OPTICS:

UNIT 2: DIFFRACTION

Rectilinear Propagation of Light

According to wave theory of light, every point on primary wave behaves as source for secondary wave and the forward envelope of secondary wavelets for certain time gives the new position of wave at that time.

Thus, light can be considered as a plane wave after a long time or at great distance.

"The phenomenon by which light wave travels in straight line is called rectilinear propagation of light.

An unexpected result is obtained when the obstacle is small size of the order of the wavelength of light. This is explained by diffraction on the basis of wave theory.

Diffraction:

The bending of light at small apertures or at the sharp edges to form a band of dark and bright fringes of varying intensities is called diffraction.

Condition: size of obstacle must be comparable to wavelength of light used.

Diffraction are of two types: Frensel and Fraunhofer diffraction.

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a) Frensel diffraction:

The diffraction phenomenon in which the source and the screen are separated by a finite distance from slits is called Frensel diffraction.

Here, a spherical wave from a point source falls upon slit and diffracts to form dark and bright fringes of varying intensities.

b) Fraunhofer diffraction

The diffraction phenomenon in which the source and the screen are separated by infinite distance from slit is called Fraunhofer diffraction.

Here, light from source at infinity falls on slit and gets diffracted resulting dark and bright fringes with different intensities.

We need lens to observe this phenomenon.

Resultant Amplitude of n waves

Consider a there are n numbers of waves with same amplitude a and frequencies as well as successive phase diffrence of δ

So,

$$y_1 = ae^{i\theta}$$

$$y_2 = ae^{i(\theta+\delta)}$$

$$y_3 = ae^{i(\theta+2\delta)}$$

$$\vdots$$

$$y_n = ae^{i(\theta+(n-1)\delta)}$$

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Now,
resultant y is given by:

$$y = y_1 + y_2 + y_3 + \cdots + y_n$$

$$= ae^{i\theta} + ae^{i(\theta+\delta)} + ae^{i(\theta+2\delta)} + ae^{i(\theta+3\delta)} + \cdots + ae^{i(\theta+(n-1)\delta)}$$

$$= ae^{i\theta}\left[1 + e^{i\delta} + e^{2i\delta} + \cdots + e^{i(n-1)\delta}\right]$$

$$= ae^{i\theta}\left[\frac{e^{in\delta}-1}{e^{i\delta}-1}\right] = ae^{i\theta}\,\frac{e^{in\delta/2}}{e^{i\delta/2}}\left[\frac{e^{in\delta/2}-e^{-in\delta/2}}{e^{i\delta/2}-e^{-i\delta/2}}\right]$$

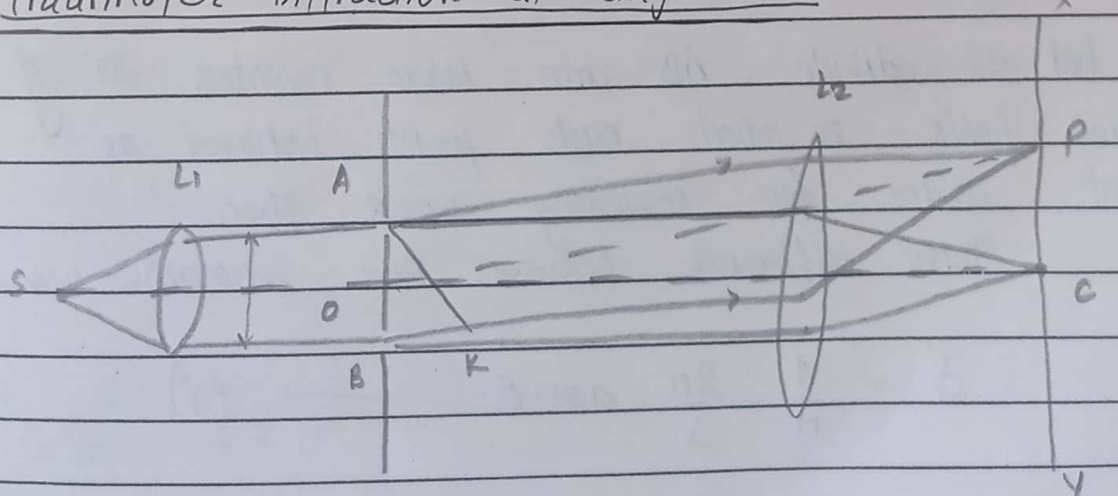
$$= ae^{i\left[\theta+\frac{(n-1)}{2}\delta\right]}\,\frac{2i\sin(n\delta/2)}{2i\sin(\delta/2)}$$

