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MECHANICS OF DEFORMABLE BODIES

~~Strength of Materials~~

A field of applied mechanics which studies the behaviour of solid bodies under the action of different types of loadings and to determine stress, strain and also deformation of structures is called mechanics of deformable bodies.

Strength of Materials

The study of the stress and strain in a body which is deformable ie, no longer rigid under the action of external forces is called strength of materials.

Load:

A load is defined as the combined effect of external forces acting on a body.

The load may be classified as:

i) Dead loads
ii) Live/ fluctuating loads
iii) Inertial ~~load~~ loads
iv) Centrifugal loads
v) Environmental loads

i) Dead loads:

The loads that are relatively constant over time is called dead loads ie, permanent loads.
Eg: weight of roofs, weight of structure, etc.

(ii) Live/ Fluctuating loads

The loads that are imposed loads are temporary and so short duration is called live/ fluctuating loads.
Eg: vibration , impact.

(iii) Inertial loads:

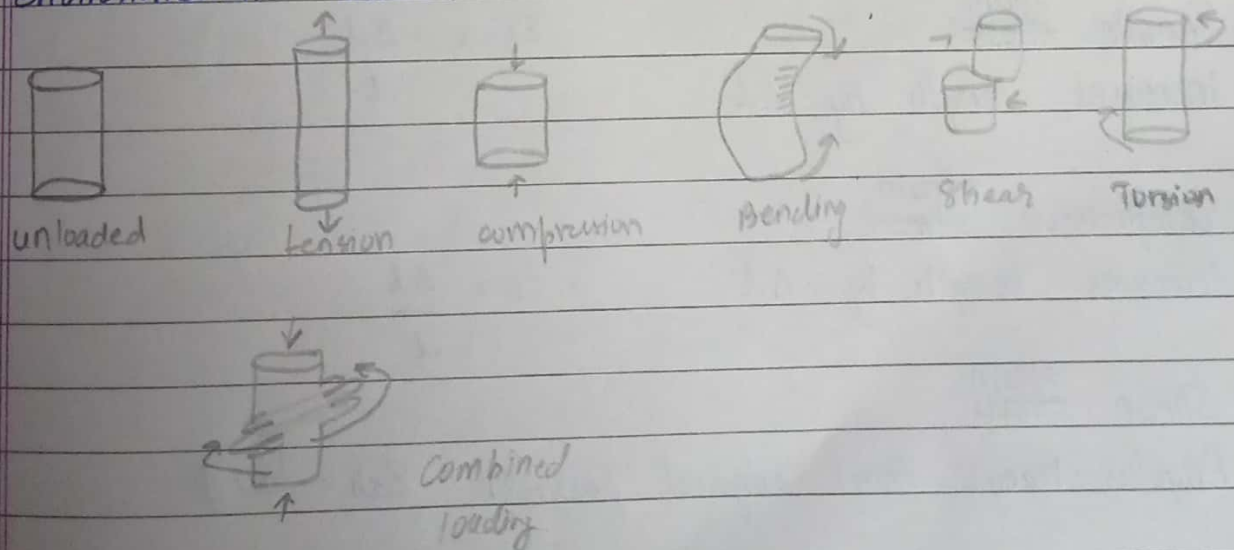
The resistance of any physical object to any change in its velocity is called inertial loads. This includes changes to the object's speed or direction of motion.

(iv) Centrifugal loads

The load that is felt by an object moving in a curved path that acts ~~normally~~ outwardly away from the center of rotation is called centrifugal loads.

(v) Environmental loads:

The load that acts as a result of weather topography and other natual phenomenon is called environmental loads.



Other classification of loads are as follows: tensile, compressive, torsional, bending, shearing & combined loads.

Stress (σ)

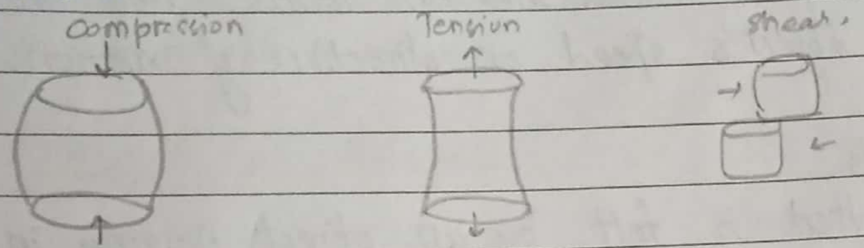
The internal ~~force~~ resistance which the body offers to meet with the loads is called stress.

