KATHMANDU UNIVERSITY

DHULIKHEL, KAVRE

Subject: PHY102

Assignment: 1

Submitted by:

Ashraya Kadel

CE I/II

Roll no: 25

Submitted to:

Ganesh Kumar Chhetri

Department of Physics.

Submission date: / /

<Q.1> Find the gradient of the following functions

a) $f(x,y,z) = x^2+y^2+z^2$

Soln.

Given,

$$f = x^2+y^2+z^2$$

We know,

$$\nabla f = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}$$

$$= \frac{\partial(x^2+y^2+z^2)}{\partial x}\hat{i} + \frac{\partial(x^2+y^2+z^2)}{\partial y}\hat{j} + \frac{\partial(x^2+y^2+z^2)}{\partial z}\hat{k}$$

$$= 2x\hat{i} + 2y\hat{j} + 2z\hat{k}$$

$$\therefore \nabla f = 2(x\hat{i} + y\hat{j} + z\hat{k})$$

b) $f(x,y,z) = e^x \ln y \sin z$

Soln.

Given,

$$f = e^x \ln y \sin z$$

We know,

$$\nabla f = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}$$

$$= \frac{\partial(e^x \ln y \sin z)}{\partial x}\hat{i} + \frac{\partial(e^x \ln y \sin z)}{\partial y}\hat{j} + \frac{\partial(e^x \ln y \sin z)}{\partial z}$$

$$\therefore \nabla f = e^x \ln y \sin z\,\hat{i} + \frac{e^x}{y}\sin z\,\hat{j} + e^x \ln y \cos z\,\hat{k}$$

<Q.2>: If $\phi(x,y,z) = 3x^2y - y^3z^2$, find $\nabla\phi$ at $(1,-2,-1)$

Soln:

Given,

$$\phi = 3x^2y - y^3z^2$$

We know,

$$\nabla\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$$

$$= \frac{\partial(3x^2y - y^3z^2)}{\partial x}\hat{i} + \frac{\partial(3x^2y - y^3z^2)}{\partial y}\hat{j} + \frac{\partial(3x^2y - y^3z^2)}{\partial z}\hat{k}$$

$$= 6xy\,\hat{i} + (3x^2 - 3y^2z^2)\hat{j} + (-2y^3z)\hat{k}$$

$$= 6xy\,\hat{i} + (3x^2 - 3y^2z^2)\hat{j} - 2y^3z\,\hat{k}$$

At $(1,-2,-1)$.

$$\nabla\phi = -12\hat{i} - 9\hat{j} - 16\hat{k}$$

<Q.3>: Let $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ be the position vector and let r be its length. Show that

a) $\nabla\left(\frac{1}{r}\right) = \frac{\hat{r}}{r^2} = \frac{\vec{r}}{r^3}$

b) $\nabla r^n = nr^{n-2}\vec{r} = nr^{n-1}\hat{r}$

Soln.

Given,

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

So,

$$r = \sqrt{x^2+y^2+z^2}$$

or, $r^2 = x^2+y^2+z^2$

Now,

$$\frac{\partial r^2}{\partial x} = \frac{\partial(x^2+y^2+z^2)}{\partial x} \quad \text{or, } \cancel{2r = 2x}\ 2r\cdot\frac{\partial r}{\partial x} = 2x$$

$$\therefore \frac{\partial r}{\partial x} = \frac{x}{r}$$

$$\frac{\partial r^2}{\partial y} = \frac{\partial(x^2+y^2+z^2)}{\partial y} \quad \text{or, } 2r\cdot\frac{\partial r}{\partial y} = 2y \quad \therefore \frac{\partial r}{\partial y} = \frac{y}{r}$$

$$\frac{\partial r^2}{\partial z} = \frac{\partial(x^2+y^2+z^2)}{\partial z} \quad \text{or, } 2r\cdot\frac{\partial r}{\partial z} = 2z \quad \therefore \frac{\partial r}{\partial z} = \frac{z}{r}$$

So,

$$\nabla\left(\frac{1}{r}\right) = \hat{i}\frac{\partial r^{-1}}{\partial x} + \hat{j}\frac{\partial r^{-1}}{\partial y} + \hat{k}\frac{\partial r^{-1}}{\partial z}$$

$$= \hat{i}\left[-1\cdot r^{-2}\cdot\frac{\partial r}{\partial x}\right] + \hat{j}\left[-1\cdot r^{-2}\cdot\frac{\partial r}{\partial y}\right] + \hat{k}\left[-1\cdot r^{-2}\frac{\partial r}{\partial z}\right]$$

$$= \frac{-1}{r^2}\left[\frac{x\hat{i}}{r} + \frac{y\hat{j}}{r} + \frac{z\hat{k}}{r}\right]$$

$$= -\frac{\hat{r}}{r^2} = -\frac{\vec{r}}{r^3}$$

$$\therefore \nabla\left(\frac{1}{r}\right) = -\frac{\vec{r}}{r^3} \quad \text{—— (a)}$$

