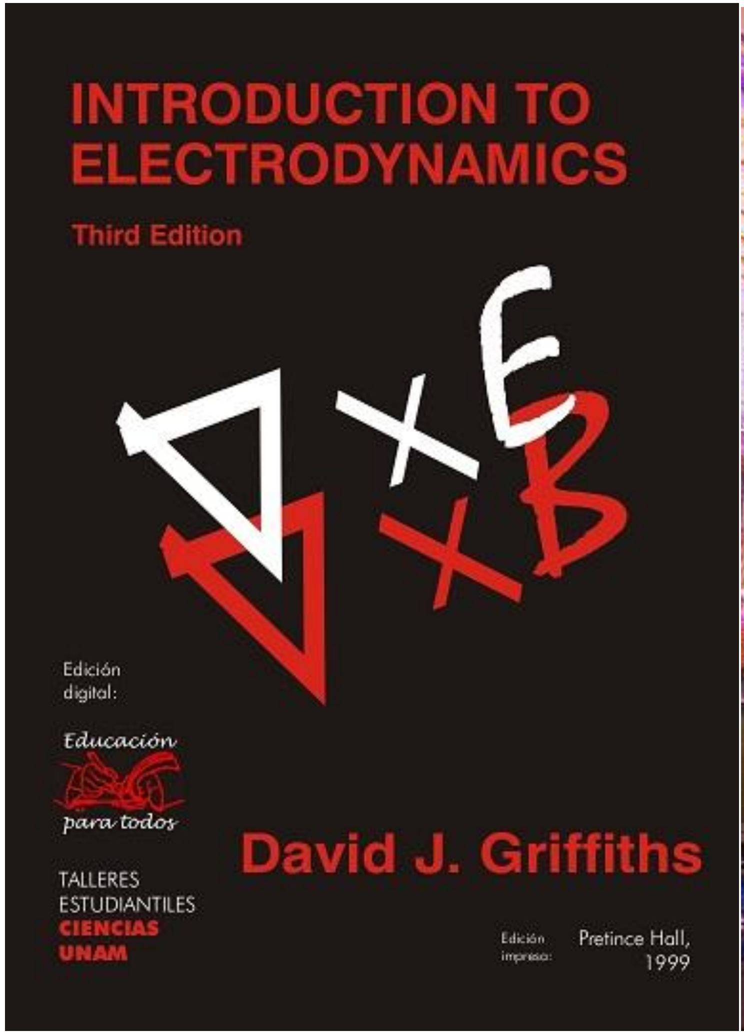
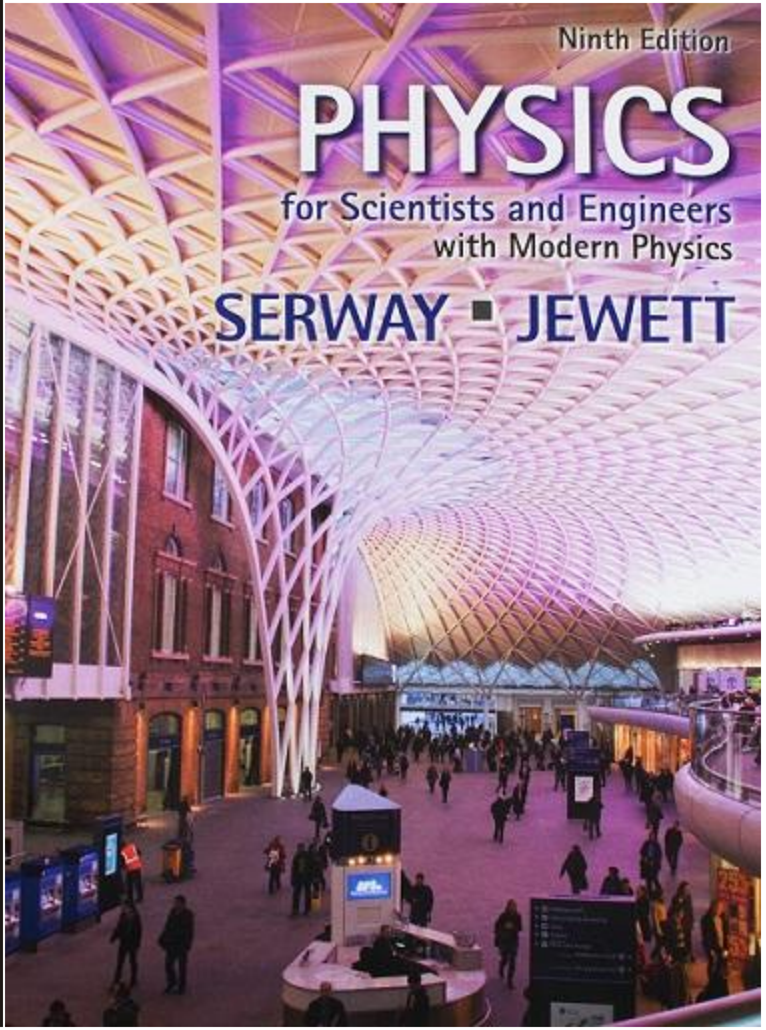
PHYSICS