The various types of stress are as follows:

a) Simple.
- → Tension
- → Compression
- → Shear

b) Indirect
- → Bending
- → Torsion

c) Combined.
- Combination of previously said forces.



Strain (ε)

The deformation produced by stress is called strain. The ratio of change in configuration to original configuration is called strain.

(i) Tensile strain ~~stress~~: $\varepsilon_t = \frac{\Delta l}{l}$
- → increases length by Δl

(ii) Compressive strain ~~stress~~: $\varepsilon_c = \frac{\Delta l}{l}$
- → decreases length by Δl

(iii) Shear strain ~~stress~~:
- Angular change from original position $\varepsilon_{sh} = \tan \phi$

(iv) Volumetric strain:
→ increases/decreases volume by ΔV
$\varepsilon_v = \Delta V / V$

Elastic body:
The body which regains its original position on the removal of force is called elastic body. It experiences elastic strains.

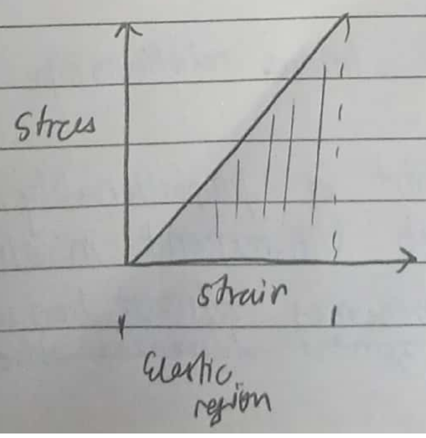
Plastic body:
The body which ~~de~~ remains deformed after the removal of external force is called plastic body.

Elastic limit:
The limiting value of load upto which the strain totally disappears on the removal of load is called elastic limit.

Hooke's law
Within elastic limit, strain is directed proportional to stress

ie, stress $\propto$ strain,

or $\frac{\text{stress}}{\text{strain}} = E$ ie, E = modulus of elasticity.



Stress-Strain Curve

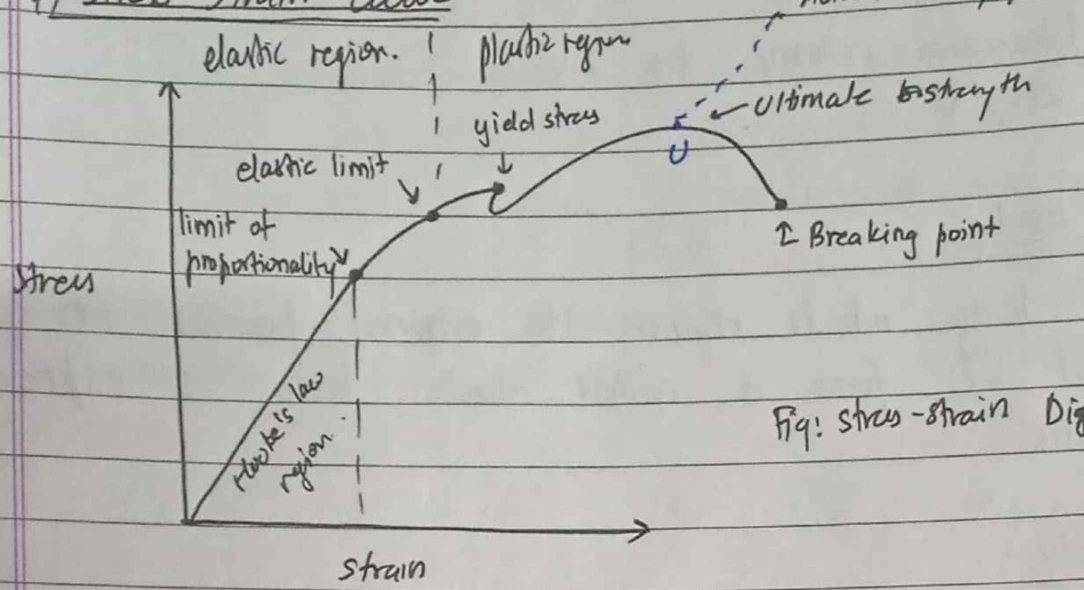
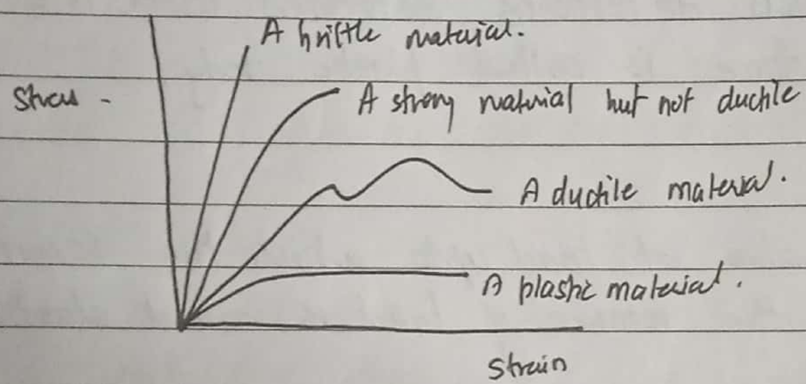