Again,

$$\nabla r^n = \frac{\partial r^n}{\partial x}\hat{i} + \frac{\partial r^n}{\partial y}\hat{j} + \frac{\partial r^n}{\partial z}\hat{k}$$

$$= \left(r^{n-1} \cdot \frac{\partial r}{\partial x}\right)\hat{i} + \left(r^{n-1} \cdot \frac{\partial r}{\partial y}\right)\hat{j} + \left(r^{n-1} \cdot \frac{\partial r}{\partial z}\right)\hat{k}$$

$$= \left(r^{n-1} \cdot \frac{x}{r}\right)\hat{i} + \left(r^{n-1} \cdot \frac{y}{r}\right)\hat{j} + \left(r^{n-1} \cdot \frac{z}{r}\right)\hat{k}$$

$$= r^{n-2}\left(x\hat{i} + y\hat{j} + z\hat{k}\right)$$

$$\therefore \nabla r^n = r^{n-2}\vec{r} \qquad = r^{n-1}\hat{r}$$

<Q.4>: Calculate the divergence of following vector functions:

a) $\vec{V_a} = x^2\hat{i} + 3xz^2\hat{j} - 2xz\hat{k}$

Soln:

Given,

$$\vec{V_a} = x^2\hat{i} + 3xz^2\hat{j} - 2xz\hat{k}$$

We know,

$$\nabla \cdot \vec{V_a} = \frac{\partial V_{ax}}{\partial x} + \frac{\partial V_{ay}}{\partial y} + \frac{\partial V_{az}}{\partial z}$$

$$= \frac{\partial x^2}{\partial x} + \frac{\partial\, 3xz^2}{\partial y} + \frac{\partial(-2xz)}{\partial z}$$

$$= 2x + 0 - 2x \qquad = 0.$$

b): $\vec{V_c} = y^2\hat{i} + (2xy + z^2)\hat{j} + 2yz\hat{k}$

Soln:

Given,

$$\vec{V_c} = y^2\hat{i} + (2xy+z^2)\hat{j} + 2yz\hat{k}$$

We know,

$$\nabla \cdot \vec{V_c} = \frac{\partial V_{cx}}{\partial x} + \frac{\partial V_{cy}}{\partial y} + \frac{\partial V_{cz}}{\partial z}$$

$$= \frac{\partial y^2}{\partial x} + \frac{\partial(2xy+z^2)}{\partial y} + \frac{\partial(2yz)}{\partial z}$$

$$= 0 + 2x + 0 + 2y$$

$$\therefore \nabla \cdot \vec{V_c} = 2(x+y)$$

<Q.5>: If $\vec{A} = x^2z\hat{i} - 2y^3z^2\hat{j} + xy^2z\hat{k}$, find $\nabla \cdot \vec{A}$ at point $(1,-2,1)$.

Soln:

Given,

$$\vec{A} = x^2z\hat{i} - 2y^3z^2\hat{j} + xy^2z\hat{k}$$

Now,

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$= \frac{\partial(x^2z)}{\partial x} + \frac{\partial(-2y^3z^2)}{\partial y} + \frac{\partial(xy^2z)}{\partial z}$$

$$= 2xz - 6y^2z^2 + xy^2$$

At $(1,-2,1)$

$$\therefore \nabla \cdot \vec{A} = -18$$

<Q.6>: Show that $\vec{A} = 3y^4z^2\hat{i} + 4x^3z^2\hat{j} - 3x^2y^2\hat{k}$ is solenoidal.

Soln:

Given,

$$\vec{A} = 3y^4z^2\hat{i} + 4x^3z^2\hat{j} - 3x^2y^2\hat{k}$$

We know,

$$\nabla\cdot\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$= \frac{\partial(3y^4z^2)}{\partial x} + \frac{\partial(4x^3z^2)}{\partial y} + \frac{\partial(-3x^2y^2)}{\partial z}$$

$$= 0 + 0 + 0 \quad \therefore \nabla\cdot\vec{A} = 0 \ \{\text{solenoidal}\}$$

<Q.7>: Calculate the curl of the following vector function.

a) $\vec{V_a} = -y\hat{i} + x\hat{j}$

Soln.

