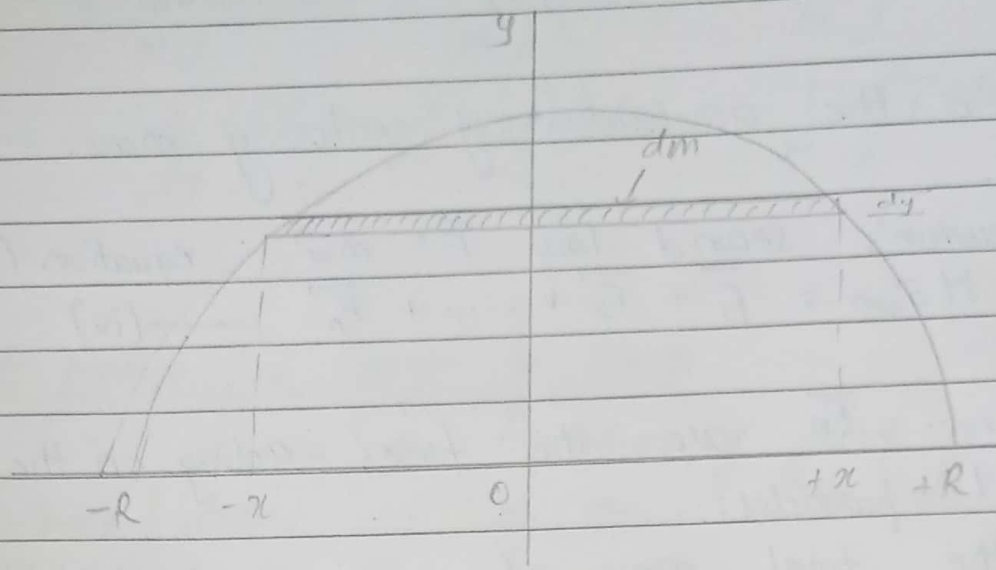
<Q>: Find the center of mass of semi-circular plate of radius R.

Soln:



Let 'σ' be the mass per unit area of the plate

ie, $\sigma = \frac{M}{\frac{\pi R^2}{2}}$ $\therefore \sigma = \frac{2M}{\pi R^2}$

The homogeneous semicircular plate has rotational symmetry about the y-axis so that the center of mass must lie on the y-axis.

Consider a thin strip of mass 'dm' of this homogeneous semicircular plane.

$\therefore$ Area of thin shaded strip $(da) = 2x\, dy$

$\therefore$ Mass of the thin strip (dm) $= \sigma \times da$

$$= \frac{2M}{\pi R^2} \times 2x\,dy$$

$$\therefore dm = \frac{4M}{\pi R^2} x\,dy.$$

Since the center of mass of all shaded strips lies on the y-axis,

ie, $X_{cm} = 0$.

$$\therefore y_{cm} = \frac{\int y\,dm}{\int dm} = \frac{1}{M}\int y\,dm$$

$$= \frac{1}{M}\int y\left[\frac{4M}{\pi R^2} x\,dy\right] = \frac{4}{\pi R^2}\int y x\,dy$$

$$= \frac{4}{\pi R^2}\int_0^R y\sqrt{R^2-y^2}\,dy$$

Put, $R^2-y^2 = t^2$

or $-2y\,dy = 2t\,dt$

$\therefore$ when $y=0$, then $t=R$.

when, $y=R$ then $t=0$.

$$\therefore y_{cm} = \frac{4}{\pi R^2}\int_0^R \sqrt{R^2-y^2}\,y\,dy = \frac{4}{\pi R^2}\int_0^R t(-t\,dt)$$

$$= \frac{4}{\pi R^2}\int_0^R t^2\,dt \qquad \therefore y_{cm} = \frac{4R}{3\pi}$$

Thus, the center of mass of the homogeneous semicircular plate lies on the y-axis at a distance of $(4R/3\pi)$ from origin ie, the co-ordinate of centre of mass $(X_{cm}, Y_{cm}) = \left(0, \frac{4R}{3\pi}\right)$

Linear Moment of System of Particles

The linear momentum $\vec{P}$ of a single particle is defined as the product of its mass 'm' and its velocity $\vec{v}$.

ie, $\vec{P} = m\vec{v}$

From Newton's second law of motion, the rate of change of momentum of a body is proportional to the resultant force acting on the body and is in the direction of force.

$$\vec{F} = \frac{d\vec{P}}{dt} = \frac{d\, m\vec{v}}{dt} = m\vec{a}$$

Now, considering a system of n-particles with the masses $m_1, m_2, \ldots, m_n$.
Let 'M' be the total mass.

$$M = \sum_{0}^{i} m_i$$

The particles interact with each other and suppose that external forces also acts on them.
∴ The total momentum $\vec{P}$ of the system is given by

$$\vec{P} = \vec{P_1} + \vec{P_2} + \dots + \vec{P_n}$$

or $$\vec{P} = m_1\vec{v_1} + m_2\vec{v_2} + \dots + m_n\vec{v_n} \quad \text{—— (i)}$$

$$\therefore \vec{P} = M\vec{v}_{cm}$$

ie, total momentum of a system of particles is equal to the product of total mass of system and the velocity of its centre of mass.