Fig: stress-strain Digagram.



~~Stre~~ Stress-strain curve displays the relationship between stress and strain of a particular material.

From O to limit of proportionality (P), stress is directly proportional to strain and behaviour of the material is linear.

Beyond P, the linear relationship between stress & strain doesn't exist.

After ~~prop~~ limit of proportionality, the strain increases more rapidly for each increment in stress. At elastic limit, the curve becomes almost horizontal ~~as the curve becomes almost horizontal decreases the~~

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From elastic limit to yield stress point, the material becomes perfectly plastic i.e. it deforms without increasing in the applied ~~for~~ load.

After large strains that occur during yielding in region EY, the material begins to strain harden. During strain hardening, the material undergoes changes in its atomic and crystalline structure.

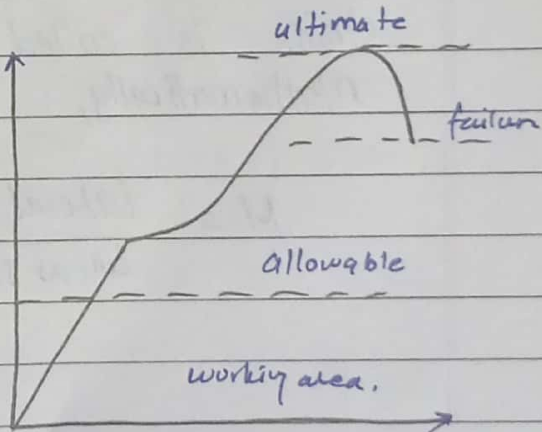
- From yield point to ultimate strength point, the stress - strain curve has positive slope. ~~A~~ The ultimate strength is maximum load value.

As stress is reduced, the bar is accompanied by reduction further stretching ~~in~~ of the ~~s~~ material.

At breaking point, the ~~th~~ material ruptures out.

*) For brittle material, the young's modulus of elasticity is very high.

Factor of safety {F.O.S}



Factor of safety (FOS) is ability of a system's structural capacity to be viable beyond its actual loads.

$$FOS = \frac{\text{Ultimate stress}}{\text{Allowable stress}}$$

$$FOS = \frac{\sigma_{ult}}{\sigma_{all}} = \frac{\tau_{ult}}{\tau_{all}}$$

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/* If a material is loaded, unloaded and reloaded within elastic limit, the body regains its original configuration.
However, if a body is loaded into plastic range, the internal structure of the material is altered and its properties change. */

Linear / Primary strain is the ratio of change of length to the original length.

Lateral strain is the ratio of change in diameter to the original length.

(*) Poission Ratio:
The ratio of Lateral strain to the linear strain is called poission's ratio.
Mathematically,

$$\mu = \frac{\text{Lateral strain}}{\text{Linear strain.}} = \frac{1}{m}$$

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<Q7: A square steel rod 20mm x 20mm in section is to carry an axial load of 100 kN. Calculate the shortening in a length of 50 mm.

$E = 2.14 \times 10^8 \ kN/m^2$

Soln:

Given,

Area (A) = $20mm \times 20mm = 400 mm^2 = 400 \times 10^{-6} m^2$

length of rod (l) = $50mm = 50 \times 10^{-3} m$

load (P) = $100 kN = 100 \times 10^3 N$

$E = 2.14 \times 10^8 \ kN/m^2$.

We know,

stress (σ) = $\frac{P}{A} = \frac{100 \times 10^3 N}{400 \times 10^{-6} m^2}$ $\sigma = 2.5 \times 10^8 N/m^2 = 2.5 \times 10^5 kN/m^2$

and

$$E = \frac{\sigma}{\varepsilon \text{ (strain)}}$$

or, $\varepsilon = \frac{2.5 \times 10^5 kN/m^2}{2.14 \times 10^8}$

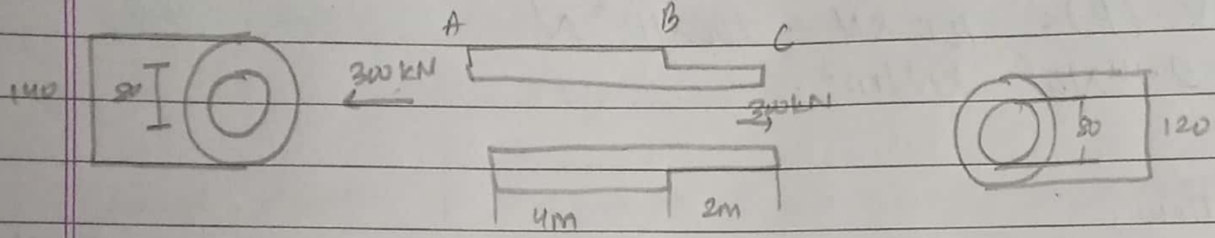
or $\frac{\Delta l}{l} = \frac{2.5 \times 10^5}{2.14 \times 10^8}$

or $\Delta l = \frac{2.5 \times 10^5}{2.14 \times 10^8} \times 50 \times 10^{-3} m$

$\therefore \Delta l = 0.0000584 m = 0.0584 mm.$

<Q>: A 6m long hollow bar of circular section has 140 mm diameter for a length of 4m, while it has 120 mm diameter for a length of 2m. The bore diameter is 80 mm throughout as shown. Find the elongation of the bar, when it is subjected to an axial tension force of 300 kN.
Y = 200 GPA.

Soln:



Given,
total length $(L) = 6m = 6 \times 10^3$ mm
Diameter 1 $(d_1) = 140$ mm.
length 1 $(l_1) = 4m = 4 \times 10^3$ mm
Diameter 2 $(d_2) = 120$ mm
length 2 $(l_2) = 2m = 2 \times 10^3$ mm
inner diagmeter $(d_1 = d_2) = 80$ mm.
load $(P) = 300$ kN $= 300 \times 10^3$ N
$\because E = 200$ GPA $= 200 \times 10^3$ N/mm^2

We know,
Area of AB,

$$A_1 = \frac{\pi}{4}\left[D^2 - d_1^2\right] = \frac{\pi}{4}\left[\{(140)^2 - (80)^2\}\right] = 3300\pi \text{ mm}^2$$

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Area of BC

$$A_2 = \frac{\pi}{4}\left[D_2^2 - d_2^2\right] = \frac{\pi}{4}\left[(120)^2 - (80)^2\right]$$

$$= 2000\pi \ mm^2$$

So,
Elongation of bar

$$\Delta l = \frac{P}{E}\left[\frac{l_1}{A_1} + \frac{l_2}{A_2}\right]$$

$$= \frac{300\times10^3}{200\times10^3} \times \left[\frac{4\times10^3}{3300\pi} + \frac{2\times10^3}{2000\pi}\right] mm$$

$$= 1.5 \times (0.385 + 0.318) = 1.054 \ mm.$$

The elongation of the bar is 1.054 mm.