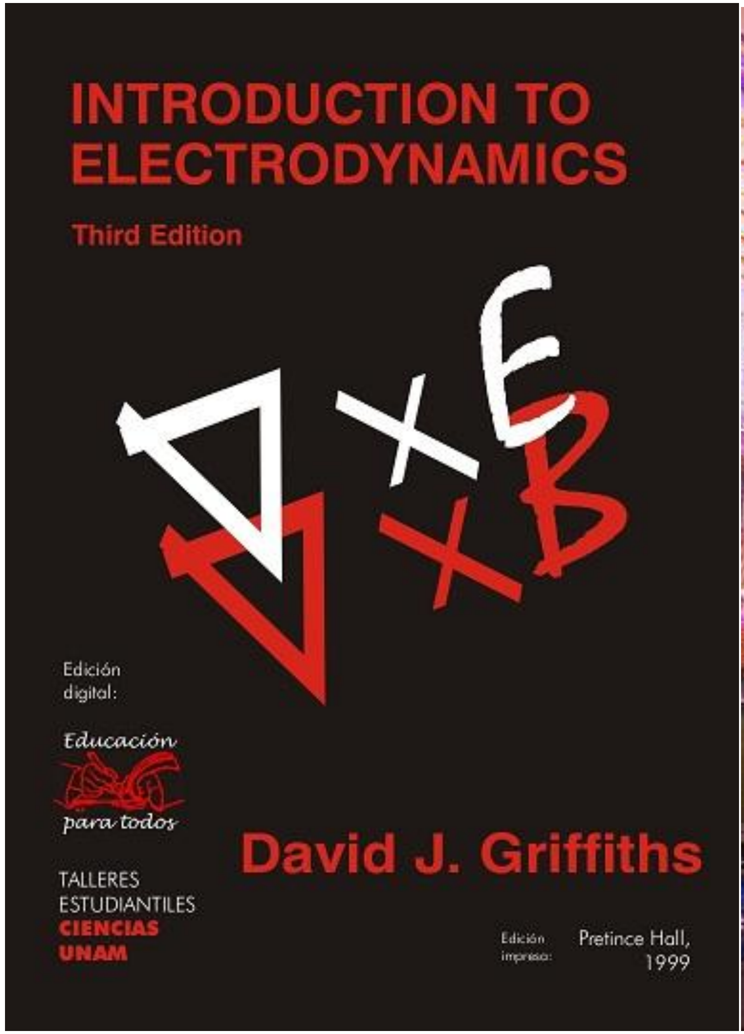
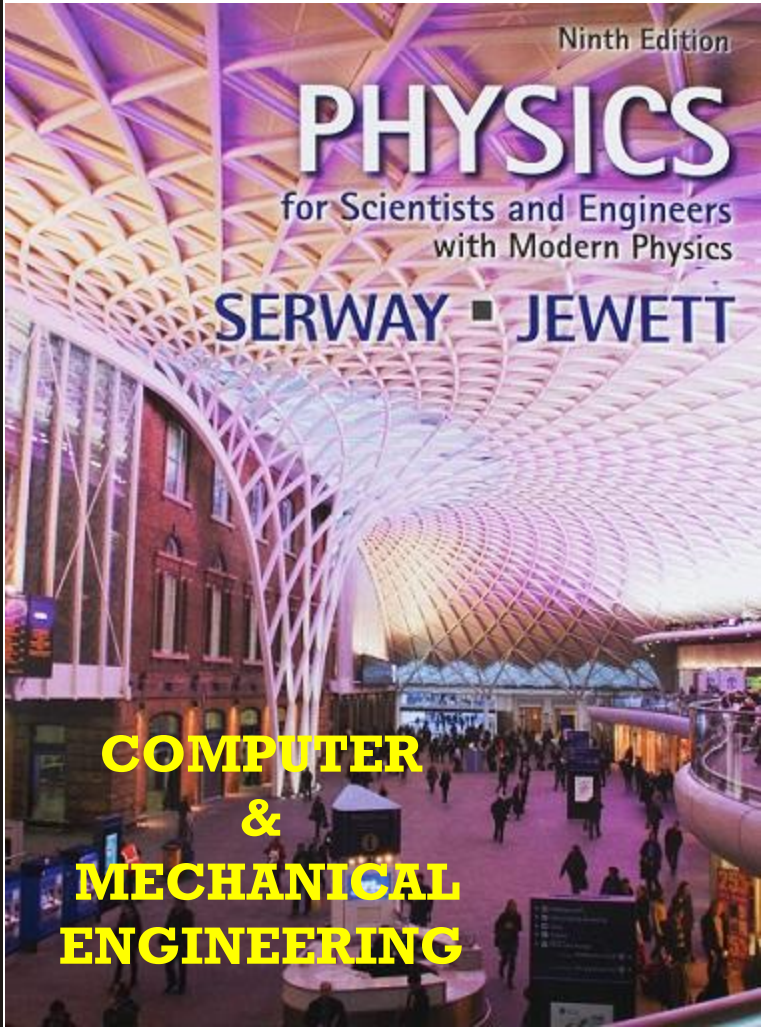
PHYSICS

