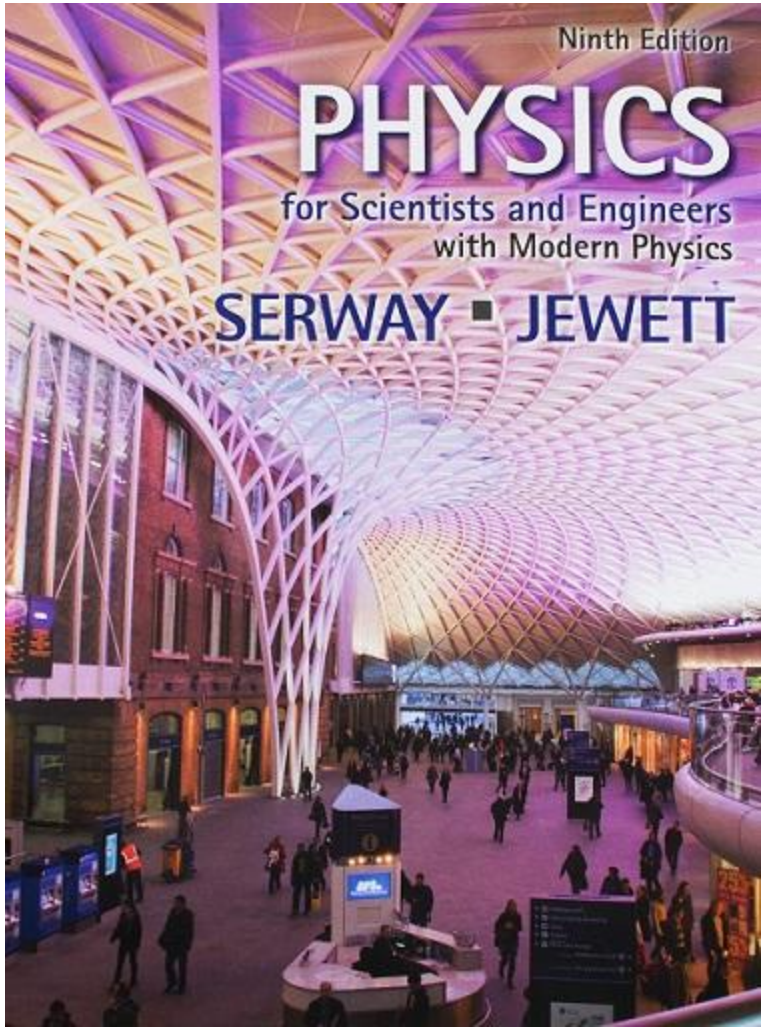
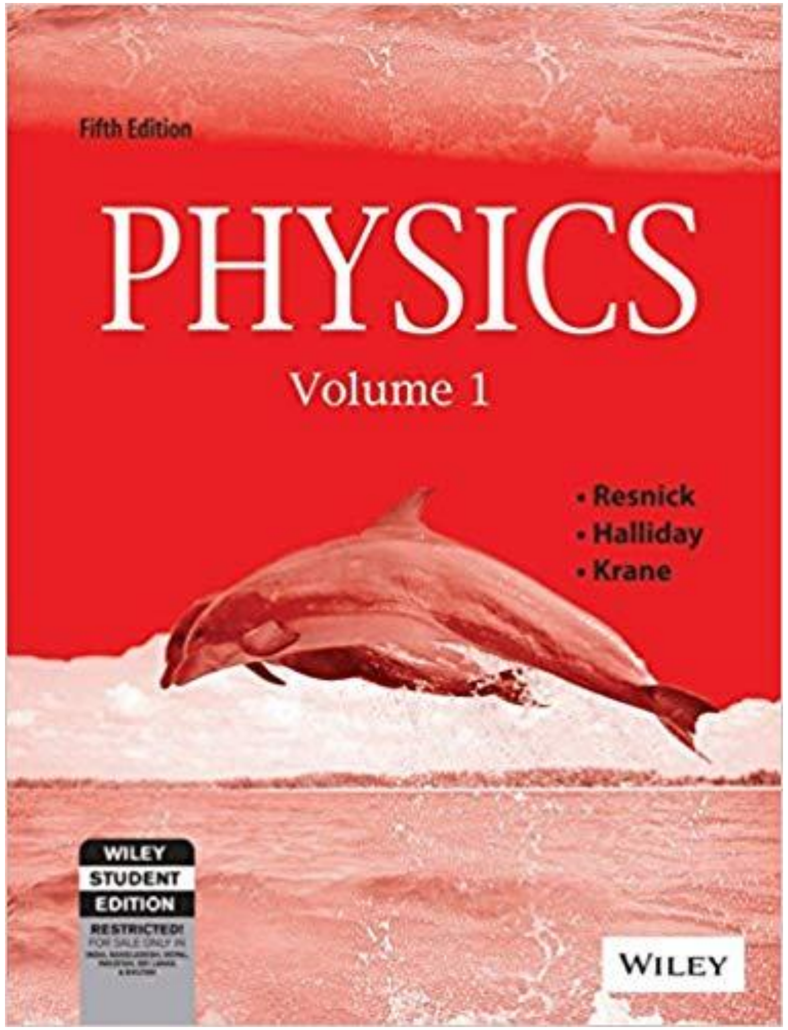
PHYSICS

General Physics I (PHYS 101)





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DIFFRACTION

Course Outline

- Diffraction
- Distinction between Fresnel and Fraunhofer Diffraction
- Diffraction at Single Slit
- Diffraction Grating

DIFFRACTION

Diffraction

- Diffraction is the bending or spreading of waves that encounter an object (a barrier or an opening or edge) in their path.
- For diffraction to occur, the size of the object must be of the order of the wavelength of the incident waves
- Diffraction patterns consist of light and dark bands similar to the interference patterns.
- By studying these patterns, we can learn about the diffracting object. For example, diffraction of x rays is an important method for the study of the structure of solids, and diffraction of gamma rays is used to study nuclei.

Figure DP-1 shows a diffraction pattern that appears on the screen when light form distant light source (or a laser beam) passes through a narrow vertical slit. The pattern consists of a broad central fringe and a series of less intense and narrower side fringes.

Figure DP-1

Figure DP-2 shows a diffraction pattern associated with light passing by the edge of an object.

