<Q.7>: The potential energy function for the force between two atoms in a diatomic molecule can be expressed approxiamety as follows:

$$U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$$

a & b = constants

x = distance betn atoms

a) At what values of x is U(x) = 0? At what values is U(x) is minimum?

b) Determine the force between the atoms.

c) What is dissociation energy of the molecule?

Soln

Given,

$$U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$$

For (a):

The value of x such that U(x) = 0.

$$0 = \frac{a}{x^{12}} - \frac{b}{x^6}$$

or, $\frac{x^6}{b} = \frac{x^{12}}{a}$ $\quad \therefore x = \left(\frac{a}{b}\right)^{1/6}$

Also, $U(x) = 0$ if $x \to \infty$.

So, $U(x)$ is zero if $x \to \infty$ or $x = \left(\frac{a}{b}\right)^{1/6}$.

We know
equilibrium occurs at condition x_m where $U(x)$ is minimum.

ie, $\left(\frac{du}{dx}\right)_{x=x_m} = 0$

So,

$$U'(x) = 0$$

Now,

$$U'(x) = \left[\frac{-12a}{x^{13}} + \frac{6b}{x^7}\right]_{x=x_m}$$

or, $-\dfrac{12a}{x_m^{13}} + \dfrac{6b}{x_m^{7}} = 0$

$\therefore x_m = \left(\dfrac{2a}{b}\right)^{1/6}$

$U(x)$ is minimum when $x_m = \left(\dfrac{2a}{b}\right)^{1/6}$

For (b):

We know,

~~Force is negative~~ The two atoms experience electrostatic force which is conservative.

So, conservative force is negative gradient of potential energy

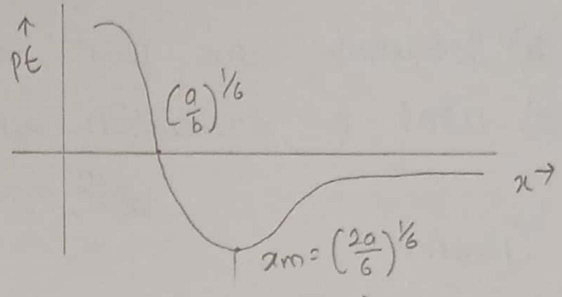


fig: a

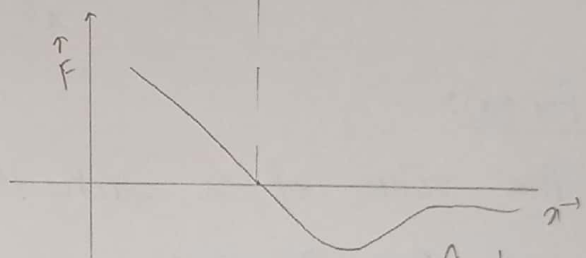


fig: b

$$F_x = -\left(\frac{dU}{dx}\right)$$

$$= -U'(x)$$

$$\therefore F_x = \frac{12a}{x^{13}} - \frac{6b}{x^{7}}$$

We plot the force as a function of separation betn the atoms in fig(b) when the force is positive from ($x=0$ to $x=x_m$), atoms are repelled from one another.

when the force is negative from ($x=x_m$ to $x=x_\infty$), the atoms are attracted to one another.

At $x=x_m$, the force is zero, that is part of stable equilibrium

For (c):

Dissociation energy = change in PE from $x=x_\infty$ to $x=x_m$

$$= U(x=\infty) - U(x=x_m)$$

$$= \left(\frac{a}{\infty^{12}} - \frac{b}{\infty^{6}}\right) - \left(\frac{a}{x_m^{12}} - \frac{b}{x_m^{6}}\right)$$

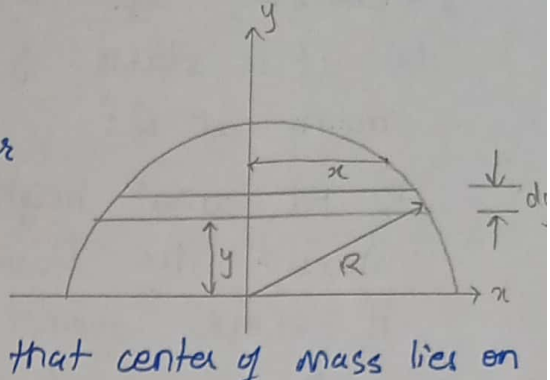
$$= \frac{b}{\left(\left(\frac{2a}{b}\right)^{1/6}\right)^{6}} - \frac{a}{\left(\left(\frac{2a}{b}\right)^{1/6}\right)^{12}}$$

$$\therefore E_d = \frac{b^2}{4a}$$

<Q.8>: Find center of mass of a homogeneous semicircular plate. Let 'R' be circle's radius.