$$\therefore y = Re^{i\left[\theta+\frac{(n-1)}{2}\delta\right]}$$

Here,

$$R = \frac{a\sin(n\delta/2)}{\sin(\delta/2)}$$

Fraunhofer Diffraction at single slit



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Let a collimating lens L_1 be placed at a distance equal to its focal length from monochromatic source S.

The collimated beam of light from L_1 falls on narrow slit AB of width a.

The converging lens L_2 on other side of slit converges the undiffracted beam at center C and diffracted beam at P on screen XY.

AK ⊥ BK is drawn and let θ be angle of diffraction.

From ~~the~~ figure, $\angle BAK = \theta$

and BK is path difference.

ie,

$$BK = AB\sin\theta = a\sin\theta .$$

So,

$$\text{phase difference} = \frac{2\pi}{\lambda} \times \text{path difference.}$$

$$= \frac{2\pi a\sin\theta}{\lambda}$$

Let us divide AB into large number 'n' of equal parts so that each point behaves as point source for secondary wave. Then,

Phase difference between two consecutive waves is

$$\delta = \frac{1}{n}\frac{2\pi}{\lambda} a\sin\theta . \quad \text{—— (i)}$$

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Now,
resultant of n waves reaching at P is.

$$R = a\frac{\sin\left(\frac{n\delta}{2}\right)}{\sin\left(\frac{\delta}{2}\right)} = a\frac{\sin\left(\frac{\pi a \sin\theta}{\lambda}\right)}{\sin\left(\frac{\pi a\sin\theta}{\lambda}\right)} = a\frac{\sin\alpha}{\sin(\alpha/n)} \quad \text{— (ii)}$$

Here, $\alpha = \frac{\pi a \sin\theta}{\lambda}$ — (iii)

For large n, $\sin\left(\frac{\alpha}{n}\right) = \frac{\alpha}{n}$.

Thus,

$$R = \frac{a\sin\alpha}{(\alpha/n)} = \frac{A\sin\alpha}{\alpha} \quad [\because A = na] \quad \text{— (iii)}$$

We know, $I \propto R^2$.

So,

$$I = \frac{A^2\sin^2\alpha}{\alpha^2} \quad \text{— (iv)}.$$

Diffrentiating eq^n(iv) w.r.t α and equating to zero,

$$\frac{dI}{d\alpha} = 0$$

or, $$\frac{d}{d\alpha}\left(\frac{A^2\sin^2\alpha}{\alpha^2}\right) = 0$$

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or, $2A^2\left(\frac{\sin\alpha}{\alpha}\right)\left(\frac{\alpha\cos\alpha-\sin\alpha}{\alpha^2}\right)=0$

ie, $\frac{\sin\alpha}{\alpha}=0$ – (v) or, $\frac{\alpha\cos\alpha-\sin\alpha}{\alpha^2}=0$

$\therefore \alpha=\tan\alpha$ – (vi).

*) Position for minimum Intensity

When $\frac{\sin\alpha}{\alpha}=0$, From eqn (iv), $I=0$

So,

For minimium intensity,

$$\frac{\sin\alpha}{\alpha}=0$$

or, $\sin\alpha=0$

$\therefore \alpha=\pm m\pi$ $\quad m\in 1,2,3\ldots$

or, $\frac{\pi a\sin\theta}{\lambda}=\pm m\pi$

$\therefore a\sin\theta=\pm m\lambda$ ——— (vii)

Eqn (vii) is condition for minimum intensity.

It occurs at $\alpha=\pm\pi,\pm2\pi,\pm3\pi$.