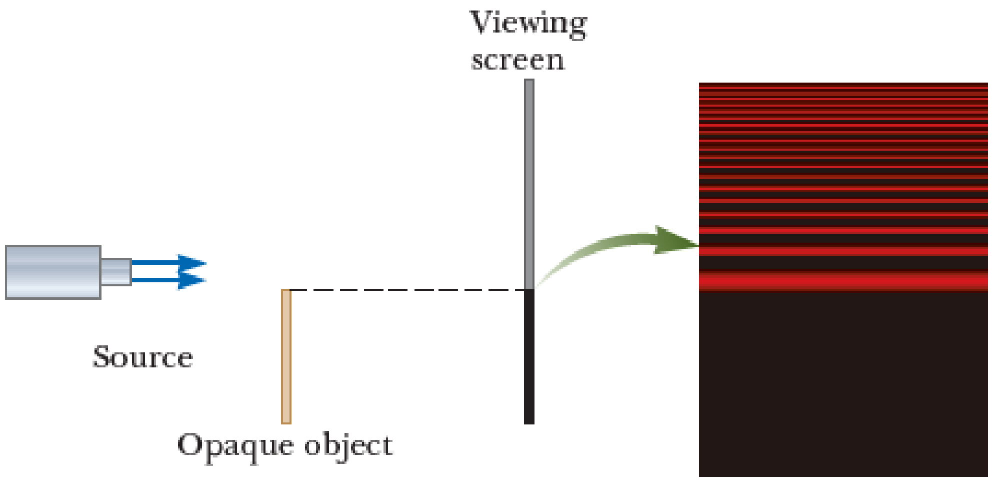


Figure DP-2

Diffraction

- **Figure DP-3** shows a diffraction pattern associated with the shadow of a penny.
- A bright spot occurs at the center, and circular fringes extend outward from the shadow's edge. We can explain the central bright spot by using the wave theory of light, which predicts constructive interference at this point. From the viewpoint of ray optics (in which light is viewed as rays traveling in straight lines), we expect the center of the shadow to be dark because that part of the viewing screen is completely shielded by the penny.

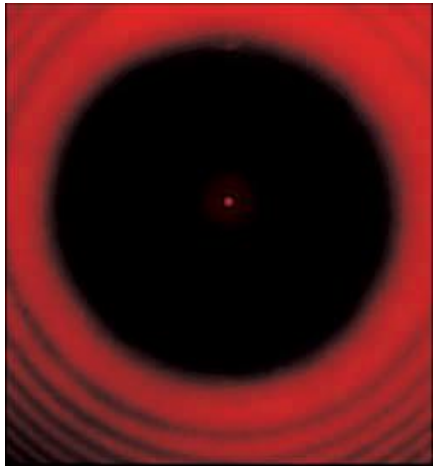


Figure DP-3
Diffraction pattern created by the illumination of a penny, with the penny positioned midway between the screen and light source.

Shortly before the central bright spot was first observed, one of the supporters of ray optics, Simeon -Denis Poisson, argued that if Augustin Fresnel's wave theory of light were valid, a central bright spot should be observed in the shadow of a circular object illuminated by a point source of light. To Poisson's astonishment, the spot was observed by Dominique Arago shortly thereafter. Therefore, Poisson's prediction reinforced the wave theory rather than disproving it.

Diffraction

- **Figure DP-3** shows a diffraction pattern produced by a razor blade in monochromatic light.

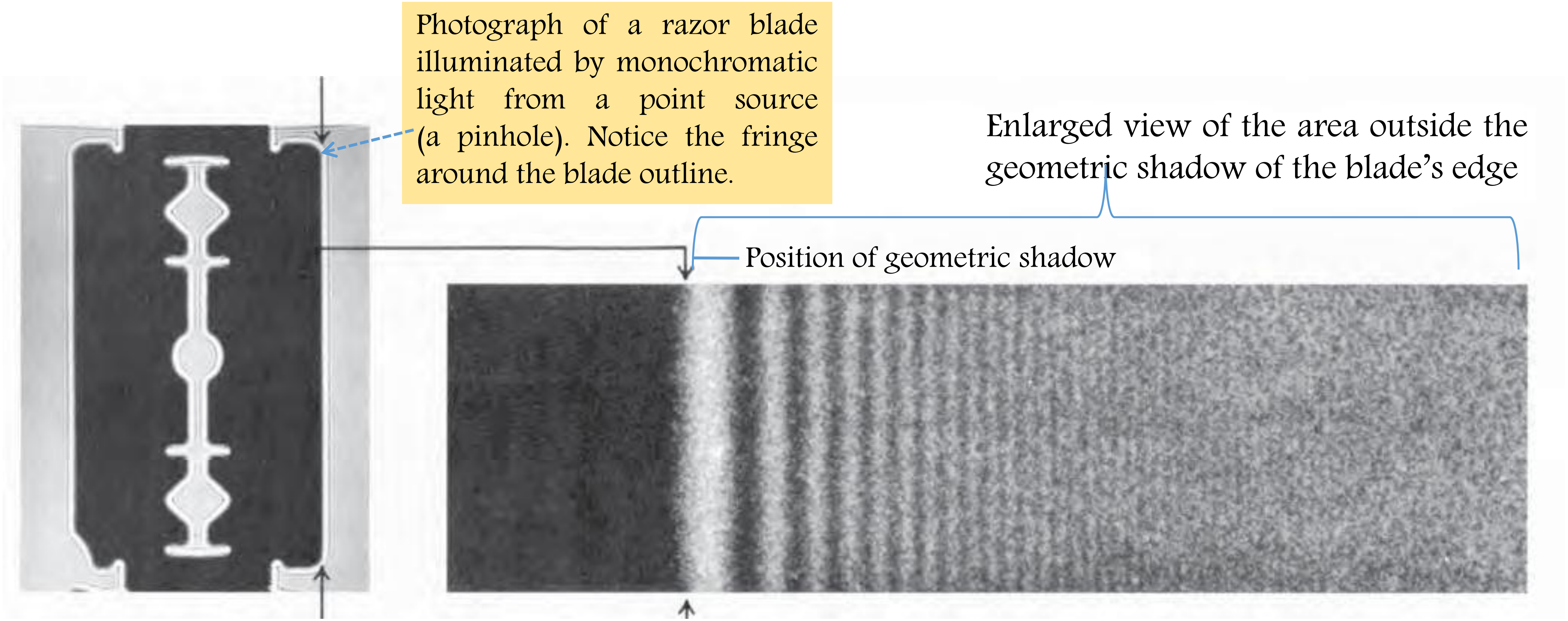


Figure DP-3
An example of diffraction

Diffraction

Diffraction

- When light waves encounter an edge, an obstacle , or an aperture the size of which is comparable to the wavelength of the waves, those waves spread out as they travel and, as a result, undergo interference. This is called **diffraction**.

  OR

  Diffraction is the deviation of light from a straight-line path when the light passes through an aperture or around an obstacle.

