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*) Note: If $r = 0$, then $\theta = \theta_0$ then $\theta = \theta_0$ represents the equation of tangent to the curve at the pole.

Find the slopes of the given equations at given points.

(i): $r = -1 + \cos\theta$, $\theta = \pm\pi/2$

Soln:

Given,

$r = f(\theta) = -1 + \cos\theta$

$\therefore f'(\theta) = \dfrac{d(-1+\cos\theta)}{d\theta} = -\sin x$

We know,

At $\theta = \theta_0$

$$m = \frac{f'(\theta_0)\sin\theta_0 + \cos\theta_0 f(\theta_0)}{f'(\theta_0)\cos\theta_0 - \sin\theta_0 f(\theta_0)}$$

So,

at # $\theta = \pi/2$,

$f'(\pi/2) = -1$

$\sin \pi/2 = 1$

$\cos \pi/2 = 0$

$f(\pi/2) = -1$

Now,

$$m = \frac{-1 \times 1 + 0 \times (-1)}{-1 \times 0 - (1 \times -1)}$$

$\therefore m = -1$.

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at $\theta = -\pi/2$.

$f'(-\pi/2) = 1$
$\sin(-\pi/2) = -1$
$\cos(-\pi/2) = 0$
$f(-\pi/2) = -1$

Now,

$$m = \frac{1 \times (-1) + 0 \times (-1)}{-1 \times 0 - (-1 \times -1)}$$

$\therefore m = 1$

$\therefore m = -1, 1.$

(ii): $r = \sin 2\theta, \quad \theta = \pm \pi/4$

Soln:

Given,
$r = f(\theta) = \sin 2\theta$

$$\therefore f'(\theta) = \frac{d \sin 2\theta}{d 2\theta} \times \frac{d 2\theta}{d\theta} = 2\cos 2\theta$$

At $\theta = \pi/4$

$f(\pi/4) = \sin 2(\pi/4) = 1$
$\cos \pi/4 = 1/\sqrt{2}$
$\sin \pi/4 = 1/\sqrt{2}$
$f'(\pi/4) = 0$

At $\theta = -\pi/4$

$f(-\pi/4) = -1$
$\cos(-\pi/4) = 1/\sqrt{2}$
$\sin(-\pi/4) = -1/\sqrt{2}$
$f'(-\pi/4) = 0$

Now

$$m = \frac{f'(\pi/4)\sin\pi/4 + f(\pi/4)\cos\pi/4}{f'(\pi/4)\cos\pi/4 - f(\pi/4)\sin\pi/4}$$
$= -1$

$$m = \frac{f'(-\pi/4)\sin -\pi/4 + f(-\pi/4)\cos -\pi/4}{f'(-\pi/4)\cos(-\pi/4) - f(-\pi/4)\sin(-\pi/4)}$$
$= 1$

$\therefore m = -1, 1.$

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Graphing Polar Curves

The steps to graphing polar curves are as follows:

(i): Check symmetry about x-axis, y-axis and origin.
(ii): Find tangent to the curve at pole.
(iii): Calculate r-θ table.

Polar Equations and their Graphs

~~(a): Circles:~~

~~Eqn: $r = \pm 2a\cos\theta$~~
~~$r = \pm 2a\sin\theta$~~
~~Here, a = radius of circle.~~

(a): Cardioids:

$r = a \pm b\cos\theta$
$r = a \pm b\sin\theta$

and $\left|\frac{a}{b}\right| = 1.$

for $\cos\theta$, it is symmetrical about x-axis
for $\sin\theta$, it is symmetrical about y-axis.

The graphs of circles are generated as the angle increases from 0 to 2π.

(b): Limacions

$r = a \pm b \cos\theta$

$r = a \pm b \sin\theta$

If $\left|\frac{a}{b}\right| < 1$, the graph is inner looped limacon.

If $1 < \left|\frac{a}{b}\right| < 2$, the graph is dimpled limacon.

If $\left|\frac{a}{b}\right| \geq 2$, the graph is convex or oval limacon.

The graphs of limacons are generated as the angle increases from 0 to 2π.

(c) Rose - petals / flowers

$r = a \sin(m\theta)$
$r = ~~a~~ b \cos(m\theta)$, the graph is rose-petal graph.

if m is odd, m petals
if m is even, $2m$ petals.

(d): Lemniscates:

$r^2 = a^2 \sin 2\theta$
$r^2 = a^2 \cos 2\theta$

The graph gives lemniscates ribbon / knot shape.

Plot the polar curve: i) $r = 2 + 2\cos\theta$.

Soln:

Given,

$r = 2 + 2\cos\theta$

(x) Check symmetry:

(a): About x-axis,

At $(r, -\theta)$, $r = 2 + 2\cos(-\theta)$
$= 2 + 2\cos\theta$ (T)

$\therefore$ $(r, -\theta)$ lies on the given curve.
The curve is symmetrical about x-axis.

(b) About y-axis

At $(-r, -\theta)$, $-r = 2 + 2\cos(-\theta)$
or $-r = 2 + 2\cos\theta$ (F)

At $(r, \pi - \theta)$

$r = 2 + 2\cos(\pi - \theta)$
$= 2 - 2\cos\theta$ (F)

$\therefore$ The curve is not symmetrical about y-axis.

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(c): About pole:

At $(-r, \theta)$, $-r = 2+2\cos\theta$ (F)

At $(r, \pi+\theta)$, $r = 2+2\cos(\pi+\theta)$

$= 2 - 2\cos\theta$ (F)

$\therefore$ The curve is not symmetrical about pole.