General Physics II (PHYS 102)





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Course Outline

- Field Lines
- Electric Flux
- Gauss's Law
- Applications of Gauss's Law
- Multiple Choice Questions

Field Lines

ELECTROSTATICS FIELD

Electric Field Lines:

- Electric field lines describe an electric field in any region of space.
- The electric field vector $\vec{E}$ is tangent to the electric field line at each point.
- The number of lines per unit area through a surface perpendicular to the lines is proportional to the magnitude of electric field in that region.

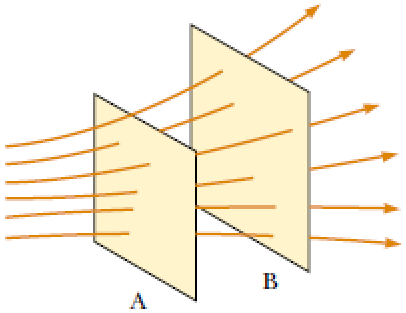
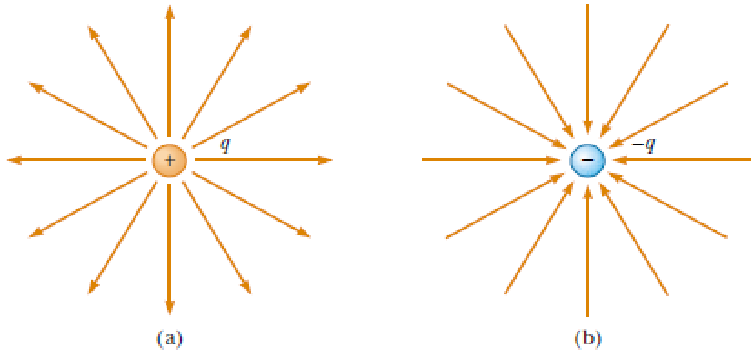


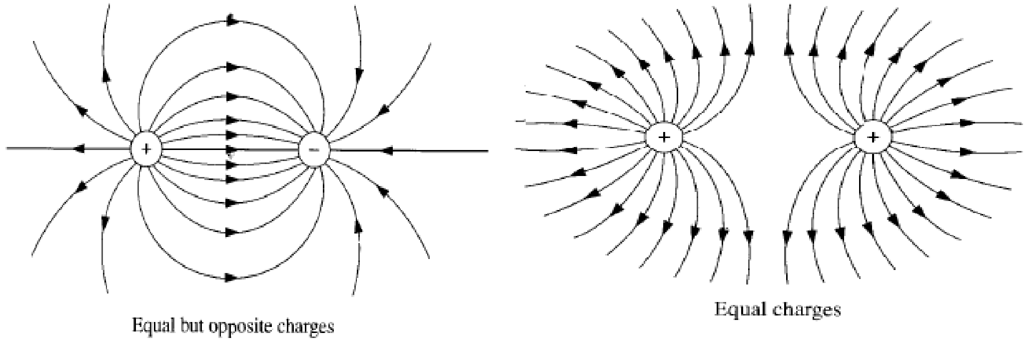
Figure F_L-1: Electric field lines penetrating two surfaces

The magnitude of the field is greater on surface A than on surface B.

