KATHMANDU UNIVERSITY

DHULIKHEL , KAVRE

Subject: PHY102

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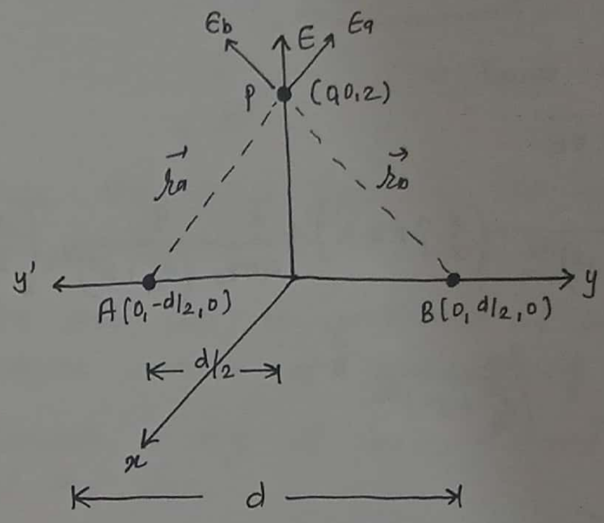
Q.1. Find the electric field a distance z above the midpoint between two charges +q and -q, distance 'd' apart.

Sol^n:



Here, figure illustrates the coordinates and the geometry to be used.

From figure,

$$\vec{r_a} = \frac{d}{2}\hat{j} + z\hat{k} \qquad \vec{r_b} = -\frac{d}{2}\hat{j} + z\hat{k}$$

So,

$$r_a = \sqrt{\left(\frac{d}{2}\right)^2 + z^2} = \left(\frac{d^2}{4} + z^2\right)^{1/2}$$

$$r_b = \sqrt{\left(-\frac{d}{2}\right)^2 + z^2} = \left(\frac{d^2}{4} + z^2\right)^{1/2}$$

Now, the electric field at P due to charge at A,

$$\vec{E_A} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_A^3}\vec{r_A} = \frac{1}{4\pi\varepsilon_0}\frac{q}{\left(\frac{d^2}{4}+z^2\right)^{3/2}}\left(\frac{d}{2}\hat{j} + z\hat{k}\right)$$

Similarly,

$$\vec{E}_b = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_B}\vec{r_B} = \frac{1}{4\pi\varepsilon_0}\frac{q}{\left(\frac{d^2}{4}+z^2\right)^{3/2}}\left(-\frac{d}{2}\hat{j}+z\hat{k}\right)$$

We know,

E_T at P is equal to

$$\vec{E_T} = \vec{E_A} + \vec{E_B}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{\left(\frac{d^2}{4}+z^2\right)^{3/2}}\left(\frac{d}{2}\hat{j}+z\hat{k}\right) + \frac{1}{4\pi\varepsilon_0}\frac{q}{\left(\frac{d^2}{4}+z^2\right)^{3/2}}\left(-\frac{d}{2}\hat{j}+z\hat{k}\right)$$

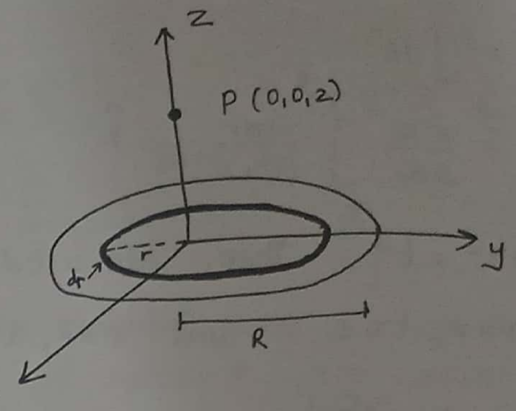
$$\therefore \vec{E_T} = \frac{1}{4\pi\varepsilon_0}\, q\, \frac{2z}{\left(\frac{d^2}{4}+z^2\right)^{3/2}}\hat{k}$$

When $z \gg d$,

$$\vec{E_T} = \frac{1}{4\pi\varepsilon_0}\, q\, \frac{2z}{z^3}\hat{k} = \frac{1}{4\pi\varepsilon_0}\, q\, \frac{2}{z^2}\hat{k}$$

<Q.2>: Find the electric field a distance z above the center of a flat circular disk of radius R, which carriers a uniform surface charge σ. What does your formula give in the limit $R \to \infty$? Also check the case $z \gg R$.

