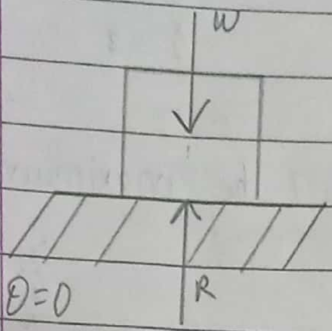
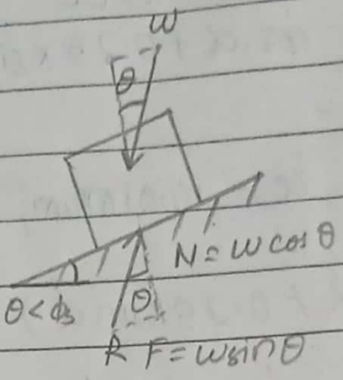
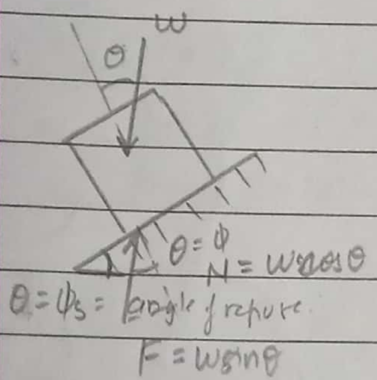
Friction in Inclined Plane

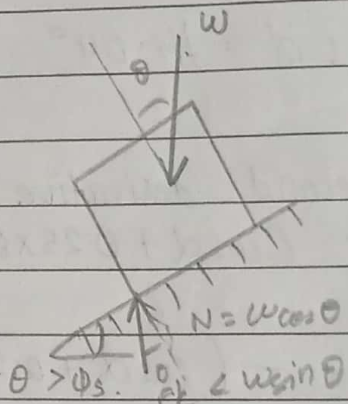




No friction ($\theta = \phi_s$)

No Motion ($\theta < \phi_s$)





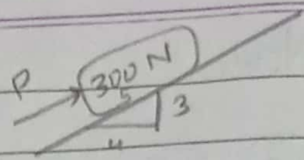
Impending motion ($\theta = \phi_s$)

Motion ($\theta > \phi_s$)

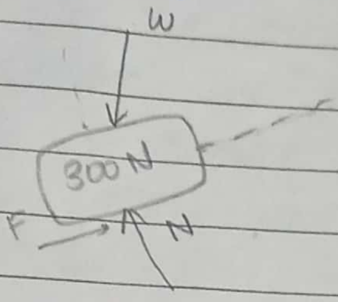
<Q>: A block 'A' rests on the inclined plane. Find the equilibrium condition of the body when applied force is:

a) $P = 0\,N$ b) $P = 100\,N$ c) $P = 120\,N$
d) $P = 140\,N$ e) $P = 200\,N$ f) $P = 240\,N$
g) $P = 280\,N$

$\mu_s = 0.25$ $\mu_k = 0.2.$



The free-body diagram is.



For (a):

when $P = 0$,

Here,

$\tan\theta = \frac{3}{4}$ $\therefore \theta = 36.87^\circ$

We know,

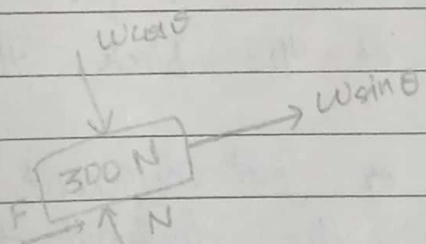
$\tan\phi_s = \mu_s$

or $\phi_s = \tan^{-1}(0.25)$ $\therefore \phi = 14.04^\circ$

Here,

angle of inclination $\geq$ angle of repose.

Hence, the body will slide down by its own weight.