- **Representative Electric Field Lines for the Field due to a Single Point Charge**



- **Electric Field Lines for Two Point Charges**



Rules for Drawing Electric Field Lines

- The lines must begin on a positive charge and terminate on a negative charge.
- No two field lines can cross.

Electric Flux

Electric Flux:

- Electric flux is proportional to the number of electric field lines that penetrate a surface.
- The electric flux through a surface S is

$$\Phi_E \equiv \int_S \vec{E} \cdot d\vec{a}$$

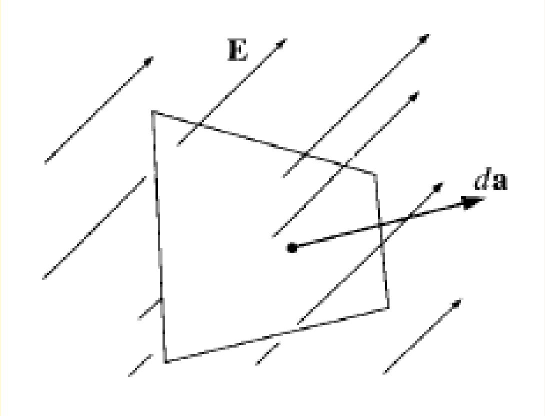


Figure E_F-1 Field lines representing an electric field penetrating an area $d\vec{a}$ that is at an angle θ to the field

- The SI unit of electric flux (Φ_E) is $\mathrm{N\ m^2\ C^{-1}}$.
- The electric flux through any closed surface is a measure of the total charged inside.

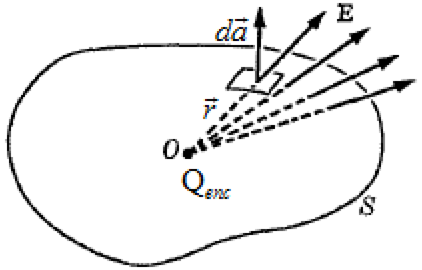
Gauss's Law

Gauss's Law:

- Formulated by **Carl Friedrich Gauss** (1777–1855), Greatest mathematicians.
- The total electric flux through any closed surface is equal to $\frac{1}{\varepsilon_0}$ times the total charge enclosed by the surface:

$$\boxed{\oint_S \vec{E}\cdot d\vec{a} = \frac{1}{\varepsilon_0}Q_{enc}}$$

- Consider an arbitrary shaped closed surface S, which encloses a point charge Q_{enc} at the origin.



The electric field at point $\vec{r}$ due to a point charge Q_{enc} located at the origin is

$$\boxed{\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Q_{enc}}{r^2}\hat{r}}$$

The total electric flux through a closed surface S is

$$\oint_S \vec{E}\cdot d\vec{a} = \oint_S \vec{E}\cdot d\vec{a} = \oint_S \left(\frac{1}{4\pi\varepsilon_0}\frac{Q_{enc}}{r^2}\hat{r}\right)\cdot\left(da_r\hat{r} + da_\theta\hat{\theta} + da_\phi\hat{\phi}\right)$$

$$= \oint_S \left(\frac{1}{4\pi\varepsilon_0}\frac{Q_{enc}}{r^2}\right)da_r = \int\left(\frac{1}{4\pi\varepsilon_0}\frac{Q_{enc}}{r^2}\right)\left(r^2\sin\theta\, d\theta\, d\phi\right)$$

