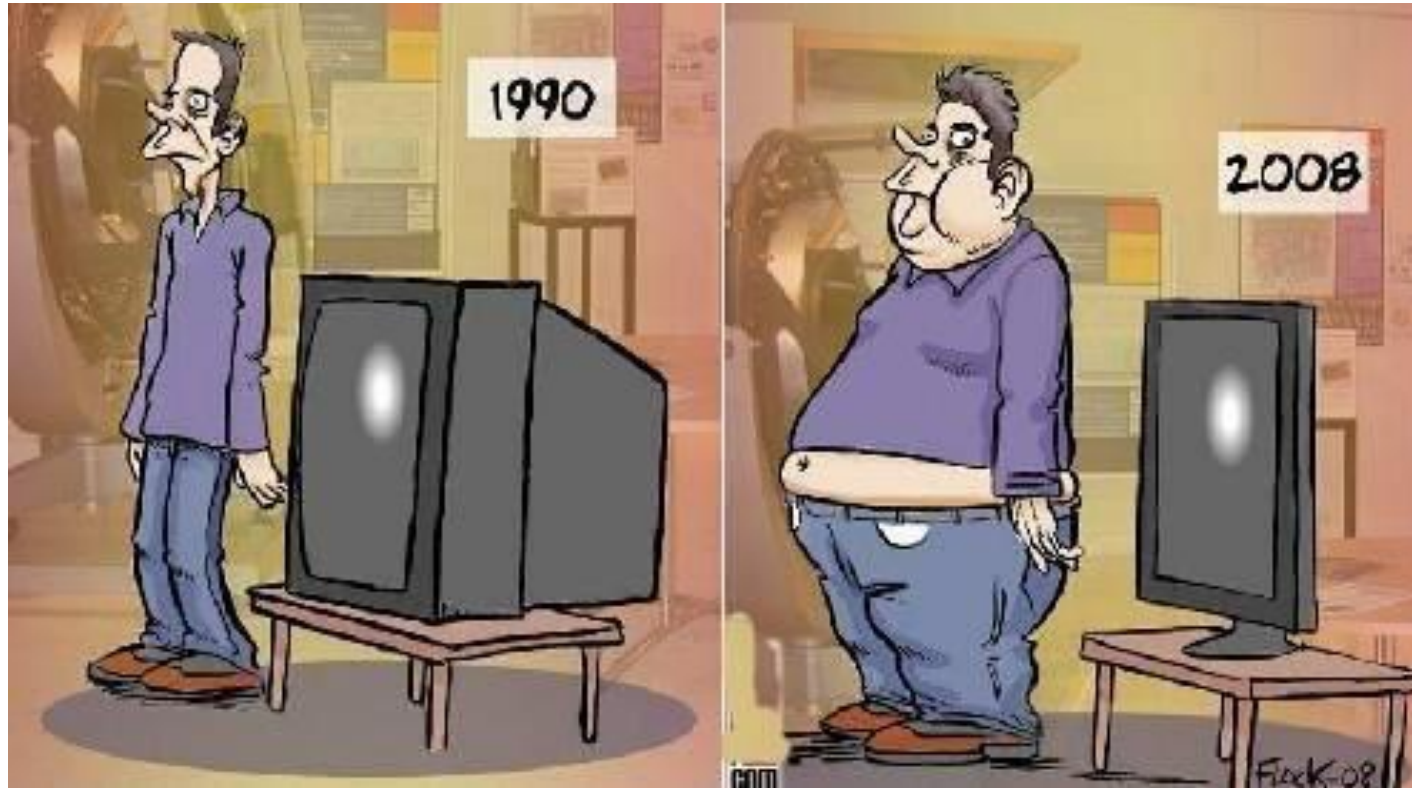


# MASS TRANSFER

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# Law of conservation of Mass



Mass is conserved in ordinary chemical and physical processes.

# Law of conservation of Mass

- Environmental engineers need to know how big to draw the “box”, for example, where to draw the line and what to include and exclude.
- If the “box” is too small, unintended consequences may result. If the “box” is too big, time and resources may be wasted.

# Mass Transfer

- ***Everything has to go somewhere*** is a simple way to express one of the most fundamental engineering principles.
- The ***law of conservation of mass*** says that mass can never be created nor destroyed.
- What this concept allows us to do is track materials, that is, pollutants, from one place to another with mass balance equations.
- In environmental systems, be it an aerobic cell participating in oxidation process or a river receiving untreated wastes, the movement of nutrients, substrate or metabolic products plays an important role.

# Mass Transfer

- Thus physical transport phenomena becomes predominant in governing the processing rate rather than the chemical or biochemical rate.
- The extent of processes such as decomposition of wastes by microorganisms or lake eutrophication is governed by either the supply of natural or artificial oxygen or availability of substrate or nutrients.
- Thus the transfer of mass from one point to another in a system becomes important.

# Mass Transfer

- There could be different mass transfer processes depending upon the underlying phases, for example:
  - ✓ Gas-liquid mass transfer: aeration or supply of oxygen for decomposing waste
  - ✓ Liquid –gas transfer: release of methane from anaerobic waste treatment
  - ✓ Liquid-liquid mass transfer: extraction of organic solvents, liquid phase oxygen transfer
  - ✓ Liquid-solid mass transfer: adsorption of pollutants on activated carbon
  - ✓ solid-gas mass transfer: release of gases from solid wastes in a landfill

# Energy Transfer

- The law of conservation of energy states that , with the exception of nuclear reactions, energy can neither be created nor destroyed.
- This energy law serves as an accounting tool in various environmental implications.
- Be it a metabolic reaction taking inside a cell or dispersion of air pollutants in the atmosphere or the different aspects of climate change processes, a knowledge of transfer of energy is essential to understand the process dynamics.
- The flow of energy can be analyzed through energy balance equations using the *first law of thermodynamics*.

# Energy Transfer

- The first law of thermodynamics states that energy can never be created nor be destroyed.

change in energy of system – change in energy of surroundings = 0

- A simple interpretation of the *second law of thermodynamics* suggests that when work is done there will always be some inefficiency; that is, some portion of the energy put into the process will end up as waste heat.
- How the waste heat affects the environment is an important consideration in the study of environmental engineering.



# Energy Transfer

Knowledge of mass and energy transfer in pollution control and remediation is essential to:

- Understand the process
- Design the preventive measures
- Design the treatment units
- Design the remediation activities

# Mass Balance

- The concept is called a *materials balance*, or a *mass balance*. This concept serves as a basis for describing and analyzing environmental engineering problems.
- Essentially, material balances are accounting procedures: total mass entering must be accounted for at the end of the process, even if it undergoes heating, mixing, drying, or any other operation within the system.
- Usually it is not feasible to measure the masses and compositions of all streams entering and leaving a system; unknown quantities can be calculated using mass-balance principles.
- Mass-balance problems have a constant theme: given the masses of some input and output streams, calculate the masses of others.

# Mass Balance

In its simplest form it may be viewed as an accounting procedure. For example, a mass balance is performed each time a check book is balanced.

$$\text{Balance} = \text{Deposit} - \text{Withdrawal} \quad \text{Eqn. 1}$$

In an environmental system or subsystem, the equation would be written as:

$$\text{Accumulation} = \text{Input} - \text{Output} \quad \text{Eqn. 2}$$

# Material Balances

- The first step a mass balance analysis is to define the particular region in the space that is to be analyzed.
- This is carried out by drawing a flow chart of the process or a conceptual diagram of the environmental subsystem termed as 'Mass Balance Diagram'.
- All of the known inputs, outputs, and accumulation are converted to the same mass units and placed on the diagram.
- This helps us define the problems. System boundaries (imaginary blocks around the process or part of the process) are drawn in such a way that calculations are made as simple as possible.
- Then material- balance equations are used to solve for unknown inputs, outputs, or accumulations.

