Lecture 13

Magnetostatics Field in Matter (Contd.)

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Outline

1. Ampere's law in magnetized materials

2. Magnetic Susceptibility (χ_m) and permeability (μ)

3. Dia-, Para- and Ferro-magnetic materials
   - Diamagnetic Materials
   - Paramagnetic Materials
     - Curie law
   - Ferromagnetic Materials
   - Domain Theory of Ferromagnetism
   - Hysteresis loop in ferromagnetic materials
   - Hysteresis loss in ferromagnetic materials

Ampere's law in magnetized materials

We know that the effect the magnetization is to produce the bound current $\vec{J}_b = \nabla \times \vec{M}$ within the material and $\vec{K}_b = \vec{M} \times \hat{n}$ on the surface. The magnetic field due to magnetization of the medium is just the field produced by those bound currents. The magnetic field within the material is the field due to bound current and the field due to everything else called free current.

The total current within the material is

$$\vec{J} = \vec{J}_f + \vec{J}_b \tag{1}$$

Now from differential form of Ampere's law reads

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

Ampere's law in magnetized materials (contd.)

$$\implies \nabla \times \vec{B} = \mu_0 \left(\vec{J}_f + \vec{J}_b \right)$$

$$\implies \nabla \times \frac{\vec{B}}{\mu_0} = \vec{J}_f + \nabla \times \vec{M}$$

$$\implies \nabla \times \left(\frac{\vec{B}}{\mu_0} - \vec{M} \right) = \vec{J}_f$$

$$\implies \nabla \times \vec{H} = \vec{J}_f \tag{2}$$

where

$$\vec{H} = \left(\frac{\vec{B}}{\mu_0} - \vec{M} \right) \tag{3}$$

is called magnetic field strength. ($\vec{B}$ should actually be called as magnetic field induction).

Ampere's law in magnetized materials (contd.)

Taking the surface integral of equation (2), we get

$$\int_S \left(\nabla \times \vec{H}\right) \cdot d\vec{a} = \int_S \vec{J}_f \cdot d\vec{a}$$

Applying the Stoke's theorem in left hand side

$$\oint_L \vec{H} \cdot d\vec{l} = I_{fenc} \tag{4}$$

where $I_{fenc} = \int_S \vec{J}_f \cdot d\vec{a}$ is the total free current enclosed by the Amperian loop drawn around the free current. Here the magnetic field strength $\vec{H}$ only refers to the free current I_f.

Ampere's law in magnetized materials (contd.)

Question:- A long copper rod of radius R carries a uniformly distributed (free) current I (Figure 1). Find H inside and outside the rod.

Solution:

To find H at a distance s from the axis of the rod, consider coaxial Amperian circle of the radius s as shown in figure 1.

For outside, $s > R$

Total free current enclosed by the loop $I_f = I$

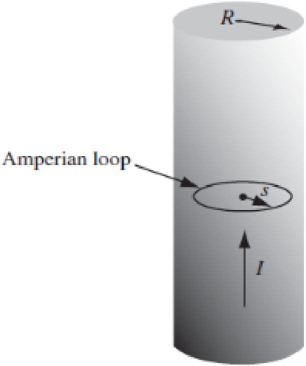

Figure 1

Ampere's law in magnetized materials (contd.)

According to Amper's law, in material medium

$$\oint \vec{H} \cdot d\vec{l} = I_{fenc} \implies H(2\pi s) = I \implies H_{out} = \frac{I}{2\pi s}\hat{l}$$

For inside , $s < R$

Total free current enclosed by the loop $\quad I_{fenc} = \frac{I\pi s^2}{\pi R^2} = \frac{Is^2}{R^2}$

Now the integral's from of Ampere's law in material medium

$$\oint \vec{H} \cdot d\vec{l} = I_{fenc} \implies H(2\pi s) = \frac{Is^2}{R^2} \implies H_{in} = \frac{Is}{2\pi R^2}\hat{l}$$

Magnetic Susceptibility (χ_m) and permeability (μ)

For most substances, magnetization is directly proportional to magnetizing field strength $\vec{H}$, provided $\vec{H}$ not too large. i.e.

$$\vec{M} \propto \vec{H} \ \Rightarrow \ \vec{M} = \chi_m \vec{H} \tag{5}$$

The materials which obey equation (5) are called linear dielectric and the constant of proportionality χ_m is called magnetic susceptibility.
If χ_m is negative, the material is diamagnetic and the magnetic induction is weakened by the presence of material.
If χ_m is small positive, the material is paramagnetic and the magnetic induction is strengthened by the presence of material.