$$= \frac{Q_{enc}}{4\pi\varepsilon_0}\left[\left\{\int_0^\pi \sin\theta\, d\theta\right\}\left\{\int_0^{2\pi} d\phi\right\}\right] = \frac{Q_{enc}}{4\pi\varepsilon_0}\left[(2)(2\pi)\right]$$

$$= \frac{Q_{enc}}{\varepsilon_0}$$

$$\boxed{\therefore \oint_S \vec{E}\cdot d\vec{a} = \frac{1}{\varepsilon_0}Q_{enc}} \Rightarrow \boxed{\oint_S \vec{E}\cdot d\vec{a} = \frac{1}{\varepsilon_0}\sum_{i=1}^{n} q_i}$$ for discrete distribution of charges

Gauss's Law:

- If there is a continuous distribution of charge with a charge density ρ , then the total charge enclosed by the surface S is $Q_{enc} = \int_V \rho \, d\tau$.

Gauss's Law

$$\boxed{\oint_S \vec{E} \cdot d\vec{a} = \frac{1}{\varepsilon_0} \int_V \rho \, d\tau}$$

By applying the divergence theorem

$$\boxed{\int_V \left(\nabla \cdot \vec{E}\right) d\tau = \int_V \left(\frac{\rho}{\varepsilon_0}\right) d\tau}$$

Since this holds any volume, the integrands must be equal:

$$\boxed{\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}}$$

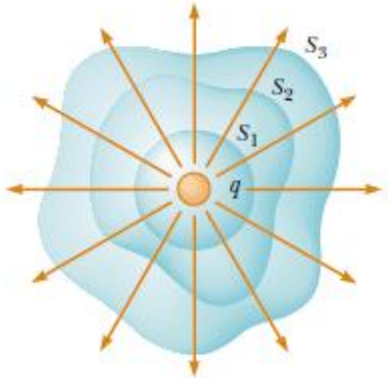
This is **a differential form of Gauss's Law**.

Gauss's Law

Notes:

- A charge q is located at the centre of a cube. The electric flux through any face is $\frac{1}{6}\left[\frac{1}{\varepsilon_0}q\right]$.

- If the closed surfaces of various shapes surrounding a charge q, then the net electric flux is the same through all surfaces.



- Two concentric imaginary spherical surfaces of radius R and $2R$ respectively surround a positive point charge Q located at the center of the surfaces. When compared to the electric flux Φ_1 through the surface of radius R, the electric flux Φ_2 through the surface of radius $2R$ is $\Phi_2 = \Phi_1$.
- When a cube is inscribed in a sphere of radius r, the length L of a side of the cube is $L = \sqrt{\frac{4}{3}}\, r$. If a positive point charge Q is placed at the center of the spherical surface, the ratio of the electric flux at the spherical surface Φ_{sphere} to the flux at the surface of the cube Φ_{cube} is 1.

Application of Gauss's Law

ELECTROSTATICS FIELD

I. Use Gauss's Law to find the electric field outside, on and inside a spherical shell of radius R, which carries a uniform surface charge density σ.

Electric Field Outside a Spherical Shell:

- Let the point P be at a distance r from the centre of a sphere so that $r > R$.

  With O as centre and, $OP = r$ as radius, a spherical Gaussian surface S is drawn as shown in Figure.

  The charged spherical shell lies completely inside the Gaussian surface S.
  So the net charge enclosed by the Gaussian surface is equal to the total charge on the spherical shell $\left(Q_{enc} = q = \sigma \times 4\pi R^2\right)$.

