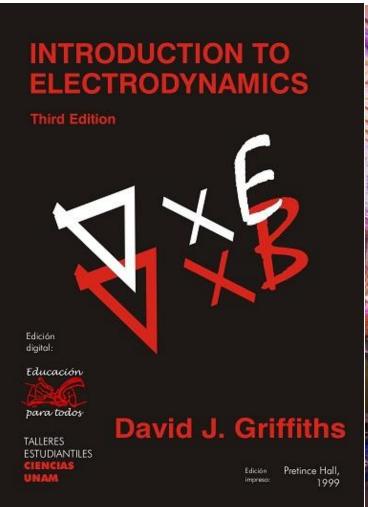
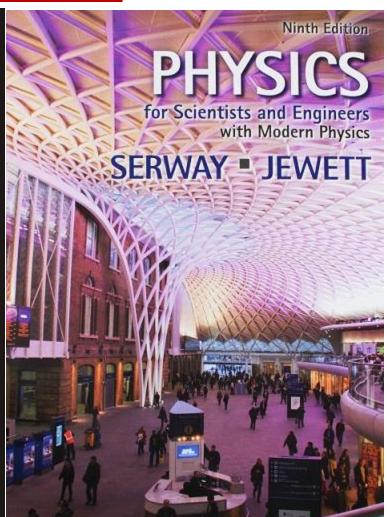
## **PHYSICS**







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# **Course Outline**





- Magnetic Dipole
- Force and Torque on Magnetic Dipole
- Energy of Magnetic Dipole in Magnetic Field
- Magnetization
- Bound Current
- Ampere's Law in Magnetization

# **Magnetic Dipole**



#### **Magnetic Dipole:**

Current loops are Magnetic Dipoles.

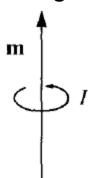


Figure S-I
Magnetic Dipole
(Ampere Model)

• Electronic orbit behaves like magnetic dipole.

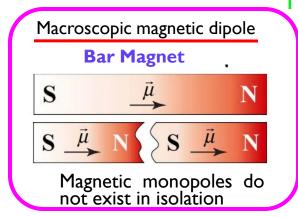
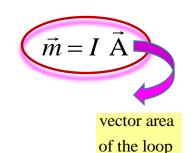




Figure S-2
Magnetic Dipole
(Gilbert Model)

#### **Magnetic Dipole Moment:**

Magnetic dipole moment is a vector pointing out of the plane of the current loop and with a magnitude equal to the product of the current and loop area:



- The area vector, and thus the direction of the magnetic dipole moment of a current-carrying loop, is given by a right-hand rule.
- The SI unit of magnetic dipole moment is

ampere meter squared  $|\mathbf{A} \cdot \mathbf{m}^2|$ 



Figure S-3
Magnetic
Dipole
a currentcarrying coil

## **Sample Problem**



Find the magnetic dipole moment of the "bookend-shaped" loop shown in Figure MD-1. All sides have length W, and it carries a current I.

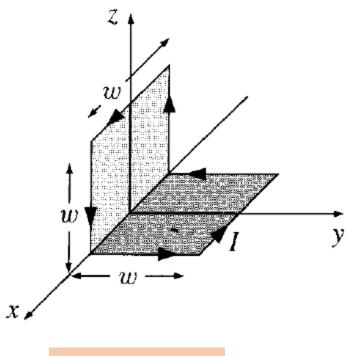
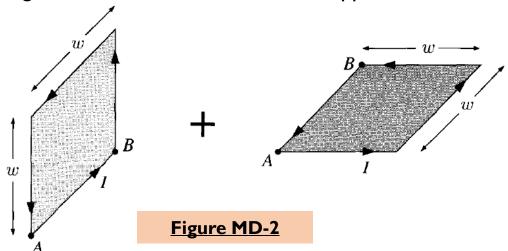


Figure MD-I

#### **Solution:**

• The wire could be considered the superposition of two plane square loops (Figure MD-I). The "extra" sides (AB) cancel when the two are put together, since the currents flow in opposite directions.