Sol^n

Let us consider a homogeneous semicircular plate of radius R and mass M.



The homogeneous semicircular plate has rotational symmetry about the y-axis so that center of mass lies on y-axis.

Consider a thin strip of mass 'dm' of this homogeneous circular plate.

Area of thin strip $(da) = 2x\,dy$

Mass of thin strip $(dm) = \dfrac{M}{\frac{\pi R^2}{2}} \times 2x\,dy = \dfrac{4M}{\pi R^2} x\,dy.$

The center of mass of homogeneous semicircular plate is given by

$$y_{cm} = \frac{1}{M}\int y\,dm = \frac{1}{M}\int \left[\frac{4M}{\pi R^2} x\,dy\right] = \frac{4}{\pi R^2}\int y\,x\,dy \quad \text{—(i)}$$

Here, ~~x²~~ $t^2 = R^2 - y^2$

or, $-2y\,dy = 2t\,dt$

$\therefore\ y\,dy = -t\cdot dt$

when $y = 0,\ t = R$ when $y = R,\ t = 0.$

From eq^n (i);

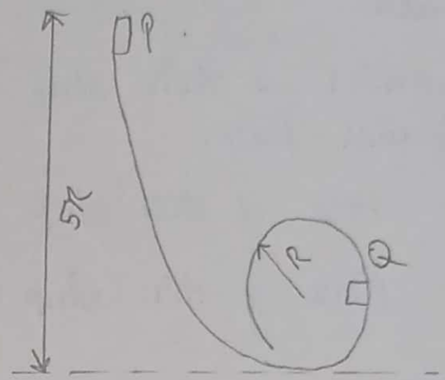
$$y_{cm} = \frac{4}{\pi R^2}\int_0^R \sqrt{R^2-y^2}\cdot y\,dy = \frac{4}{\pi R^2}\int_0^R t^2\,dt$$

$$= \frac{4}{\pi R^2} \times \frac{R^3}{3} = \frac{4R}{3\pi}$$

Thus, the center of mass of homogeneous circular plate lies on y-axis at distance $\frac{4R}{3\pi}$ from origin.

<Q.97: A small block of mass m slides along the frictionless loop-the-loop track as in figure.
(a) If it starts from rest at P, what is resultant force acting at Q?
b) At what height above the bottom of the loop should the block be released so that the force it exerts against the track at the top of the loop is equal to its weight?

Soln



For (a):

At point Q, it is at 'R' height from ground.
So, height of $PQ = (5R - R) = 4R$

We know,

$$v^2 = u^2 + 2gh$$

or, $v^2 = 2g\,4R$ $\therefore v^2 = 8gR$

At Q, centripetal force $(F_c) = \frac{mv^2}{R}$ $\therefore F_c = 8mg$

Weight of block at Q $(W) = mg$.

So, resultant force $= \sqrt{F_c^2 + \cancel{mg}\,W^2}$

$= \sqrt{8^2m^2g^2 + m^2g^2}$ $\therefore R_F = \sqrt{65}\,mg$.

For (b):

At height H, $N' + mg = \frac{mv_0^2}{R}$

By question, $N' = mg$.

So,

$2mg = \frac{mv^{o2}}{R}$ $\therefore v^o = \sqrt{2gR}$

So, $v^2 = u^2 + 2gh$

or, $2gR = 2g(h - 2R)$ $\therefore h = 3R$.

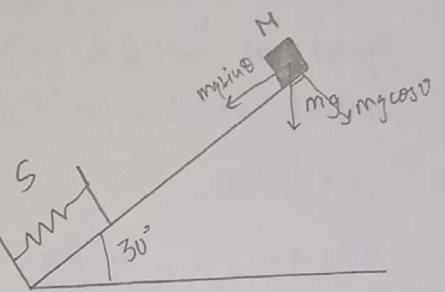
So, the required height is 3R.