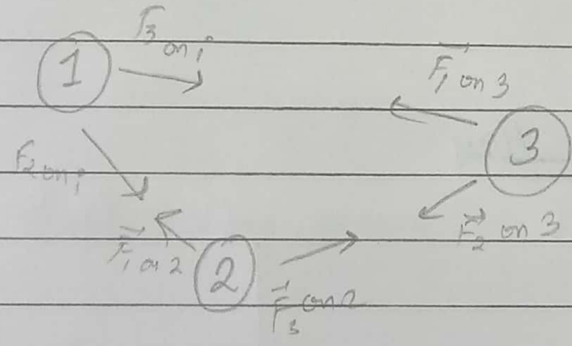
For system of particles, Newton's second law

$$\vec{F}_{ext} = M\vec{a}_{cm}. \quad \text{—— (ii)}.$$

Here,

$\vec{F}_{ext}$ = vector sum of all external force acting on system.

According to the Newton's third law, the internal forces acting between particles get cancelled in pairs.



(Fig: System of three particles.

In this system of three masses, the internal forces are cancelled with each other and only the external force provides the acceleration of center of mass.

Differentiating eq^n (i) ~~wrt~~ w.r.t. t. we get,

$$\frac{d\vec{P}}{dt} = M\frac{dV_{cm}}{dt} = M\vec{a}_{cm} \quad \text{—— (iii)}$$

From eq^n (ii) and (iii),
Newton's second law can be written as,

$$\vec{F}_{ext} = \frac{d\vec{P}}{dt} \quad \text{—— (iv)}$$

*) For conservation of linear momentum:

Suppose that the sum of the external forces acting on a system is zero. So, $F_{ext} = 0$.

So,
Eq^n (iv) becomes,

$$0 = \frac{d\vec{P}}{dt} \Rightarrow \vec{P} = \text{constant.}$$

ie, when the resultant external force is zero, the total momentum of the system remains constant.

This is called the conversation of linear momentum.

System of Variable Mass

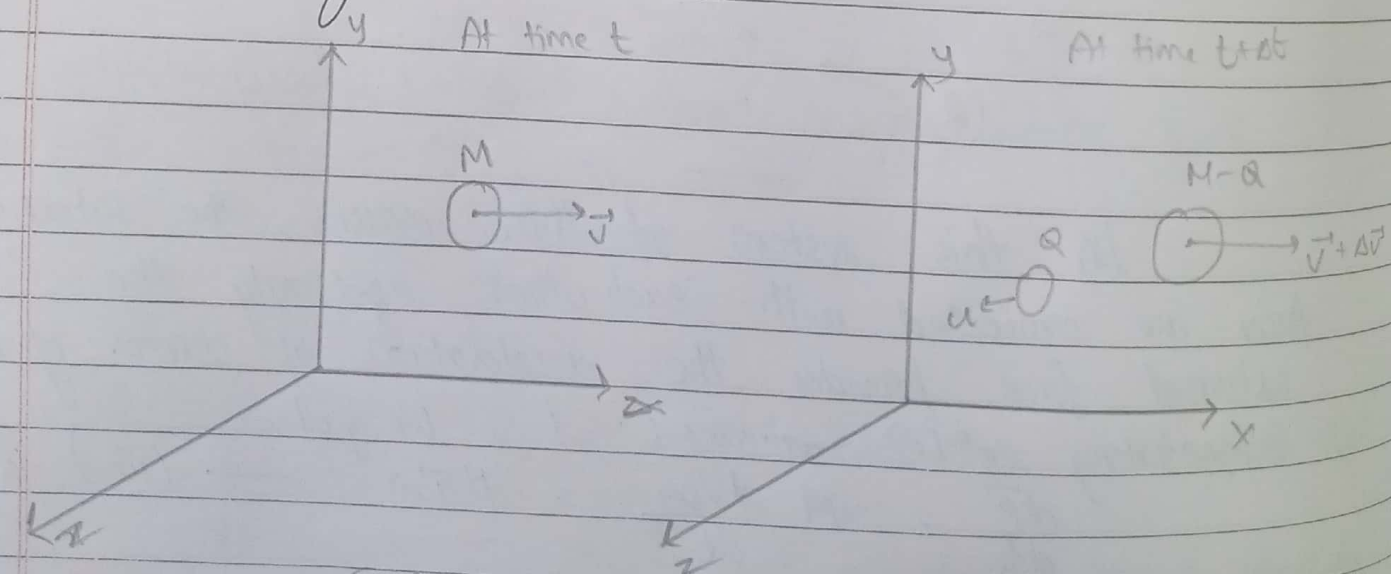


Fig: System of Variable mass.

