(ii): # $\sum_{n=1}^{\infty} \frac{1}{2^{n-1}}$

Soln.

The corresponding series is.

$1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots\ldots$

Here,

$|r| = \left|\frac{1}{2}\right| = \frac{1}{2} < 1$

Since $|r| < 1$, the series converges.

n^{th} term test for divergence:

If infinite series converges i.e, $\sum_{n=1}^{\infty} a_n$ converges,
then, $\lim_{n\to\infty} a_n = 0$

but converse may not be true.

*) Proof:

If $\sum a_n$ is convergent then, their partial sum is also convergent

i.e,

$\lim_{n\to\infty} S_n = \lim_{n\to\infty} S_{n-1} = L$

We have,

$\cancel{a_n = S_n - S_{n-1}}$

$\cancel{= L - L}$

$a_n = S_n - S_{n-1}$

Taking $\lim_{n\to\infty}$ on both sides,

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} S_n - \lim_{n\to\infty} S_{n-1}$

$= L - L$

$\therefore \lim_{n\to\infty} a_n = 0$

Hence, proved.

Converse may not be true.

$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

Soln.

Here, $a_n = \frac{1}{\sqrt{n}}$

Now,

$\lim_{n\to\infty} a_n = \cancel{\frac{1}{\sqrt{n}}} \lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$

So,

$S_n = \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots\ldots + \frac{1}{\sqrt{n}}$

$= \frac{1}{\sqrt{n}} + \frac{1}{\sqrt{n}} + \frac{1}{\sqrt{n}} + \ldots + \frac{1}{\sqrt{n}} = n \times \frac{1}{\sqrt{n}}$

$\therefore S_n = \sqrt{n}$

Now,

$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \sqrt{n} = \infty$ It is divergent.

Thus, converse of n^{th} root test is not true.