Lecture 18

Modern Physics:- Nuclear structure

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Outline

Nuclear binding energy

We know the nucleus consists of protons and neutrons. If Z and N respectively denote the number of protons and neutrons, then mass of nucleus should be $Zm_p + Nm_n$, where m_p is mass of a proton and m_n is mass of a neutron. However, measurements showed that the real mass of nucleus is less than the above value. Let the measured mass of nucleus be M then difference in mass is

$$\Delta m = (Zm_p + Nm_n) - M$$

here Δm is called the mass defect.

When Z protons and N neutrons combine to make a nucleus, some of the mass (Δm) disappears because it is converted into amount of

Nuclear binding energy (contd.)

energy $\Delta E = (\Delta m)c^2$ according to mass-energy equivalence relation. This energy is called the binding energy B. E. of nucleus. Thus expression for B. E. becomes

$$\text{B.E.} = [(Zm_p + Nm_n) - M]c^2$$

To breakup the stable nucleus in to its constituent, the minimum energy required is the binding energy. The magnitude of B. E. determines its stability against disintegration. If B.E.> 0, nucleus is stable and if B.E. < 0, the nucleus is unstable and it will disintegrate by itself. The ratio of binding energy to the total number of nucleons is called the binding energy per nucleon and it also gives the stability of atoms.

Radioactivity and half-life

The phenomenon of spontaneous emission of highly penetrating radiations from heavy elements is called natural radioactivity and such elements are called radioactive elements. The units of radioactivity are Curie (Ci), Rutherford (Rd) and Becquerel (Bq). 1 Ci is equivalent to 3.7×10^{10} disintegration per second; 1 Rd is equivalent to 10^6 disintegrations per second and 1Bq is equivalent to 1 event/sec.

For any radioactive sample, let N be the number of atoms present at time t, then it is found that the rate of decrease of atoms is proportional to N i.e.

$$\frac{dN}{dt} \propto N \implies \frac{dN}{dt} = -\lambda N$$

Radioactivity and half-life (contd.)

Upon integrating, we get

$$\ln N = -\lambda t + \ln N_0 \implies \ln\left(\frac{N}{N_0}\right) = -\lambda t$$

$$\implies N = N_0 e^{-\lambda t} \tag{1}$$

Equation (1) shows that the number of atoms in radioactive sample decreases exponentailly with time. The graph of number of atoms present and time is as shown in Figure 1

Radioactivity and half-life (contd.)

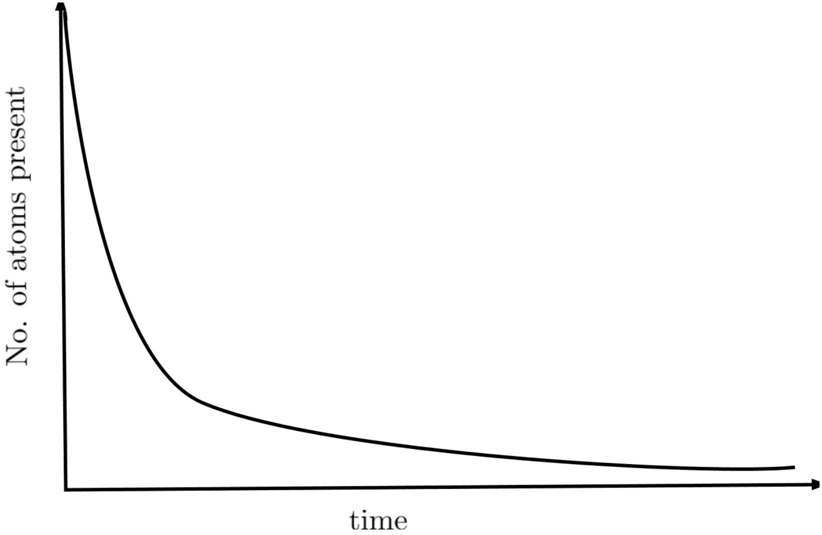

Figure 1

Radioactivity and half-life (contd.)

The half-life period of radioactive substance is defined as the time required for one half of the radioactive substance to disintegrate. It is used to compare one radioactive substance with another and is denoted by $T_{1/2}$. As we know, $N = N_0 e^{-\lambda t}$, at $t = T_{1/2}$, $N = N_0/2$. So,

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}} \implies T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

This relation gives the half live of a given radioactive sample

Nuclear reactions

Nuclear reactions are transformations in atomic nuclei brought about by their interactions with elementary particles or with one another. The general scheme of nuclear reaction is

$$A + B \rightarrow C \rightarrow P + Q$$

The projectile B strikes the target nucleus A and combines with it to form the compound nucleus C. The compound nucleus then splits in to an out going particle Q and residual product nucleus P . The same projectile may form different product elements out of the same nucleus. The bombarding agents may be neutral or charged particles. Following are some main types of nuclear reactions.

Nuclear reactions (contd.)