Soln



Here, the figure illustrates the geometry and the coordinates to be used.

The disc can be considered as combination of infinite number of infinitesimally thin rings.

Consider a ring of radius r and thickness dr of the disc.

If 'σ' is the uniform charge density, then the charge on the ring $dq = \sigma\,(2\pi r\, dr)$

The electric field at P due to charge dq on the ring is,

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\, dq\, \frac{z}{(r^2+z^2)^{3/2}}\hat{k}$$

$$= \frac{1}{4\pi\varepsilon_0}\,\sigma(2\pi r\, dr)\,\frac{z}{(r^2+z^2)^{3/2}}\hat{k}$$

$$\therefore d\vec{E} = \frac{\sigma z}{2\varepsilon_0}\frac{r\,dr}{(r^2+z^2)^{3/2}}\hat{k} \quad \text{—— (i)}$$

Hence, the total electric field due to the charge on the whole flat circular disc is given by,

$$\vec{E}_{disc} = \int d\vec{E}$$

$$= \frac{\sigma z}{2\varepsilon_0}\int_0^R \frac{r\,dr}{(r^2+z^2)^{3/2}}\,\hat{k}$$

Let $r^2+z^2 = t^2$ Then, $r\,dr = t\,dt$.

when $r=0,\ t=z$ and $r=R,\ t=\sqrt{R^2-z^2}$

So,

$$\vec{E}_{disc} = \left(\int_z^{\sqrt{R^2+z^2}} \frac{t\cdot dt}{t^3}\right)\frac{\sigma z}{2\varepsilon_0}\,\hat{k}$$

$$= \frac{\sigma z}{2\varepsilon_0}\int_z^{\sqrt{R^2+z^2}} \frac{dt}{t^2}\,\hat{k}$$

$$= \frac{\sigma z}{2\varepsilon_0}\left[-\frac{1}{t}\right]_z^{\sqrt{R^2+z^2}}\hat{k}$$

$$= \frac{\sigma z}{2\varepsilon_0}\left[\frac{1}{z} - \frac{1}{\sqrt{R^2+z^2}}\right]\hat{k}$$

$$\therefore \vec{E}_{disc} = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{R^2+z^2}}\right)\hat{k}$$

When $R\to\infty$, $\therefore \vec{E}_{disc} = \frac{\sigma}{2\varepsilon_0}\hat{k}$

For points far from the disc, $z \gg R$.

So,

$$\vec{E}_{disc} = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{1}{(R^2+z^2/z^2)^{1/2}}\right]\hat{k}$$

$$= \frac{\sigma}{2\varepsilon_0}\left[1 - \left(1+\frac{R^2}{z^2}\right)^{-1/2}\right]\hat{k}$$

Using binomial expansion,

$$= \frac{\sigma}{2\varepsilon_0}\left[1 - \left(1 - \frac{1}{2}\frac{R^2}{z^2} + \dots\right)\right]\hat{k}$$

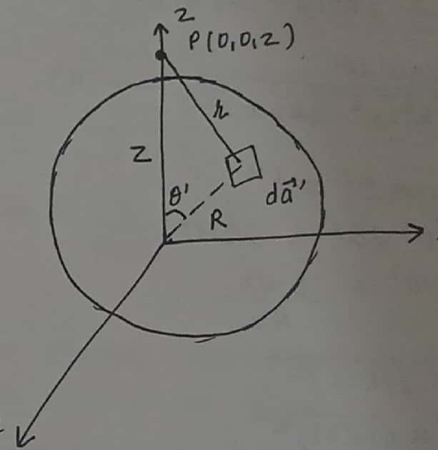
So,

$$\vec{E}_{disc} = \frac{\sigma}{2\varepsilon_0}\left(\frac{1}{2}\frac{R^2}{z^2}\right)\hat{k} = \frac{1}{4\pi\varepsilon_0}\,\frac{\sigma\pi R^2}{z^2}\,\hat{k}$$

$$\therefore \vec{E}_{disc} = \frac{1}{4\pi\varepsilon_0}\,q\,\frac{1}{z^2}\,\hat{k}$$

<Q.3>: Find the potential inside and outside a spherical shell of radius R, which carries a uniform surface charge density σ.

