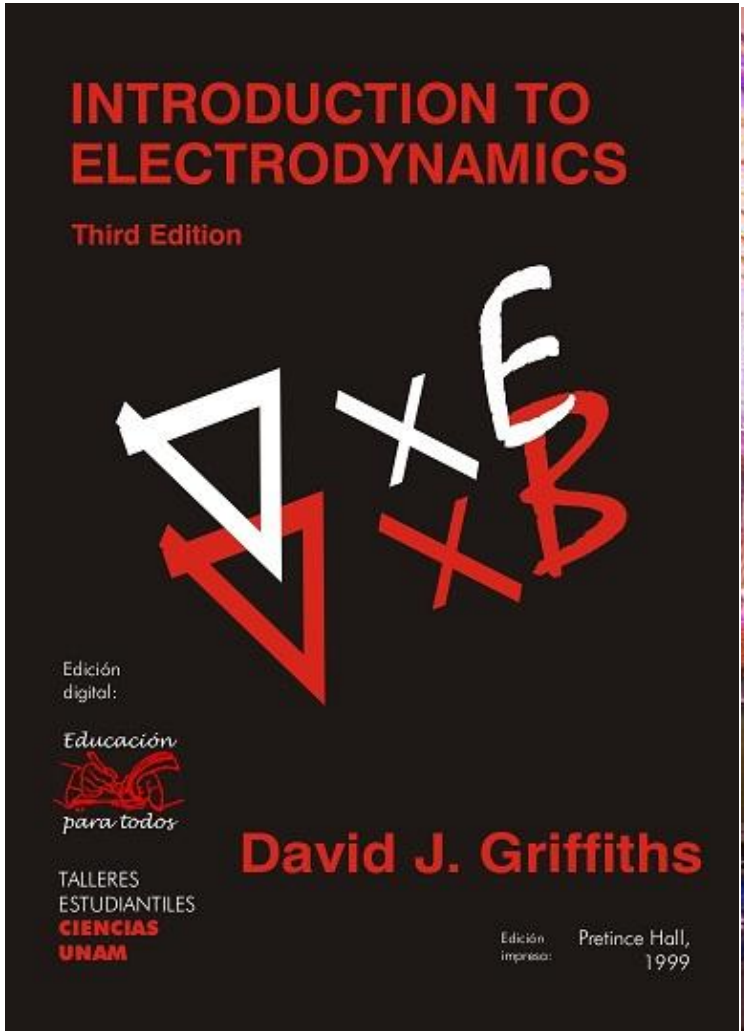
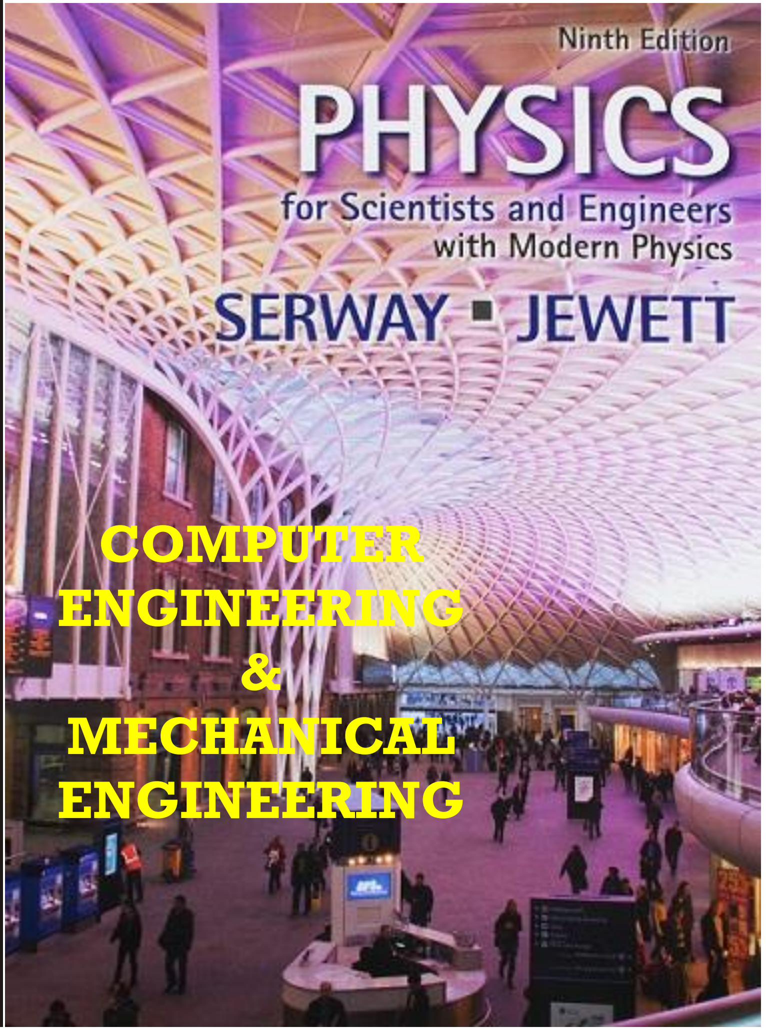
PHYSICS

General Physics II (PHYS 102)





COMPUTER ENGINEERING & MECHANICAL ENGINEERING

Ganesh Kuwar Chhetri

Dept. of Physics, School of Science, Kathmandu University

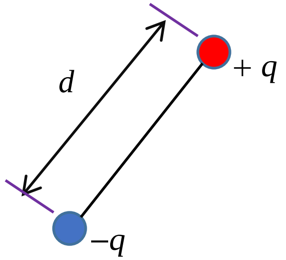
Course Outline

- Electric Dipole
- Electric Field of Dipole
- A Dipole in an Electric Field
- Polar and Non-Polar Molecules
- Induced Dipole Moment & Atomic Polarizability

Electric Dipole

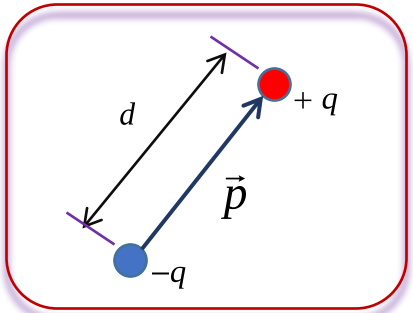
Electric Dipole:

- A pair of equal and opposite point charges separated by a small distance.