# Material balance with single material

- Material flows can be most readily understood and analyzed by using the concept of a black box.
- These boxes are schematic representations of real processes or flow junctions, and it is not necessary to specify just what this process is to be able to develop general principles about the analysis of flows.



A black box with one inflow and one outflow

# Material balance with single material

- The Figure shows a black box into which some material is flowing.
- All flows into the box are called influents and represented by the letter X. If the flow is described as mass per unit time,  $X_0$  is the mass per unit time of material X flowing in to the box. Similarly,  $X_1$  is the outflow or effluent.
- If no processes are going inside the box that will either make of more material or destroy some of it and if the flow is assumed not vary with time (that is to be at steady state), then it is possible to write a material balance around the box as:

$$[Mass \text{ per unit time } X \text{ IN}] = [Mass \text{ per unit time } X \text{ OUT}] \quad \text{Eqn. 3}$$

$$[X_0] = [X_1] \quad \text{Eqn. 4}$$

# Material balance with single material

- The black box can be used to establish a volume balance and a mass balance if the density does not change in the process.
- Because the definition of density is mass per unit volume, the conversion from a mass balance to a volume balance is achieved by dividing each term by the density (a constant).
- It is generally convenient to use the volume balance for liquids and the mass balance for solids.

# Splitting Single-Material Flow Streams

- A black box shown earlier receives flow from one feed source and separates this into two or more flow streams. The flow into the box is labeled  $X_0$  and the two flows out of the box are  $X_1$  and  $X_2$ .



- If again it is assumed that steady state conditions exist and that no material is being destroyed or produced, then the material balance is:

$$[X_0] = [X_1] + [X_2] \quad \text{Eqn. 5}$$

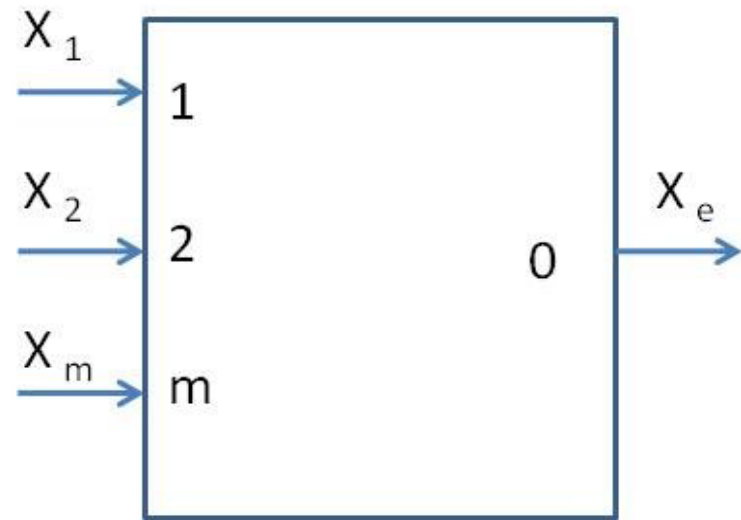
$$[X_0] = \left[ \sum_{i=1}^n X_i \right] \quad \text{where there are } n \text{ exit streams, of effluents.}$$



# Combining Single-Material Flow Streams

- A black box can also receive numerous influents and discharge one effluent, as shown.
- If the influents are labeled as  $X_1, X_2, \dots, X_m$ , the material balance would yield

$$\left[ \sum_{i=1}^m X_i \right] = [X_e]$$



A blender with several inflows and one outflow

# Conservative and non-conservative substance

- Conservative substances are not physically or chemically transformed to other substances in normal situation. Examples are salt and metals.
- Non-conservative substances are transformed to other substances through physical, chemical or biological processes in the environment.
- These include biological oxygen demand (indicator of the quantity of biologically degrading chemicals), ammonia and certain organic compounds.
- Conservative substances tend to be stable, long-lived compounds that persist within environment.
- Non-conservative substances can transform or degrade into other compounds, but the rate of transformation depends on the physical, chemical and biological conditions occurring in the environment.

# Complex process with a single material

- The preceding two sections illustrate the basic principle of material balances.
- The two assumptions used to approach the analysis above are the flows are in steady state (they do not change with time) and that no material is being destroyed (consumed) or created (produced).
- If these possibilities are included in the full material balance, the equation reads:

$$\begin{bmatrix} \text{Material per} \\ \text{unit time} \\ \text{ACCUMULATED} \end{bmatrix} = \begin{bmatrix} \text{Material per} \\ \text{unit time} \\ \text{IN} \end{bmatrix} - \begin{bmatrix} \text{Material per} \\ \text{unit time} \\ \text{OUT} \end{bmatrix} + \begin{bmatrix} \text{Material per} \\ \text{unit time} \\ \text{PRODUCED} \end{bmatrix} - \begin{bmatrix} \text{Material per} \\ \text{unit time} \\ \text{CONSUMED} \end{bmatrix}$$

# Complex process with a single material

- If the material in question is labeled A, the mass balance equation reads:

$$\left[ \begin{array}{c} \text{Mass of A per} \\ \text{unit time} \\ \text{ACCUMULATED} \end{array} \right] = \left[ \begin{array}{c} \text{Mass of A per} \\ \text{unit time} \\ \text{IN} \end{array} \right] - \left[ \begin{array}{c} \text{Mass of A per} \\ \text{unit time} \\ \text{OUT} \end{array} \right] + \left[ \begin{array}{c} \text{Mass of A per} \\ \text{unit time} \\ \text{PRODUCED} \end{array} \right] - \left[ \begin{array}{c} \text{Mass of A per} \\ \text{unit time} \\ \text{CONSUMED} \end{array} \right]$$

- Or, provided the density does not change, in volume terms as:

$$\left[ \begin{array}{c} \text{Volume of A per} \\ \text{unit time} \\ \text{ACCUMULATED} \end{array} \right] = \left[ \begin{array}{c} \text{Volume of A per} \\ \text{unit time} \\ \text{IN} \end{array} \right] - \left[ \begin{array}{c} \text{Volume of A per} \\ \text{unit time} \\ \text{OUT} \end{array} \right] + \left[ \begin{array}{c} \text{Volume of A per} \\ \text{unit time} \\ \text{PRODUCED} \end{array} \right] - \left[ \begin{array}{c} \text{Volume of A per} \\ \text{unit time} \\ \text{CONSUMED} \end{array} \right]$$

- Mass or volume per unit time can be simplified to rate, where rate simply means the flow of mass or volume. Thus, the material balance or mass balance equation for either mass or volume reads:

$$\left[ \begin{array}{c} \text{Rate of A} \\ \text{ACCUMULATED} \end{array} \right] = \left[ \begin{array}{c} \text{Rate of A} \\ \text{IN} \end{array} \right] - \left[ \begin{array}{c} \text{Rate of A} \\ \text{OUT} \end{array} \right] + \left[ \begin{array}{c} \text{Rate of A} \\ \text{PRODUCED} \end{array} \right] - \left[ \begin{array}{c} \text{Rate of A} \\ \text{CONSUMED} \end{array} \right] \quad \text{Eqn. 6}$$

# General rules in solving mass balance problems

1. Draw the system diagram, including all flows (inputs and outputs) as arrows.
2. Add the available information such as flow rates and concentrations. Assign symbols to unknown variables.
3. Draw the continuous dashed line around the component or components that are to be balanced. This could be a unit operation, a junction or a combination of these. Everything inside the dashed line becomes the black box.
4. Decide what material is to be balanced. This could be a volumetric or mass flow rate.
5. Write the material balance equation by starting with the basic equation:

$$\left[ \begin{array}{c} \text{Rate of } A \\ \text{ACCUMULATED} \end{array} \right] = \left[ \begin{array}{c} \text{Rate of } A \\ \text{IN} \end{array} \right] - \left[ \begin{array}{c} \text{Rate of } A \\ \text{OUT} \end{array} \right] + \left[ \begin{array}{c} \text{Rate of } A \\ \text{PRODUCED} \end{array} \right] - \left[ \begin{array}{c} \text{Rate of } A \\ \text{CONSUMED} \end{array} \right]$$

6. If only one variable is unknown, solve for that variable.
7. If more than one variable is unknown, repeat the procedure using a different black box or a different material for the same black box.