General Physics II (PHYS 102)





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Course Outline

ELECTROMAGNET IC WAVE PROPAGATION

- Electrodynamics Before Maxwell
- How Maxwell Fixed Ampere's Law
- Maxwell's Equations
- Maxwell's Equations in Matter
- Poynting's Theorem
- Electromagnetic Waves in Vacuum

ELECTROMAGNET IC WAVE PROPAGATION

Electrodynamics Before Maxwell

- Differential form of basic laws of electricity and magnetism are as follows:

$$\begin{array}{lll}
\text{(i)} & \nabla\cdot\vec{E}=\dfrac{\rho}{\varepsilon_0} & (\text{Gauss's law}) \\
\text{(ii)} & \nabla\cdot\vec{B}=0 & (\text{no name}) \\
\text{(iii)} & \nabla\times\vec{E}=-\dfrac{\partial\vec{B}}{\partial t} & (\text{Faraday's law}) \\
\text{(iv)} & \nabla\times\vec{B}=\mu_0\vec{J} & (\text{Ampere's law})
\end{array}$$

These equations represent the state of electromagnetic theory over century ago, when Maxwell began his work

If you apply the divergence to number (iii), everything works out:

$$\nabla\cdot(\nabla\times\vec{E})=\nabla\cdot\left(-\frac{\partial\vec{B}}{\partial t}\right)=-\frac{\partial}{\partial t}(\nabla\cdot\vec{B})$$

The left side is zero because divergence of curl is zero; the right side is zero by virtue of equation (ii).

If you apply the divergence to number (iv), you get into trouble:

$$\nabla\cdot(\nabla\times\vec{B})=\mu_0(\nabla\cdot\vec{J})$$

The left side must be zero, but the right side, in general, is *not*. *For steady currents, the* divergence of $\vec{J}$ is zero, but evidently when we go beyond magnetostatics Ampere's law cannot be right.

- Ampere's Law:

$$\oint \mathrm{B}\cdot d\vec{l}=\mu_0\, I_{\mathrm{enc}}$$

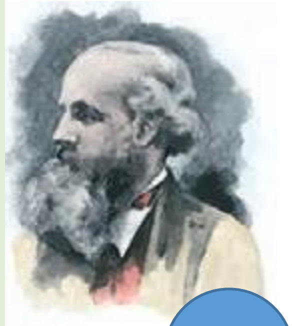
In this equation, the line integral is over any closed path through which conduction current $\left(I=\frac{dq}{dt}\right)$ passes.

- Ampère's law in this form is valid only if any electric fields present are constant in time.
- **James Clerk Maxwell** recognized this limitation and modified Ampère's law to include time-varying electric fields.

James Clerk Maxwell

Scottish physicist (1831 – 1879)

- Provided a mathematical theory that showed a close relationship between all electric and magnetic phenomena
- His equations predict the existence of electromagnetic waves that propagate through space
- Also developed and explained
  - Kinetic theory of gases
  - Color vision



How Maxwell Fixed Ampere's Law

How Maxwell Fixed Ampere's Law

- The differential form of basic laws of electricity and magnetism are as follows:

$$\begin{array}{lll} \text{(i)} & \nabla\cdot\vec{E}=\dfrac{\rho}{\varepsilon_0} & \text{(Gauss's law)} \\ \text{(ii)} & \nabla\cdot\vec{B}=0 & \text{(no name)} \\ \text{(iii)} & \nabla\times\vec{E}=-\dfrac{\partial\vec{B}}{\partial t} & \text{(Faraday's law)} \\ \text{(iv)} & \nabla\times\vec{B}=\mu_0\vec{J} & \text{(Ampere's law)} \end{array}$$

Let us take the divergence of Eq. (iii), i.e.

$$\nabla\cdot(\nabla\times\vec{E})=\nabla\cdot\left(-\frac{\partial\vec{B}}{\partial t}\right)=-\frac{\partial}{\partial t}(\nabla\cdot\vec{B})$$

The left side is zero because divergence of curl is zero; the right side is zero by virtue of equation (ii).

Again, let us take divergence of Eq. (iv), i.e.

$$\nabla\cdot(\nabla\times\vec{B})=\mu_0(\nabla\cdot\vec{J})$$

The left side must be zero, but the right side, in general, is not. For steady currents, the divergence of $\vec{J}$ is zero, but evidently when we go beyond magnetostatics Ampere's law cannot be right. So, Eq. (iv) has to be modified.