Electric Dipole Moment :

- Electric Dipole Moment, $\vec{p} = q\vec{d}$



The product of magnitude of either charge and the vector distance separating the two charges.

- The direction of electric dipole moment vector is along the line joining the two charges pointing from the negative charge toward the positive charge.
- It is a vector quantity.
- SI unit of electric dipole moment is Coulomb meter (C m) .

Electric Field of a Dipole

Approximate Potential at Points far from the Dipole:

- Consider an electric dipole lying along the z -axis with its midpoint at the origin of the co-ordinate system.

$q \rightarrow$ magnitude of each charge

$\vec{d} \rightarrow$ vector distance between the charges

Electric Dipole Moment $\Rightarrow$ $\vec{p} = q\vec{d}$

- The potential of the dipole at the point P is

$$V_{dip} = \frac{1}{4\pi\varepsilon_0}\left(\frac{+q}{r_+}\right) + \frac{1}{4\pi\varepsilon_0}\left(\frac{-q}{r_-}\right)$$

$$\therefore \boxed{V_{dip} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_+} - \frac{1}{r_-}\right]} \qquad \text{...............} \qquad (1)$$

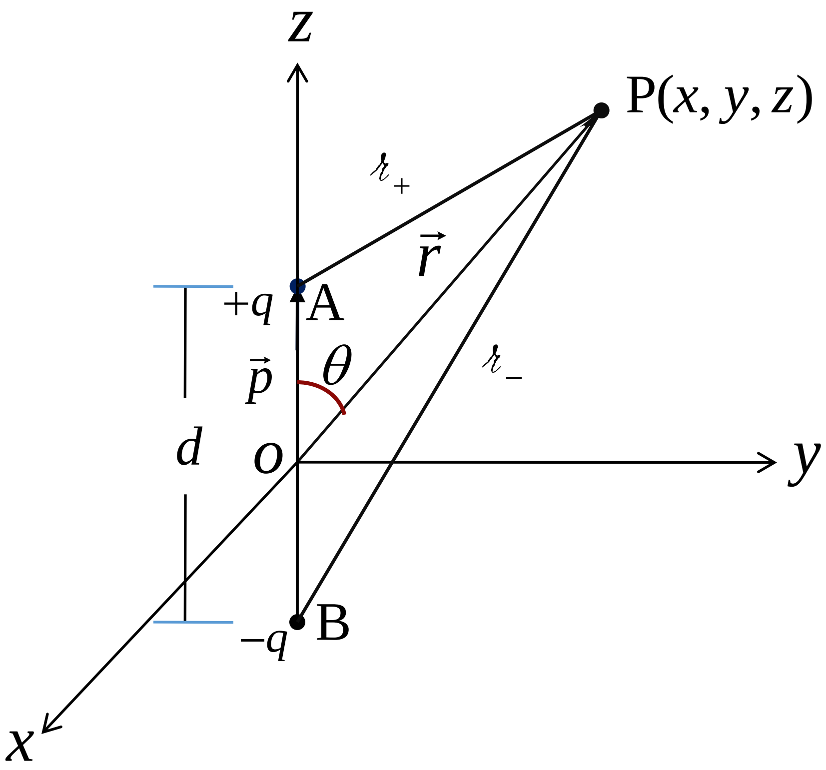


Figure Di-1

Electric Field of a Dipole

From the law of cosines,

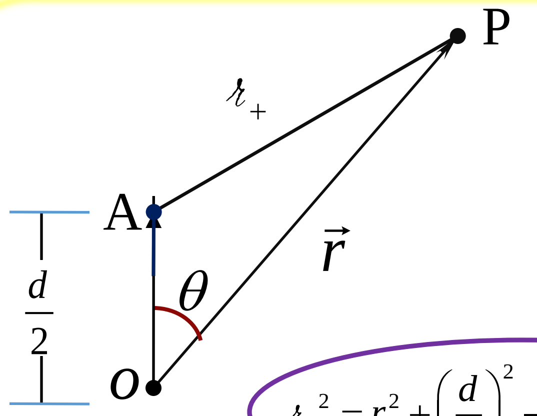
$$\mathscr{r}_+ = \sqrt{r^2 + \frac{d^2}{4} - 2\,r\,\frac{d}{2}\cos\theta}$$

$$= \sqrt{r^2\left(1 + \frac{d^2}{4r^2} - \frac{d}{r}\cos\theta\right)}$$

$$= r\left(1 - \frac{d\cos\theta}{r} + \frac{d^2}{4r^2}\right)^{\frac{1}{2}}$$

$$\cong r\left(1 - \frac{d\cos\theta}{r}\right)^{\frac{1}{2}}$$ [when the point P is far from the dipole, $r \gg d$, so the third term is negligible.]

$$\therefore \boxed{\frac{1}{\mathscr{r}_+} \cong \frac{1}{r}\left(1 - \frac{d\cos\theta}{r}\right)^{-\frac{1}{2}} \cong \frac{1}{r}\left(1 + \frac{d\cos\theta}{2r}\right)}$$ using binomial expansion



$$\mathscr{r}_+^{\,2} = r^2 + \left(\frac{d}{2}\right)^2 - 2\,r\left(\frac{d}{2}\right)\cos\theta$$

Likewise, $$\frac{1}{\mathscr{r}_-} \cong \frac{1}{r}\left(1 - \frac{d\cos\theta}{2r}\right)$$