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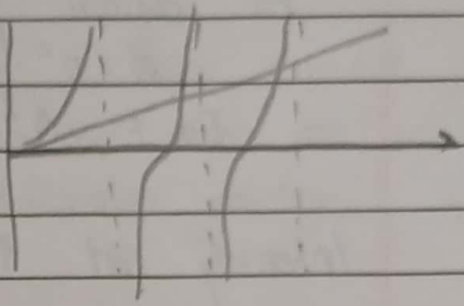
*) Position of maximum intensity



For maximum intensity,
solving $\alpha = \tan\alpha$ graphically,
ie,
$y = \alpha$ and $y = \tan\alpha$.

Hence, solving of two curves is approxiamately,

$$\alpha = 0^\circ, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots$$

Hence, points of maximum intensity are.

$$\alpha = 0^\circ, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}, \dots$$

*) For central maximum:

$$\alpha = 0$$

or, $\frac{\pi a \sin\theta}{\lambda} = 0$

$\therefore \theta = 0^\circ$

This is the condition for ~~central~~ principal maxima.

*) For secondary maxima:

Directions of secondary maxima are approximately.

$\alpha = \pm(2m+1)\frac{\pi}{2}$ or, $\frac{\pi a \sin\theta}{\lambda} = \pm(2m+1)\frac{\pi}{2}$

$\therefore a\sin\theta = \pm(2m+1)\frac{\lambda}{2}$ —(viii)

Eqn (viii) is condition for secondary maxima.

*) Intensity distribution:

At central principal maximum,

$$\alpha = 0^\circ$$

$$I_0 = A^2.$$

Intensity at 1st secondary maxima,

$$I_1 = \frac{A^2 [\sin(3\pi/2)]^2}{[3\pi/2]^2} \approx \frac{A^2}{22} = \frac{I_0}{22}$$

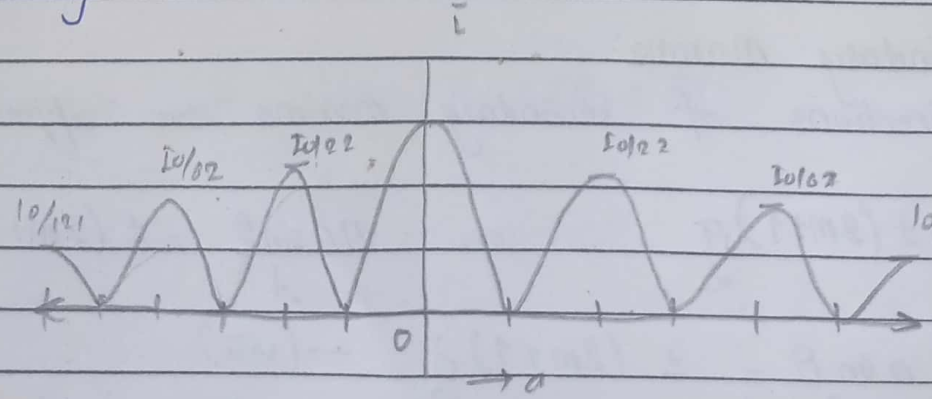
Intensity at 2nd secondary maxima,

$$I_2 = \frac{A^2 [\sin(5\pi/2)]^2}{[5\pi/2]^2} \approx \frac{A^2}{62} = \frac{I_0}{62}$$

