General Physics I (PHYS 101)

Lecture 12

Elasticity

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Outline

Elasticity

The property of a body by virtue of which it regain its original shape and size when the deforming force is removed is called the elasticity of the body. On the other hand, if the body remains deformed and shows no tendency to recover its original condition on the removable of deforming force, it is said to be perfectly plastic.

In reality, all objects are deformable and internal forces in the object resist the deformation.

Stress and Strain

Stress: The external force (deforming force) acting on an object per unit cross-sectional area is called stress. It characterizes the strength of the forces causing the deformation, on a "force per unit area" basis.

$$\text{Stress} = \frac{\text{Deforming force}}{\text{Cross-sectional area}} = \frac{F}{A} \tag{1}$$

SI unit of stress is the pascal. $\left[1 \text{ Pa} = 1\text{N} \cdot \text{m}^{-2}\right]$.

Strain: The ratio of change in dimension to the original dimension of a body is called strain of the body.

It is defined as the ratio of change in size of the elastic body under the action of stress to the original size.

$$\text{Strain} = \frac{\text{Change in dimension}}{\text{Original dimension}} \tag{2}$$

It measures of degree of deformation. It is the result of stress. It has no unit.

Normal Stress and Strain: In this stress the force acting on the body is normal to the surface. This is of three types e.g

Tensile Stress: The net force on the object is zero, but the object deforms as shown in figure 1.

Stress and Strain (contd.)

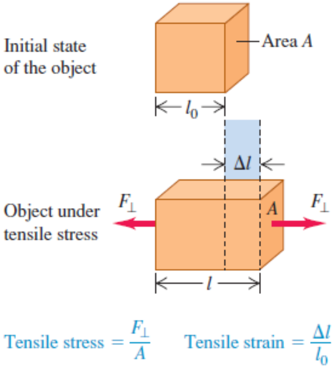

Figure 1: An example of tensile stress and strain

The tensile stress (the ratio of the force to the cross-sectional area) produces a tensile or longitudinal strain (the elongation divided by the initial length).

Compressive Stress and Strain When the forces on the ends of a bar are pushes rather than pulls (Figure 2), the bar is in compression and the stress is a compressive stress

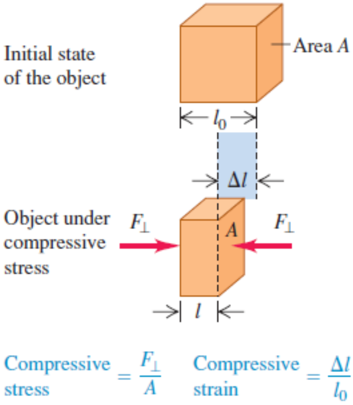

Figure 2: An example of compressional stress and strain

Stress and Strain (contd.)

The net force on the object is zero, but the object deforms. The compressive stress (the ratio of the force to the cross-sectional area) produces a compressive strain (the contraction divided by the initial length).

Bulk (Volume) stress and strain The stress is responsible to change the volume of the object as shown in figure 3. Without the stress, the cube has volume V_0 and when the stress is applied; the cube has a smaller volume V.

Stress and Strain (contd.)

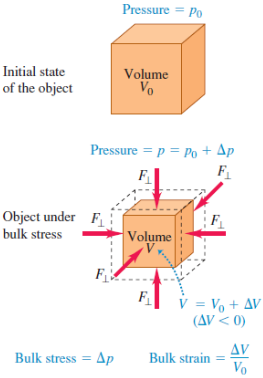

Figure 3: An example of bulk stress and strain

Stress and Strain (contd.)

Shear stress and strain The stress responsible for changing the shape of the object. The force acts tangential to a surface which is opposite to the rigid surface. As as result the surface displaces by a distance parallel to the force. The angle in radian through which a side of a body originally perpendicular to the fixed surface is turned is called shear strain.
The force per unit area of the surface is the shear stress as shown in figure 4.

Stress and Strain (contd.)

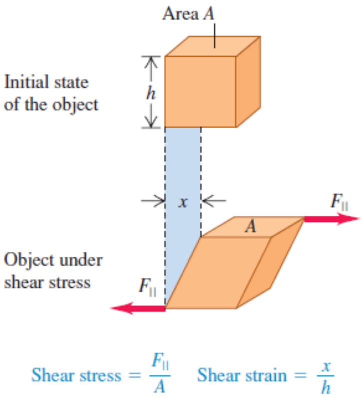

Figure 4: An example of shear stress and strain

Stress and Strain (contd.)