For short dipole
$r \gg d$

Thus

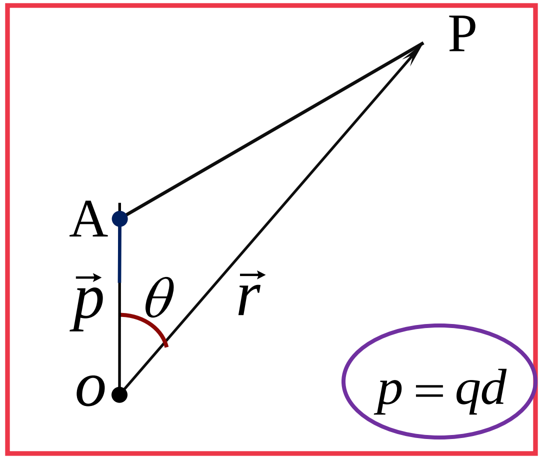
$$\boxed{\frac{1}{\mathscr{r}_+} - \frac{1}{\mathscr{r}_-} \cong \frac{1}{r}\left(1 + \frac{d\cos\theta}{2r}\right) - \frac{1}{r}\left(1 - \frac{d\cos\theta}{2r}\right) = \frac{1}{r} + \frac{d\cos\theta}{2r^2} - \frac{1}{r} + \frac{d\cos\theta}{2r^2}}$$

$$\therefore \frac{1}{\mathscr{r}_+} - \frac{1}{\mathscr{r}_-} \cong \frac{d\cos\theta}{r^2}$$

Electric Field of a Dipole

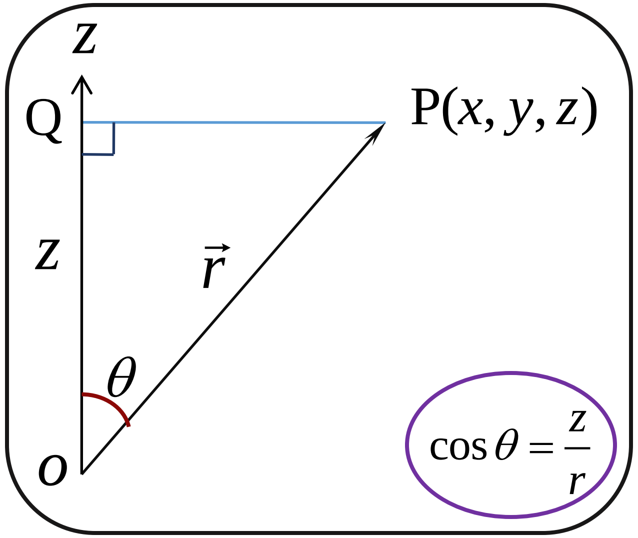
The approximate potential at points far from the dipole,

$$V_{dip}(\vec{r}) \cong \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{\mathscr{r}_+} - \frac{1}{\mathscr{r}_-}\right] \cong \frac{q}{4\pi\varepsilon_0}\left[\frac{d\cos\theta}{r^2}\right] = \frac{1}{4\pi\varepsilon_0}\left[\frac{qd\cos\theta}{r^2}\right]$$



$$\therefore\ V_{dip}(\vec{r}) \cong \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\vec{r}}{r^3} \quad \text{..............(2)}$$

$$V_{dip}(\vec{r}) \cong \frac{1}{4\pi\varepsilon_0}\frac{p\,z}{r^3} \qquad \because \cos\theta = \frac{z}{r}$$



Electric Field of a Dipole

ELECTRIC FIELDS IN MATTER

Electric Field of a Dipole:

$$\vec{E}_{dip}(\vec{r}) = -\nabla V_{dip}(\vec{r})$$

$$= -\nabla\left[\frac{1}{4\pi\varepsilon_0}\frac{p\,z}{r^3}\right] \qquad = -\frac{p}{4\pi\varepsilon_0}\left[\nabla\left(\frac{z}{r^3}\right)\right]$$

$$= -\frac{p}{4\pi\varepsilon_0}\left[z\,\left\{\nabla\left(r^{-3}\right)\right\} + \frac{1}{r^3}\left\{\nabla(z)\right\}\right]$$

$$= -\frac{p}{4\pi\varepsilon_0}\left[z\,\left(-3r^{-4}\hat{r}\right) + \frac{1}{r^3}\hat{k}\right]$$

$$= -\frac{p}{4\pi\varepsilon_0}\left[(r\cos\theta)\,\left(-3r^{-4}\hat{r}\right) + \frac{1}{r^3}\hat{k}\right]$$

$$= \frac{p}{4\pi\varepsilon_0}\left[\frac{3\cos\theta\hat{r}}{r^3} + \frac{1}{r^3}\hat{k}\right]$$

$$\therefore \boxed{\vec{E}_{dip}(\vec{r}) = \frac{p}{4\pi\varepsilon_0 r^3}\left[3\cos\theta\,\hat{r} - \hat{k}\right]} = \frac{1}{4\pi\varepsilon_0 r^3}\left[3(p\cos\theta)\,\hat{r} - p\hat{k}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{1}{r^3}\left[3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}\right]$$

Electric Field of a Dipole: [In the coordinate-free form]

$$\vec{E}_{dip}(\vec{r}) = -\nabla V_{dip}(\vec{r})$$

$$= -\frac{1}{4\pi\varepsilon_0}\left[\nabla\left(\frac{\vec{p}\cdot\vec{r}}{r^3}\right)\right]$$

$$= -\frac{1}{4\pi\varepsilon_0}\left[(\vec{p}\cdot\vec{r})\left\{\nabla\left(\frac{1}{r^3}\right)\right\} + \left(\frac{1}{r^3}\right)\left\{\nabla(\vec{p}\cdot\vec{r})\right\}\right]$$

$$= -\frac{1}{4\pi\varepsilon_0}\left[(\vec{p}\cdot\vec{r})\left(\frac{-3\vec{r}}{r^5}\right) + \left(\frac{1}{r^3}\right)(\vec{p})\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{3(\vec{p}\cdot\vec{r})\vec{r}}{r^5} - \frac{\vec{p}}{r^3}\right]$$

$$\therefore \vec{E}_{dip}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^3}\left[3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}\right]$$

$$= \frac{p}{4\pi\varepsilon_0 r^3}\left[3\cos\theta\,\hat{r} - \hat{k}\right]$$

$$\left[\because \vec{p}\cdot\hat{r} = p\cos\theta \quad \vec{p} = p\hat{k}\right]$$