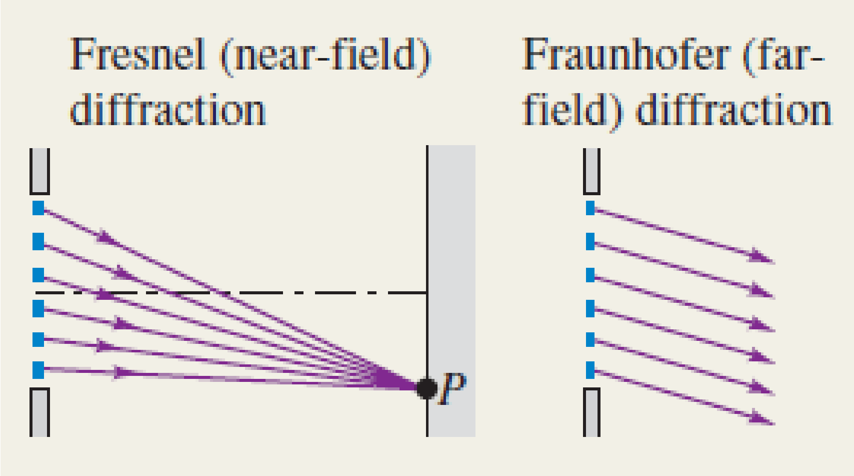
- Diffraction is due to the wave nature of light.
- The waves are diffracted only when the size of the obstacle is comparable to the wavelength of light.
- There is no fundamental distinction between interference and diffraction. The term interference for effects involving waves from a small number of sources, usually two. Diffraction usually involves a continuous distribution of Huygens's wavelets across the area of an aperture, or a very large number of sources or apertures.
- **Interference** is the result of superposition of secondary waves from two different wave fronts produced by two coherent sources.
- **Diffraction** is the result of superposition of secondary waves emitted from various points of the same wave front.
- Both interference and diffraction are consequences of superposition and Huygens's principle.

Diffraction

Fresnel and Fraunhofer Diffraction

- Diffraction occurs when light passes through an aperture or around an edge.
- When the source and the observer are so far away from the obstructing surface that the outgoing rays can be considered parallel, it is called **Fraunhofer diffraction**.
- When the source or the observer is relatively close to the obstructing surface, it is **Fresnel diffraction**.

Fresnel explained diffraction based on the wave theory, which was not widely accepted even after Thomas Young's experiments on double-slit interference.



Joseph von Fraunhofer (1787–1826)
German physicist

- Fraunhofer diffraction is usually simpler to analyze in detail (easier to handle mathematically).

Augustin Jean Fresnel (1788–1827)
French engineer & scientist
A strong proponent of the wave theory of light

In 1819, Fresnel submitted a paper on his theory of diffraction in a competition sponsored by the French Academy of Sciences.

DIFFRACTION

Diffraction from a Single Slit

Diffraction from a single slit

- Figure DFS-1 shows the diffraction pattern formed by plane-wave (parallel ray) monochromatic light when it emerges from a long, narrow slit.

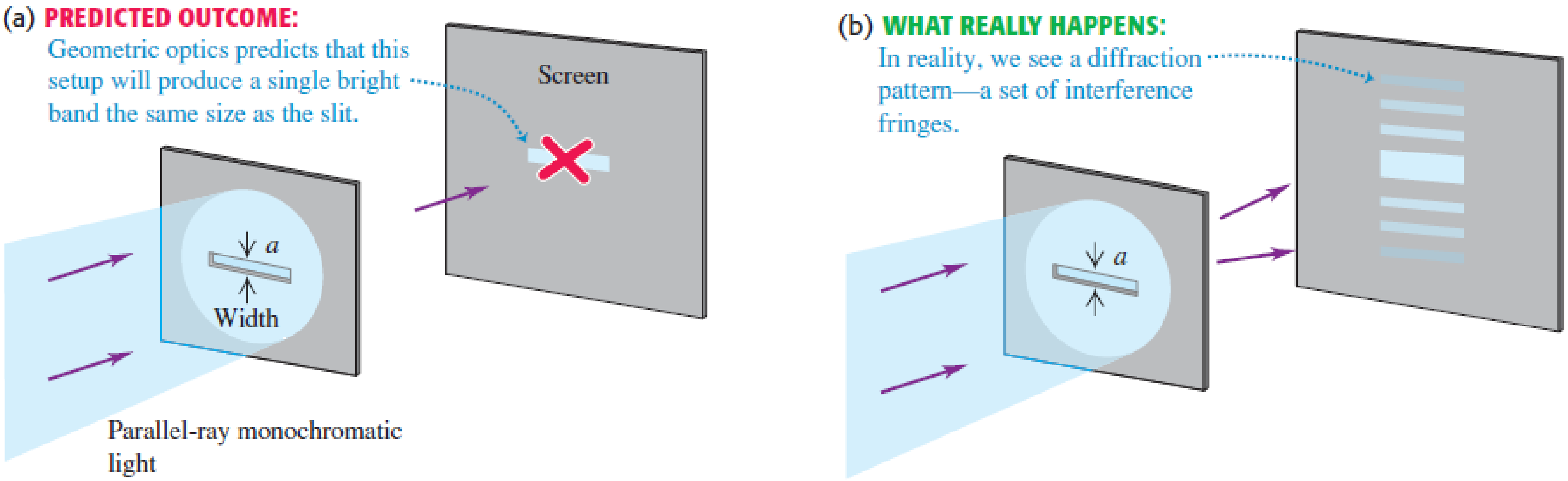


Figure DFS-1

(a) The "shadow" of a horizontal slit as incorrectly predicted by geometric optics.
(b) A horizontal slit actually produces a diffraction pattern. The slit width has been greatly exaggerated.

Diffraction Pattern from a Single Narrow Slit

Diffraction pattern from a single narrow slit

- Figure DSP-1a shows light entering a single slit from the left and diffracting as it propagates toward a screen.
- Figure DSP-1b shows the fringe structure of a Fraunhofer diffraction pattern.
- A bright fringe is observed along the axis at $\theta = 0$, with alternating dark and bright fringes on each side of the central bright fringe.

The **Fraunhofer diffraction pattern produced by a single slit of width *a*** on a distant screen consists of a central bright fringe and alternating bright and dark fringes of much lower intensities. The angles θ_{dark} at which the diffraction pattern has zero intensity, corresponding to destructive interference, are given by

$$\sin\theta_{dark} = \pm\, m\frac{\lambda}{a} \qquad m = \pm1, \pm2, \pm3 \ldots$$