- Maxwell corrected Eq. (iv) by adding extra term to $\vec{J}$. Let it be called $\vec{J}_d$, then

$$\nabla\times\vec{B}=\mu_0(\vec{J}+\vec{J}_d)$$

$$\text{or, } \nabla\cdot(\nabla\times\vec{B})=\mu_0(\nabla\cdot\vec{J}+\nabla\cdot\vec{J}_d)$$

$$\text{or, } \quad 0 = \mu_0(\nabla\cdot\vec{J}+\nabla\cdot\vec{J}_d)$$

$$\text{or, } \quad \nabla\cdot\vec{J}_d = -\nabla\cdot\vec{J}=\frac{\partial\rho}{\partial t}=\frac{\partial}{\partial t}(\varepsilon_0\nabla\cdot\vec{E})=\nabla\cdot\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

$$\therefore \boxed{\vec{J}_d=\varepsilon_0\frac{\partial\vec{E}}{\partial t}}$$

- Thus the **modified form of Ampere's law** is $\boxed{\nabla\times\vec{B}=\mu_0\vec{J}+\mu_0\varepsilon_0\dfrac{\partial\vec{E}}{\partial t}}$

Apart from curing the defect in Ampere's law, Maxwell's term has a certain aesthetic appeal:

A changing electric field induces a magnetic field.

Maxwell called his extra term the **displacement Current:** $\vec{J}_d\equiv\varepsilon_0\dfrac{\partial\vec{E}}{\partial t}$

The General Form of Ampere's Law

General Form of Ampere's Law

- Consider a capacitor being charged as illustrated in **Figure MA-1**.

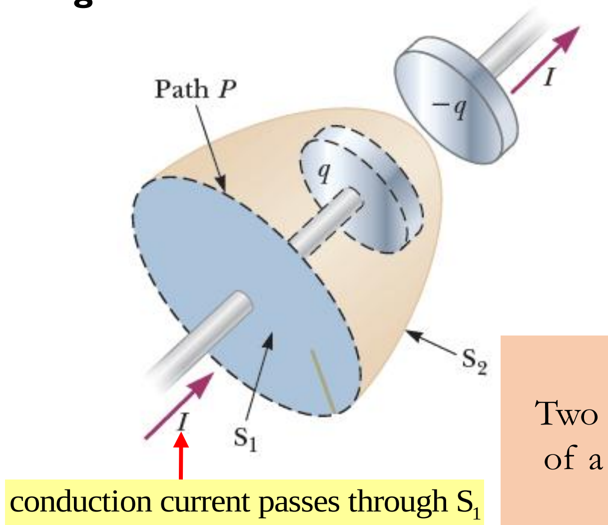


Figure MA – 1
Two surfaces S_1 and S_2 near the plate of a capacitor are bounded by the same path P.

- When a conduction current is present, the charge on the positive plate changes, but no conduction current exists in the gap between the plates because there are no charge carriers in the gap.

- Ampere's law: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$

$\downarrow$

total current through any surface bounded by the path P

When the path P is considered to be the boundary of S_1 : $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$

When the path P is considered to be the boundary of S_2 : $\oint \vec{B} \cdot d\vec{l} = 0$

Therefore, we have a contradictory situation that arises from the discontinuity of the current! Maxwell solved this problem by postulating an additional term on the right side of Ampère's law, which includes a factor called the **displacement current** I_d defined as

$$I_d = \varepsilon_0 \frac{\partial \Phi_E}{\partial t} \quad \text{..........} \quad (1)$$

As the capacitor is being charged (or discharged), the changing electric field between the plates may be considered equivalent to a current that acts as a continuation of the conduction current in the wire. When the expression for the displacement current given by Eq. (1) is added to the conduction current on the right side of Ampère's law, the difficulty represented in Figure MA-1 is resolved. No matter which surface bounded by the path P is chosen, either a conduction current or a displacement current passes through it.