.... (3)

$$\nabla(fg) = f(\nabla g) + g(\nabla f)$$

$$\nabla r^n = n r^{n-1}\hat{r}$$

Electric Field of a Dipole

Magnitude of Electric Field of a Dipole:

$$E_{dip}(\vec{r}) = \sqrt{\vec{E}_{dip}(\vec{r})\cdot\vec{E}_{dip}(\vec{r})}$$

$$= \sqrt{\left(\frac{p}{4\pi\varepsilon_0 r^3}\left[3\cos\theta\,\hat{r}-\hat{k}\right]\right)\cdot\left(\frac{p}{4\pi\varepsilon_0 r^3}\left[3\cos\theta\,\hat{r}-\hat{k}\right]\right)}$$

$$= \frac{p}{4\pi\varepsilon_0 r^3}\left[\sqrt{\left(3\cos\theta\,\hat{r}-\hat{k}\right)\cdot\left(3\cos\theta\,\hat{r}-\hat{k}\right)}\right]$$

$$= \frac{p}{4\pi\varepsilon_0 r^3}\left[\sqrt{9\cos^2\theta\,(\hat{r}\cdot\hat{r}) - 3\cos\theta\left(\hat{r}\cdot\hat{k}\right) - 3\cos\theta\left(\hat{k}\cdot\hat{r}\right) + \left(\hat{k}\cdot\hat{k}\right)}\right]$$

$$= \frac{p}{4\pi\varepsilon_0 r^3}\left[\sqrt{9\cos^2\theta\,(1) - 3\cos\theta(\cos\theta) - 3\cos\theta(\cos\theta) + (1)}\right]$$

$$\therefore \boxed{E_{dip} = \frac{p}{4\pi\varepsilon_0 r^3}\left[\sqrt{3\cos^2\theta + 1}\right]} \quad \text{...............} \quad (4)$$

The approximate potential at points far from the dipole:

$$V(\vec{r}) \cong \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{pz}{r^3}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\hat{r}\cdot\vec{p}}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\vec{r}}{r^3}$$

Magnitude of electric field of a dipole:

$$E_{dip} = \frac{p}{4\pi\varepsilon_0 r^3}\left[\sqrt{3\cos^2\theta + 1}\right]$$

Cases:

(1) At a point on the axis of a dipole $\left(\theta = 0^o\right)$

$$V_{dip}(\vec{r}) \cong \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2} \quad \& \quad E_{dip}(\vec{r}) = \frac{2p}{4\pi\varepsilon_0 r^3}$$

(2) At a point on the perpendicular bisector of a dipole $\left(\theta = 90^o\right)$

$$V_{dip} = 0 \quad \& \quad E_{dip} = \frac{p}{4\pi\varepsilon_0 r^3}$$

Electric Field of a Dipole

Electric Field of a Dipole: [In the Spherical Coordinate System]

$$\vec{E}_{dip}(r,\theta) = -\nabla V_{dip}$$

$$= -\left[\frac{\partial V_{dip}}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial V_{dip}}{\partial \theta}\hat{\theta} + \frac{1}{r\,\sin\theta}\frac{\partial V_{dip}}{\partial \phi}\hat{\phi}\right]$$

$$= -\left[\left\{\frac{\partial}{\partial r}\left(\frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}\right)\right\}\hat{r} + \frac{1}{r}\left\{\frac{\partial}{\partial \theta}\left(\frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}\right)\right\}\hat{\theta}\right]$$

$$= -\left[\frac{p\cos\theta}{4\pi\varepsilon_0}\left\{\frac{\partial}{\partial r}\left(r^{-2}\right)\right\}\hat{r} + \frac{p}{4\pi\varepsilon_0 r^3}\left\{\frac{\partial}{\partial \theta}(\cos\theta)\right\}\hat{\theta}\right]$$

$$= -\left[\frac{p\cos\theta}{4\pi\varepsilon_0}\left(\frac{-2}{r^3}\right)\hat{r} + \frac{p}{4\pi\varepsilon_0 r^3}(-\sin\theta)\hat{\theta}\right]$$

$$= -\left[\frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}(-2\cos\theta)\hat{r} + \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}(-\sin\theta)\hat{\theta}\right]$$

$$\because \boxed{\vec{E}_{dip}(r,\theta) = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}\left[2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right]} \qquad \text{...............} \qquad (5)$$

Magnitude of Electric Field of a Dipole:

$$E_{dip} = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}\left[\sqrt{(2\cos\theta)^2 + (\sin\theta)^2}\right]$$

$$= \frac{p}{4\pi\varepsilon_0 r^3}\left[\sqrt{4\cos^2\theta + \sin^2\theta}\right]$$

$$= \frac{p}{4\pi\varepsilon_0 r^3}\left[\sqrt{4\cos^2\theta + \left(1-\cos^2\theta\right)}\right]$$

$$\therefore \boxed{E_{dip} = \frac{p}{4\pi\varepsilon_0 r^3}\left[\sqrt{3\cos^2\theta + 1}\right]} \qquad \text{...............} \qquad (6)$$

Dielectrics

Dielectrics

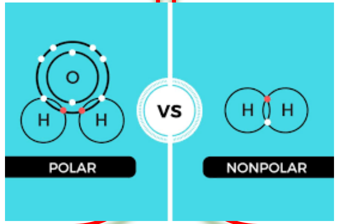
- A dielectric is a nonconducting material such as rubber, glass, or waxed paper.
- There are practically no free charges in dielectrics.
- In dielectrics (or, insulators), all charges are attached to specific atoms or molecules, and all they can do is move a bit within the atom of molecules.
- An applied field causes a displacement of charges but no a flow of charges in dielectrics.