If χ_m is large positive, the material is ferromagnetic. However in ferromagnetic materials $\vec{M}$ is not accurately proportional to $\vec{H}$ and so χ_m is not constant.

Also we have

$$\begin{aligned} \vec{H} &= \frac{\vec{B}}{\mu_0} - \vec{M} \\ \implies \vec{H} &= \frac{\vec{B}}{\mu_0} - \chi_m \vec{H} \\ \implies \vec{B} &= \mu_0 (1 + \chi_m) \vec{H} \\ \implies \vec{B} &= \mu \vec{H} \end{aligned} \tag{6}$$

Where

$$\mu = \mu_0 (1 + \chi_m) \tag{7}$$

is called permeability of the medium. Equation (6) shows that, in linear medium the magnetic field induction $\vec{B}$ is also proportional to the magnetizing field. If $\vec{B}_0$ magnetic field induction in free space, then the ratio

$$\frac{B}{B_0} = \frac{\mu}{\mu_0} = 1 + \chi_m \tag{8}$$

is called relative permeability of the medium (μ_r).

Obviously,

$$\mu_r = 1 + \chi_m \tag{9}$$

Magnetic Susceptibility (χ_m) and permeability (μ) (contd.)

We may also clarify magnetic materials in terms of relative permeability μ_r.

$$\mu_r < 1 \rightarrow \text{Diamagnetic}$$

$$\mu_r > 1 \rightarrow \text{Paramagnetic}$$

$$\mu_r \gg 1 \rightarrow \text{Ferromagnetic}$$

For vacuum there is no medium to be magnetized so $\chi_m = 0$ and $\mu = \mu_0$. Hence, the magnetic induction in vacuum is

$$\vec{B}_0 = \mu_0 \vec{H}$$

i.e. $\mu = \mu_0$. So μ_0 is called permeability of free space.

Magnetic Susceptibility (χ_m) and permeability (μ) (contd.)

Also

$$
\begin{aligned}
\vec{J}_b &= \nabla \times \vec{M} \\
&= \nabla \times \left(\chi_m \vec{H}\right) \\
&= \chi_m \left(\nabla \times \vec{H}\right) \\
\implies \vec{J}_b &= \chi_m \vec{J}_f \qquad (10)
\end{aligned}
$$

Therefore volume bound current density in a linear medium is proportional to the volume free current density. Unless free current actually flows through the material, all bound current will be at the surface.

Diamagnetic Materials

Those substances which when placed in magnetizing field are magnetized feebly in the opposite direction of applied field. The diamagnetic property arises due to the orbital motion of the electron. When the magnetic field is applied the orbital dipole moment changes in such a way that the atoms of the material acquires the net dipole moment in the direction opposite to the applied field and hence the material is said to be magnetized. The substance whose outermost orbit has an even number of electrons purely exhibits the diamagnetic behavior. Since the electrons have spins opposite to each other, the net magnetic moment of each atom in the absence of magnetic field is

Diamagnetic Materials (contd.)

zero. If these materials are brought close to the pole of a powerful electromagnet they are repelled away from a magnet.

Diamagnetic materials have small negative susceptibility which is practically independent of temperature. Relative permeability is less than unity.

For example: Copper, Silver, Bismuth, Antimony, gold, water, alcohol, quartz, H_2, CO_2, N_2 etc.

Paramagnetic Materials

Those substances which when placed in magnetizing field are magnetized feebly in the direction of magnetizing field are called paramagnetic substances. This property arises due to the spin motion of the electron. The substance whose outermost orbit has an odd number of electrons can exhibit the paramagnetic nature. If these substances are brought close to the pole of a powerful electromagnet they get attracted towards the magnet.
Paramagnetic materials have small positive susceptibility which depends on temperature and the relative permeability is slightly greater than unity.

Paramagnetic Materials (contd.)

For example: Platinum, aluminum, chromium, manganese, copper sulfate, liquid oxygen, solutions of salt of irons and nickel. The temperature dependence of many paramagnetic materials is governed by the experimentally found Curie law, which states that the susceptibility χ_m is inversely proportional to the absolute temperature T i.e.

$$\chi_m \propto \frac{1}{T} \Rightarrow \chi_m = \frac{C}{T}$$

Where C is called Curie constant in Kelvin.

Ferromagnetic Materials

Those substances which when placed in magnetizing field are strongly magnetized in the directions of magnetizing field are called ferromagnetic substances. This property is found in the substances which are generally like paramagnetic materials. If these substances are brought close to the pole of a powerful electromagnet they are strongly attracted towards the magnet.

Ferromagnetic materials have large positive susceptibility and relative permeability is much greater than unity (few thousands).

For example: Iron, Nickel, Cobalt, gadolinium, and their alloys.

Domain Theory of Ferromagnetism

A magnetic domain is a region within a magnetic material which has uniform magnetization. In 1907, Weiss proposed a domain theory to explain ferromagnetism. According to this theory, a single crystal of ferromagnetic solid consists of large number of small regions, and each region is spontaneously magnetized to saturation extent called a domain as shown in figure. The magnetization directions of different domains of the specimen are random so that the resulting magnetization of the material is zero in the absence of an external magnetic field.

Domain Theory of Ferromagnetism (contd.)

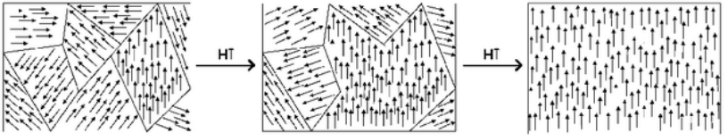

Figure 2: heading

When the magnetic field strength is applied domains align or tend to align along direction of the field and hence the material is said to be magnetized. During this process the magnetization increases in two ways: either (i) the shifting of the domain walls such domains whose magnetization already along direction of the field or (ii) the rotation of entire domain when the field is sufficiently large as shown in figure 3

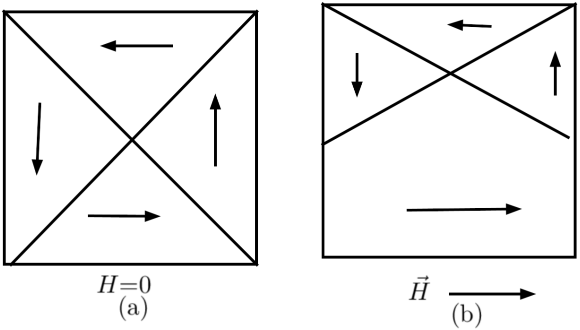

Figure 3

Dia-, Para- and Ferro-magnetic materials

Hysteresis loop in ferromagnetic materials

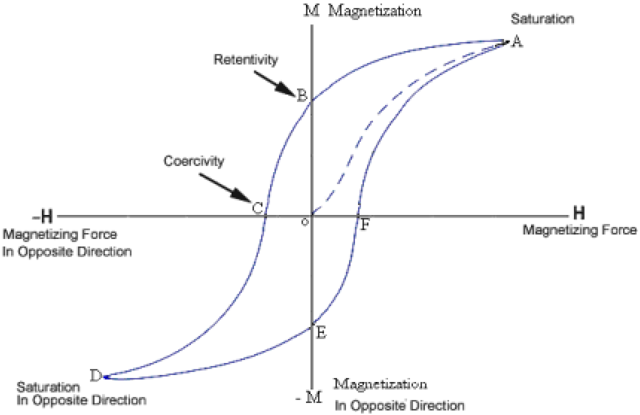

Figure 4

Hysteresis loop in ferromagnetic materials (contd.)

When a magnetic material is subjected to a gradually increasing magnetizing field, the intensity of magnetization M increase with the increase in strength of magnetizing field H along the path OA. This curve is known as virgin or initial magnetization curve.

At $H = H_0$ the intensity of magnetization assumes a steady value M_{max}. The magnetic material cannot be magnetized more strongly than this and at this stage the material is said to have reached the magnetic saturation limit.

Now if the magnetizing field H is gradually decreased the intensity of magnetization M will not decrease the same path OA, but will decrease along the path AB such that when H becomes zero M will not become zero but has a definite value M=OB.

Hysteresis loop in ferromagnetic materials (contd.)

The value of intensity of magnetization of the magnetic material even when the magnetizing field is reduced to zero is called its **retentivity** or **remanence** or **residual magnetism**.

Now if the direction of magnetizing field is reversed the intensity of magnetization takes along the path BC till it become zero at C. Thus to reduce the residual magnetism to zero, a magnetizing field is equal to the value OC has to be applied in reverse direction. The value of reverse magnetizing field require to reduce the residual magnetism to zero is called the **coercive force** or **coercivity**.

When the magnetizing field is further increased in reverse direction, the intensity of magnetization increases along the path CD and acquires the magnetic saturation limit at point D. If the magnetizing field H is now reduced to zero, the intensity of magnetization M follow the path DE. Finally if H is increased in the original direction M follow the path EFA and a closed curve ABCDEFA is obtained as shown in figure 4. This closed curve is known as **hysteresis loop**. On repeating the process the same closed curve is obtained again and again but never the portion OA. It is seen that M always lags behind H. This lagging of M behind H is called **hysteresis**.

The shape of this loop varies from one material to another. Some ferrites have an almost rectangular hysteresis loop. These ferrites are used in digital computers as magnetic information storage device. The area of a hysteresis

Hysteresis loop in ferromagnetic materials (contd.)

loop gives energy loss (hysteresis loss) per unit volume during one cycle of the periodic magnetization of the ferromagnetic materials. This energy loss is in the form of heat. It is therefore desirable that materials used in electronic generators, motors and transformers should have tall but narrow hysteresis loops so that hysteresis losses are minimal.

Permanent magnets (hard magnetic materials) are device which retain their magnetic field indefinitely i.e. coercivity and area of hysteresis loop are large.

Hysteresis loss in ferromagnetic materials

The amount of energy lost (in the form of heat) per unit volume of ferromagnetic substance when the substance undergoes one cycle of magnetization is known as **hysteresis loss**.

Consider a unit volume of ferromagnetic substance. Let $\vec{m}_i$ be the magnetic moment of i^{th} atomic dipole which makes an angle θ_i with the field H. Only the component of $\vec{m}_i$ along the direction of field contributes to the magnetization. If N be the number of atomic dipoles in the given unit volume of substance, then magnetization is

$$M = \sum_{i=1}^{N} m_i \cos\theta_i \tag{11}$$

Hysteresis loss in ferromagnetic materials (contd.)

Differentiating both sides

$$dM = -\sum_{i=1}^{N} m_i \sin\theta_i \, d\theta_i \tag{12}$$

Each dipole experiences a torque and work done by torques on all dipoles in one complete cycle of magnetization is hysteresis loss. Therefore

$$\begin{aligned} \text{hysteresis loss} &= \oint \left(\sum_{i=1}^{N} \tau_i d\theta_i \right) = \oint \left(\sum_{i=1}^{N} m_i B \sin\theta_i d\theta_i \right) \\ &= -\oint B \left(-\sum_{i=1}^{N} m_i \sin\theta_i d\theta_i \right) = -\oint B dM \end{aligned}$$

Hysteresis loss in ferromagnetic materials (contd.)

We know that

$$H = \frac{1}{\mu_0}B - M \implies dH = \frac{1}{\mu_0}dB - dM \implies dM = \frac{1}{\mu_0}dB - dH.$$

Therefore

$$\begin{aligned}\text{hysteresis loss} &= -\oint B\left(\frac{1}{\mu_0}dB - dH\right)\\ &= -\frac{1}{\mu_0}\oint B\,dB + \oint B\,dH\end{aligned}$$

Since $\oint B\,dB = 0$ $\quad$ ($\because$ gives the line so area enclosed =0)

$$\text{hysteresis loss} = \oint BdH = \text{area enclosed by } B - H \text{ loop}$$

End of Lecture 13

Thank you