ELASTICITY

(Q17): Castor oil which has density of 0.96×10^3 kg/m^3 at room temperature is forced through a pipe of circular cross-section by pump that maintains a gauge pressure of 950 Pa. The pipe has diameter of 2.6 cm and length 65 cm. The castor oil emerges from free end of pipe is collected at atmospheric pressure. After 90 s, a total of 1.23 kg has been collected. What is the coefficient of viscosity of castor oil at that temperature.

Soln:

Given,

density of castor oil (ρ) = 0.96×10^3 kg/m^3

Gauge pressure (P) = 950 N/m^2

radius of pipe (r) = ~~2.7~~ 1.3 cm = 1.3×10^{-2} m

Length of pipe (L) = 65 cm = 65×10^{-2} m

mass of oil collected (m) = 1.23 kg

Time interval (t) = 90 s

Flow rate (V) = ?

Coefficient of viscosity of castor oil (η) = ?

We know,

$$\text{Flow rate } (V) = \frac{m \rho^{-1}}{t} = \frac{1.23 \times (0.96 \times 10^3)^{-1}}{90}$$

$$\therefore V = 13.12 \text{ m}^3/\text{sec}$$

Using Poiseuille's law,

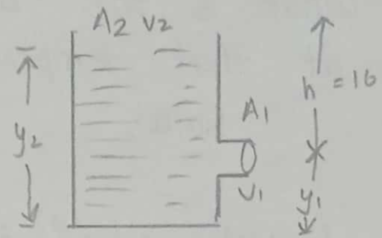
$$V = \frac{\pi P r^4}{8 \eta L} \quad \text{or,} \quad \eta = \frac{\pi \times 950 \times (1.3 \times 10^{-2})^4}{8 \times 13.12 \times 65 \times 10^{-2}}$$

$$\therefore \eta = 1.15 \text{ N s/m}^2$$

<Q.2>: A large storage tank, open at the top and filled with water, develops a small hole in its side at a point 16.0 m below water level. The rate of flow from the leak is $2.50 \times 10^{-3} m^3/min$.
Determine:
a) the speed at which the water leaves the hole.
b) the diameter of the table.

Soln:

Given,



Rate of flow from leak (V) = $2.5 \times 10^{-3} m^3/min$

$= 4.16 \times 10^{-5} m^3/sec$

Height of small hole (h) = 16 m.

As $A_2 \gg A_1$, so $V_2 \approx 0$ ie, water at top of the tank remains at rest.

Now, using Bernoulli's equation,

$$P_0 + \frac{\rho v_1^2}{2} + \rho g y_1 = P_0 + \frac{\rho v_2^2}{2} + \rho g y_2$$

or, $\frac{v_1^2}{2} + g y_1 = g y_2$

or, $v_1 = \sqrt{2g(y_2 - y_1)}$

$= \sqrt{2gh}$

$= \sqrt{2 \times 9.81 \times 16}$

$\therefore V_1 = 17.7 m/s.$

Now,
from eqn. of continuity,
$Av = V$

or, $\frac{\pi d^2}{4} \times 17.7 = 4.16 \times 10^{-5}$

or, $d = \sqrt{\frac{4 \times 4.16 \times 10^{-5}}{17.7 \times \pi}}$

$\therefore d = 1.7 \times 10^{-3}$ m $= 1.729$ mm

<Q.3> In ideal flow, a liquid of density 850 kg/m^3 moves from a horizontal tube of radius 1 cm into a second horizontal tube of radius 0.500 cm. A pressure difference ΔP exists between the tubes.

a) Find the volume flow rate as function of ΔP.

b) Evaluate the volume flow rate for $\Delta P = 6.00$ kPa

c) State how the volume flow rate depends on ΔP.

Soln:

Given,

density of liquid (ρ) = 850 kg/m^3

radius of 1st horizontal tube (r_1) = 1 cm = 10^{-2} m

radius of 2nd horizontal tube (r_2) = 0.5×10^{-2} m

Now, we know,

Flow rate (V) = $A_1 v_1 = A_2 v_2$.