# Material balance with multiple materials

- Mass and volume balances can be developed with multiple materials flowing in a single system.
- In some cases the process is one of mixing, where several inflow streams are combined to produce a single outflow stream, while in other cases a single inflow stream is split into several outflow streams according to some material characteristics.
- Because the mass balance and volume balance equations are actually the same equations for a black box, unless there is more than one material involved in the flow.

# Steady state condition

For many environmental problems, time is an important factor in establishing the degree of severity of the problem or in the designing a solution. In these instances, Equation 6 is modified to the following form:

$$\text{Rate of accumulation} = \text{Rate of input} - \text{Rate of output} \quad \text{Eqn. 7}$$

where rate is used to mean per unit of time. In the calculus this may be written as:

$$\frac{dM}{dt} = \frac{d(In)}{dt} - \frac{d(Out)}{dt} \quad \text{Eqn. 8}$$

In most systems of environmental interest, transformation occur within the system: byproducts are formed (e.g. sludge, CO may be oxidized to CO<sub>2</sub>) or compounds are destroyed (e.g. ozone). Because many environmental reactions do not occur instantaneously, the time dependence of the reaction must be taken into account.

# Steady state condition

Thus the mass balance equation may be written as follows:

$$\text{Accumulation rate} = \text{Input rate} - \text{Output rate} \pm \text{Transformation rate} \quad \text{Eqn. 9}$$

Time-dependent reactions are called *kinetic reactions*. The rate of transformation, or reaction rate ( $r$ ), is used to describe the rate of formation (ex. growth of microorganisms) or disappearance (utilization of substrate) of a substance or chemical species.

In the calculus:

$$\frac{dM}{dt} = \frac{d(In)}{dt} - \frac{d(Out)}{dt} \pm r \quad \text{Eqn.10}$$

The reaction rate is often some complex function of temperature, pressure, the reacting components, and/or product of reaction.



# Steady state condition

Frequently, Eq. 10 can be simplified. The most common simplification results when *steady state* or *equilibrium* conditions can be assumed. This implies that there is no accumulation in the system or

$$dM/dt = 0$$

## ***Steady-State with Conservative Systems***

A conservative system implies that there would be no transformation inside the system (no growth, no decomposition, no conversion ). In this case, if steady state condition is assumed then Eq. 9 is reduced to:

$$\text{Input rate} = \text{Output rate}$$

# Steady-State System with Non-conservative Pollutants

Many environmental pollutants undergo chemical, biological, or nuclear reactions at a rate sufficient to require us to treat them as non-conservative substances. Thus an assumption of steady state condition would reduce Eq. 9 to:

$$\text{Input rate} - \text{Output rate} \pm \text{transformation rate} = 0 \quad \text{Eqn.11}$$

If the transformation rate is simply the decomposition or decay of the substance (various pollutants are decomposed in the environment ) then Eq. 11 is further reduced to:

$$\text{Input rate} = \text{Output rate} + \text{decay rate} \quad \text{Eqn.12}$$

# Steady-State System with Non-conservative Pollutants

The decay of non conservative substances is frequently modeled as a first-order reaction ; that is, it is assumed that the rate of loss of the substance is proportional to the amount of substance that is present at any given time. That is,

$$\frac{dC}{dt} = -k C \quad \text{Eqn.13}$$

where k is the reaction rate coefficient with dimension of (1/time), the negative sign implies a loss of substance with time, and C is the pollutant concentration (mass per unit volume). The differential equation may be integrated to yield either

$$\ln \frac{C}{C_o} = -k t \quad \text{Eqn.14}$$

# Steady-State System with Non-conservative Pollutants

$$\ln \frac{C}{C_o} = -k t \quad \text{Eqn.14}$$

$$C = C_o e^{-kt} \quad \text{Eqn.15}$$

indicates the rate of change of concentration of the substance. If we assume the substance is uniformly distributed throughout the volume  $V$ . Thus the total mass ( $M$ ) of the substance is equal to the product of the concentration and volume ( $V$ ). The total rate of decay of the amount of a non-conservative substance is thus

$$\text{Decay rate} = \frac{dM}{dt} = \frac{d(CV)}{dt} = V \frac{(dC)}{dt}$$

Therefore, decay rate =  $k C V$

# Steady-State System with Non-conservative Pollutants

Eq. 12 can be re-written as

$$\text{Input rate} = \text{Output rate} + k C V \quad \text{Eqn.16}$$

Implicit in Eq. 16 is the assumption that the concentration  $C$  is uniform throughout the volume  $V$ .

This complete mixing assumption is common in the analysis of chemical tanks, called *reactors*, and in such cases the idealization is referred to as a *continuously stirred tank reactor (CSTR)* model. In other contexts, such as modeling air pollution, the assumption is referred to as a *complete mix box model*.

# Steady-State System with Non-conservative Pollutants

- Idealized models involving non-conservative pollutants in completely mixed, steady-state systems are used to analyze a variety of commonly encountered water pollution problems such as mixing of sewage in receiving waters.
- The same simple models can be applied to certain problems involving air quality.

# Problem (Water)

A stream, (shown in the following figure) flowing at 150 L/s and 20 mg/L suspended solids, receives wastewater from three separate sources:

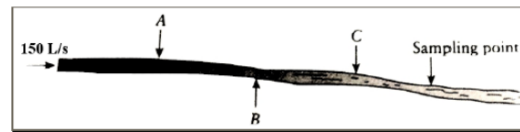
Source	Quantity (L/s)	Solids concentration (mg/L)
A	100	200
B	300	50
C	50	200



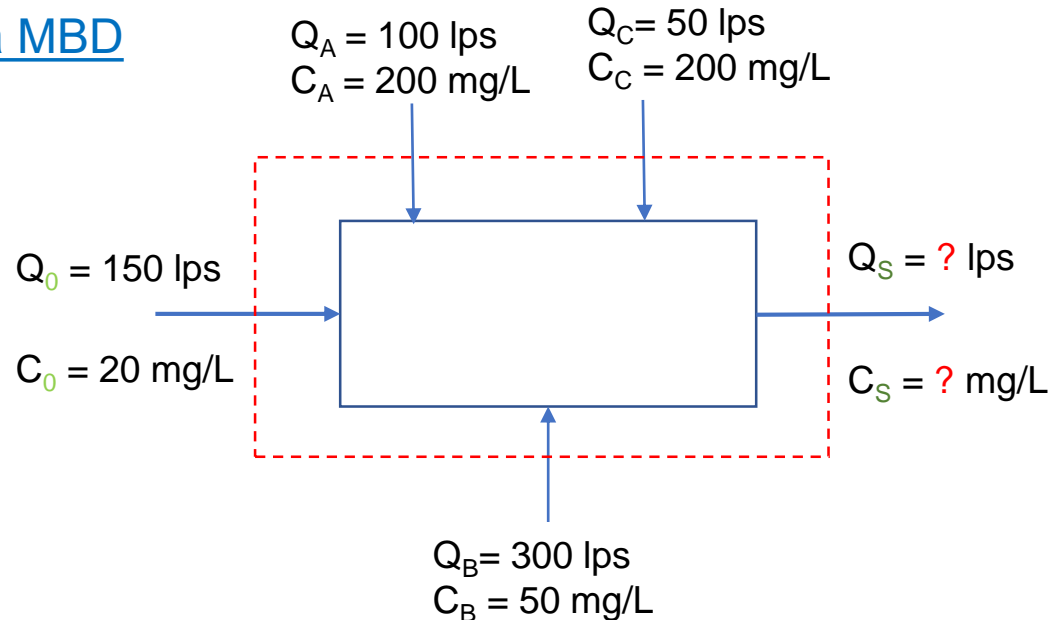
What are the flow rate and suspended solids concentration downstream at the sampling point?

