Unit: 1:

FUNCTIONS, LIMITS AND CONTINUITY

Function:

A function $y = f(x)$ from set A to set B is a relation which associates every element of set A to the unique (only one) element of set B.

The relations that give one-one and ~~may~~ many-one relation is called function.

A complete function is a function which is diffrentiable.

A function can also be taken as a machine.

$$y = f(x)$$

Here,
x is input $y = f(x)$ is machine y is output.

Domain and Range

The elements of set A is called domain and the elements of set B is called range.

Domain is independent whereas range is dependent.

In $y = f(x)$ → Range
Domain ↵

Eg:(i) Domain and Range of $y = \sqrt{x^2 - 3x}$:

Here, $x^2 - 3x \geq 0$

or $x(x-3) \geq 0$

$\therefore x \geq 0 \; , \; x \geq 3$

$D_f = (-\infty, 0] \cup [3, \infty)$

$R_f = [0, \infty)$

(ii) Domain and Range of $y = \sqrt{4 - x^2}$

Here,

$4 - x^2 \geq 0$

So, $D_f = [-2, 2]$

~~$R_f = [0,$~~ Again,

$y^2 = 4 - x^2$

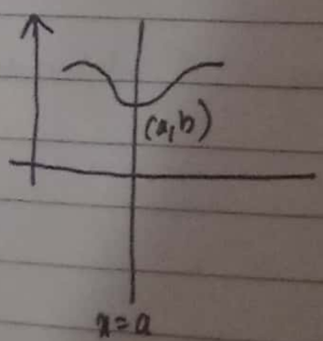
or $x^2 + y^2 = 4$

$\therefore R_f = [0, 2]$

Test for Function:

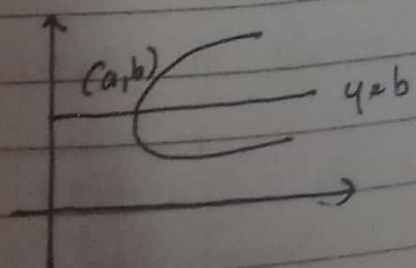
(i): Vertical line test:

- checking function of x



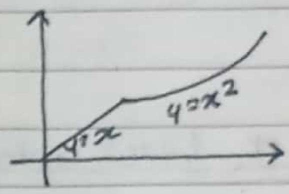
ii) Horizontal line test

-checking function of y.



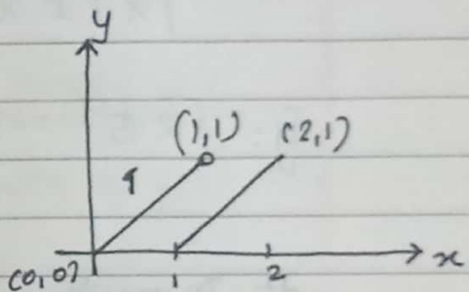
Piecewise Defined Function:

Piece-wise defined function is a function whose domain is divided into parts and each part is defined by a different function rule.



Eg: $f(x) = \begin{cases} x & 0 \le x \le 1 \\ x^2 & 1 < x \le 2 \end{cases}$

- The functions are represented togethes in coordinate axes.

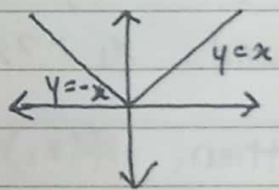


Eg: Find the piecewise function for:

Here;

$f(x) = \begin{cases} x & 0 \le x < 1 \\ x-1 & 1 \le x \le 2 \end{cases}$

Absolute function:



$f|x| = \begin{cases} x & \text{if } x \ge 0 \\ -x & \text{if } x < 0 \end{cases}$

Signum Function:

$f(x) = \begin{cases} x & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ x & \text{if } x > 0 \end{cases}$

it returns negative for negative value and positive for positive values.

Greatest Integer Function

$$\lfloor x \rfloor \le x$$

Eg: $\lfloor 2\cdot5 \rfloor = 2$ $\qquad \lfloor -2\cdot8 \rfloor = -3$

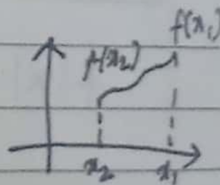
Least integer Function:

$$\lceil x \rceil \ge x$$

Eg: $\lceil 2\cdot5 \rceil = 3$ $\qquad \lceil -2\cdot8 \rceil = -2$

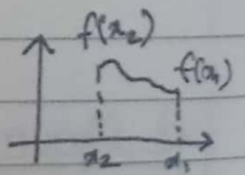
Increasing and Decreasing function:

If $x_1, x_2 \in [a,b]$
and $x_1 > x_2$



then, $f(x_1) > f(x_2)$ is increasing function
and

$f(x_1) < f(x_2)$ is decreasing function:



Even and Odd function:

*) Symmetry: the transformation that leaves the graph unchanged.