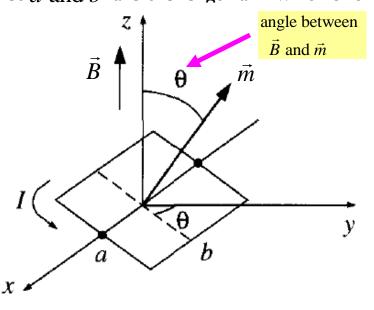
• The net magnetic dipole moment is  $\vec{m} = I \ w^2 \ \hat{j} + I \ w^2 \hat{k}$ ; its magnitude is  $m = \sqrt{2}I \ w^2$ , and it points along the line  $45^0 \ \text{line} \ z = y$ .

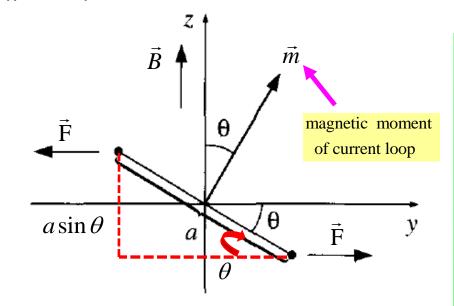
## Force and Torque on Magnetic Dipole



#### Force and Torque on Magnetic Dipole

• Consider a rectangular loop of wire carrying a current I placed in a uniform magnetic field B. Let a and b are the length and width of the rectangular loop.





The forces on the two slopping sides cancel (they tend to stretch the loop, but they don't rotate it).

The equal and opposite forces on the "horizontal" sides generate a torque.

Magnetic force on a current carrying conductor  $\vec{F}_{P} = I \ \vec{L} \times \vec{B}$ 

#### Figure MD-3

In a uniform field, the **net force** on a current loop is zero:

(a)

$$\vec{F} = I \oint (d\vec{l} \times \vec{B}) = I (\oint d\vec{l}) \times \vec{B} = 0$$

In a uniform field, the **net torque** on a current loop is

(b)

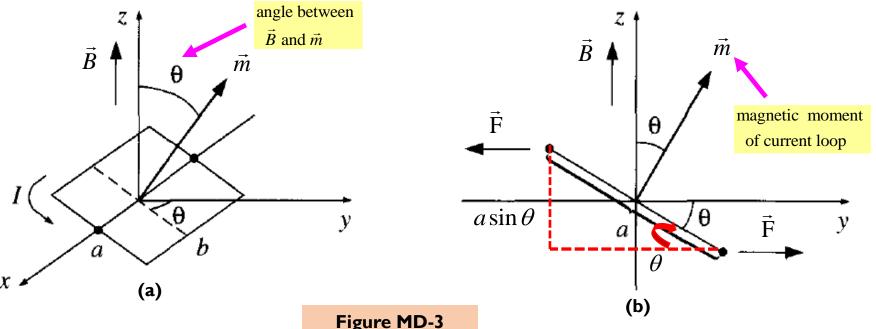
$$\vec{\tau} = F(a\sin\theta)\hat{i}$$
 =  $(BIb)(a\sin\theta)\hat{i}$   
=  $[(Iab)(B)\sin\theta]\hat{i}$  =  $[mB\sin\theta]\hat{i}$   $\vec{\tau} = \vec{m} \times \vec{B}$ 

# Potential Energy of a Magnetic Dipole in a Magnetic Field



#### Potential Energy of a Magnetic Dipole in a Magnetic Field

• Consider a rectangular loop of wire carrying a current I placed in a uniform magnetic field B. Let a and b are the length and width of the rectangular loop.



The torque on a magnetic dipole in a uniform magnetic field:

$$\vec{\tau} = \vec{m} \times \vec{B}$$

#### Figure MD-3

The work done by the external field in turning the dipole from  $\theta_0$  to a final angle  $\theta$  is

# Potential Energy of a Magnetic Dipole in a Magnetic Field



### Potential Energy of a Magnetic Dipole in a Magnetic Field

The change in potential energy of the system of field + dipole,

$$\Delta U \equiv U(\theta) - U(\theta_0) = -\mathbf{W}$$

$$=-mB(\cos\theta-\cos\theta_0)$$

• We arbitrary define the reference angle  $\theta_0$  to be 90° and potential energy  $U(\theta_0)$  to be zero at that angle. At any angle  $\theta$ , the potential energy is then

$$|U(\theta) = -\vec{m} \cdot \vec{B}| \qquad \dots (3)$$

Therefore the potential energy II of an electric dipole with dipole moment  $\vec{m}$  in a uniform external electric field

$$ec{B}$$
 is

$$U = -\vec{m} \cdot \vec{B}$$

The potential energy of the dipole is least when  $\theta = 0$ .

