ASSIGNMENT-II (2023)
MATH 104

1. Find the function's domain, range, level curve, boundary of the function's domain, determine if the domain is open or closed and decide if the domain is bounded or unbounded if the function is defined by the equation $f(x, y) = \ln(x^2 + y^2)$.

2. Evaluate the following limits (if exist)

   a. $\lim\limits_{(x,y)\to(0,0)} \dfrac{x - y + 2\sqrt{x} - 2\sqrt{y}}{\sqrt{x} - \sqrt{y}}$

   b. $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$

   c. $\lim\limits_{(x,y)\to(\pi/2,0)} \dfrac{\cos y + 2}{y - \sin x}$

   d. $\lim\limits_{P_0\to(\pi,0,3)} ze^{-2y}\cos 2x$

   e. $\lim\limits_{(x,y)\to(0,0)} \dfrac{2xy^2}{x^2 + y^4}$ along the curve $y^2 = x$.

3. Show that limits of the given functions do not exist at origin.

   a. $f(x, y) = \dfrac{x}{\sqrt{x^2 + y^2}}$ b. $f(x, y) = \dfrac{x^4 - y^2}{x^4 + y^2}$.

4. Prove that the function defined by $f(x, y) = \begin{cases} \dfrac{x^2}{x^2 + y^2} & \text{for } (x, y) \neq (0, 0) \\ 0 & \text{for } (x, y) = (0, 0) \end{cases}$

   is discontinuous at (0, 0).

5. Define $f(0, 0)$ in a way that extends $f(x, y) = xy\dfrac{x^2 - y^2}{x^2 + y^2}$ to be continuous at the origin.

6. Define first partial derivative of the function $f(x, y)$ with respect to x and y and give its geometrical meaning.

7. Calculate the first order partial derivatives f_x , f_y and f_z of the following of the following functions.

   a. $f(x, y) = (x^2 - 1)(y + 2)$

   b. $f(x, y) = \dfrac{x + y}{xy - 1}$

   c. $f(x, y) = e^{x+y+z}$

   d. $f(x, y) = \ln(x + y)$

   e. $f(x, y, z) = \sin^{-1}(xyz)$

   f. $f(x, y, z) = yz\ln(xy)$

8. Calculate the second order partial derivatives:

   a. $f(x, y) = \sin(xy)$ b. $h(x, y) = xe^y + y + 1$ c. $r(x, y) = \ln(x + y)$.

9. Use the limit definition of derivative to compute the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ of the following functions at the specified points.
   a. $f(x,y) = 1 - x + y - 3x^2y$ at (1,2) b. $f(x,y) = 4 + 2x - 3y - xy^2$ at (-2,1).

10. Find $\frac{dy}{dx}$ if $x^2 + \sin y - 2y = 0$ and find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if $z^3 - xy + yz + y^3 - 2 = 0$.

11. Draw a tree diagram and write a chain rule formula for $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ for $w = f(x,y,z)$, $x = g(r,s), y = h(r,s)$ and $z = k(r,s)$.

12. Evaluate $\frac{dw}{dt}$ at the given value of t.
   a. $w = x^2 + y^2, x = \cos t + \sin t, y = \cos t - \sin t$ at $t = 0$.
   b. $w = \ln(x^2 + y^2 + z^2), x = \cos t, y = \sin t, z = 4t$ at $t = 3$.

13. Define directional derivative of a function $f(x,y)$ at the point $P_0(x_0,y_0)$ in the direction of the unit vector $\hat{u}$. What is the difference between the partial derivatives and directional derivative of a function $f(x,y)$ at the point $P_0(x_0,y_0)$.

14. Find the gradient of the given functions at the given points.
   a. $f(x,y) = \ln(x^2 + y^2)$ at $(1,1)$.
   b. $f(x,y,z) = e^x + y\cos z + (y+1)\sin^{-1} x$ at $(0,0,\pi/6)$.

15. Find the derivative of the following functions at the point P_0 in the direction of the vector $\vec{A}$.
   a. $f(x,y) = 2xy - 3y^2$ at $P_0(5,5), \vec{A} = 4\vec{i} + 3\vec{j}$.
   b. $f(x,y,z) = xe^y + yz$ at $P_0(2,0,0), \vec{A} = \vec{i} + 2\vec{j}$.

16. Find the directions in which the functions increase and decrease most rapidly at P_0. Then find the derivatives of the functions in these directions.
   a. $f(x,y) = x^2 + xy + y^2, P_0(-1,1)$ b. $h(x,y,z) = \ln xy + \ln yz + \ln xz, P_0(1,1,1)$.

17. Find the equations for the (a) tangent plane and (b) normal line at the point P_0 on the given surfaces.
   a. $x^2 + y^2 + z^2 = 3, P_0(1,1,1)$. b. $x^2 - xy - y^2 - z = 0, P_0(1,1,-1)$.

** The End **