(x): Tangent at pole:

At pole, $r = 0$.

i.e,

$2+2\cos\theta = 0$

or $\cos\theta = -1$

$\therefore \theta = \pi$

$\therefore \theta = \pi$ gives tangent to the curve at pole.

(x) r-θ table:

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	$5\pi/6$	π
$r = 2+2\cos\theta$	4	3.73	3.41	3	2	1	0.5	0.25	0

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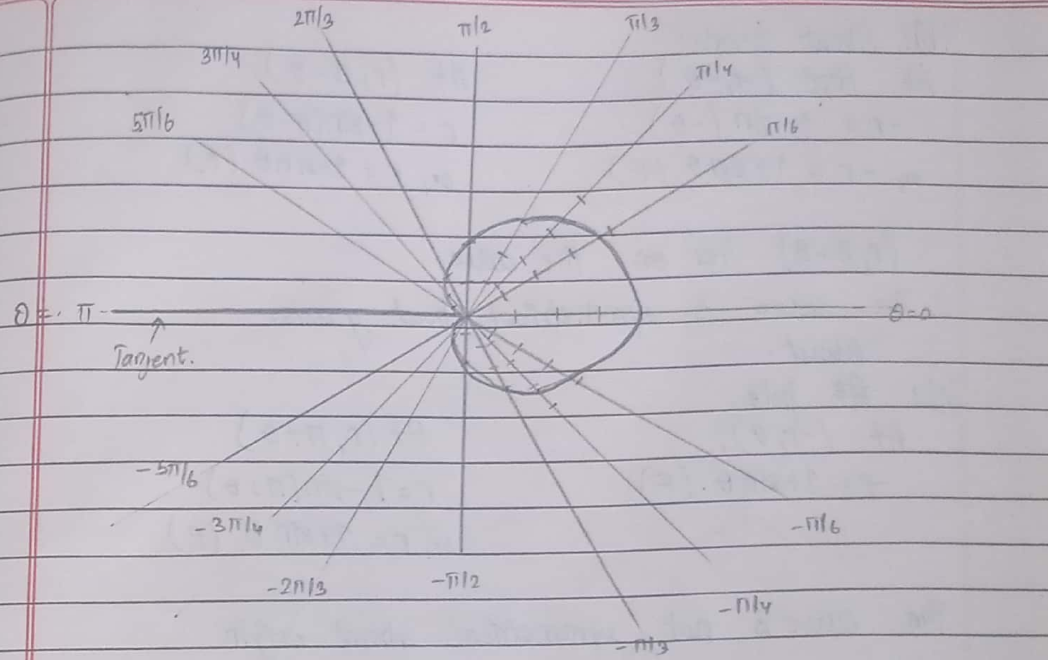


(ii) $r = 1 - \sin\theta$

Soln:

Given curve,

$r = 1 - \sin\theta$

(x) Symmetry:

a) About x-axis:

At $(r, -\theta)$

$r = 1 - \sin(-\theta)$

or, $r = 1 + \sin\theta$ (F)

At $(-r, \pi-\theta)$

$-r = 1 - \sin(\pi-\theta)$

$-r = 1 - \sin\theta$ (F)

The curve is not symmetrical about x-axis.

(b): About y-axis:

At ~~(r, θ)~~ $(-r, -\theta)$

$-r = 1 - \sin(-\theta)$

or, $-r = 1 + \sin\theta$ (F)

At $(r, \pi - \theta)$

$r = 1 - \sin(\pi - \theta)$

or, $r = 1 - \sin\theta$ (F)

$(r, \pi - \theta)$ lies on the curve

∴ The curve is symmetrical about y-axis.

About.

(c): ~~At~~ pole.

At $(-r, \theta)$.

$-r = 1 + \sin\theta$ (F)

At $(r, \pi + \theta)$

$r = 1 - \sin(\pi + \theta)$

or, $r = 1 + \sin\theta$ (F)

The curve is not symmetrical about origin.

(x) Tangent at pole:

At pole, $r = 0$,

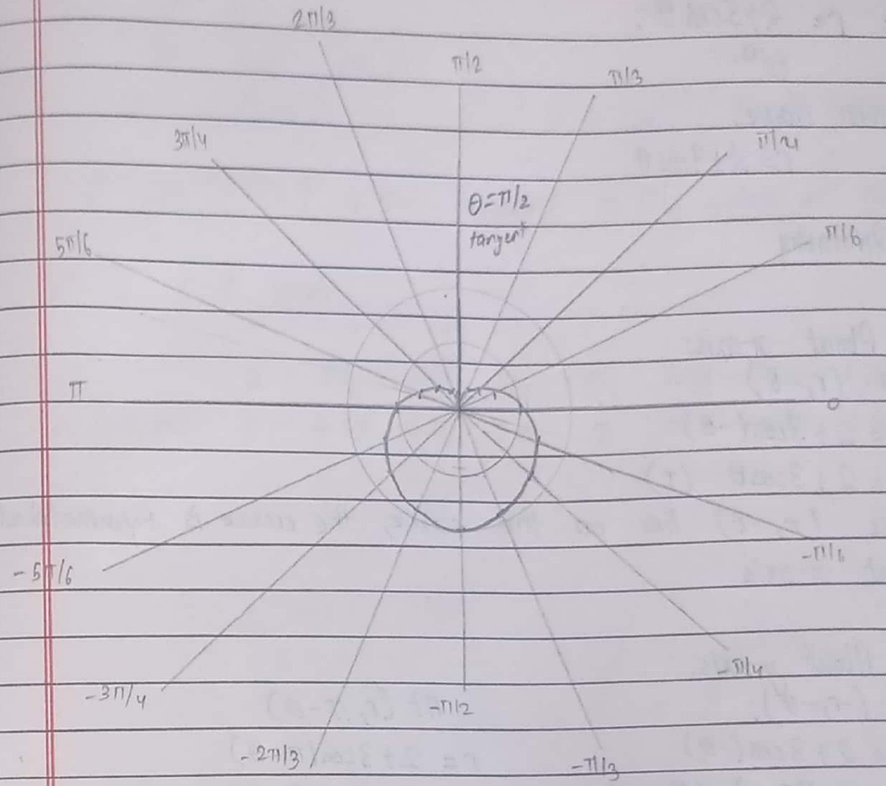
$\sin\theta = 1$

∴ $\theta = \pi/2$

∴ $\theta = \pi/2$ gives the equation of the tangent to the curve at pole.