[2+2]

Source	Quantity (L/s)	Solids concentration (mg/L)
A	100	200
B	300	50
C	50	200



In the beginning, let's draw a MBD



Firstly, MBE is:

$$\left[ \begin{array}{c} \text{Rate of SS} \\ \text{ACCUMULATED} \end{array} \right] = \left[ \begin{array}{c} \text{Rate of SS} \\ \text{IN} \end{array} \right] - \left[ \begin{array}{c} \text{Rate of SS} \\ \text{OUT} \end{array} \right] + \left[ \begin{array}{c} \text{Rate of SS} \\ \text{PRODUCED} \end{array} \right] - \left[ \begin{array}{c} \text{Rate of SS} \\ \text{CONSUMED} \end{array} \right]$$

Balancing the volumetric rate:(i.e. Discharge):

$$\left[ \begin{array}{c} \text{Volume rate} \\ \text{of water} \\ \text{ACCUMULATED} \end{array} \right] = \left[ \begin{array}{c} \text{Volume rate} \\ \text{of water} \\ \text{IN} \end{array} \right] - \left[ \begin{array}{c} \text{Volume rate} \\ \text{of water} \\ \text{OUT} \end{array} \right] + \left[ \begin{array}{c} \text{Volume rate} \\ \text{of water} \\ \text{PRODUCED} \end{array} \right] - \left[ \begin{array}{c} \text{Volume rate} \\ \text{of water} \\ \text{CONSUMED} \end{array} \right]$$



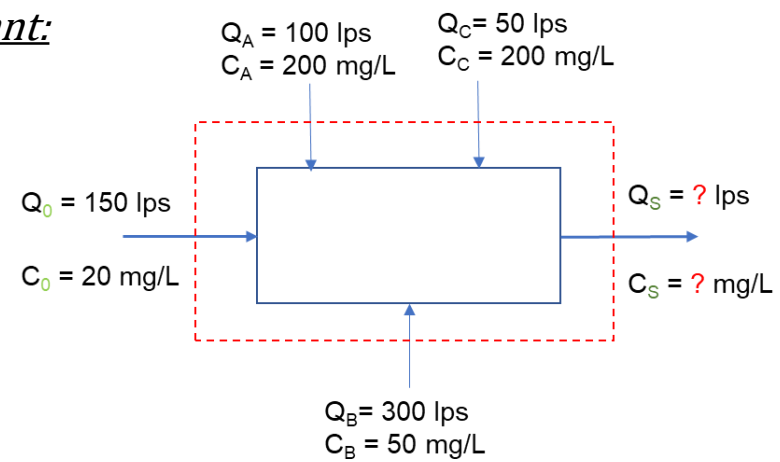
Assuming the steady state condition and conservative pollutant:

$$0 = \left[ \begin{array}{c} \text{Volume of water per} \\ \text{unit time} \\ \text{IN} \end{array} \right] - \left[ \begin{array}{c} \text{Volume of water per} \\ \text{unit time} \\ \text{OUT} \end{array} \right] + 0 - 0$$

$$\text{or, } \left[ \begin{array}{c} \text{Volume of water per} \\ \text{unit time} \\ \text{IN} \end{array} \right] = \left[ \begin{array}{c} \text{Volume of water per} \\ \text{unit time} \\ \text{OUT} \end{array} \right]$$

or,  $Q_0 + Q_A + Q_B + Q_C = Q_S$

or,  $Q_S = 150 + 100 + 300 + 50$   $\therefore Q_S = 600 \text{ lps}$



Balancing the Mass flow rate (i.e. Rate of Suspended Solid (SS)) in the system:

Assuming same as above:

$$0 = \left[ \begin{array}{c} \text{Rate of SS} \\ \text{IN} \end{array} \right] - \left[ \begin{array}{c} \text{Rate of SS} \\ \text{OUT} \end{array} \right] + 0 - 0$$

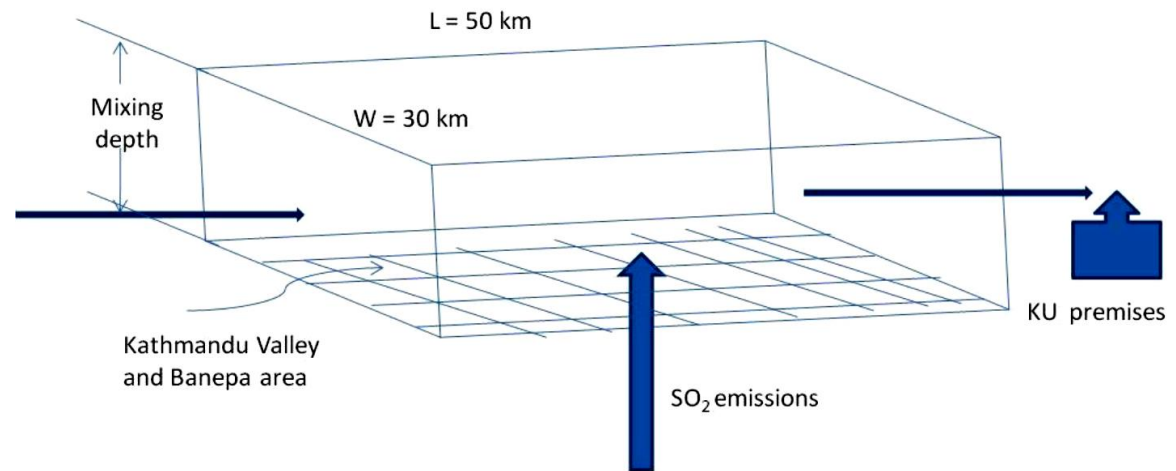
or,  $Q_0 C_0 + Q_A C_A + Q_B C_B + Q_C C_C = Q_S C_S$

or,  $C_S = \frac{Q_0 C_0 + Q_A C_A + Q_B C_B + Q_C C_C}{Q_S} = \frac{(150 \times 20 + 100 \times 200 + 300 \times 50 + 50 \times 200) \frac{L}{s} \times \frac{mg}{L}}{600 \frac{L}{s}}$

$\therefore C_S = 80 \text{ mg/L}$

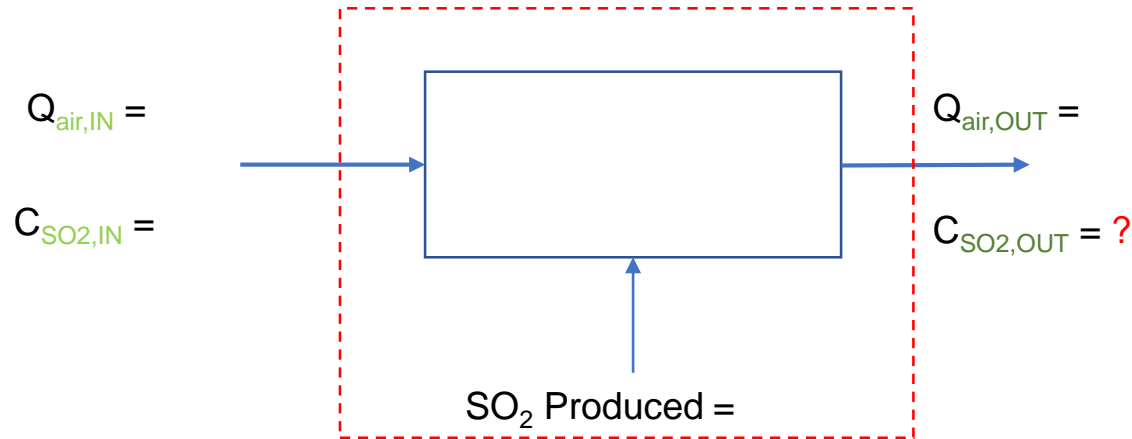
# Problem (Air)

Estimate the concentration of  $\text{SO}_2$  in the urban air above Kathmandu University premises at Dhulikhel. The mixing height above Kathmandu Valley and Banepa urban areas is 1 km. The length and width of the box representing the mixing zone (as shown in Figure) is 50 km by 30 km. The average annual wind speed is 10,000 m/h. There are 200 brick kilns in the area.  $\text{SO}_2$  release rate is 0.2 kg/brick produced and the annual brick production rate is 10 million bricks/year.