Now,

$(+\uparrow)\ \sum F_y = 0$

or, $N - W\cos\theta = 0$

or, $N = \overset{60}{\cancel{300}} \times \frac{4}{\cancel{5}}$

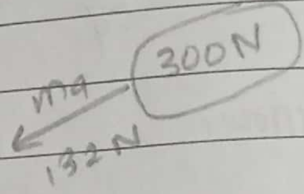
$\therefore N = 240\ N \nwarrow$

Again,

$(+)\ \sum F_x = F_k - W\sin\theta$

$= \mu_k N - W\sin\theta$

$= 0.2 \times 240 - 300 \times \frac{3}{5}$



$\therefore \sum F_x = 132\ N \swarrow$

Here, body slides down with force of 132 N.

/* As we move from 0 N to ~~[illegible]~~ 280 N, the direction of forces remains constant for all forces except frictional force F_f.

If P is not sufficient or just sufficient to balance the weight of the block, the frictional force acts upward.

If P is large or just sufficient to overcome weight of the block, the frictional force acts downward.

Initial eqⁿ: $(\overset{+}{\nearrow})\ \sum F_x = P \pm F_f - W\sin\theta$

$= P \pm 60 - 180$

According to the tendency of motion of the body, the value of direction and ~~friction~~ friction of changes.

As P increases, it is not sufficient but to prevent body from sliding down,

But for certain level of P, the motion of block is just balanced but the tendency of motion is still downward.

After P is increased again, the tendency of motion of the body changes but the ~~place is~~ block is still balanced.

After further increasing P, the block accelerates ~~down the~~ up the block. */

For various ~~forward~~ forces:

P	Initial Equation $\sum F_x = P \pm 60 - 180$	Motion Remarks	Actual Equation $\sum F_x = P \pm F - 180$	Actual Resultant.
0	$= 0 + 60 - 180 = -120$	slides down	$\sum F_x = P + F_k - 180$	$= 0 + 48 - 180 = -132$
100	$= 100 + 60 - 180 = -20$	slides down	$\sum F_x = P + F_k - 180$	$= 100 + 48 - 180 = -32$
120	$= 120 + 60 - 180 = 0$	prevented from sliding down	$\sum F_x = P + F_s - 180$	$= 120 + 60 - 180 = 0$
140	$= 140 - 60 - 180 = -100$	static equilibrium	$\sum F_x = P + F - 180$	$= 140 + 40 - 180 = 0$
200	$= 200 - 60 - 180 = -40$	static equilibrium	$\sum F_x = P - F - 180$	$= 200 - 20 - 180 = 0$
240	$= 240 - 60 - 180 = 0$	Just sliding up the plane.	$\sum F_x = P - F_s - 180$	$= 240 - 60 - 180 = 0$
280	$= 280 - 60 - 180 = 40$	slides up.	$\sum F_x = P - F_k - 180$	$= 280 - 48 - 180 = 52$

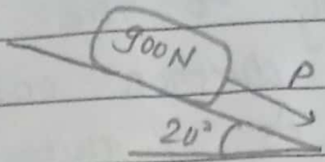
Here, $P < 120$, block slides down.

$P = 120$, just prevented from sliding down; $120 < P < 180$, static equilibrium, upward F_f

$P = 180$, block has no F_f; $180 < P < 240$, static eqm., downward F_f.

$P = 240$, just sliding up, $P > 240 \rightarrow$ block slides up.

<Q>: Find the force 'P' required to move the block down the plane. The force 'P' is applied parallel to the plane.



Weight of block = 900 N.
$\mu = 0.5$.

Solⁿ.

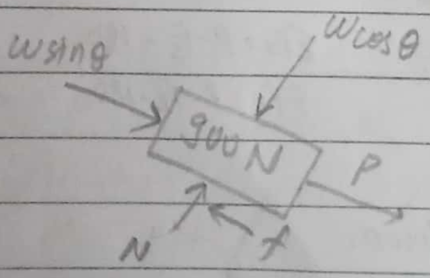
Here,
angle of ~~repose~~ inclination $(\theta) = 20°$

Given,
$\mu = 0.5$ $\qquad$ $W = 900\ N$

~~We k~~ Let angle of repose be ϕ.
We know,
$\mu = \tan\phi$
or, $\phi = \tan^{-1}(\mu) = \tan^{-1}(0.5)$
$\therefore \phi = 26.56$.

