General Physics I (PHYS 101)

Lecture 10

Wave and Oscillation

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Outline

The Compound (Physical) Pendulum

The physical pendulum (also called a compound pendulum) is a rigid body of any shape capable to oscillate in a vertical plane about a horizontal axis passing through it.

The point of intersection of vertical plane and horizontal axis is called point of suspension S in figure 1. C is the center of mass of the pendulum. The distance between the point of suspension S and center of mass C is called length of the pendulum. It is denoted by l.

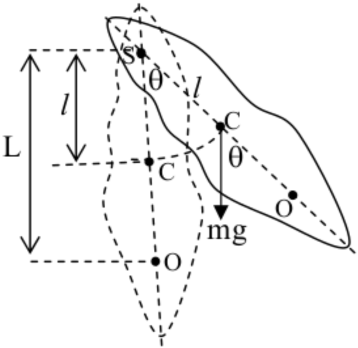

Figure 1: Oscillation of compound pendulum

The compound pendulum can be converted into a simple pendulum by concentrating the whole mass of the pendulum at a point. When the mass of the compound pendulum is concentrated at a point to form a simple pendulum such that the time period of the resulting simple pendulum is equal to that of the compound pendulum, then the point of concentration is called point of oscillation O in figure 1. The distance between the point of suspension S and point of oscillation O is called length of equivalent simple pendulum and denoted by L.

Now the rigid body (physical pendulum) be displaced from equilibrium position by a small angle θ at any time t. The restoring torque τ for an angular displacement θ is given by

$$\tau = -mgl\sin\theta$$

Negative sign shows couple is oppositely directed to the displacement θ.

Now, if I be the moment of inertia of a body about an axis of rotation and $\frac{d^2\theta}{dt^2}$ is its angular acceleration.

The Compound (Physical) Pendulum (contd.)

From Newton's second law of motion, the restoring torque is

$$\tau_{res} = I\frac{d^2\theta}{dt^2}$$

Therefore

$$\begin{aligned} I\frac{d^2\theta}{dt^2} &= -mgl\sin\theta \\ \implies \frac{d^2\theta}{dt^2} &= -\frac{mgl}{I}\sin\theta \end{aligned} \tag{1}$$

For a sufficiently small angular displacement, $\sin\theta \approx \theta$ (in radian).

Then equation (1) is reduced to

$$\frac{d^2\theta}{dt^2} \approx -\frac{mgl}{I}\theta \tag{2}$$

Comparing the equation (2) with the equation of simple harmonic oscillation $a = -\omega_0^2 x$, the angular frequency $\omega_0 = \sqrt{\frac{mgl}{I}}$.
Hence the time period of the compound pendulum is given by

$$T = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{I}{mgl}}$$

Now, if I_G is the moment of inertia and K is the radius of gyration of the compound pendulum about the horizontal axis passing through the center of mass are parallel to the axis passing through the center of suspension. Then using the theorem of parallel axes

$$I = I_G + ml^2 = mK^2 + ml^2 = m\left(K^2 + l^2\right)$$

Hence the period of oscillation is

$$T = 2\pi\sqrt{\frac{m\left(K^2+l^2\right)}{mgl}} = 2\pi\sqrt{\frac{\left(K^2+l^2\right)}{gl}} = 2\pi\sqrt{\frac{K^2/l+l}{g}}$$

$$\implies T = 2\pi\sqrt{\frac{L}{g}} \tag{3}$$

Thus the time period of pendulum is same as that of simple pendulum of length $L = \dfrac{K^2}{l} + l$. This length is therefore called the length of equivalent simple pendulum or reduced length.

Now consider a point O on the other side C on a line with SC produced such that $SO = l + \dfrac{K^2}{l}$ or $CO = \dfrac{K^2}{l}$. The point O is called the center of oscillation or the point of oscillation corresponding to the center of suspension S.

Now if the pendulum is inverted and made to oscillate about the centre of oscillation, then the new time period T' will be obtained by substituting $\dfrac{K^2}{l}$ in place of l in equation (3)

Hence,

$$T' = 2\pi\sqrt{\frac{\dfrac{K^2}{K^2/l} + K^2/l}{g}} = 2\pi\sqrt{\frac{l + K^2/l}{g}} = T$$

Thus the time period about the center of oscillation is same as the time period about the center of suspension. Hence the point of suspension and point of oscillation are interchangeable or reciprocal to each other. Rearranging equation (3) we get

$$l^2 - \frac{T^2 g}{4\pi^2} l + K^2 = 0 \tag{4}$$

This equation (4) is quadratic in l, that means for each value of time period l has two roots l_1 and l_2 (say), such that $l_1 + l_2 = \frac{T^2 g}{4\pi^2}$ and $l_1 l_2 = K^2$. That means $T = 2\pi\sqrt{\frac{l_1 + l_2}{g}}$.

The Compound (Physical) Pendulum (contd.)

Both sum and product of length l_1 and l_2 are positive. Therefore for any value of time period T there are two points at distances l_1 and l_2 from the center of gravity and on the same side of it. There mus be two other points on the other side of the center of gravity for which the time period will be same. Hence there are four points collinear with center of gravity about which the time period is the same. The graph of T versus l is as shown in figure 2.

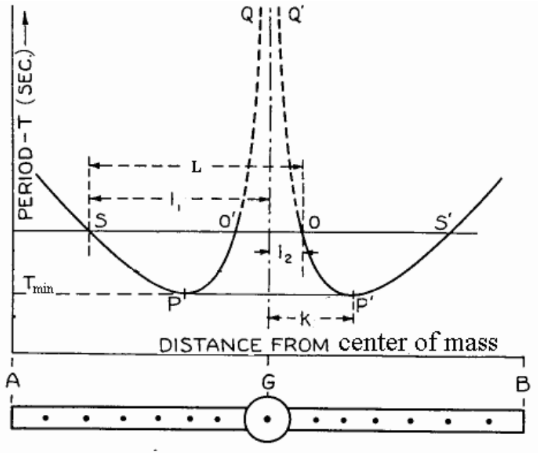

Figure 2: The time period versus length of compound pendulum

Condition for minimum time period:

From equation (3)

$$T^2 = \frac{4\pi^2}{g}\left(\frac{K^2}{l} + l\right)$$

Differentiating T with respect to l we get,

$$2T\frac{dT}{dl} = \frac{4\pi^2}{g}\left(-\frac{K^2}{l^2} + 1\right)$$

For T be minimum $\dfrac{dT}{dl} = 0$

Which gives

$$-\frac{K^2}{l^2} + 1 = 0$$

$$\implies \quad l^2 = K^2$$

$$\implies \quad l = \pm K$$

i.e. the time period will be minimum if $l = \pm K$.

Then

$$T_{min} = 2\pi\sqrt{\frac{2K}{g}}$$

But the time period will be maximum i.e. infinite when $l = 0$.