(x) Note:

(i): If charge 'q' is located at the centre of a cube, the electric flux through any face is

$$\phi_{face} = \frac{1}{6}\left[\frac{q}{\varepsilon_0}\right]$$

(ii) If the closed surfaces of various shapes surrounding a ~~charge~~ charge 'q', the net electric flux is same through all surfaces.

(iii) When a cube is inscribed in a sphere of radius 'r', the length 'L' of a side of cube is

$$L = \sqrt{\frac{4}{3}}\, r.$$

If a positive point charge Q is placed at the center of the spherical surface, the ratio.

$$\frac{\phi_{sphere}}{\phi_{cube}} = 1.$$

Application of Gauss's Theorem

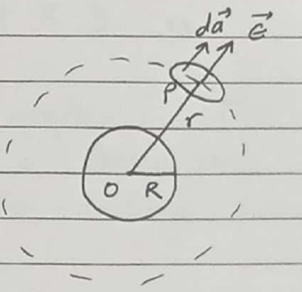
(1): Use Gauss's law to find the electric field outside, on and inside a spherical shell of radius R, which carries a uniform surface charge density 'σ'

Sol^n:

Let the radius of the spherical shell be R and the mass be M.

Let the uniform surface charge density be σ.

Let P be a point at r distance from the centre of sphere i.e, $r > R$.



We draw a Gaussian surface of radius r.

Since the charged spherical shell lies completely inside the Gaussian surface, The net charge enclosed by the Gaussian surface is equal to the total charge on spherical shell.

$$(Q_{enc} = q = \sigma \times 4\pi R^2)$$

For every point on the Gaussian surface S, the magnitude of electric field E is same and electric field $\vec{E}$ is directed radially outward as does $d\vec{a}$.

From Gauss's law,

$$\oint_s \vec{E}\cdot d\vec{a} = \frac{1}{\varepsilon_0} Q_{enc}$$

or, $$\oint_s E\, da = \frac{1}{\varepsilon_0} Q_{enc}$$

or, $$E \oint_s da = \frac{1}{\varepsilon_0}\left(\sigma \times 4\pi R^2\right)$$

or, $$E \times \cancel{4\pi} r^2 = \frac{1}{\varepsilon_0} \times \sigma \times \cancel{4\pi} R^2$$

or $$E = \frac{1}{\varepsilon_0}\,\sigma\,\frac{R^2}{r^2}$$

$$\frac{\frac{q}{4\pi R^2} \times R^2}{\varepsilon_0\, r^2}$$

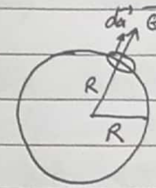
$$\therefore E = \frac{1}{4\pi\varepsilon_0}\,\frac{q}{r^2}$$

Therefore, the field at a point outside the shell is equivalent to a point charge q located at the center.

Now,

Let P be the point at the surface of the spherical shell.

and then we draw a gaussian surface of radius OP.



Here, we know, $r = R$.

Since the spherical shell ~~and~~ lies on the Gaussian surface, the net charge enclosed by the Gaussian surface is equal to total charge of spherical shell.

$$Q_{enc} = \sigma \times 4\pi R^2$$

for every point on the Gaussian surface S, the magnitude of the electric field E is same and Electric ~~filer~~ field $\vec{E}$ is directed radially outward as does $d\vec{a}$

From gauss's law,

$$\oint_s \vec{E}\cdot d\vec{a} = \frac{1}{\varepsilon_0} Q_{enc}.$$

or, $$E \oint_s da = \frac{1}{\varepsilon_0}\left(\sigma \times 4\pi R^2\right)$$

or, $$E \times 4\pi R^2 = \frac{1}{\varepsilon_0}\,\sigma\, 4\pi R^2$$

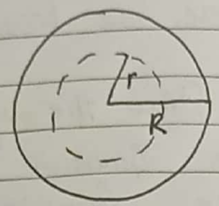
or $$E = \frac{\cancel{q}\,\sigma}{\varepsilon_0}$$

$$\therefore E = \frac{\sigma}{\varepsilon_0} = \frac{1}{4\pi R^2} \times \frac{q}{R^2}$$

Again,

Let P be the point inside the spherical shell surface with radius ~~R~~ r and then draw Gaussian surface of radius OP.