(  $\vec{m}$  and  $\vec{B}$  are in the same direction)

The potential energy is greatest when  $\theta = 180^{\circ}$ (  $\vec{m}$  and B are in opposite directions).

Potential energy can be associated with the orientation of an magnetic dipole in a magnetic field.

If an applied torque ( due to an "external agent") rotates a magnetic dipole rotates from an initial orientation  $\theta_i$  to another orientation  $\theta_f$ , the work  $W_a$  done on the dipole by the applied torque is

$$\mathbf{W}_{a} = -\mathbf{W} = U_{f} - U_{i} = U(\theta_{f}) - U(\theta_{i})$$

$$U(\theta) = -mB\cos\theta$$

## **Sample Problem**



#### Sample Problem

#### Rotating a magnetic dipole in a magnetic field

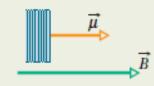
Figure 28-21 shows a circular coil with 250 turns, an area A of  $2.52 \times 10^{-4}$  m<sup>2</sup>, and a current of 100  $\mu$ A. The coil is at rest in a uniform magnetic field of magnitude B = 0.85 T, with its magnetic dipole moment  $\vec{\mu}$  initially aligned with  $\vec{B}$ .

(a) In Fig. 28-21, what is the direction of the current in the coil?

**Right-hand rule:** Imagine cupping the coil with your right hand so that your right thumb is outstretched in the direction of  $\vec{\mu}$ . The direction in which your fingers curl around the coil is the direction of the current in the coil. Thus, in the wires on the near side of the coil—those we see in Fig. 28-21—the current is from top to bottom.

B How much work would the torque applied by an external agent have to do on the coil to rotate it 90° from its ini-

Fig. 28-21 A side view of a circular coil carrying a current and oriented so that its magnetic dipole moment is aligned with magnetic field  $\vec{B}$ .



tial orientation, so that  $\vec{\mu}$  is perpendicular to  $\vec{B}$  and the coil is again at rest?

#### **KEY IDEA**

The work  $W_a$  done by the applied torque would be equal to the change in the coil's orientation energy due to its change in orientation.

**Calculations:** From Eq. 28-39 ( $W_a = U_f - U_i$ ), we find

$$W_a = U(90^\circ) - U(0^\circ)$$
=  $-\mu B \cos 90^\circ - (-\mu B \cos 0^\circ) = 0 + \mu B$   
=  $\mu B$ .

Substituting for  $\mu$  from Eq. 28-35 ( $\mu = NiA$ ), we find that

$$W_a = (NiA)B$$
= (250)(100 × 10<sup>-6</sup> A)(2.52 × 10<sup>-4</sup> m<sup>2</sup>)(0.85 T)
= 5.355 × 10<sup>-6</sup> J ≈ 5.4  $\mu$ J. (Answer)

# **Magnetization**



## **Magnetization:**

- All magnetic phenomena are due to electric charges in motion.
- All matter consists ultimately of atoms, and each atom consists of electrons in motion.

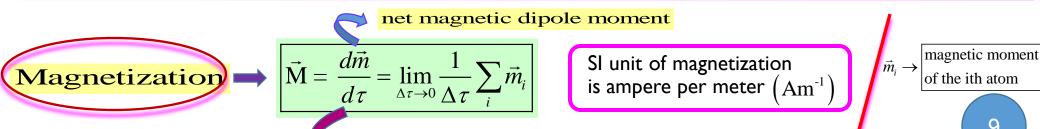
If you could examine a piece of magnetic material on an atomic scale you would find tiny currents: electrons orbiting around nuclei and electrons spinning about their axes. For macroscopic purposes, these current loops are so small that we may treat them as magnetic dipoles. Ordinarily, they cancel each other out because of the random orientation of the atoms. But when a magnetic field is applied, a net alignment of these magnetic dipoles occurs, and the medium becomes magnetically polarized, or magnetized.

In the presence of a magnetic field, matter becomes magnetized; that is, upon microscopic examination it will be found to contain many tiny dipoles, with a net alignment along some direction.

