CHAPTER: 3

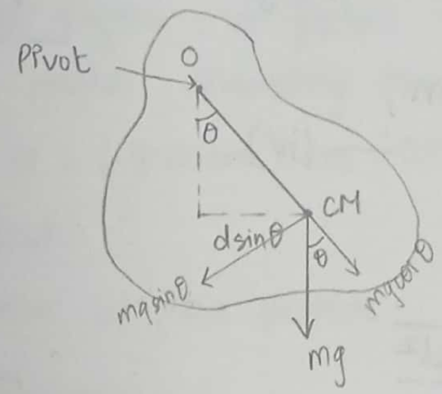
WAVE AND OSCILLATIONS

<Q.1> Consider a physical pendulum as shown in figure below. Representing its moment of inertia about an axis passing through its center of mass and parallel to the axes passing through its pivot point as I_{cm}. Show that its period is

$$T = 2\pi\sqrt{\frac{I_{cm}+md^2}{mgd}}$$

Here, d = distance between the pivot point and center of mass. Show that time period has minimum value when d satisfies $md^2 = I_{cm}$.

Soln:



Given figure is compound pendulum,
This oscillates about O as axis of rotation and distance between pivot and center of mass be 'd'.
Let the moment of inertia of pendulum about O is I.

Using law of motion, we get,

$$\sum \tau_z = I\alpha_z$$

or, $-mgd\sin\theta = I \cdot \frac{d^2\theta}{dt^2}$ —— (i) [∵ -ve sign indicates torque about O tends to decrease θ.]

For small angle θ,

$$\sin\theta \approx \theta$$

So,

$$-mgd\theta = I\frac{d^2\theta}{dt^2}$$

$$\text{or, } \frac{d^2\theta}{dt^2} = \left(\frac{-mgd}{I}\right)\theta \quad \text{— (ii)}$$

Above eqn (ii) is in the same form as $\frac{d^2x}{dt^2} = -\omega_0^2 x$.

This means the motion is SHM and hence,

angular frequency $(\omega_0) = \frac{mgd}{I}$

So,

Time period of pendulum $= 2\pi\sqrt{\frac{I}{mgd}} = 2\pi\sqrt{\frac{I_0}{mgd}}$ — (iii)

Using parallel axes theorem,

$$I_0 = I_{cm} + md^2 \quad \text{— (iv)}$$

Hence, eqn (iii) becomes,

$$T = 2\pi\sqrt{\frac{I_{cm} + md^2}{mgd}}$$

This is solved.

For T be minimum, $\frac{dT}{dd} = 0$

$$\text{or, } \frac{d}{dd}\left(2\pi\sqrt{\frac{I_{cm} + md^2}{mgd}}\right) = 0$$

$$\text{or, } 2\pi\frac{d}{dd}\left(\frac{I_{cm}}{mgd} + \frac{d}{g}\right)^{1/2} = 0$$

$$\text{or, } \cancel{2}\pi \times \frac{1}{\cancel{2}}\left(\frac{I_{cm}}{mgd} + \frac{d}{g}\right)^{-1/2} * \left[\frac{-I_{cm}}{mgd^2} + \frac{1}{g}\right] = 0$$

or, $\left(\frac{I_{cm}}{mgd}+\frac{d}{g}\right)^{-1/2}\left[\frac{-I_{cm}}{mgd}+\frac{1}{g}\right]=0$

either,

$\left(\frac{I_{cm}}{mgd}+\frac{d}{g}\right)^{-1/2}=0$ $\therefore I_{cm}=-md^2$ (rejected)

or,

$\frac{-I_{cm}}{mgd^2}=-\frac{1}{g}$ $\therefore I_{cm}=md^2$

Hence, when $md^2 = I_{cm}$, T has minimum value.

<Q.27>: In an engine, a piston oscillates with SHM so that its position varies according to the expression

$$x=(5.00\,cm)\cos\left(2t+\frac{\pi}{6}\right)$$

At t = 0, find.

a) the position of the particle
b) its velocity
c) its acceleration
d) the period and amplitude of the motion

Soln:

Given,

$$x=(5.00)\cos\left(2t+\frac{\pi}{6}\right)$$

At t = 0,

For (a);

$x = 5\times\cos\left(2\times 0+\frac{\pi}{6}\right)$ $\therefore x = 4.33$ cm.

$\therefore$ Position of particle ~~after~~ at t = 0 is 4.33 cm.

For (b):

We know,

$$v = \frac{dx}{dt}$$

$$= \frac{d}{dt}\left(5 \cancel{x} \cos\left(\frac{\pi}{6}\right)\right) \quad [\because \text{At } t = \cancel{0}]$$

$$= \cancel{5 \frac{d}{}} \; \frac{d}{dt}\left(5 \cdot \cos\left(2t + \pi/6\right)\right)$$

$$= -5 \cdot 2 \sin\left(\frac{\pi}{6}\right) \quad [\text{At } t = 0] = -5.00 \text{ cm/s}$$

$\therefore$ Velocity $= -5.00$ cm/s.