**Air quality box model**

## Drawing MBD:

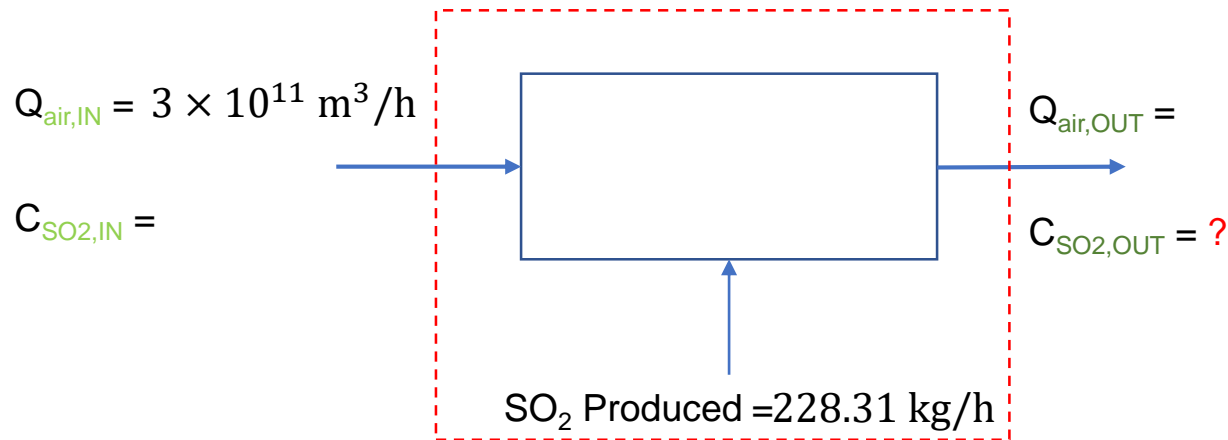


The volume of the air moving into the box per unit time is calculated as the velocity times the area through which the flow occurs.

$$\text{or, } Q_{\text{air,IN}} = A \times v$$

where  $v$  = wind velocity and

$A$  = area of the side of the box (mixing depth times width).

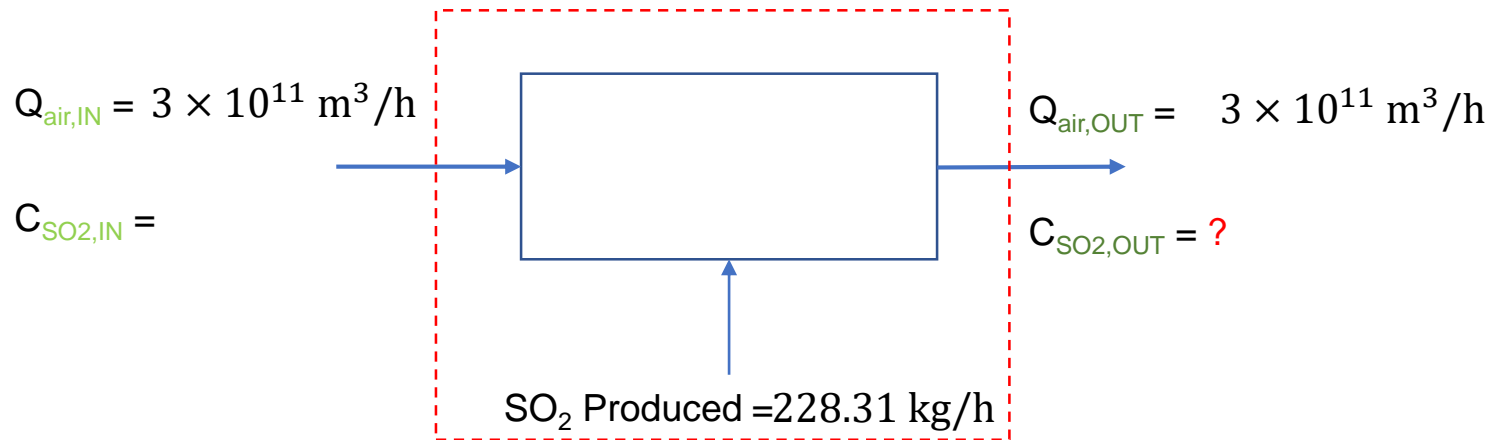


$$\text{or, } Q_{\text{air,IN}} = A \times v$$

$$\text{or, } Q_{\text{air,IN}} = 30 \text{ km} \times 1 \text{ km} \times \frac{1000 \text{ m}}{1 \text{ km}} \times \frac{1000 \text{ m}}{1 \text{ km}} \times 10000 \frac{\text{m}}{\text{h}}$$

$$\text{or, } Q_{\text{air,IN}} = 3 \times 10^{11} \text{ m}^3/\text{h}$$

$$\begin{aligned}
 \text{Average SO}_2 \text{ produced} &= 0.2 \frac{\text{kg}}{\text{brick}} \times 10 \times 10^6 \frac{\text{bricks}}{\text{year}} \times \frac{1 \text{ year}}{365 \text{ days}} \times \frac{1 \text{ day}}{24 \text{ h}} \\
 &= 228.31 \text{ kg/h}
 \end{aligned}$$



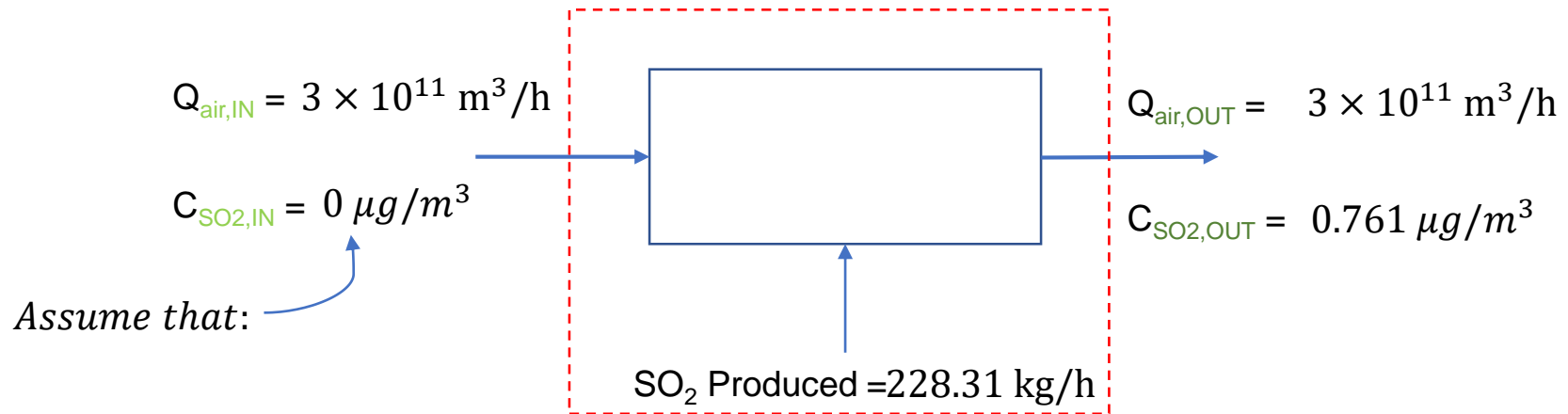
Firstly, writing MBE:

$$\left[ \begin{array}{c} \text{Rate of } \text{SO}_2 \\ \text{ACCUMULATED} \end{array} \right] = \left[ \begin{array}{c} \text{Rate of } \text{SO}_2 \\ \text{IN} \end{array} \right] - \left[ \begin{array}{c} \text{Rate of } \text{SO}_2 \\ \text{OUT} \end{array} \right] + \left[ \begin{array}{c} \text{Rate of } \text{SO}_2 \\ \text{PRODUCED} \end{array} \right] - \left[ \begin{array}{c} \text{Rate of } \text{SO}_2 \\ \text{CONSUMED} \end{array} \right]$$

It is clear that a simple application of the volume balance equation would show:

$$Q_{\text{air,IN}} = Q_{\text{air,OUT}}$$

Thus, the flow of air at the outlet is  $3 \times 10^{10} \text{ m}^3/\text{h}$ .