Here, $r < R$.



Since the Gaussian surface lies inside the spherical cell, it doesn't enclose any charge.
Thus, $Q_{enc} = 0$.

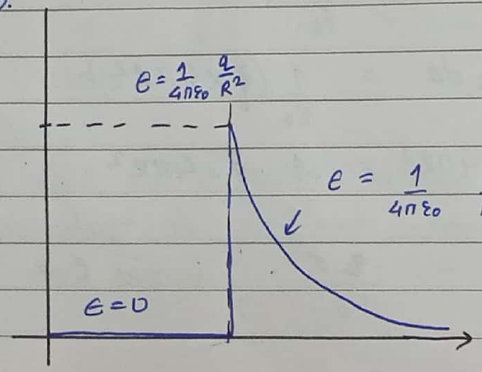
From Gauss's law,

$$\oint \vec{E} \cdot d\vec{a} = \frac{1}{\varepsilon_0} Q_{enc}$$

$$\therefore E = 0$$

Therefore, the electric field inside a spherical shell is zero.

Graph for spherical shell.

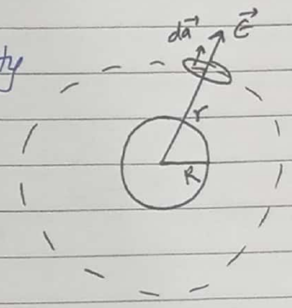


⟨Q⟩: A hollow metallic sphere of radius 0.1 m has 10^{-8} C of charge uniformly spread over it. The electric field intensity at point 7 cm away from the center is 0.

⟨2⟩ Use Gauss's law, to find the electric field outside, on and inside ~~of~~ a ~~st~~ uniformly charged solid sphere of radius R and volume charge density ρ.

Soln.

Let us consider a uniform charged solid sphere having radius R and mass M.



Let the uniform volume charge density be ρ.

Let us consider a point P at distance r from the center of the solid sphere.

Let us draw a Gaussian surface S of radius r. Since the Gaussian surface encloses the solid sphere entirely, the net charge enclosed by the Gaussian surface $(Q_{enc}) = \rho \times \frac{4}{3}\pi R^3$

From Gauss's law,

$$\oint_S \vec{E}\cdot d\vec{a} = \frac{1}{\varepsilon_0} Q_{enc}$$

or, $$E \oint_S da = \frac{1}{\varepsilon_0} \times \rho \times \frac{4}{3}\pi R^3$$

or $$E \times 4\pi r^2 = \frac{1}{\varepsilon_0} \times \rho \times \frac{4}{3}\pi R^3$$

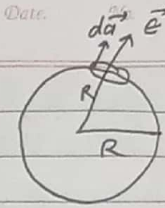
$$\therefore E = \frac{\rho R^3}{3\varepsilon_0 r^2}$$

$$= \frac{\frac{q}{\frac{4\pi R^3}{3}} \times R^3}{3\varepsilon_0 r^2}$$

$$\therefore E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

Now,
Let us consider point P at the surface of the solid sphere.

Let us draw a Gaussian surface of radius R.
Since the Gaussian surface encloses the solid sphere entirely, the total charge enclosed by the Gaussian surface $(Q_{enc}) = \rho \times \frac{4}{3}\pi R^3$.



