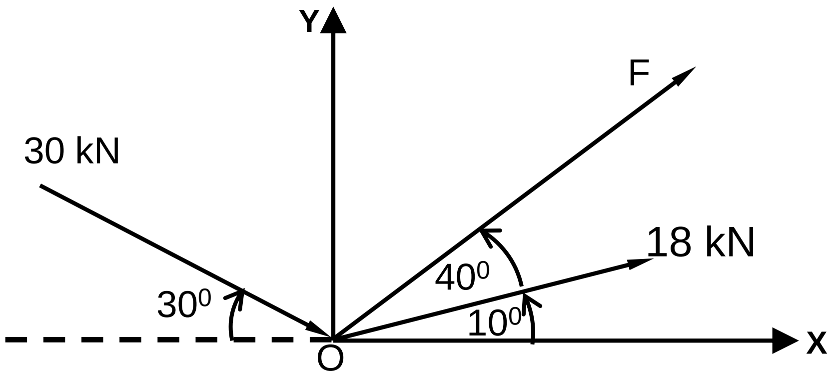


$$R=30\cos30^0+ 18\cos100+F\cos50^0=53.67 \text{ kN.}$$

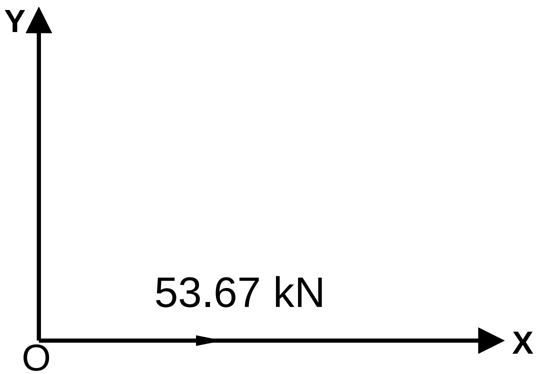
If E be the equilibrant to make $\sum F_X = 0$ then,

$$R+E=0$$
$$53.67+E=0$$
$$E=-53.67\text{kN.}$$

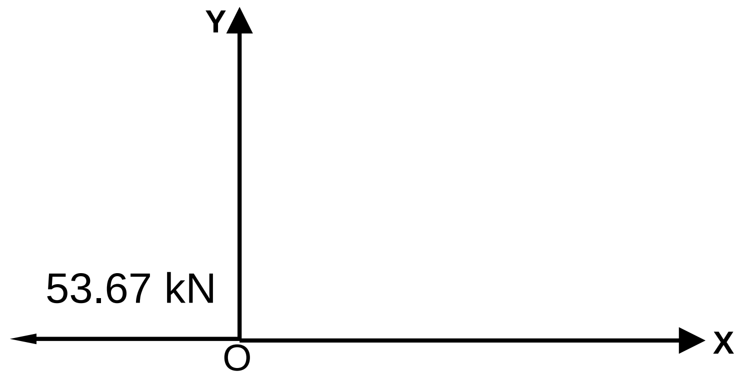
The summary of the system is as shown below:



Given System



Resultant



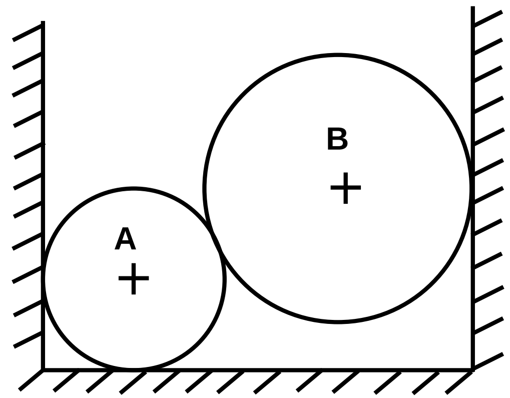
Equilibrant

FREE BODY DIAGRAM

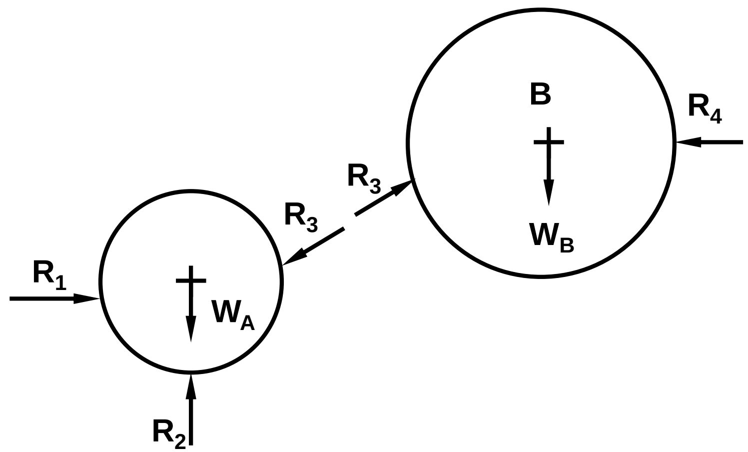
- A diagram of a body (or a part of it) which shows all the forces and couples applied on it, and which has all the forces and couples labeled for use in the solution of the problem is called a free-body diagram.

- Follow these steps to draw a free-body diagram.

  1. Select the body (or part of a body) that you want to analyze, and draw it.
  2. Identify all the forces and couples that are applied onto the body and draw them on the body.
  3. Place each force and couple at the point that it is applied.
  4. Label all the forces and couples with unique labels for use during the solution process.
  5. Add any relevant dimensions onto your picture

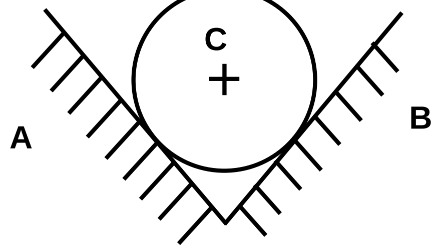
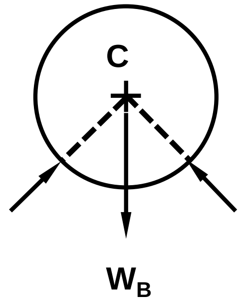
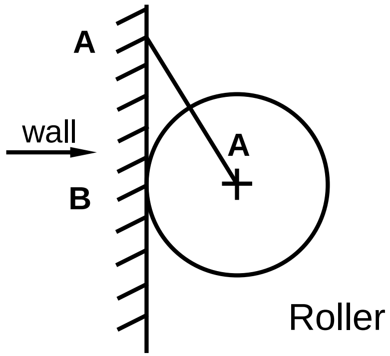
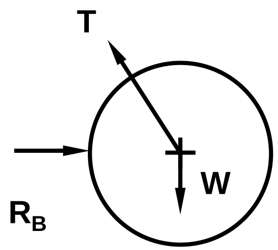


Two spheres in equilibrium

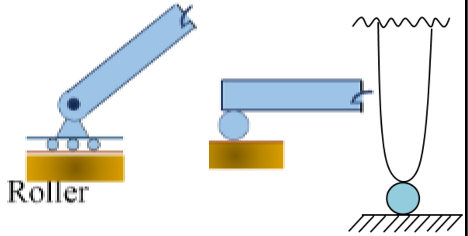
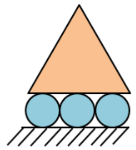
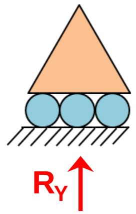
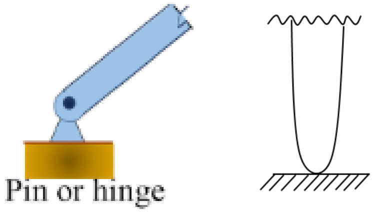
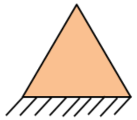
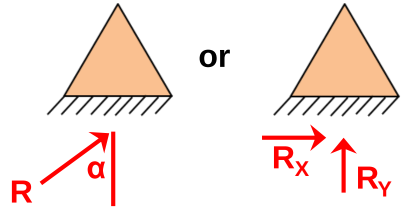
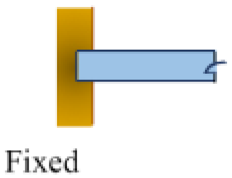
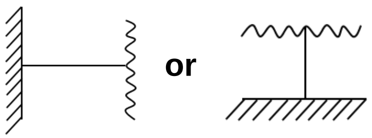
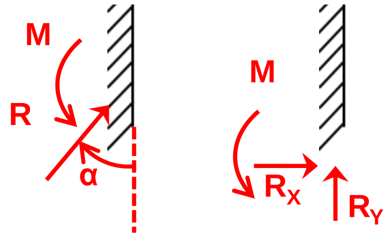


Free body Diagram (FBD)

FREE BODY DIAGRAM

Diagram	Free body diagram
 Sphere in groove	
	

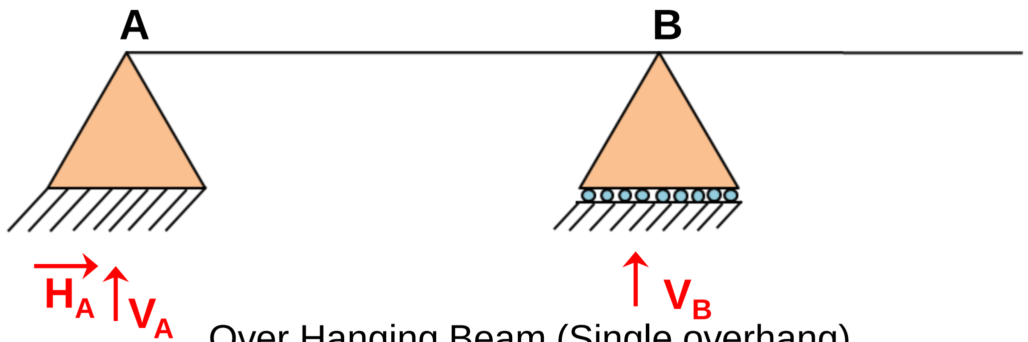
TYPES OF SUPPORT OR CONNECTION

Name	Example	Representation	Reaction	Unknowns	Remarks
Roller support Smooth Surface				One (R_Y)	Displacement in vertical direction is prevented
Hinge/pin support Rough Surface				Two (R, α) or (R_X,R_Y)	Displacement in both vertical and horizontal direction is prevented
Fixed support				Three (R, α,M) or (R_X,R_Y ,M)	Displacement in both vertical and horizontal direction as well as rotation is prevented

BEAM

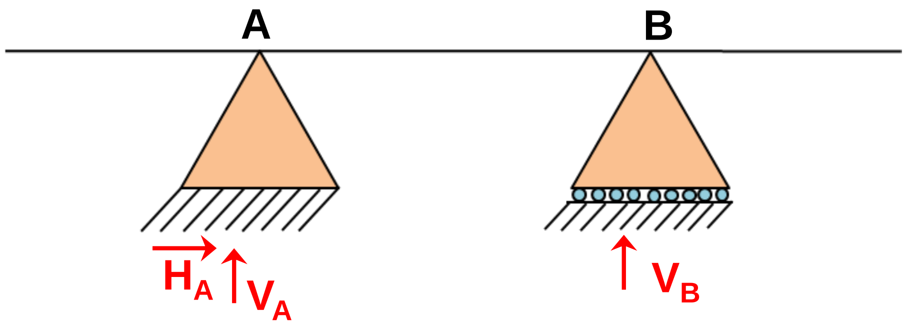
- A member which bends when subjected to load applied transverse to the long dimensions (axis of the member) is known as beam. Eg. In structures and machines.
- **Simple supported beam:** beam supported by hinge or roller on the smooth surface at the ends having one span
- **Overhanging beam:** Both ends project beyond the support.
- **Continuous beam**: More than two supports.
- **Cantilever beam:** One end is built into a wall or other support so that it cannot rotate or move transversely.

Simply Supported Beam

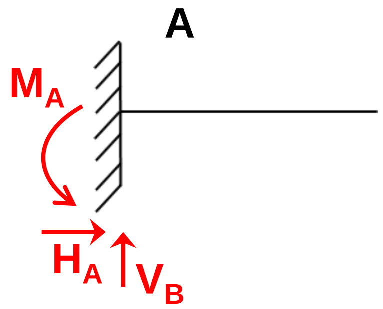


Over Hanging Beam (Single overhang)

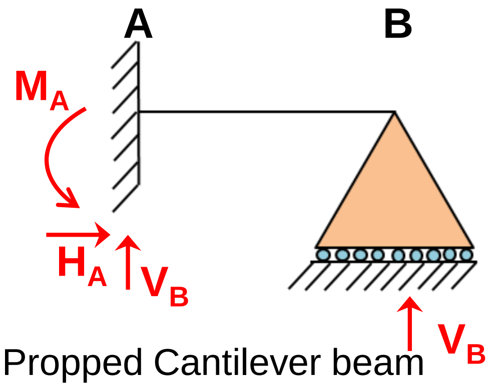
BEAM



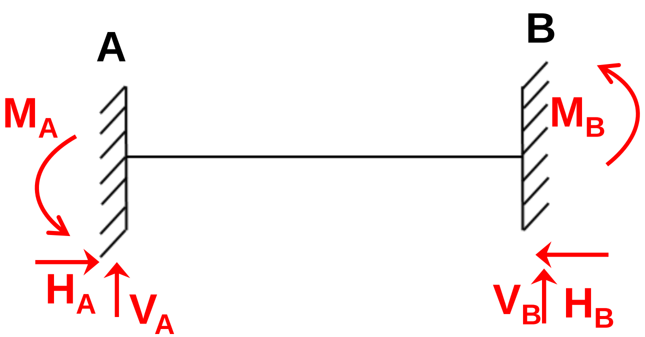
Over hanging Beam (Double over hang)



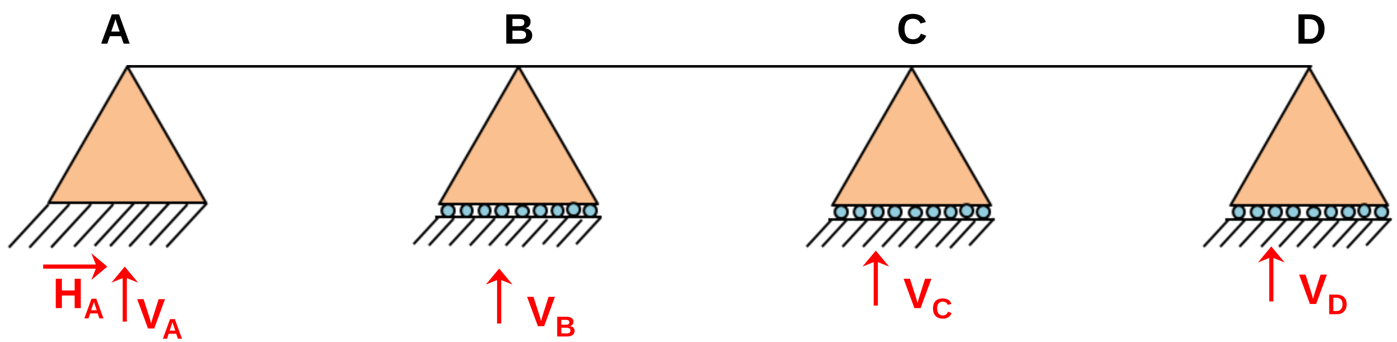
Cantilever beam



Propped Cantilever beam

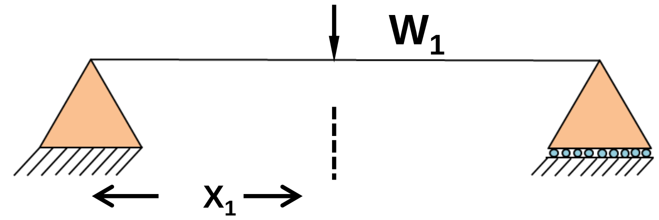
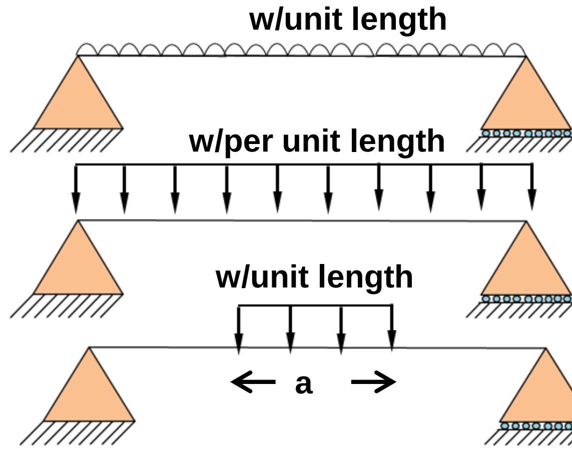
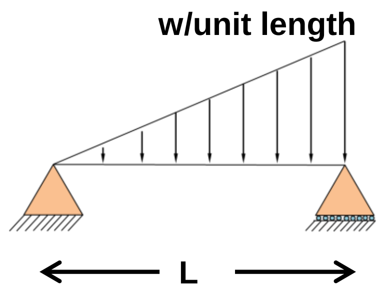
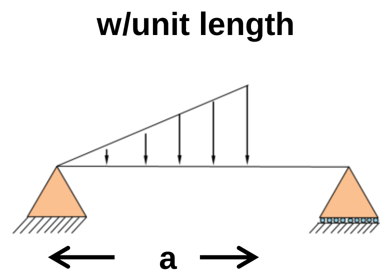


Fixed Beam

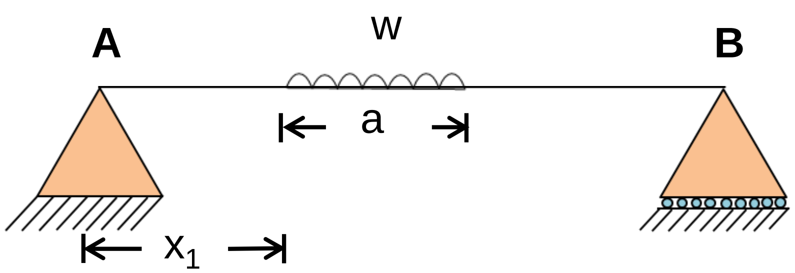
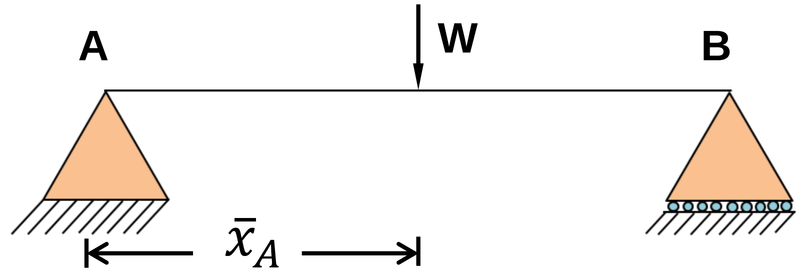
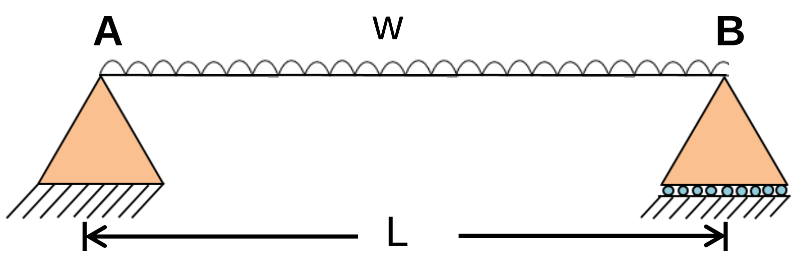
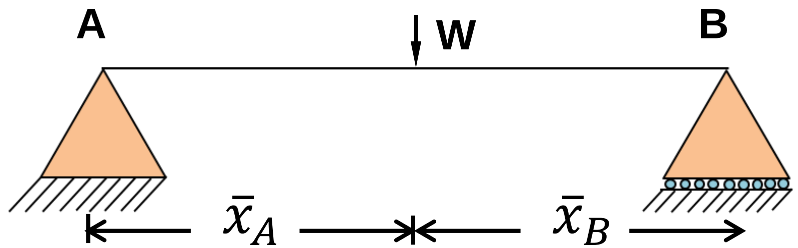
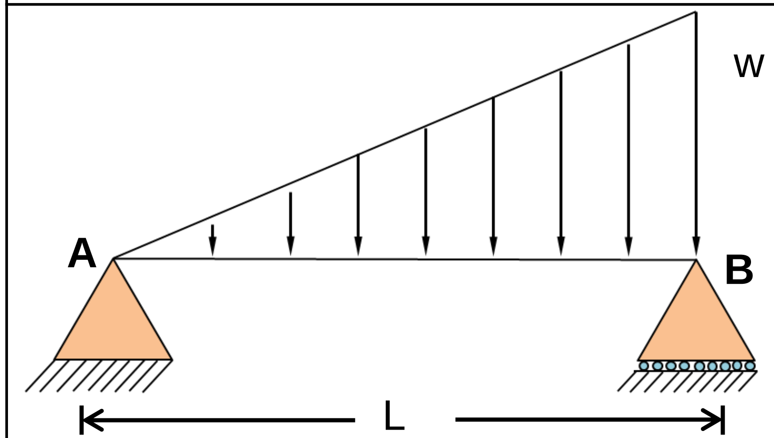
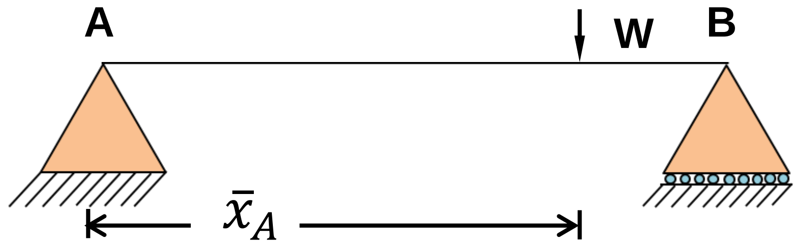