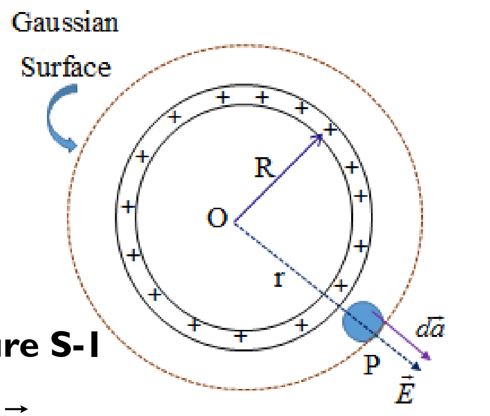


Figure S-I

- For every point of Gaussian surface S, the magnitude of the electric field E is same and Electric filed $\vec{E}$ is directed radially outward as does $d\vec{a}$.
- The total electric flux through the Gaussian surface S is given by $\oint_S \vec{E}\cdot d\vec{a} = \oint_S E\ da = E\oint_S da = E\ (4\pi r^2)$.
- From Gauss's Law:

$$\oint_S \vec{E}\cdot d\vec{a} = \frac{1}{\varepsilon_0}Q_{enc} \Rightarrow E\ (4\pi r^2) = \frac{1}{\varepsilon_0}\left(\sigma \times 4\pi R^2\right)$$

$$\therefore \quad \boxed{E = \frac{\sigma}{\varepsilon_0}\frac{R^2}{r^2}} \qquad \left[E = \frac{\left(\frac{q}{4\pi R^2}\right)R^2}{\varepsilon_0\ r^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\right]$$

Therefore, the field at a point outside the shell is equivalent to a point charge q located at the centre.

Application of Gauss's Law

ELECTROSTATICS FIELD

I. Use Gauss's Law to find the electric field outside, on and inside a spherical shell of radius *R* , which carries a uniform surface charge density σ .

Electric Field on the Surface of a Spherical Shell $(r = R)$:

$$\boxed{E = \frac{\sigma}{\varepsilon_0}}$$

$$\left[E = \frac{\left(\frac{q}{4\pi R^2}\right)}{\varepsilon_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}\right]$$

Electric field inside a spherical shell:

- Let the point P be at a distance r from the centre of a sphere so that $r < R$.

  With O as centre and $OP = r$, as radius, a spherical Gaussian surface S is drawn as shown in Figure S-2.

  The total charge enclosed by the Gaussian surface S is $Q_{enc} = 0$.

- From Gauss's Law,

$$\oint_S \vec{E} \cdot d\vec{a} = \frac{1}{\varepsilon_0} Q_{enc}$$

$$\therefore \boxed{E = 0}$$

Therefore, the electric field inside a spherical shell is zero.

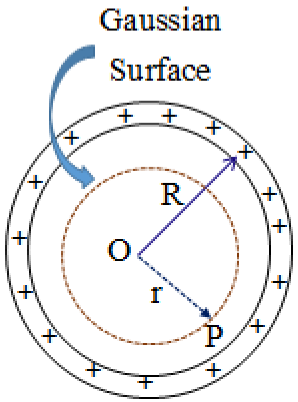


Figure S-2

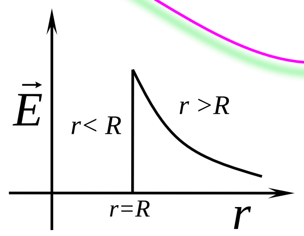


Figure S-3 : A plot of E versus r

Application of Gauss's Law

2. Electric Field outside a uniformly charged solid sphere of radius *R* and total charge *q* .

Electric Field outside a uniformly charged solid sphere

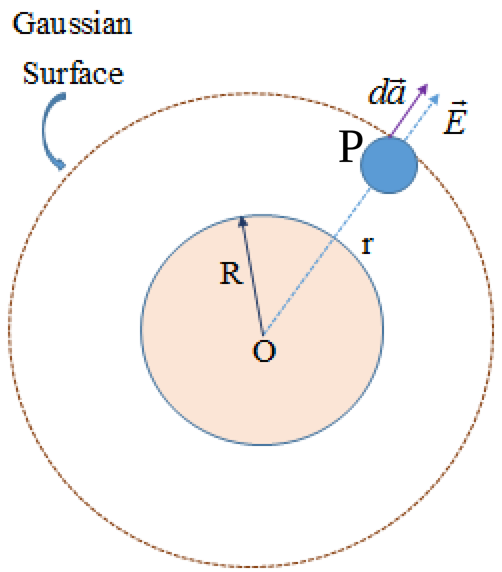
From Gauss's Law:

$$\oint_S \vec{E} \cdot d\vec{a} = \frac{1}{\varepsilon_0} Q_{enc}$$

$$\Rightarrow E\ (4\pi r^2) = \frac{1}{\varepsilon_0} q$$

$$\Rightarrow \qquad E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}$$

$$\therefore \qquad \boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r}}$$