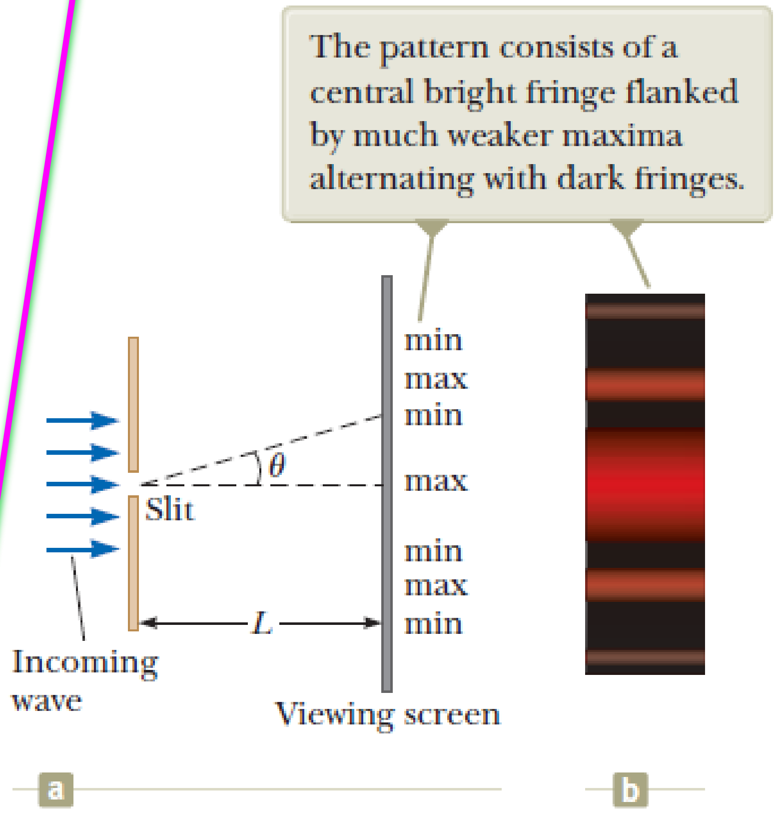


Figure DSP-1
(a) Geometry for analyzing the Fraunhofer diffraction pattern of a single slit. (Drawing not to scale.)
(b) Simulation of a single-slit Fraunhofer diffraction pattern.

Fraunhofer's Diffraction at a Single Slit

Diffraction at a single slit

- Let a parallel beam of monochromaticc light of wavelength λ be incident normally upon a long, narrow slit of width a , as shown in Figure 1.

- Let the diffracted light be focused by convex lens L_2 . The diffraction pattern obtained on the screen consists of a central bright band, having alternate dark and weak bright bands of decreasing intensity on either side of central bright band.

- According to Hygen's theory, a plane wave front is incident normally on the slit AB and each point in AB sends out secondary wavelets in all directions. The rays proceeding in the same direction as the incident rays are focused at O; while those diffracted an angle θ are focused at P.

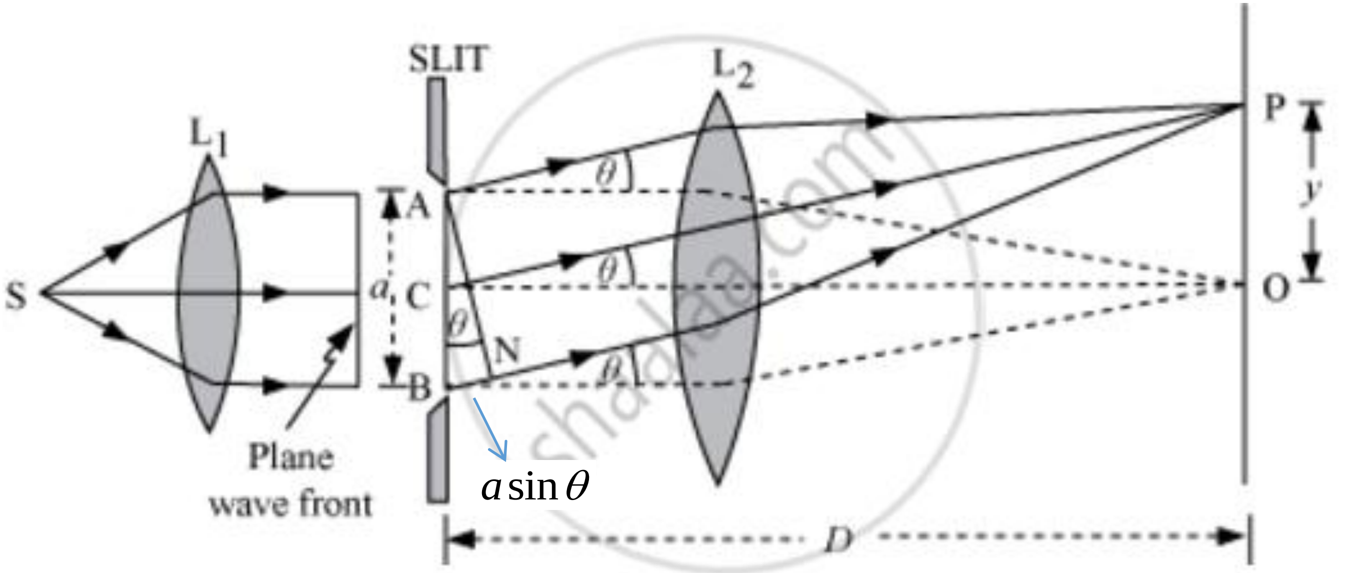


Figure DSS-1
Fraunhofer's diffraction at a single slit

DIFFRACTION

Fraunhofer's Diffraction at a Single Slit

- Let the slit AB be divided into *n* equal parts, each part being the source of secondary wavelets. The amplitude at the point P due to the waves obtained from each point will be equal.
- The phase difference between the waves obtained at point P from any two consecutive parts is $\delta = \frac{1}{n}\frac{2\pi}{\lambda}(a\sin\theta)$
- Resultant amplitude of *n* waves, each of amplitude A_0, at point P is given by

$$R = A_0 \frac{\sin\left(\frac{n\delta}{2}\right)}{\sin\left(\frac{\delta}{2}\right)} = A_0 \frac{\sin(\alpha)}{\sin\left(\frac{\alpha}{n}\right)} \qquad \text{where } \alpha = \frac{\pi a \sin\theta}{\lambda}$$

For large value of *n*, $\sin\left(\frac{\alpha}{n}\right) \approx \frac{\alpha}{n}$

$$\therefore R = nA_0 \frac{\sin\alpha}{\alpha} = A\frac{\sin\alpha}{\alpha} \qquad \text{where, } A = nA_0$$

The resultant intensity at P is given by $\boxed{I = A^2 \frac{\sin^2\alpha}{\alpha^2}}$.

DIFFRACTION

Fraunhofer's Diffraction at a Single Slit

Position of central or principle maximal:

- For central point O on the screen (Figure 1)

$$\alpha = 0$$

$$\therefore \lim_{\alpha \to 0} \frac{\sin \alpha}{\alpha} = 1$$

Hence intensity at O

$$I = A^2 \frac{\sin^2 \alpha}{\alpha^2} = A^2 = I_0$$

This is maximum as all waves reach at O in phase.

Again, $\alpha = 0$

$$\therefore \frac{\pi a \sin \theta}{\lambda} = 0$$

or, $\theta = 0$

This shows that the waves are travelling normal to the slit and O gives the position of the central maximum.