1. *Elastic scattering:-* In this case incident particle strikes the target nucleus and leaves without loss of energy but its direction may change e.g.

$$_{79}Au^{179} + {_2He^4} \rightarrow {_2He^4} + {_{79}Au^{179}}$$

2. *Inelastic scattering:-* In this case the incident particle loses some of its energy and excites the target nucleus. The excited nucleus later decays radiating a γ-ray photon.

$$_3Li^7 + {_1H^1} \rightarrow \left({_3Li^7}\right)^* + {_1H^1}$$

$$\left({_3Li^7}\right)^* \rightarrow {_3Li^7} + \gamma$$

Nuclear reactions (contd.)

3. *Radioactive capture:-* In this case the incident particle is captured by the target nucleus. The new nucleus then decays with the emission of one or more -rays photon e.g.

$$_{6}C^{12} + {}_{1}H^{1} \rightarrow \left({}_{7}N^{13}\right)^{*} \rightarrow {}_{7}N^{13} + \gamma$$

4. *Disintegration:-* The incident particle is absorbed by the target nucleus and the ejected particle is a different one. The composition of product nucleus is different from parent nucleus.

$$_{4}Be^{9} + {}_{2}He^{4} \rightarrow {}_{6}C^{12} + {}_{0}n^{1}$$

Nuclear reactions (contd.)

5. *Photodisintegration:-* When target materials are bombarded with radiations, the compound nucleus decays with formation of neutrons.

$$_1H^2 + \gamma \rightarrow \, _1H^1 + \, _0n^1$$

Q-value and threshold enrgy in nuclear reaction

The general scheme of nuclear is

$$x + X \to Y + y$$

where x is projectile, X is target nucleus, Y is product nucleus and y is product particle. The above equation can also be written as $X(x,y)Y$. Suppose, m_x , M_X , M_Y , m_y denote the masses of x, X, Y and y and E_x , E_Y , E_y be their respective energies, then for target at rest, from energy conservation,

$$E_x + m_x c^2 + M_X c^2 = E_Y + M_Y c^2 + E_y + m_y c^2$$

$$\implies E_Y + E_y - E_x = (M_X + m_x - M_Y - m_y)c^2$$

Q-value and threshold enrgy in nuclear reaction (contd.)

The left hand side of this equation i.e. the difference in energy is called Q-value of reaction given by

$$Q = E_Y + E_y - E_x = (M_X + m_x - M_Y - m_y)c^2$$

If Q is positive the reaction is called exothermic or exoergic and if it is negative then the reaction is called endothermic or endoergic. The term E_Y represents the recoil (kinetic) energy of the product nucleus. It is very small and hard to measure. Consider the product nucleus and particle scattered at angles ϕ and θ as shown Figure 2

Q-value and threshold enrgy in nuclear reaction (contd.)

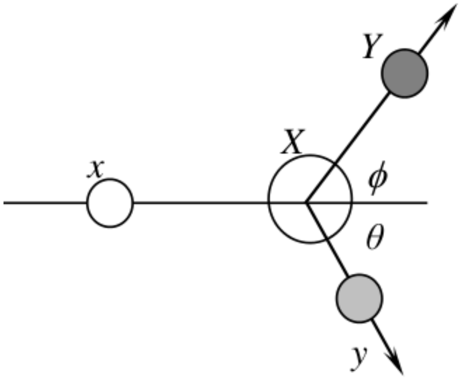

Figure 2: Collision of target and projectile nuclii

Then from momentum conservation,

$$M_Y V_Y \sin\phi = m_y v_y \sin\theta \tag{2}$$

$$m_x v_x = M_Y V_Y \cos\phi + m_y v_y \cos\theta$$

$$M_Y V_Y \cos\phi = m_x v_x - m_y v_y \cos\theta \tag{3}$$

Squaring and adding Equations (2) and (3)

$$(M_Y V_Y)^2 = (m_y v_y)^2 + (m_x v_x)^2 - 2 m_x m_y v_x v_y \cos\theta \tag{4}$$

Setting $E_x = \frac{1}{2} m_x v_x^2$, $E_y = \frac{1}{2} m_y v_y^2$, $E_Y = M_Y V_Y^2$, we get

$$E_Y = \frac{m_x}{M_Y} E_x + \frac{m_y}{M_Y} E_y - \frac{2}{M_Y} \sqrt{E_x E_y m_x m_y \cos\theta} \tag{5}$$

Q-value and threshold enrgy in nuclear reaction (contd.)

So, the Q-value is

$$Q = \frac{m_x}{M_Y}E_x + \frac{m_y}{M_Y}E_y - \frac{2}{M_Y}\sqrt{E_x E_y m_x m_y \cos\theta} + E_y - E_x$$
$$\implies Q = \left(1 + \frac{m_y}{M_Y}\right)E_y - \left(1 - \frac{m_x}{M_Y}\right)E_x - \frac{2}{M_Y}\sqrt{E_x E_y m_x m_y \cos\theta} \tag{6}$$

For $\theta = 90^\circ$

$$Q = \left(1 + \frac{m_y}{M_Y}\right)E_y - \left(1 - \frac{m_x}{M_Y}\right)E_x \tag{7}$$

If masses are not known accurately, the mass numbers may be used in equations to obtain the value of Q.