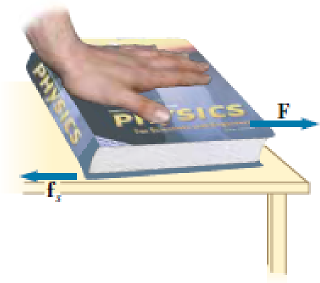

Figure 5: A book under shear stress

The force acting tangent to the surface divided by the areaon which it acts:

$$\text{Shear stress} = \frac{F_{\parallel}}{A}$$

with shear Strain given by

$$\text{Shear strain}(\theta) = \frac{x}{h}$$

where x is the horizontal distance that the sheared face moves and h is the height of the object.
The concepts of shear stress, shear strains apply to solid materials only.

Hooke's Law and Modulus of elasticity

[Robert Hooke (1635–1703), a contemporary of Newton]

Within the elastic (proportional limit), i.e. Elastic Modulus $= \dfrac{\text{stress}}{\text{strain}}$

For example, the extension is proportional to the load or tension in the wire when the proportional limit is not exceeded.

Elastic modulus depends on the material being deformed and on the nature of deformation. Elastic modulus, in general, relates what is done to a solid object (a force is applied) and how that object responds (it deforms to some extent).

It is not really a general law but an experimental finding that is valid only over a limited range. There are are three types of modulus of elasticity

Young's Modulus: Elasticity in Length

It measures the resistance of a solid to a change in its length.

$$\text{Young's Modulus (Y)} = \frac{\text{Tensile stress}}{\text{Tensile strain}} = \frac{F_\perp/A}{\Delta l/l_0}$$

SI unit of Young's Modulus is the pascal. $\left[1\text{Pa} = 1 \text{ N m}^{-2}\right]$

Young's modulus is typically used to characterize a rod or wire stressed under either tension or compression.

It measures the resistance of solids or fluids to changes in their volume.

$$\text{Bulk Modulus (B)} = \frac{\text{Bulk stress}}{\text{Bulk strain}} = -\frac{\Delta p}{\Delta V / V_0}$$

Negative sign indicatres volume decreases as pressure increases.

SI unit of Bulk Modulus is Pascal (Pa).

Compressibility

The reciprocal of the bulk modulus.

$$\text{Compressibility (k)} = \frac{1}{\text{B}} = -\frac{\Delta V / V_0}{\Delta p} = -\frac{1}{V_0}\frac{\Delta V}{\Delta p}$$

Bulk Modulus: Volume Elasticity (contd.)

Compressibility is the fractional decrease in volume $-\frac{\Delta V}{V_0}$, per unit increase Δp in pressure.

The unit of compressibility is Pa^{-1} .

It measures the resistance of motion of the planes within the solid parallel to each other.

$$\text{Shear Modulus (S)} = \frac{\text{Shear stress}}{\text{Shear strain}} = \frac{\mathrm{F}_{\parallel}/\mathrm{A}}{x/\mathrm{h}}$$

It is also defined

$$\text{Modulus of rigidity}(\eta) = \frac{\text{Shear stress}}{\text{Shear strain}} = \frac{\mathrm{F}_{\parallel}/\mathrm{A}}{\theta}$$

SI unit of Shear Modulus is $\mathrm{N{\cdot}m^{-2}}$

Relation between elastic constants: Relation between Young's modulus (Y), Bulk modulus (K) and Modulus of rigidity(η):

$$\frac{9}{Y} = \frac{3}{\eta} + \frac{1}{K}$$

Stress-versus-strain curve for elastic solid

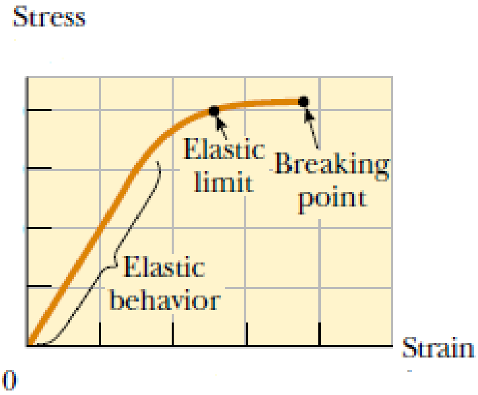

Figure 6: Stress versus strain graph

Figure 6 shows the stress-versus-strain curve for elastic solid. Initially, a stress–strain curve is a straight line. As the stress increases, however, the curve is no longer straight. When the stress exceeds the elastic limit, the object is permanently distorted and does not return to its original shape after the stress is removed. Hence, the shape of the object is permanently changed. As the stress is increased even further, the material ultimately breaks. The proportional limit is the maximum stress for which stress and strain are proportional. Beyond the proportional limit, Hooke's law is not valid. The elastic limit is the stress beyond which irreversible deformation occurs. The breaking stress, or ultimate strength, is the stress at which the material breaks.