Here, $\theta < \phi_s$, so ~~thu~~ some force has to be applied for block to slide down.

The free body diagram;



impending motion

Here,

(+→) $\sum F_x = 0$

or, $P - f + W\sin\theta = 0$

$\therefore P = f - W\sin\theta$ — (i)

Also.

(+↑) $\sum F_y = 0$

or, $N - W\cos\theta = 0$

$\therefore N = W\cos\theta$ — (ii).

For impending motion; $f = \mu N$.

So, eq^n(i) becomes

$P = \mu N - W\sin\theta$

$= \mu W\cos\theta - W\sin\theta$

$= W(\mu\cos 20^\circ - \sin 20^\circ)$

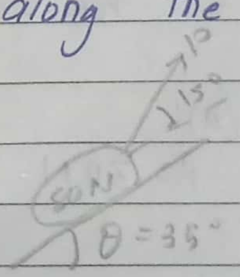
$= 900\,(0.25 \times \cos 20^\circ - \sin 20^\circ)$

$\therefore P = 115.04\ N$

<Q7>: A block 80 N is pulled up the smooth plane. Determine the acceleration along the plane

a) $P = 30\ N$

b) $P = 75\ N$.



Sol^n:

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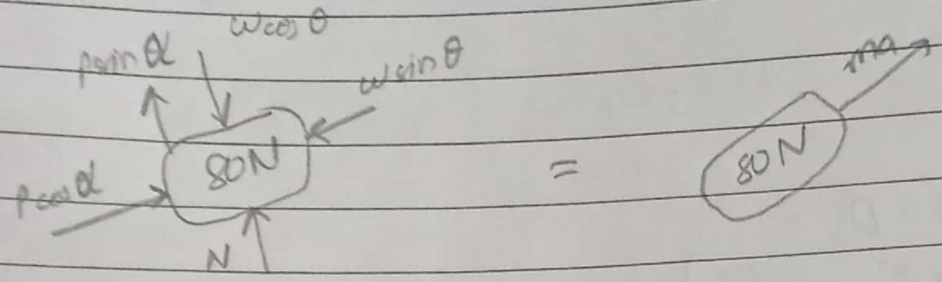
Given;

$W = 80N$

Since the plane is smooth, $f = 0$.

$m = \frac{80}{9.81} = 8.15$ kg

The free body diagram;



Let the ~~body of the~~ acceleration of the body be a.

~~For (a)~~: From free body diagram;

~~P= 30 N~~.

$\sum F_y = 0$

or, $N + P\sin\alpha - W\cos\theta = 0$

or $N = W\cos\theta - P\sin\alpha$ ——— (i)

$\sum F_x = ma.$

$P\cos\alpha - W\sin\theta = ma.$

or $a = \frac{P\cos\alpha - W\sin\theta}{m}$ — (ii)

For (a):

· When $P = 30N$

Date :

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$$a = \frac{\cancel{P}30\cos 15^\circ - 80\times\sin 35^\circ}{8.15} = -2.07 \text{ m/s}$$

$\therefore a = 2.07$ m/s ($\swarrow$)

$N = 80\times\cos 35^\circ - 30\times\sin 15^\circ = 57.77$ N.

For (b): $P = \cancel{3}70$ N

$$a = \frac{70\cos 15^\circ - 80\times\sin 35^\circ}{8.15}$$

$\therefore a = 3.26$ m/s^2 ($\longrightarrow$)

$N = 80\cos 35^\circ - 75\sin 15^\circ$

$\therefore$ ~~N = 46.132N~~ $\therefore N = 46.12$ N.