Application of Gauss's Law

ELECTROSTATICS FIELD

3. Use Gauss's Law to find the electric field outside, on and inside a uniformly charged solid sphere of radius *R*, which carries a volume charge density ρ .

Electric Field Outside a Uniformly Charged Solid Sphere:

- From Gauss's Law:

$$\oint_S \vec{E}\cdot d\vec{a} = \frac{1}{\varepsilon_0} Q_{enc} \qquad \Rightarrow E\ (4\pi r^2) = \frac{1}{\varepsilon_0}\left(\rho \times \frac{4}{3}\pi R^3\right)$$

$$\therefore \qquad \boxed{E = \frac{\rho}{3\varepsilon_0}\frac{R^3}{r^2}} \qquad \left[E = \frac{\left(\dfrac{Q}{\frac{4}{3}\pi R^3}\right)}{3\varepsilon_0}\frac{R^3}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \right]$$

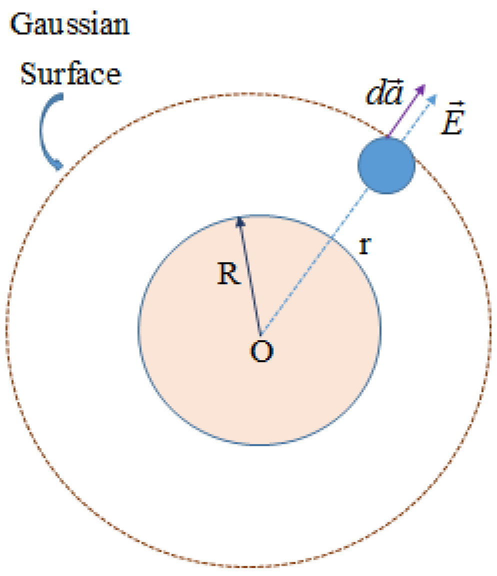


Figure Ss-2

Electric Field On the Surface of a Uniformly Charged Solid Sphere

$$\boxed{E = \frac{\rho R}{3\varepsilon_0}} \qquad \left[E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2} \right]$$

Application of Gauss's Law

ELECTROSTATICS FIELD

3. Use Gauss's Law to find the electric field outside, on and inside a uniformly charged solid sphere of radius *R*, which carries a volume charge density ρ.

Electric Field Inside a Uniformly Charged Solid Sphere:

- **From Gauss's Law**:

$$\oint_S \vec{E}\cdot d\vec{a} = \frac{1}{\varepsilon_0} Q_{enc}$$

$$\Rightarrow E\ (4\pi r^2) = \frac{1}{\varepsilon_0}\left(\rho\times\frac{4}{3}\pi r^3\right)$$

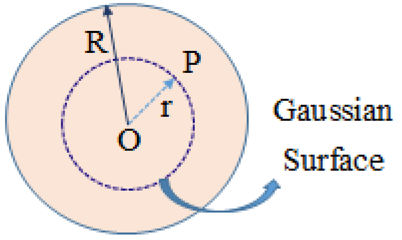


Figure Ss-3

$$\therefore \quad \boxed{E = \frac{\rho r}{3\varepsilon_0}}$$

$$\left[E = \frac{\left(\dfrac{Q}{\frac{4}{3}\pi R^3}\right) r}{3\varepsilon_0} = \frac{1}{4\pi\varepsilon_0}\frac{Qr}{R^3}\right]$$

Thus, the electric field inside a uniformly charged sphere is directly proportional to the distance of field point from the centre of sphere.

