

# Lecture 16

## Electromagnetic Wave Propagation

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# Displacement current and Maxwell's equations

The basic laws of electricity and magnetism can be summarized in differential form as follows

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} \quad (\text{Gauss's law}) \quad (1a)$$

$$\nabla \cdot \vec{B} = 0 \quad (\text{No name}) \quad (1b)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (\text{Faraday's law}) \quad (1c)$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} \quad (\text{Ampere's law}) \quad (1d)$$

Let us take the divergence of equation (1c), i.e.

$$\nabla \cdot (\nabla \times \vec{E}) = \nabla \cdot \left( -\frac{\partial \vec{B}}{\partial t} \right) = -\frac{\partial}{\partial t} (\nabla \cdot \vec{B})$$

## Displacement current and Maxwell's equations (contd.)

Here left hand side is zero because divergence of curl is always zero and right hand side is zero by virtue of equation (1b).

Similarly, let us take divergence of equation (1d), i.e.

$$\nabla \cdot (\nabla \times \vec{B}) = \nabla \cdot (\mu_0 \vec{J}) = \mu_0 (\nabla \cdot \vec{J})$$

Here left side is zero as it is divergence of curl is always zero but the right side in general is not equal to zero. So for steady current the divergence of  $\vec{J}$  is zero, but when we go beyond magneto statics Ampere's law cannot be right.

Therefore for time dependent field equation (1d) is not correct and should be modified. Maxwell suggested that the definition of total

## Displacement current and Maxwell's equations (contd.)

current density is incomplete and advice to add something to  $\vec{J}$ . Let it be called  $\vec{J}'$ , then equation (1d) becomes

$$\nabla \times \vec{B} = \mu_0(\vec{J} + \vec{J}')$$

if we take divergence of this equation, we get

$$\begin{aligned}\nabla \cdot (\nabla \times \vec{B}) &= 0 = \mu_0(\nabla \cdot \vec{J} + \nabla \cdot \vec{J}') \\ \implies \nabla \cdot \vec{J}' &= -\nabla \cdot \vec{J} = \frac{\partial \rho}{\partial t} = \frac{\partial}{\partial t}(\epsilon_0 \nabla \cdot \vec{E}) = \nabla \cdot \left( \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right) \\ \therefore \vec{J}' &= \epsilon_0 \frac{\partial \vec{E}}{\partial t}\end{aligned}\tag{2}$$

# Displacement current and Maxwell's equations (contd.)

Thus, the modified form of Ampere's law is

$$\nabla \times \vec{B} = \mu_0 \left( \vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right) \quad (3)$$

or

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \quad (4)$$

The added term  $\vec{J}' = \epsilon_0 \frac{\partial \vec{E}}{\partial t}$  is called the *displacement current* or *displacement current density*. To check the modified equation, let us take divergence of equation (4) i.e.

$$\begin{aligned} \nabla \cdot (\nabla \times \vec{B}) &= \mu_0 \nabla \cdot \vec{J} + \mu_0 \epsilon_0 \frac{\partial}{\partial t} (\nabla \cdot \vec{E}) \\ \implies \nabla \cdot (\nabla \times \vec{B}) &= \mu_0 \nabla \cdot \vec{J} + \mu_0 \frac{\partial}{\partial t} (\epsilon_0 \nabla \cdot \vec{E}) \end{aligned}$$

## Displacement current and Maxwell's equations (contd.)

$$\implies \nabla \cdot (\nabla \times \vec{B}) = \mu_0 \nabla \cdot \vec{J} + \mu_0 \frac{\partial \rho}{\partial t}$$

$$\implies \nabla \cdot (\nabla \times \vec{B}) = \mu_0 \left[ \nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} \right]$$

Here left hand term is zero as it is divergence of curl of  $\vec{B}$ . And as we know from continuity equation  $\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$ ; so the right hand term is in general equal to zero. Thus, equation (4) is the correct form of Ampere's law. Maxwell's equations Faraday's law and Ampere's law after Maxwell's correction shows the connection between electricity and magnetism. So, there is complete set of four equations; called the *Maxwell's equations or electromagnetic field equations*. In absence of dielectrics or magnetic materials, Maxwell's equations are

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

(Gauss's law) (5a)

# Displacement current and Maxwell's equations (contd.)

$$\nabla \cdot \vec{B} = 0 \quad (\text{No name}) \quad (5b)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (\text{Faraday's law}) \quad (5c)$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \quad (\text{Ampere's law with Maxwell's correction}) \quad (5d)$$

**Example:-** Imagine thin wires that connect to the centers of the parallel plates (capacitor) as shown in figure 1 . The constant current  $I$  is maintained to flow. The radius of the capacitor is  $a$ , and the separation of the plates is  $w \ll a$ . Assume that the the current flows out over



# Displacement current and Maxwell's equations (contd.)

the plates in such a way that the surface charge is uniform, at any given time, and is zero at  $t = 0$ .

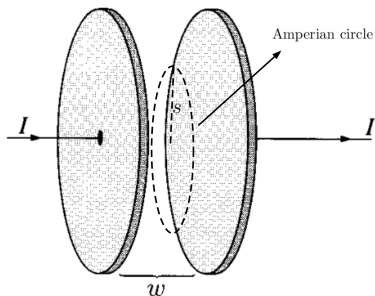


Figure 1

## Displacement current and Maxwell's equations (contd.)

- a Find the electric field between the plates, as a function of  $t$ .
- b Find the displacement current through a circle of radius  $S$  in the plane midway between the plates.
- c Find the magnetic field at a distance  $s$  from the axis.

**Hint:-**

Since the current  $I$  is kept constant, the charge over the surface at any time  $t$  is  $q = It$ .

The surface charge density is  $\sigma = \frac{q}{\pi a^2}$

# Displacement current and Maxwell's equations (contd.)

As  $w \ll a$ , the electric field between the plates is

$$E = \frac{\sigma}{\epsilon_0} = \frac{It}{\epsilon_0 \pi a^2}$$

Since the electric field is time varying, it produces the displacement current as

$$J_d = \epsilon_0 \frac{\partial E}{\partial t} = \frac{I}{\pi a^2}$$

This displacement current appears to flow in the region between the plate and parallel to the axis of the plates. To find the magnetic field due to the current, let's

# Displacement current and Maxwell's equations (contd.)

construct coaxial Amperian circle of radius  $s$  such that  $s < a$ . The current inclosed by the circle is

$$I_{\text{enc}} = J_d \pi s^2 = \frac{Is^2}{a^2}$$

Using Ampere's law,

$$B(2\pi s) = \mu_0 I_{\text{enc}} \implies B = \frac{\mu_0 Is}{2\pi a^2}$$

# Maxwell's equations in material medium

In electric fields in matter the electric polarization  $\vec{P}$  results in an accumulation of bound charge density given by

$$\rho_b = -\nabla \cdot \vec{P}$$

Likewise, in magnetic fields in matter, the magnetic polarization (or magnetization)  $\vec{M}$  results in a bound current density,  $\vec{J}_b$  given by

$$\vec{J}_b = \nabla \times \vec{M}$$

When the polarization  $\vec{P}$  varies with time, then the volume bound charge density,  $\rho_b = -\nabla \cdot \vec{P}$ , also varies with time. The current results

# Maxwell's equations in material medium (contd.)

from this is called polarization current  $\vec{J}_p$  and satisfies the continuity equation as

$$\begin{aligned}\nabla \cdot \vec{J}_p &= -\frac{\partial \rho_b}{\partial t} \\ \Rightarrow \nabla \cdot \vec{J}_p &= -\frac{\partial}{\partial t} (-\nabla \cdot \vec{P}) \\ \Rightarrow \nabla \cdot \vec{J}_p &= \nabla \cdot \frac{\partial \vec{P}}{\partial t}\end{aligned}$$

Therefore, the polarization current density can be written in term of polarization  $\vec{P}$  as

$$\vec{J}_p = \frac{\partial \vec{P}}{\partial t}$$

# Maxwell's equations in material medium (contd.)

For material medium, the total charge density can be written as

$$\rho = \rho_f + \rho_b$$

Where,  $\rho_f$  is charge density due to free charges.

$$\therefore \rho = \rho_f - \nabla \cdot \vec{P} \quad (6)$$

And total current density is

$$\begin{aligned} \vec{J} &= \vec{J}_f + \vec{J}_b + \vec{J}_p \\ \implies \vec{J} &= \vec{J}_f + (\nabla \times \vec{M}) + \frac{\partial \vec{P}}{\partial t} \end{aligned} \quad (7)$$

Now, from Gauss's law in electrostatics,

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

# Maxwell's equations in material medium (contd.)

$$\implies \epsilon_0 \nabla \cdot \vec{E} = \rho$$

$$\implies \nabla \cdot (\epsilon_0 \vec{E}) = \rho_f - \nabla \cdot \vec{P} \quad \text{Using (6)}$$

$$\implies \nabla \cdot (\epsilon_0 \vec{E}) + \nabla \cdot \vec{P} = \rho_f$$

$$\implies \nabla \cdot [\epsilon_0 \vec{E} + \vec{P}] = \rho_f$$

$$\implies \nabla \cdot \vec{D} = \rho_f$$

Where  $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$ .

Similarly, Ampere's law is

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\implies \frac{1}{\mu_0} (\nabla \times \vec{B}) = \vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$



## Maxwell's equations in material medium (contd.)

$$\Rightarrow \frac{1}{\mu_0}(\nabla \times \vec{B}) = \vec{J}_f + \vec{J}_b + \vec{J}_p + \frac{\partial(\epsilon_0 \vec{E})}{\partial t}$$

$$\Rightarrow \frac{1}{\mu_0}(\nabla \times \vec{B}) = \vec{J}_f + \nabla \times \vec{M} + \frac{\partial \vec{P}}{\partial t} + \frac{\partial}{\partial t}(\epsilon_0 \vec{E})$$

$$\Rightarrow \nabla \times \frac{\vec{B}}{\mu_0} - \nabla \times \vec{M} = \vec{J}_f + \frac{\partial}{\partial t}[\vec{P} + \epsilon_0 \vec{E}]$$

$$\Rightarrow \nabla \times \left( \frac{\vec{B}}{\mu_0} - \vec{M} \right) = \vec{J}_f + \frac{\partial \vec{D}}{\partial t}$$

$$\therefore \nabla \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t} \quad \because \vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

where,  $\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$ .

# Maxwell's equations in material medium (contd.)

Hence, Maxwell's equations in material medium become

$$\nabla \cdot \vec{D} = \rho_f \quad (8a)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (8b)$$

$$\nabla \cdot \vec{B} = 0 \quad (8c)$$

$$\nabla \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t} \quad (8d)$$

# Energy in electromagnetic field (Poynting theorem / Poynting's vector)

The energy stored in electric field  $\vec{E}$  is given by

$$W_e = \frac{1}{2} \epsilon_0 \int_V E^2 d\tau$$

And the energy stored in magnetic field  $\vec{B}$  is given by

$$W_m = \frac{1}{2\mu_0} \int_V B^2 d\tau$$

So, the total energy stored in electromagnetic field is

$$W_{em} = W_e + W_m = \frac{1}{2} \int_V \left( \epsilon_0 E^2 + \frac{1}{\mu_0} B^2 \right) d\tau \quad (9)$$

# Energy in electromagnetic field (Poynting theorem / Poynting's vector) (contd.)

The energy stored per unit volume in the electromagnetic field is

$$u_{em} = \frac{1}{2} \left( \epsilon_0 E^2 + \frac{1}{\mu_0} B^2 \right) \quad (10)$$

Suppose a system of point charges  $q_1, q_2, \dots, q_n$  are moving with velocities  $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n$  respectively, in combined electric field  $\vec{E}$  and magnetic field  $\vec{B}$ . The Lorentz force experienced by the  $i^{\text{th}}$  charge is

$$\vec{F}_i = q_i \left( \vec{E} + \vec{v}_i \times \vec{B} \right)$$

# Energy in electromagnetic field (Poynting theorem / Poynting's vector) (contd.)

Within elemental time  $dt$  the  $i^{\text{th}}$  charge covers an elemental displacement  $d\vec{l}_i = \vec{v}_i dt$ . The amount of work done by the force  $\vec{F}_i$  on  $q_i$  within time  $dt$  is

$$\begin{aligned}dW_i &= \vec{F}_i \cdot d\vec{l}_i \\ \implies dW_i &= q_i \left( \vec{E} + \vec{v}_i \times \vec{B} \right) \cdot \vec{v}_i dt \\ \implies \frac{dW_i}{dt} &= q_i \left( \vec{E} \cdot \vec{v}_i + \underbrace{(\vec{v}_i \times \vec{B}) \cdot \vec{v}_i}_0 \right)\end{aligned}$$

# Energy in electromagnetic field (Poynting theorem / Poynting's vector) (contd.)

$$\therefore \vec{v} \cdot (\vec{v} \times \vec{B}) = \begin{vmatrix} v_x & v_y & v_z \\ v_x & v_y & v_z \\ B_x & B_y & B_z \end{vmatrix} = 0$$

$$\therefore \frac{dW_i}{dt} = q_i (\vec{E} \cdot \vec{v}_i)$$

This gives the power delivered by the electromagnetic field on the point charge  $q_i$ . The total power delivered to all the point charge is given by

$$\frac{dW}{dt} = \sum_{i=1}^n \frac{dW_i}{dt} = \sum_{i=1}^n q_i (\vec{E} \cdot \vec{v}_i)$$

# Energy in electromagnetic field (Poynting theorem / Poynting's vector) (contd.)

For the continuous charge distribution,

$$\frac{dW}{dt} = \int dq (\vec{E} \cdot \vec{v})$$

Here  $\vec{v}$  is the velocity of the elemental charge  $dq$  of the system of continuous charge. For the region charge distribution with volume charge density  $\rho$ , we can have  $dq = \rho d\tau$ , and the net power delivered is

$$\frac{dW}{dt} = \int_V \rho d\tau (\vec{E} \cdot \vec{v}) = \int_V \vec{E} \cdot (\rho \vec{v}) d\tau$$

# Energy in electromagnetic field (Poynting theorem / Poynting's vector) (contd.)

The integration is taken in the region of volume  $V$ . Since  $\vec{J} = \rho \vec{v}$ , we have

$$\frac{dW}{dt} = \int_V (\vec{E} \cdot \vec{J}) d\tau \quad (11)$$

From Maxwell's modification of Ampere's law (i.e. Maxwell's fourth law)

$$\begin{aligned} \nabla \times \vec{B} &= \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \\ \implies \vec{E} \cdot (\nabla \times \vec{B}) &= \mu_0 \vec{E} \cdot \vec{J} + \mu_0 \epsilon_0 \vec{E} \cdot \frac{\partial \vec{E}}{\partial t} \\ \implies \vec{E} \cdot \vec{J} &= \frac{1}{\mu_0} \vec{E} \cdot (\nabla \times \vec{B}) - \frac{\epsilon_0}{2} \frac{\partial E^2}{\partial t} \end{aligned} \quad (12)$$



# Energy in electromagnetic field (Poynting theorem / Poynting's vector) (contd.)

Here we have used

$$\vec{E} \cdot \frac{\partial \vec{E}}{\partial t} = \frac{1}{2} \left( \vec{E} \cdot \frac{\partial \vec{E}}{\partial t} + \frac{\partial \vec{E}}{\partial t} \cdot \vec{E} \right) = \frac{1}{2} \frac{\partial}{\partial t} (\vec{E} \cdot \vec{E}) = \frac{1}{2} \frac{\partial E^2}{\partial t}$$

We know the vector identity,

$$\begin{aligned} \nabla \cdot (\vec{E} \times \vec{B}) &= \vec{B} \cdot (\nabla \times \vec{E}) - \vec{E} \cdot (\nabla \times \vec{B}) \\ \implies \vec{E} \cdot (\nabla \times \vec{B}) &= \vec{B} \cdot (\nabla \times \vec{E}) - \nabla \cdot (\vec{E} \times \vec{B}) \end{aligned}$$

So, equation (12) becomes

$$\vec{E} \cdot \vec{J} = \frac{1}{\mu_0} \vec{B} \cdot (\nabla \times \vec{E}) - \frac{1}{\mu_0} \nabla \cdot (\vec{E} \times \vec{B}) - \frac{\epsilon_0}{2} \frac{\partial E^2}{\partial t} \quad (13)$$

# Energy in electromagnetic field (Poynting theorem / Poynting's vector) (contd.)

Also, from Faraday's law

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (14)$$

$$\implies \vec{B} \cdot (\nabla \times \vec{E}) = -\vec{B} \cdot \frac{\partial \vec{B}}{\partial t} = -\frac{1}{2} \frac{\partial B^2}{\partial t} \quad (15)$$

So, equation (13) becomes

$$\begin{aligned} \vec{E} \cdot \vec{J} &= -\frac{1}{2\mu_0} \frac{\partial B^2}{\partial t} - \frac{\epsilon_0}{2} \frac{\partial E^2}{\partial t} - \frac{1}{\mu_0} \nabla \cdot (\vec{E} \times \vec{B}) \\ &= -\frac{1}{2} \frac{\partial}{\partial t} \left( \epsilon_0 E^2 + \frac{1}{\mu_0} B^2 \right) - \frac{1}{\mu_0} \nabla \cdot (\vec{E} \times \vec{B}) \end{aligned}$$

# Energy in electromagnetic field (Poynting theorem / Poynting's vector) (contd.)

and the equation (11) reduces to

$$\frac{dW}{dt} = \int_V \left[ -\frac{1}{2} \frac{\partial}{\partial t} \left( \epsilon_0 E^2 + \frac{1}{\mu_0} B^2 \right) \right] d\tau - \frac{1}{\mu_0} \int_V \nabla \cdot (\vec{E} \times \vec{B}) d\tau$$
$$\therefore \frac{dW}{dt} = -\frac{1}{2} \frac{d}{dt} \int_V \left( \epsilon_0 E^2 + \frac{1}{\mu_0} B^2 \right) d\tau - \frac{1}{\mu_0} \oint_S (\vec{E} \times \vec{B}) \cdot d\vec{a} \quad (16)$$

where S is a surface which encloses the volume V. This is called Poynting's theorem. It is also called the work-energy theorem of electrodynamics. The first integral on the right is the rate of change of total energy stored in the fields. The second term represents the rate at

# Energy in electromagnetic field (Poynting theorem / Poynting's vector) (contd.)

which energy is carried out of  $V$  across its boundary surface by the electromagnetic fields.

The Poynting's theorem states that “the work done on the charges by the electromagnetic field is equal to the decrease in energy stored in the fields less the energy which is flowed out through the surface”.

The energy per unit time, per unit area, transported by the electromagnetic fields is called the Poynting's vector denoted by  $\vec{S}$

$$\therefore \vec{S} = \frac{1}{\mu_0} (\vec{E} \times \vec{B})$$

# Energy in electromagnetic field (Poynting theorem / Poynting's vector) (contd.)

In equation (16),  $\vec{S} \cdot d\vec{a}$  is the energy per unit time crossing the infinitesimal surface  $d\vec{a}$  i.e. the energy flux. So equation (16) may be written as

$$\frac{dW}{dt} = -\frac{dW_{em}}{dt} - \oint_S \vec{S} \cdot d\vec{a}$$

The work  $W$  done on the charges will increase their mechanical energy (kinetic, potential or whatever). If  $u_{\text{mech}}$  denotes mechanical energy density, such that

$$\frac{dW}{dt} = \frac{d}{dt} \int_V u_{\text{mech}} d\tau$$

# Energy in electromagnetic field (Poynting theorem / Poynting's vector) (contd.)

Therefore,

$$\begin{aligned}\frac{d}{dt} \int_V (u_{\text{mech}} + u_{\text{em}}) d\tau &= - \oint_S \vec{S} \cdot d\vec{a} = \int_V (\nabla \cdot \vec{S}) d\tau \\ \implies \int_V \frac{\partial}{\partial t} (u_{\text{mech}} + u_{\text{em}}) d\tau &= - \int_V (\nabla \cdot \vec{S}) d\tau \\ \implies \frac{\partial}{\partial t} (u_{\text{mech}} + u_{\text{em}}) &= -\nabla \cdot \vec{S}\end{aligned}$$

This is differential version of Poynting theorem. This analogous to the continuity equation  $\frac{\partial \rho}{\partial t} = -\nabla \cdot \vec{J}$  which infers that the energy transported is due to the expense of the total (mechanical plus electromagnetic ) energy of the system of moving charges in the electromagnetic field.

# Electromagnetic wave equations in vacuum

For free space where there is no free charge and current i.e.  $\rho = 0$  and  $\vec{J} = 0$ ; the Maxwell's equations reads

$$\nabla \cdot \vec{E} = 0 \quad (17a)$$

$$\nabla \cdot \vec{B} = 0 \quad (17b)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (17c)$$

$$\nabla \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \quad (17d)$$

Taking curl of equation (17c), we get

$$\nabla \times (\nabla \times \vec{E}) = -\nabla \times \frac{\partial \vec{B}}{\partial t}$$

## Electromagnetic wave equations in vacuum (contd.)

$$\begin{aligned}\implies \nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E} &= -\frac{\partial(\nabla \times \vec{B})}{\partial t} \\ \implies \nabla^2 \vec{E} &= \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}\end{aligned}\tag{18}$$

In component form, equation (18) can be written as

$$\boxed{\begin{aligned}\nabla^2 E_x &= \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2} \\ \nabla^2 E_y &= \mu_0 \epsilon_0 \frac{\partial^2 E_y}{\partial t^2} \\ \nabla^2 E_z &= \mu_0 \epsilon_0 \frac{\partial^2 E_z}{\partial t^2}\end{aligned}}\tag{19}$$

Again taking curl of equation (17d) yields

$$\nabla \times (\nabla \times \vec{B}) = \mu_0 \epsilon_0 \nabla \times \frac{\partial \vec{E}}{\partial t}$$



## Electromagnetic wave equations in vacuum (contd.)

$$\begin{aligned}\implies \nabla(\nabla \cdot \vec{B}) - \nabla^2 \vec{B} &= \mu_0 \epsilon_0 \frac{\partial(\nabla \times \vec{E})}{\partial t} \\ \implies \nabla^2 \vec{B} &= \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}\end{aligned}\tag{20}$$

In component form

$$\boxed{\begin{aligned}\nabla^2 B_x &= \mu_0 \epsilon_0 \frac{\partial^2 B_x}{\partial t^2} \\ \nabla^2 B_y &= \mu_0 \epsilon_0 \frac{\partial^2 B_y}{\partial t^2} \\ \nabla^2 B_z &= \mu_0 \epsilon_0 \frac{\partial^2 B_z}{\partial t^2}\end{aligned}}\tag{21}$$

The equation (19) and (21) satisfy the wave equation

$$\nabla^2 f = \frac{1}{c^2} \frac{\partial^2 f}{\partial t^2}\tag{22}$$

# Electromagnetic wave equations in vacuum (contd.)

where  $c$  is velocity of wave. Thus, equations (18) and (20) are equations of electromagnetic waves as shown in figure 2 with speed

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \approx 3 \times 10^8 \text{ m} \cdot \text{s}^{-1}$$

This is equal to the experimentally determined velocity of electromagnetic waves.

# Electromagnetic wave equations in vacuum (contd.)

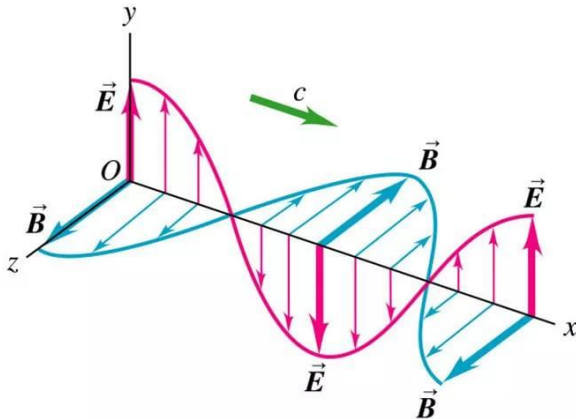


Figure 2: An electromagnetic wave

**End of Lecture 16**

**Thank you**