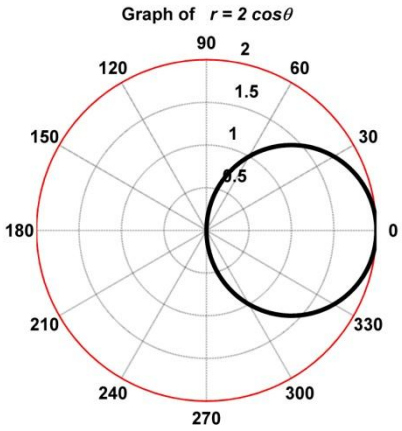
Polar Equations and their Graphs

Equations of Circles:

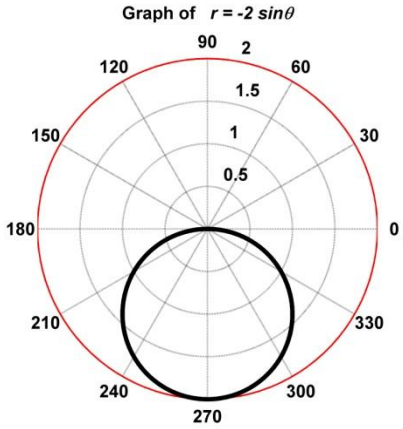
- $r = \pm 2a\cos\theta$, $r = \pm 2a\sin\theta$: Circles with center along one of the coordinate axes and radius a.
- $r = a$: Circle with center at the origin and radius a.

NOTE: a is NOT equal to **0**.The graphs of Circles are generated as the angle increases from **0** to 2π.

➢ $r = 2\cos\theta$: Equation of a **Circle** with center at **(1, 0)** and radius **1.**



➢ $r = -2\sin\theta$: Equation of a **Circle** with center at $(1, -\frac{\pi}{2})$ and radius **1**.

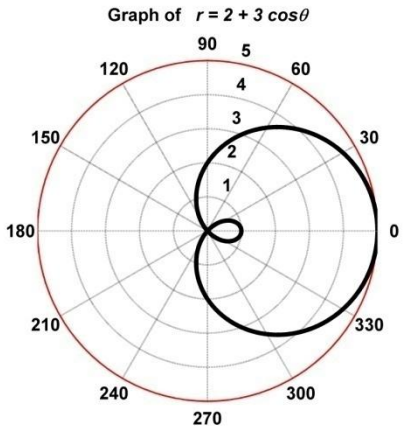


Equations of Limacons : $r = a \pm b\cos\theta$, $r = a \pm b\sin\theta$, a and b are NOT equal to **0**.

- If $\left|\frac{a}{b}\right| < 1$, the graph of the Limacon has an **inner loop**!
- If $1 < \left|\frac{a}{b}\right| < 2$, the graph of the Limacon is "**dimpled**".
- If $\left|\frac{a}{b}\right| \geq 2$, the graph of the Limacon is considered "**convex (oval)**".

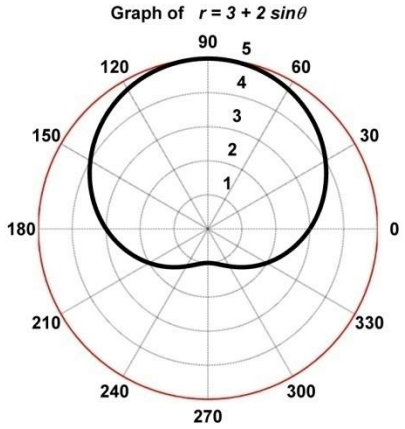
The graphs of Limacons are generated as the angle increases from **0** to 2π.

➢ **Limacon with inner loop**: $r = 2 + 3\cos\theta$



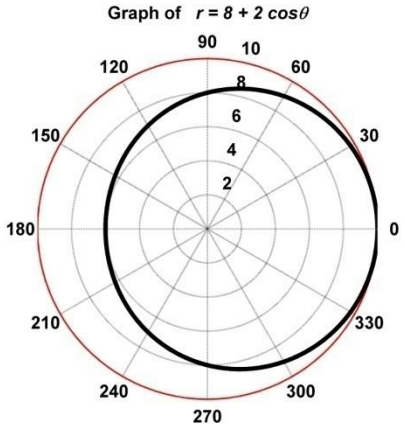
Note that: $\boldsymbol{a = 2, b = 3}$ and $\left|\frac{2}{3}\right| < 1$.

- **Dimpled Limacon: $\boldsymbol{r = 3 + 2sin\theta}$**



Note that: $\boldsymbol{a = 3, b = 2}$ and $\left|\frac{3}{2}\right| < 2$.

- **Convex (Oval) Limacon: $\boldsymbol{r = 8 + 2cos\theta}$**



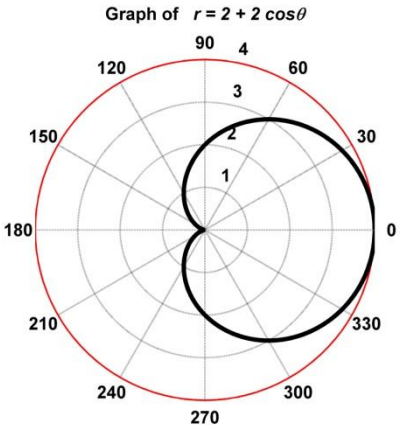
Note that: $\boldsymbol{a = 8, b = 2}$ and $\left|\frac{8}{2}\right| \geq 2$.