Molecules of Dielectrics

Polar Molecules

- A polar molecule has a permanent dipole moment, even in the absence of a polarizing filed.
- In polar molecules, the "centers of gravity" of the positive and negative charge distributions do not coincide.

Examples: H_2O, HCl, NH_3 etc.



Nonpolar Molecules

- A nonpolar molecule does not have a permanent dipole moment.
- In nonpolar molecules, the "centers of gravity" of the positive and negative charge distributions coincide.

Examples: H_2, N_2, O_2, He, Ne, Ar etc.

Polar Molecules

Polar Molecules

- Molecules are said to be polarized when a separation exists between the average position of the negative charges and the average position of the positive charges in the molecule.
- In some molecules such as **water**, this condition is always present; such molecules are called **polar molecules**.
- Molecules that do not possess a permanent polarization are called **nonpolar molecules**.

A Molecule of Water

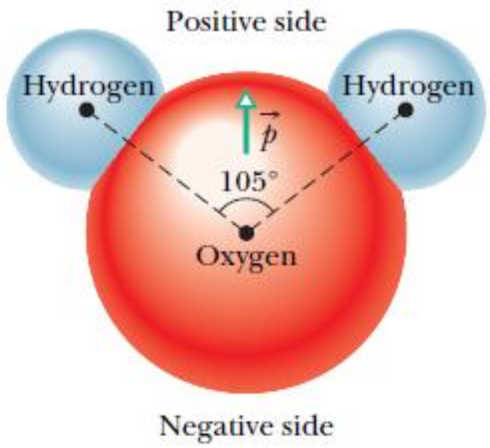


Figure Dm-1: The water molecule, H_2O, has a permanent polarization resulting from its nonlinear geometry.

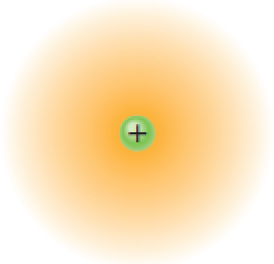
- A molecule of water H_2O, showing the three nuclei (represented by dots) and the regions in which the electrons can be located.
- The oxygen atom in the water molecule is bonded to the hydrogen atoms such that an angle of 105^0 is formed between the two bonds. As a result, the molecule has a definite "oxygen side" and "hydrogen side."
- Moreover, the 10 electrons of the molecule tend to remain closer to the oxygen nucleus than to the hydrogen nuclei. This makes the oxygen side of the molecule slightly more negative than the hydrogen side and creates an electric dipole moment $\vec{P}$ that points along the symmetry axis of the molecule as shown in Figure Dm-1.

Induced Dipole Moment & Atomic Polarizability

Induced Dipole Moment & Atomic Polarizability:

- In nonpolar molecules and in every isolated atom, the centers of the positive and negative charges coincide and thus no dipole moment is set up.
- However, if we place an atom or a nonpolar molecule in an external electric field, the field distorts the electron orbits and separates the centers of positive and negative charge.

  Because the electrons are negatively charged, they tend to be shifted in a direction opposite the field. This shift sets up a dipole moment that points in the direction of the field. This dipole moment is said to be *induced* by the field, and the atom or molecule is then said to be *polarized* by the field.



The electric field shifts the positive and negative charges, creating a dipole

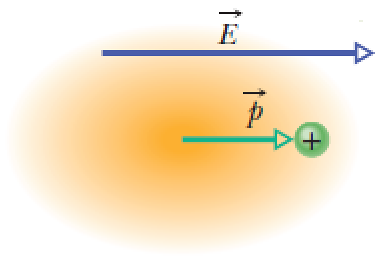


Figure I_D-1

An atom, showing the positively charged nucleus(green) and the negatively charged electrons(gold shading). The centers of positive and negative charge coincide.

Figure I_D-2

If the atom is placed in an external electric field ,the electron orbits are distorted so that the centers of positive and negative charge no longer coincide. An induced dipole $\vec{p}$ moment appears.

Typically, this induced dipole moment is approximately proportional to the field(as long as the electric field is not too strong): $\vec{p} = \alpha \vec{E}$

The constant of proportionality α is called atomic polarizability.

Atomic Polarizability:

- The electric dipole moment induced in the atom by an electric field of unit strength.

Atomic Polarizability → $\alpha = \dfrac{p}{E}$

p: Induced Dipole Moment

E: Electric Field

Unit of Atomic Polarizability

$$\frac{Cm}{Vm^{-1}} = Cm^2V^{-1}$$
$$= \left(CV^{-1}\right)m^2$$
$$= Fm^2 \text{(Farad meter squared).}$$

Example:

Calculate the induced dipole moment per unit volume of He gas if placed in a field of 6000 volts /cm. The atomic polarizability of $\text{He} = 0.18\times10^{-40}$ Farad m^{-2} **and density of He is** 2.6×10^{25} atoms m^{3} **.**

Hint:

- **Dipole moment of He atom:** $p = \alpha E = \left(0.18\times10^{-40}\right)\times\left(6\times10^{5}\right) = 1.08\times10^{-35}$ C m
- **The induced dipole moment per unit volume:**

$$P = Np = \left(2.6\times10^{25}\right)\times\left(1.08\times10^{-35}\right) = 2.81\times10^{-10} \text{ C m}^{-2}$$

Atomic Polarizability

Example

A primitive model for an atom consists of a point nucleus $(+q)$ surrounded by a uniformly charged spherical cloud $(-q)$ of radius a [Figure A-1]. Calculate the atomic polarizability of such an atom.