If $(x, y) \to (-x, y)$; symmetry on y-axis

If $(x, y) \to (x, -y)$; symmetry on x-axis.

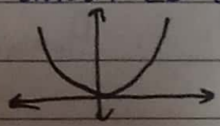
*) Even function:

Condition:

$$f(-x) = f(x)$$

- it is symmetric about co-ordinate axes.

Eg: $f(x) = x^2$



Here, $-x$ also gives $+y$

ie, symmetry on y.

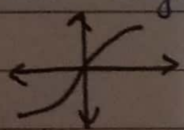
*) Odd function:

Condition:

$$f(-x) = -f(x)$$

- it is symmetric about origin.

Eg: $f(x) = x^3$



Here, $-x$ gives $-y$

ie, symmetry on origin.

Eg: Find if $f(x) = x+1$ is odd or even.

$f(x) = x+1$

Replacing x by $-x$.

$f(-x) = -x+1 \neq f(x)$

and $\neq -f(x)$.

$\therefore$ It is neither odd nor even.

Combining functions:

Let f & g be two functions then they can be combined in the following ways:

$f \pm g$, $f \cdot g$, f/g.

Here, Domain of combined function ie, $D_f \Rightarrow x \in D_{fx} \cap D_{gx}$

Eg: (i) $f(x) = x$ & $g(x) = \sqrt{1-x}$, find domain:

Here,

$D_f = (-\infty, \infty)$

$D_g = (-\infty, 1]$

So, $D_{combination} = (-\infty, 1]$

(ii) If $f(x) = \sqrt{x}$ & $g(x) = \sqrt{1-x}$, find domain.

Here,

$D_f = [0, \infty)$

$D_g = (-\infty, 1]$

$\therefore D_{combination} = [0, 1]$

Composite function:

$fog(x) = f(g(x))$

$gof(x) = g(f(x))$

Eg: $f(x) = x^2$

$g(x) = \sqrt{x}$

$fog(x) = f(g(x))$

$= (\sqrt{x})^2$

$= x$

Here, it looks like domain of $f \circ g(x)$ is $(-\infty, \infty)$ but $g(x)$ is not defined for interval $(-\infty, \infty)$.

Here,

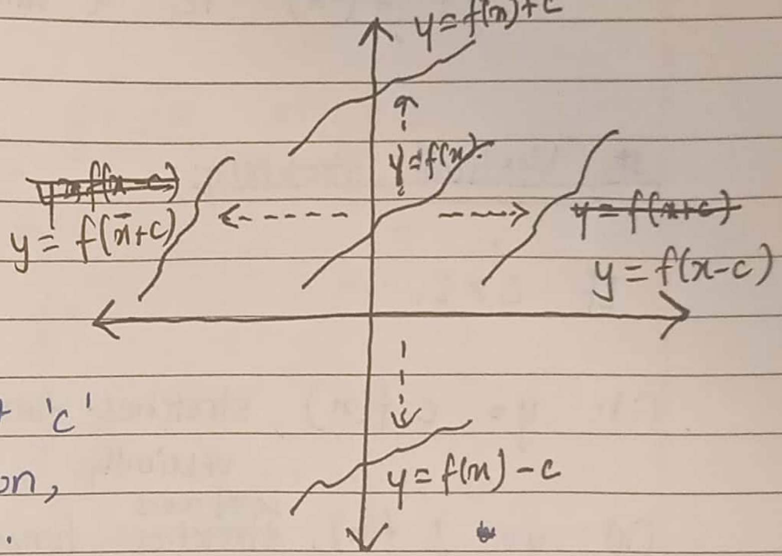
$D_f = (-\infty, \infty)$

$D_g = [0, \infty)$

So,

$D_{composite} / D_{fog} = [0, \infty)$

Shifting of a Function:



~~If a~~

(i): Vertical shifting:

If a certain constant 'c' is added to the function, the function shifts up.