Now, Balancing the Mass flow rate (i.e. Rate of Sulphur dioxide( $\text{SO}_2$ )) in the system:  
Assuming the steady state condition and conservative pollutant:

$$0 = \left[ \begin{matrix} \text{Rate of } \text{SO}_2 \\ IN \end{matrix} \right] - \left[ \begin{matrix} \text{Rate of } \text{SO}_2 \\ OUT \end{matrix} \right] + \left[ \begin{matrix} \text{Rate of } \text{SO}_2 \\ PRODUCED \end{matrix} \right] - 0$$

$$0 = Q_{air,IN} C_{SO2,IN} - Q_{air,OUT} C_{SO2,OUT} + 228.31 \text{ kg}/\text{h}$$

$$\therefore C_{SO2,OUT} = \frac{228.31 \text{ kg}/\text{h}}{Q_{air,OUT}} = \frac{228.31 \text{ kg}/\text{h}}{3 \times 10^{11} \text{ m}^3/\text{h}} = \frac{228.31 \text{ kg}}{3 \times 10^{11} \text{ m}^3} \times \frac{10^9 \mu\text{g}}{1 \text{ kg}} = 0.761 \frac{\mu\text{g}}{\text{m}^3}$$

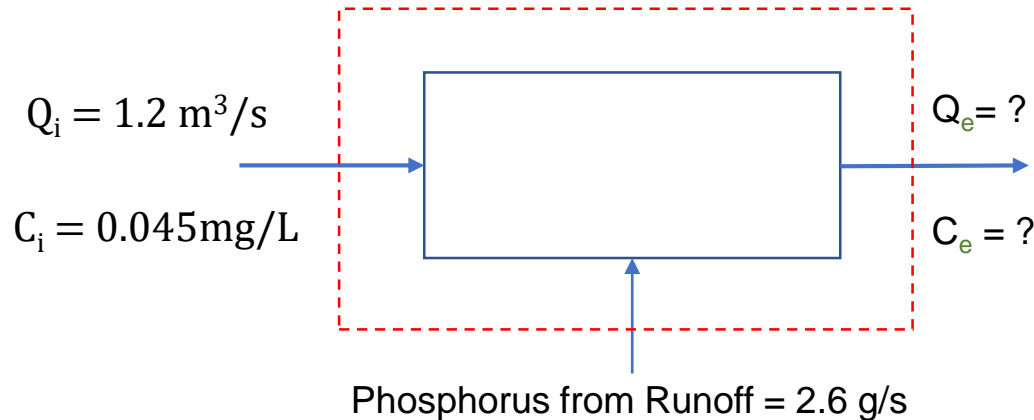
# Problem (Water)

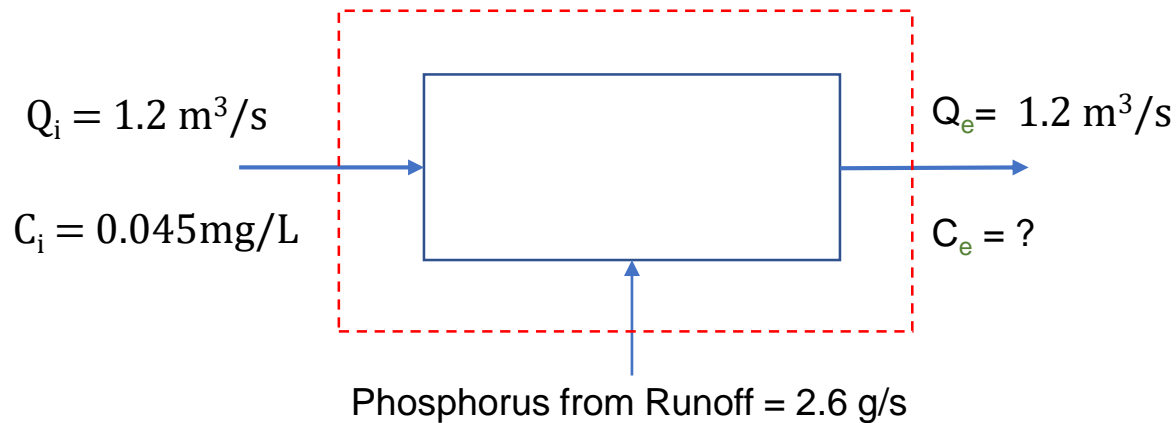
A lake has a surface area of  $2.6 \times 10^6 \text{ m}^2$ . The average depth is 12 m. The lake is fed by a stream having flow rate of  $1.2 \text{ m}^3/\text{s}$  and phosphorus concentration of  $0.045 \text{ mg/L}$ . Runoff from the homes along the lake adds phosphorus at an average annual rate of  $2.6 \text{ g/s}$ . The degradation rate of the lake is  $0.36 \text{ day}^{-1}$ . A river flow from the lake at a flowrate of  $1.2 \text{ m}^3/\text{s}$ . What is the steady state concentration of phosphorus in the lake?

## Solution:

Volume of water in the lake ( $V$ ) =  $2.6 \times 10^6 \text{ m}^2 \times 12 \text{ m} = 3.12 \times 10^7 \text{ m}^3$

## Let us draw MBD:





Firstly, writing MBE:

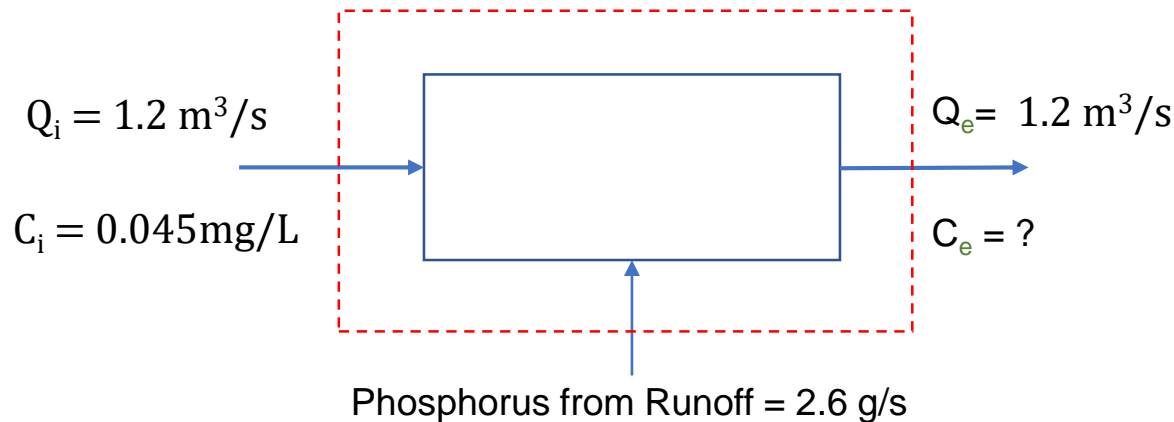
$$\left[ \begin{array}{c} \text{Rate of } P \\ \text{ACCUMULATED} \end{array} \right] = \left[ \begin{array}{c} \text{Rate of } P \\ \text{IN} \end{array} \right] - \left[ \begin{array}{c} \text{Rate of } P \\ \text{OUT} \end{array} \right] + \left[ \begin{array}{c} \text{Rate of } P \\ \text{PRODUCED} \end{array} \right] - \left[ \begin{array}{c} \text{Rate of } P \\ \text{CONSUMED} \end{array} \right]$$

It is clear that a simple application of the volume balance equation would show:

$$Q_i = Q_e$$

Thus, the flow of stream leaving the lake is also 1.2 m³/s.