Position of minima:

The intensity is minimum (I=0),

when $\dfrac{\sin \alpha}{\alpha} = 0$

or, $\sin \alpha = 0 \left(\text{but } \alpha \neq 0\right)$

or, $\alpha = \pm\, \mathrm{m}\pi$ where m=1,2 ...

or, $\dfrac{\pi a \sin \theta}{\lambda} = \pm\, \mathrm{m}\pi$

$$\therefore \boxed{a \sin \theta = \pm\, \mathrm{m}\lambda}$$

where m =1,2,3,... gives the direction of first, second, third, ... minima respectively.

DIFFRACTION

Fraunhofer's Diffraction at a Single Slit

Secondary maxima

- To determine the position of secondary maxima, differentiate the equation of intensity with respect to α and equate to zero.

$$\frac{dI}{d\alpha}=0 \Rightarrow A^2\frac{d}{d\alpha}\left(\frac{\sin^2\alpha}{\alpha^2}\right)=0$$

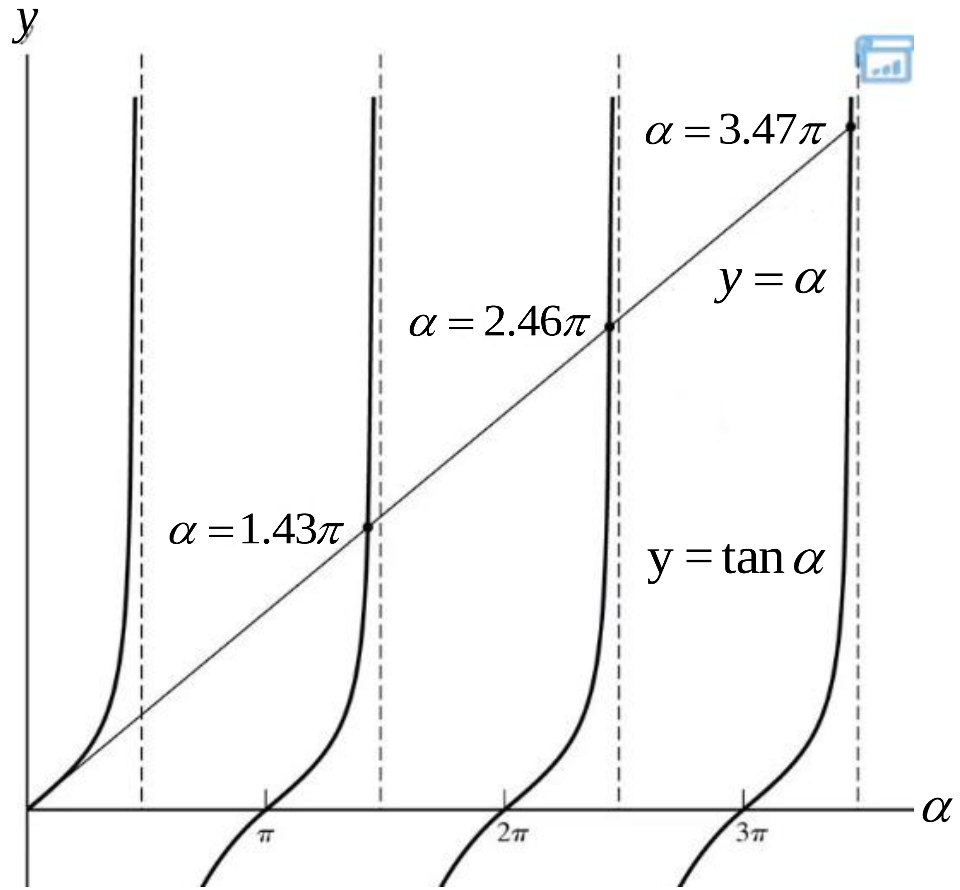
$$\Rightarrow A^2\left(\frac{2\sin\alpha\cos\alpha\ \alpha^2-\sin^2\alpha\ 2\alpha}{\alpha^4}\right)=0$$

$$\Rightarrow 2A^2\left(\frac{\sin\alpha}{\alpha}\right)\left(\frac{\alpha\cos\alpha-\sin\alpha}{\alpha^2}\right)=0$$

$$\Rightarrow \frac{\alpha\cos\alpha-\sin\alpha}{\alpha^2}=0$$

$$\Rightarrow \boxed{\alpha=\tan\alpha}$$

This equation can be solved by plotting the graphs for $y=\alpha$ and $y=\tan\alpha$.



The points on intersection of the curves y = tanα with the straight line y = α give the positions of secondary maxima. The positions are α=1.43π, 2.46π ,3.47π, …

The principal (central) maxima occurs at α = 0, minima occur at α = ±π, ±2π, …, Secondary maxima occur at α=1.43π, 2.46π ,3.47π, …

Fraunhofer's Diffraction at a Single Slit

Intensity Distribution

Intensity of a single slit diffraction pattern

$$\boxed{I = I_0\left[\frac{\sin(\alpha)}{\alpha}\right]^2 = I_0\left[\frac{\sin\left(\dfrac{\pi a\sin\theta}{\lambda}\right)}{\dfrac{\pi a\sin\theta}{\lambda}}\right]^2}$$

- Intensity of central maximum $I_0 = A^2$
- Intensity of first secondary maxima:

$$I_1 = A^2\left[\frac{\sin(1.43\pi)}{1.43\pi}\right]^2 \approx \frac{A^2}{21.2} = \frac{I_0}{21.2} = 0.0472 I_0$$

- Intensity of second secondary maxima :

$$I_2 = A^2\left[\frac{\sin(2.46\pi)}{2.46\pi}\right]^2 \approx \frac{A^2}{60.7} = \frac{I_0}{60.7} = 0.0165 I_0$$