<Q.10>: An ideal spring S can be compressed 1.0 m by force of 100N. this same spring is placed at bottom of a frictionless inclined plane which makes angle 30° with ~~a rot~~ horizontal. A 10 kg mass M is released form rest at top of incline and brought to rest momentarily after compressing spring 2 meter.

a) Through what distance does the mass slide before coming to rest?

b) what is speed of mass just before reaching spring?

Soln:



~~Her~~ Given,

Force (F) = 100 N

compression (x) = 1 m

∴ Spring constant (K) = F/x = 100N/m.

Fur (a):

Let 'd' be required distance through which mass slides before coming to rest. According to WE theorem,

$$W_{net} = \Delta K$$

or, $W_g + W_s + W_n = \Delta K$

or, $mg\sin\theta\, d - \frac{1}{2}kx^2 + 0 = 0$

$$\therefore d = \frac{\frac{1}{2} \times 100 \times 2^2}{10 \times 9.8 \times \sin 30^\circ} = 4m.$$

The mass slides 4m before coming to rest.

Fur (b):

Let 'v_f' be speed of mass just before reaching spring.

According to WE theorem, $W_{net} = K_f - K_i$

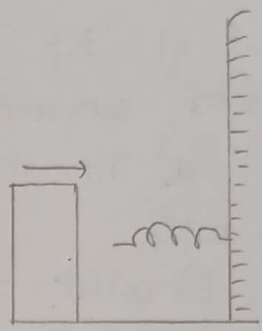
or, $W_g + W_N = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

or, $mg\sin\theta \times (d-x) + 0 = \frac{1}{2}mv_f^2 + 0$

So, $v_f = \sqrt{2 \times 9.8 \times \sin\theta \times (4-2)}$

$\therefore v_f = 4.5$ m/s. is the speed just before reaching spring.

Q.117: A 1.0 kg block collides with horizontal weightless spring of force constant 2.0 N/m as in figure. The block compresses the spring 4.0 m from rest position. Assuming coefficient of K.F. is 0.25, what was speed of block at collision?



Soln:

Given,

mass of block (m) = 1 kg

spring constant (K) = 2.0 N/m

compression (x) = 4.0 m.

coefficient of KF (μ) = 0.25

speed at collision (v) = ?

initial velocity (u) = 0.

We know,

Energy in spring when compressed 4 meters $(E) = \frac{1}{2} kx^2$

$= \frac{1}{\cancel{2}} \times \cancel{2} \times \cancel{8}16$

$\therefore E = 16\ J$

Workdone to move the block $(W) = F \times x$

$= \mu R x = \mu m g x$

$= 0.25 \times 1 \times 9.8 \times 4$

$\therefore W = 9.8\ J$

According to work-energy theorem,

$W_{net} = \Delta K$

or, $(W + E) = K_f - K_i \quad [\because K_i = 0]$

or, $9.8 + 16 = \frac{1}{2} m v^2$

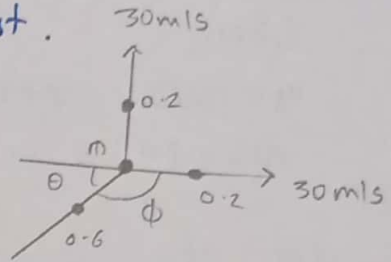
$v = \sqrt{2 \times (9.8 + 16)} \quad \therefore v = 7.18\ m/s.$

The speed of block at collision is 7.18 m/s.

<Q·12>: A vesset at rest explodes, breaking into three pieces. Two pieces, having equal mass, fly off $\perp^r$ to one another with same speed of 30 m/s. The third piece has three times with mass of each other piece. What is the direction and magnitude of its velocity immediately after explosion?

Soln:

Let 'm' be the initial mass of body at rest.



By question,

$$x + x + 3x = m$$

$$\therefore x = 0.2\,m.$$

Thus, the masses of three fragments be 0.2m, 0.2m ~~and~~ and 0.6 m.

Let the third fragment fly off with velocity 'v' at angle ϕ.

We know, momentum is conserved when the body exploded.