Sol^n:



Let us consider a uniformly charged ~~solid~~ spherical shell having radius R and surface charge density (σ).

Let us consider an elemental area $d\vec{a}'$ on the surface that produces electric potential at $P(0,0,z)$

Now, we know, the electric potential for a surface charge is

$$V = \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma\,d\vec{a}'}{r}$$

From the law of cosines, $r = \sqrt{R^2+z^2-2Rz\cos\theta'}$

We know,

$$d\vec{a}' = R^2\sin\theta'\,d\theta'\,d\phi'$$

Now, eqn (i) can be written as.

$$V = \frac{1}{4\pi\varepsilon_0} \int \frac{\sigma R^2 \sin\theta' \, d\theta' \, d\phi'}{\sqrt{R^2+z^2-2Rz\cos\theta'}}$$

$$= \frac{\sigma R^2}{4\pi\varepsilon_0} \int \frac{\sin\theta' \, d\theta' \, d\phi'}{\sqrt{R^2+z^2-2Rz\cos\theta'}}$$

$$= \frac{\sigma R^2}{4\pi\varepsilon_0} \left[\left\{ \int_0^{\pi} \frac{\sin\theta' \, d\theta'}{\sqrt{R^2+z^2-2Rz\cos\theta'}} \right\} \left\{ \int_0^{2\pi} d\phi' \right\} \right]$$

$$= 2\pi \times \frac{\sigma R^2}{4\pi\varepsilon_0} \int_0^{\pi} \frac{\sin\theta' \, d\theta'}{\sqrt{R^2+z^2-2Rz\cos\theta'}} \quad \text{—} \cancel{(ii)}$$

$$\text{or, } V = \frac{\sigma R^2}{2\varepsilon_0} \int_0^{\pi} \frac{\sin\theta' \, d\theta'}{\sqrt{R^2+z^2-2Rz\cos\theta'}} \quad \text{— (ii)}$$

Put $R^2+z^2-2Rz\cos\theta' = t^2$

Then, $2Rz \sin\theta' \, d\theta' = 2t\,dt.$

or, $\sin\theta' \, d\theta' = \frac{t \cdot dt}{Rz}$

When $\theta' = 0$, $t = \sqrt{(R-z)^2}$

When $\theta' = \pi$, $t = \sqrt{(R+z)^2}$

So eqn (ii) can be written as,

$$V = \frac{\sigma R^2}{2\varepsilon_0} \int_{\sqrt{(R-z)^2}}^{\sqrt{(R+z)^2}} \frac{t \cdot dt}{t \cdot Rz}$$

$$= \frac{\sigma R}{2\varepsilon_0 z} \int_{\sqrt{(R-z)^2}}^{\sqrt{(R+z)^2}} 1 \cdot dt$$

$$\text{or, } V = \frac{\sigma R}{2\varepsilon_0 z} \left[\sqrt{(R+z)^2} - \sqrt{(R-z)^2} \right]$$

For points, outside the spherical shell, $z > R$

$$V_{outside} = \frac{\sigma R}{2\varepsilon_0 z} \left[(R+z) - (z-R) \right]$$

$$= \frac{\sigma R^2}{\varepsilon_0 z} = \frac{\frac{q}{4\pi R^2} \times R^2}{\varepsilon_0 \, z}$$

$$\therefore V_{outside} = \frac{1}{4\pi\varepsilon_0} \frac{q}{z}$$

For points inside the spherical surface, $z < R$

$$V_{inside} = \frac{\sigma R}{2\varepsilon_0 z} \left[(R+z) - (R-z) \right]$$

$$= \frac{\sigma R \cdot 2z}{2\varepsilon_0 z} = \frac{\sigma R}{\varepsilon_0} = \frac{\frac{q}{4\pi R^2} \times R}{\varepsilon_0}$$