Q-value and threshold enrgy in nuclear reaction (contd.)

In an endoergic reaction, the energy Q is needed to excite the compound nucleus sufficiently so that it will break up. This energy must be supplied in the form of kinetic energy of the incoming particle. The amount of energy needed for an endoergic reaction is called the threshold energy.

Let M_c and V_c denote the mass and velocity of the compound nucleus, then from conservation of momentum, $m_x v_x = M_c V_c$ or $V_c = \dfrac{m_x v_x}{M_c}$.

The part of kinetic energy of the incident particle needed for excitation of the compound nucleus is

$$-Q = \frac{1}{2}m_x v_x^2 - \frac{1}{2}M_c V_c^2 = \frac{1}{2}m_x v_x^2 \left(1 - \frac{m_x}{M_c}\right)$$

Q-value and threshold enrgy in nuclear reaction (contd.)

But, $M_c = M_X + m_x$, therefore

$$(-Q) = \frac{1}{2} m_x v_x^2 \left(\frac{M_X}{M_X + m_x} \right)$$

The threshold energy is then

$$E_{\text{th}} = \frac{1}{2} m_x v_x^2 = (-Q) \left(1 + \frac{m_x}{M_X} \right)$$

This is the expression for threshold energy and for the reaction induced by γ-rays, $m_x = 0$, $E\text{th} = -Q$

Nuclear fission and fusion

The process of breaking up the nucleus of a heavy atom into two, more or less equal fragments with the release of large amount of energy is known as nuclear fission. When Uranium ${}_{92}\mathrm{U}^{235}$ is bombarded with slow neutrons, the uranium nucleus breaks up into two nuclei ${}_{56}\mathrm{Ba}^{141}$ and ${}_{36}\mathrm{Kr}^{92}$ long with three neutrons and a large amount of energy as shown by the reactiona below

$$ {}_{92}\mathrm{U}^{235} + {}_{0}\mathrm{n}^{1} \rightarrow \left({}_{92}\mathrm{U}^{235}\right)^{*} \rightarrow {}_{56}\mathrm{Ba}^{141} + {}_{36}\mathrm{Kr}^{92} + 3\,{}_{0}\mathrm{n}^{1} + Q $$

here Q is the energy produced in the reaction. The value of Q is about 200 MeV for this reaction.

Nuclear fission and fusion (contd.)

The process by which a heavy nucleus is formed by the combination of two or more light nuclei with the release of large amount of energy is called nuclear fusion. For example, a helium nucleus is formed by the fusion of two deuterium nuclei as given by the reaction

$$ {}_{1}\mathrm{H}^{2} + {}_{1}\mathrm{H}^{2} \rightarrow {}_{2}\mathrm{He}^{4} + Q $$

here Q is amount of energy liberated in the process and for this reaction, the value of Q is about 24 MeV. Fusion is the main source of Sun's energy.

Problems

1. Calculate the binding energy per nucleon (MeV/nucleon) for tritium, ($_1H^3$) a radioactive isotope of hydrogen.

   Assume: $m_p = 1.007825$ u, $m_n = 1.008665$ u, $m_t = 3.01605$ u, 1u = 1.66×10^{-27} kg.

   [Ans: $\dfrac{E_B}{A} = 2.8$ **MeV/nucleon]**

2. How much energy (in MeV) is released when a $_{92}U^{235}$ fissions to $_{56}Ba^{141}$ and $_{36}Kr^{92}$ in the reaction

   $\mathrm{n} + {}_{92}\mathrm{U}^{235} \rightarrow {}_{56}\mathrm{Ba}^{141} + {}_{36}\mathrm{Kr}^{92} + 3\mathrm{n}$?

   Given: $m(\mathrm{n}) = 1.008665$ u, $m(\mathrm{U}) = 235.043915$ u, $m(\mathrm{Ba}) = 140.9139$ u, $m(\mathrm{Kr}) = 91.8973$ u, 1u = 1.66×10^{-27} kg.

   [Ans: $Q\,(\textbf{MeV}) = 200$ **MeV]**

Problems (contd.)

3. How long does it take for 60% of a sample of radon to decay? Half life of radon is 3.8 days. **[Ans:** $t = 5.02$ **days]**

4. The half life of radon is 3.8 days. After how many days will only 1/20 of a radon sample be left over? **[Ans:** $t = 16.43$ **days]**

5. A superconducting tin has a critical temperature of 3.7 K in zero magnetic field and a critical field of 0.0306 T at 0 K. Find the critical field at 2 K. **[Ans:** $H_C\left(2\mathbf{K}\right) = 0.0217\mathbf{T}$**]**

End of Lecture 18

Thank you