- **At the centre of the sphere r =0, Electric Field is Zero.**

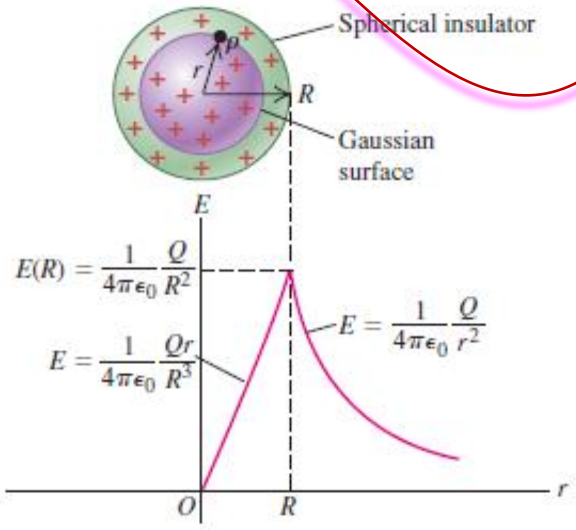


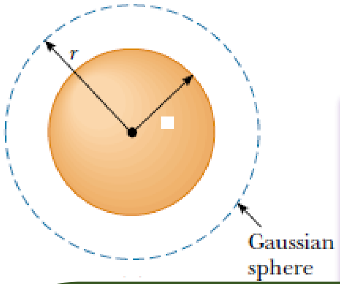
Figure Ss-3 : A plot of E versus r

Application of Gauss's Law

ELECTROSTATICS FIELD

4. Find the electric field outside and inside a sphere, which carries a charge density proportional to the distance from the origin $\rho = kr$, for some constant k.

Electric Field Outside a Sphere:



Let the point P be at a distance r from the centre of a sphere so that $r > R$ or $r < R$. With O as centre and, $OP = r$ as radius, a spherical Gaussian surface is drawn as shown in Figure.

By symmetry, the magnitude of the electric field is constant everywhere on the spherical Gaussian surface and is normal to the surface at each point.

The total electric flux through the Gaussian surface is given by

$$\oint_S \vec{E}\cdot d\vec{a} = \oint_S E\ da = E\oint_S\ da = E\ (4\pi r^2)$$

From Gauss's Law:

$$\oint_S \vec{E}\cdot d\vec{a} = \frac{1}{\varepsilon_0}\left[\int_V \rho d\tau\right]$$

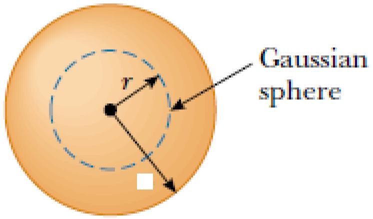
$$\Rightarrow E\ (4\pi r^2) = \frac{1}{\varepsilon_0}\left[\int_V (kr) r^2 \sin\theta dr d\theta d\phi\right]$$

$$= \frac{k}{\varepsilon_0}\left[\left\{\int_0^R r^3 dr\right\}\left\{\int_0^\pi \sin\theta d\theta\right\}\left\{\int_0^{2\pi} d\phi\right\}\right]$$

$$= \frac{k}{\varepsilon_0}\left(\frac{R^4}{4}\right)(2)(2\pi)$$

$$\therefore \boxed{E = \frac{k}{4\varepsilon_0}\frac{R^4}{r^2}}$$

Electric Field Inside a Sphere:



From Gauss's Law:

$$\oint_S \vec{E}\cdot d\vec{a} = \frac{1}{\varepsilon_0}\left[\int_V \rho d\tau\right]$$

$$\Rightarrow E(4\pi r^2) = \frac{1}{\varepsilon_0}\left[\int_V (kr) r^2 \sin\theta dr d\theta d\phi\right]$$

$$= \frac{k}{\varepsilon_0}\left[\left\{\int_0^r r^3 dr\right\}\left\{\int_0^\pi \sin\theta d\theta\right\}\left\{\int_0^{2\pi} d\phi\right\}\right]$$

$$= \frac{k}{\varepsilon_0}\left(\frac{r^4}{4}\right)(2)(2\pi)$$

$$\therefore \boxed{E = \frac{k\ r^2}{4\varepsilon_0}}$$

Summary

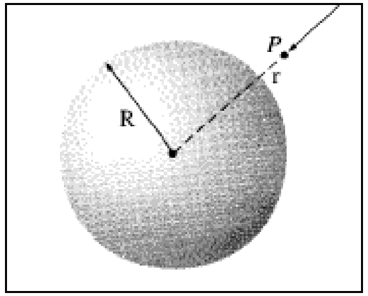
- 1. **Spherical Shell:**

  - For a spherical shell of radius R which carries a uniform surface charge density σ :

Electric field: $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{\sigma}{\varepsilon_0}\frac{R^2}{r^2}$; Outside the spherical shell

$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2} = \frac{\sigma}{\varepsilon_0}$; On the surface of spherical shell

$E = 0$; Inside the spherical shell



Total charge on the spherical shell, $q = \sigma\left(4\pi R^2\right)$

2. **Uniformly Charged Solid Sphere**

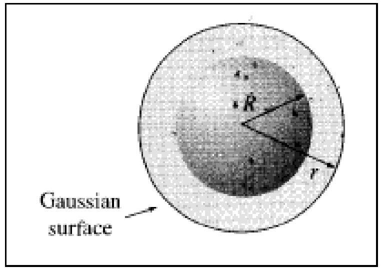
- For a uniformly charged solid sphere of radius which carries a volume charge density ρ:

Electric field: $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{\rho}{3\varepsilon_0}\frac{R^3}{r^2}$; Outside the sphere

$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2} = \frac{\rho R}{3\varepsilon_0}$; On the surface of sphere

$E = \frac{1}{4\pi\varepsilon_0}\frac{qr}{R^3} = \frac{\rho r}{3\varepsilon_0}$ Inside the spherical sphere

$E = 0$ At the centre



Total charge on the sphere, $q = \rho\left(\frac{4}{3}\pi R^3\right)$

Electrostatic Field

ELECTROSTATICS FIELD

Notes:

- Two charges, each of , q separated by a distance . The net electric field at a distance x from a charge and on the line joining them is
$$\boxed{E = \frac{1}{4\pi\varepsilon_0} q\left[\frac{1}{x^2} - \frac{1}{(d-x)^2}\right]}$$

- A charge of 0.80 nC is placed at the center of a cube that measures 4.0 m along each edge. What is the electric flux through one face of the cube?

The electric flux through one face of the cube:
$$\frac{1}{6}\left[\oint_S \vec{E}\cdot d\vec{a}\right] = \frac{1}{6}\left[\frac{1}{\varepsilon_0}Q_{enc}\right] = \frac{1}{6}\left[\frac{1}{8.85\times10^{-12}}\times 0.80\times10^{-9}\right] = 15\ N\cdot m^2/C$$

- A point charge (5.0 pC) is located at the center of a spherical surface (radius = 2.0 cm), and a charge of 3.0 pC is spread uniformly upon this surface. Determine the magnitude of the electric field 1.0 cm from the point charge.
$$\left[E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{1}{4\pi\varepsilon_0}\left[\frac{5\times10^{-12}}{\left(1\times10^{-2}\right)^2}\right] = 450\ N/C\right]$$

- A hollow metallic sphere of radius 0.1 m has 10^{-8} C of charge uniformly spread over it. The electric field intensity at point 7 cm away from the centre is $zero$.

Text Books & References

1. **David J. Griffith**, Introduction to Electrodynamics
2. **R.A. Serway and J.W. Jewett**, Physics for Scientist and Engineers with Modern Physics
3. **Halliday and Resnick**, Fundamental of Physics
4. **D. Halliday, R. Resnick, and K. Krane** , Physics, Volume 2, Fourth Edition


Thank you