Intensity of 3rd secondary maxima,

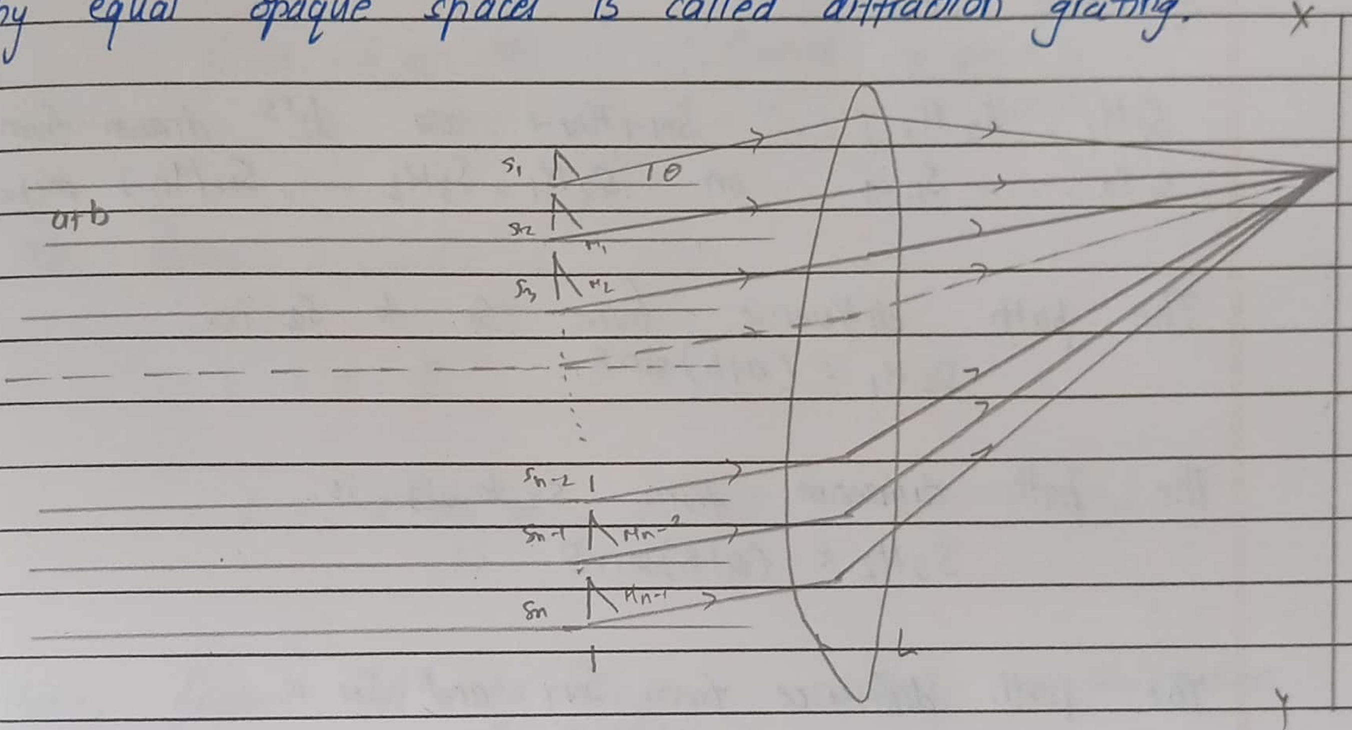
$$I_3 = \frac{A^2 [\sin(7\pi/2)]^2}{(7\pi/2)^2} \approx \frac{A^2}{121} = \frac{I_0}{121}$$

Thus, it is seen that intensity of secondary maxima goes on decreasing when number of secondary maxima increases.



Fraunhofer Diffraction Due to N parallel Slits / Theory of Diffraction Grating

An arrangement consisting of a large number of parallel slits of equal width and separated from one another by equal opaque spaces is called diffraction grating.



When the wave-front reaches the plane of the slits, each point on the slit sends out secondary wavelets in all directions.

From the theory of Fraunhofer diffraction at single slit, the wavelets proceeding from all points in a slit of direction θ are equivalent to a single wave of amplitude

$$R = \frac{A\sin\alpha}{\alpha}$$

starting from center of slits where $\alpha = \frac{\pi a \sin\theta}{\lambda}$.

Thus, the waves diffracted from all slits in direction θ are equivalent to N parallel waves.

Each wave starting from middle point $S_1, S_2, S_3, \ldots S_{n-1}, S_n$ of the slits.

$S_1M_1, S_2M_2, \ldots S_{N-1}M_{N-1}$ are $\perp^{rs}$ drawn from, $S_1 S_2, \ldots S_{n-1}$ on $S_2M_1, S_3M_2, \ldots, S_N M_{n-1}$ respectively

The path difference from S_1 to S_2 is.

$$S_2M_1 = (a+b)\sin\theta.$$

The path difference from S_2 to S_3 is.

$$S_3M_2 = (a+b)\sin\theta$$

The path difference from S_{N-1} and S_N is.

$$S_N M_{N-1} = (a+b)\sin\theta.$$

Thus,

corresponding phase difference is $= \frac{2\pi}{\lambda}(a+b)\sin\theta$

When we pass from one vibration to another, the phase goes on increasing by same amount.

$$\frac{2\pi}{\lambda}(a+b)\sin\theta$$

Thus, to find amplitude in direction θ, resultant amplitude of N waves, each having amplitude R and common phase difference.