The General Form of Ampere's Law

General Form of Ampere's Law

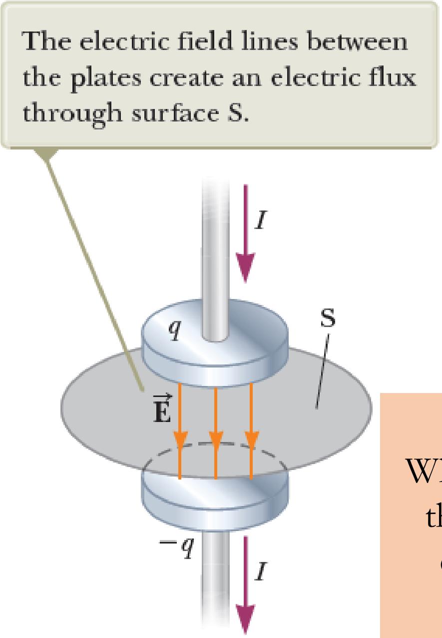
- The general form of Ampere's law:

$$\oint \vec{B}\cdot d\vec{l} = \mu_0\left(I + I_d\right)$$

$$\therefore \boxed{\oint \vec{B}\cdot d\vec{l} = \mu_0 I + \mu_0\varepsilon_0 \frac{d\Phi_E}{dt}}$$

Ampère–Maxwell law

- We can understand the meaning of this expression by referring to **Figure MA-2**



q → charge on the plate at any instant

A → area of the capacitor plate

Figure MA – I
When a conduction current exists in the wires, a changing electric field exists between the plates of the capacitor

The electric flux through surface S : $\Phi_E = \int \vec{E}\cdot d\vec{a} = EA = \dfrac{q}{\varepsilon_0}$

The magnitude of uniform electric field between the plates $\rightarrow E = \dfrac{q}{\varepsilon_0 A}$

Hence, the **displacement current** through S is

$$I_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \frac{d}{dt}\left(\frac{q}{\varepsilon_0}\right) = \frac{dq}{dt}$$

That is, the displacement current I_d through S is precisely equal to the conduction current I in the wires connected to the capacitor!

The magnetic fields are produced both by conduction currents and by time-varying electric fields.
This result was a remarkable example of theoretical work by Maxwell, and it contributed to major advances in the understanding of electromagnetism.

Ampere-Maxwell Law

Ampere – Maxwell Law

- Ampere's Law with Maxwell's Correction:

$$\oint \vec{B}\cdot d\vec{l} = \mu_0\left(I + I_d\right)$$

$$\therefore \quad \boxed{\oint \vec{B}\cdot d\vec{l} = \mu_0 I + \mu_0\varepsilon_0 \frac{d\Phi_E}{dt}}$$

Ampère–Maxwell law

- Ampere's Law with Maxwell's Correction:

$$\boxed{\nabla\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}}$$

The statement established by Maxwell:

Time varying electric field induces a magnetic field.

The modified form of Ampere Law:

$$\nabla\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t} \qquad .... \quad (A)$$

To check this modified equation,

Let us take the divergence of Eq. (A):

$$\nabla\cdot(\nabla\times\vec{B}) = \mu_0\nabla\cdot\vec{J} + \mu_0\varepsilon_0\frac{\partial}{\partial t}(\nabla\cdot\vec{E})$$

$$\text{or,}\quad \nabla\cdot(\nabla\times\vec{B}) = \mu_0\nabla\cdot\vec{J} + \mu_0\frac{\partial}{\partial t}(\varepsilon_0\nabla\cdot\vec{E})$$

$$\text{or,}\quad \nabla\cdot(\nabla\times\vec{B}) = \mu_0\nabla\cdot\vec{J} + \mu_0\frac{\partial\rho}{\partial t}$$

$$\therefore\quad \nabla\cdot(\nabla\times\vec{B}) = \mu_0\left[\nabla\cdot\vec{J} + \frac{\partial\rho}{\partial t}\right]$$

The left side is zero as it is divergence of curl of $\vec{B}$.

And as we know from continuity equation $\nabla\cdot\vec{J} + \frac{\partial\rho}{\partial t} = 0$,

so the right hand term is in general equal to zero.

Thus Eq. (A) is the correct form of Ampere's Law.

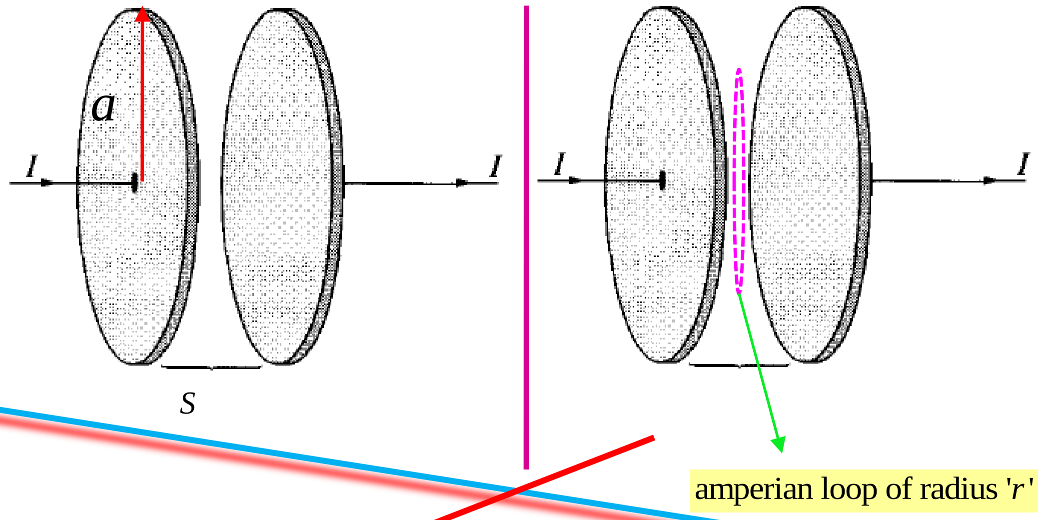
Ampere-Maxwell Law : $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc} + \mu_0\varepsilon_0\int\left(\frac{\partial\vec{E}}{\partial t}\right)\cdot d\vec{a}$