(x) r-θ table:

θ	$-\pi/2$	$-\pi/3$	$-\pi/4$	$-\pi/6$	0	$\pi/6$	$\pi/3$	$\pi/2$	$\pi/3$	$\pi/2$
$r = 1 - \sin\theta$	2	1.866	1.707	1.5	1	0.5	0.292	0.292	0.133	0



(X): Note:

For cardioids,

(i) $r = a + b\cos\theta$ ⇒ symmetry about x, tail towards +ve x-axis

(ii) $r = a - b\cos\theta$ ⇒ symmetry about ~~y~~ x, tail towards -ve x-axis

(iii) $r = a + b\sin\theta$ ⇒ symmetry about y, tail towards +ve y-axis

(iv) $r = a - b\sin\theta$ ⇒ symmetry about y, tail towards -ve y-axis.

(iii): $r = 2+3\cos\theta$.

Soln.

Given curve,

$r = 2+3\cos\theta$

x) Symmetry

(a): About x-axis:

At $(r,-\theta)$

$r = 2+3\cos(-\theta)$

$= 2+3\cos\theta$ (T)

Since $(r,-\theta)$ lies on the curve, the curve is symmetrical about x-axis.

(b): About y-axis.

At $(-r,-\theta)$,

$-r = 2+3\cos(-\theta)$

or, $-r = 2\cancel{+}-3\cos\theta$ (F)

At $(r,\pi-\theta)$

$r = 2+3\cos(\pi-\theta)$

$= 2-3\cos\theta$ (F)

The curve is not symmetrical on y-axis.

(c): About pole.

At $(-r,\theta)$

$-r = 2+3\cos\theta$ (F)

At $(r,\pi+\theta)$

$r = 2+3\cos(\pi+\theta)$

$= 2-3\cos\theta$ (F)

The curve is not symmetrical ~~on~~ about origin.

x) Tangent at pole:

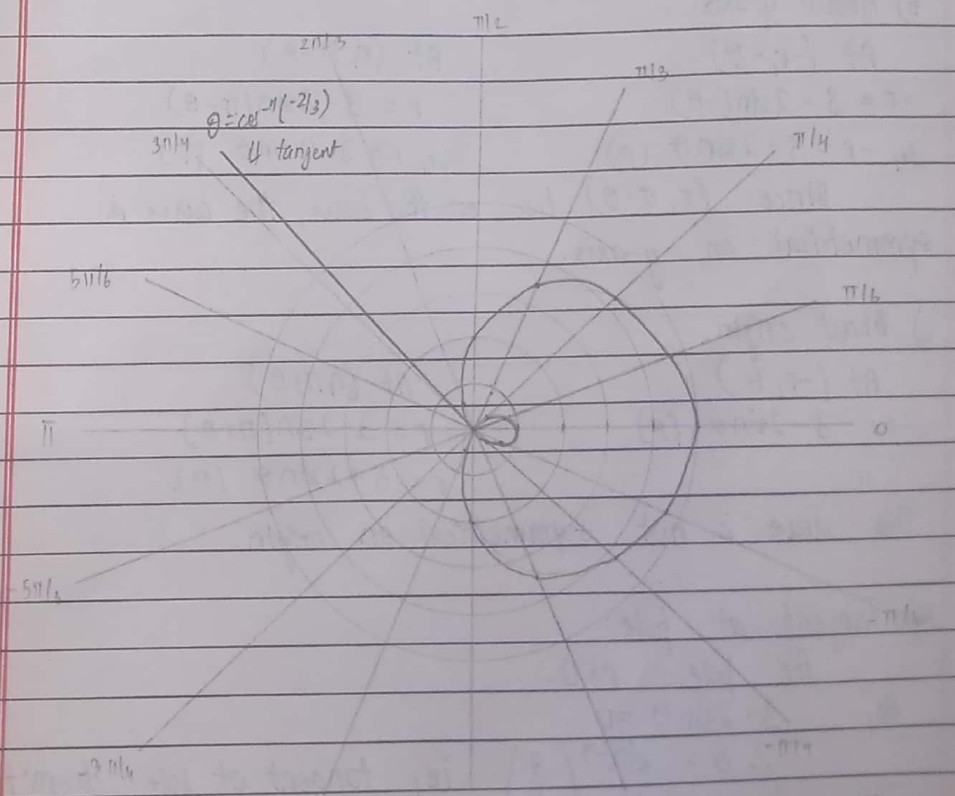
At pole, $r=0$.

$0 = 2+3\cos\theta$ $\quad \therefore \theta = \cos^{-1}(-2/3) = 131.81°$

$\therefore \theta = \cos^{-1}(-2/3)$ is the tangent to the curve at pole.

x) In r-θ table:

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	$5\pi/6$	π
$r = 2+3\cos\theta$	5	4.59	4.121	3.5	2	0.5	-0.121	-0.59	-1



Limacon with inner loop

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(iv): $r = 3 - 2\sin\theta$.