So,

Conservation of momentum in y-direction:

$$0.6\,m\,(v\sin\theta) = 0.2\,m\,(30) \quad \text{--- (i)}$$

Conservation of momentum in x-direction,

$$0.6\,m\,(v\cos\theta) = 0.2\,m\,(30) \quad \text{--- (ii)}$$

Dividing (i) and (ii), we get: $\tan\theta = 1$

So, $\theta = 45°$

Hence, from figure $\theta + \phi = 180°$ $\therefore \phi = 135°$

Squaring and adding (i) and (ii);

$$(0.6\,mv)^2(\sin^2\theta + \cos^2\theta) = [0.2m \times 30 + 0.2m \times 30]^2$$

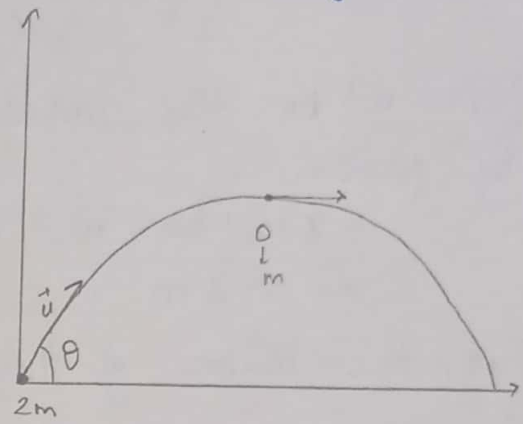
or, $0.6\,mv = 0.2\,m\,(30) \times \sqrt{2}$

$$\therefore v = 10\sqrt{2}\ \text{m/s}.$$

The direction of velocity is 135° and magnitude is $10\sqrt{2}$ m/s

<Q.13>: A projectile is fired from a gun at angle 45° with horizontal and muzzle speed of 457.2 m/s. At heighest point, the projectile explodes into two fragments of equal mass. one fragment, whose initial speed is zero, falls vertically. How far from the gun does other fragment land, assuming level terrain?

Soln:



Given,

~~u~~ = muzzle speed $(u) = 457.2$ m/s

angle $(\theta) = 45°$

Let the mass of the projectile be 2m.

Since one fragment falls down vertically with zero initial velocity, the ~~project~~ other fragments follows the true trajectory.

We have,

$$x_{cm} = \frac{m_A x_A + m_B x_B}{m_A + m_B} \quad \text{or,} \quad 2x_{cm} = x_A + x_B.$$

$$\therefore x_B = \cancel{x_A - 2x_{cm}} \; 2x_{cm} - x_A \quad \text{— (i).}$$

Since center of mass follows true trajectory. $x_{cm} = \text{Range} = \frac{u^2 \sin 2\theta}{g}$

and $x_A = u_A t = u\cos\theta \left(\frac{u\sin\theta}{g}\right)$

So, in eqn (i)

$$x_B = 2 \times \frac{u^2 \sin 2\theta}{g} - u\cos\theta\left(\frac{u\sin\theta}{g}\right)$$

$$= \frac{2 \times (457.2)^2 \times \sin 90°}{9.8} - 457.2\cos 45°\left(\frac{457.2\sin 45°}{9.8}\right)$$

$$\therefore x_B = 4.27 \times 10^{4} - 1.11 \times 10^4 \text{ m}$$

$$\therefore x_B = 3.16 \times 10^4 \text{ m}.$$

The other fragment lands 3.16×10^4 m from the fired points

<Q.147: A 6000 kg rocket is set for a vertical firing. If exhaust speed is 1000 m/s, how much gas must be ejected per second to supply the thrust needed

a) to overcome the weight of rocket.

b) to give the rocket an initial upward acceleration of 19.6 m/s^2?

Soln:

Given,

mass (m) = 6000 kg.

velocity at exhaust (v) = 1000 m/s

<u>For (a):</u>

We know,

$$F_{thrust} = F_{relative} \times \frac{dm}{dt}$$

For the rocket to overcome its weight,

$$F_{thrust} = F_{weight}$$

or, $1000 \times \frac{dm}{dt} = 6000 \times 9.8$ $\therefore \frac{dm}{dt} = 58.8$ kg/s.

<u>For (b):</u>

$\frac{\Delta v}{\Delta t} = 19.6$ m/s^2

We know,

$$m.\frac{\Delta v}{\Delta t} = v.\frac{\Delta m}{\Delta t} - mg$$

or, $6000 \times 19.6 = 1000 \times \frac{\Delta m}{\Delta t} - 6000 \times 9.8$ $\therefore \frac{\Delta m}{\Delta t} = 176.4$ kg/s

To overcome weight, ~~gas~~ 58.8 kg of gas ejected/second and to give initial upward acceleration of 19.6 m/s^2, 176.4 kg/s gas is ejected.