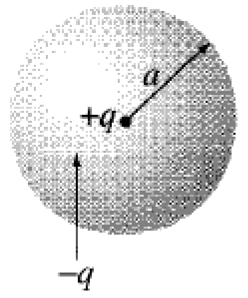


Figure A-1

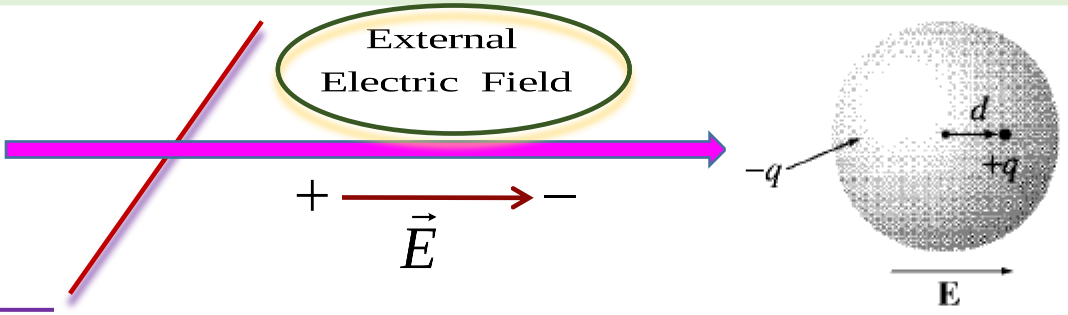


Figure A-2

In the presence of an external field $\vec{E}$, the nucleus will be shifted slightly to the right and the electron cloud to the left, as shown in Figure A-2.

Say that equilibrium occurs when the nucleus is displaced a distance *d* from the centre of the sphere.

At equilibrium,

$$\begin{bmatrix} \text{The external filed pushing} \\ \text{the nucleus to the right} \\ (E) \end{bmatrix} = \begin{bmatrix} \text{The internal field (field produced by the} \\ \text{electron cloud) pulling the nucleus to the left} \\ (\mathrm{E}_e) \end{bmatrix}$$

$$\mathrm{E} = \frac{1}{4\pi\varepsilon_0}\frac{qd}{a^3}$$

Field a distance *d* from the centre of a uniformly charged sphere

$$\mathrm{E} = \frac{1}{4\pi\varepsilon_0}\frac{p}{a^3} \Rightarrow \boxed{p = \left(4\pi\varepsilon_0 a^3\right)E}$$

$$p = \alpha E$$

The atomic polarizability is therefore

$$\alpha = 4\pi\varepsilon_0 a^3 = 3\varepsilon_0\left[\frac{4}{3}\pi a^3\right] = 3\varepsilon_0 v$$

$\therefore \alpha = 3\varepsilon_0 v$ where v is the volume of the atom.

Dipole in an Electric Field

- The electric dipole consists of two charges $+q$ and - q separated by a distance d as shown in Figure E_D-1.

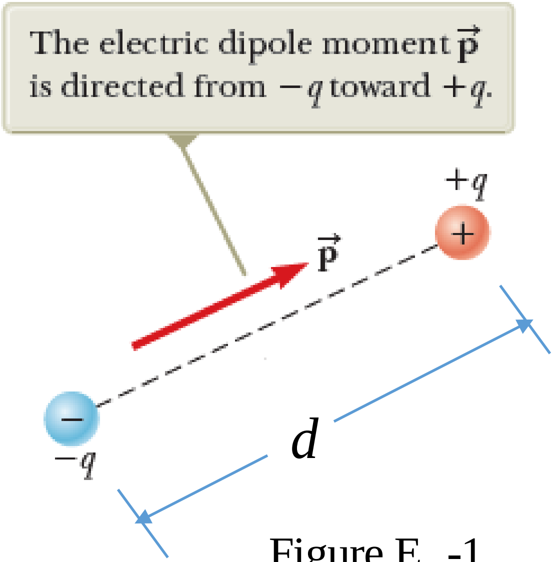


Figure E_D-1

- Now suppose an electric dipole is placed in a uniform external electric field $\vec{E}$ and makes an angle θ with the field as shown in Figure E_D-2.

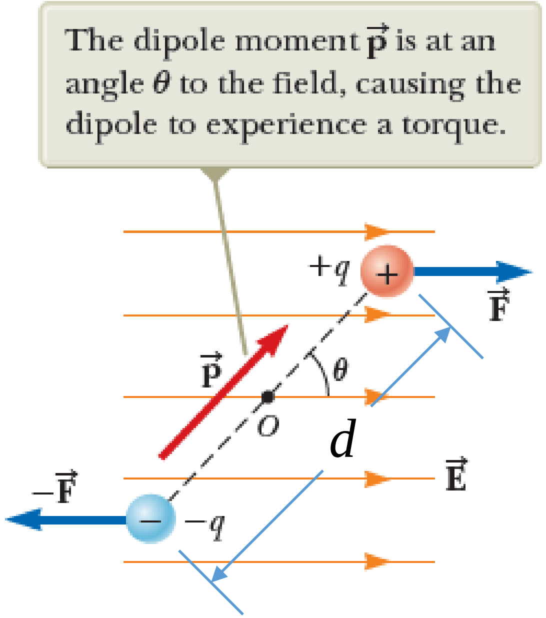


Figure E_D-2

Dipole in an Electric Field

ELECTRIC FIELDS IN MATTER

Net Force on the Dipole in a uniform electric filed

- Each of the charges is modeled as a particle in an electric field.