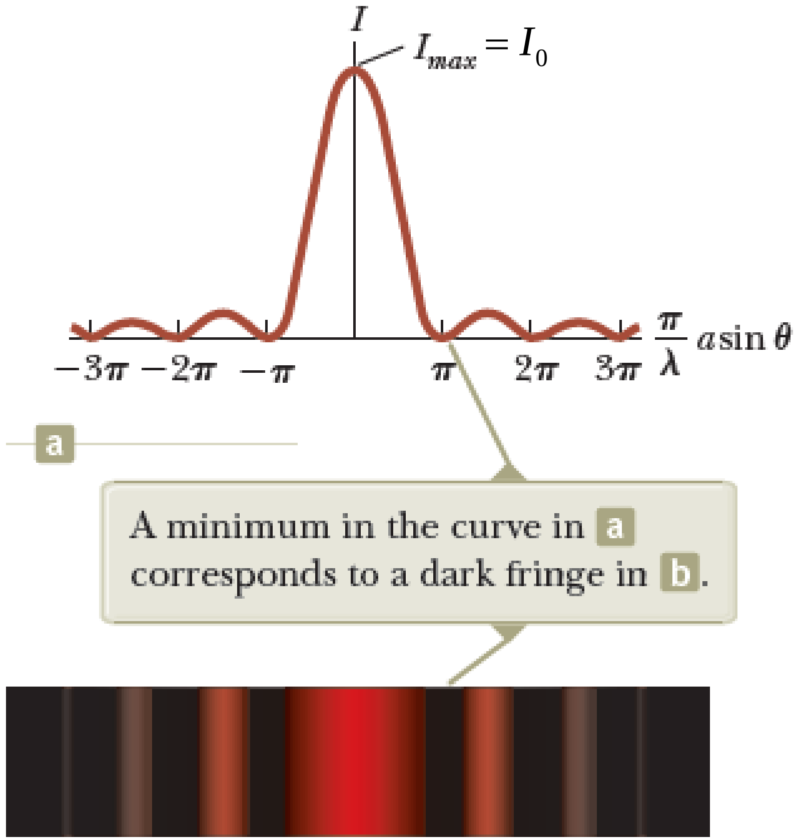


Figure DSS-2

(a) A plot of light intensity *I versus* (π/λ)*a sinθ for* the single-slit Fraunhofer diffraction pattern.
(b) Simulation of a single-slit Fraunhofer diffraction pattern.

Summary of Single Slit Diffraction

Single Slit Diffraction

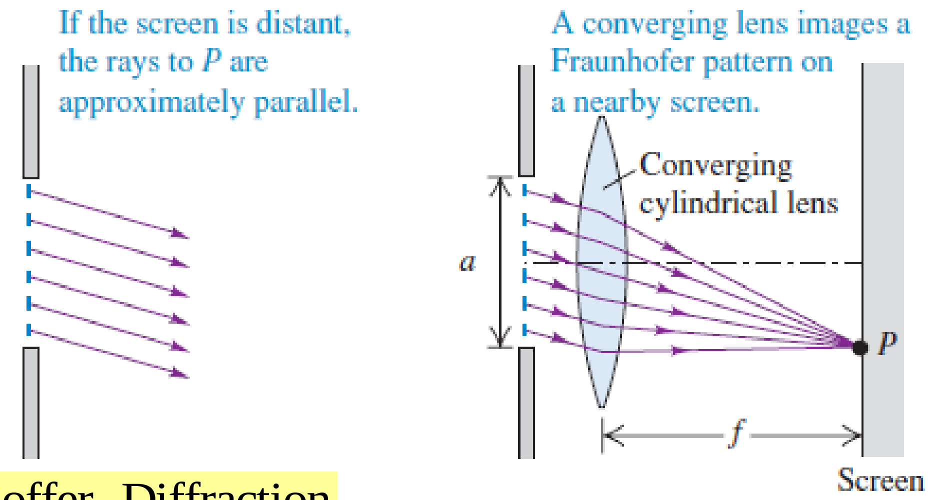
- Monochromatic light sent through a narrow slit of width a produces a diffraction pattern on a distant screen.
- Equation (S-1) gives the condition for destructive interference (a dark fringe) at a point in the pattern at angle θ .

$$\sin\theta_{dark} = m\frac{\lambda}{a} \qquad (m = \pm1, \pm2, \ldots) \qquad \ldots\ldots\ldots\ (S\text{–}1)$$

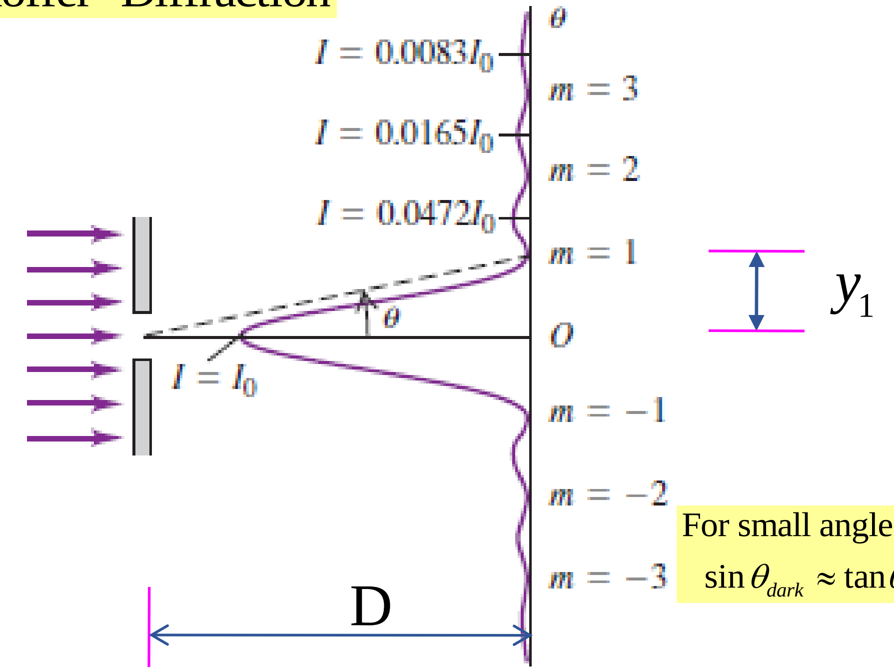
- Equation (S-2) gives the intensity in the pattern as a function of θ .

$$I = I_0\left[\frac{\sin\left(\frac{\pi a \sin\theta}{\lambda}\right)}{\frac{\pi a \sin\theta}{\lambda}}\right]^2 \qquad \ldots\ldots\ldots\ (S\text{-}2)$$

- Width of Central Bright Fringe $= 2y_1 = 2\left(D\sin\theta_{dark}\right) = 2D\frac{\lambda}{a}$



Fraunhoffer Diffraction



For small angle θ

$\sin\theta_{dark} \approx \tan\theta_{dark}$

Diffraction Grating

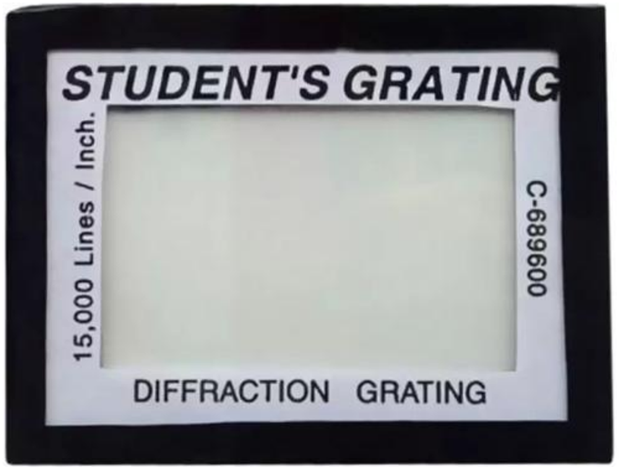
Diffraction Grating

- **Diffraction Grating**--the most useful tool in the study of light and of objects that emit and absorb light
- An array of a large number of parallel slits, all with the same width *a* and spaced equal distances *d* between centers, is called a **diffraction grating.**
- The **first** one was constructed by **Fraunhofer** using fine wires.
- Grating can be made by using a diamond point to scratch many equal spaced grooves on a glass or metal surface.
- A **transmission grating** can be made by cutting parallel grooves on a glass plate with a precession ruling machine. The spaces between the grooves are transparent to the light and hence act as separate slits.
- Current technology can produce gratings that have very small slit spacing.
- **Diffraction grating** has a much greater number N of slits perhaps as many as several thousand per millimeter.