Sample Problem

- **A capacitor made from parallel circular plates, of radius a and separation S, is inserted into a long straight wire carrying current I. As the capacitor charges up, find the induced magnetic field midway between the plates, at a distance $r\left(r<a\right)$ from the center.**

$Q \downarrow$ charge on the plate at any instant

$A \downarrow$ area of the capacitor plate



Solution:

As the capacitor charges up, the magnitude of uniform electric field between the capacitor plates is

$$E=\frac{\sigma}{\varepsilon_0}=\frac{1}{\varepsilon_0}\frac{Q}{A}=\frac{1}{\varepsilon_0}\frac{Q}{\pi a^2}$$

The displacement current density is

$$J_d=\varepsilon_0\frac{\partial E}{\partial t}=\varepsilon_0\frac{\partial}{\partial t}\left(\frac{1}{\varepsilon_0}\frac{Q}{\pi a^2}\right)=\frac{1}{\pi a^2}\frac{dQ}{dt}=\frac{I}{\pi a^2}$$

Ampere-Maxwell Law:

$$\oint\vec{B}\cdot d\vec{l}=\mu_0 I+\mu_0\varepsilon_0\int\left(\frac{\partial\vec{E}}{\partial t}\right)\cdot d\vec{a}=\mu_0 I+\mu_0\int\vec{J}_d\cdot d\vec{a}$$

$= 0$ ($\because$ loop doesn't enclose conduction current)

$$\Rightarrow B(2\pi r)=0+\mu_0 J_d\pi r^2$$

$$\Rightarrow B=\frac{1}{2\pi r}\mu_0\left[\frac{I}{\pi a^2}\right]\pi r^2$$

$$\therefore\ B=\frac{\mu_0 I}{2\pi a^2}r$$

Maxwell's Equations

Maxwell's Equations

(i)	$\nabla\cdot\vec{E}=\frac{\rho}{\varepsilon_0}$	(Gauss's law)	(i)	$\oint\vec{E}\cdot d\vec{a}=\frac{q}{\varepsilon_0}$	(Gauss's law)
(ii)	$\nabla\cdot\vec{B}=0$	(no name)	(ii)	$\oint\vec{B}\cdot d\vec{a}=0$	(Gauss's law in magnetism)
(iii)	$\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$	(Faraday's law)	(iii)	$\oint\vec{E}\cdot d\vec{l}=-\frac{d\Phi_B}{dt}$	(Faraday's law)
(iv)	$\nabla\times\vec{B}=\mu_0\vec{J}+\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$	(Ampere-Maxwell law)	(iv)	$\oint\vec{B}\cdot d\vec{l}=\mu_o I+\mu_o\varepsilon_o\frac{d\Phi_E}{dt}$	(Ampere-Maxwell law)

In his unified theory of electromagnetism, Maxwell showed that electromagnetic waves are a natural consequence of the fundamental laws expressed in above four equations.

These four equations are regarded as the basis of all electrical and magnetic phenomena. These equations, developed by Maxwell, are as fundamental to electromagnetic phenomena as Newton's laws are to mechanical phenomena.

Electromagnetic Waves in Vacuum

The Wave Equations for $\vec{E}$ and $\vec{B}$

ELECTROMAGNET IC WAVE PROPAGATION

In regions of space where there is no charge or current, Maxwell's equations read

$$\nabla \cdot \vec{E} = 0 \qquad .... \quad (1)$$

$$\nabla \cdot \vec{B} = 0 \qquad .... \quad (2)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad .... \quad (3)$$

$$\nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \qquad .... \quad (1)$$

Taking curl of Eq. (3), we get

$$\nabla \times (\nabla \times \vec{E}) = -\nabla \times \frac{\partial \vec{B}}{\partial t}$$

$$\text{or, } \nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = -\frac{\partial (\nabla \times \vec{B})}{\partial t}$$

$$\therefore \quad \boxed{\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}}$$