Given,

$$\vec{V_a} = -y\hat{i} + x\hat{j}$$

We know,

$$\nabla\times\vec{V_a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -y & x & 0 \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial\cdot 0}{\partial y} - \frac{\partial x}{\partial z}\right) - \hat{j}\left(\frac{\partial(-y)}{\partial z} - \frac{\partial\cdot 0}{\partial x}\right) + \hat{k}\left(\frac{\partial x}{\partial x} - \frac{\partial(-y)}{\partial y}\right)$$

$$= 0 + 0 + 2\hat{k} \quad \therefore \nabla\times\vec{V_a} = 2\hat{k}$$

b) $\vec{V_b} = x^2y\hat{i} + (x-y)\hat{k}$

Soln:

Given,

$$\vec{V_b} = x^2y\hat{i} + (x-y)\hat{k}$$

We know,

$$\nabla\times\vec{V_b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2y & 0 & (x-y) \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial(x-y)}{\partial y} - \frac{\partial\cdot 0}{\partial z}\right) - \hat{j}\left(\frac{\partial(x-y)}{\partial x} - \frac{\partial(x^2y)}{\partial z}\right)$$

$$+ \hat{k}\left(\frac{\partial\cdot 0}{\partial x} - \frac{\partial x^2y}{\partial y}\right)$$

$$= \hat{i}(-1) - \hat{j}(1-0) + \hat{k}(-x^2)$$

$$\therefore \nabla\times\vec{V_b} = -\hat{i} - \hat{j} - x\hat{k}$$

<Q.8>: If $\vec{A} = xz^3\hat{i} - 2x^2yz\hat{j} + 2yz^4\hat{k}$, then find $\nabla\times\vec{A}$ at the point $(1,-1,1)$.

Soln:

Given,

$$\vec{A} = xz^3\hat{i} - 2x^2yz\hat{j} + 2yz^4\hat{k}$$

We know,

$$\nabla\times\vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ xz^3 & -2x^2yz & 2yz^4 \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial\, 2yz^4}{\partial y} - \frac{\partial(-2x^2yz)}{\partial z}\right) - \hat{j}\left(\frac{\partial\, 2yz^4}{\partial x} - \frac{\partial}{\partial z} xz^3\right)$$

$$+ \hat{k}\left(\frac{\partial(-2x^2yz)}{\partial x} - \frac{\partial}{\partial y}(xz^3)\right)$$

$$= \hat{i}\left(2z^4 + 2x^2y\right) - \hat{j}\left(0 - 3z^2\right) + \hat{k}\left(-4xyz - 0\right)$$

$$\therefore \nabla \times \vec{A} = (2z^4 + 2x^2y)\hat{i} + 3z^2\hat{j} - 4xyz\,\hat{k}$$

At point $(1,-1,1)$

$$\therefore \nabla \times \vec{A} = 3\hat{j} + 4\hat{k}$$

<Q.9>: Calculate the Laplacian of following functions

a) $T = \sin x \sin y \sin z$

Soln:

Given,

$T = \sin x \sin y \sin z$

we know,

$$\nabla^2 T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}$$

$$= \frac{\partial^2(\sin x \sin y \sin z)}{\partial x^2} + \frac{\partial^2(\sin x \sin y \sin z)}{\partial y^2} + \frac{\partial^2(\sin x \sin y \sin z)}{\partial z^2}$$

$$= -\sin x \sin y \sin z - \sin x \sin y \sin z - \sin x \sin y \sin z$$

$$\therefore \nabla^2 T = -3\sin x \sin y \sin z$$

b): $T = e^{-5x} \sin 4y \cos 3z$

Soln:

Given,

$T = e^{-5x} \sin 4y \cos 3z$

We know,

$$\nabla^2 T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}$$

$$= \frac{\partial^2(e^{-5x}\sin 4y \cos 3z)}{\partial x^2} + \frac{\partial^2(e^{-5x}\sin 4y\cos 3z)}{\partial y^2} + \frac{\partial^2(e^{5x}\sin 4y \cos 3z)}{\partial z^2}$$

$$= 25e^{-5x}\sin 4y\cos 3z - 16e^{-5x}\sin 4y \cos 3z - 9e^{-5x}\sin 4y \cos 3z$$

$$\therefore \nabla^2 T = 0.$$

c) $\vec{v} = x^2\hat{i} + 3xz^2\hat{j} - 2xz\,\hat{k}$

Soln:

Given,

$\vec{v} = x^2\hat{i} + 3xz^2\hat{j} - 2xz\,\hat{k}$

we know,

$$\nabla^2\vec{v} = (\nabla^2 v_x)\hat{i} + (\nabla^2 v_y)\hat{j} + (\nabla^2 v_z)\hat{k}$$

$$= (\nabla^2 x^2)\hat{i} + (\nabla^2 3xz^2)\hat{j} + (\nabla^2(-2xz))\hat{k}$$

$$= \left(\frac{\partial^2 x^2}{\partial x^2} + \frac{\partial^2 x^2}{\partial y^2} + \frac{\partial^2 x^2}{\partial z^2}\right)\hat{i} + \left(\frac{\partial^2(3xz^2)}{\partial x^2} + \frac{\partial^2(3xz^2)}{\partial y^2} + \frac{\partial^2(3xz^2)}{\partial z^2}\right)\hat{j}$$

$$+ \hat{k}\left(\frac{\partial^2(-2xz)}{\partial x^2} + \frac{\partial^2(-2xz)}{\partial y^2} + \frac{\partial^2(-2xz)}{\partial z^2}\right)$$

$$= (2+0+0)\hat{i} + (0+0+6x)\hat{j} + (0+0+0)\hat{k} = 2\hat{i} + 6x\hat{j}$$

<Q.10>: Prove that the divergence of a curl is always zero. Check it for the function

$$\vec{V} = x^2\hat{i} + 2xz^2\hat{j} - 2xz\hat{k}$$

Soln:

We know,

$$\nabla \times \vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ V_x & V_y & V_z \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right) - \hat{j}\left(\frac{\partial V_z}{\partial x} - \frac{\partial V_x}{\partial z}\right) + \hat{k}\left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right)$$

Now,

$$\nabla\cdot(\nabla\times\vec{V}) = \left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right]\left[\hat{i}\left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right) - \hat{j}\left(\frac{\partial V_z}{\partial x} - \frac{\partial V_x}{\partial z}\right) + k\left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right)\right]$$

$$= \frac{\partial}{\partial x}\left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right) - \frac{\partial}{\partial y}\left(\frac{\partial V_z}{\partial x} - \frac{\partial V_x}{\partial z}\right) + \frac{\partial}{\partial z}\left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right)$$

$$\therefore \nabla\cdot(\nabla\times\vec{V}) = 0$$

Given,

$$\vec{V} = x^2\hat{i} + 2xz^2\hat{j} - 2xz\hat{k}$$

So,

$$\nabla\times\vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2 & 2xz^2 & -2xz \end{vmatrix}$$

$$= \hat{i}\left(-\frac{\partial\, 2xz^2}{\partial z} + \frac{\partial(-2xz)}{\partial y}\right) - \hat{j}\left(\frac{\partial(-2xz)}{\partial x} - \frac{\partial x^2}{\partial z}\right) + \hat{k}\left(\frac{\partial\, 2xz^2}{\partial x} - \frac{\partial x^2}{\partial y}\right)$$

$$\cancel{= \hat{i}(4xz - 0) - \hat{j}} \quad = \hat{i}(-4xz + 0) - \hat{j}(-2z - 0) + \hat{k}(2z^2 - 0)$$

$$= -4xz\hat{i} + 2z\hat{j} + 2z^2\hat{k}$$

Now,

$$\nabla\cdot(\nabla\times\vec{V}) = \frac{\partial(\nabla\times\vec{V})_x}{\partial x} + \frac{\partial(\nabla\times\vec{V})_y}{\partial y} + \frac{\partial(\nabla\times\vec{V})_z}{\partial z}$$

$$= \frac{\partial(-4xz)}{\partial x} + \frac{\partial\, 2z}{\partial y} + \frac{\partial\, 2z^2}{\partial z}$$

$$= -4z + 4z = 0$$

$$\therefore \nabla\cdot(\nabla\times\vec{V}) = 0.$$

<Q.11>: Prove that the curl of a gradient is always zero. Check it for the function $f(x,y,z) = x^2y^3z^4$.

Soln:

~~Given, f =~~ Now,

$$\nabla\times(\nabla T) = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\times\left(\hat{i}\frac{\partial T}{\partial x} + \hat{j}\frac{\partial T}{\partial y} + \hat{k}\frac{\partial T}{\partial z}\right)$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \partial T/\partial x & \partial T/\partial y & \partial T/\partial z \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial^2 T}{\partial y\partial z} - \frac{\partial^2 T}{\partial z\partial y}\right) - \hat{j}\left(\frac{\partial^2 T}{\partial x\partial z} - \frac{\partial^2 T}{\partial x\partial z}\right) + \hat{k}\left(\frac{\partial^2 T}{\partial x\partial y} - \frac{\partial^2 T}{\partial x\partial y}\right)$$