From Gauss's law,

$$\oint_S \vec{E}\cdot d\vec{a} = \frac{1}{\varepsilon_0} Q_{enc}.$$

or, $$E \oint_S da = \frac{1}{\varepsilon_0}\,\frac{4\pi R^3}{3} \times \rho$$

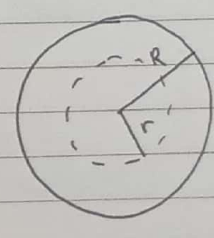
or $$E \times 4\pi R^2 = \frac{4\pi R^3 \rho}{3\varepsilon_0}$$

$$\therefore E = \frac{\rho R}{3\varepsilon_0} = \frac{\frac{q}{\frac{4}{3}\pi R^3} \times R}{3\varepsilon_0}$$

$$\therefore E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$$

Again,
Let us consider point P inside the surface of solid sphere.

Let us draw ~~of~~ a Gaussian surface of radius r.
Since the Gaussian surface encloses certain region of solid sphere entirely, the total charge enclosed by the Gaussian surface, the total charge enclosed by Gaussian surface $(Q_{enc}) = \rho \times \frac{4}{3}\pi r^3$.



The electric field magnitude E and $\vec{E}$ is directed radially ~~downward~~ outward as $d\vec{a}$ at every point.

From Gauss's law,

$$\oint_S \vec{E}\cdot d\vec{a} = \frac{1}{\varepsilon_0} Q_{enc}$$

or $$E \oint_S da = \frac{1}{\varepsilon_0} \times \frac{4}{3}\pi r^3$$

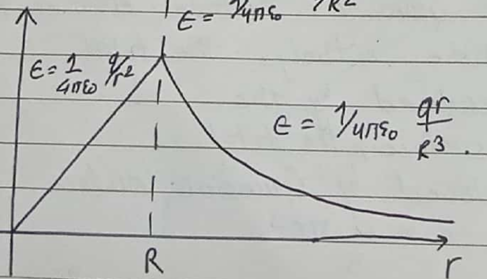
or $$E \times \cancel{4}\cancel{\pi} r^2 = \frac{1}{\varepsilon_0} \times \frac{\cancel{4}}{3}\cancel{\pi} r^3$$

$$\therefore E = \frac{\rho r}{3\varepsilon_0} = \frac{\left(\frac{q}{\frac{4}{3}\pi R^3}\right) r}{3\varepsilon_0}$$

$$\therefore E = \frac{1}{4\pi\varepsilon_0}\frac{qr}{R^3}$$

Thus, the electric field inside uniformly charged sphere is directly proportional to the distance of field point from the center of the sphere.

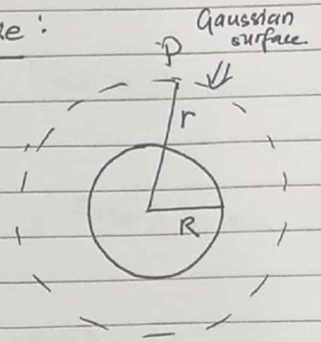
At center of sphere, $r=0$, Electric field is zero.



<3>: Use Gauss's law to find electric field outside, ~~on~~ and inside a ~~uniformly charged solid~~ sphere ~~of radius~~ which carries a charge density proportional to distance from the origin $\rho = kr$, for some constant k.

Sol^n.

(A): For electric field outside a sphere:



Let us consider a sphere of radius R with O as centre.

A point P is r distance from center of sphere such that $r > R$.

Let us draw a gaussian surface ~~to enclosing~~ of radius r as in figure.

By symmetry, the magnitude of electric field is constant everywhere on the Gaussian spherical surface and is normal to the surface at each point.