Eg: $y = f(x) + c$

If a certain constant 'c' is ~~added~~ subtracted from the function, the function shifts down. $y = f(x) - c$

(ii): Horizontal shifting:

If a certain constant 'c' is added to x in function, function shifts horizontally ~~right~~ left. Eg: $y = f(x+c)$ {from our p.o.v}

If a certain constant 'c' is subtracted from x in function, function shifts horizontally right ~~left~~. Eg: $y = f(x-c)$ {from our p.o.v}

(*): Note:

(i): Adding / Subtracting inside = Horizontal

(ii) Adding / subtracting outside = vertical.

~~(iii): Adding → positive shift.~~

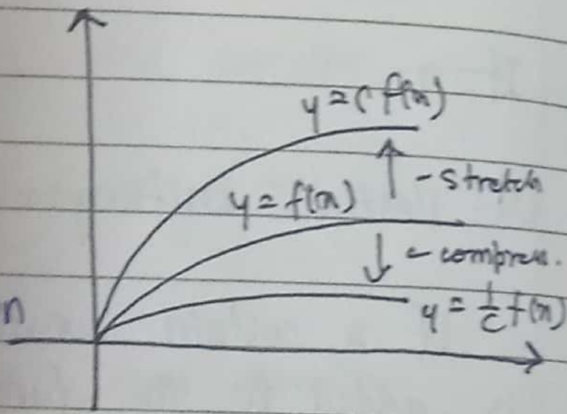
~~(iv) Subtracting → negative shift~~

Reflection of a function:

(i): For reflection about x-axis,
$y = -f(x)$. ie, y value ~~interchanged~~ changed.

(ii): For reflection about y-axis.
$y = f(-x)$ ie, x value changed.

Vertical Stretching:



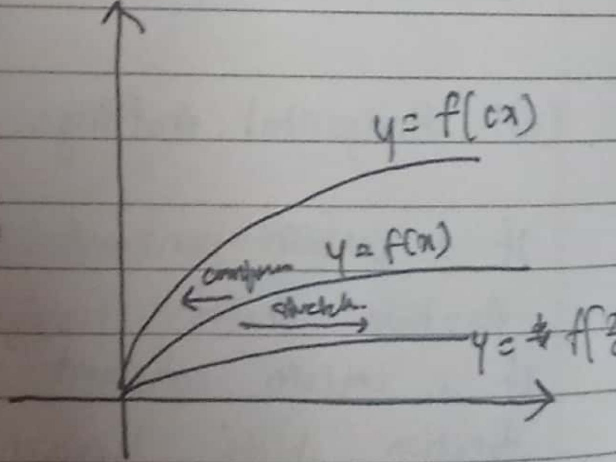
If $c > 1$,

(i): $y = c\,f(x)$, stretches function vertically.

(ii) $y = \frac{1}{c} f(x)$, compresses ~~stretches~~ function horizontally.

Horizontal Stretching:

If $c > 1$,



(i): $y = f(cx)$, compressed function horizontally.

(ii) $y = f\left(\frac{x}{c}\right)$, stretches function horizontally.

(*): Note:

(i): Operation outside function: affects vertically

(ii) operation inside function: affects ~~vertr~~ horizontally.

(iii) Vertical stretching means horizontal compressing (product)

(iv) Vertical compressing means horizontal stretching (division)

Eg: Stretch the function $y = \sqrt{x}$ at $c = 3$.

Ans:

Verticall stretching ; $y = 3\sqrt{x}$

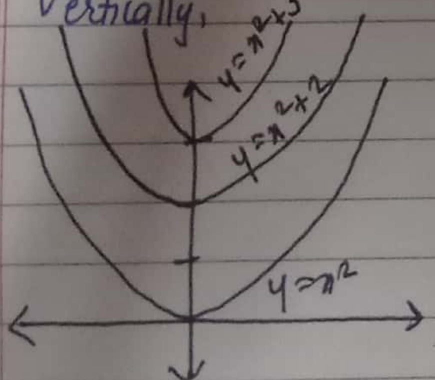
Vertical compressing, $y = \frac{1}{3}\sqrt{x}$

Horizontal stretching; $y = \sqrt{x/3}$

Horizontal compressing; $y = \sqrt{3x}$

Eg: Shifting function $y = x^2$ at $c = 2$ and 3.

Vertically,



Horizontally, at $c=2$ and $c=-2$.