<Q.15>: A rocket moving in free space has velocity 3.0×10^3 m/s relative to earth. It's engines are turned on, and fuel is ejected in a direction opposite the rocket's motion at speed of 5.0×10^3 m/s relative to rocket.

a) what is the speed of the rocket relative to the earth once the rocket's mass is reduced to one ~~third~~ half its mass before ignition?

b) what is thrust if it burns at rate 50 kg/s?

Sol:

velocity of rocket (v_0) = 3×10^3 m/s.

velocity of fuel relative to rocket (V_{rel}) = 5×10^3 m/s.

Since mass is halved, $M_f = 0.5 M_i$

For:
(a)

$$V_f = V_0 + V_{rel} \ln\left(\frac{M_i}{M_f}\right)$$

$$= 3\times10^3 + 5\times10^3 \times \ln\left(\frac{M_i}{0.5 M_i}\right)$$

$$= 6.5\times10^3 \text{ m/s.}$$

For (b):

~~Thrust~~ $\frac{dm}{dt} = 50$ kg/s.

So,

$$\text{thrust} = \left|V_{rel}\frac{dM}{dt}\right| = 5\times10^3 \times 50$$

$$= 2.5\times10^5 \text{ N}$$

The speed of rocket when mass is halved is 6.5×10^3 m/s and the thrust if it burns at 50 kg/s is 2.5×10^5 N

<Q.167: A bullet of mass 10 gm strikes a ballistic pendulum of mass 2.0 kg. The center of mass of pendulum rises to vertical distance of 12 cm. Assuming that the bullet is embedded, calculate initial speed.

Soln:

Given,

mass of bullet $(m) = 10\,gm = 10 \times 10^{-3}$ kg

mass of pendulum $(M) = 2$ kg.

height of rise $(h) = 12$ cm $= 12 \times 10^{-2}$ m

Let 'u' be the initial velocity and 'v' be the final velocity.

We know,

From conservation of linear momentum,

$$mu = (M+m)v \qquad \therefore u = \frac{(M+m)}{m} v \quad \text{—(i)}$$

In ballistic pendulum, only gravitational force contributes to total workdone, the total mechanical energy is conserved.

$$\frac{1}{2}(\cancel{M}\, m+M)v^2 = (M+m)gh$$

$$\therefore v = \sqrt{2gh} \quad \text{—(ii)}$$

So, eqn (i) becomes;

$$u = \frac{M+m}{m} \times \sqrt{2gh}$$

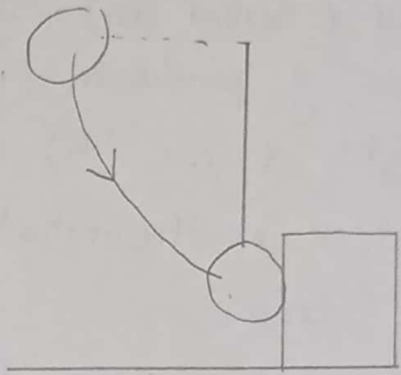
$$= \frac{2 + 10\times10^{-3}}{10\times10^{-3}} \times \sqrt{2\times9.8\times12\times10^{-12}}.$$

$$\therefore u = 308.3 \text{ m/s}.$$

The initial speed of bullet is 308.3 m/s

<Q.17>: A steel ball weighing 453.59 gm fastened to a cord 68.6 cm, long and is released when the cord is horizontal. At the bottom of its path, the ball strikes a 2.268 kg steel block initially at rest on a frictionless surface as shown in figure. The collision is elastic. Find the speed of ball and the speed of the block initially at just after collision.

