Lecture 09

Electrostatic Field in Matter (Contd.)

16 September, 2020

Keshav Raj Sigdel

Assistant Professor

Department of Physics

School of Science

Kathmandu University

Outline

- Electric field at the center of cavity sphere inside a polarized medium
- Clausius –Mossotti Equation

Question:- Derive an expression for the electric field at the centre of a spherical cavity inside a polarized dielectric due to the charges on the wall of the cavity

Solution:-

When a spherical cavity of radius r is made inside a uniformly polarized dielectric medium with polarization $\vec{P}$ directed from left to right as shown in Figure 1, then negative bound charges induce on the right hemisphere and positive bound charges on the left hemisphere.

Electric field at the center of cavity sphere inside a polarized medium (contd.)

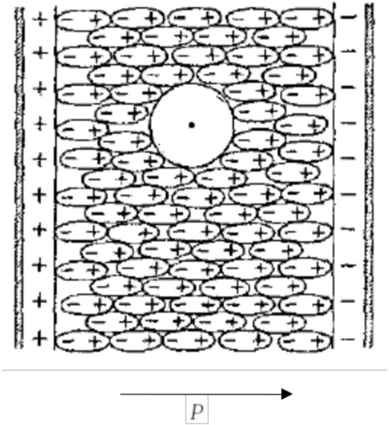

Figure 1

Electric field at the center of cavity sphere inside a polarized medium (contd.)

Figure 2 is the magnified view of the cavity sphere with induced bound charges.

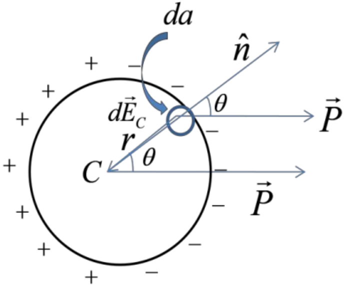

Figure 2

Electric field at the center of cavity sphere inside a polarized medium (contd.)

Let us take an elemental area da on the surface of cavity sphere such that the unit normal vector $\hat{n}$ makes an angle θ with $\vec{P}$.

The charge on an elemental area da is

$$dq = \sigma_b da = \left(\vec{P} \cdot \hat{n}\right) da = P\cos\theta \left(r^2 \sin\theta \; d\theta \; d\phi\right)$$

The electric field at the centre of the cavity due to the charge on dq is

$$d\vec{E}_C = \frac{1}{4\pi\varepsilon_0} \frac{dq}{r^3} \vec{r}$$

Electric field at the center of cavity sphere inside a polarized medium (contd.)

where $\vec{r}$ is the vector from the surface to the centre of the sphere

$$d\vec{E}_C = \frac{1}{4\pi\varepsilon_0}\frac{\left(P\cos\theta\; r^2\sin\theta\; d\theta\; d\phi\right)}{r^3}\vec{r}$$

$$\therefore \boxed{d\vec{E}_C = \frac{P}{4\pi\varepsilon_0}\left(\cos\theta\sin\theta\; d\theta\; d\phi\right)\hat{n}}$$

The component of $d\vec{E}_C$ along the direction of $\vec{P}$ is
$dE_C\cos\theta = \dfrac{P}{4\pi\varepsilon_0}\cos^2\theta\sin\theta\; d\theta\; d\phi$.
Due to symmetry of the cavity, the components of $d\vec{E}_C$ along the direction perpendicular to $\vec{P}$ is zero.

Electric field at the center of cavity sphere inside a polarized medium (contd.)

Therefore, the electric field at the centre C of the spherical cavity due to the entire surface charge on the cavity surface is

$$
\begin{aligned}
E_C &= \int \frac{1}{4\pi\varepsilon_0} P\cos^2\theta \sin\theta \; d\theta \; d\phi \\
&= \frac{P}{4\pi\varepsilon_0}\left\{\int_0^{\pi} \cos^2\theta \sin\theta d\theta\right\}\left\{\int_0^{2\pi} d\phi\right\} \\
&= -\frac{P}{4\pi\varepsilon_0}\left\{\int_0^{\pi} \cos^2\theta d(\cos\theta)\right\}\{2\pi\} \\
&= -\frac{P}{2\varepsilon_0}\left[\frac{\cos^3\theta}{3}\right]_0^{\pi} = \frac{P}{3\varepsilon_0} \qquad \therefore \boxed{\vec{E}_C = \frac{\vec{P}}{3\varepsilon_0}}
\end{aligned}
$$

Clausius –Mossotti Equation

Clausius and Mossotti established a relation between the dielectric constant and the molecular polarizability of a dielectric. This relation is known as Clausius-Mossotti Equation.

Clausius and Mossotti assumed that each molecule of a uniformly polarized dielectric medium lies at the centre of the cavity sphere. Therefore, the net electric field experienced by the molecule (also called molecular field) is the sum of electric field due to the bound charge on the cavity surface and resultant of all other fields except due to the bound charges on the cavity surface.

$$\vec{E}_m = \vec{E}_C + \vec{E} \tag{1}$$

Clausius –Mossotti Equation (contd.)

The dipole moment of each molecule is proportional to the molecular field $\vec{E}_m$, i.e. $\vec{p}_m = \alpha_m \vec{E}_m$

Where α_m be the molecular polarizability of the molecules.

If there are N molecules per unit volume, then the polarization $\vec{P}$ is

$$\vec{P} = N\vec{p}_m = N\alpha_m \left(\vec{E}_C + \vec{E}\right) = N\alpha_m \left[\frac{\vec{P}}{3\varepsilon_0} + \vec{E}\right]$$

In a microscopic point of view the polarization is directly proportional to total electric field. i.e. $\vec{P} = \varepsilon_0 \chi_e \vec{E} \implies \vec{E} = \dfrac{\vec{P}}{\varepsilon_0 \chi_e}$

$$\text{Thus,} \quad \vec{P} = N\alpha_m \left[\frac{\vec{P}}{3\varepsilon_0} + \frac{\vec{P}}{\chi_e \varepsilon_0}\right]$$

$$\text{or, } 1 = N\alpha_m \left[\frac{1}{3\varepsilon_0} + \frac{1}{\chi_e \varepsilon_0}\right] = N\alpha_m \left[\frac{\chi_e + 3}{3\varepsilon_0 \chi_e}\right]$$

$$\text{or, } \alpha_m = \frac{3\varepsilon_0}{N}\left[\frac{\chi_e}{\chi_e + 3}\right] = \frac{3\varepsilon_0}{N}\left[\frac{K-1}{(K-1)+3}\right]$$

$$\therefore \boxed{\alpha_m = \frac{3\varepsilon_0}{N}\left[\frac{K-1}{K+2}\right]}$$

This is known as Clausius-Mossotti Equation.

The Clausius-Mossotti relation connects the dielectric constant K to the polarizability α_m of the atoms or molecules constituting the dielectric. The dielectric constant is a bulk (macroscopic) property and polarizability is a microscopic property of matter; hence the relation bridges the gap between a directly-observable macroscopic property with a microscopic molecular property.

End of Lecture 09

Thank you