Soln:

Given,

$r = 3 - 2\sin\theta$

x) Symmetry

a) About x-axis:

At $(r, -\theta)$

$r = 3 - 2\sin(-\theta)$

$= 3 + 2\sin\theta$ (F)

At $(-r, \pi - \theta)$

$-r = 3 - 2\sin(\pi - \theta)$

or, $-r = 3 - 2\sin\theta$ (F)

The curve is not symmetrical on x-axis

b) About y-axis:

At $(-r, -\theta)$

$-r = 3 - 2\sin(-\theta)$

or, $-r = 3 + 2\sin\theta$ (F)

At $(r, \pi - \theta)$

$r = 3 - 2\sin(\pi - \theta)$

or, $r = 3 - 2\sin\theta$ (T)

Since $(r, \pi - \theta)$ lies on the curve, the curve is symmetrical on y-axis.

c) About origin.

At $(-r, \theta)$

$-r = 3 - 2\sin\theta$ (F)

At $(r, \pi + \theta)$

$r = 3 - 2\sin(\pi + \theta)$

$r = 3 + 2\sin\theta$ (F).

The curve is not symmetrical on origin.

x) Tangent at pole:

At pole, $r = 0$.

So, $3 - 2\sin\theta = 0$

$\therefore \theta = \sin^{-1}\left(\frac{3}{2}\right)$ ie, tangent at pole doesn't exist.

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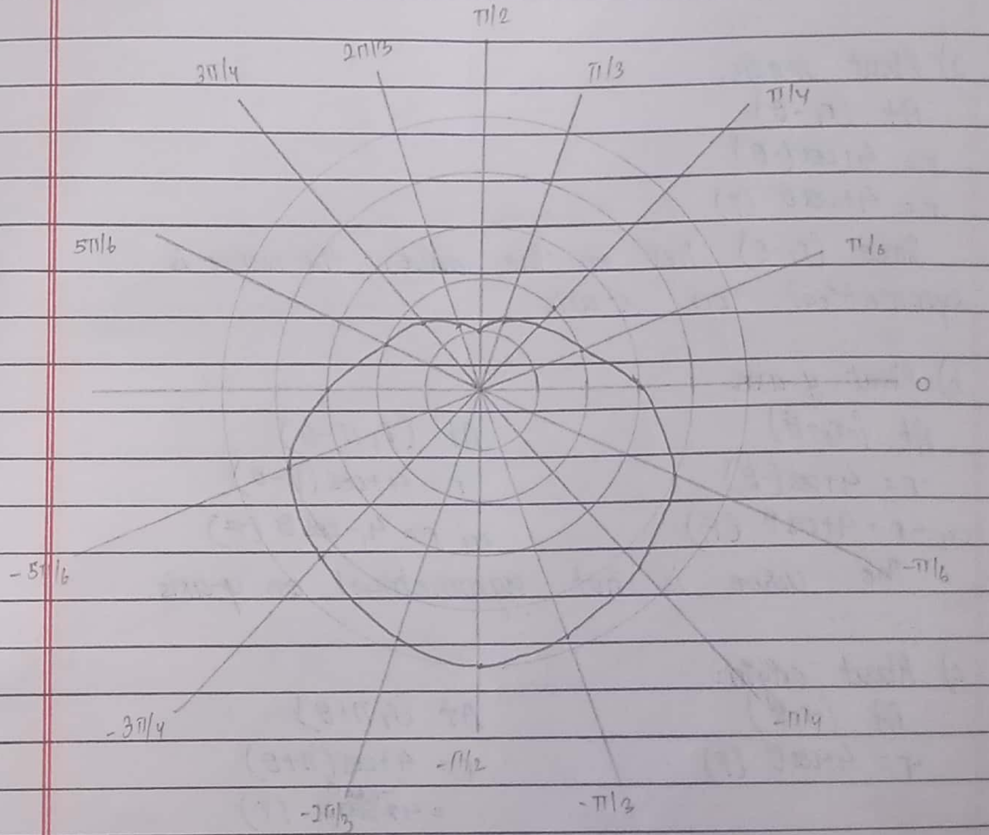
x) r-θ table.

θ	$-\pi/2$	$-\pi/3$	$-\pi/4$	$-\pi/6$	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$r = 3 - 2\sin\theta$	5	4.73	4.41	4	3	2	1.585	1.267	1.



Dimpled limacon.

(v) $r = 4+\cos\theta$

Soln.

Given,

$r = 4+\cos\theta$

x) Symmetry:

a) About x-axis.

At $(r, -\theta)$

$r = 4+\cos(-\theta)$

$r = 4+\cos\theta$ (T)

Since $(r,-\theta)$ lies on the curve, the curve is symmetrical on x-axis.

b) About y-axis.

At $(-r,-\theta)$

$-r = 4+\cos(-\theta)$

or, $-r = 4+\cos\theta$ (F)

At $(r, \pi-\theta)$

$r = 4+\cos(\pi-\theta)$

or, $r = 4-\cos\theta$ (F)

The curve is not symmetrical on y-axis.

c) About origin:

At $(-r,\theta)$

$-r = 4+\cos\theta$ (F)

At $(r, \pi+\theta)$

$r = 4+\cos(\pi+\theta)$

$= 4 - \cos\theta$ (F)

(x): Tangent at pole.