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We consider a system in which mass enters or leaves the system while we are observing it, (dM/dt) is positive in the former case and negative in the later case.

Consider a system of mass 'M' whose center of mass is moving with velocity '$\vec{v}$') at time 't'.

An external force F_{ext} acts on the system. At a time Δt later, a mass ΔM has been ejected from the system, its center of mass moving with a velocity $\vec{u}$.

The system is reduced to $M - \Delta M$ and the velocity '$\vec{v}$ of the center of mass of the system is changed to $\vec{v} + \Delta\vec{v}$.

We have,

$$\vec{F}_{ext} = \frac{d\vec{P}}{dt}$$

For finite time interval Δt,

$$\vec{F}_{ext} \approx \frac{\Delta P}{\Delta t} = \frac{\vec{P_f} - \vec{P_i}}{\Delta t}$$

Here,

$\vec{P_f}$ = final system momentum $\qquad$ $\vec{P_i}$ = initial system momentum

Since,

$$\vec{P_f} = (M-\Delta M)(\vec{v}+\Delta v) + \Delta M\vec{u} \quad \text{and} \quad \vec{P_i} = M\vec{v}$$

The above equation becomes,

$$F_{ext} = \frac{[(M-\Delta M)(\vec{v}+\Delta\vec{v}) + \Delta M\vec{u}] - [M\vec{v}]}{\Delta t}$$

$$= M\frac{\Delta\vec{v}}{\Delta t} + [\vec{u} - (\vec{v}+\Delta\vec{v})]\frac{\Delta M}{\Delta t} \qquad \text{——— (i)}$$

If Δt approaches to zero, $\frac{\Delta \vec{v}}{\Delta t}$ approaches $\frac{d\vec{v}}{dt}$ ie, the acceleration of the body. The quantity ΔM is the mass ejected in Δt; this leads to the decrease in mass 'M' of the original body.

Since the change in mass of the body with time is negative, $\left(\frac{\Delta M}{\Delta t}\right)$ is replaced by $-\frac{dM}{dt}$ as Δt approaches zero.

Finally, $\Delta \vec{v}$ goes to zero as Δt approaches 0. Hence, above eq^n becomes,

$$\vec{F}_{ext} = M\frac{d\vec{v}}{dt} + \vec{v}\frac{dM}{dt} - \vec{u}\frac{dM}{dt} \quad \text{— (ii)}$$

or, $$\vec{F}_{ext} = \frac{d}{dt}(M\vec{v}) - \vec{u}\frac{dM}{dt}$$

This is the Newton's second law for a system of variable mass.

For a constant mass $(dM/dt) = 0$, eq^n (ii) becomes

$$\vec{F}_{ext} = M\cdot\frac{d\vec{v}}{dt} = M\vec{a} \quad \text{— (iii)}$$

So, eq^n(i) becomes,

$$M\cdot\frac{d\vec{v}}{dt} = \vec{F}_{ext} + (\vec{u}-\vec{v})\frac{dM}{dt}$$

$$= \vec{F}_{ext} + \vec{v}_{rel}\frac{dM}{dt} \quad \text{— (iv)}$$

Here, $\vec{V}_{rel}$ = velocity of the ejected mass relative to the main body.

Best example is Rocket.

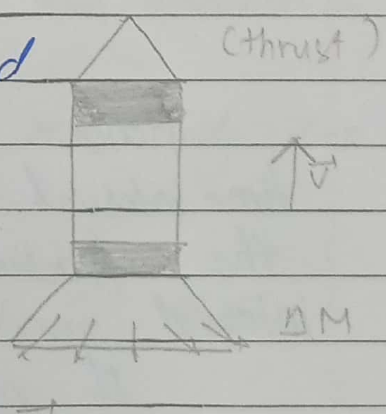
Rocket:

A rocket carries both the fuel and the oxidizer which ~~born~~ burn in a combustion chamber within the rocket. When the rocket is fired then, the exhaust gases rushes downwards at a high speed and pushes the rocket upward. ~~Thus, the~~

Thus, the thrust on the rocket is supplied by the reaction forces of the high speed gases exhausted at the rear.

In Eq^n(4) $(\vec{V}_{rel}\, dM/dt)$ is the rate at which momentum is being transferred into the system by the mass that system has ejected. It is the reaction force exerted on the system by the mass that leaves it.

The reaction force of this Rocket is called thrust.



So,
eq^n(iv) can be written as.

$$\vec{M}\cdot\frac{dv}{dt} = \vec{F}_{ext} + \vec{F}_{rxn}.$$

If all the external forces is neglected, $F_{rxn} = \vec{V}_{rel}\cdot\frac{dM}{dt}$

$$M\frac{d\vec{v}}{dt} = \vec{F}_{rxn}$$

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or, $M \cdot \frac{d\vec{v}}{dt} = \cancel{F_{ext}}\ \vec{V}_{rel} \frac{dM}{dt}$

$\therefore d\vec{v} = \vec{V}_{rel} \cdot \frac{dM}{M} \quad \text{— (v)}$

Integrating eq^n (v) from ~~an~~ instance of mass M_0 and velocity V_0 to the mass of M and V. We get,

$$\int_{\vec{V_0}}^{V} d\vec{v} = \vec{V}_{rel} \int_{M^0}^{M} \frac{dM}{M}$$

or $\vec{V} - \vec{V_0} = \vec{V}_{rel} \log_e \left(\frac{M}{M_0}\right)$

$\therefore \vec{V} = \vec{V_0} + \vec{V}_{rel} \log_e \left(\frac{M}{M_0}\right)$

So,

$\vec{V} = \vec{V_0} - \vec{V}_{rel} \log_e \left(\frac{M_0}{M}\right) \quad \text{— (vi}$

So, for a rocket moving upward,

$\vec{V} = \vec{V_0} + \vec{V}_{rel} \log_e \left(\frac{M_0}{M}\right) \quad \text{— (vii)}$

Hence, the change in speed of rocket at any time interval depends on the exhaust velocity and on the fraction of mass exhausted during that time interval.

If rocket starts from rest ($\vec{V_0} = 0$) with an initial mass M_0 and reaches final velocity $\vec{V_f}$ at burn out when its mass is M_f, then eq^n(vi) becomes,

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$$V_f = V_{rel} \ln\left(\frac{M_0}{M_f}\right)$$

or, $$\frac{V_f}{V_{rel}} = -\ln\left(\frac{M_f}{M_0}\right)$$

or, $$\frac{-V_f}{V_{rel}} = \ln\left(\frac{M_f}{M_0}\right)$$

$$\therefore M_f = M_0 e^{\left(\frac{-V_f}{V_{rel}}\right)} \quad \text{—(viii)}$$

In the case, the weight of a rocket is taken into account, eq^n (vii) for a rocket moving vertically upwards becomes,

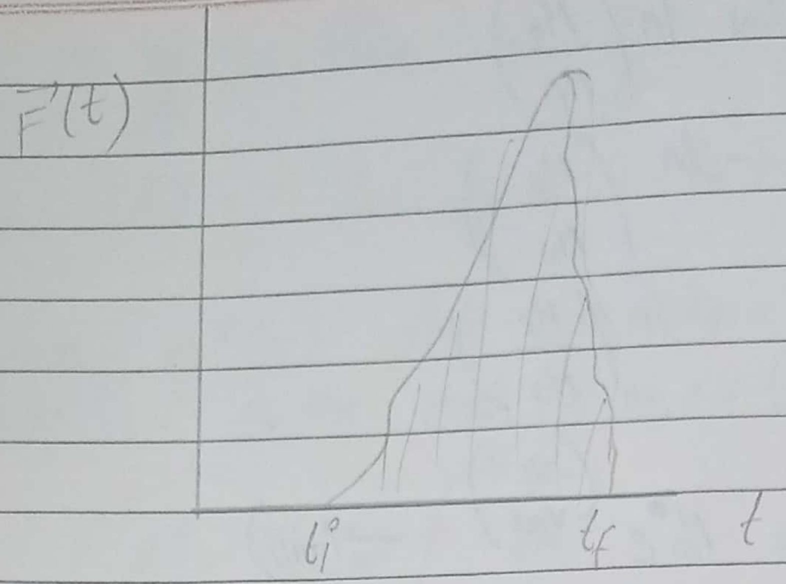
$$\vec{v} = \vec{v_0} - \vec{v}_{rel} \ln\left(\frac{M_0}{M}\right) - gt$$

Impact Force and Impulse

When ~~the~~ two bodies are brought in contact for a ~~cont~~ short interval of time and they transfer the momentum to one another, then the situation is called collision.

The force experienced by any one body due to another body is called impact force.

From the given graph, we can observe that :



The impact force starts to act on a body when collision starts, it increases to a maximum value and then decreases to zero as collision ends.

Impulse is defined as the time integral of impact force during the impact time.

Let the collision starts at 't_i' and ends 't_f'. Let the impact force $F(t)$ is experienced by a body at any time t.

Thus, impulse

$$\vec{J} = \int_{t_i}^{t_f} \vec{F}(t)\,dt \quad -(i)$$

Graphically, impulse is equal to the area under the impact force versus time graph. So, from Newton's second law of motion,

$$\vec{F} = \frac{d\vec{P}}{dt}$$

Therefore,

$$\vec{J} = \int_{t_i}^{t_f} \frac{d\vec{P}}{dt} \cdot dt = \vec{P_f} - \vec{P_i}$$

$$\therefore \vec{J} = \vec{P_f} - \vec{P_i} \quad \text{--- (ii)}$$

Hence, impulse on a body is also equal to the change in linear momentum of the body.