The state of magnetic polarization is described by the vector quantity

 $M \equiv$  magnetic dipole moment per unit volume.

M) is called the magnetization; it plays a role analogous to the polarization  $\vec{P}$  in electrostatics.



an elemental volume of the material

## **Bound Current**

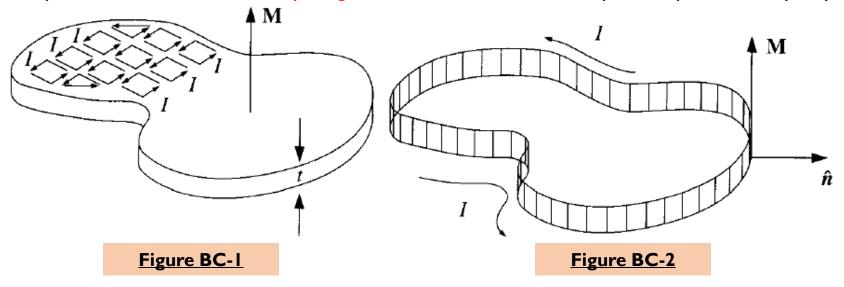


#### **Bound Currents:**

- The field of a magnetized object is equivalent to the field produced by a volume current  $\vec{J}_b = \nabla \times \vec{M}$  throughout the material, plus a surface current  $\vec{K}_b = \vec{M} \times \hat{n}$ , on the boundary.
- Bound currents result from the contribution of all (aligned) magnetic dipoles.

#### **Physical Interpretation of Bound Currents**

Figure BC-I depicts a thin slab of uniformly magnetized material, with the dipoles represented by tiny current loops.



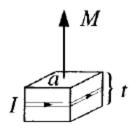
- All the "internal" currents cancel. At the edge, there is no adjacent loop to do the canceling.
- The whole thing, then, is equivalent to a single ribbon of current flowing around the boundary [Figure BC-2].

## **Physical Interpretation of Bound Current**



#### **Surface Bound Current:**

• Let's consider that each of tiny loops has area a and thickness t [Figure BC-3].

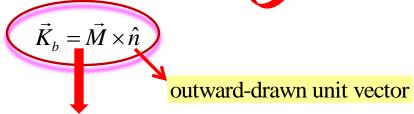


#### Figure BC-3

Surface Bound Current

magnetic dipole moment

$$K_b = \frac{I}{t} = \frac{m}{a} = \frac{m}{at} = M$$
Magnetization



This expression also records the fact that there is no current on the top or bottom surface of the slab.

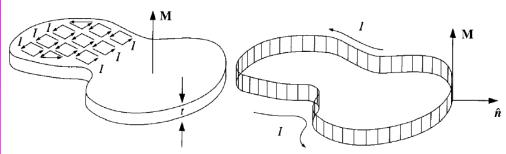


Figure BC-I depicts a thin slab of uniformly magnetized material, with the dipoles represented by tiny current loops.

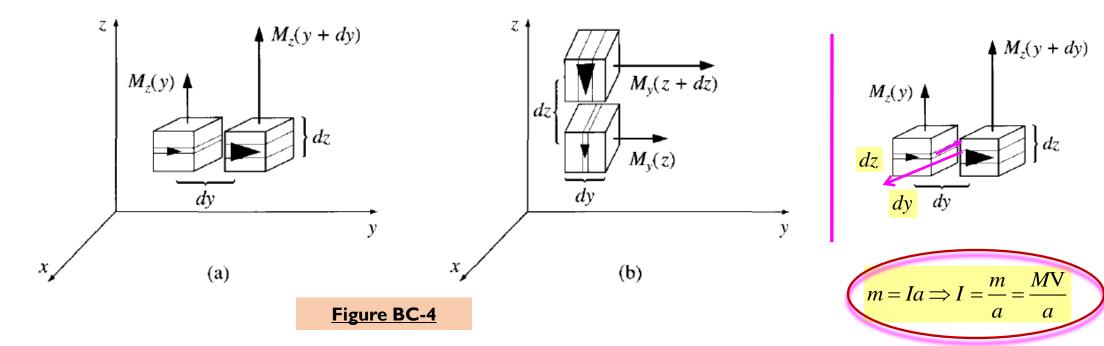
It is a peculiar kind of current, in the sense that no single charge makes the whole trip on the contrary, each charge moves only in a tiny loop within a single atom. Nevertheless, the net effect is a macroscopic current flowing over the surface of the magnetized object. We call it a "bound" current to remind ourselves that every charge is attached to a particular atom, but it's a perfectly genuine current, and produces a magnetic field in the same way any other current does.