Again taking curl of Eq. (4), we get

$$\nabla \times (\nabla \times \vec{B}) = \mu_0 \varepsilon_0 \nabla \times \frac{\partial \vec{E}}{\partial t}$$

$$\text{or, } \nabla(\nabla \cdot \vec{B}) - \nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial (\nabla \times \vec{E})}{\partial t}$$

$$\therefore \quad \boxed{\nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}}$$

In vacuum, then, each Cartesian component of $\vec{E}$ and $\vec{B}$ satisfies the **three-dimensional wave equation**,

$$\nabla^2 f = \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2}$$

So Maxwell's equations imply that empty space supports the propagation of electromagnetic waves, travelling at a speed

$$\boxed{v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \approx 3 \times 10^8 \, \mathrm{m \cdot s^{-1}}}$$

This is velocity of electromagnetic waves.

Maxwell's Equations in Matter

When we are working with materials that are subject to electric and magnetic polarization there is a more convenient way to write Maxwell's equations.

An electric polarization $\vec{P}$ produces a bound charge density

$$\rho_b = -\nabla \cdot \vec{P}$$

Likewise, a magnetic polarization $\vec{M}$ results in a bound current

$$\vec{J}_b = \nabla \times \vec{M}$$

The time rate of change of electric polarization $\vec{P}$ is called the polarization current density denoted by $\vec{J}_p$ and given by

$$\vec{J}_p = \frac{\partial \vec{P}}{\partial t}$$

In matter, the total charge density can be written as

$$\rho = \rho_f + \rho_b$$

$$\therefore \quad \rho = \rho_f - \nabla \cdot \vec{P} \qquad \text{.........} \quad (1)$$

And total current density is

$$\vec{J} = \vec{J}_f + \vec{J}_b + \vec{J}_p$$

$$\therefore \quad \vec{J} = J_f + (\nabla \times \vec{M}) + \frac{\partial \vec{P}}{\partial t} \qquad \text{..........} \quad (2)$$

Maxwell's Equations in Matter

ELECTROMAGNETIC WAVE PROPAGATION

Maxwell's Equations in Matter

Now from Gauss's law in electrostatics,

$$\nabla\cdot\vec{E}=\frac{\rho}{\varepsilon_0}$$

$$\text{or, }\varepsilon_0\nabla\cdot\vec{E}=\rho$$

$$\text{or, }\nabla\cdot(\varepsilon_0\vec{E})=\rho_f-\nabla\cdot\vec{P};\text{using (1)}$$

$$\text{or, }\nabla\cdot(\varepsilon_0\vec{E})+\nabla\cdot\vec{P}=\rho_f$$

$$\text{or, }\nabla\cdot[\varepsilon_0\vec{E}+\vec{P}]=\rho_f$$

$$\therefore \boxed{\nabla\cdot\vec{D}=\rho_f} \qquad \left[\text{where }\vec{D}=\varepsilon_0\vec{E}+\vec{P}\right]$$

Similarly, Ampere law (with Maxwell's term) is

$$\nabla\times\vec{B}=\mu_0\vec{J}+\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

$$\text{or, }\frac{1}{\mu_0}(\nabla\times\vec{B})=\vec{J}+\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

$$\text{or, }\frac{1}{\mu_0}(\nabla\times\vec{B})=\vec{J}_f+\vec{J}_b+\vec{J}_p+\frac{\partial(\varepsilon_0\vec{E})}{\partial t}$$

$$\text{or, }\frac{1}{\mu_0}(\nabla\times\vec{B})=\vec{J}_f+\nabla\times\vec{M}+\frac{\partial\vec{P}}{\partial t}+\frac{\partial}{\partial t}(\varepsilon_0\vec{E})$$

$$\text{or, }\nabla\times\frac{\vec{B}}{\mu_0}-\nabla\times\vec{M}=\vec{J}_f+\frac{\partial}{\partial t}[\vec{P}+\varepsilon_0\vec{E}]$$

$$\text{or, }\nabla\times\left(\frac{\vec{B}}{\mu_0}-\vec{M}\right)=\vec{J}_f+\frac{\partial\vec{D}}{\partial t}$$

$$\therefore \boxed{\nabla\times\vec{H}=\vec{J}_f+\frac{\partial\vec{D}}{\partial t}} \qquad \left[\because \vec{H}=\frac{\vec{B}}{\mu_0}-\vec{M}\text{ and }\vec{D}=\varepsilon_0\vec{E}+\vec{P}\right]$$

Hence,

Maxwell's equations in matter become

$$\nabla\cdot\vec{D}=\rho_f \qquad \nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$$

$$\nabla\cdot\vec{B}=0 \qquad \nabla\times\vec{H}=\vec{J}_f+\frac{\partial\vec{D}}{\partial t}$$

Poynting's Theorem

ELECTROMAGNETIC WAVE PROPAGATION

Poynting's Theorem

The total energy stored in electromagnetic fields is

$$U_{em} = \int_V \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2\right) d\tau \text{ .... (1)}$$

Suppose we have some charge and current configuration which, at time *t*, produces fields $\vec{E}$ and $\vec{B}$ and In the next interval *dt*, , the charges move around a bit.