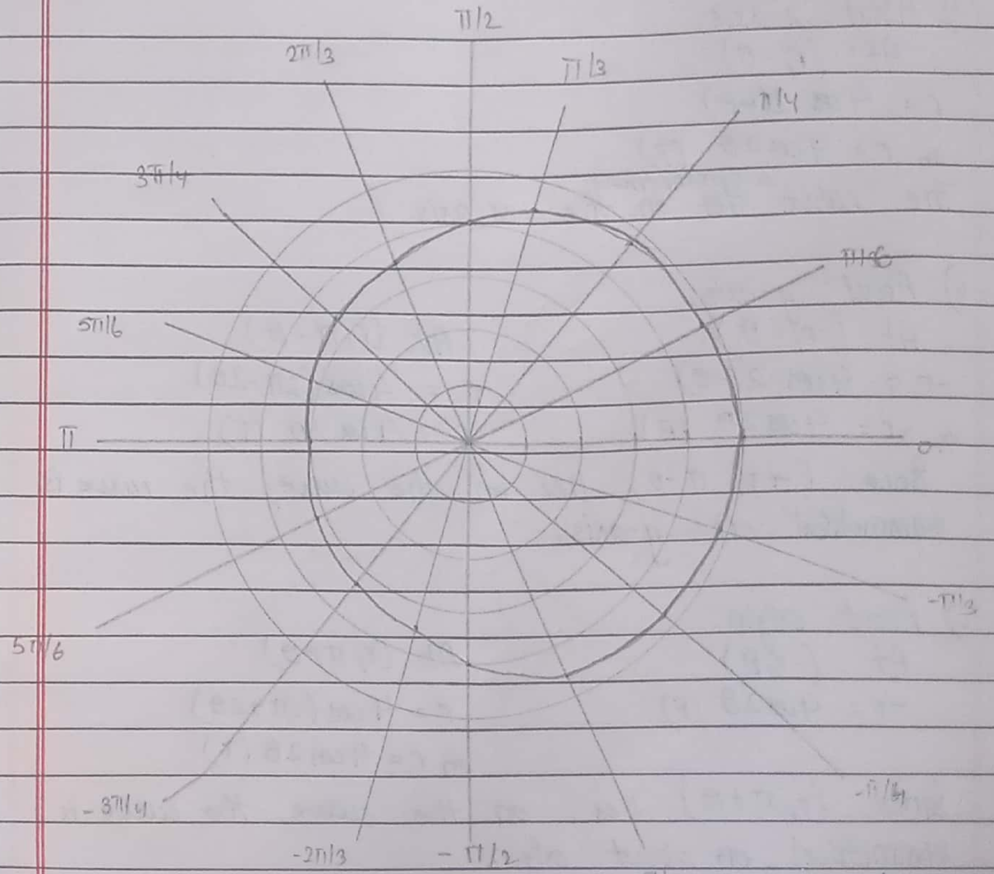
At pole, $r=0$.

$4+\cos\theta = 0$

$\theta = \cos^{-1}(4)$ Tangent at pole doesn't exist.

x) r-θ table.

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	$5\pi/6$	π
$r = 4+\cos\theta$	5	4.86	4.707	4.5	4	3.5	3.29	3.13	3



oval limacon.

(vi): $r = 4\cos 2\theta$.

Soln.

Given, $r = 4\cos 2\theta$

x) Symmetry:

a) About x-axis,

At $(r, -\theta)$

$r = 4\cos 2(-\theta)$

or $r = 4\cos 2\theta$ (T)

The curve is symmetrical on the x-axis.

b) About y-axis,

At $(-r, -\theta)$

$-r = 4\cos 2(-\theta)$

or $-r = 4\cos 2\theta$ (F)

At $(r, \pi-\theta)$

$r = 4\cos(2\pi - 2\theta)$

$r = 4\cos 2\theta$ (T)

Since $(r, \pi-\theta)$ lies on the curve, the curve is symmetrical on y-axis.

c) About origin.

At $(-r, \theta)$

$-r = 4\cos 2\theta$ (F)

At $(r, \pi+\theta)$

$r = 4\cos(2\pi + 2\theta)$

or $r = 4\cos 2\theta$ (T)

Since $(r, \pi+\theta)$ lies on the curve, the curve is symmetrical ~~on~~ about origin.

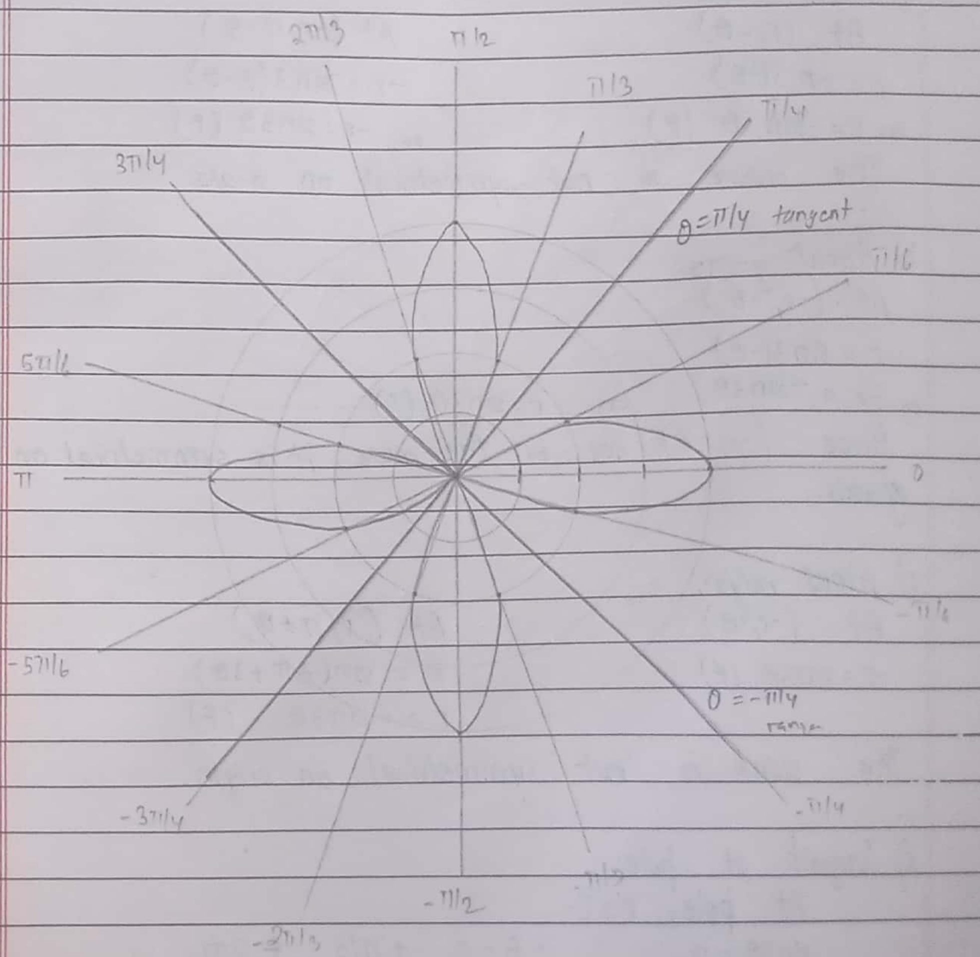
x) Tangent at pole:

At pole, $r = 0$.

$4\cos 2\theta = 0$ or, $\cos 2\theta = 0$ $\therefore 2\theta = \pi/2$ $\therefore \theta = \pm\pi/4$

x) r-θ table

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	$5\pi/6$	π
$r = 4\cos 2\theta$	4	2	0	-2	-4	-2	0	2	4



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(vii). $r=\sin 3\theta$

Soln:

Given, $r=\sin 3\theta$

x) Symmetry

a) About x-axis:

At $(r,-\theta)$

$r=\sin 3(-\theta)$

or, $r=-\sin 3\theta$ (F)

At $(-r,\pi-\theta)$

$-r=\sin 3(\pi-\theta)$

or, $-r=\sin 3\theta$ (F)

The curve is not symmetrical on x-axis.

b) About y-axis:

At $(-r,-\theta)$

$-r=\sin 3(-\theta)$

or $-r=-\sin 3\theta$ or, $r=\sin 3\theta$ (T)

Since $(-r,-\theta)$ lies on the curve, it is symmetrical on y-axis.

c) About origin.

At $(-r,\theta)$

$-r=\sin 3\theta$ (F)

At $(r,\pi+\theta)$

$r=\sin(3\pi+3\theta)$

$r=-\sin 3\theta$ (F)

The curve is not symmetrical on origin.

x) Tangent at pole:

At pole, $r=0$.

$\sin 3\theta=0$ $\therefore \theta=0, \pm\pi/3, \pm 2\pi/3$

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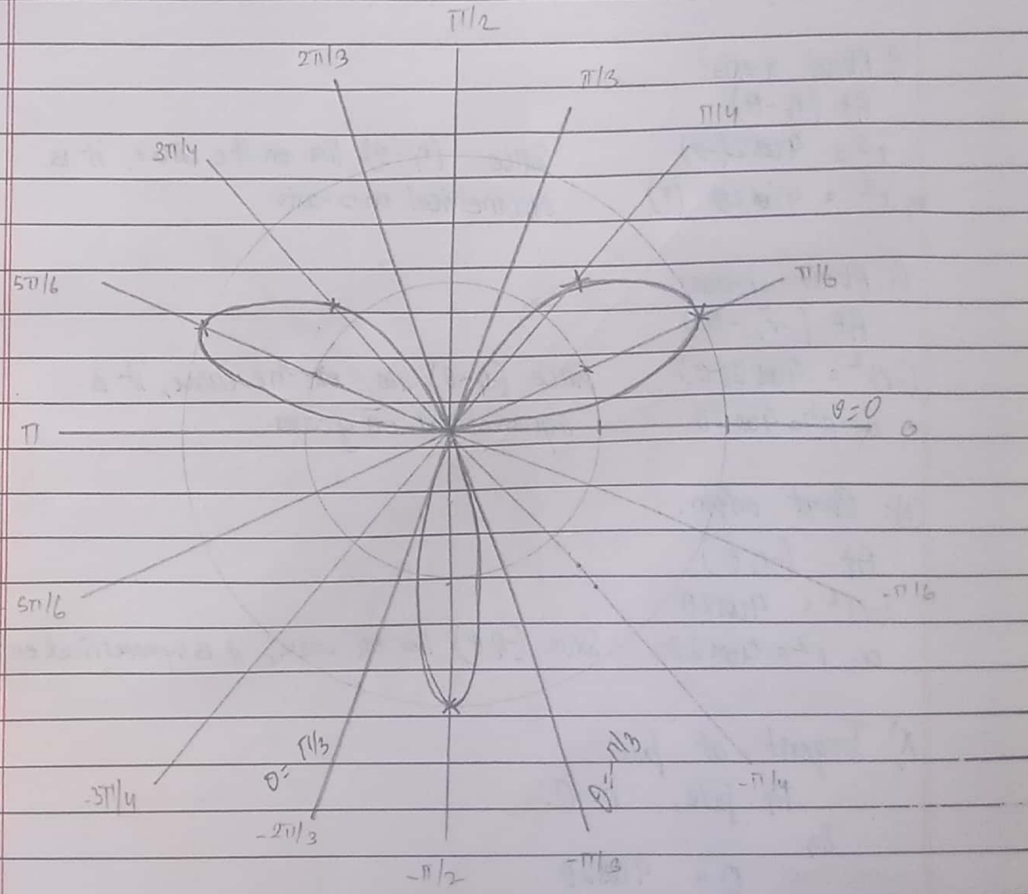
x) In r-θ table

