# General Physics I (PHYS 101)

# Lecture 14

# Interference


Keshav Raj Sigdel

Assistant Professor

Department of Physics

Kathmandu University


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# Outline

# Coherent Source

Two light sources are said to be coherent if they emit continuous light waves of the same frequency, nearly equal or equal amplitude and same or constant phase difference.

The two sources of light must emit radiations of the same color (wavelength). The phase relation between the waves at the time of emission rapidly changes with time, not only in different sources but even in different parts of the same source. As a result there is rapid change in brightness and darkness, which produce general illumination. It is not possible to produce interference with two independent sources which cannot be coherent. Two virtual sources

# Coherent Source (contd.)

formed from a single source can act as coherent sources. Coherent sources are generally practiced in the following ways

1. Two virtual images of the same source produced by reflection as in Fresnel's Bi-prism.

2. One real source and its virtual image produced by reflection as in Lloyd's mirror.

3. Two real images of the same source produced from refraction as in Billet's split lens.

4. By dividing the amplitude of a portion of wave front either by reflection or by refraction as in Newton's ring and Michelson's interferometer.

# Interference

The phenomenon of getting dark and bright fringes due to superposition of two coherent light sources is called interference. There are two types of interference, constructive interference and destructive interference.

*Constructive interference:* The phenomenon of getting bright fringes due to superposition of crest of one wave to crest of the other is called constructive interference.

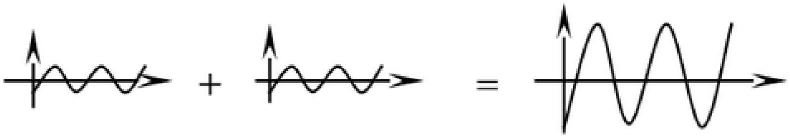

Figure 1

# Interference (contd.)

*Destructive interference:* The phenomenon of getting dark fringes due to superposition of trough of one wave to crest of the other is called destructive interference.

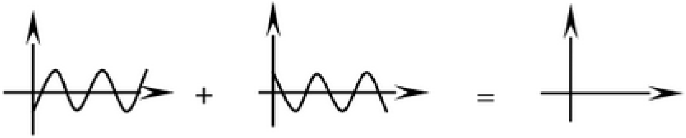

Figure 2

# Phase difference and path difference:

For phase difference of $2\pi$ path difference is $\lambda$.

Therefore, for phase difference of $\delta$ path difference is $\frac{\lambda}{2\pi}\delta$.

Also, for path difference of $\lambda$ phase difference is $2\pi$.

So for path difference of $x$ phase difference is $\frac{2\pi}{\lambda}x$.

# Superposition of two waves

Let us consider two waves with amplitude $a_1$ and $a_2$ having constant phase difference of $\delta$ and frequency $\omega$. In complex form, these waves are represented as

$$y_1 = a_1 e^{i\omega t}$$
$$y_2 = a_2 e^{i(\omega t+\delta)}$$

After superposition, the resultant wave take the form

$$\begin{aligned} y &= y_1 + y_2 \\ &= a_1 e^{i\omega t} + a_2 e^{i(\omega t+\delta)} \\ \implies y &= \left(a_1 + a_2 e^{i\delta}\right) e^{i\omega t} \end{aligned} \tag{1}$$

# Superposition of two waves (contd.)

Let the resultant wave has the amplitude $R_0$ and phase difference with respect to the first wave is $\phi$, then it can be written as $y = R_0 e^{i(\omega t+\phi)} = R_0 e^{i\phi} e^{i\omega t}$. Comparing this with equation (1), we get

$$R_0 e^{i\phi} = a_1 + a_2 e^{i\delta}$$

$$R_0 \cos\phi + iR_0 \sin\phi = a_1 + a_2 \cos\delta + ia_2 \sin\delta \tag{2}$$

Equating real and imaginary part of equation (2) we get

$$R_0 \cos\phi = a_1 + a_2 \cos\delta \tag{3}$$

$$R_0 \sin\phi = a_2 \sin\delta \tag{4}$$

## Superposition of two waves (contd.)

Squaring and adding equation (3) and (4), we get

$$R_0^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos\delta \tag{5}$$

Dividing equation (4) by (3), we get

$$\frac{\sin\phi}{\cos\phi} = \frac{a_2 \sin\delta}{a_1 + a_2\cos\delta} \implies \phi = \tan^{-1}\left(\frac{a_2\sin\delta}{a_1 + a_2\cos\delta}\right) \tag{6}$$

Since the intensity of a wave is directly proportional to the square of the amplitude, then in term of intensity equation (5) can be written as

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta \tag{7}$$

where, $I \propto R_0^2$ is the intensity of the resultant wave, $I_1 \propto a_1^2$ is the intensity of first and $I_2 \propto a_2^2$ is that of second waves.