Continuous Beam

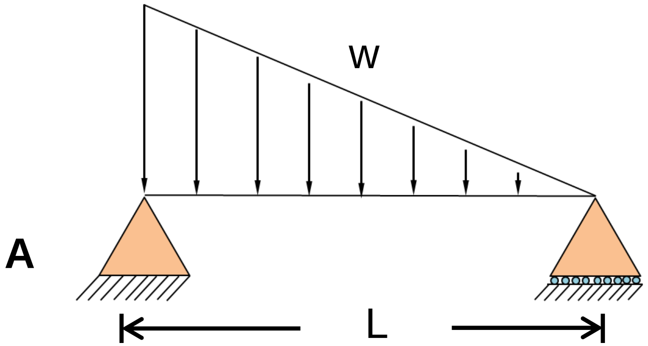
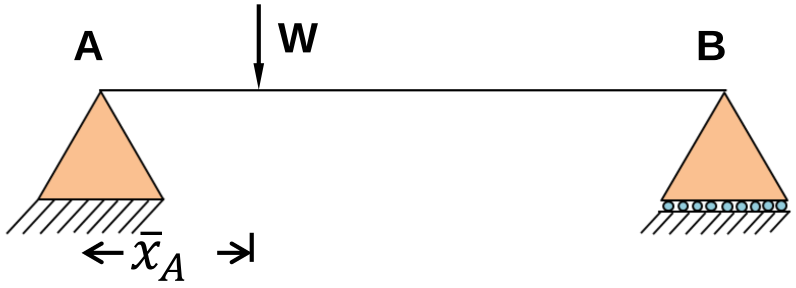
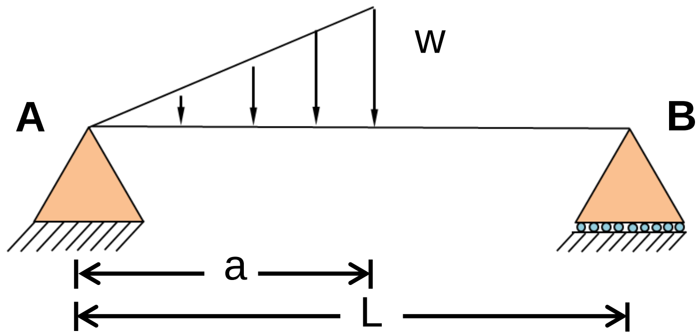
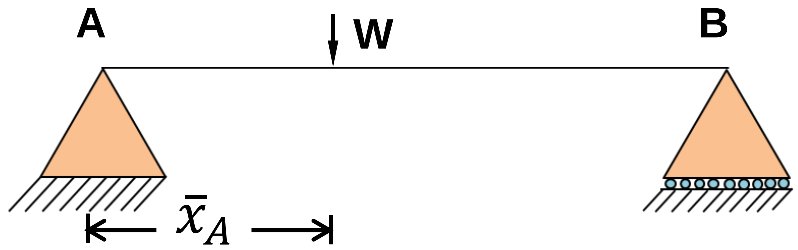
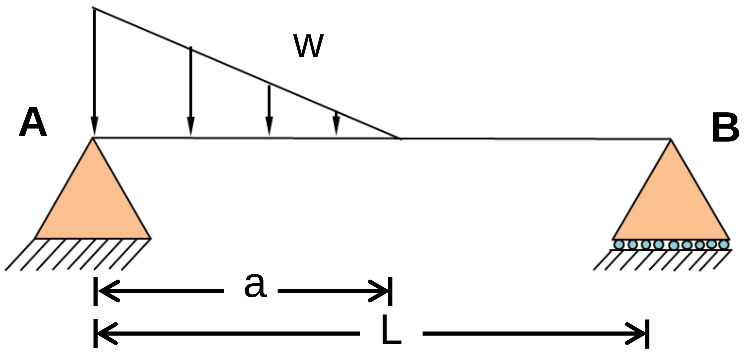
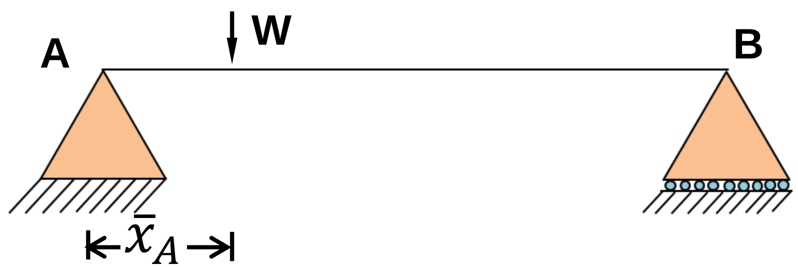
TYPES OF LOAD

S.N.	Types	Definations	Example
1	Concentrated load	Acting at the mid-point, point load Unit=kN	
2	Uniformly distributed load	Load is constant, Unit=kN/m, Eg.Water load acting on the floor of a tank when it contains water to a height ‘h’	
3	Uniformly varying load:	Varying linearly over considerable length, Eg. The water pressure acting on the side wall of tank	 

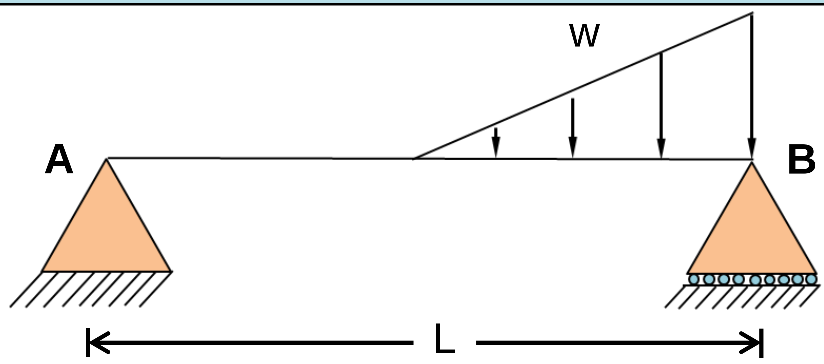
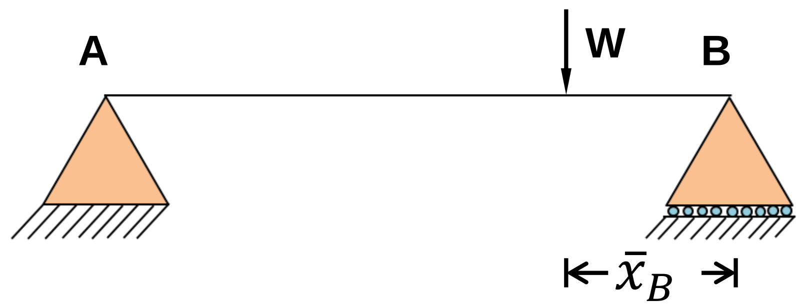
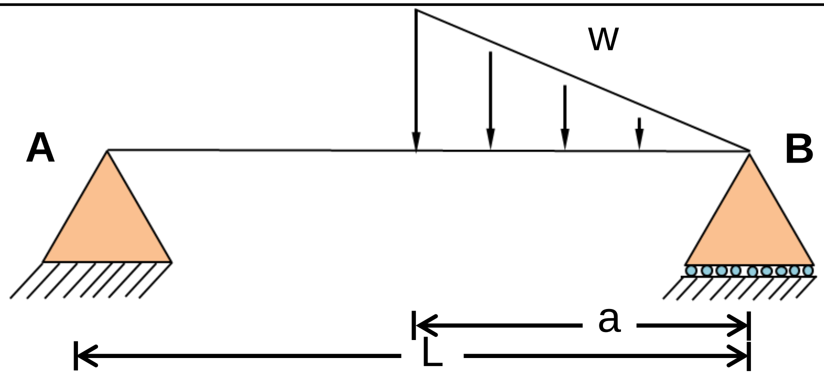
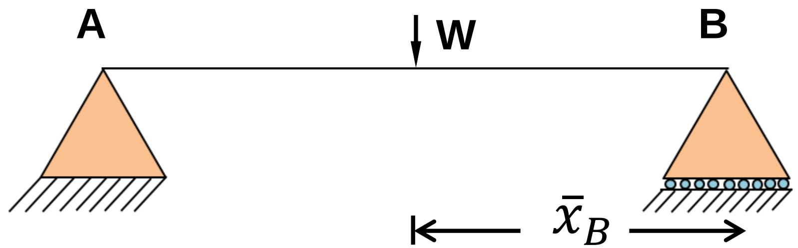
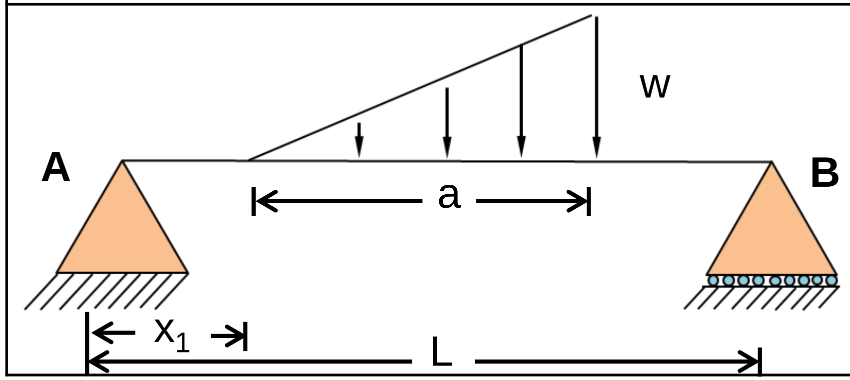
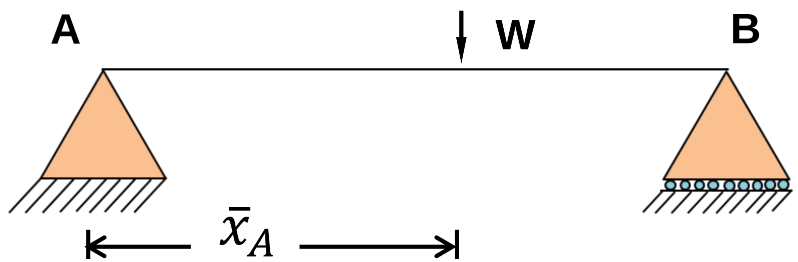
EQUIVALENT LOAD

Load	Equivalent W	Remarks
		W=wa $\bar{x}_A$=x_1+a/2
		W=wL $\bar{x}_A$=(L/2) $\bar{x}_B$=(L/2)
		W=wL/2 $\bar{x}_A$=(2L/3) $\bar{x}_B$=(L/3)

EQUIVALENT LOAD

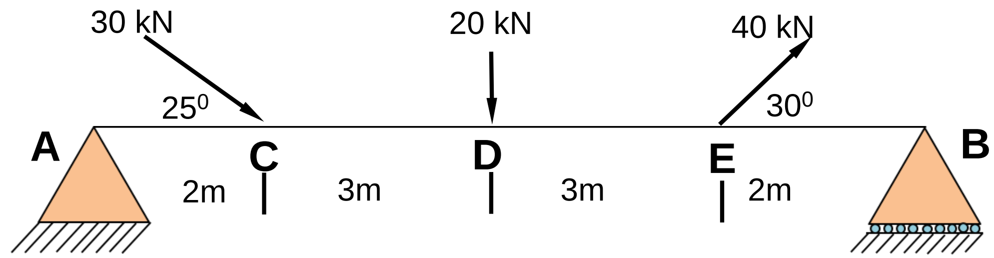
Load	Equivalent W	Remarks
		W=wL/2 $\bar{x}_A$=L/3 $\bar{x}_B$=2L/3
		W=wa/2 $\bar{x}_A$=(2a/3)
		W=wa/2 $\bar{x}_A$=(a/3)

EQUIVALENT LOAD

Load	Equivalent W	Remarks
		$W=wa/2$ $\bar{x}_B=a/3$
		$W=wa/2$ $\bar{x}_B=(2a/3)$
		$W=wa/2$ $\bar{x}_A=(x_1+2a/3)$

SAMPLE PROBLEM 1

Find the reaction at A and B so that the system is in equilibrium.

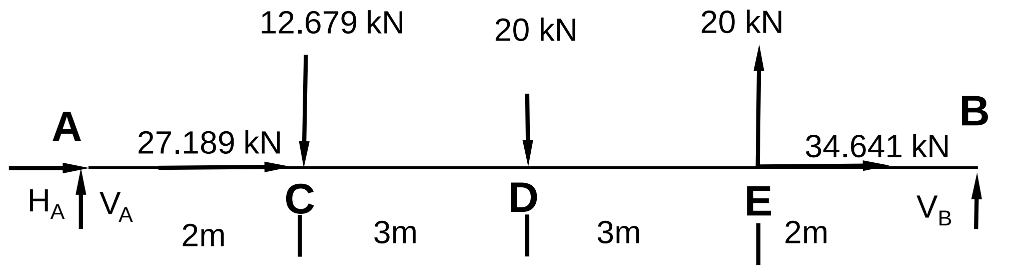


Solution:

$30 \sin 25^0 = 12.679$; $30 \cos 25^0 = 27.189$

$40 \sin 30^0 = 20$; $40 \cos 30^0 = 34.641$

The free body of above system can be drawn as:



$\overset{+}{\rightarrow}$ $\sum F_X = 0$ $H_A + 27.189 + 34.641 = 0$

$H_A = -61.83$ kN.

Here the negative sign for H_A indicated that the assumed direction is wrong and it acted towards left

SAMPLE PROBLEM 1

$\uparrow +$ $\sum F_Y = 0$,V_A-12.679-20+20+V_B=0

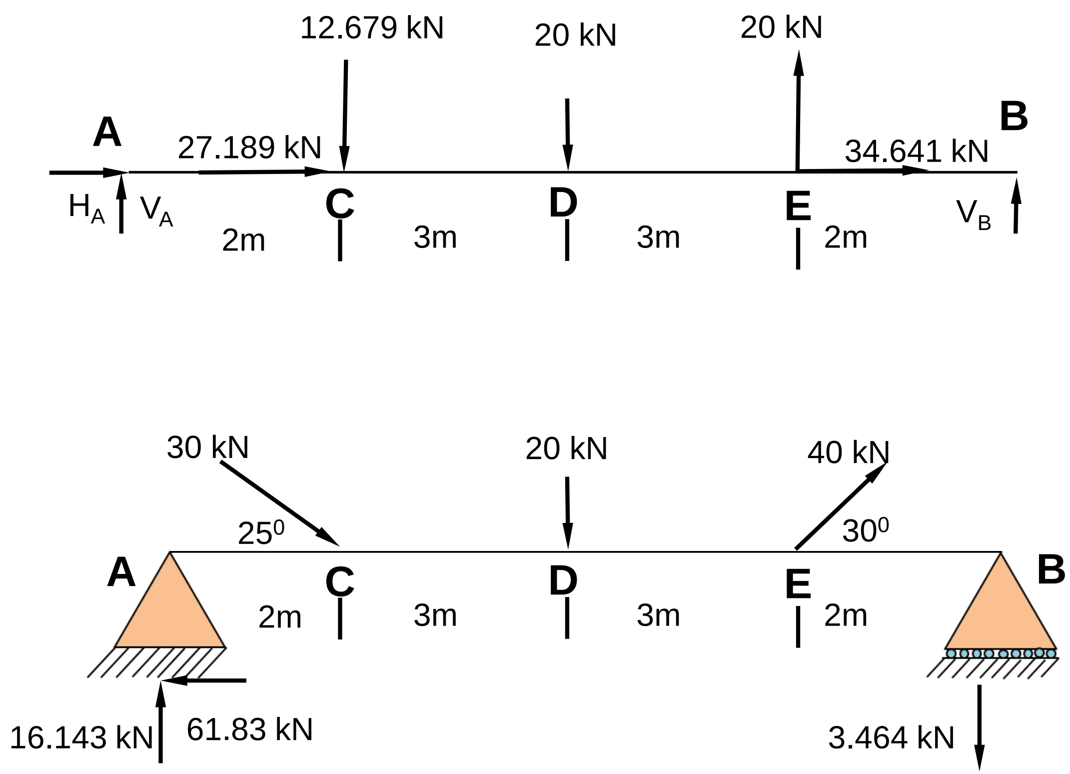
$V_A+V_B=12.679$

$\circlearrowleft +$ $\sum M_A = 0$, -12.679(2)-20(5)+20(8)+V_B(10)=0

V_B=-3.464 kN
V_A=12.679-V_B=12.679-(-3.464)=16.143 kN

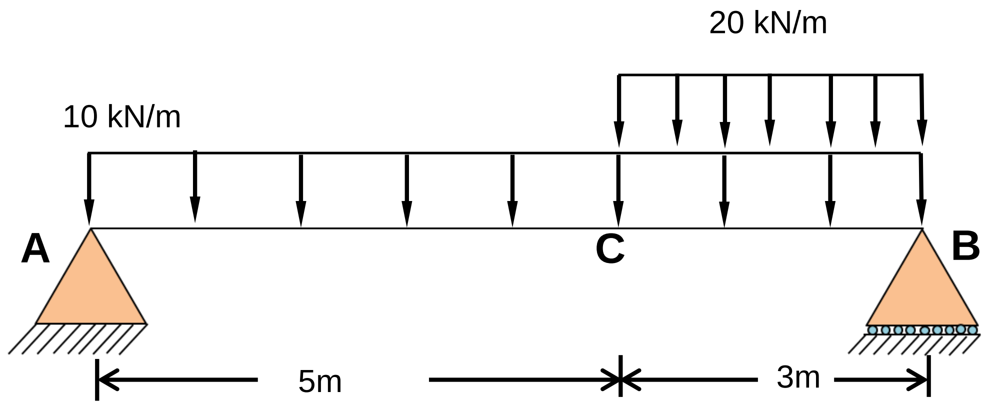
The negative sign for V_B indicates that the assumed direction is wrong and it acts in downward direction.
So,
H_A=61.83 kN(←)
V_A=16.143 kN(↑)
V_B=3.464 kN (↓)

The final equilibrium is as shown in figure.



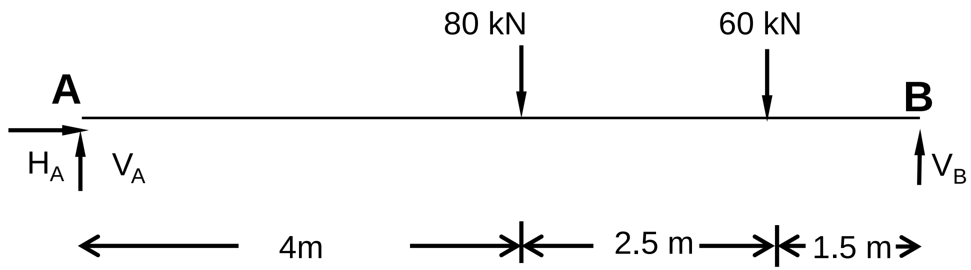
SAMPLE PROBLEM 2

Find the reaction at A and B so that the system is in equilibrium.



Solution:
The equivalent loads are calculated as:
W_1=10(8)=80 kN at 4 m from A.
W_2=20(3)=60 kN at 1.5 m from B.

The free body of above system can be drawn as:



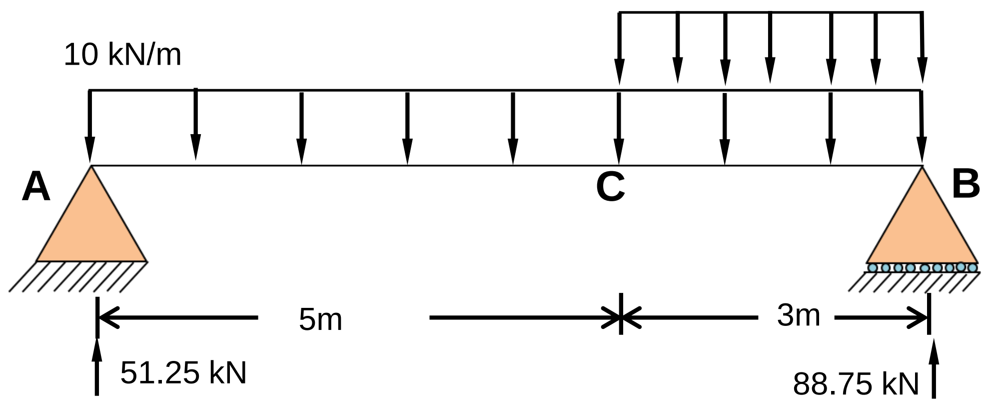
$$\overset{+}{\rightarrow} \sum F_X = 0 : H_A = 0$$

$$\uparrow + \sum F_Y = 0 : V_A - 80 - 60 + V_B = 0$$

$$V_A + V_B = 140 \text{ kN}$$

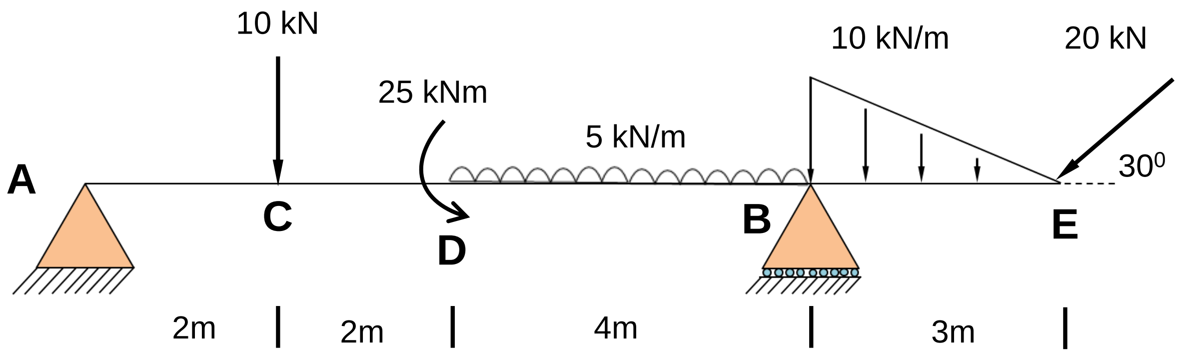
$$\circlearrowleft + \sum M_A = 0 : -80(4) - 60(6.5) + V_B(8)$$

V_B=88.75 kN (↑) ; V_A=140-88.75=51.25 kN (↑)



SAMPLE PROBLEM 3

Find the reaction at A and B so that the system is in equilibrium.



