General Physics I (PHYS 101)

Lecture 09

Harmonic Oscillation

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Outline

Wave and Oscillation

If a motion repeats itself in equal interval of time then the motion is said to be periodic motion. The displacement of the particle in such a motion can always be expressed in terms of sine or cosine. Hence the periodic motion often called harmonic motion.

If a particle in periodic motion moves back and forth over the same path the motion is said to be oscillatory or vibratory.

Simple Harmonic Oscillation:- Loaded spring

The to and fro motion of a body in which the acceleration of the body is directly proportional to the displacement from mean position and always toward the mean position is called simple harmonic oscillation.

Consider a horizontal spring of force constant *k* whose one end is attached to a rigid support and the next end is attached to a body of mass m rest on a friction-less level surface as shown in figure 1.

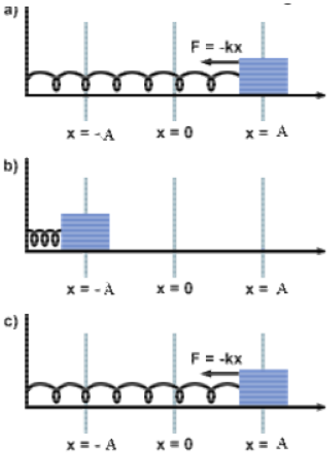

Figure 1: Oscillation in loaded spring

Simple Harmonic Oscillation:- Loaded spring (contd.)

Consider the spring is stretched to a distance A from its mean position and then released. The force exerted by the spring on the body at any position x from the mean position is given by Hook's law as

$$F_{spring} = -kx \tag{1}$$

This is the resultant force exerted on the body. This force is also called restoring force or elastic force.

But from Newton's second law of motion

$$F_{resultant} = m\frac{d^2x}{dt^2} \tag{2}$$

Hence

$$m\frac{d^2x}{dt^2} = -kx$$
$$\implies \frac{d^2x}{dt^2} = -\frac{k}{m}x \tag{3}$$

Let $\omega_0 = \sqrt{\frac{k}{m}}$ angular velocity of the particle, the equation (3) becomes

$$\frac{d^2x}{dt^2} = -\omega_0^2 x \tag{4}$$

Equation (4) represents the differential equation of the motion of the body attached to the spring.

Simple Harmonic Oscillation:- Loaded spring (contd.)

Here, $\dfrac{dx}{dt} = v$ is the velocity of the body at any time t.

So, $\dfrac{d^2x}{dt^2} = \dfrac{dv}{dt} = \dfrac{dv}{dx}\dfrac{dx}{dt} = \dfrac{dv}{dx}v = v\dfrac{dv}{dx}$

and equation (4) becomes

$$v\frac{dv}{dx} = -\omega_0^2 x \implies vdv = -\omega_0^2 xdx \tag{5}$$

Integrating both sides, we get

$$\frac{1}{2}v^2 = -\frac{1}{2}\omega_0^2 x^2 + c$$

Where c is constant of integration.

Simple Harmonic Oscillation:- Loaded spring (contd.)

Now when the displacement x is maximum, the velocity v will be zero. The maximum displacement is called the amplitude and represented by A. Hence the displacement x is maximum. i.e. at extreme position $x = \pm A$ and $v = 0$, so that

$$0 = -\frac{1}{2}\omega_0^2 A^2 + c \implies c = \frac{1}{2}\omega_0^2 A^2$$

Hence

$$\frac{1}{2}v^2 = -\frac{1}{2}\omega_0^2 x^2 + \frac{1}{2}\omega_0^2 A^2$$

Simple Harmonic Oscillation:- Loaded spring (contd.)

$$v = \omega_0 \sqrt{A^2 - x^2} \tag{6}$$

$$\implies \frac{dx}{dt} = \omega_0 \sqrt{A^2 - x^2}$$

$$\implies \frac{dx}{\sqrt{A^2 - x^2}} = \omega_0 dt \tag{7}$$

Integrating equation (7), we get

$$\sin^{-1}\left(\frac{x}{A}\right) = \omega_0 t + \phi$$

$$\implies x = A \sin\left(\omega_0 t + \phi\right) \tag{8}$$

Where ϕ is another constant of integration called the phase constant or initial phase.

Equation (8) is the solution of equation (4). When time t of equation (8) is replaced by $t + \frac{2\pi}{\omega_0}$, then the new displacement is

$$x' = A\sin(\omega_0 t + 2\pi + \phi) = A\sin(\omega_0 t + \phi) = x$$

That means the body's position is repeated after every $\dfrac{2\pi}{\omega_0}$ time. So this is called time period of oscillation and given by

$$T = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{m}{k}} \tag{9}$$

The maximum displacement A is called the amplitude of the oscillation and ϕ is called the phase angle.

Simple Harmonic Oscillation:- Loaded spring (contd.)

The velocity in term of time is given by

$$v = \omega_0 A \cos(\omega_0 t + \phi) \tag{10}$$

The kinetic energy and potential energy of the body in term of position are

$K(x) = \frac{1}{2}mv^2 = \frac{1}{2}m\omega_0^2\left(A^2 - x^2\right)$ and $U(x) = \frac{1}{2}kx^2$

The kinetic energy and potential energy of the body in term of time are

$K(t) = \frac{1}{2}mv^2 = \frac{1}{2}m\omega_0^2 A^2 \cos^2(\omega_0 t + \phi) = \frac{1}{2}kA^2 \cos^2(\omega_0 t + \phi)$

and $U(t) = \frac{1}{2}kx^2 = \frac{1}{2}kA^2 \sin^2(\omega_0 t + \phi)$

The total energy $E_0 = K + U = \frac{1}{2}kA^2$

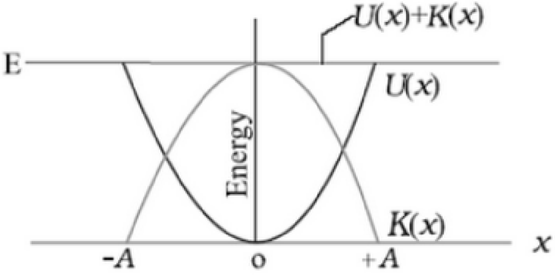

Figure 2: Energy versus position graph

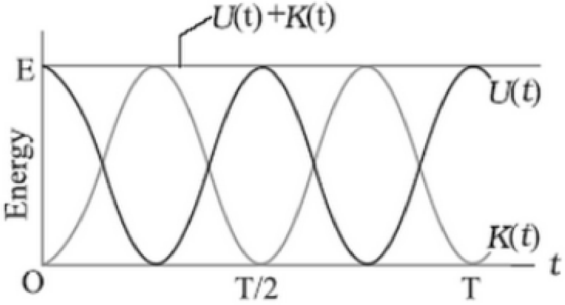

Figure 3: Energy versus time graph

Simple Harmonic Oscillation:- Loaded spring (contd.)

The kinetic energy is maximum at mean position and minimum at extreme position. The maximum kinetic energy $K_{max} = \frac{1}{2}kA^2$ and the minimum kinetic energy $K_{min} = 0$. The average kinetic energy

$$K_{ave} = \frac{K_{min} + K_{max}}{2} = \frac{1}{4}kA^2.$$

The potential energy is maximum at extreme position and minimum at mean position. The maximum potential energy $U_{max} = \frac{1}{2}kA^2$ and the minimum potential energy $U_{min} = 0$. The average potential energy

$$U_{ave} = \frac{U_{min} + U_{max}}{2} = \frac{1}{4}kA^2.$$

The graphs of K and U versus x and t are as shown in figures 2 and 3.