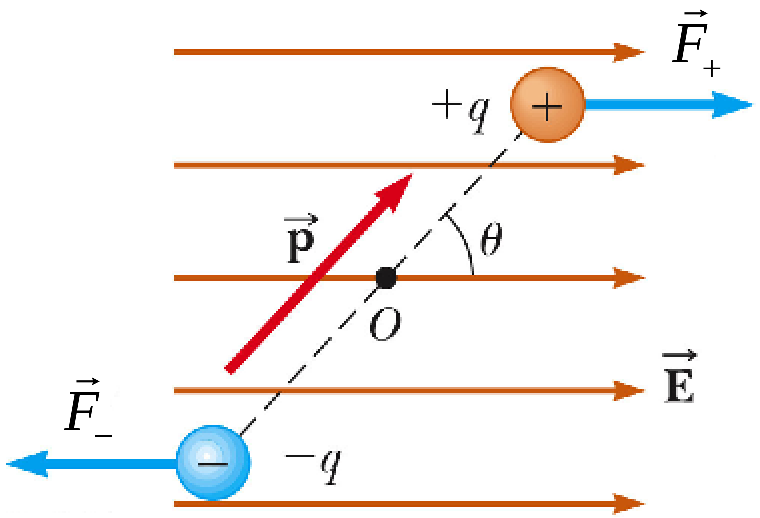


Figure E_D-1

Net force on the Dipole

The Force acting on the charge $+q$, $\vec{F}_+ = q\vec{E}$

The Force acting on the charge $-q$, $\vec{F}_- = -q\vec{E}$

Therefore, the net force

$$\vec{F}_{net} = \vec{F}_+ + \vec{F}_-$$

$$= q\vec{E} + \left(-q\vec{E}\right)$$

$$\therefore\ \vec{F}_{net} = 0 \qquad \ldots\ldots\ldots\ldots\ldots \qquad (1)$$

Hence, the net force on an electric dipole in a uniform external electric field is zero.

Dipole in an Electric Field

Torque on the dipole in a uniform electric field

- Each of the charges is modeled as a particle in an electric field.
- As the two forces acting at the two ends of the dipole are unlike parallel forces, they form a couple which rotates the dipole in clockwise direction, tending it in the direction of field

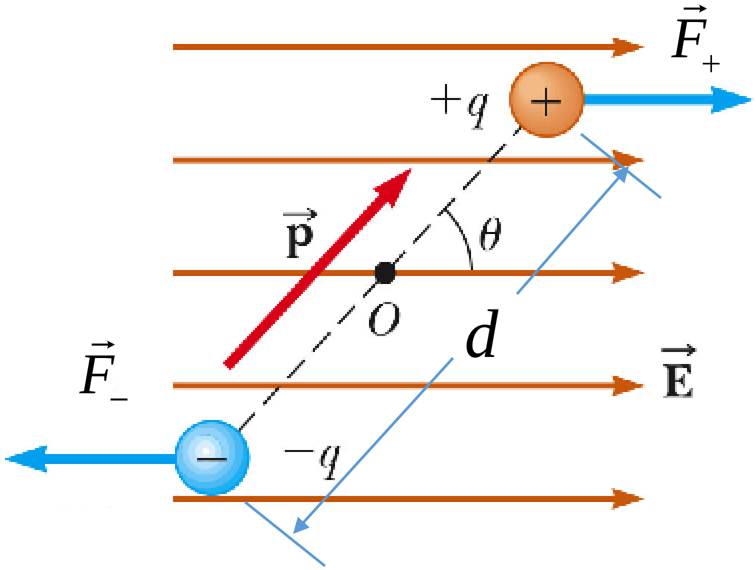


Figure E_D-2

Torque on the Dipole

$$\vec{\tau} = \left(\vec{r}_+ \times \vec{F}_+\right) + \left(\vec{r}_- \times \vec{F}_-\right)$$

$$= \left[\left(\frac{\vec{d}}{2}\right) \times \left(q\vec{E}\right)\right] + \left[\left(-\frac{\vec{d}}{2}\right) \times \left(-q\vec{E}\right)\right]$$

$$= q\vec{d} \times \vec{E}$$

$$\therefore \boxed{\vec{\tau} = \vec{p} \times \vec{E}} \quad \ldots\ldots\ldots\ldots\ldots \quad (2)$$

Thus a dipole $\vec{p} = q\vec{d}$ in a uniform field $\vec{E}$ experience a torque

$$\vec{\tau} = \vec{p} \times \vec{E}$$

Dipole in an Electric Field

Potential Energy of a dipole in a uniform electric field

- Let us consider a dipole of dipole moment $\vec{p}$ oriented an angle θ with the direction of a uniform electric field $\vec{E}$ as shown in Figure E_D-3.

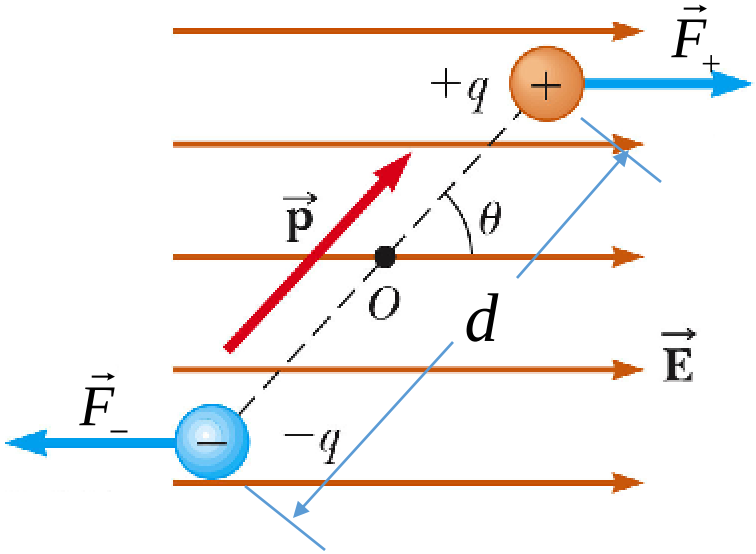


Figure E_D-2 A elecric dipole in a uniform external electric field

The work done by the external field in turning the dipole from an initial angle θ_0 to a final angle θ is