Solution:

$\xrightarrow{+}\sum F_X = 0$;H_A -17.32=0 H_A=17.32 kN($\rightarrow$)

$\uparrow + \sum F_Y = 0$; V_A-10-20-15-10+V_B=0

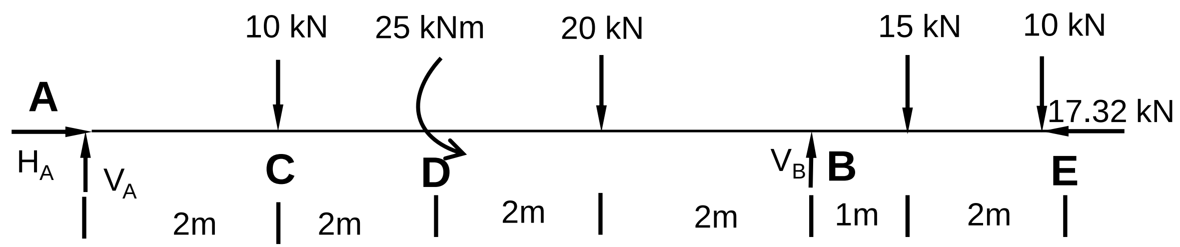
V_A+V_B=55 kN

$\circlearrowleft + \sum M_A = 0$;

-10(2)+25-20(6)+V_B(8)-15(9)-10(11)=0

V_B=45kN ($\uparrow$)

V_A=55-45=10 kN ($\uparrow$)



Free body diagram

FRICTION

- In practical application, perfect frictionless or smooth surface never exists.
- When two surfaces are in contact with each other, tangential forces will always developed when one surface tends to move with respect to other. The tangential forces are called frictional forces.
- However, these friction forces are limited in magnitude and do not prevent motion if you apply sufficiently large forces.
- There are two types of friction: **dry friction**, sometimes called Coulomb friction, and **fluid friction or viscosity.**

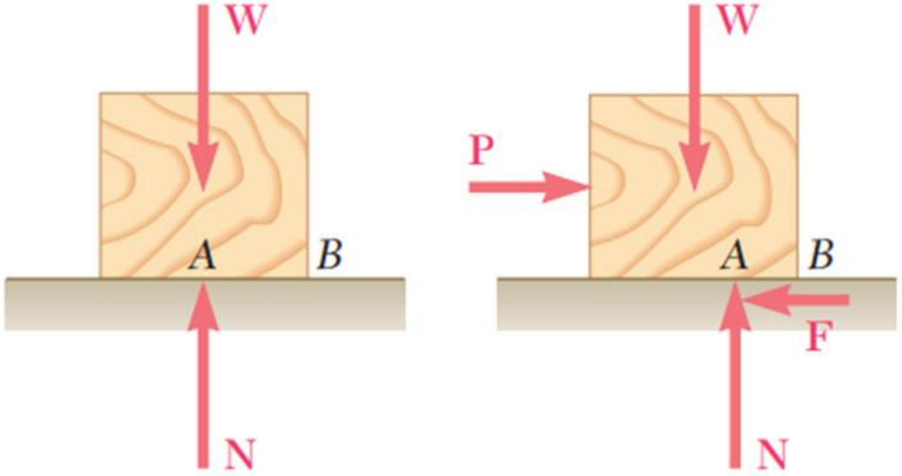
FRICTION

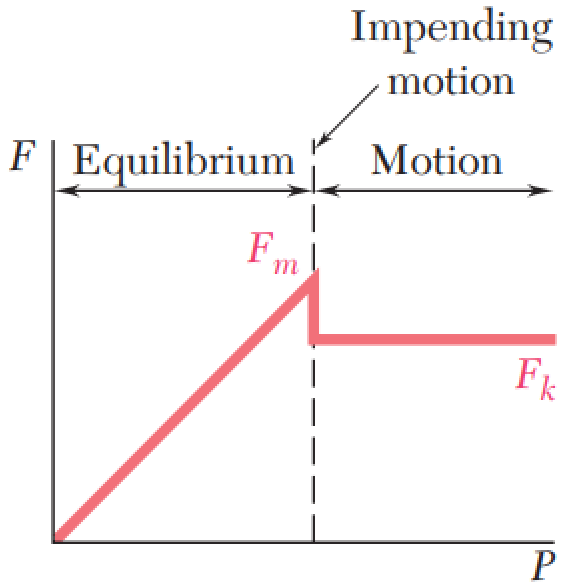
- Block of weight W placed on horizontal surface. Forces acting on block are its weight and reaction of surface N.
- Small horizontal force P applied to block. For block to remain stationary, in equilibrium, a horizontal component F of the surface reaction is required. F is a static-friction force.
- As P increases, the static-friction force F increases as well until it reaches a maximum value F_m.

$$F_m = \mu_s N$$

- Further increase in P causes the block to begin to move as F drops to a smaller kinetic-friction force F_k.

$$F_k = \mu_k N$$





FRICTION

- Static friction force:

$$F_m = \mu_s N$$

- Kinetic friction force:

$$F_k = \mu_k N$$

- In general for practical application

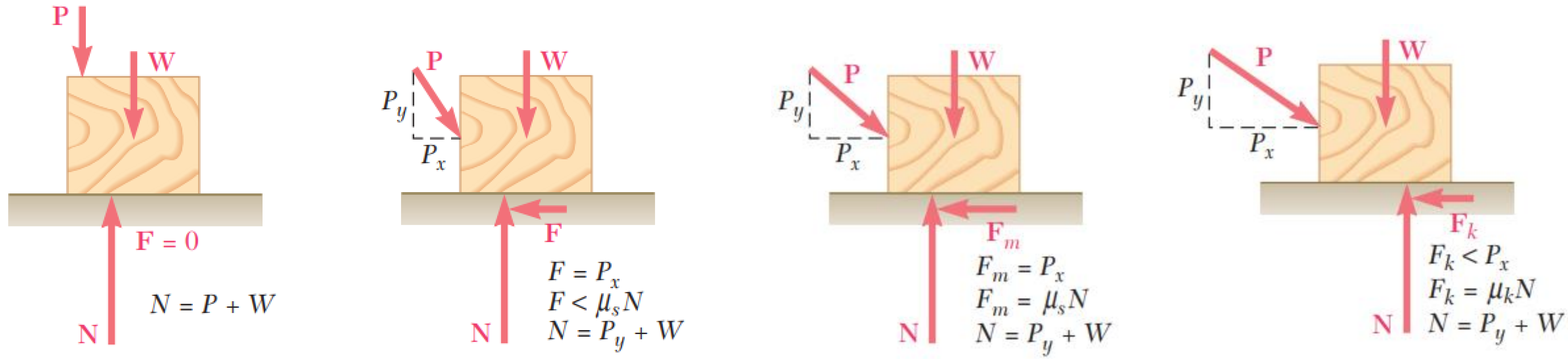
$$\mu_k \cong 0.75\mu_s$$

Approximate value of coefficient of static friction for Dry surfaces

Metal on metal	0.15–0.60
Metal on wood	0.20–0.60
Metal on stone	0.30–0.70
Metal on leather	0.30–0.60
Wood on wood	0.25–0.50
Wood on leather	0.25–0.50
Stone on stone	0.40–0.70
Earth on earth	0.20–1.00
Rubber on concrete	0.60–0.90

FRICTION

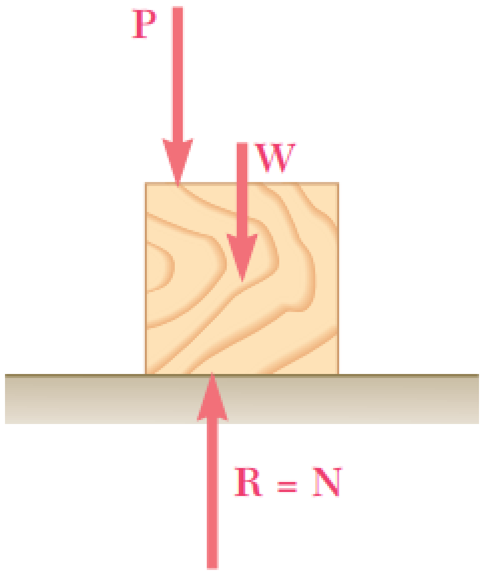
- Four situations can occur when a rigid body is in contact with a horizontal surface:



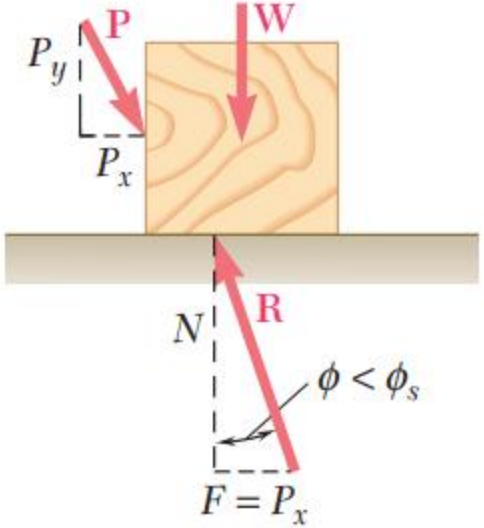
No friction(P_x=0) **No motion(P_x<F_m)** **Impending motion(P_x=F_m)** **Motion (P_x>F_k)**

FRICTION

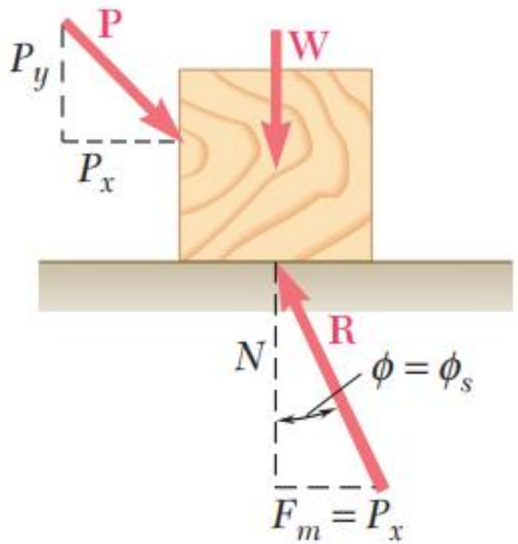
- It is sometimes convenient to replace normal force N and friction force F by their resultant R:



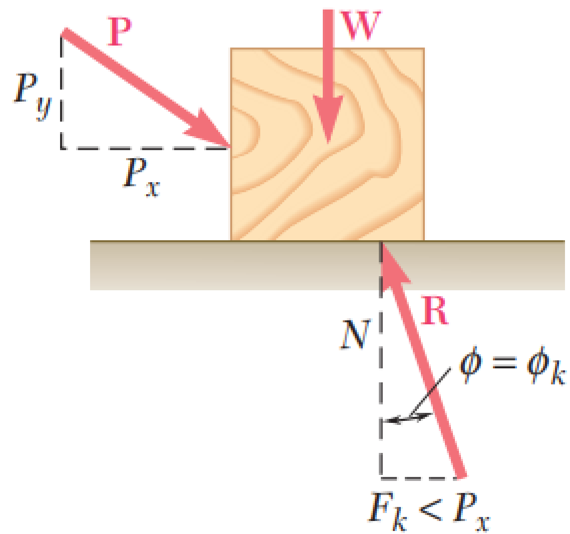
No friction(P_x=0)



No motion(P_x<F_m)



Impending motion(P_x=F_m)



Motion (P_x>F_k)

Note: The coefficients of friction μ_s and μ_k do not depend upon the area of the surfaces in contact. Both coefficients, however, depend strongly on the *nature* of the surfaces in contact.

$$\tan\phi_s = \frac{F_m}{N} = \frac{\mu_s N}{N}$$

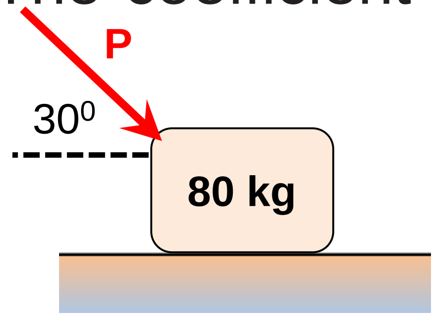
$$\tan\phi_s = \mu_s$$

$$\tan\phi_k = \frac{F_k}{N} = \frac{\mu_k N}{N}$$

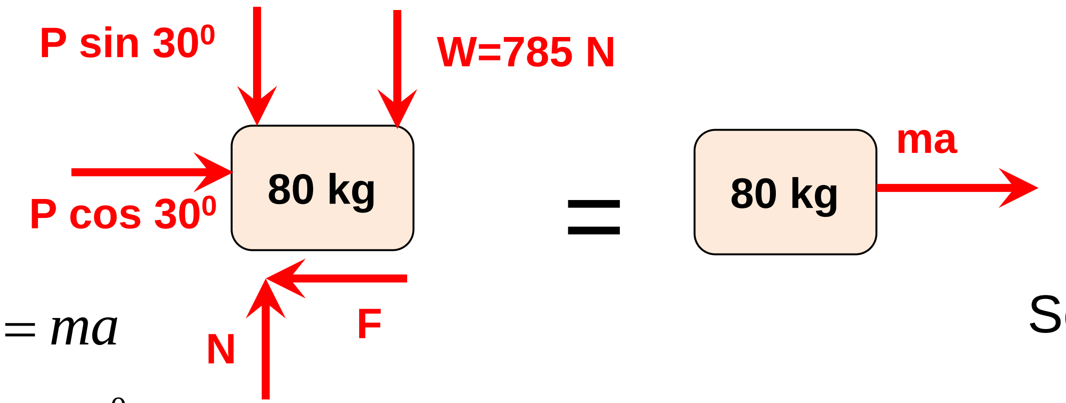
$$\tan\phi_k = \mu_k$$

SAMPLE PROBLEM 1

- An 80-kg block rests on a horizontal plane. Find the magnitude of the force **P** required to give the block an acceleration of 2.5 m/s^2 to the right. The coefficient of kinetic friction between the block and the plane is μ_k=0.25



Solution:



$$(\xrightarrow{+}) \sum F_X = ma$$

$$\Rightarrow P\cos 30^0 - F = 80(2.5)$$

$$\Rightarrow P\cos 30^0 - 0.25N = 80(2.5)$$

$$\Rightarrow P\cos 30^0 - 0.25N = 200 \quad (1)$$

$$(\uparrow +) \sum F_Y = 0$$

$$\Rightarrow N - P\sin 30^0 - 785 = 0$$

$$\Rightarrow N = P\sin 30^0 + 785 \quad (2)$$

Solving equation (1) and (2) for P gives:

$$P\cos 30^0 - 0.25(P\sin 30^0 + 785) = 200$$

$$\Rightarrow P\cos 30^0 - 0.25(P\sin 30^0) - 0.25(785) = 200$$

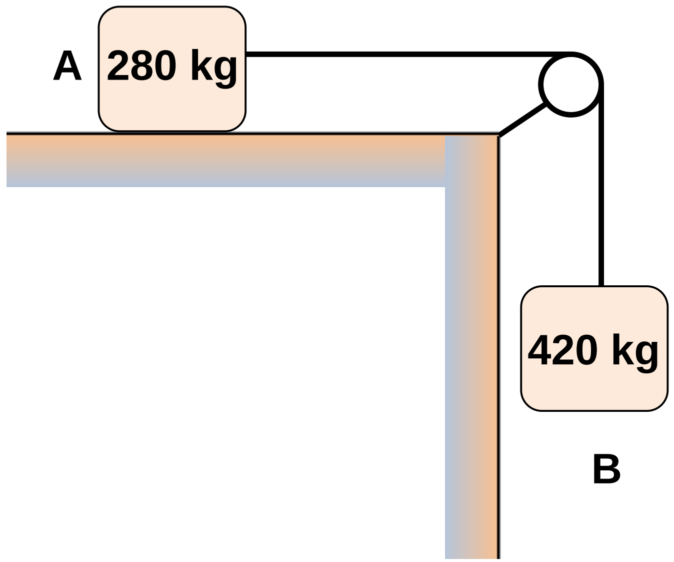
$$\Rightarrow P = \frac{200 + 0.25\times 785}{\cos 30^0 - 0.25\times \mathrm{Sin}\, 30^0}$$

$$\Rightarrow P = 535N$$

SAMPLE PROBLEM 2

Two blocks ‘A’ and ‘B’ of masses 280 kg and 420 kg respectively are joined by an inextensible cable as shown. Assume that the pulley is frictionless and μ=0.3 between block A and surface. The system is initially at rest. Determine:

1. Acceleration of the block.
2. Velocity after it has moved 3.5 m
3. Velocity after 1.5 s.



Solution:

$$m_A = 280kg; m_B = 420kg$$

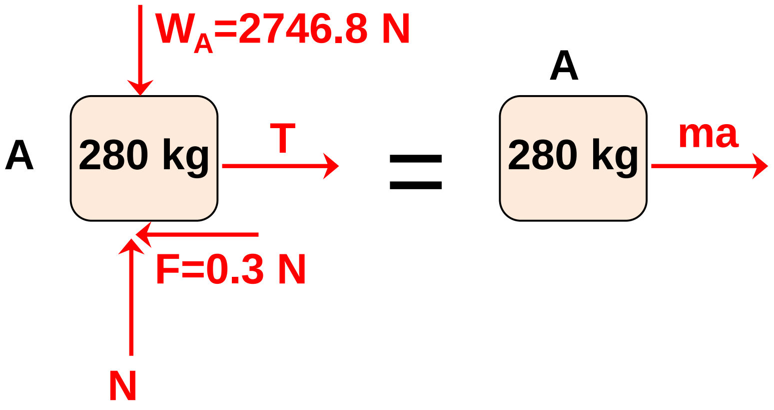
$$W_A = m_A g = 2746.8N$$

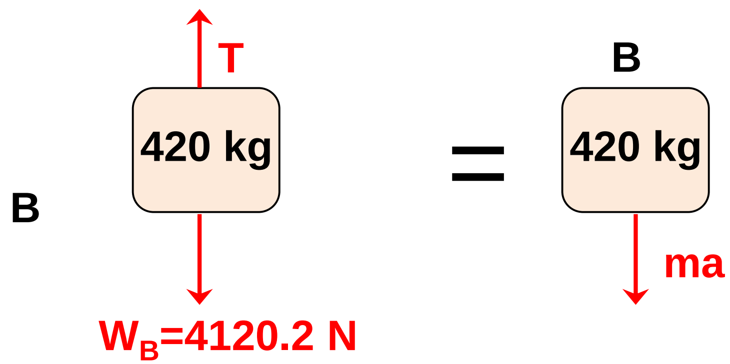
$$W_B = m_B g = 4120.2N$$

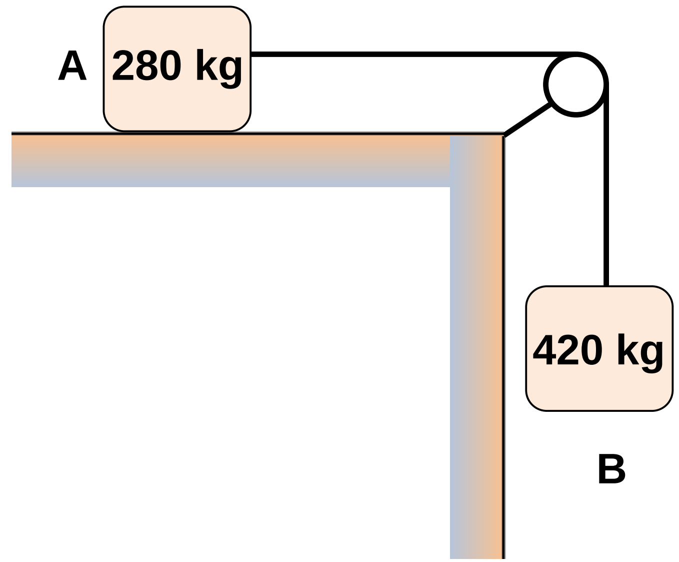
$$g = 9.81m/s^2$$

SAMPLE PROBLEM 2

The free body diagram for both block is as shown;







$$(+\uparrow)\sum F_Y = 0 \Rightarrow N = 2746.8$$

$$(\xrightarrow{+})\sum F_X = m_A a$$

$$\Rightarrow T - 0.3(2746.8) = 280a$$

$$\Rightarrow T - 280a = 824.04 \quad (1)$$

$$(+\uparrow)\sum F_Y = -m_B a$$

$$\Rightarrow T - 4120.2 = -m_B a$$

$$\Rightarrow T + 420a = 4120.2 \quad (2)$$

Solving above two equations gives:

$$700a = 3296.16 \Rightarrow a = 4.709 m/s^2$$

$$T = 280(4.709) + 824.04 = 2142.56N$$

Velocity after 3.5 m:

$$v^2 = v_0^{\ 2} + 2as$$

$$v^2 = (0)^2 + 2(4.709)(3.5)$$

$$v = 5.741 m/s$$

Velocity after 1.5 s:

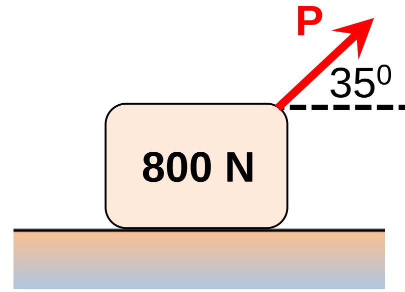
$$v = u + at$$

$$v = 0 + (4.709)(1.5)$$

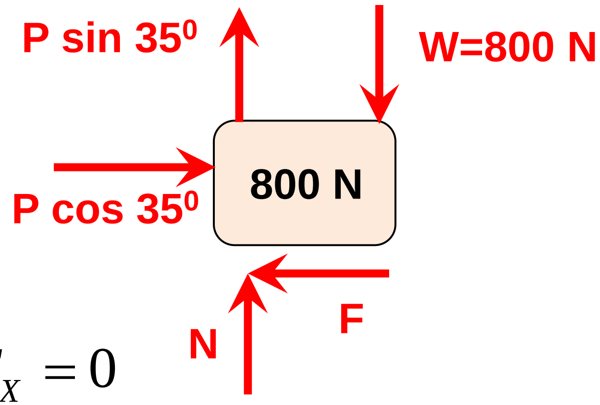
$$v = 7.064 m/s$$

SAMPLE PROBLEM 3

A block weighting 800N, lying on a horizontal floor is just dragged by a force inclined at 35^0 to the floor. Find the a) the value of 'P' b) the inclination of 'P' with horizontal so that 'P' is minimum. c) the value of Pmin *(μ =0.25)*



Solution:



$$\circlearrowright\!\!\overrightarrow{+}\ \sum F_X = 0$$

$$\Rightarrow P\cos 35^0 - F = 0$$

$$\Rightarrow P\cos 35^0 - 0.25N = 0 \qquad (1)$$

$$\uparrow + \ \sum F_Y = 0$$

$$\Rightarrow N + P\sin 35^0 - 800 = 0$$

$$\Rightarrow N = 800 - P\sin 35^0 \qquad (2)$$

Solving equation (1) and (2) for P gives:

$$P\cos 35^0 - 0.25N = 0$$

$$\Rightarrow P\cos 30^0 - 0.25(800 - P\sin 35^0) = 0$$

$$\Rightarrow P = \frac{0.25 \times 800}{\cos 35^0 + 0.25 \times \mathrm{Sin}\, 35^0}$$

$$\Rightarrow P = 207.782N$$

SAMPLE PROBLEM 3

If the angle between force P and horizontal is α, then:

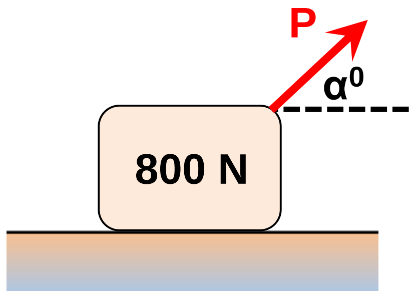
$$P = \frac{0.25 \times 800}{\cos\alpha + 0.25 \times \mathrm{Sin}\,\alpha}$$

To be P minimum, the denominator must be maximum.

$$\frac{d}{d\alpha}(\cos\alpha + 0.25 \times \sin\alpha) = 0$$
$$\Rightarrow 0.25\cos\alpha - \sin\alpha = 0$$
$$\Rightarrow \tan\alpha = 0.25$$
$$\Rightarrow \alpha = 14.04^0$$

Now taking second derivative test :

$$\frac{d^2}{d\alpha^2}(\cos\alpha + 0.25 \times \sin\alpha)$$
$$= -(\cos\alpha + 0.25\sin\alpha)$$
$$= -(\cos 14.04^0 + 0.25\sin 14.04^0)$$
$$= -1.0377 < 0$$



This shows $\cos\alpha + 0.25 \times \sin\alpha$ has maximum value at α=14.04⁰

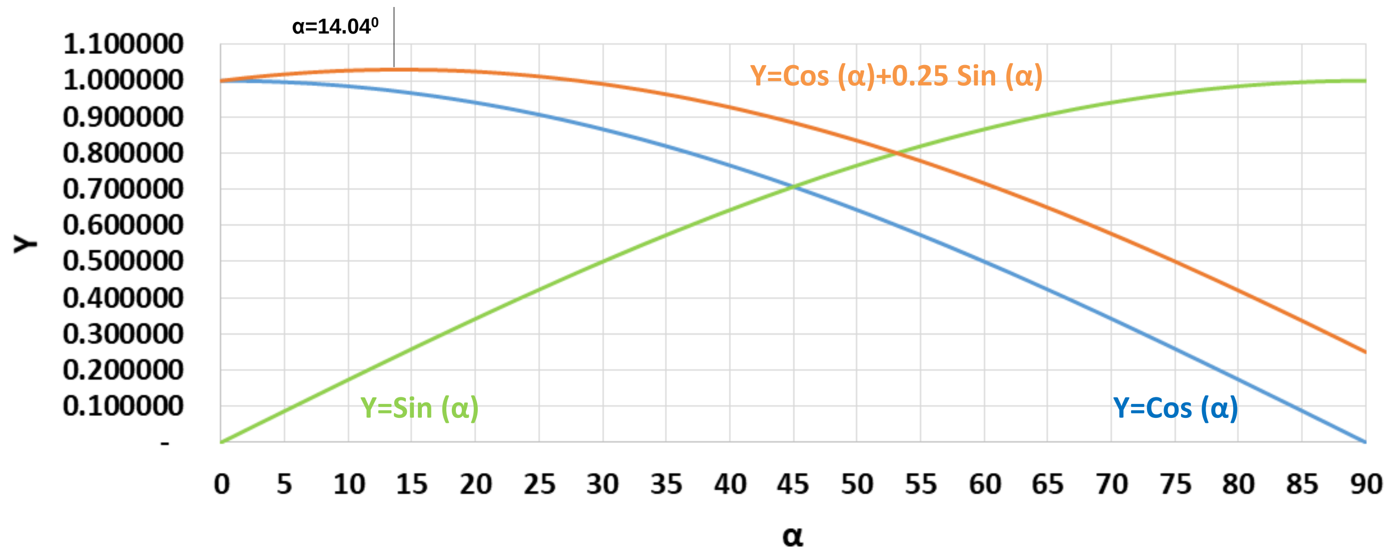
$$P_{\min} = \frac{0.25 \times 800}{\cos 14.04^0 + 0.25 \times \sin 14.04^0}$$
$$\Rightarrow P_{\min} = 194.03N$$

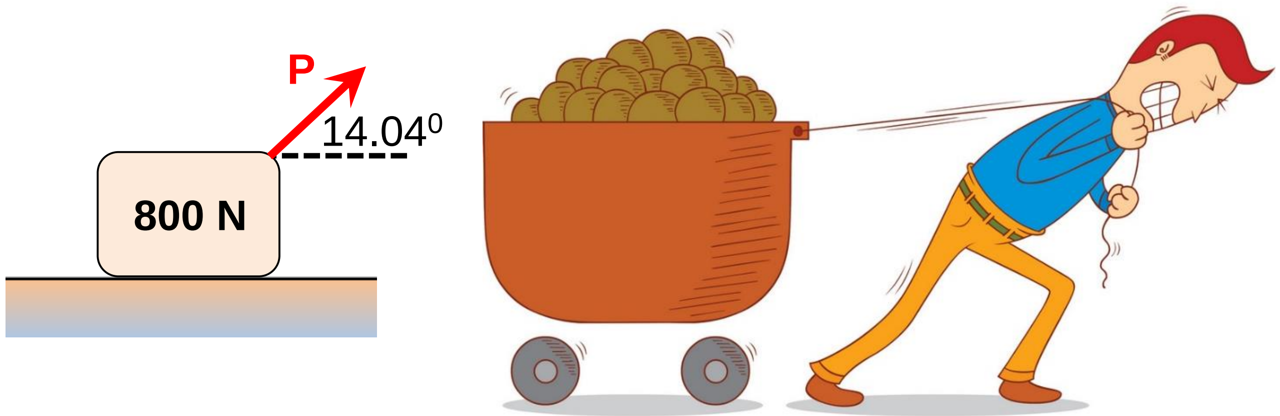
SAMPLE PROBLEM 3 (PHYSICAL MEANING)

- Due to the presence of friction force, the minimum force should come in certain optimum angle.

- This is because of blending of both sin and cos function with some factor in front of sin function.

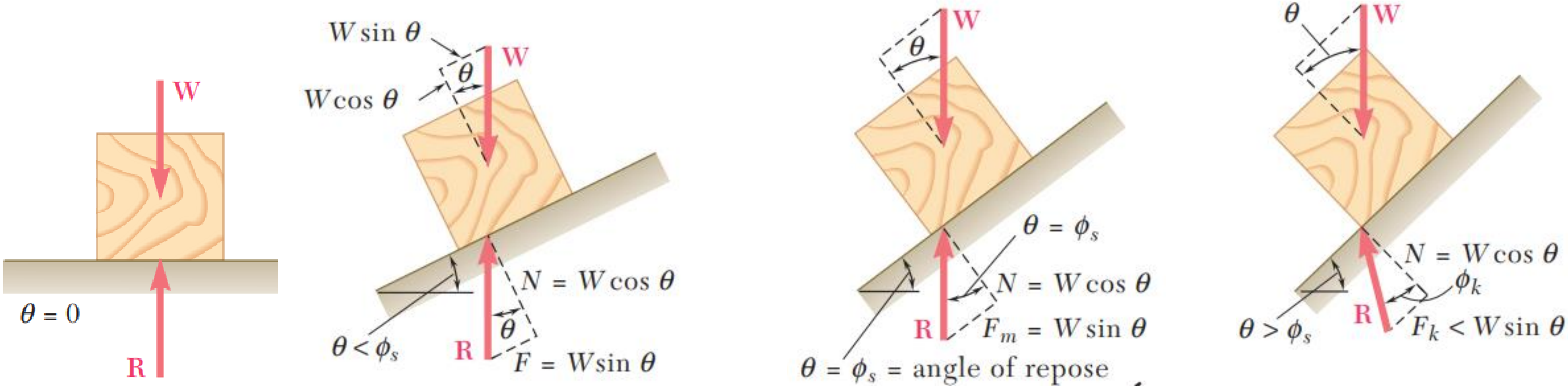
- So, if a man was pulling a cart of 800 N and the friction coefficient between wheel and surface was 0.25 , then he will feel most comfortable at 14.04^0





FRICTION IN INCLINED PLANE

- Four situations can occur when a rigid body is in contact with a inclined surface:

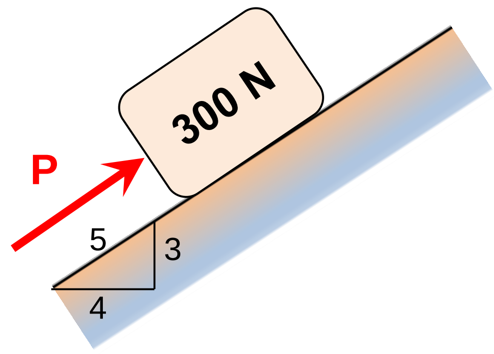


No friction(θ=ϕ_s) **No motion(θ<ϕ_s)** **Impending motion(θ=ϕ_s)** **Motion (θ>ϕ_s)**

FRICTION IN INCLINED PLANE

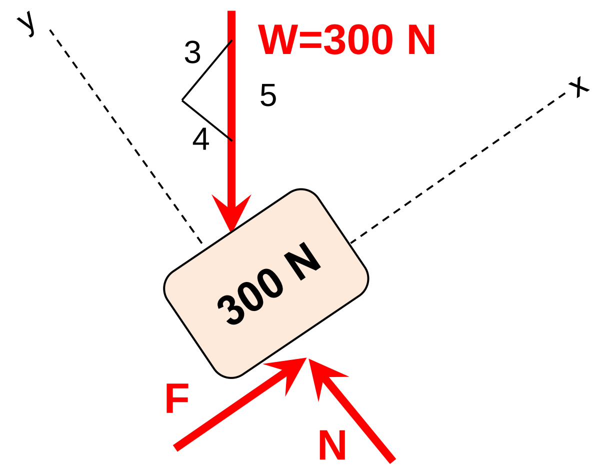
- A block 'A' rests on the inclined plane. Find the equilibrium condition of the body when applied force is :

a) P= 0 N b) P=100 N c) P=120 N d) P=140 N
e) P=200 N f) P=240 N g) P=280 N
(μ_s=0.25, μ_k=0.2)



Solution:
a) When P=0,

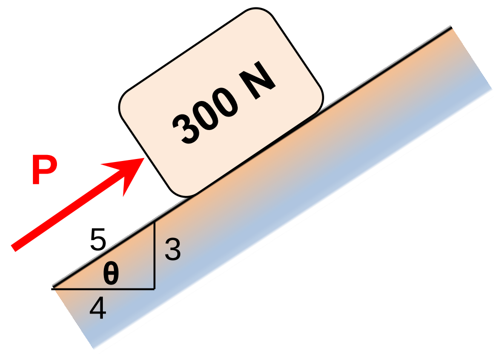
$$\tan\theta = (3/4)$$
$$\Rightarrow \theta = \tan^{-1}(3/4) = 36.87^0$$
$$\tan\phi_s = \mu_s$$
$$\Rightarrow \phi_s = \tan^{-1}(\mu_s) = \tan^{-1}(0.25) = 14.04^0$$
$$\therefore \theta > \phi_s$$



Since the angle of inclination is greater than angle of repose, the block will slide down due to its self weight.

FRICTION IN INCLINED PLANE

Now, Lets resolve the force in two direction, X along the plane and Y perpendicular to plane:



Since there is no motion in Y direction Cos component of W is balanced by normal Reaction N

$$\sum F_Y = 0$$

$$\Rightarrow N - W\cos\theta = 0$$

$$\Rightarrow N = 300(4/5) = 240N \nwarrow$$

$$\sum F_X = F_k - W\,\mathrm{Sin}\,\theta$$

$$\Rightarrow \sum F_X = 48 - 300(3/5)$$

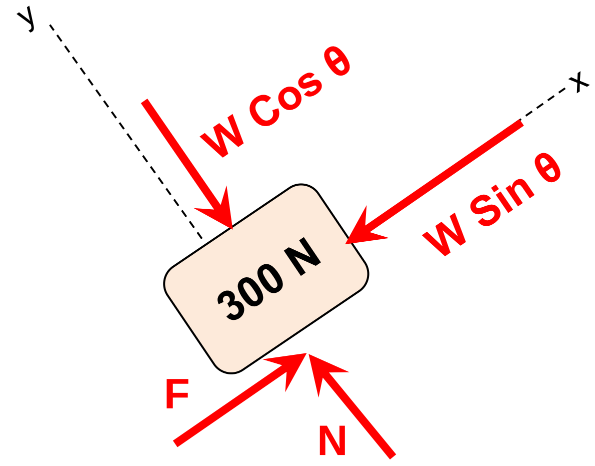
$$\Rightarrow \sum F_X = 48 - 180$$

$$\Rightarrow \sum F_X = 132N \swarrow$$

Values of frictional forces are:

$$F_s = \mu_s N = 0.25 \times 240 = 60N$$

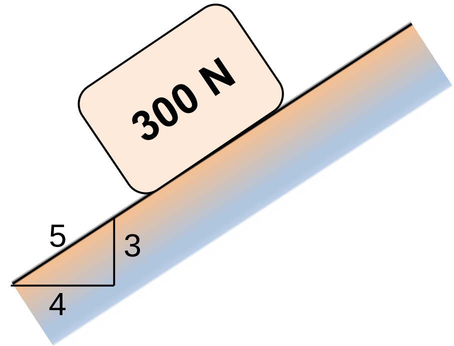
$$F_k = \mu_k N = 0.2 \times 240 = 48N$$



Note: Here we choose F_k,as we already know that the block will slide down and dynamic friction will come in action

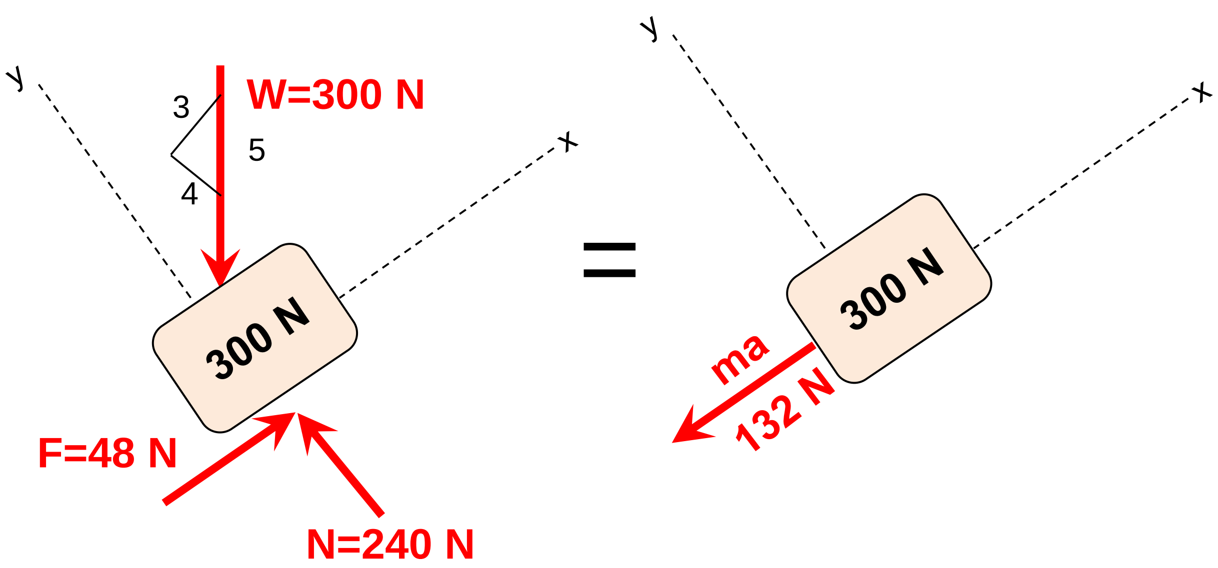
FRICTION IN INCLINED PLANE

So the force acting of the plane are not balanced. Their resultant is:



$$\sum F_X = 132N \swarrow$$

$$\therefore R = 132N \swarrow$$



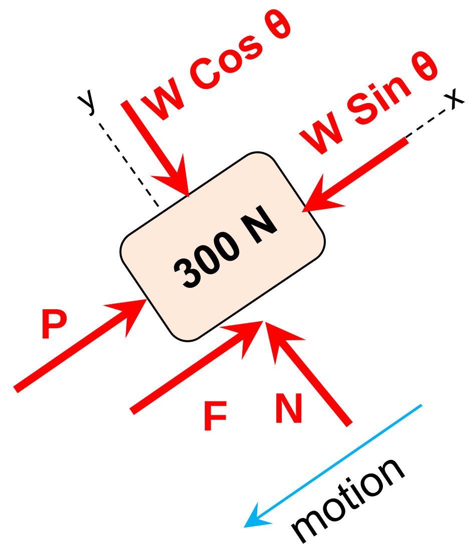
FRICTION IN INCLINED PLANE

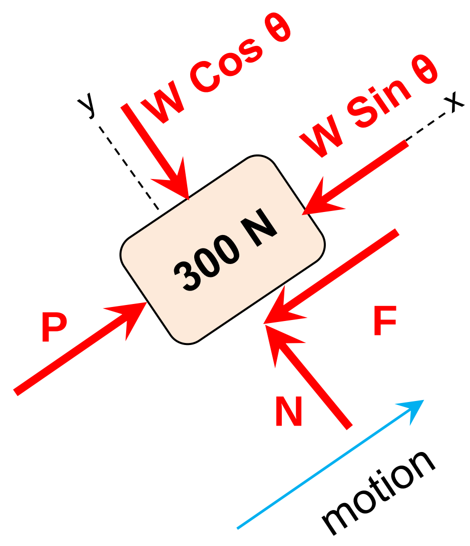
- For condition b to g, the direction of forces are constant for P, N,W cos θ , W sin θ , except for the frictional force F_f.
- The direction of frictional force may change based upon the effect created by applied force P.
- If the force P is not sufficient or just sufficient to balance the weight of block down the plane, the block will slide down or have tendency to slide down and the frictional force will act in upward direction
- If the force P is large or just sufficient to overcome the weight of block up the plane, the block will slide up or have tendency to slide up and the frictional force will act in upward direction.

$$\sum F_X = P \pm F_f - W\sin\theta$$

$$\Rightarrow \sum F_X = P \pm 60 - 180$$

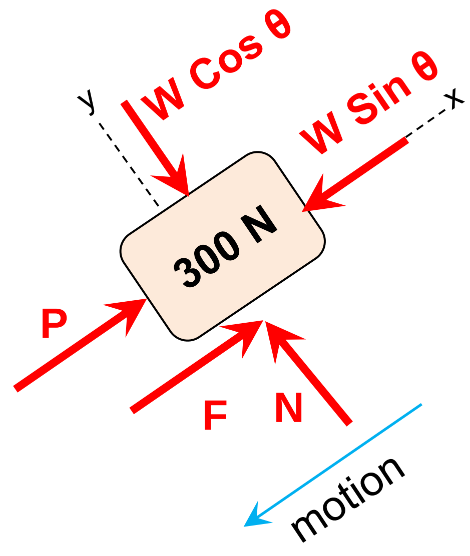
- Here we put F_f as frictional force. It may be in variou form and value depeding upon the tendency of motion of body.