## Superposition of two waves (contd.)

**Maximum intensity:-** The intensity is maximum when $\cos\delta = 1 \implies \delta = 2n\pi$ for $n = 0, 1, 2, \cdots$. That means the intensity is maximum when the phase difference between superposing waves is equal to the even integral multiplication of $\pi$. The maximum intensity is

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2} \tag{8}$$

$$\text{or, } I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 \tag{9}$$

# Superposition of two waves (contd.)

Minimum intensity:- The intensity is minimum when $\cos\delta = -1 \implies \delta = (2n+1)\pi$ for $n = 0, 1, 2, \cdots$. That means the intensity is minimum when the phase difference between the superposing waves is odd integral multiple of $\pi$. The minimu intensity is

$$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2} \tag{10}$$

$$\text{or, } I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 \tag{11}$$

We also have,

$$\frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{I_1}+\sqrt{I_2}\right)^2}{\left(\sqrt{I_1}-\sqrt{I_2}\right)^2} = \left(\frac{a_1+a_2}{a_1-a_2}\right)^2 \tag{12}$$

**Analytical treatment of interference**

Let us consider two interfering waves $y_1 = a\sin\omega t$ and $y_2 = a\sin(\omega t+\delta)$. Then the resultant wave after superstition is

$$\begin{aligned} y &= y_1 + y_2 \\ &= a\sin\omega t + a\sin(\omega t+\delta) \\ &= a[\sin\omega t + \sin(\omega t+\delta)] \end{aligned}$$

# Superposition of two waves (contd.)

$$= a[2\sin\frac{1}{2}(\omega t + \omega t + \delta)\cos\frac{1}{2}(\omega t - \omega t - \delta)]$$

$$= 2a\sin(\omega t + \phi)\cos\left(\frac{\delta}{2}\right) \;; \text{where}, \phi = \frac{\delta}{2}$$

$$= R\sin(\omega t + \phi)$$

Here, $R = 2a\,cos\left(\frac{\delta}{2}\right)$ is amplitude of resultant wave.

Since intensity of resultant wave is proportional to square of the resultant amplitude, the intensity I of resultant wave is given by

$$I \propto R^2$$

$$\text{or}, I = 4a^2\cos^2\phi = 4a^2\cos^2\left(\frac{\delta}{2}\right)$$

# Superposition of two waves (contd.)

*Bright fringes*

The maximum intensity of the resultant wave is $I = 4I_0$ which occurs for phase difference, $\delta = 0, 2\pi, 4\pi, ..., 2n\pi$ or path difference equal to $0, \lambda, 2\lambda, 3\lambda, ..., n\lambda$. Thus, for bright fringes the path difference between the waves should be equal to integral multiple of wavelength i.e. $n\lambda$

*Dark fringes*

The minimum intensity of the resultant wave is $I = 0$ which occurs when phase difference, $\delta = \pi, 3\pi, 5\pi, ..., (2n+1)\pi$ or path difference equal to $\frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, ..., (2n+1)\dfrac{\lambda}{2}$. Thus, for dark fringes the path

# Superposition of two waves (contd.)

difference between the waves should be equal to half odd integral multiple of wavelength i.e. $\left((2n+1)\dfrac{\lambda}{2}\right)$.

<u>*Intensity distribution*</u>

The intensity of resultant wave is given by
$I = 4I_0\cos^2\left(\frac{\delta}{2}\right) = 4a^2\cos^2\left(\dfrac{\delta}{2}\right)$ with $a$ is the amplitude of a superposing wave. The values of maximum and minimum intensities are $4a^2$ and 0 respectively. This also confirms that the intensity of each bright fringe is same and same is true for dark fringes. The intensity distribution curve is as shown.

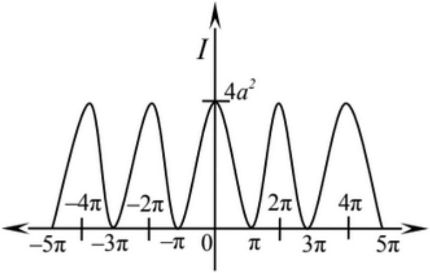


Figure 3