## **Physical Interpretation of Bound Current**



## **Volume Bound Current Density:**

- When the magnetization is nonuniform, the internal currents no longer cancel.
- Figure BC-4 shows two adjacent chunks of magnetized material, with a larger arrow on the one to the right suggesting greater magnetization at that point.



• On the surface where they join [Figure BC-4 (a)], there is a net current in the positive x-direction, given by

$$I_{+x} = \frac{M_z(y + dy)dxdydz - M_z(y)dxdydz}{dxdy} = [M_z(y) + \frac{\partial M_z}{\partial y}dy - M_z(y)]dz = \frac{\partial M_z}{\partial y}dydz$$

## **Physical Interpretation of Bound Current**



## **Volume Bound Current Density:**

• Therefore, the corresponding volume current density is  $(J_b)_{+x} = \frac{\partial M_z}{\partial v}$ .  $(J_b)_{+x} = \frac{I_{+x}}{dvdz}$ 

$$\left[ \because \left( J_b \right)_{+x} = \frac{I_{+x}}{dydz} \right]$$

Again, on the surface where they join [Figure BC-4(b)], there is a net current in the negative x-direction, given by

$$I_{-x} = \frac{M_{y}(z+dz)dxdydz - M_{y}(z)dxdydz}{dxdz} = [M_{y}(z) + \frac{\partial M_{y}}{\partial z}dz - M_{y}(z)]dy = \frac{\partial M_{y}}{\partial z}dydz$$

The corresponding volume current density is therefore  $(J_b)_{-x} = \frac{\partial M_y}{\partial z}$ .  $(J_b)_{-x} = \frac{I_{-x}}{\partial y \partial z}$ 

$$\left[ \because \left( J_b \right)_{-x} = \frac{I_{-x}}{dydz} \right]$$

So

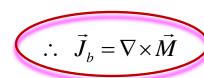
$$(J_b)_x = (J_b)_{+x} - (J_b)_{-x} = \frac{\partial M_z}{\partial y} - \frac{\partial M_y}{\partial z}.$$

The volume current Density:  $\vec{J}_b = (\vec{J}_b)_x \hat{i} + (\vec{J}_b)_y \hat{j} + (\vec{J}_b)_z \hat{k} = (\frac{\partial M_z}{\partial v} - \frac{\partial M_y}{\partial z}) \hat{i} + (\frac{\partial M_x}{\partial z} - \frac{\partial M_z}{\partial v}) \hat{j} + (\frac{\partial M_y}{\partial z} - \frac{\partial M_z}{\partial v}) \hat{j} + (\frac{\partial M_y}{\partial z} - \frac{\partial M_z}{\partial v}) \hat{k}$ 

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M_x & M_y & M_z \end{vmatrix} = \nabla \times \vec{M}$$

$$\therefore \vec{J}_b = \nabla \times \vec{M}$$

$$\therefore \vec{J}_b = \nabla \times \vec{M}$$



$$\nabla \cdot \vec{J}_b = \nabla \cdot \left( \nabla \times \vec{M} \right) = 0$$

# **Ampere's Law in Magnetized Materials**

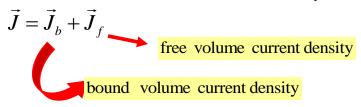


### **Ampere's Law in Magnetized Materials**

- The effect of magnetization is to establish bound currents  $\vec{J}_b = \nabla \times \vec{M}$  within the material and  $\vec{K}_b = \vec{M} \times \hat{n}$  on the surface. The field due to the magnetization of the medium is just the field produced by these bound currents.
- Differential form of Ampere's law:

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

• In the magnetized materials, the total volume current density can be written as