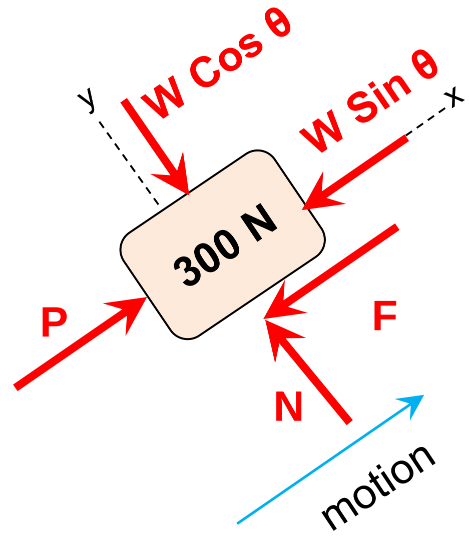




FRICTION IN INCLINED PLANE

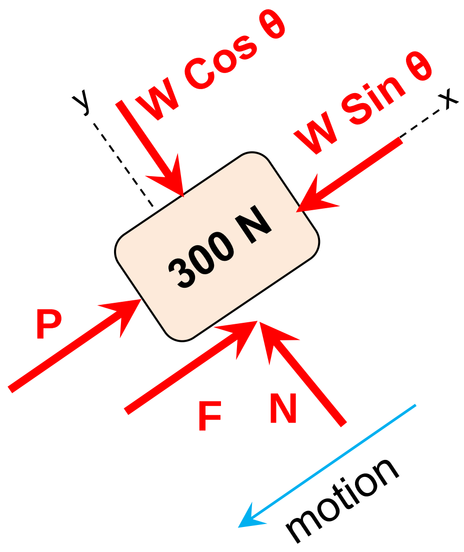
- We know that in case of P=0, the block will slide down as inclination angle is greater then angle of repose.
- Now when we start to increase P firstly it tries overcome the weight of the body down the plane, but the value of P may not be sufficient to balance the weight of block.
- Gradually increasing P there comes the point where the weight of body down the plane is just balance by P and F.
- After further increasing P beyond this value, the tendency of block to slide down will change such that P force tries to move the block up the plane.
- But up to some point the value of P will not be sufficient to move the block up the plane due to frictional force.
- The block will just start to move up the plane when weight of body down the plane and friction force is just equal to P.
- Further increasing P will accelerate the block up the plane.





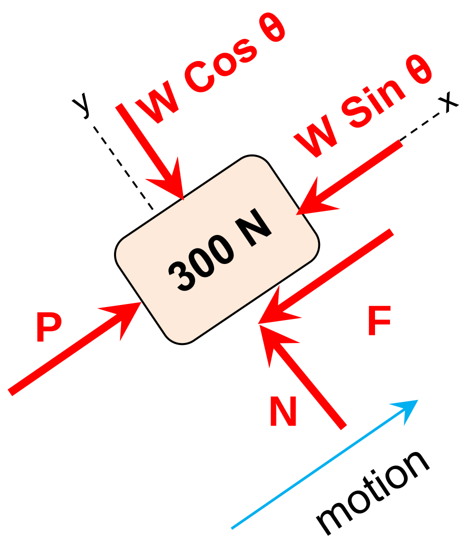
FRICTION IN INCLINED PLANE

P	Initial Equation	Motion Remarks	Actual Equation	Actual Resultant
	$\sum F_X = P \pm 60 - 180$		$\sum F_X = P \pm F - 180$	
0	=0+60-180=-120	Slides down	$\sum F_X = P + F_k - 180$	=48-180=-132
100	=100+60-180=-20	Slides down	$\sum F_X = P + F_k - 180$	=100+48-180=-32
120	=120+60-180=0	Just prevented to slide down	$\sum F_X = P + F_s - 180$	=120+60-180=0
140	=140-60-180=-100	No motion, static equilibrium	$\sum F_X = P + F - 180$	=140+40-180=0
200	=200-60-180=-40	No motion, static equilibrium	$\sum F_X = P - F - 180$	=200-20-180=0
240	=240-60-180=0	Just to slide up the plane	$\sum F_X = P - F_s - 180$	=240-60-180=0
280	=280-60-180=40	Slides up	$\sum F_X = P - F_k - 180$	=280-48-180=52



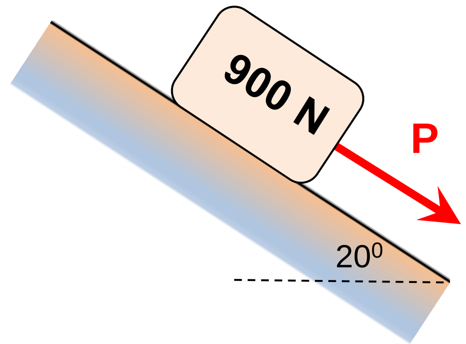
Summary:

- For P<120, block slides down.
- For P=120, block is just prevented to slide down.
- For 120<P<180, block is at rest, static equilibrium, upward Frictional reaction.
- For P=180, block is at rest, static equilibrium, No frictional reaction
- For 180<P<240, block is at rest, static equilibrium, downward Frictional reaction.
- For P=240 N, block just start to slides up.
- For 240<P, block slides up.



SAMPLE PROBLEM 1

- Find the force 'P' required to move the block down the plane. The force 'P' is applied parallel to the plane. Weight of the block is 900N and μ= 0.5.



Solution:

Here,

$$\theta = 20^0$$

$$But,$$

$$\tan\phi = \mu$$

$$\Rightarrow \phi = \tan^{-1}(\mu) = \tan^{-1}(0.5) = 26.56^0$$

$$\therefore \theta < \phi$$

Since the angle of inclination is smaller than angle of repose, the block will not slide down due to its self weight. So some force P should come in action in order to move it.

SAMPLE PROBLEM 1

- Here

$$\sum F_X = 0$$

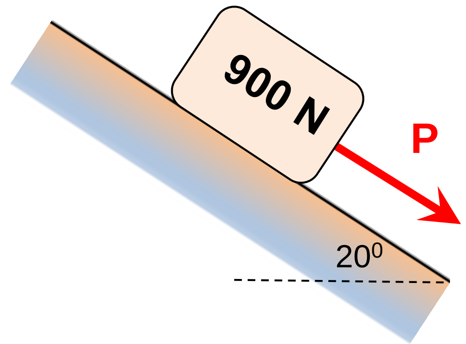
$$\Rightarrow P - F + W\,\mathrm{Sin}\,\theta = 0$$

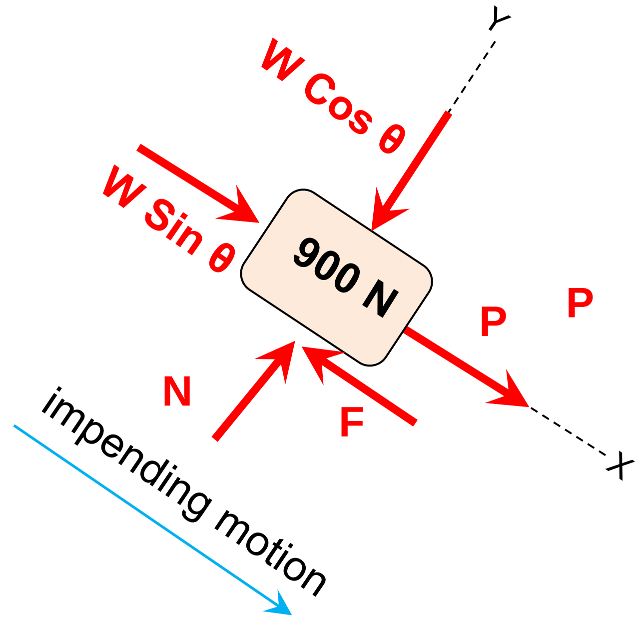
$$\Rightarrow P = F - W\,\mathrm{Sin}\,\theta$$

$$\sum F_Y = 0$$

$$\Rightarrow N - W\,\mathrm{Cos}\,\theta = 0$$

$$\Rightarrow N = W\,\mathrm{Cos}\,\theta$$





For Impending motion, $F = \mu N$

$$\therefore P = \mu N - W\,\mathrm{Sin}\,\theta$$

$$\Rightarrow P = \mu W\,\mathrm{Cos}\,\theta - W\,\mathrm{Sin}\,\theta$$

$$\Rightarrow P = W(\mu\,\mathrm{Cos}\,\theta - \mathrm{Sin}\,\theta)$$

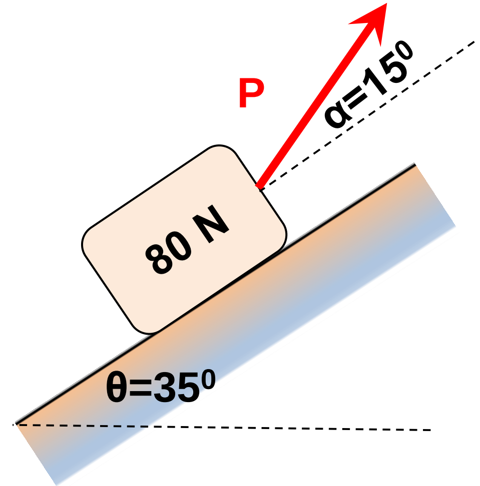
$$\Rightarrow P = 900(0.5 \times \mathrm{Cos}\,20^0 - \mathrm{Sin}\,20^0)$$

$$\therefore P = 115.04N$$

SAMPLE PROBLEM 2

- A block 80 N is pulled up the smooth plane. Determine the acceleration along the plane when:

a) P= 30 N b) P=75 N



Solution:

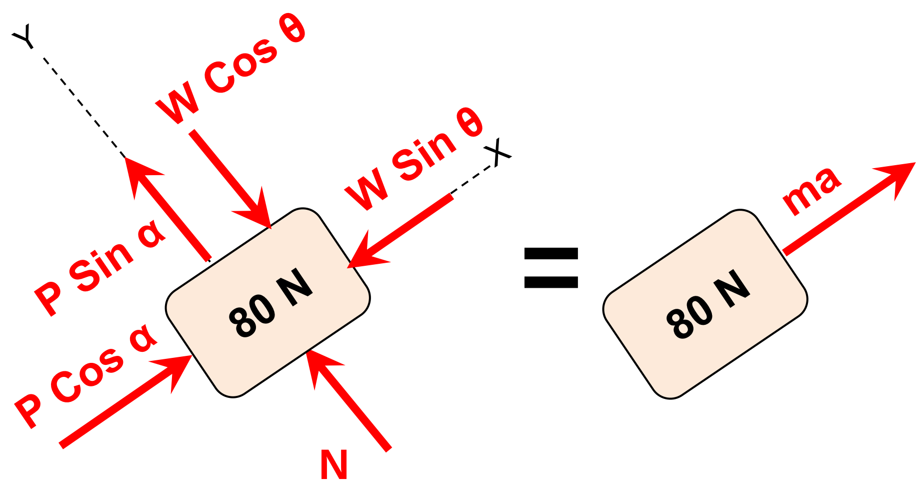
Let body slides up with acceleration 'a'.

So from Free body diagram

$$\sum F_Y = 0$$

$$\Rightarrow N + P\,\mathrm{Sin}\,\alpha - W\,\mathrm{Cos}\,\theta = 0$$

$$\Rightarrow N = W\cos\theta - P\,\mathrm{Sin}\,\alpha$$



$$\sum F_X = ma$$

$$\Rightarrow P\,\mathrm{Cos}\,\alpha - W\,\mathrm{Sin}\,\theta = ma$$

$$\Rightarrow a = \frac{P\,\mathrm{Cos}\,\alpha - W\,\mathrm{Sin}\,\theta}{m}$$

SAMPLE PROBLEM 2

$$\because a = \frac{P\,\mathrm{Cos}\,\alpha - W\,\mathrm{Sin}\,\theta}{m}$$

When

$P = 30N$

$$\Rightarrow a = \frac{30\,\mathrm{Cos}15^0 - 80\,\mathrm{Sin}\,35^0}{8.15}$$

$\Rightarrow a = -2.07 m/s^2$

When

$P = 75N$

$$\Rightarrow a = \frac{75\,\mathrm{Cos}15^0 - 80\,\mathrm{Sin}\,35^0}{8.15}$$

$\Rightarrow a = 3.26 m/s^2$

$\because N = W\cos\theta - P\,\mathrm{Sin}\,\alpha$

When

$P = 30N$

$N = 80\,\mathrm{Cos}\,35^0 - 30\,\mathrm{Sin}\,15^0$

$N = 57.77N$

When

$P = 75N$

$N = 80\,\mathrm{Cos}\,35^0 - 75\,\mathrm{Sin}\,15^0$

$N = 46.12N$

- Negative sign indicates that the body slides down with acceleration of 2.07 m/s^2
- When P=30 N the block slides down with acceleration of 2.07 m/s^2
- When P=75 the block slides up with acceleration of 3.26 m/s^2

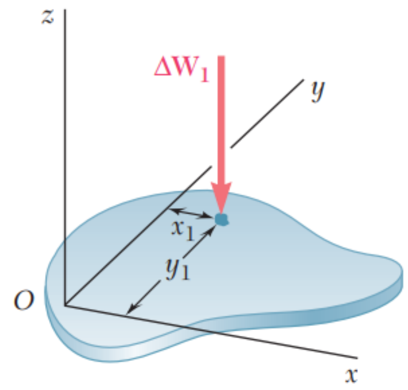
CENTRE OF GRAVITY, CENTER OF MASS, CENTROID

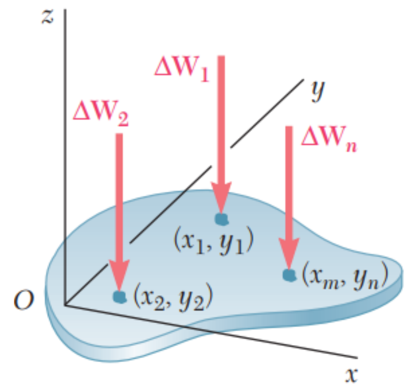
- The line of action of resultant and point where it is assumed to be acting or concentrated is defined as centroid , center of mass, center of gravity.
- Centre of mass is the point where the mass of body is supposed to be concentrated. A line passing through center of mass of body have equal moment on both side.
- Centre of gravity is the point where the resultant gravitation force act on the body.
- Centroid is defined as the geometric center of the body. For a body having uniform density center of mass and centroid coincide.
- For a body having, uniform density, centroid and center of mass will be same.
- For body in constant gravitational field and uniform density, center of mass, centroid and center of gravity coincide.

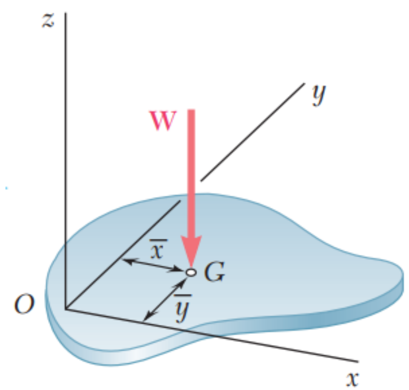
CENTRE OF GRAVITY

- Let us first consider a flat horizontal plate where we can divide the plate into n small elements.
- We denote the coordinates of the first element by x_1 and y_1, those of the second element by x_2 and y_2, etc.
- The forces exerted by the earth on the elements of the plate are denoted, respectively, by ΔW_1, ΔW_2, . . . , ΔW_n. These forces or weights are directed toward the center of the earth; however, for all practical purposes, we can assume them to be parallel.
- The magnitude W of this force is obtained by adding the magnitudes of the elemental weights.

$$W = \Delta W_1 + \Delta W_2 + \dots\dots\dots + \Delta W_n.$$







CENTRE OF GRAVITY

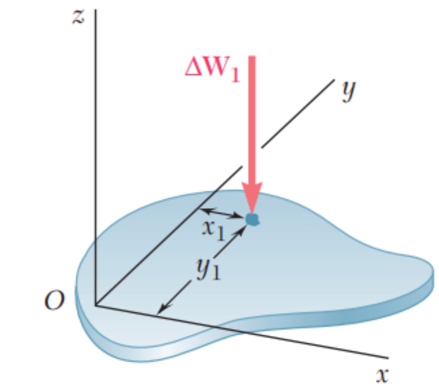
- To obtain the coordinates $\bar{x}$ and $\bar{y}$ of point G where the resultant W should be applied, we note that the moments of W about the y and x axes are equal to the sum of the corresponding moments of the elemental weights.

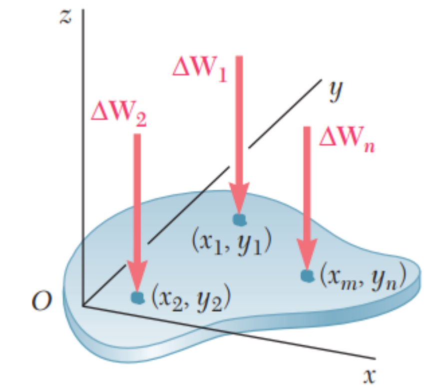
$$\sum M_y;\ \bar{x}W = x_1\Delta W_1 + x_2\Delta W_2 + \ldots\ldots\ldots + x_n\Delta W_n.$$

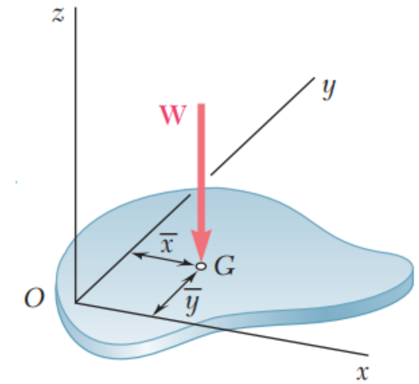
$$\sum M_x;\ \bar{y}W = y_1\Delta W_1 + y_2\Delta W_2 + \ldots\ldots\ldots + y_n\Delta W_n.$$

$$\bar{x} = \frac{x_1\Delta W_1 + x_2\Delta W_2 \ldots\ldots\ldots + x_n\Delta W_n}{W}$$

$$\bar{y} = \frac{y_1\Delta W_1 + y_2\Delta W_2 \ldots\ldots\ldots + y_n\Delta W_n}{W}$$







CENTROID OF HOMOGENEOUS BODY

- In case of, flat homogeneous plate of uniform thickness, we can express the magnitude of ΔW as:

$$\Delta W = \gamma\, t\, \Delta A$$

Where,

γ specific weight (weight per unit volume) of the material,

t thickness of the plate,

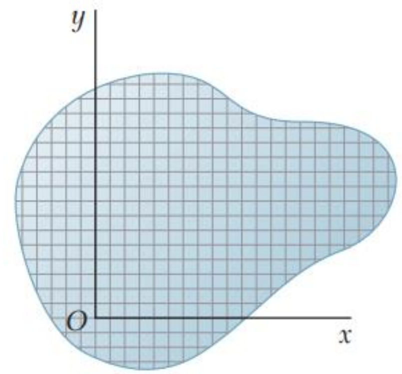
ΔA area of the element

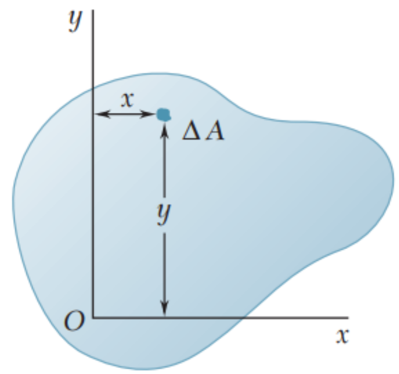
- Similarly, we can express the magnitude W of the weight of the entire plate as:

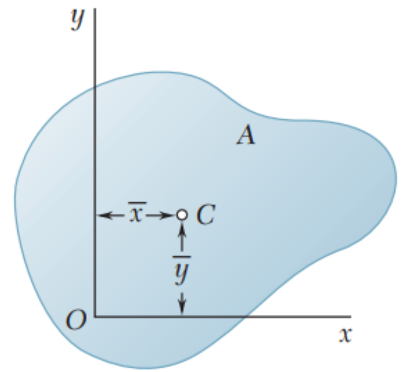
$$W = \gamma\, t\, A$$

where A is the total area of the plate.