Soln:



Given,

mass of ball $(m_1) = 453.59$ gm

$= 453.59 \times 10^{-3}$ gm

length of chord $(l) = 68.6$ cm

$= 68.6 \times 10^{-2}$ m

mass of block $(m_2) = 2.268$ kg

We know, total mechanical energy remains conserved.

$$PE = KE$$

$$\text{or, } mgh = \frac{1}{2}mv^2 \qquad \text{or, } v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 0.686}$$

$$\therefore v = 3.66 \text{ m/s}$$

Now,

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2} \times u_1 \qquad v_2 = \frac{2m_1}{m_1 + m_2} \times u_1$$

$$= \frac{0.45351 - 2.268}{0.45351 + 2.268} \times 3.66 \qquad = \frac{2 \times 453.59 \times 10^{-3}}{(0.45351 + 2.268)} \times 3.66$$

$$\therefore v_1 = 2.44 \text{ m/s} \qquad \therefore v_2 = 1.21 \text{ m/s.}$$

The speed of ball before collision is -2.44 m/s and ~~after collision is~~ speed of the block initially is 1.22 m/s. {taking motion of block as positive}

<Q.18>: A block of mass $m_1 = 2.0$ kg slides along frictionless table with speed of 10 m/s. Directly in front of it, and moving in same direction is the block $m_2 = 5.0$ kg moving at 3 m/s. A massless spring has ~~mass~~ spring constant of $k = 1120$ N/m is attached to backside of m_2 as in figure. When the blocks collide, what is maximum compression of spring? Spring obeys Hooke's law.

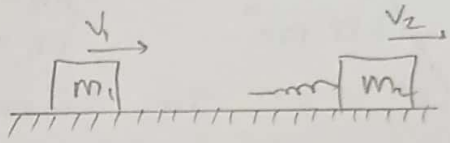
Soln:

Given,

$m_1 = 2$ kg $\qquad v_1 = 10$ m/s

$m_2 = 5$ kg $\qquad v_2 = 3$ m/s

$k = 1120$ N/m.



We know,

when spring is compressed to its maximum, the amount of velocity of both blocks become same as ~~ea~~ each other. So,

$$m_1 v_1 + m_2 v_2 = mV$$

$$\text{or, } 2 \times 10 + 5 \times 3 = (5+2) \times V$$

$$\therefore V = 5 \text{ m/s}$$

Although the collision is inelastic i.e, KE is lost in spring but total mechanical energy is conserved.

$$KE_{1\,before} + KE_{2\,before} = PE_{spring} + KE_{after}$$

$$\text{or, } \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = PE_s + \frac{1}{2} m V^2$$

$$\text{or, } PE_s = 35$$

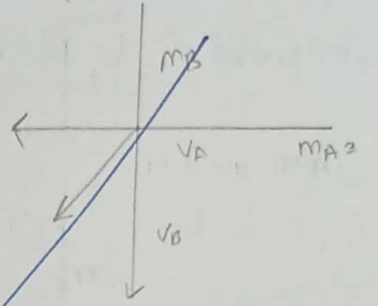
$$\text{or } \frac{1}{2} \times 1120 \times x^2 = 35$$

$$\therefore x = 0.25 \text{ m}$$

The maximum compression of ~~ea~~ spring is 0.25 m

<Q.19>: Two vehicles A and B are travelling west and south respectively towards same intersection where they collide and lock together. Before the collision A (m = 408.233 kg) is moving with the speed of 17.88 m/s and B(m = 544.310 kg) has a speed of 26.82 m/s. Find the magnitude and direction of the velocity of the vehicles after collision.

Soln:



Given,

For A,

$m_A = 408.233$ kg

$v_A = 17.88$ m/s

For B,

$m_B = 544.310$ kg

$v_B = 26.82$ m/s.

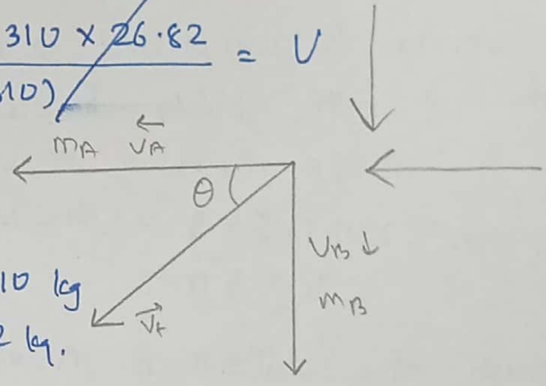
Let V be the common velocity after collision.