So

$$\nabla \times \vec{B} = \mu_0 (\vec{J}_b + \vec{J}_f)$$

$$or, \qquad \nabla \times \frac{1}{\mu_0} \vec{B} = \vec{J}_b + \vec{J}_f$$

or, 
$$\nabla \times \frac{1}{\mu_0} \vec{B} = \nabla \times \vec{M} + \vec{J}_f$$

or, 
$$\nabla \times \left(\frac{1}{\mu_0} \vec{B} - \vec{M}\right) = \vec{J}_f$$
  

$$\therefore \nabla \times \vec{H} = \vec{J}_f \quad \text{where}$$

 $\vec{H} = \frac{1}{\mu_0} \vec{B} - \vec{M}$  is the auxiliary field

In Integral Form,

$$\int_{s} \left( \nabla \times \vec{H} \right) . d\vec{a} = \int_{s} \vec{J}_{f} \cdot d\vec{a}$$

 $\therefore \oint \vec{H} \cdot d\vec{l} = I_{f_{enc}}$ 

total free current passing through the Amperian loop

## **Linear Media**



#### **Magnetic Susceptibility and Permeability:**

• For most substances, the magnetization is proportional to the field, provided the field is not too strong.

The constant of proportionality is called the **magnetic susceptibility**; it is a dimensionless quantity that varies from one substance to another.

- Materials that obey Eq. (1) are called a linear media.
- For Linear Media,

$$\vec{B} = \mu_0 (\vec{H} + \vec{M}) = \mu_0 (\vec{H} + \chi_m \vec{H}) = \mu_0 (1 + \chi_m) \vec{H}$$

$$\vec{B} = \mu \vec{H}$$

where  $\mu \equiv \mu_0 (1 + \chi_m)$ 

permeability of the material



permeability

the free space

<u>In a homogeneous linear material</u>, the volume bound current density is proportional to the free current density:

$$ec{J}_{b} = 
abla imes ec{M} = 
abla imes \left( \chi_{m} ec{H} 
ight) = \chi_{m} ec{J}_{f}$$

# Diamagnetic, Paramagnetic & Ferromagnetic Substances



## Diamagnetic Substances:

 Diamagnetic substances are those in which the magnetic moment is weak and opposite the applied magnetic field.

Examples: - Bismuth, Gold, Silver, Copper

- Diamagnetic substances are weakly repelled by a magnet.
- The magnetic susceptibility is negative for diamagnets.

#### **Paramagnetic Substances:**

 Paramagnetic substances are those in which the magnetic moment is weak and in the same direction as the applied magnetic field.

Examples:- Aluminum, platinum, oxygen

- They are feebly attracted by magnets.
- The magnetic susceptibility is positive and low for paramagnetic substances

#### Ferromagnetic Substances:

- Ferromagnetic substance exhibit strong magnetic effects.
- In ferromagnetic substances, interaction between atoms cause magnetic moments to align and create a strong magnetization that remains after the external field is removed.

Examples: - Iron, Nickel, Cobalt

- They are strongly attracted by magnets.
- The magnetic susceptibility is positive and high for ferromagnetic substances.
- Ferromagnetism, like Paramagnetism, occurs in materials in which the atoms have permanent magnetic dipole moments. What distinguishes ferromagnetic materials from paramagnetic materials is that in ferromagnetic materials there is a strong interaction between neighboring atoms that keeps their dipole moments aligned even when the external magnetic field is removed.
- The temperature at which a ferromagnetic material becomes paramagnetic is called its Curie temperature. The Curie temperature of iron, for instance, is 770°C; above this temperature, iron is paramagnetic.

## **Ferromagnetic Domains**

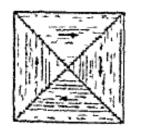


## **Ferromagnetic Domains**

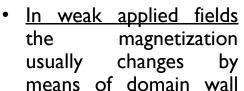
- The presence of domains was first postulated by Weiss in 1907.
- All ferromagnetic materials are made up of microscopic regions called domains.

The domain is an area within which all magnetic moments are aligned.

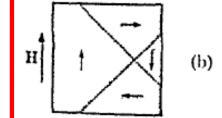
- The boundaries between various domains having different orientations are called domain walls.
- In an unmagnetized sample, each domain contains billions of dipoles, all lined up, but the domains themselves are randomly oriented so that the net magnetic moment is zero.
- The increase in magnetization resulting from the action of an applied magnetic field takes place by two independent processes:
  - (a) By an increase in the volume of domains that are favorably oriented relative to the field at the expense of domains that are unfavorably oriented (domain wall motion)
  - (b) By rotation of the domain magnetization toward the field direction.