Now, Balancing the Mass flow rate (i.e. Rate of Phosphorus) in the system:  
Assuming the steady state condition and no production of Phosphorus in the system.

$$\text{or, } 0 = \left[ \begin{array}{c} \text{Rate of P} \\ \text{IN} \end{array} \right] - \left[ \begin{array}{c} \text{Rate of P} \\ \text{OUT} \end{array} \right] + 0 - \left[ \begin{array}{c} \text{Rate of P} \\ \text{CONSUMED} \end{array} \right]$$

$$\text{or, } 0 = Q_i C_i + 2.6 \frac{\text{g}}{\text{s}} - Q_e C_e - kCV \quad \text{Where, } k = \text{degradation rate} = 0.36 \text{ day}^{-1} \text{ (given)}$$

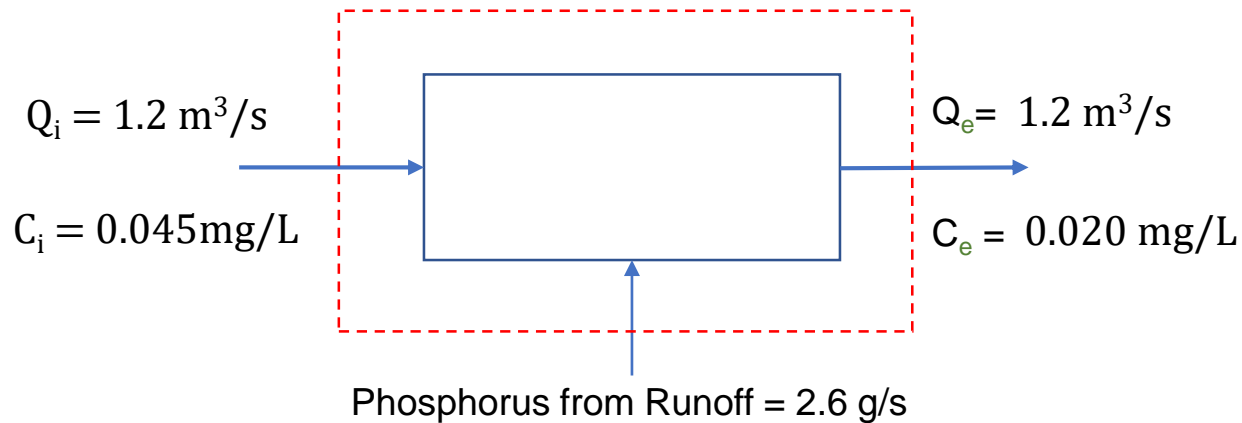
$C = \text{concentration of Phosphorus in the Lake}$   
 $V = \text{volume of the Lake}$

Assume that the Lake is completely mixed.

In completely stirred tank reactor (CSTR): Concentration of pollutant in the reactor is equal to concentration of pollutant in the effluent. (i.e.  $C = C_e$ )

Then,

$$\text{or, } 1.2 \frac{\text{m}^3}{\text{s}} \times 0.045 \frac{\text{mg}}{\text{L}} + 2.6 \frac{\text{g}}{\text{s}} = \left[ 1.2 \frac{\text{m}^3}{\text{s}} \times C_e \right] + [0.36 \text{ day}^{-1} \times C_e \times 3.12 \times 10^7 \text{ m}^3]$$



$$\text{or, } 1.2 \frac{\text{m}^3}{\text{s}} \times 0.045 \frac{\text{mg}}{\text{L}} + 2.6 \frac{\text{g}}{\text{s}} = \left[ 1.2 \frac{\text{m}^3}{\text{s}} \times C_e \right] + \left[ 0.36 \text{ day}^{-1} \times C_e \times 3.12 \times 10^7 \text{ m}^3 \right]$$

$$\text{or, } 1.2 \frac{\text{m}^3}{\text{s}} \times 0.045 \frac{\text{mg}}{\text{L}} \times \frac{10^3 \text{ L}}{1 \text{ m}^3} + 2.6 \frac{\text{g}}{\text{s}} \times \frac{10^3 \text{ mg}}{1 \text{ g}} = C_e \left[ 1.2 \frac{\text{m}^3}{\text{s}} + \frac{0.36}{\text{day}} \times \frac{1 \text{ day}}{86400 \text{ s}} \times 3.12 \times 10^7 \text{ m}^3 \right]$$

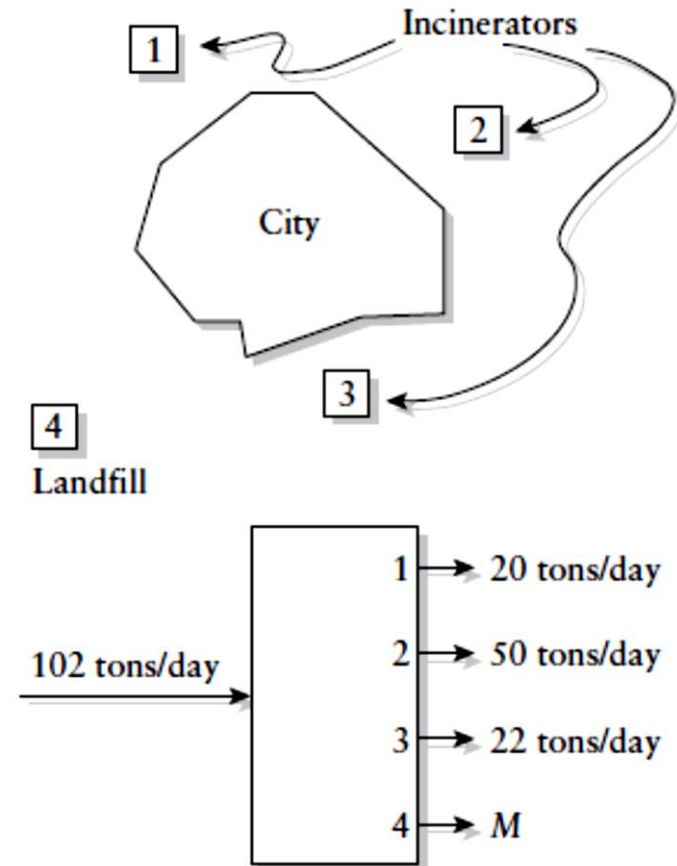
$$\text{or, } 54 \frac{\text{mg}}{\text{s}} + 2600 \frac{\text{mg}}{\text{s}} = C_e \left[ 1.2 \frac{\text{m}^3}{\text{s}} + 130 \frac{\text{m}^3}{\text{s}} \right]$$

$$\text{or, } C_e = \frac{2654 \frac{\text{mg}}{\text{s}}}{131.2 \frac{\text{m}^3}{\text{s}} \times \frac{10^3 \text{ L}}{1 \text{ m}^3}}$$