$$W = \int dW$$

$$= \int_{\theta_0}^{\theta} \vec{\tau} \cdot d\vec{\theta} \quad \left[\vec{\tau} \rightarrow \text{the torque exerted by the external electric field}\right]$$

$$= \int_{\theta_0}^{\theta} -\tau d\theta \quad \left[\text{The minus sign is necessary because the torque t tends to decrease } \theta\right]$$

$$= \int_{\theta_0}^{\theta} -\left(pE\sin\theta\right)d\theta$$

$$= pE\int_{\theta_0}^{\theta}\left(-\sin\theta\right)d\theta$$

$$\therefore \boxed{W = pE\left(\cos\theta - \cos\theta_0\right)}$$

Dipole in an Electric Field

Potential Energy of a dipole in a uniform electric field

- The change in potential energy of the system of field + dipole,

$$\Delta U \equiv U(\theta) - U(\theta_0) = -\mathrm{W}$$
$$= -pE\left(\cos\theta - \cos\theta_0\right)$$

- We arbitrary define the reference angle θ_0 to be 90^0 and potential energy $U(\theta_0)$ to be zero at that angle. At any angle θ, the potential energy is then

$$\boxed{U(\theta) = -\vec{p}\cdot\vec{E}} \qquad \ldots\ldots\ldots\ldots\ldots \qquad (3)$$

- Therefore the potential energy U of an electric dipole with dipole moment $\vec{p}$ in a uniform external electric field $\vec{E}$ is

$$U = -\vec{p}\cdot\vec{E}$$

- **The potential energy of the dipole is least when θ = 0.**
  ($\vec{p}$ and $\vec{E}$ are in the same direction)
- **The potential energy is greatest when θ =180°**
  ($\vec{p}$ and $\vec{E}$ are in opposite directions).

Potential Energy of a Dipole in an Electric Field

Potential Energy of a dipole in a uniform electric field is $U = -\vec{p} \cdot \vec{E}$

- Potential energy can be associated with the orientation of an electric dipole in an electric field.
- The dipole has its least potential energy when it is in its equilibrium orientation, which is when its dipole moment $\vec{p}$ is lined up with the field $\vec{E}$ (then $\vec{\tau} = \vec{p} \times \vec{E} = 0$).
- It has greater potential energy in all other orientations.

- When a dipole rotates from an initial orientation θ_i to another orientation θ_f, the work W done on the dipole by the electric field is

$$\boxed{W = -\Delta U_E = -\left(U_f - U_i\right)}$$

where U_f and U_i are calculated with $U = -\vec{p} \cdot \vec{E} = -pE\cos\theta$.

- If the change in orientation is caused by an applied torque (commonly said to be due to an external agent), then the work W_a done on the dipole by the applied torque is the negative of the work done on the dipole by the field; that is,

$$\boxed{W_a = -W = \left(U_f - U_i\right)}$$

The work results in an increase in potential energy.

Torque and Energy of an Electric Dipole in an Electric Field

Torque and energy of an electric dipole in an electric field

A neutral water molecule (H_2O) in its vapor state has an electric dipole moment of magnitude 6.2×10^{-30} C·m.

(a) How far apart are the molecule's centers of positive and negative charge?

KEY IDEA

A molecule's dipole moment depends on the magnitude q of the molecule's positive or negative charge and the charge separation d.

Calculations: There are 10 electrons and 10 protons in a neutral water molecule; so the magnitude of its dipole moment is

$$p = qd = (10e)(d),$$

in which d is the separation we are seeking and e is the elementary charge. Thus,

$$d = \frac{p}{10e} = \frac{6.2 \times 10^{-30}\ \text{C}\cdot\text{m}}{(10)(1.60 \times 10^{-19}\ \text{C})}$$
$$= 3.9 \times 10^{-12}\ \text{m} = 3.9\ \text{pm}. \qquad \text{(Answer)}$$

This distance is not only small, but it is also actually smaller than the radius of a hydrogen atom.

(b) If the molecule is placed in an electric field of 1.5×10^4 N/C, what maximum torque can the field exert on it? (Such a field can easily be set up in the laboratory.)

KEY IDEA

The torque on a dipole is maximum when the angle θ between $\vec{p}$ and $\vec{E}$ is 90°.

Calculation: Substituting $\theta = 90°$ in Eq. 22-33 yields

$$\tau = pE \sin\theta$$
$$= (6.2 \times 10^{-30}\ \text{C}\cdot\text{m})(1.5 \times 10^4\ \text{N/C})(\sin 90°)$$
$$= 9.3 \times 10^{-26}\ \text{N}\cdot\text{m}. \qquad \text{(Answer)}$$

(c) How much work must an *external agent* do to rotate this molecule by 180° in this field, starting from its fully aligned position, for which $\theta = 0$?

KEY IDEA

The work done by an external agent (by means of a torque applied to the molecule) is equal to the change in the molecule's potential energy due to the change in orientation.

Calculation: From Eq. 22-40, we find

$$W_a = U_{180°} - U_0$$
$$= (-pE\cos 180°) - (-pE\cos 0)$$
$$= 2pE = (2)(6.2 \times 10^{-30}\ \text{C}\cdot\text{m})(1.5 \times 10^4\ \text{N/C})$$
$$= 1.9 \times 10^{-25}\ \text{J}. \qquad \text{(Answer)}$$

Text Books & References

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2. **R.A. Serway and J.W. Jewett**, Physics for Scientist and Engineers with Modern Physics
3. **Halliday and Resnick**, Fundamental of Physics
4. **D. Halliday, R. Resnick, and K. Krane** , Physics, Volume 2, Fourth Edition


Thank
you