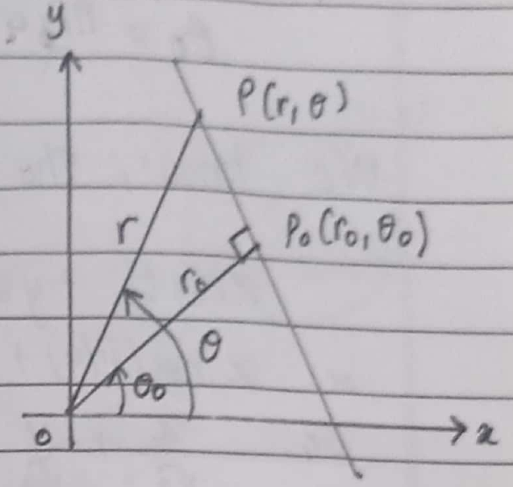
Straight Line In Polar Form



If the point $P_0(r_0, \theta_0)$ is the foot of the perpendicular from the origin to the line L and $r_0 \geq 0$, then

Let $P(r, \theta)$ be a point on the line.

In ΔPOP_0,

$$\cos(\theta - \theta_0) = \frac{r_0}{r}$$

So,

$$r\cos(\theta - \theta_0) = r_0 \quad \text{—(i)}$$

This the standard polar equation for lines.

Expanding,

$$x \cos\theta_0 + y \sin\theta_0 = r_0 \quad \text{—(ii)}$$

Eqn (ii) gives normal form of a straight line.

<Q7! Rewrite the equation in cartesion form.

$\theta_0 = \pi/4 \qquad r_0 = \sqrt{2}$.

Soln:

We know, the eqn of st. line in polar form is.

$$x\cos\theta_0 + y\sin\theta_0 = r_0$$

or, $x\cos(\pi/4) + y\sin(\pi/4) = \sqrt{2}$

or, $\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = \sqrt{2}$

or $x + y = 2$

⊗ If asked for three marks,

~~ff~~ $r\cos(\theta-\theta_0) = r_0$

or, $r\cos\left(\theta - \frac{\pi}{4}\right) = \sqrt{2}$

or, $r\left[\cos\theta . \cos\frac{\pi}{4} + \sin\theta\sin\frac{\pi}{4}\right] = \sqrt{2}$

or, $r\cos\theta \times \frac{1}{\sqrt{2}} + \frac{r\sin\theta}{\sqrt{2}} = \sqrt{2}$

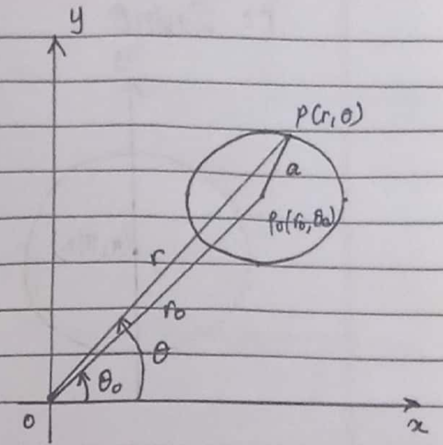
or, $r\cos\theta + r\sin\theta = 2$

We know,

$x = r\cos\theta$ and $y = r\sin\theta$

$\therefore x + y = 2$. is the reqd eqn.

Circle:



Let $P_0(r_0, \theta_0)$ be the centre of the circle and $P(r,\theta)$ be any point on its circumference.

If a is its radius,

Using law of cosines on ΔPOP_0,

$$\cos(\theta-\theta_0) = \frac{r^2 + r_0^2 - a^2}{2rr_0}$$

or, $a^2 = r_0^2 + r^2 - 2r_0 r\cos(\theta-\theta_0)$ —(i)

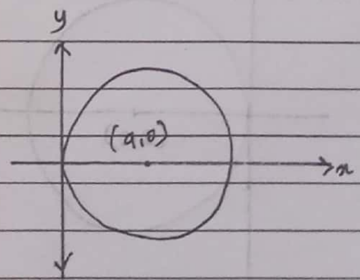
*) Special case.