$$\therefore C_e = 0.020 \text{ mg/L}$$

## Problem (Solid waste)

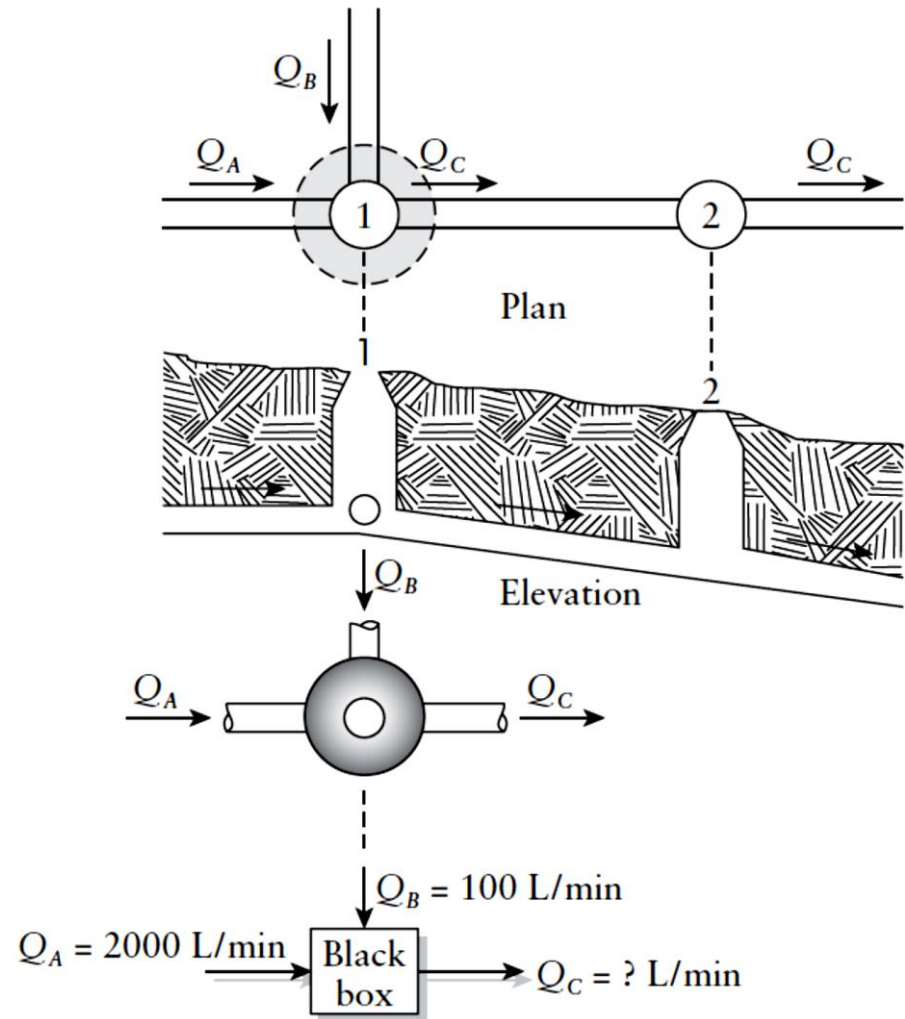
A city generates 102 tons/day of refuse, all of which goes to a transfer station. At the transfer station the refuse is split into four flow streams headed for three incinerators and one landfill. If the capacity of the incinerators is 20, 50, and 22 tons/day, how much refuse must go to the landfill?



**Answer :10 tons/day**

## Problem (Wastewater)

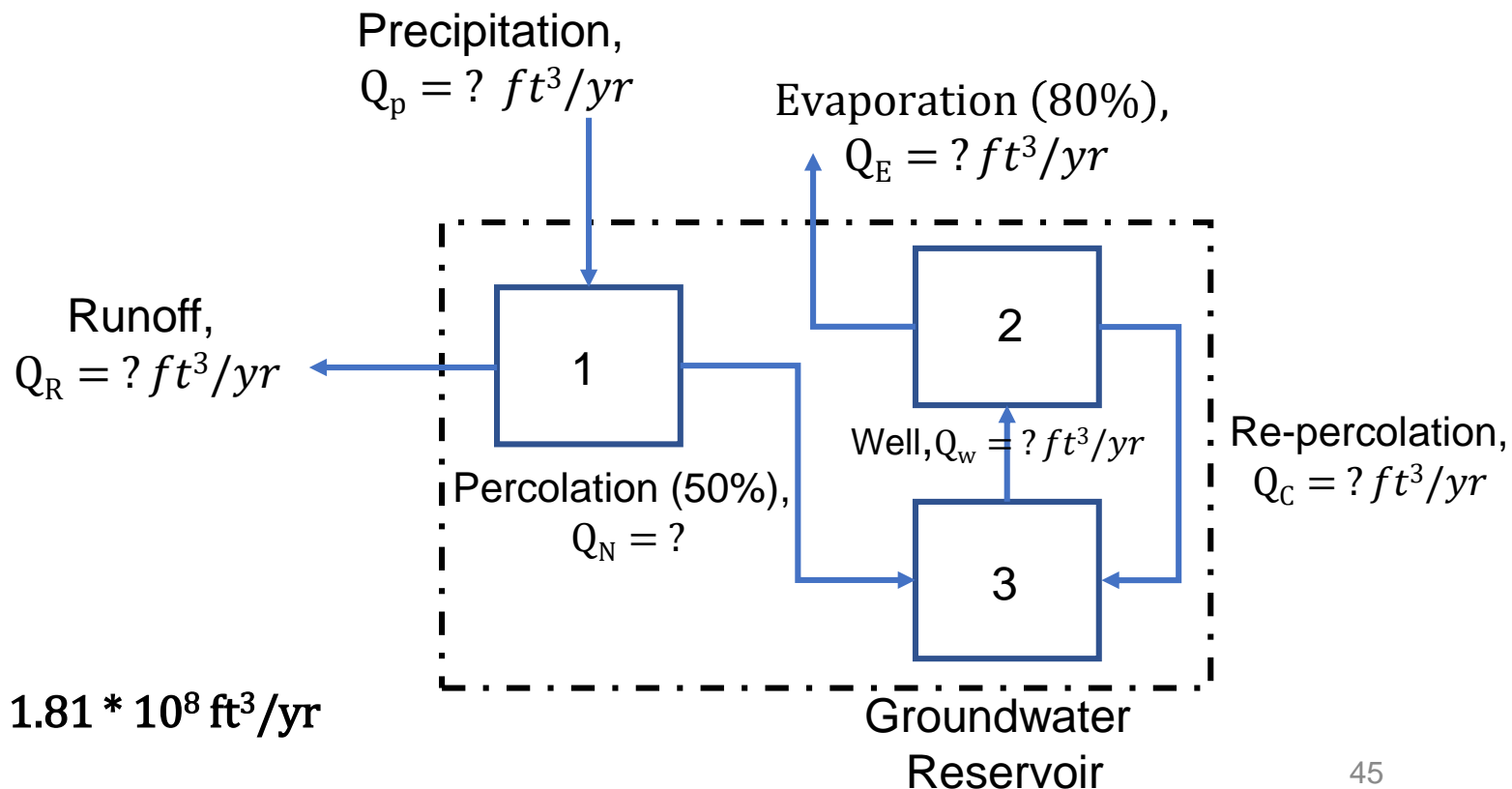
A sewer carrying stormwater to Manhole 1 (Figure) has a constant flow of 2000 L/min ( $Q_A$ ). At Manhole 1 it receives a constant lateral flow of 100 L/min ( $Q_B$ ). What is the flow to Manhole 2 ( $Q_C$ )?



**Answer: 2100 L/min**

# Problem

Suppose the rainfall is 40 in./yr, of which 50% percolates into the ground. The farmer irrigates crops using well water. Of the extracted well water, 80% is lost by evapotranspiration; the remainder percolates back into the ground. How much groundwater could a farmer on a 2000 acres farm extract from the ground per year without depleting the groundwater reservoir volume?



**Answer:**  $Q_w = 1.81 \times 10^8 \text{ ft}^3/\text{yr}$

# References

- Bailey, J. E. and Ollis, D. F. (1986). *Biochemical Engineering Fundamentals*, McGraw-Hill International, New York
- Vesilind, P. A., Morgan, S. M., & Heine, L. G. (2010). *Introduction to environmental engineering-SI version*. Cengage Learning.