$$= 0.$$

Given,

$$f = x^2y^3z^4$$

We know,

$$\nabla f = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}$$

$$= \frac{\partial (x^2y^3z^4)}{\partial x}\hat{i} + \frac{\partial (x^2y^3z^4)}{\partial y}\hat{j} + \frac{\partial (x^2y^3z^4)}{\partial z}\hat{k}$$

$$= 2xy^3z^4\,\hat{i} + 3x^2y^2z^4\hat{j} + 4x^2y^3z^3\hat{k}$$

Now,

$$\nabla \times (\nabla f) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2xy^3z^4 & 3x^2y^2z^4 & 4x^2y^3z^3 \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial\, 4x^2y^3z^3}{\partial y} - \frac{\partial (3x^2y^2z^4)}{\partial z}\right) - \hat{j}\left(\frac{\partial (4x^2y^3z^3)}{\partial x} - \frac{\partial\, 2xy^3z^4}{\partial z}\right)$$

$$+\hat{k}\left(\frac{\partial (3x^2y^2z^4)}{\partial x} - \frac{\partial\, 2xy^3z^4}{\partial y}\right)$$

$$= \hat{i}\left(12x^2y^2z^3 - 12x^2y^2z^3\right) - \hat{j}\left(8xy^3z^3 - 8xy^3z^3\right)$$

$$+\hat{k}\left(6xy^2z^4 - 6xy^2z^4\right)$$

$$= 0$$

$$\therefore \nabla \times (\nabla f) = 0$$

⟨Q.12⟩: Let $\vec{F_1} = x^2\hat{k}$ and $\vec{F_2} = x\hat{i} + y\hat{j} + z\hat{k}$. Calculate the divergence and curl of $\vec{F_1}$ and $\vec{F_2}$. Which one can be written as gradient of scalar? Which one can be written as curl of a vector.

Soln.

Given,

$$\vec{F_1} = x^2\hat{k} \qquad \vec{F_2} = x\hat{i} + y\hat{j} + z\hat{k}$$

Now,

$$\nabla \cdot \vec{F_1} = \frac{\partial V_z}{\partial z} = \frac{\partial x^2}{\partial z} = 0$$

$$\nabla \cdot \vec{F_2} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = \frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial z}{\partial z} = 3$$

Since divergence of $\vec{F_1}$ is 0, it can be written as curl of a vector.

Again,

$$\nabla \times \vec{F_1} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & 0 & x^2 \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial x^2}{\partial y} - \frac{\partial \cdot 0}{\partial z}\right) - \hat{j}\left(\cancel{\frac{\partial 0}{\partial z}}\,\frac{\partial x^2}{\partial x} - \frac{\partial \cdot 0}{\partial z}\right) + \hat{k}\left(\frac{\partial 0}{\partial x} - \frac{\partial \cdot 0}{\partial y}\right)$$

$$= \hat{i}(0-0) - \hat{j}(2x) + \hat{k}(0-0)$$

$$= -2x\hat{j}$$

$$\nabla \times \vec{F_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x & y & z \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial z}{\partial y} - \frac{\partial y}{\partial z}\right) - \hat{j}\left(\frac{\partial z}{\partial x} - \frac{\partial x}{\partial z}\right) + \hat{k}\left(\frac{\partial y}{\partial x} - \frac{\partial x}{\partial y}\right)$$

$$= 0$$

Since curl of $\vec{F_2}$ is 0, it can be written as gradient of scalar.

(Q.13): Show that $\vec{F} = yz\hat{i} + zx\hat{j} + xy\hat{k}$ can be written both as gradient of a scalar and as the curl of a vector.

Soln:

Given,

$$\vec{F} = yz\hat{i} + zx\hat{j} + xy\hat{k}$$

Now,

$$\text{curl } \vec{F} = \nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & zx & xy \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial xy}{\partial y} - \frac{\partial zx}{\partial z}\right) - \hat{j}\left(\frac{\partial xy}{\partial x} - \frac{\partial yz}{\partial z}\right) + \hat{k}\left(\frac{\partial zx}{\partial x} - \frac{\partial yz}{\partial y}\right)$$

$$= \hat{i}(x - x) - \hat{j}(y - y) + \hat{k}(z - z)$$

$$= 0$$

Again,

$$\text{Div. } \vec{F} = \nabla \cdot \vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

$$= \frac{\partial yz}{\partial x} + \frac{\partial zx}{\partial y} + \frac{\partial xy}{\partial z}$$

$$= 0$$

Here, divergence and curl of vector $\vec{F}$ is 0, the $\vec{F}$ can be written as both the curl of a vector and gradient of a scalar.