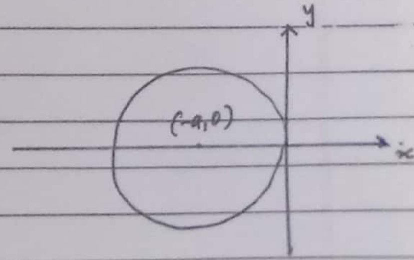
i) If circle passes through origin/pole then, $r_0 = a$ and eqn (i) becomes.

$$r = 2a\cos(\theta-\theta_0).$$

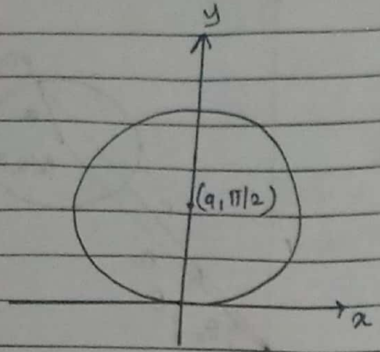
ii) If circle is on a axis and passing through origin.

$r = 2a\cos\theta$ $\qquad\qquad$ $r = -2a\cos\theta$

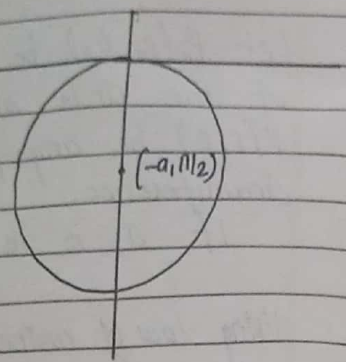




$r = 2a\sin\theta$



$r = -2a\sin\theta$



<Q7: Sketch the circles, give polar coordinates for their centers and find the radii.

(i): $r = 4\cos\theta$.

Sol^n:

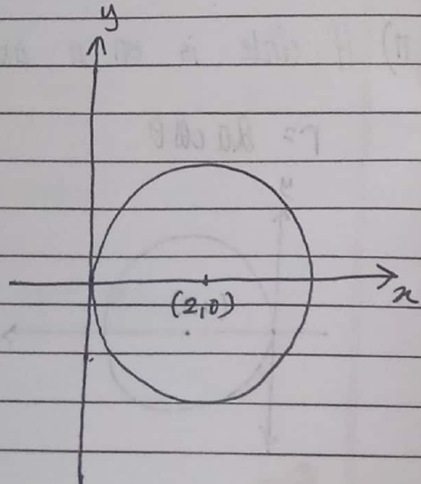
Given,

$r = 2\cdot 2\cdot\cos(\theta - 0)$

Comparing with $r = 2a\cos(\theta - \theta_0)$

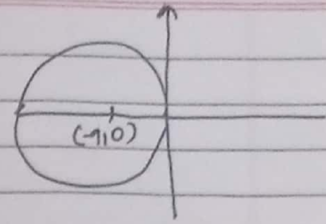
$a = 2$ $\quad\quad \theta_0 = 0$

$\therefore$ centre $(a, \theta_0) = (2, 0)$



(ii) $r = -2\,\cancel{\sin}\cos\theta$.

Sol^n,



Given,

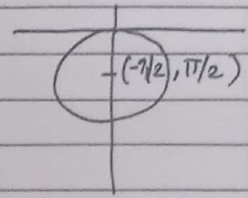
$r = -2\cdot 1\cdot\cos(\theta - 0)$

Comparing with $r = 2a\cos(\theta - \theta_0)$

$\therefore a = 1 \quad\quad \theta_0 = 0$ in negative x-axis.

(iii): $r = -\sin\theta$.

Sol^n:



Given,

$r = 2\cdot\left(\frac{-1}{2}\right)\cdot\sin(\theta - 0)$

Here, $a = -1/2$ in negative y-axis.