Stress-versus-strain curve for elastic solid (contd.)

Ductile Material

A ductile material is one that can be stressed well beyond its elastic limit without breaking [a large amount of plastic deformation takes place between the elastic limit and the fracture point]

A soft iron wire – Ductile material

Brittle Material

A brittle material is one that breaks soon after the elastic limit is reached.

A steel piano string – Brittle material

Energy Stored in a Stretched Wire

In order to deform a body work must be done by the applied force. The energy so used is stored up the body in the form of potential energy and is called energy of strain.

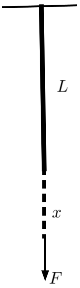

Figure 7

Suppose a wire of original length L, Young's modulus Y, and cross-sectional area A, suspended vertically with upper end is attached to a rigid support and is stretched elastically by an amount x when a normal force F is applied at lower end [Figure 7].

Energy Stored in a Stretched Wire (contd.)

$$\text{Tensile stress} = \frac{F}{A}$$

$$\text{Tensile strain} = \frac{x}{L}$$

$$\begin{aligned}
\text{Young's Modulus (Y)} &= \frac{\text{Tensile stress}}{\text{Tensile strain}} \\
&= \frac{F/A}{x/L} \\
&= \frac{FL}{Ax}
\end{aligned}$$

Energy Stored in a Stretched Wire (contd.)

$$\therefore F = \frac{YAx}{L}$$

The work done in stretching the wire by an amount x from its original position 0 is

$$W = \int_0^x F dx$$

$$= \int_0^x \frac{YAx}{L} dx = \frac{YA}{L} \int_0^x x dx$$

$$\therefore W = \frac{1}{2}\frac{YA}{L}x^2$$

$$\text{i.e. } W = \frac{1}{2}\left(\frac{YAx}{l}\right)x$$

$$\implies W = \frac{1}{2}Fx = \frac{1}{2} \times \text{force} \times \text{extension}$$

Energy Stored in a Stretched Wire (contd.)

Energy stored in the wire (U) = Work done in stretching the wire [W]

$\therefore$ Energy stored in the wire $(U) = \dfrac{1}{2} YA \dfrac{(x)^2}{L}$

Energy Density = $\dfrac{U}{V} = \dfrac{\frac{1}{2} YA \frac{(x)^2}{L}}{AL} = \dfrac{1}{2}\dfrac{F}{A}\dfrac{x}{L} = \dfrac{1}{2} \times \text{stress} \times \text{strain}$

$\therefore$ Energy density $= \frac{1}{2} \times \text{stress} \times \text{strain}$

NOTES:

Elasticity decreases as the temperature increases. Steel is more elastic than rubber: For a given stress, the strain produced in steel is much smaller than that produced in the rubber. This implies that Young's modulus Y for steel is greater than that for rubber.

Poisson's Ratio (σ)

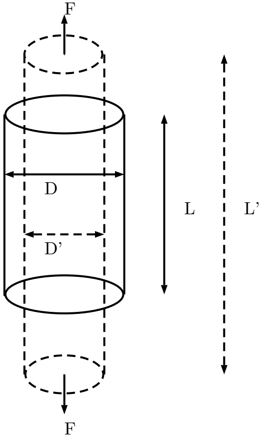

Figure 8

Poisson's Ratio (σ) (contd.)

Let a wire of original length L and diameter D is subjected to equal and opposite force F along its length. If the length increases and become L' and the diameter decreases and becomes D'. Then

$$\text{Longitudinal strain}(\alpha) = \frac{L - L'}{L}$$

in the direction of applied force and

$$\text{Lateral strain}(\beta) = \frac{D - D'}{D}$$

in the direction perpendicular to the applied force.

$$\text{Poisson's Ratio } (\sigma) = \frac{\text{Lateral strain}\,(\beta)}{\text{Longitudional strain}\,(\alpha)}$$

It is a pure number.It is constant for given material.