So, from the conservation of linear momentum,

$$m_A v_A + m_B v_A = (m_A + m_B) V$$

or, $$\frac{408.233 \times 17.88 + 544.310 \times 26.82}{(408.233 + 544.310)} = V$$

$\therefore V =$



For A,

$m_A = 408.233$ kg

$v_A = 17.88$ m/s

For B,

$m_B = 544.310$ kg

$v_B = 26.82$ kg.

Let v_f be common velocity after wreckage.

Here,

momentum along x-axis $(\Sigma p_x) = 408.233 \times 17.88$
$= 7299.20$ kg m/s —— (i)

If the wreckage moves with angle θ, v_f for x-direction.

$$\Sigma p_{x\,v_f} = (952.632) \times v_f \times \cos\theta \quad \text{—— (ii)}$$

Equating (i) and (ii), we get.

$$7299.20 = 952.632 \times v_f \times \cos\theta \quad \text{—— (a)}$$

Similarly, along y-direction $(\Sigma p_y) = 544.310 \times 26.82$
$= 14598.40$ —— (iii)

For wreckage, momentum along y-axis $(\Sigma p_{v_f\, y}) = (952.632) \times v_f \times \sin\theta$ —— (iv)

Equating (ii) and (iv),

$$14598.40 = 952.632 \times V_f \times \sin\theta \quad \text{—— (b)}$$

Dividing (a) from (b),

$$\frac{14598.40}{7299.20} = \frac{952.632}{952.632} \times \frac{V_f}{V_f} \times \frac{\sin\theta}{\cos\theta}$$

or, $2 = \tan\theta$ $\quad \therefore \theta = 63.43°$

Putting $\theta = 63.43°$ in eq^n (a), we get

$$V_f = 17.13 \text{ m/s.}$$

<Q.20> A gas molecule having a speed of 300 m/s collides elastically with another molecule of same mass at rest. After collision, the first molecule moves at angle 30° to initial direction. Find the speed of each molecule after collision and the angle made with the incident direction by recoiling target molecule.

Sol^n:

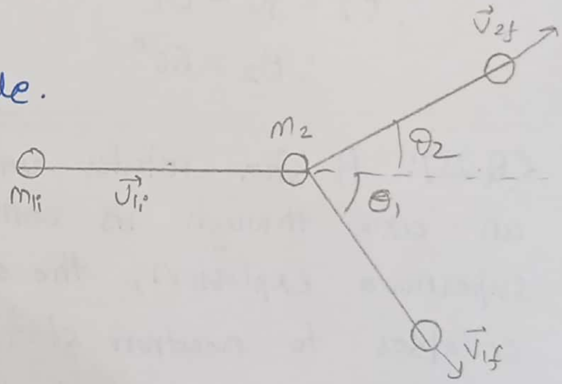
Let 'm' be the mass of gas molecule.

Here,

$V_{1i} = 300$ m/s $\qquad V_{2i} = 0$ m/s

$\theta_1 = 30°$ $\qquad \theta_2 = ?$

$V_{1f} = ?$ $\qquad V_{2f} = ?$



By conservation of momentum,

$$V_{1i} - V_{1f}\cos\theta_1 = V_{2f}\cos\theta_2 \quad \text{—— (i)}$$

$$V_{1f}\sin\theta_1 = V_{2f}\sin\theta_2 \quad \text{—— (ii).}$$

By conservation of KE,

$$V_{1i}^2 - V_{1f}^2 = V_{2f}^2 \quad \text{—— (iii)}$$

Squaring and adding (i) and (ii), we get.

$$(V_{1i} - V_{1f}\cos\theta_1)^2 + (V_{1f}\sin\theta_1)^2 = (V_{2f}\cos\theta_2)^2 + (V_{2f}\sin\theta_2)^2$$

or, $V_{1i}^2 - 2V_{1i}V_{1f}\cos\theta_1 + \cos\theta_1 + V_{1f}^2\cos^2\theta_1 + V_{1f}^2\sin^2\theta_1 = V_{2f}^2\cos^2\theta_2 + V_{2f}\sin^2\theta_2$

or, $V_{1i}^2 - 2V_{1i}V_{1f}\cos\theta_1 + V_{1f}^2 = V_{2f}^2$

Using eq^n(iii),

$$V_{1i}^2 - 2V_{1i}V_{1f}\cos\theta_1 + V_{1f}^2 = V_{1i}^2 - V_{1f}^2$$

or, $2V_{1f}^2 = 2V_{1i}V_{1f}\cos\theta_1$

or, $V_{1f} = V_{1i}\cos\theta_1$

$\therefore V_{1f} = 300 \times \cos 30^\circ = 260$ m/s.