According to the Lorentz force law,

the work done, dw , by the electromagnetic forces acting on a charge *dq* is

$$dw = \vec{F}\cdot d\vec{l} = dq(\vec{E} + \vec{v}\times\vec{B})\cdot d\vec{l}$$

$$= dq(\vec{E} + \vec{v}\times\vec{B})\cdot\vec{v}\,dt = \vec{E}\cdot\vec{v}\,dq\,dt$$

The rate of total work done on all the charges in a volume *V*:

$$\frac{dW}{dt} = \int_V \vec{E}\cdot\vec{J}\,d\tau \qquad \left[\because dq = \rho d\tau \text{ and } \rho\vec{v} = \vec{J}\right]$$

.... (2)

From Ampere-Maxwell law

$$\nabla\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

$$\Rightarrow \vec{J} = \frac{1}{\mu_0}\left(\nabla\times\vec{B}\right) - \varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

Now,

$$\vec{E}\cdot\vec{J} = \vec{E}\cdot\frac{1}{\mu_0}(\nabla\times\vec{B}) - \varepsilon_0\vec{E}\cdot\frac{\partial\vec{E}}{\partial t}$$

$$= \frac{1}{\mu_0}\left[\vec{B}\cdot(\nabla\times\vec{E}) - \nabla\cdot(\vec{E}\times\vec{B})\right] - \varepsilon_0\vec{E}\cdot\frac{\partial\vec{E}}{\partial t}$$

$$\left[\because \nabla\cdot(\vec{E}\times\vec{B}) = \vec{B}\cdot(\nabla\times\vec{E}) - \vec{E}\cdot(\nabla\times\vec{B})\right]$$

$$= \frac{1}{\mu_0}\left[\vec{B}\cdot\left(-\frac{\partial\vec{B}}{\partial t}\right) - \nabla\cdot(\vec{E}\times\vec{B})\right] - \varepsilon_0\vec{E}\cdot\frac{\partial\vec{E}}{\partial t}$$

$$\left[\because \nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t};\ \text{Faraday's law}\right]$$

$$= \frac{1}{\mu_0}\left(-\frac{1}{2}\frac{\partial B^2}{\partial t}\right) - \frac{1}{\mu_0}\nabla\cdot(\vec{E}\times\vec{B}) - \varepsilon_0\frac{1}{2}\frac{\partial E^2}{\partial t}$$

$$\therefore \boxed{\vec{E}\cdot\vec{J} = -\frac{1}{2}\frac{\partial}{\partial t}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) - \frac{1}{\mu_0}\nabla\cdot(\vec{E}\times\vec{B})} \text{ .... (3)}$$

Poynting's Theorem

ELECTROMAGNETIC WAVE PROPAGATION

Poynting's Theorem

Putting $\vec{E}\cdot\vec{J}$ into Eq. (2), we get

$$\frac{dW}{dt}=\int_V\left[-\frac{1}{2}\frac{\partial}{\partial t}\left(\varepsilon_0E^2+\frac{1}{\mu_0}B^2\right)-\frac{1}{\mu_0}\nabla\cdot(\vec{E}\times\vec{B})\right]d\tau$$

$$=-\frac{1}{2}\frac{d}{dt}\int_V\left(\varepsilon_0E^2+\frac{1}{\mu_0}B^2\right)d\tau-\frac{1}{\mu_0}\int_V\nabla\cdot(\vec{E}\times\vec{B})d\tau$$

$$=-\frac{1}{2}\frac{d}{dt}\int_V\left(\varepsilon_0E^2+\frac{1}{\mu_0}B^2\right)d\tau-\frac{1}{\mu_0}\oint_S(\vec{E}\times\vec{B})\cdot d\vec{a}$$

$$\therefore\ \boxed{\frac{dW}{dt}=-\frac{dU_{\text{em}}}{dt}-\oint_S\vec{S}\cdot d\vec{a}}\quad\left[\because\ \vec{S}=\frac{1}{\mu_0}(\vec{E}\times\vec{B})\right]$$

.... (4) Poynting's vector

where S is the surface bounding V.

This is **Poynting's theorem**; it is "**the work-energy theorem" of electrodynamics**.