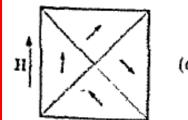






motion.





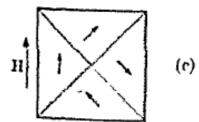


Figure FD-I

In stronger fields the magnetization proceeds by irreversible motion, and finally by domain rotation: these circumstances the substance remains magnetized when the external magnetic field is removed.

Magnetization of a ferromagnetic material:

- (a) unmagnetized,
- (b) magnetization by domain wall motion,
- (c) magnetization by domain rotation

## **Hysteresis Loop**

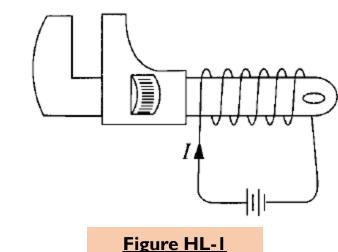


### **Hysteresis Loop**

- Wrap a coil of wire around the object to be magnetized (Figure HL-I).
- Run a current I through the coil; this provides the external magnetic field (pointing to the left in the diagram).
- As you increase the current, the field increases, the domain boundaries move, and the magnetization grows. Eventually, you reach the saturation point, with all the dipoles aligned, and a further increase in current has no effect on (Figure HL-2, point b).
- If you reduce the current, M decreases, but even with the current off there is some residual magnetization (point c).

The wrench is now a permanent magnet.

- If you want to eliminate the remaining magnetization, you'll have to run a current backwards through the coil (a negative I). Now the external field points to the right, and as you increase I (negatively), M drops down to zero (point d). If you turn I still higher, you soon reach saturation in the other direction-all the dipoles now pointing to the right (point e). At this stage switching off the current will leave the wrench with a permanent magnetization to the right (point f).
- To complete the story, turn I on again in the positive sense: Mreturn to zero (point g), and eventually to the forward saturation point (b).



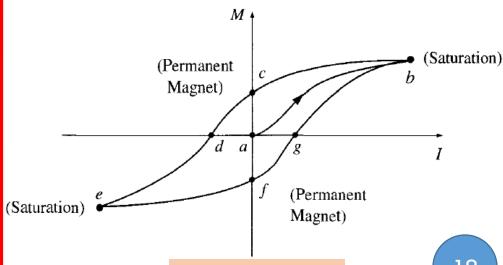


Figure HL-2

## **Hysteresis Loop**



## **Hysteresis Loop**

• If the material is initially unmagnetized at 'o' it will reach saturation at 'b' as H is increased. As the applied field is reduced and again increased the loop 'abcdefa' is formed.

The curve of Figure HL-3 is called hysteresis loop of the material.

#### **Retentivity or Remanence:-**

The value of magnetization of the material, when the magnetizing field is reduced to zero.

#### **Coercive force or Coercivity:-**

The value of reverse magnetizing field needed to reduce residual magnetization to zero.

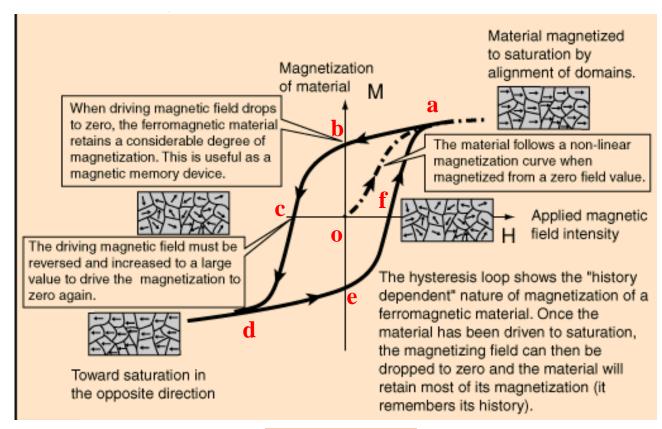


Figure HL-3

## **Hysteresis Loop**



## **Hysteresis Loop**

• Actually, it is customary to draw hysteresis loops as plots of B against H, rather than M against I or H

If our coil is approximated by a long solenoid, with n turns per unit length, then H=n I, so H and I are proportional. Meanwhile,  $\vec{B}=\mu_0 \left(\vec{H}+\vec{M}\right)$  but in practice M is huge compared to H, so to all intents and purposes B is proportional to M.

