# Advanced Calculus - Polar Coordinates


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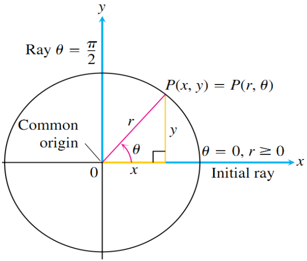
# Relating Polar and Cartesian Coordinates




1. Convert to Cartesian coordinates: i. $(\sqrt{2}, \pi/4)$, ii. $(5, \tan^{-1} 4/3)$.

2. Convert to polar coordinates: i. $(3, 4)$ ii. $(-2, 0)$.

## Relation

$x = r\cos\theta$, $y = r\sin\theta$ (Polar - Cartesian)

$r^2 = x^2 + y^2$, $\tan\theta = \dfrac{y}{x}$ (Cartesian - Polar)

## Write equivalent Cartesian equations.

**a.** $r\sin\theta = 2$ **b.** $r = -3\sec\theta$

**c.** $r^2 \sin 2\theta = 2$ **d.** $r = 1 - \cos\theta$

**e.** $r = 1 + 2r\cos\theta$ **f.** $r = 2\cos\theta - \sin\theta$

**g.** $r\sin\left(\theta + \dfrac{\pi}{6}\right) = 2$ **h.** $r\sin\theta = \ln r + \ln\cos\theta$

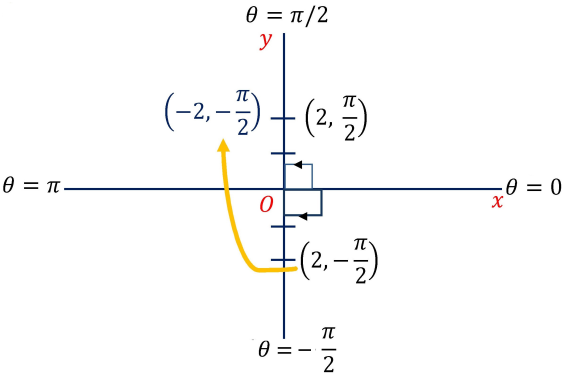
## Write equivalent Polar equations.

**a.** $x = 1$ **b.** $x^2 + y^2 = 4$ **c.** $x^2 + xy + y^2 = 1$

# Deceptive Coordinates

**Does the point $(2, \pi/2)$ lie on the curve $r = 2\cos 2\theta$?**

At $(2, \pi/2)$, $r = 2\cos 2\theta$ becomes $2 = 2\cos\pi$ i.e., $2 = -2$ (False)

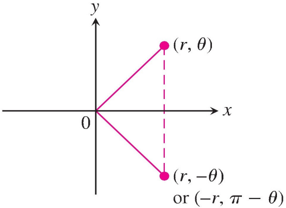



Note: $(2, \pi/2)$ and $(-2, -\pi/2)$ label the same point.

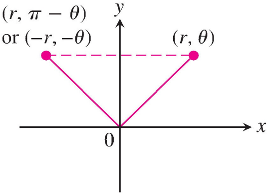
At $(-2, -\pi/2)$, $r = 2\cos 2\theta$ becomes $-2 = 2\cos(-\pi)$ i.e., $-2 = -2$(True)

### Symmetry Tests for Polar Graphs

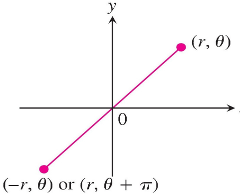
1. *Symmetry about the x-axis:* If the point $(r, \theta)$ lies on the graph, then the point $(r, -\theta)$ or $(-r, \pi - \theta)$ lies on the graph
2. *Symmetry about the y-axis:* If the point $(r, \theta)$ lies on the graph, then the point $(r, \pi - \theta)$ or $(-r, -\theta)$ lies on the graph
3. *Symmetry about the origin:* If the point $(r, \theta)$ lies on the graph, then the point $(-r, \theta)$ or $(r, \theta + \pi)$ lies on the graph



About the x-axis



About the y-axis



About the origin

# Slope of a curve $r = f(\theta)$

$$x = r\cos\theta = f(\theta)\cos\theta, \qquad y = r\sin\theta = f(\theta)\sin\theta.$$

If $f$ is a differentiable function of $\theta$, then so are $x$ and $y$ and, when $dx/d\theta \neq 0$, we can calculate $dy/dx$ from the parametric formula

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\frac{d}{d\theta}(f(\theta)\cdot \sin\theta)}{\frac{d}{d\theta}(f(\theta)\cdot \cos\theta)} = \frac{\frac{df}{d\theta}\sin\theta + f(\theta)\cos\theta}{\frac{df}{d\theta}\cos\theta - f(\theta)\sin\theta}$$

Therefore we see that $dy/dx$ is not the same as $df/d\theta$.

**Slope of the Curve $r = f(\theta)$**

$$\left.\frac{dy}{dx}\right|_{(r,\theta)} = \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta},$$

provided $dx/d\theta \neq 0$ at $(r, \theta)$.

If the curve $r = f(\theta)$ passes through the origin at $\theta = \theta_0$, then $f(\theta_0) = 0$, and the slope equation gives

$$\left.\frac{dy}{dx}\right|_{(0,\theta_0)} = \frac{f'(\theta_0)\sin\theta_0}{f'(\theta_0)\cos\theta_0} = \tan\theta_0.$$

Note: If $r = 0 \implies \theta = \theta_0$, then $\theta = \theta_0$ represents the equation of tangent to the curve at the pole???????

Problems: Find the slopes of the given curves at the given points

1. $r = -1 + \cos\theta$, $\theta = \pm\pi/2$.

   Ans; -1, 1.

2. $r = \sin 2\theta$, $\theta = \pm\pi/4$

   Ans: -1, 1

Graph the polar curve $r = 1 + \cos\theta$.

1. symmetry: $(r, \theta)$ on the graph

$\implies$ $r = 1 + \cos\theta$

$\implies$ $r = 1 + \cos(-\theta)$

$\implies$ $(r, -\theta)$ on the graph.

The curve is symmetrical about $x$ - axis.

**2.** At origin $(r = 0)$,
$1 + \cos\theta = 0 \implies \theta = \pi$.

$\theta = \pi$ is tangent to the curve at pole.

Also, the slope of the curve
$\tan\theta_0 = \tan\pi = 0$.

**3.** $r$ - $\theta$ table.

| $\theta$ | $r = 1 + \cos\theta$ |
|---|---|
| 0 | 2 |
| $\pi/6$ | 1.87 |
| $\pi/4$ | 1.71 |
| $\pi/3$ | 1.5 |
| $\pi/2$ | 1 |
| $2\pi/3$ | 0.50 |
| $3\pi/4$ | 0.29 |
| $5\pi/6$ | 0.13 |
| $\pi$ | 0 |

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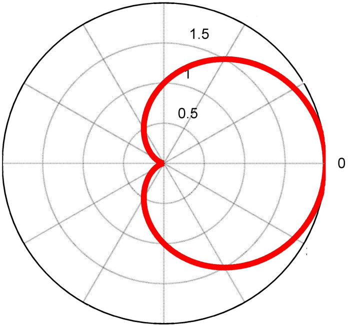


Figure: Graph of $r = 1 + \cos\theta$ (Cardiod)

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