So,

$$v_1 = \frac{A_1}{V} = \frac{A_1}{\pi r_1} \quad \text{— (i)}$$

and

$$v_2 = \frac{A_2}{V} = \frac{A_2}{\pi r_2} \quad \text{— (ii)}$$

Using Bernoulli's equation, we get.

$$P_1 + \frac{\rho v_1^2}{2} = P_2 + \frac{\rho v_2^2}{2} \quad [\because \text{Tube is horizontal}]$$

$$\text{or, } (P_1 - P_2) = \frac{\rho}{2}(v_2^2 - v_1^2)$$

$$\text{or, } \frac{2\Delta P}{850} = \frac{V^2}{\pi^2}\left(\frac{1}{r_2^4} - \frac{1}{r_1^4}\right)$$

$$\text{or, } \frac{2\Delta P \pi^2}{850} = V^2\left(\frac{1}{(0.5\times10^{-2})^4} - \frac{1}{(10^{-2})^4}\right)$$

So,

$$V = \sqrt{\frac{2.3\times10^{-2}}{10^8\times1.5}\Delta P} \qquad \therefore V = 3.93\times10^{-6}\sqrt{\Delta P}\ m^3/s.$$

If $\Delta P = 6.00$ kPa

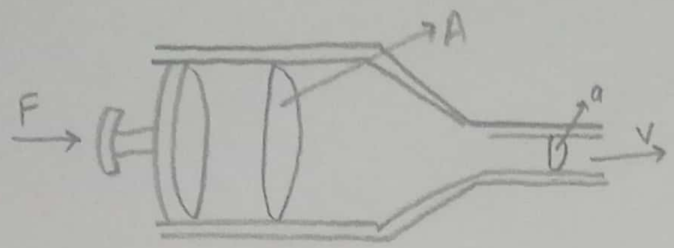
So,

$$\text{Flow rate } (V) = 3.93\times10^{-6}\times\sqrt{6\times10^{-3}}$$

$$= 3.04\times10^{-4}\ m^3/s.$$

Here, we see that $V \propto \sqrt{\Delta P}$.

<Q.4>: A hypodermic syringe contains a medicine having density of water as shown in figure. The barrel of the syringe has a cross-sectional area $A = 2.50\times10^{-2} m^2$ and the needle has cross-sectional area $a = 1\times10^{-8} m^2$. In the absence of force on the plunger, the pressure is 1 atm everywhere. A force $\vec{F}$ magnitude 2.00 N acts on plunger making medicine squirt horizontally form the needle. Determine the speed of the medicine as it leaves needle's tip.



Given,

Gauge pressure $(\Delta P) = \dfrac{2.00}{2.5\times10^{-5}} = 8\times10^{4}\ N/m^2$

from ~~of~~ eq^n of continuity,

$$A_1V_1 = A_2V_2$$

$$V_1 = \frac{A_2}{A_1}\times V_2 \qquad \text{or,}\quad V_1 = \frac{r_2^2}{r_1^2}\times V_2$$

$$\therefore V_1 = 4.00\times10^{-4}\ V_2$$

Using Bernoulli's theorem,

$$P_1 + \frac{\rho v_1^2}{2} + \rho g y_1 = P_2 + \frac{\rho v_2^2}{2} + \rho g y_2$$

Since $y_1 = y_2$,

$$P_1 - P_2 = \frac{\rho}{2}(v_2^2 - v_1^2)$$

$$\text{or,}\quad \Delta P = \frac{1000}{2}(v_2^2 - v_1^2)$$

Since, $v_1^2 <<< v_2^2$. v_1^2 is negligible.

So,

$$\Delta P = 500\times V_2^2$$

$$\text{or}\quad V_2 = \sqrt{\frac{\Delta P}{500}} \qquad \therefore V_2 = 12.6\ m/s.$$