Diffraction Grating: Slits → rulings or lines

↓

Interference grating



Diffraction Grating

Diffraction Grating

- An idealized grating consisting of only five slits is represented in Figure DG-1.

DIFFRACTION

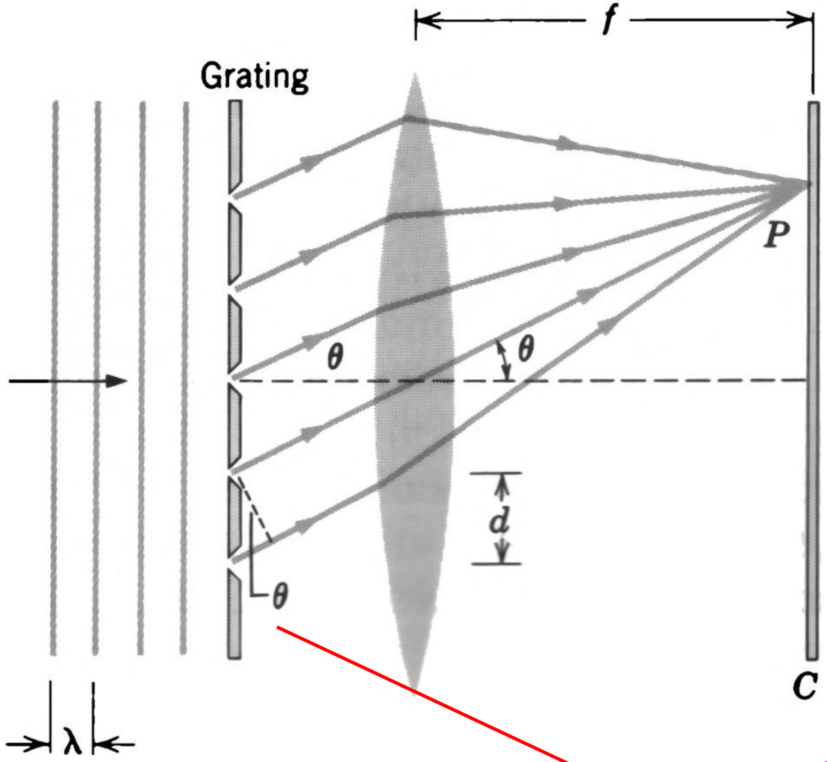
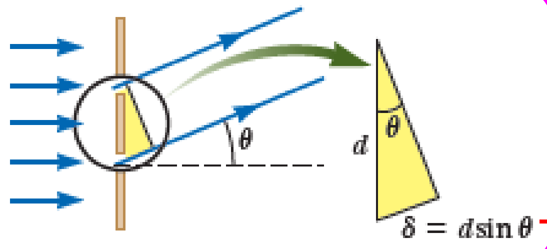


Figure DG-1
An idealized diffraction grating, consisting of only five rulings, that produces an interference pattern on a viewing screen *C*.

The pattern observed on the screen is the result of the combined effects of interference and diffraction. Each slit produces diffraction, and the diffracted beams interfere with one another to produce the final pattern.



path length difference between adjacent rays

Grating element

↓

the spacing between centers of adjacent slits of diffraction grating

↓

$d = a + b$

b → width of an opacity

a → width of each slit or tranparancy

↓

$$a + b = \frac{1}{N}$$

N → Number of lines per inch of grating

Fraunhofer's Diffraction at N Parallel Slits

Plane Transmission Diffraction Grating

- Plane wave of light of wavelength λ is incident from left, normal to the plane of the plane transmission grating. The pattern observed on the screen to the right of the grating is the result of the combined effects of interference and diffraction. Each slit produces diffraction, and the diffracted beams interfere with one another to produce the final pattern.

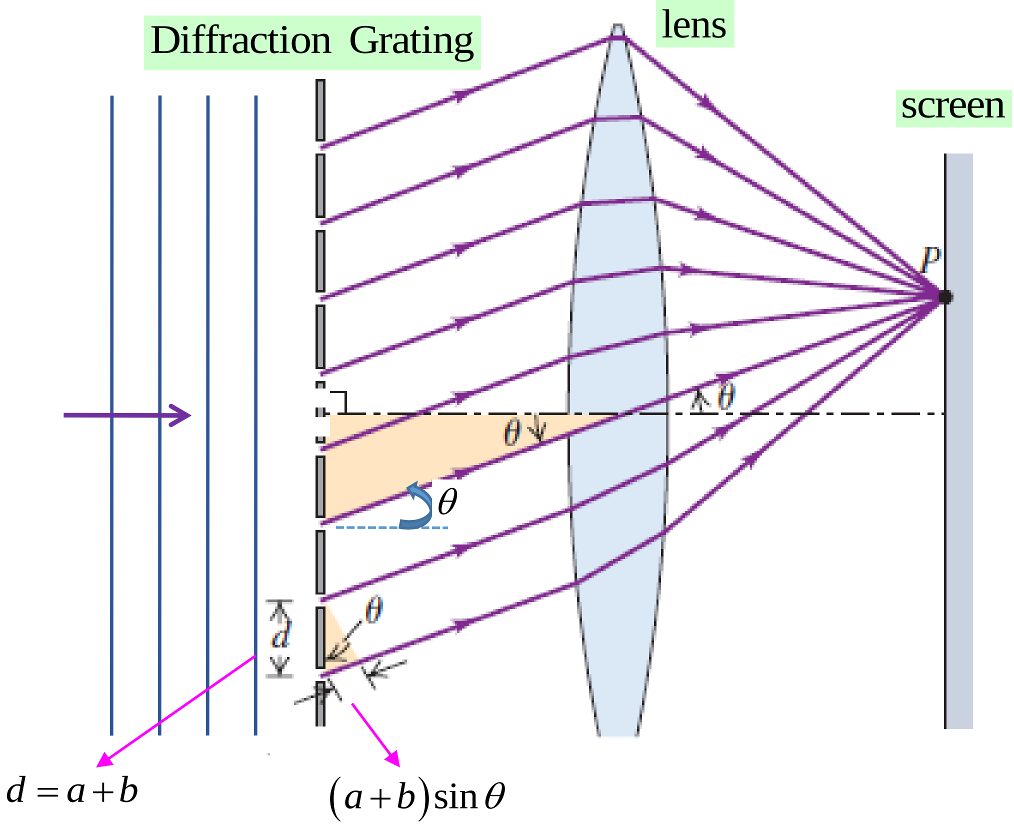


Figure DG-1 Fraunhofer's diffraction at N parallel slits

At an angle θ on the right, we have N waves each of amplitude $R_0 = A\dfrac{\sin\alpha}{\alpha}\left(\text{where } \alpha = \dfrac{\pi}{\lambda}a\sin\theta\right)$

and successive phase difference $\delta = \dfrac{2\pi}{\lambda}(a+b)\sin\theta = 2\beta$ (say)

The superposition of N diffracted waves give rise to the intensity on the screen.