Conservation ~~Converstation~~ of Linear Momentum During Collision

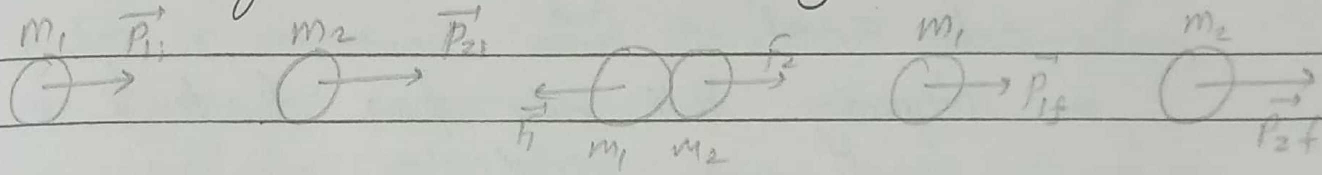


Fig: Conservation of Linear Momentum

Consider two bodies of masses m_1 and m_2 moving with momentum $\vec{P_{1i}}$ and $\vec{P_{2i}}$ respectively collide after certain time interval.

Collision begins from t_i to t_f.

After collision, the bodies moves momentum $\vec{P_{1f}}$ and P_{2f}.

During collision, the first body experiences a time varying impact force F_1 and the second body experiences a time varying impact force $\vec{F_2}$.

According to Newton's third law of motion, at any time during collision,

$$\vec{F_1} = -\vec{F_2} \quad \text{--- (i)}$$

from definition of impulse,

$$\vec{J_1} = \int_{t_i}^{t_f} F_1 dt = \vec{P_{2f}} - \vec{P_{1i}} \quad \text{— (ii)}$$

$$\vec{J_2} = \int_{t_i}^{t_f} F_2 dt = \vec{P_{2f}} - \vec{P_{1f}} \quad \text{— (iii)}$$

~~Fr~~ Taking eq^n (i),

$$\vec{F_1} = -\vec{F_2}$$

~~or,~~ Integrating both sides from t_i to t_f.

$$\int_{t_i}^{t_f} F_1 = - \int_{t_i}^{t_f} \vec{F_2}$$

or, $\vec{J_1} = -\vec{J_2}$

or, $\vec{P_{1f}} - \vec{P_{1i}} = -(\vec{P_{2f}} - \vec{P_{2i}})$

or, $\vec{P_{1f}} - \vec{P_{1i}} = -\vec{P_{2f}} + \vec{P_{2i}}$

or, $\vec{P_{1f}} + \vec{P_{2f}} = \vec{P_{1i}} + \vec{P_{2i}}$

or, $\vec{P_1} + \vec{P_2} = \text{constant}$.

ie, $m_1\vec{v_1} + m_2\vec{v_2} = \text{constant}$.

Here, $\vec{J_1}$ and $\vec{v_2}$ are the velocities of the colliding bodies.

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Hence, the total momentum of two colliding bodies remains conserved during collision.

Elastic collision:

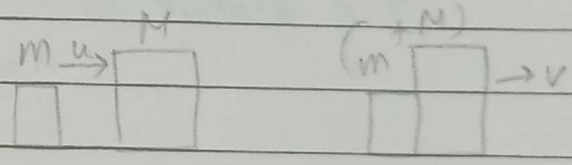
The collision in which the total momentum and the total kinetic energy of is conserved is called elastic collision.

Inelastic collision:

The collision in which the total momentum of the system is conserved ~~and~~ but the total kinetic energy is not conserved is called inelastic collision.

If the total kinetic energy after collision is always less than that before collision, then the collision is said to the perfectly inelastic collision.

Eg: (i): Sticking collision:



Consider a body of mass 'm' moving with speed 'u' collides with another body of mass M at rest.

After collision, both the bodies move together with common velocity 'V'.

From conservation of linear momentum, we have:

$mu = (m+M)V$ $\therefore V = \frac{mu}{(M+m)}$

(K_i)

Initial total KE $= \frac{1}{2} m u^2$

Final total KE $^{(K_f)} = \frac{1}{2} (M+m) v^2 = \frac{1}{2} \frac{m^2 u^2}{M+m}$

ie, $K_f = \frac{m}{m+M} K_i$.

Here,

$K_i > K_f$. So, sticking collision is example of perfectly elastic collision.

Decrease in KE $(\Delta KE) = K_i - K_f$

$= K_i - \frac{m}{M+m} K_i$

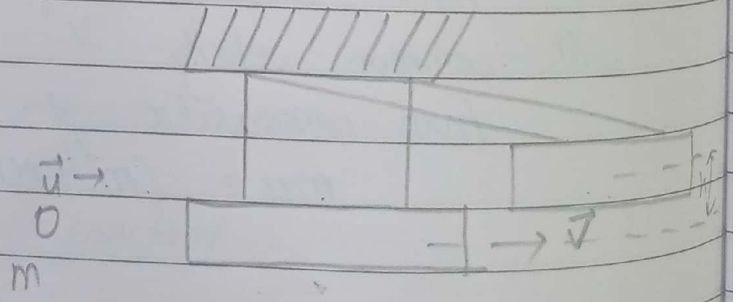
$\therefore \Delta K = \frac{M}{M+m} K_i$

% of decrease in KE $(\Delta KE\%) = \frac{\Delta K}{K_i} \times 100\%$

$\therefore \Delta KE\% = \frac{M}{M+m} \times 100\%$

(ii) Ballistic pendulum:

A heavy wooden block vertically suspended by light string is called ballistic pendulum.



Consider the mass of the block and bullet are M and m respectively.

Let 'u' be the muzzle velocity of the bullet fired.

As bullet is fired very closely to block, it embeds into the block and the system acquires common velocity which causes the block to attain in maximum height.

From conservation of linear momentum, we have.

$$mu = (M+m)V$$

or $V = \frac{m}{(M+m)} u$ or $u = \frac{(M+m)V}{m}$

Only the force of gravity contribute to the workdone on the system, the total mechanical energy of system is conserved. Hence,

$$\frac{1}{2}(m+M)v^2 = (M+m)gh$$

$$\therefore V = \sqrt{-2gh}$$

Then,

$$u = \frac{M+m}{m} \times \sqrt{2gh}$$

Now, the total KE initially $(K_i) = \frac{1}{2} mu^2 = \frac{1}{2} \frac{(M+m)^2}{m} (2gh)$

total KE finally $(K_f) = \frac{1}{2} \cdot (M+m)(2gh)$

Decrease in KE $(\Delta KE) = K_i - K_f$

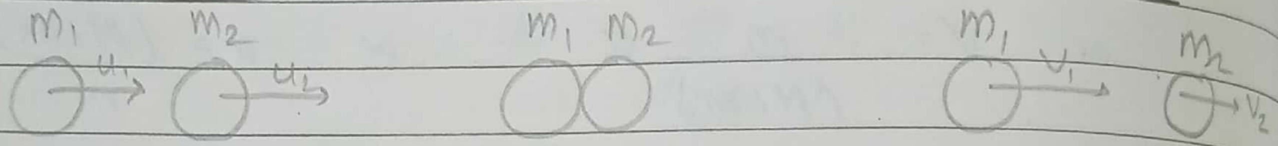
$$= \frac{1}{2} \frac{M(M+m)}{m} (2gh)$$

Decrease in percentage

$$= \frac{\text{Decreased KE}}{K_i} \times 100\% = \frac{M}{M+m} \times 100\%$$

This shows collision is perfectly inelastic.

One Dimensional Elastic Collision / Head-ON collision



Consider a body of mass 'm_1' with velocity 'v_1' collides ~~perfle~~ elastically a body of mass 'm_2' with velocity 'v_2' along same direction.
If $u_1 > u_2$, the bodies move with velocities v_1 and v_2 respectively after collision.

Head-on collision : if both bodies move along same initial direction, the collision is called head-on.

from conservation of linear momentum,

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

or $\quad m_1(u_1 - v_1) = m_2(v_2 - u_2) \quad \text{——} (i)$

Since the collision is elastic, the total KE remains conserved.

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$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

or, $m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$

or $m_1(u_1+v_1)(u_1-v_1) = m_2(v_2+u_2)(v_2-u_2)$ — (ii)

Dividing eq^n (ii) by (i), we get,

$u_1 + v_1 = v_2 + u_2$

or, $u_1 - u_2 = -(v_1 - v_2)$ — (iii)

Here, velocity of approach = velocity of reproach

Here, velocity before collision and after collision is equal and opposite.

Substituting the value of v_1 from eq^n(iii) to eq^n (i),

$m_1u_1 + m_2u_2 = m_1(v_2 + u_2 - u_1) + m_2v_2$

or, $(m_1+m_2)v_2 = 2m_1u_1 + (m_2 - m_1)u_2$

~~So~~ $\therefore v_2 = \frac{2m_1}{m_1+m_2}u_1 + \frac{m_2-m_1}{m_1+m_2}u_2$ — (iv)

Substituting the value of v_2 in eq^n (i),

$v_1 = \frac{m_1-m_2}{m_1+m_2}u_1 + \frac{2m_2}{m_1+m_2}u_2$ — (v)

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Case I: If $m_1 = m_2$ i.e, equal colliding mass

$$v_1 = u_2$$
$$v_2 = u_1.$$

When two bodies of equal masses elastically collides leading one-dimensional collision then, after collision they exchange their velocity.

Case II: If $u_2 = 0$.

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2} u_1$$

$$v_2 = \frac{2m_1}{m_1 + m_2} u_1$$

S.case a: If $m_1 = m_2$.

$$v_1 = 0$$
$$v_2 = u_1$$

When a body elastically collides another body at rest, the first body comes to rest and the second body moves with the initial velocity of the first body.

S.case b: $m_2 >> m_1$.

$$v_1 \approx -u_1$$
$$v_2 \approx 0$$

When a light body elastically collides with massive body at rest, the light body rebounds with approxiametely remains at rest.

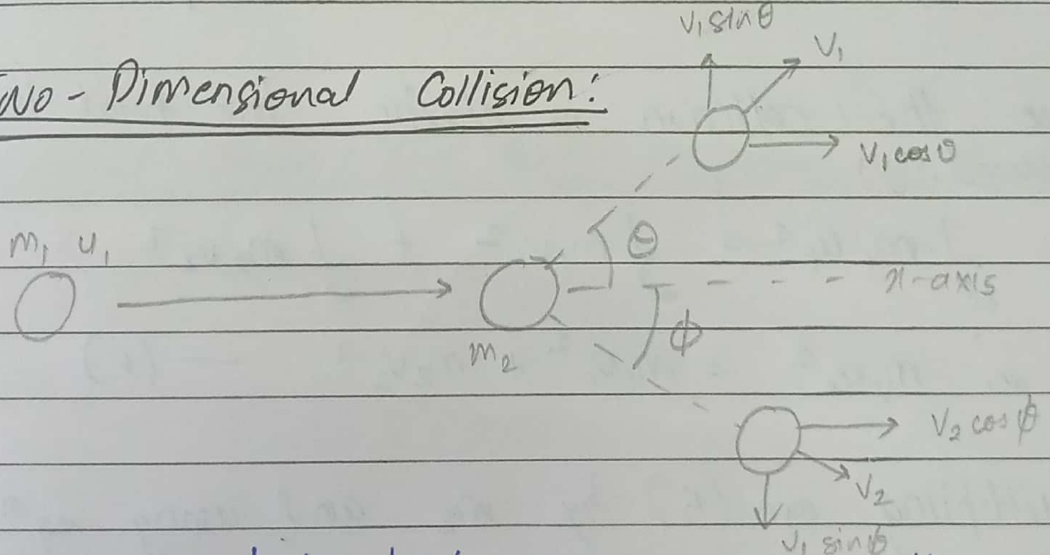
S.case: C: If $m_1 >> m_2$.

$v_1 \approx u_1$

$v_2 \approx 2u_1$.

When massive body elastically collides with light body at rest, the massive body continues its motion with same initial velocity ~~of~~ along same direction and the light body scatters with approxiamety double ~~in~~ the initial velocity of the massive body along same direction.

Two-Dimensional Collision:



Consider a body 'm_1' is moving with velocity 'u_1' and collides with 'm_2' at rest elastically. After collision, body 'm_1' moves with velocity 'v_1' making angle θ from x-axis and body with mass 'm_2' moves with velocity 'v_2' opposite direction of x-axis making angle ϕ.

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From the conservation of linear momentum.

$$m_1\vec{u_1} = m_1\vec{v_1} + m_2\vec{v_2} \quad \text{— (i)}$$

For eqn (i), taking x-component,

$$m_1 u_1 = m_1 v_1 \cos\theta + m_2 v_2 \cos\phi$$

or, $$m_2 v_2 \cos\phi = \cancel{m_1 u_1}\ m_1 u_1 - m_1 v_1 \cos\theta \quad \text{— (ii)}$$

For eqn (ii), taking y-componenet.

$$0 = m_1 v_1 \sin\theta - m_2 v_2 \sin\phi$$

or, $$m_1 v_1 \sin\theta = m_2 v_2 \sin\phi \quad \text{— (iii)}$$

Squaring and adding eqn (ii) and (iii), we get

$$m_2^2 v_2^2 = m_1^2 u_1^2 + m_1^2 v_1^2 - 2m_1^2 u_1 v_1 \cos\phi \quad \text{— (iv)}$$

Since the collision is elastic, the total KE is conserved.

$$\frac{1}{2} m_1 u_1^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

or, $$m_1 u_1^2 = m_1 v_1^2 + m_2 v_2^2 \quad \text{— (v)}$$

Multipling eqn (5) by m_2 and using eqn (iv),

$$m_1 m_2 u_1^2 = m_1 m_2 v_1^2 + m_1^2 u_1^2 + m_1^2 v_1^2 - 2 m_1^2 u_1 v_1 \cos\theta$$

or, $$(m_1 + m_2) v_1^2 - 2m_1 u_1 v_1 \cos\theta + (m_1 - m_2) u_1^2 = 0$$

$$\text{— (vi)}$$

Here, using eq^n (vi) we can measure scattering angle and v_1 from initial information. After putting v_1 in eq^n (5), v_2 can be determined and using eq^n(2)/(3), recoil angle can be calculated.

If the masses are equal ie, $m_1 = m_2$ then, eq^n (6) becomes,

$$v_1 = u_1 \cos\theta \quad \text{—— (vii)}$$

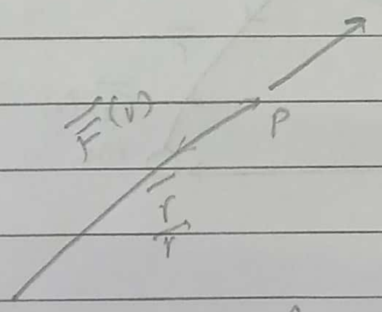
Using eq^n (5),

$$v_2 = u_1 \sin\theta \quad \text{—— (viii)}$$

and from eq^n (iii)

$$\sin\phi = \cos\theta \qquad \therefore \phi + \theta = 90^\circ.$$

Central Force / Two Body Diagram and Reduced Mass



A central force.

A force at a point is said to be a central force whose magnitude is the function of distance from a fixed point and lies ~~on the~~ on the lies joining the point and fixed point.

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Let 'O' be the fixed point and 'P' be the position vector '$\vec{r}$'.

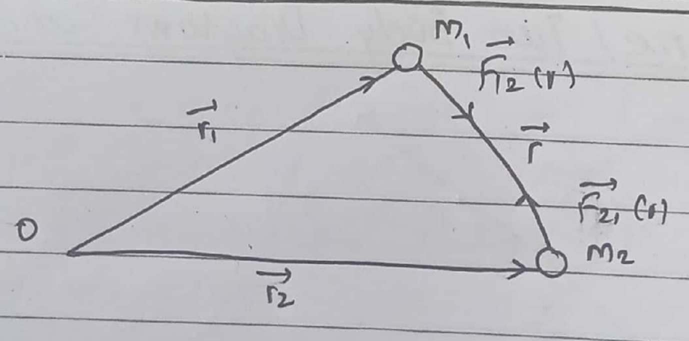
The force $\vec{F}(r)$ is said to be the central force if it's the function of r and whose direction is either toward or away from the fixed point O ie,

$$\vec{F}(r) = \pm F(r) \cdot \hat{r} \quad \text{— (i)}$$

Here, F(r) is magnitude of central force.

+ve sign: C.F. directed away from fixed point
-ve sign: C.F directed towards fixed point.

Two-Body Problems and Reduced Mass



Consider two bodies of masses m_1 and m_2 with positive vectors $\vec{r}_1$ and $\vec{r}_2$ w.r.t origin

$\vec{r}$ is the separation vector between the directed from first body to the second. There two bodies are acted by the force due to their mutual interaction.

$\vec{F_{12}}(r)$ is force on first due to second body and $\vec{F_{21}}(r)$ is force on second due to first body.

These are function of separation distance $\vec{r}$ and lie on line joining two bodies so that these C.F's w.r.t the fixed as the position of either body.

$\vec{f_{12}}(r)$ is c.f from m_2 and $\vec{F_{21}}(r)$ is c.f. from m_1.

From Newton's third law,

$$\vec{F_{12}}(r) = -\vec{F_{21}}(r) = F(r)\hat{r} \quad \text{— (i)}$$

Here, $F(r)$ is magnitude of force of mutual interaction.

Applying Newton's 2nd law for 1st body,

$$m_1 \frac{d^2\vec{r_1}}{dt^2} = \vec{F_{12}}(r) = \frac{d^2\vec{r_1}}{dt} = \frac{1}{m_1} F(r)\hat{r} \quad \text{— (ii)}$$

Applying Newton's 2nd law for 2nd body,

$$m_2 \frac{d^2\vec{r_2}}{dt^2} = -\vec{F_{21}}(r) = \frac{d^2\vec{r_2}}{dt^2} = -\frac{1}{m_2} F(r)\hat{r} \quad \text{— (iii)}$$

Subtracting (ii) from (iii); we get

$$\frac{d^2(\vec{r_2}-\vec{r_1})}{dt^2} = -\left(\frac{1}{m_2} + \frac{1}{m_1}\right) . F(r)\hat{r} \quad \text{— (iv)}$$

From triangle law of vector addition,

$$\vec{r_1} + \vec{r} = \vec{r_2}$$

or $$\vec{r_2} - \vec{r_1} = r \qquad \text{— (v)}$$

$$\frac{d^2\vec{r}}{dt^2} = -\frac{1}{\mu} F(r)\,\hat{r} \qquad \text{— (vi)}$$

or, $$\mu \frac{d^2\vec{r}}{dt^2} = -F(r)\,\hat{r} \qquad \text{— (vii)}$$

where, $$\frac{1}{\mu} = \left(\frac{1}{m_1} + \frac{1}{m_2}\right)$$

$$\mu = \frac{m_1 m_2}{m_1 + m_2} \qquad \text{— (viii)}$$

This is the reduced mass of the system.

According to eq^n (viii), the two body problem can be reduced to a single body problem under the central force of mutual attraction with the reduced mass μ placed at position m_2 setting position m_1 as fixed point or vice-versa.