**Retentivity**: The value of B at point b.

**Coercivity**: The magnitude of H at point c.

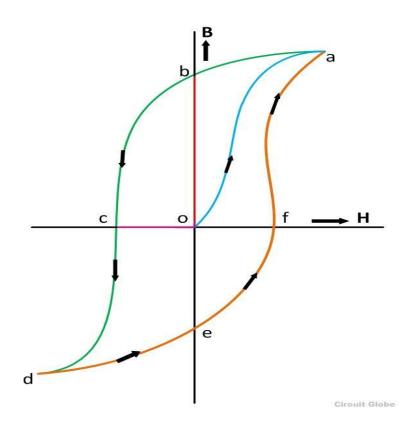


Figure HL-3
Typical hysteresis loop of a ferromagnetic material

# **Hysteresis Loss**



## **Hysteresis Loss**

- The amount of energy lost (in the form of heat) per unit volume of ferromagnetic substance when the substance undergoes one cycle of magnetization is known as **hysteresis loss**.
- Consider a unit volume of ferromagnetic substance.

Let  $\vec{m}_i$  is the magnetic moment of  $i^{th}$  atomic dipole which makes an angle  $\theta_i$  with the field H .

Only the component of  $\vec{m}_i$  along the direction of field contributes to the magnetization.

$$\therefore \text{ Magnetization, } M = \sum_{i=1}^{N} m_i \cos \theta_i$$

where  $N \rightarrow$  the number of atomic dipoles in the given unit volume of substance

• Differentiating both sides, we get

$$dM = -\sum_{i=1}^{N} m_i \sin \theta_i d\theta_i \qquad \dots (1)$$

Each dipole experiences a torque and work done by torques on all dipoles in one complete cycle of magnetization is hysteresis loss.

# **Hysteresis Loss**



### **Hysteresis Loss**

Therefore,

$$\therefore \text{ Hysteresis loss} = \oint B_0 \left( \frac{dB}{\mu_0} - dH \right)$$

$$= \oint \mu_0 H \left( \frac{dB}{\mu_0} - dH \right) = \oint H dB - \mu_0 \oint H dH$$

$$= \oint H dB \qquad \left[ \because \oint H dH = 0 \right]$$

$$= \text{Area of B} - H \text{ curve} \qquad ......(5)$$

Also,  $H = \frac{B}{\mu_0} - M$ [The net magnetic field,  $\vec{B} = \vec{B}_0 + \vec{B}_M = \mu_0 \vec{H} + \mu_0 \vec{M}$ ]  $\vec{B}_M = \text{field produced by the dipoles}$   $\Rightarrow dH = \frac{dB}{\mu_0} - dM$   $\Rightarrow dM = \frac{dB}{\mu_0} - dH \dots (4)$ 

#### **Notes:**

- The word hysteresis derives from a Greek verb meaning "to lag behind".
- Domain formation is the necessary feature of ferromagnetism.
- The area of the *B-H* hysteresis loop is an indication of the energy dissipated per cycle.

# **Sample Problems**



- Which of the following statement is NOT CORRECT?
  - [a] Ferromagnetic material becomes paramagnetic above Curie temperature.
  - [b] Domain formation is the necessary feature of ferromagnetism.
  - [c] The area of the B-H hysteresis loop is an indication of the energy dissipated per cycle.
  - [d] The magnetic susceptibility is positive and low for diamagnets.

Ans: [d]

- The area of M-H hysteresis loop is an indication of the
  - [a] retentivity of the materials.
  - [b] permeability of material.
  - [c] energy dissipated per unit volume of the substance per cycle.
  - [d] susceptibility of materials.

Ans: [c]

- The magnetization left on a ferromagnetic material after the removal of magnetizing field once the saturation has been reached is called
  - [a] hysteresis loss.

[b] retentivity.

[c] coercive force.

[d] magnetic susceptibility.

# **Text Books & References**



- I. David J. Griffith, Introduction to Electrodynamics
- 2. R.A. Serway and J.W. Jewett, Physics for Scientist and Engineers with Modern Physics
- 3. Halliday and Resnick, Fundamental of Physics
- 4. D. Halliday, R. Resnick, and K. Krane, Physics, Volume 2, Fourth Edition