The resultant amplitude in a direction θ is given by

$$R = R_0\frac{\sin\left(\frac{N\delta}{2}\right)}{\sin\left(\frac{\delta}{2}\right)} = A\frac{\sin\alpha}{\alpha}\frac{\sin N\beta}{\sin\beta}$$

Fraunhofer's Diffraction at N Parallel Slits

Plane Transmission Diffraction Grating

- The resultant intensity in a direction θ is given by

$$\boxed{I=R^2=A^2\frac{\sin^2\alpha}{\alpha^2}\frac{\sin^2 N\beta}{\sin^2\beta}} \quad .... \quad (1)$$

Hence the intensity distribution is the product of two terms. The first term $A^2\frac{\sin^2\alpha}{\alpha^2}$ represents the diffraction pattern due to a single slit. The secondary term $\frac{\sin^2 N\beta}{\sin^2\beta}$ represents the interference pattern due to N Slits. Both effects combine together give the intensity pattern of the light diffracted by plane transmission grating.

Principal Maxima:

Intensity would be maximum ,

when $\sin\beta=0$

$\Rightarrow \beta=\pm n\pi$ where n = 0,1,2,3....

Also, $\sin N\beta=0$

Thus, $\frac{\sin N\beta}{\sin\beta}=\frac{0}{0}$ (Undefined)

So,

$$\lim_{\beta\to\pm n\pi}\frac{\sin N\beta}{\sin\beta}=\lim_{\beta\to\pm n\pi}\frac{N\ \cos N\beta}{\cos\beta}=\pm N$$

$\therefore$ Intensity at principal maxima,

$$\boxed{I=A^2\frac{\sin^2\alpha}{\alpha^2}N^2} \quad .... \quad (2)$$

$$\because \quad \beta=\pm n\pi \Rightarrow \frac{\pi}{\lambda}(a+b)\sin\theta=\pm n\pi$$

$$\therefore \quad (a+b)\sin\theta=\pm n\lambda$$

For n= 0, maximum is zero order maximum and for n=±1, ±2, …etc. are called first, second … etc order principal maxima respectively.

Fraunhofer's Diffraction at N Parallel Slits

Plane Transmission Diffraction Grating

- **Minima**:

The intensity is minimum (I=0),

$$\text{when} \quad \sin N\beta = 0, \text{ but } \sin\beta \neq 0$$

$$\Rightarrow N\beta = \pm m\pi \qquad \text{where m =1,2,3....(N-1),(N+1),(N+2),...,(2N-1)....}$$

$$\Rightarrow N\frac{\pi}{\lambda}(a+b)\sin\theta = \pm m\pi$$

$$\therefore \quad \boxed{N(a+b)\sin\theta = \pm m\lambda}$$

For m=0 , it gives principal maxima and m=N also gives principal maxima, and m=1,2,…,N-1 gives minima. There is (N-1) minima between two principal maxima.

- **Secondary Maxima**:

Since there are (N-1) minima between two maxima, there must be (N-2) secondary maxima between two principal maxima.

For the secondary maxima

$$\frac{dI}{d\beta} = 0 \Rightarrow A^2 \frac{\sin^2\alpha}{\alpha^2} \frac{d}{d\beta}\left[\frac{\sin^2 N\beta}{\sin^2\beta}\right] = 0$$

Fraunhofer's Diffraction at N Parallel Slits

DIFFRACTION

Plane Transmission Diffraction Grating

- **Secondary Maxima**:

$$\Rightarrow A^2 \frac{\sin^2 \alpha}{\alpha^2} \frac{d}{d\beta}\left[\frac{\sin^2 N\beta}{\sin^2 \beta}\right] = 0$$

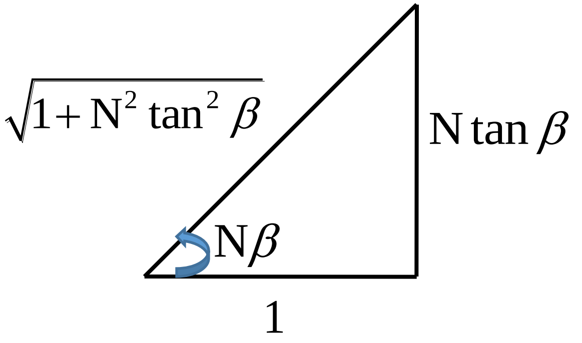
$$\Rightarrow A^2 \frac{\sin^2 \alpha}{\alpha^2}\left[\frac{(2\sin N\beta \cos N\beta\ N)\sin^2 \beta - \sin^2 N\beta\ (2\sin \beta \cos \beta)}{\sin^4 \beta}\right] = 0$$

$$\Rightarrow 2A^2 \frac{\sin^2 \alpha}{\alpha^2} \frac{\sin N\beta}{\sin \beta}\left[\frac{N\sin \beta \cos N\beta - \sin N\beta \cos \beta}{\sin^2 \beta}\right] = 0$$

$$\Rightarrow N\sin \beta \cos N\beta - \sin N\beta \cos \beta = 0$$

$$\Rightarrow \tan N\beta = N \tan \beta$$

Then From Figure



$$\frac{\sin^2 N\beta}{\sin^2 \beta} = \frac{\frac{N^2 \tan^2 \beta}{1+N^2 \tan^2 \beta}}{\sin^2 \beta} = \frac{N^2}{\cos^2 \beta\left(1+N^2 \tan^2 \beta\right)}$$

$$= \frac{N^2}{\cos^2 \beta + N^2 \sin^2 \beta} = \frac{N^2}{1+\left(N^2-1\right)\sin^2 \beta}$$

- Thus, Intensity at secondary maxima is given by

$$I = A^2 \frac{\sin^2 \alpha}{\alpha^2} \frac{N^2}{1+\left(N^2-1\right)\sin^2 \beta} \qquad \text{..........} \quad (3)$$

$$\boxed{\frac{\text{Intensity of secondary maxima } (I')}{\text{Intensity of Principal maxima } (I)} = \frac{1}{1+\left(N^2-1\right)\sin^2 \beta}}$$

If N is large, then the intensity of secondary maxima is less. Since in the grating, the number of slits is very large, the secondary maxima are not visible in grating spectra.


Thank you