Poynting's theorem says that "the work done on the charges by the electromagnetic field is equal to the decrease in energy stored in the field, less the energy which is flowed out through the surface".

Poynting's theorem:

$$\frac{dW}{dt}=-\frac{dU_{\text{em}}}{dt}-\oint_S\vec{S}\cdot d\vec{a}$$

$$or,\ \frac{d}{dt}\int_V u_{\text{mech}}\,d\tau=-\frac{d}{dt}\int_V u_{em}\,d\tau-\oint_S\vec{S}\cdot d\vec{a}$$

$$\begin{bmatrix} u_{\text{mech}} = \text{mechanical energy density} \\ u_{em} = \text{energy density of the fields}\end{bmatrix}$$

$$or,\ \frac{d}{dt}\int_V\left(u_{\text{mech}}+u_{em}\right)d\tau=-\int_V\left(\nabla\cdot\vec{S}\right)d\tau$$

$$\therefore\ \frac{\partial}{\partial t}\left(u_{\text{mech}}+u_{em}\right)=-\nabla\cdot\vec{S}$$

This is the differential version of **Poynting's theorem**.

Poynting's vector: $\vec{S}=\frac{1}{\mu_0}(\vec{E}\times\vec{B})$

The energy per unit time, per unit area, transported by the fields

The SI unit of $\vec{S}$ are $J\ s^{-1}\ m^{-2}$ or $W\ m^{-2}$.

Multiple Choice Questions

- **The correct form of Ampere's law for circuits with gaps in them is**

  [a] $\oint \vec{B}\cdot d\vec{l} = 0$ **[b]** $\oint \vec{B}\cdot d\vec{l} = I_{enclosed}$

  [c] $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enclosed} + \mu_0\varepsilon_0 \frac{\partial E}{dt}$ **[d]** $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enclosed} + \mu_0\varepsilon_0 \frac{d\Phi_E}{dt}$

  Ans: [d]

- **The speed of the light is given by the value of**

  [a] $\varepsilon_0\mu_0$ **[b]** $\frac{1}{\sqrt{\varepsilon_0\mu_0}}$ **[c]** $\sqrt{\varepsilon_0\mu_0}$ **[d]** $\frac{1}{\varepsilon_0\mu_0}$

  Ans: [b]

- **Maxwell's equations can be written in the form shown below. If magnetic monopole exists, which of these equations will have to be changed?**

  I. $\nabla\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$ II. $\nabla\cdot\vec{B} = 0$

  III. $\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$ IV. $\nabla\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$

  [a] I only **[b] IV only** **[c] II and III** **[c] I and IV**

  Ans: [c]

- **Magnetic fields are produced by**

  [a] constant electric currents. **[b] electric currents that vary sinusoidally with time.**

  [c] time-varying electric fields. **[d] all of the above.**

  Ans: [d]

Multiple Choice Questions & Fill in the Blanks

ELECTROMAGNET IC WAVE PROPAGATION

- The unit of $\frac{1}{\mu_0}(\vec{E}\times\vec{B})$ is … $Js^{-1}m^{-2}$.
- The speed of electromagnetic wave in vacuum in term of permeability and permittivity is given by … $\frac{1}{\sqrt{\varepsilon_0\mu_0}}$
- The wave equation for $\vec{E}$ and $\vec{B}$ in vacuum are … $\nabla^2\vec{E}=\mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}$ & $\nabla^2\vec{B}=\mu_0\varepsilon_0\frac{\partial^2\vec{B}}{\partial t^2}$
- The unit of $\left(\vec{B}\cdot\vec{H}\right)$ is … $\mathrm{J\cdot m^{-3}}$
- The term $\frac{1}{\mu_0}(\vec{E}\times\vec{B})$ is … the energy per unit time, per unit area, transported by the electromagnetic fields.
- The rate of transfer of energy by an electromagnetic wave is described by a vector $\vec{S}$, called the Poynting vector, which is defined by the expression … $\frac{1}{\mu_0}(\vec{E}\times\vec{B})$
- The statement established by Maxwell is … A changing electric field induces a magnetic field.
- James Clerk Maxwell added an extra term on the right hand side of ampere's law to include time –varying electric fields. He called his extra term the displacement current : … $\vec{J}_d\equiv\varepsilon_0\frac{\partial\vec{E}}{\partial t}$

Text Books & References

1. **David J. Griffith**, Introduction to Electrodynamics
2. **R.A. Serway and J.W. Jewett**, Physics for Scientist and Engineers with Modern Physics
3. **Halliday and Resnick**, Fundamental of Physics
4. **D. Halliday, R. Resnick, and K. Krane** , Physics, Volume 2, Fourth Edition
5. **Hugh D.Young, Roger A. Freedman,** University Physics with Modern Physics, 13TH Edition


Thank you