θ	$-\pi/2$	$-\pi/3$	$-\pi/4$	$-\pi/6$	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$r=\sin 3\theta$	1	0	-0.707	-1	0	1	0.707	0	-1



(viii): $r^2 = 4\cos 2\theta$

Soln:

Given,

$r = 4\cos 2\theta$

i) Symmetry

a) About x-axis:

At $(r, -\theta)$

$r^2 = 4\cos 2(-\theta)$ — Since $(r, -\theta)$ lies on the curve, it is symmetrical on x-axis

or, $r^2 = 4\cos 2\theta$ (T)

b) About y-axis:

At $(-r, -\theta)$

$(-r)^2 = 4\cos 2(-\theta)$ — Since $(-r, -\theta)$ lies on the curve, it is symmetrical on y-axis.

or $r^2 = 4\cos 2\theta$

(c): About origin.

At $(-r, \theta)$

$(-r)^2 = 4\cos 2\theta$

or, $r^2 = 4\cos 2\theta$ — Since $(-r, \theta)$ lies on curve, it is symmetrical on origin.

ii) Tangent at pole:

At pole, $r = 0$.

So

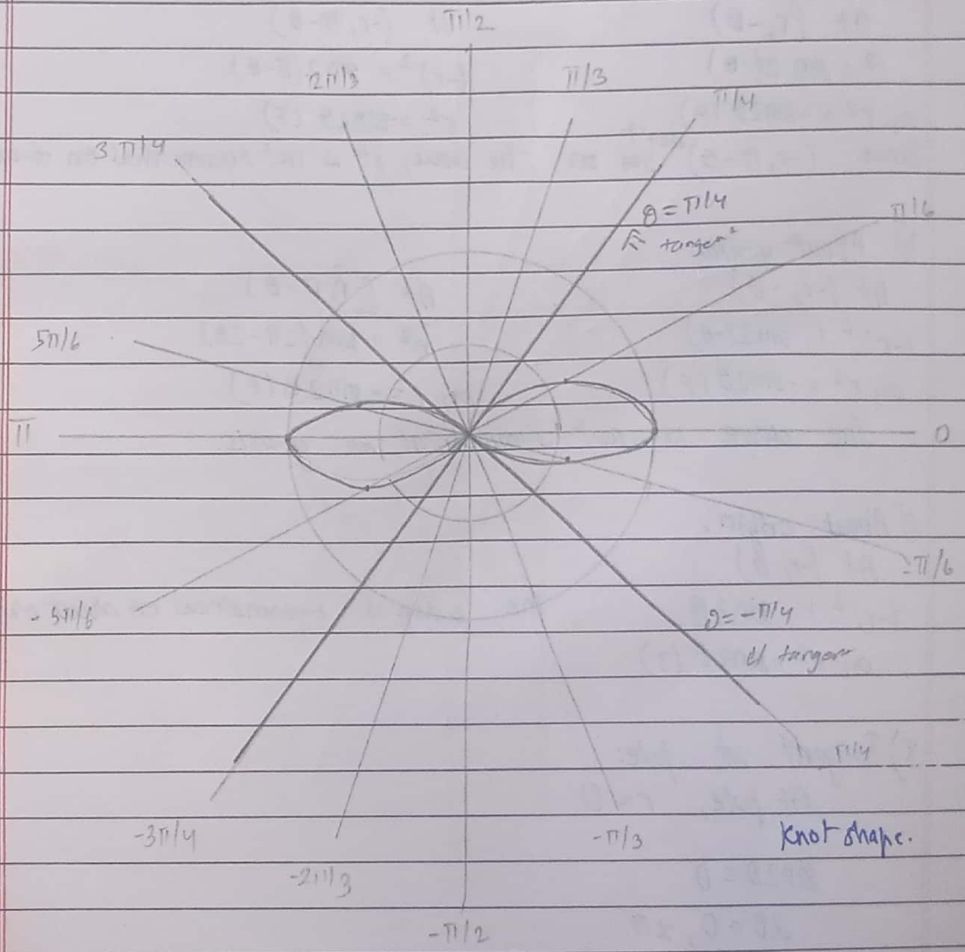
$0 = 4\cos 2\theta$

or $2\theta = \pm\pi/2$

$\therefore \theta = \pm\pi/4, \pm 3\pi/4$

x) r-θ table.

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$r = \sqrt{4\cos 2\theta}$ $= \pm 2\sqrt{\cos\theta}$	± 2	± 1.41	0	-	-



(ix) $r^2 = \sin 2\theta$

Soln:

Given, $r^2 = \sin 2\theta$

x) Symmetry:

a) About x-axis:

At $(r, -\theta)$ | At $(-r, \pi-\theta)$
$r^2 = \sin 2(-\theta)$ | $(-r)^2 = \sin 2(\pi-\theta)$
or, $r^2 = -\sin 2\theta$ (F) | $r^2 = -\sin 2\theta$ (F)

Since $(-r, \pi-\theta)$ don't lies on the curve, it is not symmetrical on x-axis.

b) About y-axis:

At $(-r, -\theta)$ | At $(r, \pi-\theta)$
$(-r)^2 = \sin 2(-\theta)$ | $r^2 = \sin(2\pi - 2\theta)$
or, $r^2 = -\sin 2\theta$ (F) | or $r^2 = -\sin 2\theta$ (F)

The curve is not symmetrical on y-axis.

c) About origin.