- Now putting the value of W and ΔW in center of gravity equation, and dividing through out by γ t we get,







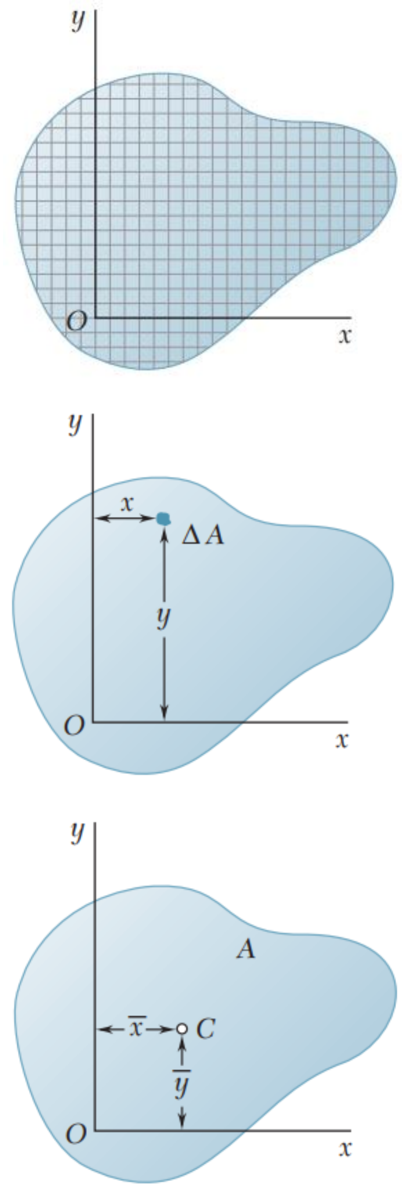
CENTROID OF HOMOGENEOUS BODY

$$\sum M_y; \ \bar{x}A = x_1\Delta A_1 + x_2\Delta A_2 + \dots\dots\dots + x_n\Delta A_n.$$

$$\sum M_x; \ \bar{y}A = y_1\Delta A_1 + y_2\Delta A_2 + \dots\dots\dots + y_n\Delta A_n.$$

$$\bar{x} = \frac{x_1\Delta A_1 + x_2\Delta A_2 \dots\dots\dots + x_n\Delta A_n}{A}$$

$$\bar{y} = \frac{y_1\Delta A_1 + y_2\Delta A_2 \dots\dots\dots + y_n\Delta A_n}{A}$$



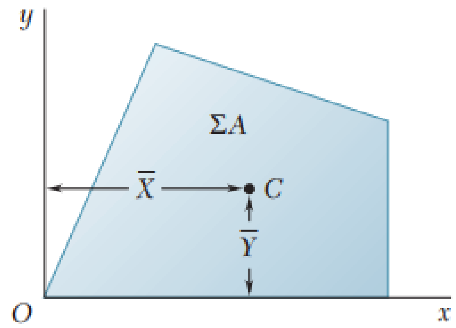
- The term $\bar{x}A$ and $\bar{y}A$ are also denoted by first moment of area about Y axis and X axis respectively and denoted by:

$$Q_y = \bar{x}A : Q_x = \bar{y}A$$

- The point whose coordinates are $\bar{x}$ and $\bar{y}$ is also known as the centroid C of the area A of the plate.
- If the plate is not homogeneous, you cannot use these equations to determine the center of gravity of the plate however they still define, the centroid of the area

CENTROID OF COMPOSITE AREA

- If the plate is homogeneous and of uniform thickness, the center of gravity coincides with the centroid C of its area.
- We can determine the abscissa $\overline{X}$ of the centroid of the area by noting that we can express the first moment Qy of the composite area with respect to the y axis as (1) the product of $\overline{X}$ and the total area and (2) as the sum of the first moments of the elementary areas with respect to the y axis.

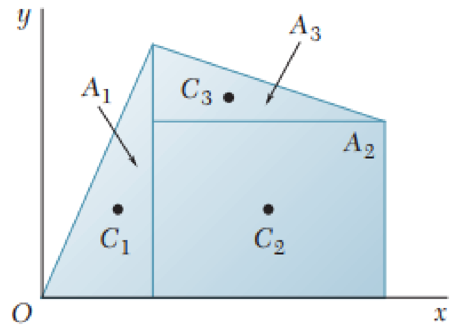


$$Q_y = \overline{X}(A1+A2+\ldots..+An) = \bar{x}_1 A_1 + \bar{x}_2 A_2 + \ldots\ldots.. + \bar{x}_n A_n = \sum \bar{x}A$$

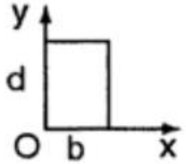
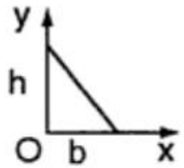
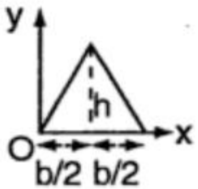
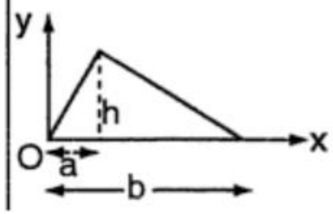
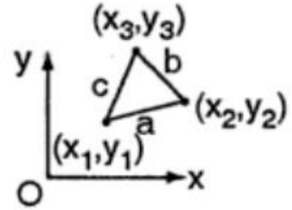
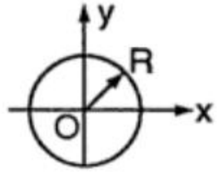
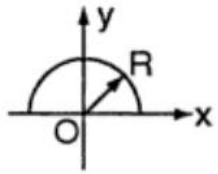
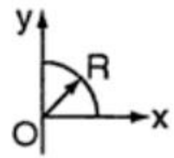
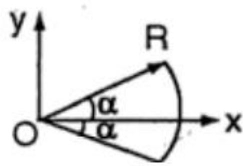
$$Q_x = \overline{Y}(A1+A2+\ldots..+An) = \bar{y}_1 A_1 + \bar{y}_2 A_2 + \ldots\ldots.. + \bar{y}_n A_n = \sum \bar{y}A$$

$$\overline{X} = \frac{\overline{x_1}A_1 + \overline{x_2}A_2 \ldots\ldots\ldots + \overline{x_n}A_n}{A} = \frac{\sum \bar{x}A}{\sum A}$$

$$\overline{Y} = \frac{\overline{y_1}A_1 + \overline{y_2}A_2 \ldots\ldots\ldots + \overline{y_n}A_n}{A} = \frac{\sum \bar{y}A}{\sum A}$$

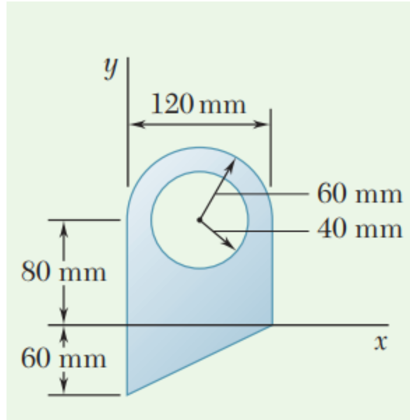


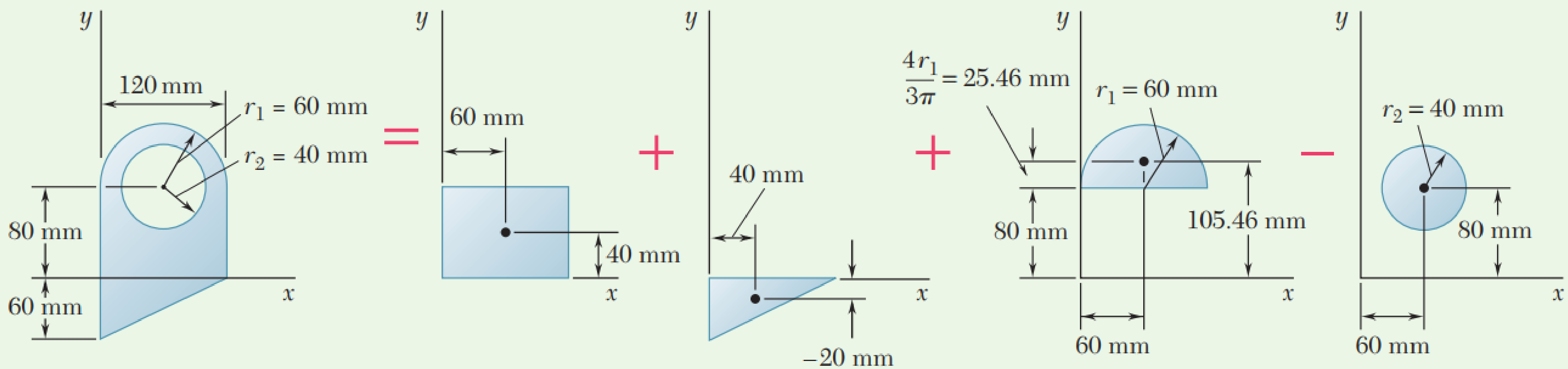
CENTROID OF COMPOSITE AREA

Shape	*Figure*	*Centroid* $G(\bar{x}, \bar{y})$
Rectangle		$\bar{x} = (b/2)$ $\bar{y} = (d/2)$
Triangle (right angled)		$\bar{x} = (b/3)$ $\bar{y} = (h/3)$
Triangle (isoceles)		$\bar{x} = (b/2)$ $\bar{y} = (h/3)$
Triangle (unsymmetric)		$\bar{x} = (a+b)/3$ $\bar{y} = (h/3)$
Triangle (General)		$\bar{x} = \dfrac{(x_1 + x_2 + x_3)}{3}$ $\bar{y} = \dfrac{(y_1 + y_2 + y_3)}{3}$
Circle		$\bar{x} = 0$ $\bar{y} = 0$
Semicircle		$\bar{x} = 0$ $\bar{y} = (4R/3\pi)$
Quadrant of a circle		$\bar{x} = (4R/3\pi)$ $\bar{y} = (4R/3\pi)$
Circular sector		$\bar{x} = (4R/3\pi)$ $\bar{y} = (4R/3\pi)$

SAMPLE PROBLEM

- For the plane area shown, determine (a) the first moments with respect to the x and y axes; (b) the location of the centroid.





Component	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
Rectangle	$(120)(80) = 9.6 \times 10^3$	60	40	$+576 \times 10^3$	$+384 \times 10^3$
Triangle	$\frac{1}{2}(120)(60) = 3.6 \times 10^3$	40	−20	$+144 \times 10^3$	-72×10^3
Semicircle	$\frac{1}{2}\pi(60)^2 = 5.655 \times 10^3$	60	105.46	$+339.3 \times 10^3$	$+596.4 \times 10^3$
Circle	$-\pi(40)^2 = -5.027 \times 10^3$	60	80	-301.6×10^3	-402.2×10^3
	$\Sigma A = 13.828 \times 10^3$			$\Sigma\bar{x}A = +757.7 \times 10^3$	$\Sigma\bar{y}A = +506.2 \times 10^3$

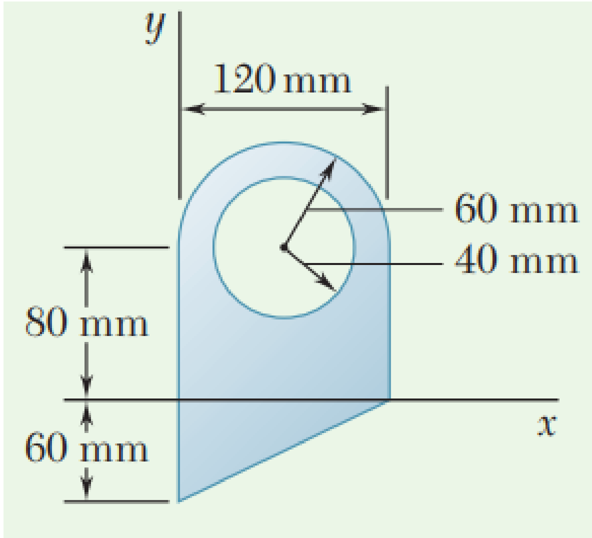
SAMPLE PROBLEM

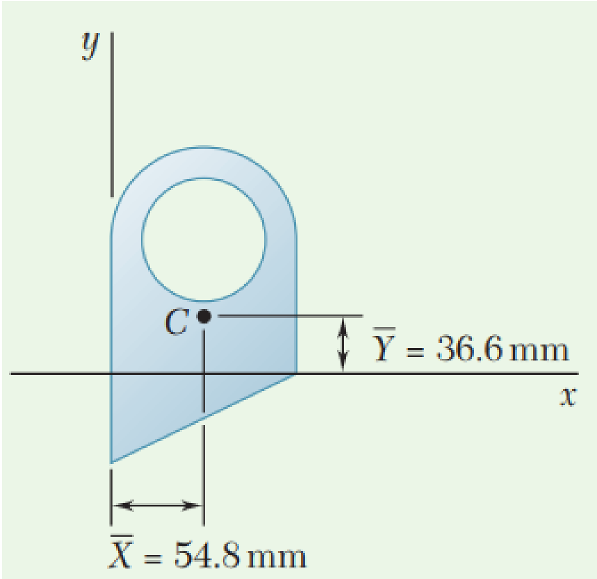
- The first moment of area is calculated as:

$$Q_x = \sum \bar{y}A = 506.2\times10^3\,mm^3$$
$$Q_y = \sum \bar{x}A = 757.7\times10^3\,mm^3$$

- And the location of centroid is calculated as:

$$\bar{X} = \frac{\sum \bar{x}A}{\sum A} = \frac{506.2\times10^3}{13.828\times10^3}\,mm$$
$$\Rightarrow \bar{X} = 54.8mm$$
$$\bar{Y} = \frac{\sum \bar{y}A}{\sum A} = \frac{757.7\times10^3}{13.828\times10^3}\,mm$$
$$\Rightarrow \bar{Y} = 36.6mm$$





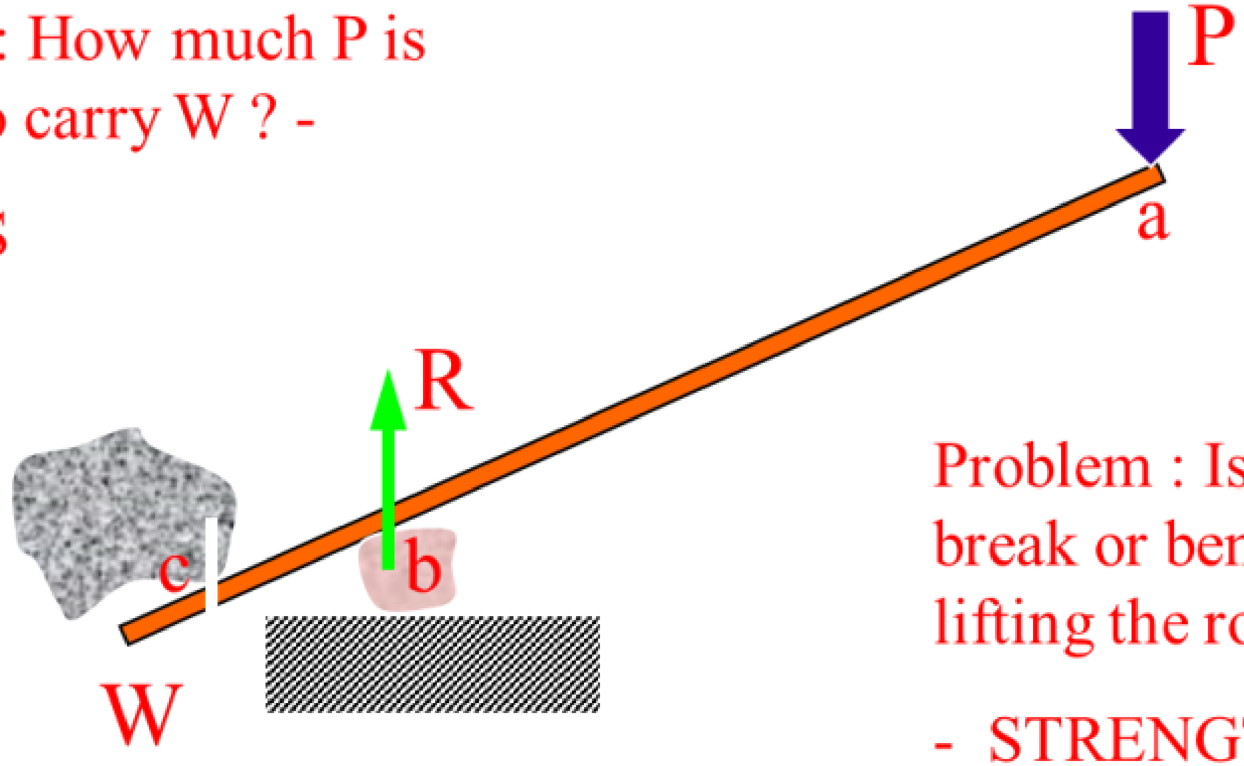
STRENGTH OF MATERIAL

- A field of applied mechanics which studies the behavior of solid bodies under the actions different types of loadings.
- To determine stress, strain and also deformation of structures
- Information about stress, strain and deformation are essential in the process of designing a structure which is safe and economical.
- Other names: mechanics of materials; mechanics of deformable bodies.
- Statics and Dynamics: to study forces acting on particles or rigid bodies and also the motion of particles and rigid bodies.
- Strength of Materials: to study stress and strain in a body which is deformable (no longer rigid) under the action of external forces

STRENGTH OF MATERIALS and STATICS & DYNAMICS

Problem : How much P is needed to carry W ? -

STATICS



Problem : Is the crowbar going to break or bend excessively when lifting the rock ?

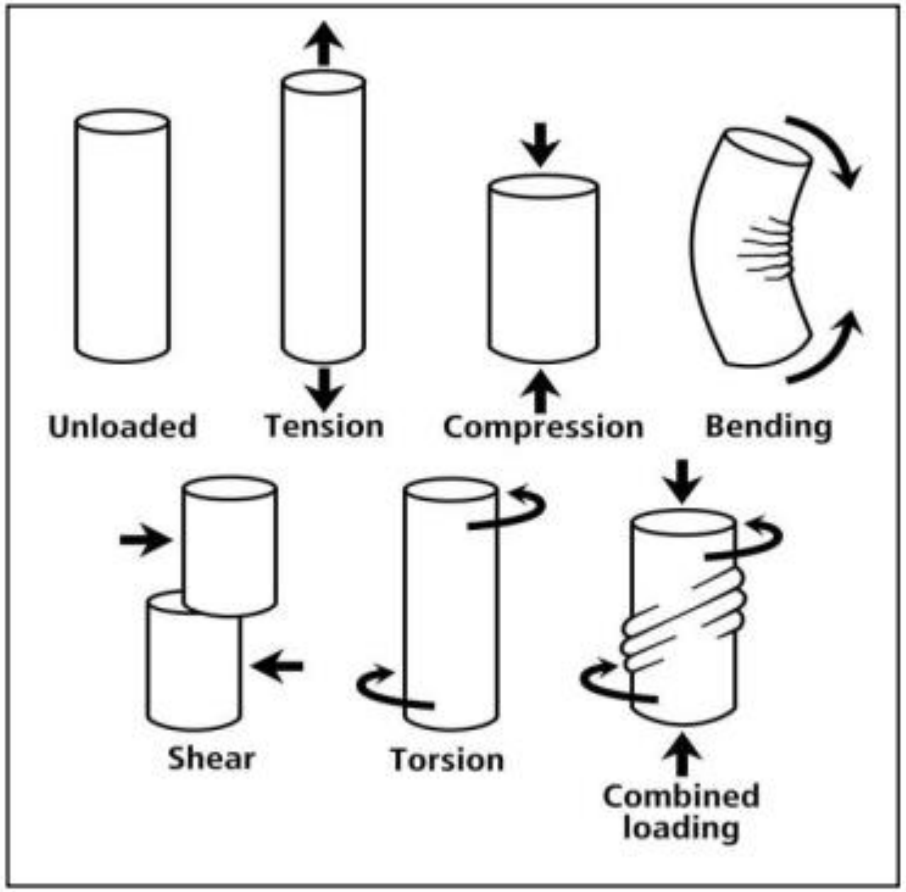
- STRENGTH of MATERIALS

Example : Lifting of a rock with a crowbar

CONCEPTS AND DEFINATION

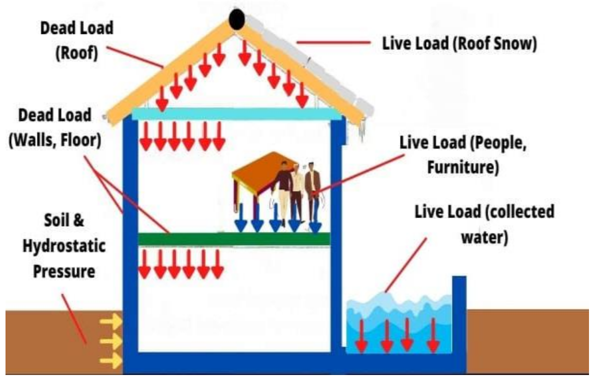
- A load is defined as the combined effect of external forces acting on a body. The load can be a ‘point’ (or concentrated) or ‘distributed’
- The loads may be classified as:
  1. Dead loads
  2. Live or fluctuating loads
  3. Inertial loads or forces
  4. Centrifugal loads or forces
  5. Environmental Loads

- The other way of classification is
  1. Tensile loads
  2. Compressive loads
  3. Torsional loads
  4. Bending loads and
  5. Shearing loads
  6. Combined load

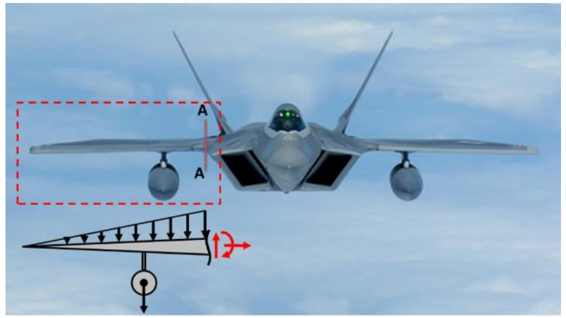


CONCEPTS AND DEFINATION

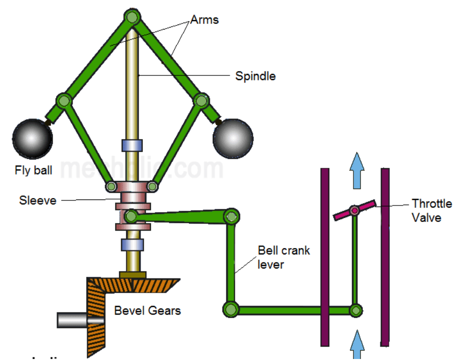
1. Dead loads:
   - Loads that are relatively constant over time. e.g. self weight of structure, immovable fixtures such as walls, Roofs
   - Dead loads are also known as permanent loads.
2. Live or fluctuating loads:
   - Live loads or imposed loads are temporary of short duration E.g. Vibration, impact
3. Inertial loads:
   - Inertia load is induced as the resistance of any physical object to any change in its velocity. This includes changes to the object's speed, or direction of motion
4. Centrifugal loads:
   - The force that is felt by an object moving in a curved path that acts outwardly away from the center of rotation.



Combined loads



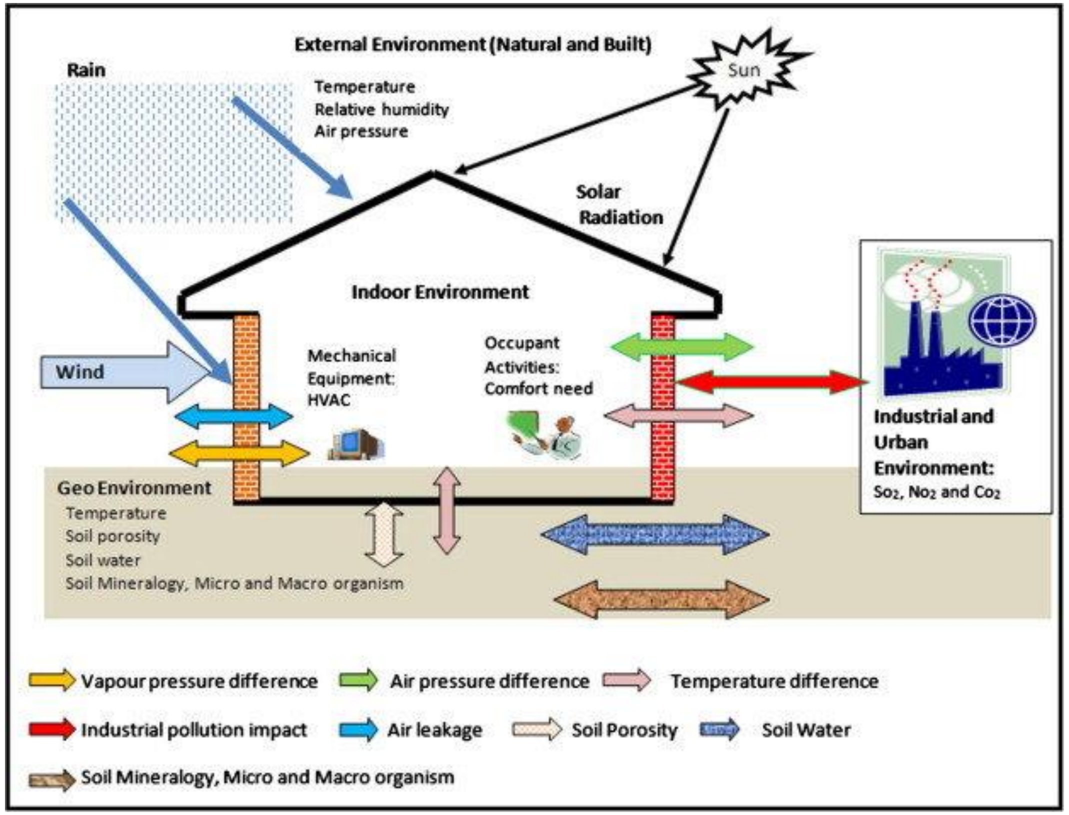
Inertial loads



Centrifugal loads

5. Environmental Loads

- These loads act as a result of weather topography and other natural phenomenon.
- Wind load, snow load, rain, Dust load.
- Temperature changes leading to thermal expansion causes thermal loads.

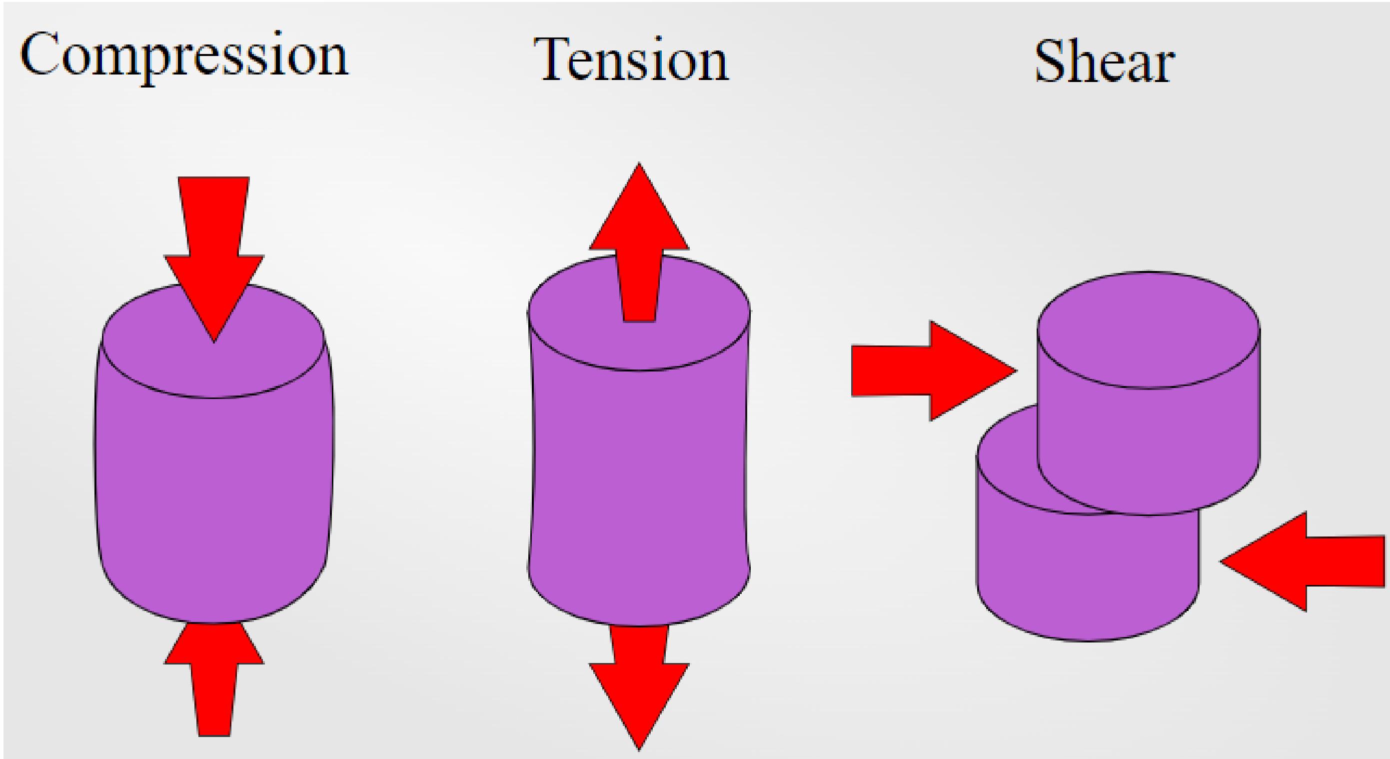


CONCEPTS AND DEFINATION

Stress ($\boldsymbol{\sigma}$)

- The internal resistance which the body offers to meet with the loads is called stress.
- Units: N, kN or MN

The various types of stress are:



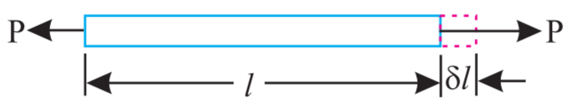
1. Simple or direct stress
   i. Tension
   ii. Compression
   iii. Shear
2. Indirect stress
   i. Bending
   ii. Torsion
3. Combined stress
   - Any possible combination of 1 and 2.

CONCEPTS AND DEFINATION

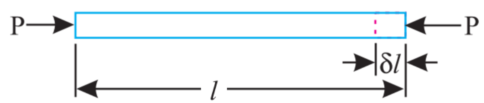
Strain (e or ϵ)

- Any element in a material subjected to stress is said to be strained. The strain is the deformation produced by stress.
- The various types of strains are explained below:

1. Tensile strain:
   - Subjected to a uniform axial tensile stress will increase its length from l to l+ δl
   - where δl is the actual deformation of the material.
   - Fractional deformation or tensile strain is given by $e_t = \delta l/l$



2. Compressive strain:
   - Under compressive forces, a similar piece of material would be reduced in length from l to $l - \delta l$
   - Fractional deformation or tensile strain is given by $e_c = \delta l/l$



CONCEPTS AND DEFINATION

3. Shear Strain ($\boldsymbol{e\ or\ \epsilon}$)
   - In case of a shearing load, a shear strain will be produced which is measured by the angle through which the body distorts.
   - The shear strain (e_s) is given by:

$$e_s = \frac{NN'}{NP} = \tan\phi$$

4. Volumetric Strain
   - The ratio between change in volume and original volume of the body, and is denoted by e_v.

$$e_v = \frac{Change\ in\ Volume}{Original\ Volume} = \frac{\delta V}{V}$$

CONCEPTS AND DEFINATION

- The strain which disappears with the removal of loads are termed as elastic strains and the body which regains its original position on the removal of force is called **elastic body.**

- The body is said to be **plastic** if the strains exist even after the removal of external force.

- **Elastic limit:** There is always a limiting value of load up to which the strain totally disappears on the removal of load, the corresponding to this load is called elastic limit.

CONCEPTS AND DEFINATION

Hooke's Law

- Robert Hooke discovered experimentally that within elastic limit, stress varies directly as strain.

i.e, Stress α Strain

Or, $\frac{Stress}{Strain} = Const. = E$

- This constant E is termed as Modulus of Elasticity
- An elastic modulus or modulus of elasticity is a quantity that measures an object or substance's resistance to being deformed elastically (i.e., non-permanently) when a stress is applied to it.
- The elastic modulus of an object is defined as the slope of its stress–strain curve in the elastic deformation region.
- A stiffer material will have a higher elastic modulus.

$$Stress \alpha Strain$$

$$\sigma \alpha \varepsilon$$

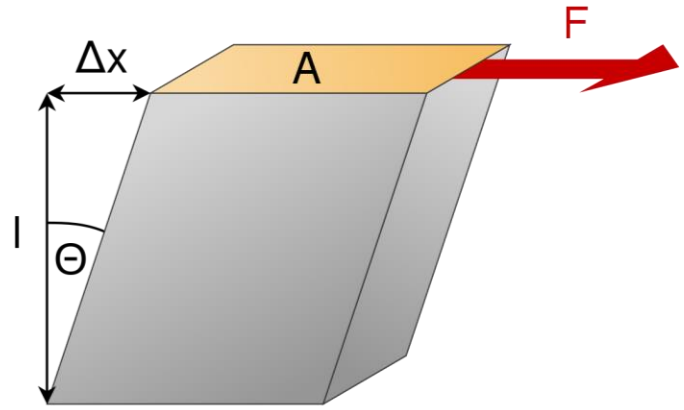
$$\sigma = E\varepsilon$$

$$E = \frac{\sigma}{\varepsilon}$$

CONCEPTS AND DEFINATION

Shear Modulus

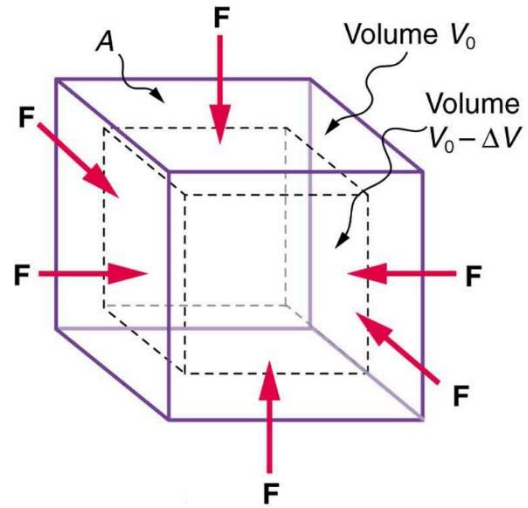
- The shear modulus or modulus of rigidity, denoted by G, or sometimes S or μ, is defined as the ratio of shear stress to the shear strain:



$$G = \frac{\tau_{xy}}{\gamma_{xy}} = \frac{F/A}{\Delta x/l} = \frac{Fl}{A\Delta x}$$

Bulk Modulus:

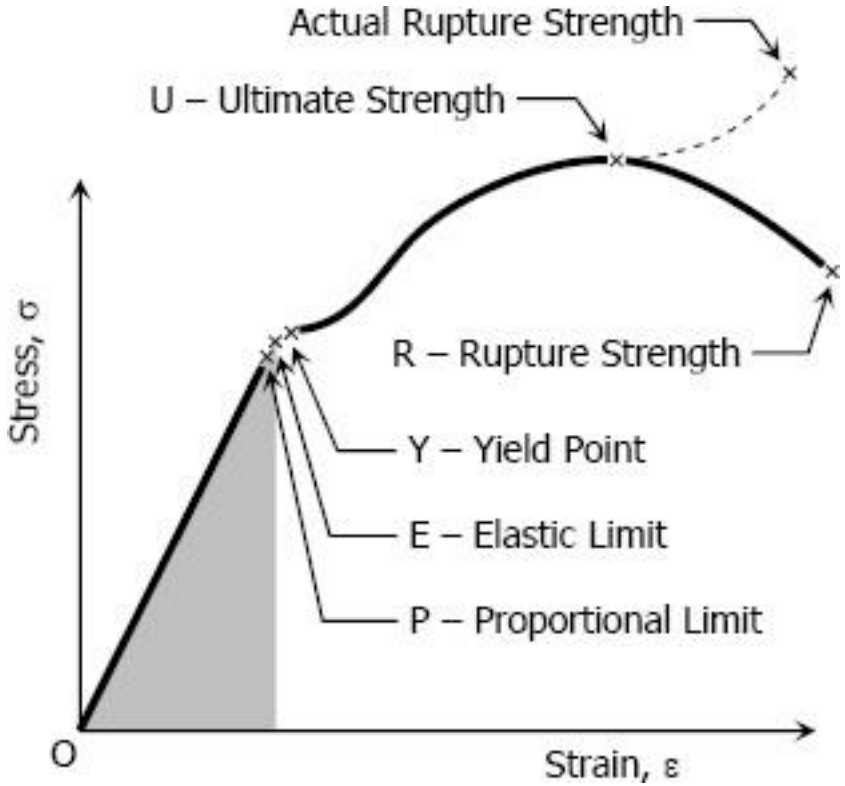
- The bulk or volume modulus (K or B) of a substance is a measure of how resistant to compression that substance is.
- It is defined as the ratio of the pressure increase to the resulting relative decrease of the volume



- Bulk Modulus K = $\frac{Bulk\ Stress}{Bulk\ Strain}$ $\quad K = \frac{(Force/Area)}{(\Delta V/V)} = -\frac{\Delta p}{\Delta V/V}$

STRESS STRAIN CURVE

- The relationship between the stress and strain that a particular material displays is known as the material's stress–strain curve.
- It is unique for each material and is found by recording the amount of deformation (strain) at distinct intervals of tensile or compressive loading (stress).
- These curves reveal many of the properties of a material (including data to establish the Modulus of Elasticity, E).
- Strains are plotted on the horizontal axis and stresses on the vertical axis.
- The diagram begins with a straight line from O to P. In this region, the stress and strain are directly proportional, and the behavior of the material is said to be linear.



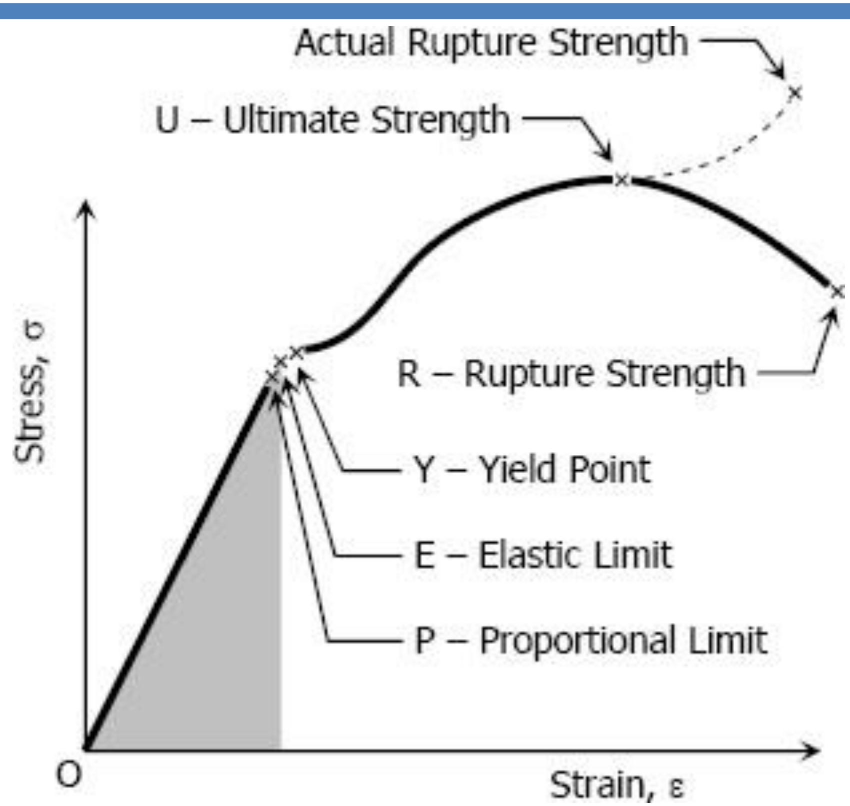
Typical Stress-Strain Diagram

STRESS STRAIN CURVE

- Beyond point P, the linear relationship between stress and strain no longer exists; hence, the stress at P is called the proportional limit.

- With an increase in the load beyond the proportional limit, the strain begins to increase more rapidly for each increment in stress. The stress-strain curve then has a smaller and smaller slope, until, at point E, the curve becomes almost horizontal.

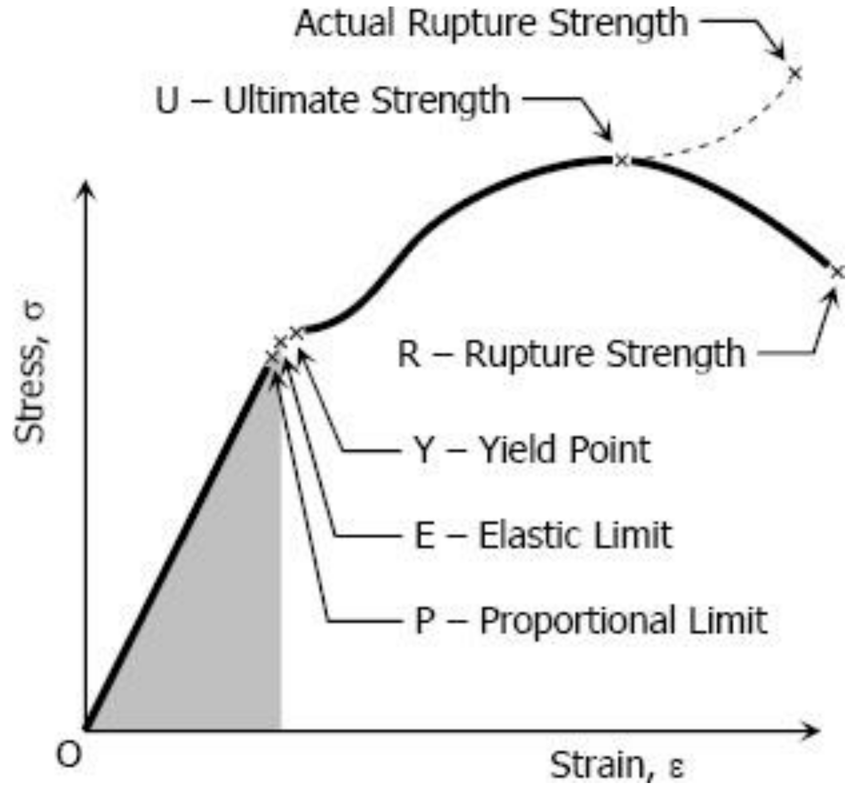
- Beginning at this point, considerable elongation occurs, with no noticeable increase in the tensile force (from E to Y on the diagram). This phenomenon is known as yielding of the material, and the stress at point Y is called the yield stress, or yield point.



Typical Stress-Strain Diagram

STRESS STRAIN CURVE

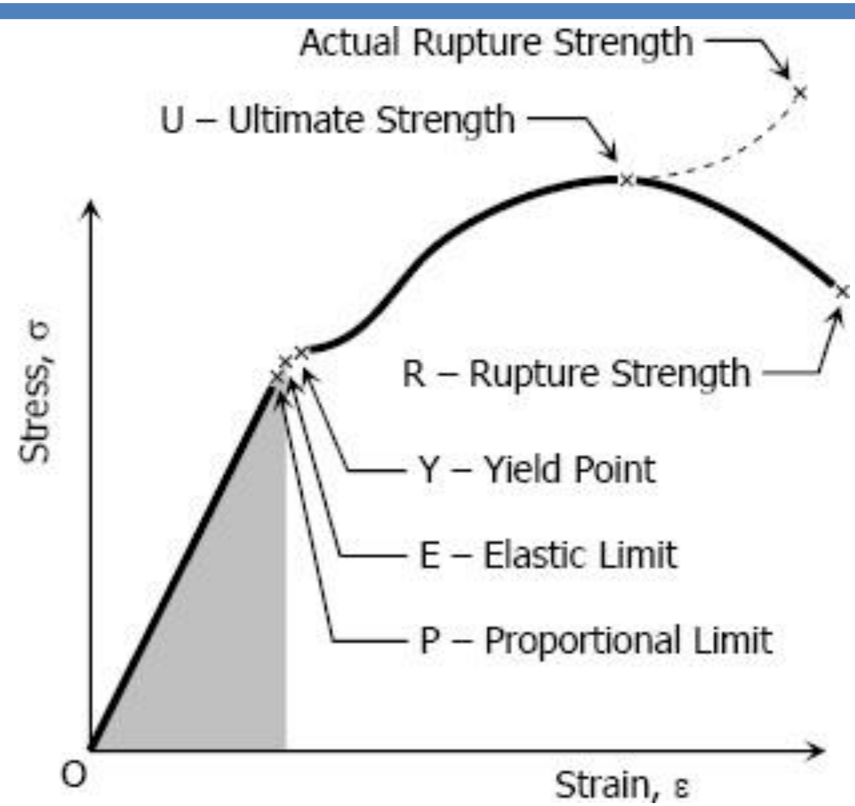
- In the region from E to Y, the material becomes perfectly plastic, which means that it can deform without an increase in the applied load.
- The elongation of a mild-steel specimen in the perfectly plastic region is typically 10 to 15 times the elongation that occurs between the onset of loading and the proportional limit.
- After the large strains that occur during yielding in the region EY, the steel begins to strain harden.
- During strain hardening, the material undergoes changes in its atomic and crystalline structure, resulting in increased resistance of the material to further deformation.



Typical Stress-Strain Diagram

STRESS STRAIN CURVE

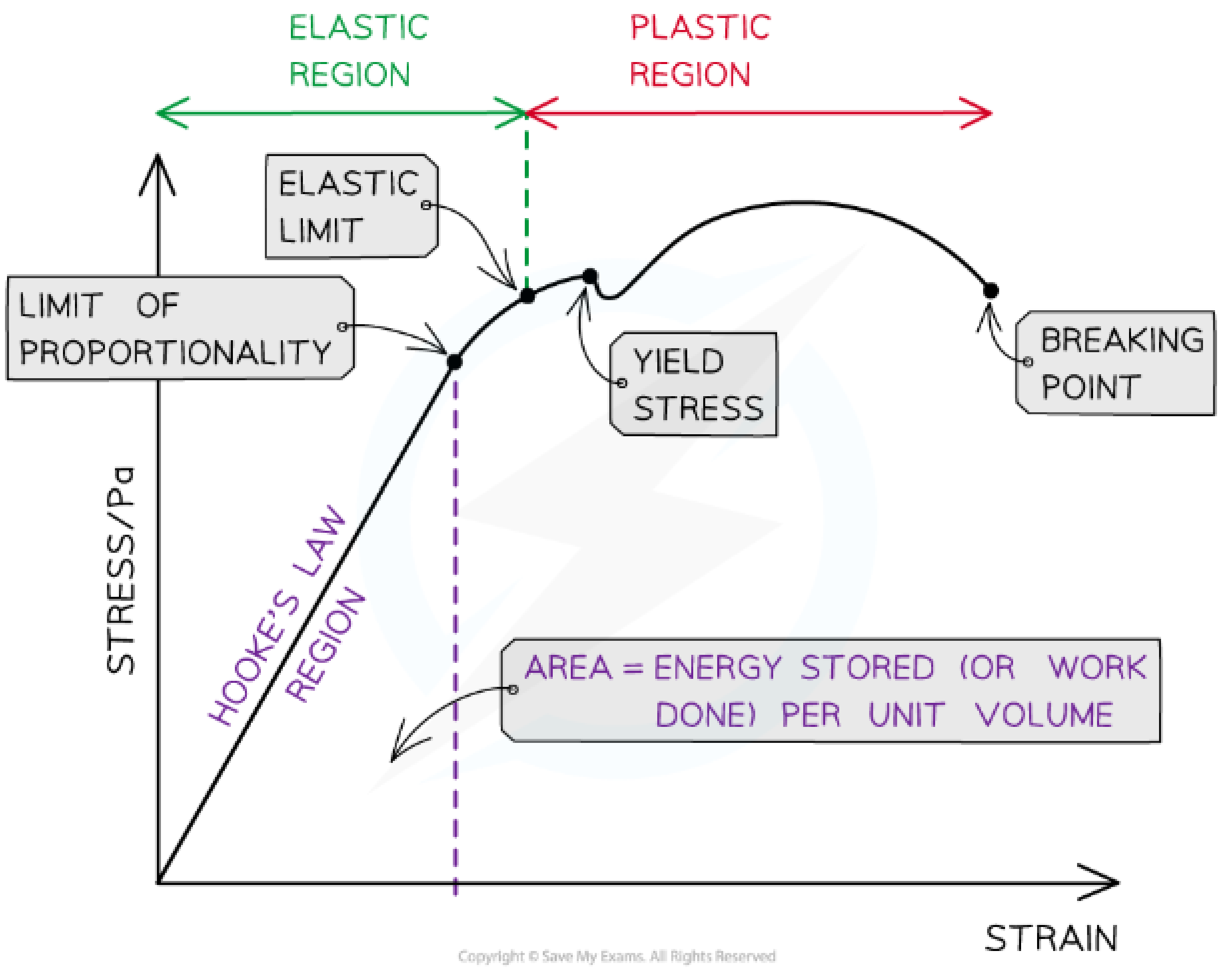
- Thus, additional elongation requires an increase in the tensile load, and the stress-strain diagram has a positive slope from Y to U The load eventually reaches its maximum value, and the corresponding stress at point U is called the ultimate stress.
- Further stretching of the bar is actually accompanied by a reduction in the load, and rupture finally occurs at a point such as R on the diagram.
- For a brittle material (like glass) there the rupture occurs as soon as the proportional limit P is crossed.
- Since the brittle material cannot undergo the plastic deformation there is no chance for any elongation above the proportional limit. However the slope of stress–strain curve (i.e. Young's modulus of elasticity) for the brittle material is very high.



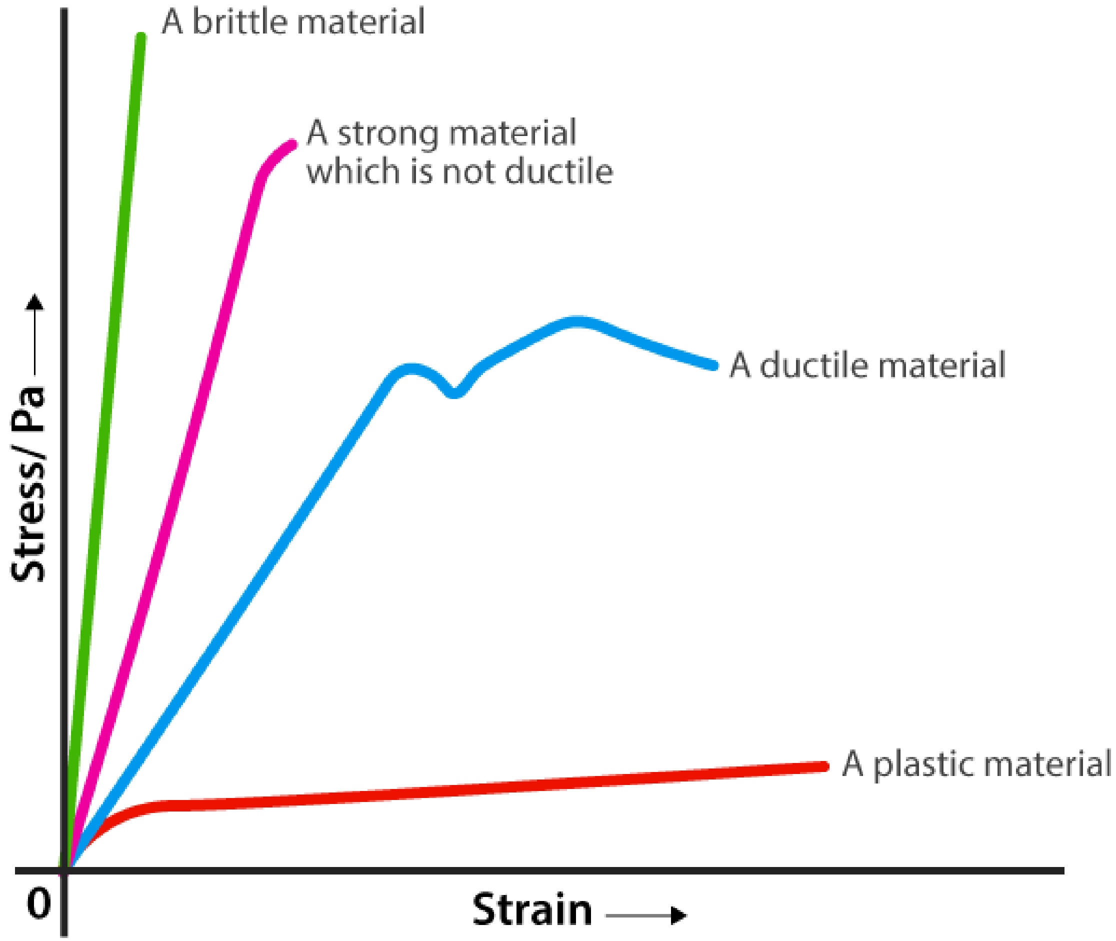
Typical Stress-Strain Diagram

STRESS STRAIN CURVE

- The stress-strain curve of various material is as shown.

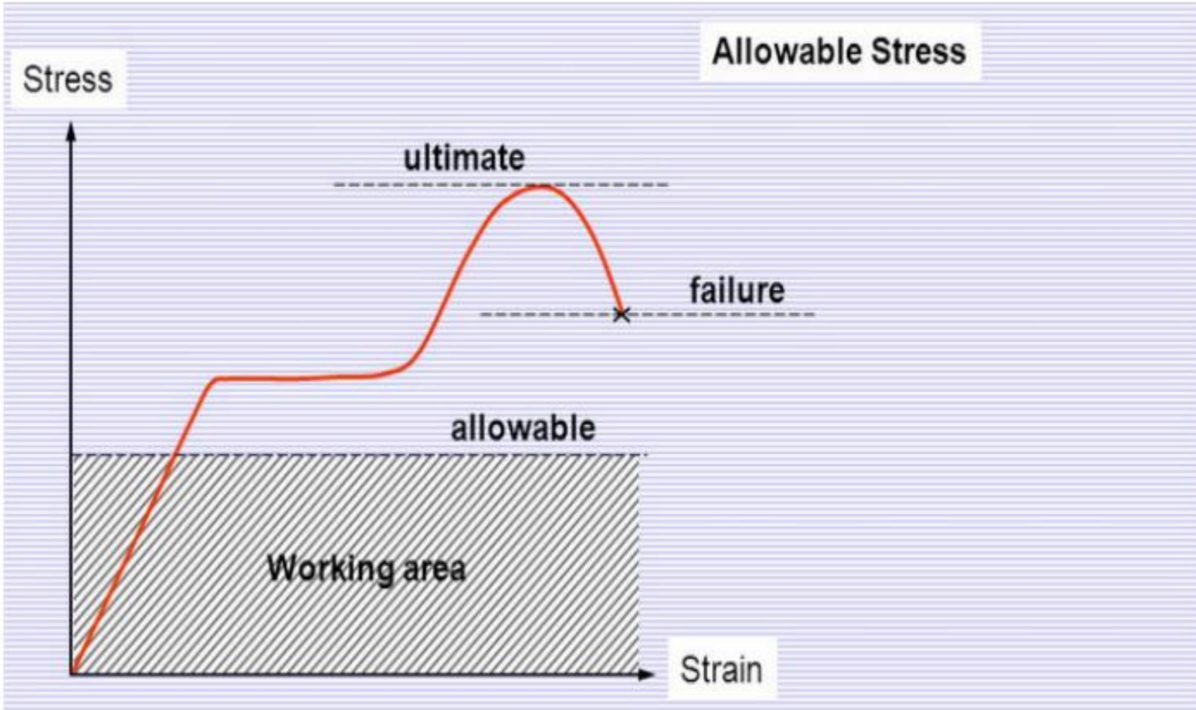


Typical Stress-Strain Diagram



FACTOR OF SAFETY

- Factor of safety (F.O.S.) is ability of a system's structural capacity to be viable beyond its expected or actual loads.
- F.O.S. may be expressed as a ratio that compares absolute strength to actual applied load.
- Design and engineering standards usually specify the allowable stress, or ultimate strength of a given material divided by the factor of safety.



Working Stress Diagram

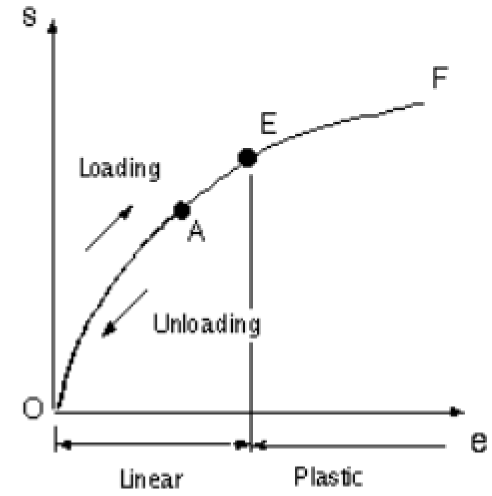
$$Factor\ of\ safety = F.O.S. = \frac{Ultimate\ Stress}{Allowable\ Stress}$$

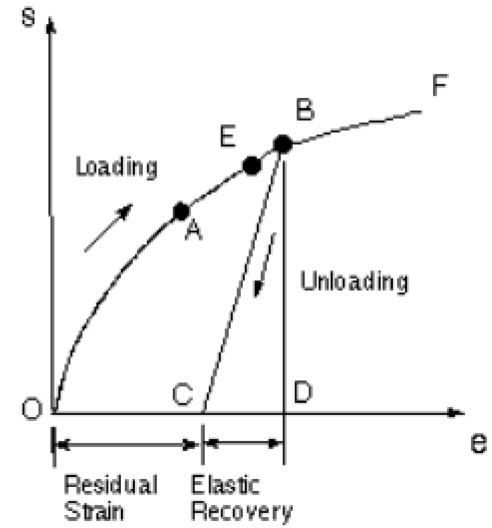
$$F.O.S. = \frac{\sigma_{ult}}{\sigma_{all}}$$

$$F.O.S. = \frac{\tau_{ult}}{\tau_{all}}$$

ELASTICITY VS PLASTICITY

- The characteristic of a material by which it undergoes inelastic strains beyond those at the elastic limit is known as plasticity.
- Thus, on the stress-strain curve in Figure, we have an elastic region followed by a plastic region.
- When large deformations occur in a ductile material loaded into the plastic region, the material is said to undergo plastic flow.
- If the material remains within the elastic range, it can be loaded, unloaded, and loaded again without significantly changing the behavior
- However, when loaded into the plastic range, the internal structure of the material is altered and its properties change
- For instance, we have already observed that a permanent strain exists in the specimen after unloading from the plastic region.

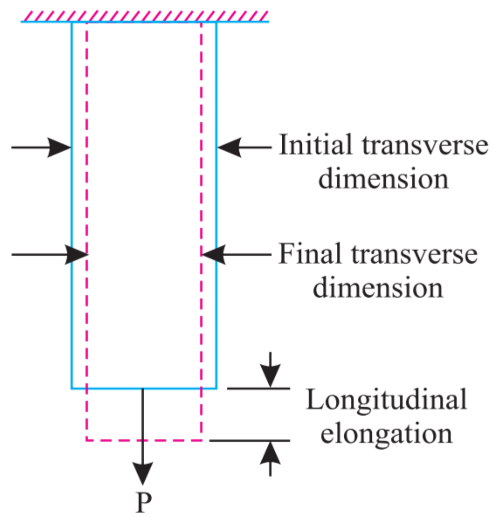




POISSON'S RATIO

- If a body is subjected to a load, its length changes; ratio of this change in length to the original length is known as linear or primary strain.
- Due to this load, the dimensions of the body change; in all directions at right angles to its line of application the strains thus produced are called lateral or secondary or transverse strains and are of nature opposite to that of primary strains.
- For example, if the load is tensile, there will be an increase in length and a corresponding decrease in cross-sectional area of the body. In this case,e, linear or primary strain will be tensile and secondary or lateral or transverse strain compressive
- The ratio of lateral strain to linear strain is known as Poisson's ratio.
- i.e., Poisson's ratio,

$$\mu = \frac{Lateral\ strain\ or\ transverse\ strain}{Linear\ or\ primary\ strain} = \frac{1}{m}$$



SAMPLE PROBLEM 1

- A square steel rod 20mm×20mm in section is to carry an axial load (compressive) of 100kN. Calculate the shortening in a length of 50 mm. $E = 2.14 \times 10^8$ kN/m^2.

Solution.

Area, $A = 0.02 \times 0.02 = 0.0004 \text{ m}^2$

Length, $l = 50 \text{ mm or } 0.05 \text{ m}$

Load, $P = 100 \text{ kN}$

$E = 2.14 \times 10^8 \text{ kN/m}^2$

Shortening of the rod δl:

Stress, $\sigma = \dfrac{P}{A}$

$\therefore$ $\sigma = \dfrac{100}{0.0004} = 250000 \text{ kN/m}^2$

Also, $E = \dfrac{\text{Stress}}{\text{Strain}}$

or, $\text{Strain} = \dfrac{\text{Stress}}{E} = \dfrac{250000}{2.14 \times 10^8}$

or, $\dfrac{\delta l}{l} = \dfrac{250000}{2.14 \times 10^8}$

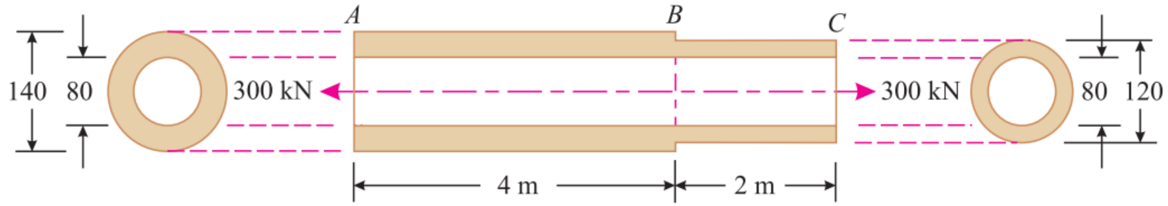
$\therefore$ $\delta l = \dfrac{250000}{2.14 \times 10^8} \times l = \dfrac{250000}{2.14 \times 10^8} \times 0.05$

$= 0.0000584 \text{ m or } 0.0584 \text{ mm}$

Hence, the shortening of the rod **= 0.0584 mm (Ans.)**

SAMPLE PROBLEM 2

- A 6m long hollow bar of circular section has 140 mm diameter for a length of 4m, while it has 120 mm diameter for a length of 2m. The bore diameter is 80 mm throughout as shown. Find the elongation of the bar, when it is subjected to an axial tensile force of 300 kN. Take modulus of elasticity for the bar material as 200 GPA.



SOLUTION. Given : Total length $(L) = 6\text{ m} = 6 \times 10^3$ mm ; Diameter of section 1 (D_1)= 140 mm; Length of section 1 $(l_1) = 4\text{ m} = 4 \times 10^3$ mm ; Diameter of section 2 $(D_2) = 120$ mm ; Length of section 2 $(l_2) = 2\text{ m} = 2 \times 10^3$ mm ; Inner diameter $(d_1) = d_2 = 80$ mm ; Axial tensile force $(P) = 300$ kN = 300 $\times$ 10 N and modulus of elasticity $(E) = 200$ GPa $= 200 \times 10^3$ N/mm^2.

We know that area of portion *AB*,

$$A_1 = \frac{\pi}{4} \times [D_1^2 - d_1^2] = \frac{\pi}{4} \times [(140)^2 - (80)^2] = 3300\,\pi\text{ mm}^2$$

and area of portion *BC*.

$$A_2 = \frac{\pi}{4} \times [D_2^2 - d_2^2] = \frac{\pi}{4} \times [(120)^2 - (80)^2] = 2000\,\pi\text{ mm}^2$$

$\therefore$ Elongation of the bar,

$$\delta l = \frac{P}{E}\left[\frac{l_1}{A_1} + \frac{l_2}{A_2}\right] = \frac{300 \times 10^3}{200 \times 10^3} \times \left[\frac{4 \times 10^3}{3300\,\pi} + \frac{2 \times 10^3}{2000\,\pi}\right]\text{mm}$$

$$= 1.5 \times (0.385 + 0.318) = 1.054\text{ mm} \qquad \textbf{Ans.}$$