For (c):

We know,

$$a = -\omega^2 x$$

From q, $\omega = 2$

So

$$a = -(2)^2 \times 4.33$$

$$\therefore a = -17.32 \text{ cm/s}^2.$$

For (d):

Amplitude of motion $(A) = 5.00$ cm

Time period $(T) = \frac{2\pi}{\omega_0} = \frac{2\pi}{2} = \pi = 3.14$ sec.

<Q·37>: A 0·500 kg cart connected to a light spring from which the force constant is 20 N/m oscillates on a horizontal, frictionless air track.

a) Calculate the total energy of the system and the maximum speed of the cart if amplitude is 3·00 cm.

b) what is the total velocity of the cart when position is 2·00 cm?

c) Compute PE and KE of system at 2·00 cm.

Soln:

Given,

mass of cart (m) = 0·5 kg

Force constant (k) = 20 N/m.

For (a):

amplitude (A) = 3cm = 3×10^{-2} cm

Total energy (E) = ?

We know,

Total energy for a loaded spring $= \frac{1}{2} kA^2$

$= \frac{1}{2} \times 20 \times (3 \times 10^{-2})^2$

$= 9 \times 10^{-3}$ J

~~For (b):~~

~~amplitude (A) = 2·00cm = 2×10^{-2} cm~~

We know,

Maximum velocity in SHM (V_{max}) = $A\omega$

$= 3 \times 10^{-2} \times \sqrt{\frac{k}{m}}$ $\left(\because \omega = \sqrt{\frac{k}{m}}\right)$

$= 3 \times 10^{-2} \times \sqrt{\frac{20}{0 \cdot 5}}$

$= 0 \cdot 189$ m/s

For (b):

Amplitude $(A) = 3.00\,cm = 3\times10^{-2}\,m$

position of cart $(x) = 2.00\,cm = 2\times10^{-2}\,m$.

Total velocity $(V) = \omega\sqrt{A^2 - x^2}$

$$= \sqrt{\frac{k}{m}} \times \sqrt{(3\times10^{-2})^2 - (2\times10^{-2})^2}$$

$$= \sqrt{\frac{20}{0.5}} \times \sqrt{(3\times10^{-2})^2 - (2\times10^{-2})^2}$$

$\therefore V = \pm\, 0.141\ m/s^2$

For (c):

position of cart $(x) = 2.00\,cm$.

We know,

$KE = \frac{1}{2}k(A^2 - x^2)$ and $PE = \frac{1}{2}kx^2$

$= \frac{1}{2}\times 20 \times \{(3\times10^{-2})^2 - (2\times10^{-2})^2\}$ $\quad = \frac{1}{2}\times 20 \times (2\times10^{-2})^2$

$\quad = 4\times10^{-3}\ J$

$= 5\times10^{-3}\ J$

<Q.4>: A 10.6 kg object oscillates at the end of the vertical spring has spring constant of $2.05\times10^4\,N/m$. The effect of air resistance is represented by damping coefficient $b = 3.00\ Ns/m$. Calculate frequency of damped oscillation.

Soln:

Given,

mass of object $(m) = 10.6\ kg$

spring constant $(k) = 2.05\times10^4\ N/m$

damping coefficient $(b) = 3.00\ Ns/m$.

We know,

Frequency of damped oscillation $(f') = \frac{\text{Angular frequency } (\omega)}{2\pi}$

$$= \frac{\sqrt{\omega_0^2 - \gamma^2}}{2\pi}$$

$$= \frac{\sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}}{2\pi}$$

$$= \frac{\sqrt{\frac{\cancel{2.0\ 5\times10^4}\ 2.05\times10^4}{10^{-6}} - \left(\frac{3}{2\times10^{-6}}\right)^2}}{2\pi}$$

$\therefore f' = 7$ Hz

<Q.57>: A 2.00 kg object attached to a spring moves without friction and is driven by external force given by $F = (3.00\,N)\sin(2\pi t)$. The force constant is 20.0 N/m. Determine

a) the period
b) the amplitude of motion.

Soln:

Given,
mass of object (m) = 2.00 kg
force constant (k) = 20 N/m

Force (F) = $3.00\sin(2\pi t)$ —— (i)

Comparing eqn (i) with $F_{ext} = F_0 \sin(\omega t)$

so,
$F_0 = 2.00$ kg and $\omega = 2\pi$

~~We~~ For (a):

We know,

Time period $(T) = \frac{2\pi}{\omega'} = \frac{2\pi}{2\pi} = 1$ sec.

For (b):

We know,

Amplitude of motion $= \frac{f_0}{\sqrt{(\omega_0^2 - \omega'^2)^2 + 4\gamma^2\omega^2}}$

Since the body moves without friction; $\gamma = 0$.

Amplitude of motion $= \frac{F_0}{m\left(\frac{k}{m} - (2\pi)^2\right)}$

$$= \frac{3}{2\left(\frac{20}{2} - 4\pi^2\right)}$$

$$= 0.0509 \text{ m}$$

$\therefore A = 5.09$ cm