The total electric flux through the Gaussian surface is given by

$$\oint_S \vec{E}\cdot d\vec{a} = E\oint_S d\vec{a}$$
$$= \cancel{E(4\pi R^2)}\quad E\times(4\pi r^2)$$

From Gauss's law,

$$\oint_s \vec{E}\cdot d\vec{a} = \frac{1}{\varepsilon_0}\int_V \rho \, d\tau$$

or, $$E(4\pi r^2) = \frac{1}{\varepsilon_0}\int_V (kr)\, r^2 \sin\theta \, d\theta \, d\phi \, dr$$

or, $$E \times 4\pi r^2 = \frac{k}{\varepsilon_0}\left[\left\{\int_0^R r^3 dr\right\}\left\{\int_0^\pi \sin\theta \, d\theta\right\}\left\{\int_0^{2\pi} d\phi\right\}\right]$$

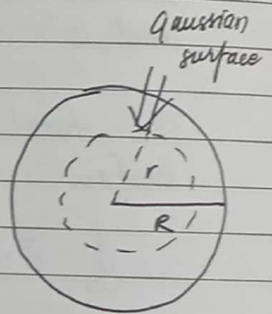
or, $$E \times 4\pi r^2 = \frac{k}{\varepsilon_0}\left(\frac{R^4}{4}\right)(2)(2\pi)$$

or, $$E \times \cancel{4\pi} r^2 = \frac{k}{\varepsilon_0}\,\frac{R^4}{4} \times \cancel{4\pi}$$

$$\therefore E = \frac{kR^4}{4\varepsilon_0 r^2}$$

(B) For electric field inside a sphere.

Let us consider a sphere of radius R with O as center.



A point P is at ~~radius~~ r distance from center such that r<R.

Let us draw a Gaussian surface of radius r as in figure.

By symmetry, the magnitude of electric field is constant everywhere on the spherical Gaussian surface and is normal to the surface at each point.

The total electric flux through the Gaussian surface is given by,

$$\oint_s \vec{E}\cdot d\vec{a} = E\oint_s da = E \times (4\pi r^2)$$

From Gauss's law,

$$\oint_s \vec{E}\cdot d\vec{a} = \frac{1}{\varepsilon_0}\int_V \rho \cdot d\tau$$

or, $$E \times (4\pi r^2) = \frac{1}{\varepsilon_0}\left[\int_V (kr)\, r^2 \sin\theta \, dr \, d\theta \, d\phi\right]$$

or, $$E \times (4\pi r^2) = \frac{k}{\varepsilon_0}\left[\left\{\int_0^r r^3 dr\right\}\left\{\int_0^\pi \sin\theta \, d\theta\right\}\left\{\int_0^{2\pi} d\phi\right\}\right]$$

or, $$E \times (\cancel{4\pi} r^{\cancel{2}}) = \frac{k}{\varepsilon_0}\left(\frac{r^{\cancel{4}2}}{4}\right)\cancel{2} \times \cancel{2\pi}$$

$$\therefore E = \frac{kr^2}{4\varepsilon_0}.$$

Date. No.

Q: Note:

(i) Two charges, each of q separated by a distance. The net electric field at a distance 'x' from a charge and on the line joining them is

$$E = \frac{1}{4\pi\varepsilon_0} q \left[\frac{1}{x^2} - \frac{1}{(d-x)^2} \right]$$

(ii) A charge of 0.80 nC is placed at the center of a cube that measures 4.0 m along each edge. What is the electric flux through one face of the cube?

Sol.

Given,

$q_{enc} = 0.80\,nC = 0.80 \times 10^{-9}$ C

$\varepsilon_0 = 8.85 \times 10^{-12}$ N/m^2C^2

Now,

$$\phi = \frac{1}{6}\left[\frac{1}{\varepsilon_0} \times q_{enc}\right]$$

$$= \frac{1}{6} \times \left[\frac{1}{8.85 \times 10^{-12}} \times 0.80 \times 10^{-9}\right] = 15\ N/m^2/C$$

Date. No.

(iii) A point charge (5.0 pC) is located at the center of a spherical surface (radius = 2.0 cm) and a charge 3.0 pC is spread uniformly upon this surface. Determine the magnitude of electric field 1.0 cm from the point charge.

Sol:

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}$$

$$= 9 \times 10^9 \times \frac{5.0 \times 10^{-12}}{(10^{-2})^2}$$

$\therefore E = 450\ N/C.$