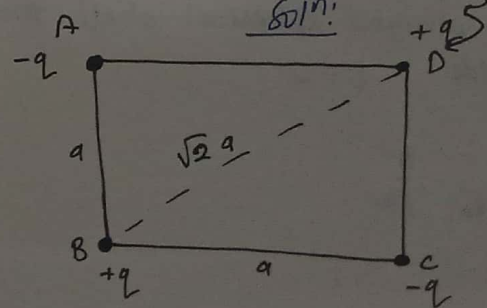
$$\therefore V_{inside} = \frac{1}{4\pi\varepsilon_0} \frac{q}{R}$$

Q.47: Three charges are situated at the corners of a square (side a) as in figure.

a) How much work does it take to bring in another charge (+q) from far away and place it in the fourth corner?

b) How much work does it take to assemble the whole configuration of four charges?

Soln:



a) Now, at point D,

$$W_D = (+q)V$$

$$= (+q)\left[\frac{1}{4\pi\varepsilon_0}\left\{\frac{-q}{a} + \frac{q}{\sqrt{2}a} - \frac{q}{a}\right\}\right]$$

$$\therefore W_D = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{a}\left[-2+\frac{1}{\sqrt{2}}\right]$$

b): Workdone to assemble the whole configuration of four charges (w) = U

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_1q_4}{r_{14}} + \frac{q_2q_3}{r_{23}} + \frac{q_2q_4}{r_{24}} + \frac{q_3q_4}{r_{34}}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[-\frac{q^2}{a} + \frac{q^2}{\sqrt{2}a} - \frac{q^2}{a} - \frac{q^2}{a} + \frac{q^2}{\sqrt{2}a} - \frac{q^2}{a}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[-\frac{4q^2}{a} + \frac{2q^2}{\sqrt{2}a}\right]$$

$$= \frac{q^2}{4\pi\varepsilon_0 a}\left[-4+\frac{2}{\sqrt{2}}\right]$$

$$\therefore U = 2\times\frac{1}{4\pi\varepsilon_0}\times\frac{q^2}{a}\left[-2+\frac{1}{\sqrt{2}}\right]$$

<Q.5>: Find the energy of a uniformly charged spherical shell of total charge 'q' and radius 'R'.

Soln:

For an uniformly charged spherical shell, the electric field inside $E=0$

and

outside $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

We know,

$$W_{total} = \int_{\text{all space}} \frac{\varepsilon_0}{2} E^2 d\tau$$

$$= \frac{\varepsilon_0}{2}\int_{\text{all space}} E^2 d\tau$$

We know, $d\tau = r^2 \sin\theta\, dr\, d\theta\, d\phi$

$$= \frac{\varepsilon_0}{2}\left[\left(\int_0^\infty E^2 r^2 dr\right)\left(\int_0^\pi \sin\theta\, d\theta\right)\left(\int_0^{2\pi} d\phi\right)\right]$$

$$= \frac{\varepsilon_0}{2}\times 2\times 2\pi\left[\int_R^\infty E_{out}^2\, r^2\, dr\right]$$

$$= \frac{4\pi\varepsilon_0}{2}\left[\int_R^\infty \left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\right)^2 r^2 dr\right]$$

$$= \frac{4\pi\varepsilon_0}{2}\left[\int_R^\infty \left(\frac{1}{4\pi\varepsilon_0}\right)^2 \times \frac{q^2}{r^2}\, dr\right]$$

$$= \frac{q^2}{(4\pi\varepsilon_0)^2}\times\frac{(4\pi\varepsilon_0)}{2}\times\left[\int_R^\infty \frac{1}{r^2}dr\right]$$

$$= \frac{1}{2}\times\frac{1}{4\pi\varepsilon_0}\times q^2\times\frac{1}{R}$$

$$\therefore W_{total} = \frac{q^2}{8\pi\varepsilon_0 R}$$