Equations of Cardioids:

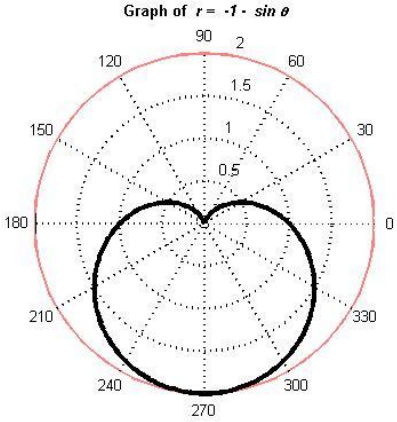
$\boldsymbol{r = a \pm bcos\theta}$, $\boldsymbol{r = a \pm bsin\theta}$, where $\left|\frac{a}{b}\right| = 1$.

The graphs of Cardioids are generated as the angle increases from $\mathbf{0}$ to $\boldsymbol{2\pi}$.

- $\boldsymbol{r = 2 + 2cos\theta}$: Cardioid



- $\boldsymbol{r = -1 - sin\theta}$: Cardioid



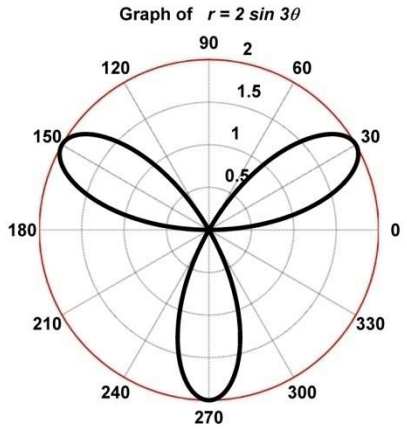
Note that: $\boldsymbol{a = 2, b = 2}$ and $\left|\frac{2}{2}\right| = 1$.

<u>Equations of Rose Curves:</u>

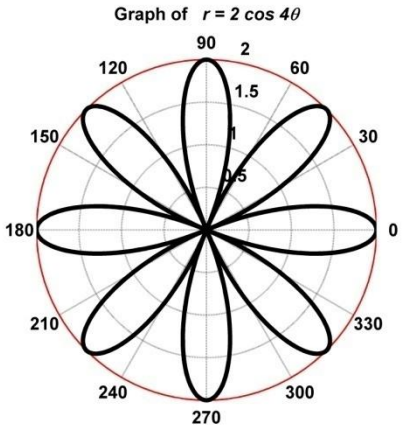
$\boldsymbol{r = asin\, n\theta}$, $\boldsymbol{r = acos\, n\theta}$, where $\boldsymbol{a}$ and $\boldsymbol{n}$ are NOT equal to $\mathbf{0}$.

- When $\boldsymbol{n}$ is odd, the entire curve is generated as $\boldsymbol{\theta}$ increases from $\mathbf{0}$ to $\boldsymbol{\pi}$. The curve has $\boldsymbol{n}$ petals.
- When $\boldsymbol{n}$ is even, the entire curve is generated as $\boldsymbol{\theta}$ increases from $\mathbf{0}$ to $\boldsymbol{2\pi}$. The curve has $\boldsymbol{2n}$ petals.

- $\boldsymbol{r = 2\,sin\,3\theta}$: Equation of a Rose Curve with 3 petals



- $\boldsymbol{r = 2\,cos\,4\theta}$: Equation of a Rose Curve with 8 petals



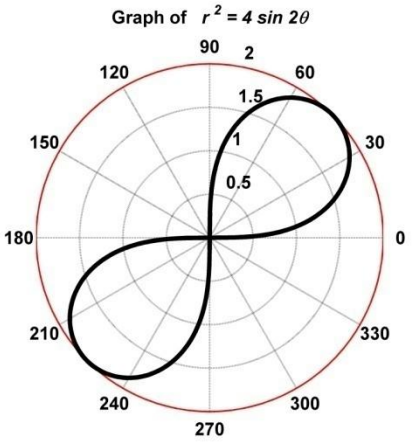
Note that: $\boldsymbol{n = 3}$ is odd, therefore the rose curve has **3** petals.

Note that: $\boldsymbol{n = 4}$ is even, therefore the rose curve has $\boldsymbol{2(4) = 8}$ petals.

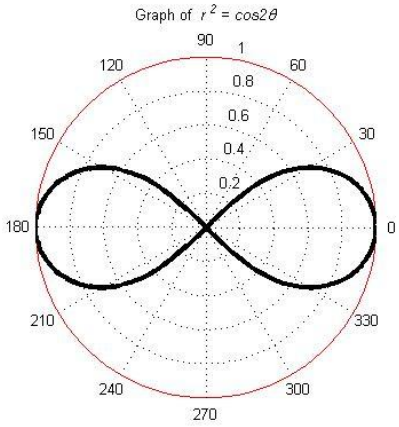
<u>Equations of Lemniscates:</u> $\boldsymbol{r^2 = a^2 sin2\theta}$, $\boldsymbol{r^2 = a^2 cos2\theta}$, where $\boldsymbol{a}$ is NOT equal to **0**.

The graphs of Lemniscates are generated as the angle increases from **0** to $\boldsymbol{2\pi}$.

- $\boldsymbol{r^2 = 4sin2\theta}$: Equation of a Lemniscate



- $\boldsymbol{r^2 = cos2\theta}$: Equation of a Lemniscate



THE END