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$$\frac{2\pi}{\lambda}(a+b)\sin\theta = 2\beta \quad (\text{Let})$$

Thus, resultant amplitude in direction θ is.

$$R' = \frac{R\sin N\beta}{\sin\beta} = \frac{A\sin\alpha}{\alpha} \times \frac{\sin N\beta}{\sin\beta}.$$

We know, $I \propto R^2$

So,

$$I = R'^2$$

$$\therefore I = \frac{A^2\sin^2\alpha}{\alpha} \cdot \frac{\sin^2 N\beta}{\sin^2\beta} \quad \text{———} \quad (1)$$

Here, factor $\frac{A^2\sin^2\alpha}{\alpha^2}$ gives intensity in diffraction pattern due to single slit.

It has minima at $\alpha = \pm\pi, \pm 2\pi, \pm 3\pi, \ldots$

and

maxima at $\alpha = 0, \pm 3\pi/2, \pm 5\pi/2, \ldots$

~~He~~ Here,

$\alpha = 0 \rightarrow$ principal maximum

and

$\alpha = \pm(2m+1)\pi/2 \rightarrow$ secondary maxima.

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Factor $\left(\frac{\sin^2 N\beta}{\sin^2\beta}\right)$ gives the distribution of intensity in the pattern due to interference betn the waves from all N slits.

Both effects combine together give the pattern of the light differected by plane transmission grating.

*) Principal maxima

When $\sin\beta = 0$ ie, $\beta = \pm n\pi$, $n = 0, 1, 2 \ldots$. we have $\sin N\beta = 0$. so, $\sin N\beta / \sin\beta$ becomes indeterminant.

Now, $\frac{\sin N\beta}{\sin\beta}$ at $\beta \to \pm n\pi$.

$$\lim_{\beta \to n\pi} \frac{\sin N\beta}{\sin\beta} = \lim_{\beta \to \pm n\pi} \frac{N\cos N\beta}{\cos\beta} = N.$$

Substituting value in eqn(1), we get.

$$I = \frac{A^2 \sin^2\alpha}{\alpha^2} N^2 \quad \text{———} \quad (2).$$

The maxima with resultant intensity as in eqn(2) are called principal maxima.

Hence, if we increase the number of slits, the intensity of principal maxima increases.

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Thus, direction of principal maxima are given by.

$$\sin\beta = 0$$

$$\therefore \beta = \pm n\pi .$$

or, $$\frac{\pi (a+b)\sin\theta}{\lambda} = \pm n\pi$$

$$\therefore (a+b)\sin\theta = \pm n\lambda \quad \text{——— (iii)}$$

Putting $n = 0, 1, 2, 3, \ldots$
we obtain central maximum, first order principal maxima, second order principal maximum, ...

*) Minima:

From eq^n (1),

$$\sin N\beta = 0 \quad \text{and} \quad \sin\beta \neq 0.$$

or, $$N\beta = \pm m\pi$$

$$\therefore N(a+b)\sin\theta = \pm m\lambda \quad \text{——— (iii)}.$$

Here, m can have values 0, N, 2N corresponding to $\sin\beta = 0$ which gives position of principal maxima.

From eq^n (4), $m=0$ gives principal maximum of zero order and $m = 1, 2, 3, \ldots, N-1$ give minima and $m=N$ gives principal maxima of first order.

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*) Secondary maxima

As there are $N-1$ minima between two consecutive principal maxima, there must be $N-2$ secondary maxima betn them.