At $(-r, \theta)$
$(-r)^2 = \sin 2\theta$ | The curve is symmetrical ~~on~~ about origin.
or, $r^2 = \sin 2\theta$ (T)

x) Tangent at pole:

At pole, $r = 0$

$\sin 2\theta = 0$

$2\theta = 0, \pm\pi$

$\therefore \theta = 0, \pm\pi/2$

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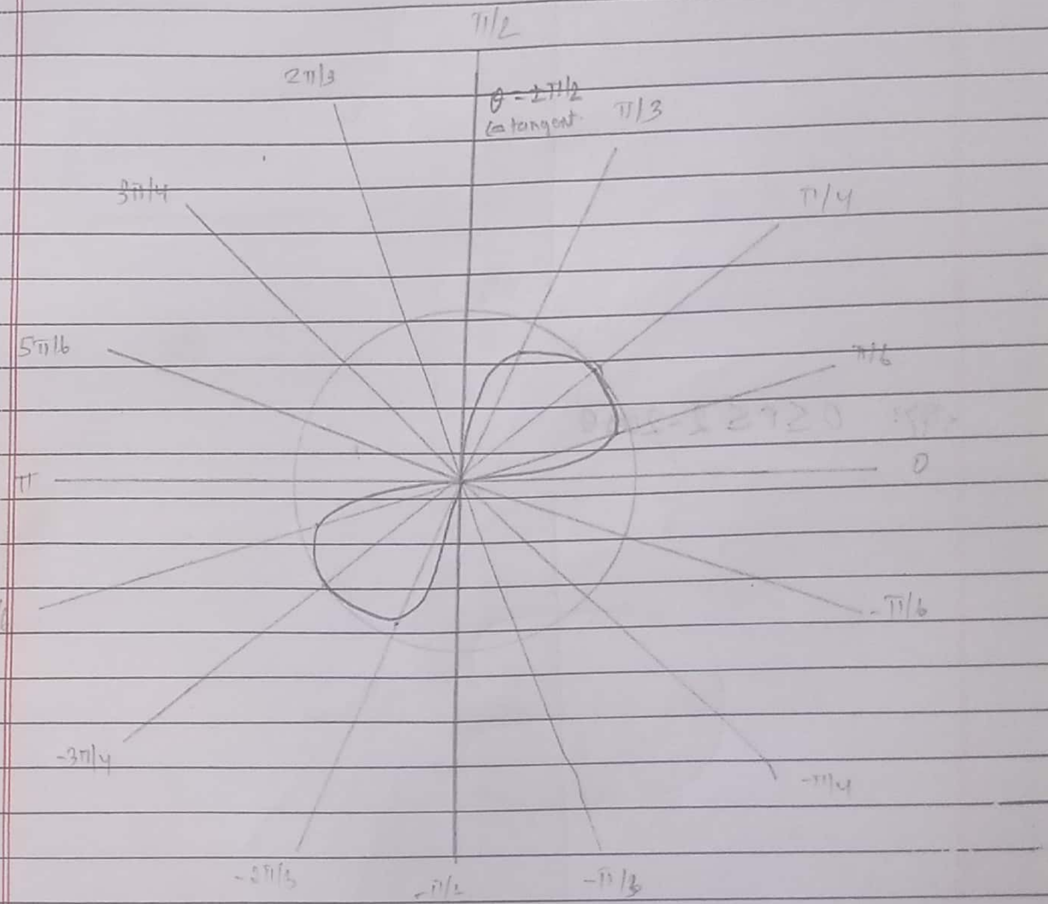
(x)! r-θ table:

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	$5\pi/6$	π
$r = \sqrt{\sin 2\theta}$	0	0.93	1	0.93	0	0.93i	i	0.93i	0



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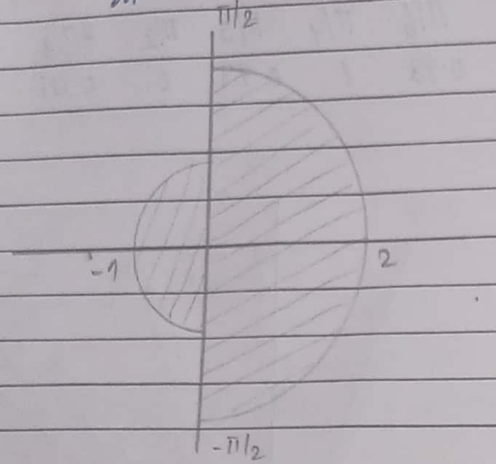
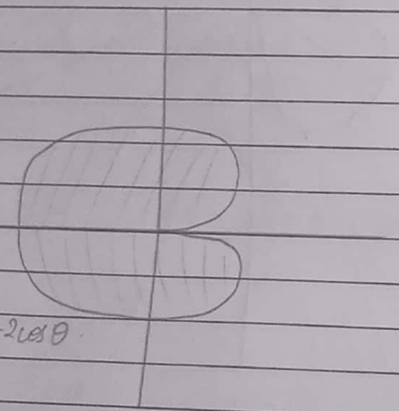
<Q>: $-1 \leq r \leq 2$, $-\pi/2 \leq \theta \leq \pi/2$

Sol^n:



<Q>: $0 \leq r \leq 2 - 2\cos\theta$



$0 \leq r = 2-2\cos\theta$

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<Q>: Sketch $0 \leq r \leq 2\sec\theta$, $-\pi/4 \leq \theta \leq \pi/4$.

Sol^n: