DIFFRACTION

<Q.1>: Monochromatic light of helium-neon laser ($\lambda_1 = 632.8\,nm$) is incident on a diffraction grating containing 6000 groves per centimeter. Find the angles at which first and second order maxima are observed. Would third maxima exists?

Soln:

Given,

Wavelength (λ) = $632.8\,nm = 632.8 \times 10^{-9}\,m$

No of grooves (N) = $6000\,/cm = 6 \times 10^5/m$

Grating element $(a+b) = \frac{1}{N} = \frac{1}{6 \times 10^5}$

For first order principal maxima,

$$(a+b)\sin\theta_1 = n\lambda$$

or, $\sin\theta_1 = \frac{n\lambda}{a+b}$ $\quad \therefore \theta_1 = 22.31^\circ$

For second order principal maxima,

$$(a+b)\sin\theta_2 = n\lambda$$

or, $\sin\theta_2 = \frac{n\lambda}{a+b}$ $\quad \therefore \theta_2 = 49.40^\circ$

For third order principal maxima,

$$\sin\theta_3 = \frac{3 \times 632.8 \times 10^{-9}}{\frac{1}{6 \times 10^5}} = 1.13$$

which is not in range of $\sin\theta$.

Thus, we cannot find third order principal maxima.

<Q2>: A helium-neon laser ($\lambda = 632.8\,nm$) is used to calibrate a diffracting. If the first order maximum occurs at 20.5°, what is the spacing between adjacent grooves in the grating?

Soln

Given,

wavelength of ~~hot~~ light (λ) = 632.8 nm

$= 632.8 \times 10^{-9}$ m

$\theta_1 = 20.5°$

Grating element $(a+b) = ?$

We know,

$$(a+b) \sin\theta_n = n\lambda$$

$$\text{or } (a+b) = \frac{1 \times 632.8 \times 10^{-9}}{\sin 20.5}$$

$$\therefore (a+b) = 1.80 \times 10^{-6} \text{ m} = 1.8 \text{ µm}$$

<Q.3>: A plane transmission grating has 6000 lines/cm. It is used to obtain a spectrum of light from sodium lamp in second order. Calculate the angular separation between two sodium lines 5890 Å and 5896 Å.

Soln:

Given,

N = 6000 lines/cm $= 6 \times 10^5$ m

Grating element $(a+b) = \frac{1}{N} = \frac{1}{6 \times 10^5}$ m^{-1}

$\lambda_1 = 5890$ Å $= 5890 \times 10^{-10}$ m $\quad \lambda_2 = 5896$ Å $= 5896 \times 10^{-10}$ m

Now,

$(a+b) \sin\theta_1 = n_2 \lambda_1 \quad\quad (a+b) \sin\theta_2 = n_2 \lambda_2$

$\text{or, } \sin\theta_1 = \frac{n_2 \lambda_1}{(a+b)} \quad\quad \text{or, } \sin\theta_2 = \frac{n_2 \lambda_2}{(a+b)}$

$\therefore \theta_1 = 44.975° \quad\quad \therefore \theta_2 = 45.033°$

Thus, angulas separation $= \theta_2 - \theta_1$

$= 45.033^\circ - 44.975^\circ$

$= 0.0585^\circ$

<Q.4>: A diffraction grating is used at normal incidence gives a line $\lambda_1 = 6000 A^\circ$ in a certain order superimposed on another line $\lambda_2 = 4500 \mathring{A}$ of the next higher order. If the angle of diffraction is 30°, how many lines per cm are there in the grating.

Soln:

Given,

angle of diffraction $(\theta) = 30^\circ$

Grating element $(a+b) = ?$

According to question,

$$(a+b) \sin\theta = n\lambda_1 \quad \text{— (i)}$$

and

$$(a+b) \sin\theta = (n+1)\lambda_2 \quad \text{— (ii)}$$

Using eqn (i) and (ii), we get

$$\frac{n+1}{n} = \frac{6000}{\cancel{4800}\ 4500} \qquad \text{or, } n+1 = 1.33n$$

$$\therefore n = 3$$

From eqn (i),

$$(a+b) \sin 30^\circ = 3 \times 6 \times 10^{-7}$$

$$\therefore (a+b) = 3.6 \times 10^{-6} \text{ m.}$$

So,

$$N = 27777.77 \ /\text{m}$$

$$\therefore N = 2778 \text{ lines/cm}$$

<Q.5>: Assume that the limits of the visible spectrum are arbitrary chosen as 4300 Å and 6800 Å. Design a grating that will spread the first order spectrum through angular range of 20°?

Soln:

Given,

Angular range $(\theta_1 - \theta_2) = 20°$

$\lambda_1 = 4300 \times 10^{-10}$ m $\qquad \lambda_2 = 6800 \times 10^{-10}$ m

Now, we know,

$(a+b)\sin\theta_1 = \lambda_1$ — (i) $\qquad$ and $\qquad (a+b)\sin(\theta+20) = \lambda_2$ — (ii)

Using eqn(i) and eqn(ii), we get.

$$\frac{(a+b)\sin\theta}{(a+b)\sin(\theta+20)} = \frac{4300\times10^{-10}}{6800\times10^{-10}}$$

or, $\sin\theta = 0.632353(\sin\theta\cdot\cos 20° + \cos\theta\cdot\sin 20°)$

or, $\sin\theta = 0.5942\sin\theta + 0.21627\cos\theta$

or, $\tan\theta = 0.5329 \qquad \therefore \theta = 28.055°$.

From eqn(i),

$(a+b)\sin(28.055°) = 4300\times10^{-10}$

$\therefore (a+b) = \cancel{3.7}\ 9.14\times10^{-7}$ m

$= 9.14\times10^{-5}$ cm

$= 3.6\times10^{-5}$ inch.

So,

Number of lines per inch $= \frac{1}{3.6\times10^{-5}} = 27780$ lines/inch.