For secondary maxima,

$$\frac{dI}{d\beta} = 0$$

or, $$\frac{d}{d\beta}\left[\frac{A^2\sin^2\alpha}{\alpha^2} \cdot \frac{\sin^2 N\beta}{\sin^2\beta}\right] = 0$$

Solving we get,

$$\therefore \tan N\beta = N\tan\beta.$$

Then,

$$\sin N\beta = \frac{\tan N\beta}{\sec N\beta} = \frac{\tan N\beta}{\sqrt{1+\tan^2 N\beta}} = \frac{N\tan\beta}{\sqrt{1+N^2\tan^2\beta}}$$

and

$$\frac{\sin^2 N\beta}{\sin^2\beta} = \frac{N^2\tan^2\beta}{\sin^2\beta(1+N^2\tan^2\beta)} = \frac{N^2}{\cos^2\beta(1+N^2\tan^2\beta)}$$

$$\therefore \frac{\sin^2 N\beta}{\sin^2\beta} = \frac{N^2}{1+(N^2-1)\sin^2\beta}$$

Thus, intensity of secondary maxima is.

$$I' = \frac{A^2\sin^2\alpha}{\alpha^2} \frac{N^2}{1+(N^2-1)\sin^2\beta}$$

Now, the ratio of intensities of secondary maxima to principal maxima is.

$$\frac{I'}{I} = \frac{\text{intensity of secondary maxima}}{\text{intensity of principal maxima}}$$

$$= \frac{\frac{A^2 \sin^2\alpha}{\alpha^2} \cdot \frac{N^2}{1+(N^2-1)\sin^2\beta}}{\frac{A^2 \sin^2\alpha}{\alpha^2} \cdot N^2}.$$

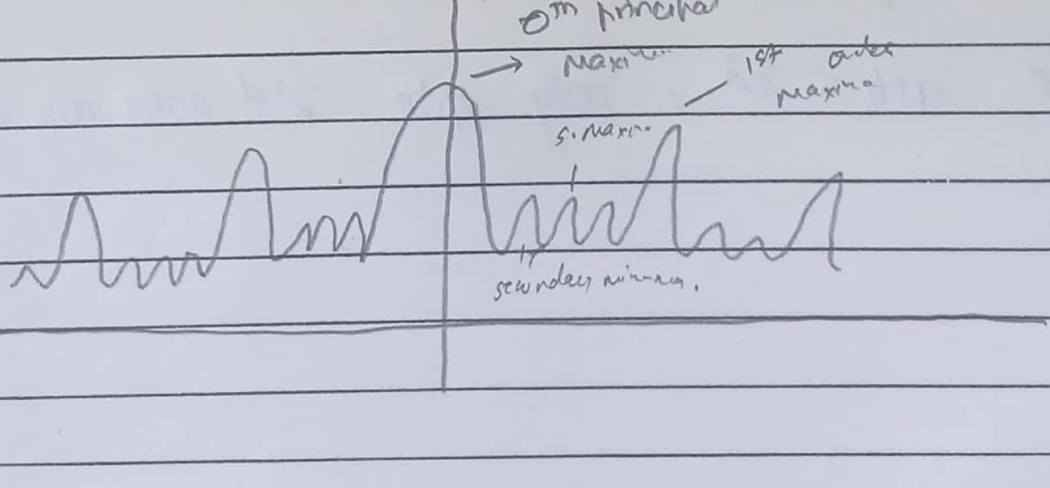
$$\therefore \frac{I'}{I} = \frac{1}{1+(N^2-1)\sin^2\beta}$$

Hence, as N increases, the intensity of secondary maxima decreases.

If N is large, the secondary maxima are not visible.

In such cases, there is uniform darkness betn any two principal maxima.

The intensity distribution is as shown.



*) Maximum number of orders with diffraction grating

For diffraction grating,

$$(a+b)\sin\theta = n\lambda.$$

Here,
a = width of slit
b = width of opaque portion.

$$\therefore n = \frac{(a+b)\sin\theta}{\lambda}$$

For normal incidence, maximum possible value of angle of diffraction is $\theta = 90^\circ$.
So,
maximum number of possible orders is.

$$n_{max} = \frac{(a+b)\sin 90^\circ}{\lambda}$$

$$\therefore n_{max} = \frac{a+b}{\lambda} \quad \text{— (a)}$$

Eq^n(a) gives the number of orders with diffraction grating.

For $a+b < 3\lambda$, only upto 2nd order are observed.