So, from eqn (iii),

$$V_{2f}^2 = V_{1i}^2 - V_{1f}^2 = V_{1i}^2 - (V_{1i}\cos\theta_1)^2 = V_{1i}^2\sin^2\theta_1$$

$\therefore V_{2f} = V_{1i}\sin\theta_1 = 300 \times \sin 30^\circ = 150$ m/s.

We have,

$V_{1f} = V_{1i}\cos\theta_1$ — (a)

$V_{2f} = V_{1i}\sin\theta_1$ — (b)

So, $\theta_1 + \theta_2 = 90^\circ$

$\therefore \theta_2 = 90^\circ - \theta_1$

$\therefore \theta_2 = 60^\circ$

<Q.217: A star rotates with a period of 30 days about an axis through its center. After the star undergoes supernova explosion, the stellar core, with radius 1×10^4 km collapses to neutron star of radius 3.0 km. Determine the period of rotation of the neutron star.

Soln:

Given,

Time period of star = 30 days

radius of stellar core $(r_1) = 1.0 \times 10^4$ km

radius of neutron star $(r_2) = 3.0$ km.

Period of neutron star = ?

Let 'T' = time period of star

T_i = initial time period of star

T_f = final time period of star

The angular speed of the star,

$$\omega = \frac{2\pi}{T}$$

From the conservation of angular momentum,

$$I_i \omega_i = I_f \omega_f$$

or, $$K M R_i^2 \frac{2\pi}{T_i} = K M R_f^2 \frac{2\pi}{T_f}$$

or, $$\frac{T_i}{T_f} = \left(\frac{R_f}{R_i}\right)^2$$

or, $$T_f = \left(\frac{3.0 \times 1000}{1.4 \times 10^4 \times 1000}\right)^2 \times 30 \times 24 \times 60 \times 60$$

$\therefore T_f = 0.23$ seconds.

<Q.22>: A horizontal platform in shape of circular disk rotates in a horizontal plane about frictionless vertical axle. The platform has mass $M = 100$ kg and radius $R = 2.0$ m. A student whose mass is 60 kg walks slowly from the rim of disk towards center. If angular speed is 2.0 rad/s when student is at the rim, what is angular speed when he is 0.50 m from the center.

Soln.

Given,

$m_1 = 100$ kg $\qquad$ $m_2 = 60$ kg

$R_1 = 2.0$ m $\qquad$ $R_2 = 0.50$ m

$\omega_i = 2.0$ rad/s at rim $\qquad$ $\omega_f = ?$ (at r distance)

For the conservation of angular momentum,

$$I_i \omega_i = I_f \omega_f$$

or, $$\left(\frac{1}{2} MR^2 + MR^2\right) \times \omega_i = \left(\frac{1}{2} MR^2 + Mr^2\right) \omega_f$$

or, $$\left(\frac{1}{2} \times 100 \times 2^2 + 60 \times 2^2\right) \times 2 = \left(\frac{1}{2} \times 100 \times 2^2 + 60 \times (0.5)^2\right) \omega_f$$

or, $800 = 215\, w_f$

$\therefore w_f = 4.1$ rad/s.

The angular speed will be 4.1 rad/s. at r distance.

<Q.23>: Calculate the reduced mass of Hydrogen atom.

Soln:

let m_e = mass of electron

m_p = mass of proton

m_n = mass of neutron = 0 in H-atom.

We know,

$$\frac{1}{\mu\,(\text{Reduced mass})} = \frac{1}{m_e} + \frac{1}{m_p}$$

or, $$\frac{1}{\mu} = \frac{m_p + m_e}{m_e m_p}$$

or $$\mu = \frac{m_e m_p}{m_e + m_p}$$

$$= m_e\left[1 + \frac{m_e}{m_p}\right]^{-1} \approx m_e\left[1 - \frac{m_e}{m_p}\right]$$

We know, $$\frac{m_e}{m_p} = \frac{1}{1836}.$$

So,

$$\mu = m_e\left[1 - \frac{1}{1836}\right] \approx m_e.$$

Hence, the reduced mass of ~~electron~~ hydrogen is nearly equal to the mass of electron.