ASSIGNMENT-I (2023)
MATH 104

1. Define polar, Cylindrical and spherical coordinates of a point in space. Establish the relation among Cartesian, cylindrical and spherical coordinates in three dimensional coordinates.

2. Replace the following Cartesian equations by equivalent polar equations.
   (a) $y = -5$ (b) $\dfrac{x^2}{4} + \dfrac{y^2}{4} = 1$ (c) $(x-1)^2 + (y-2)^2 = 9$ (d) $xy = 4$

3. (a) Find the rectangular coordinates for $(2, \pi/2, \pi/3)$ and $(2, 2\pi/3, 1)$.
   (b) Find the spherical coordinates for $(0, 2\sqrt{3}, -2)$ and the cylindrical coordinates for $(3, -3, -7)$.
   (c) Find the equation in spherical coordinate system.
      (i) $x^2 + y^2 + z^2 = 1$.
      (ii) $z = \sqrt{x^2 + y^2}$.
   (d) Find the equation in cylindrical polar coordinates for $x^2 + (y-3)^2 = 9$.
   (e) Find the Cartesian and cylindrical coordinate equations for the equation $\phi = 5\pi/6,\ 0 \leq \rho \leq 2$ with proper ranges for z in Cartesian and r in cylindrical coordinates.

4. Find the pair of polar coordinates that label the same point.
   (a) $(3, 0)$ (b) $(-3, 0)$ (c) $(2, 2\pi/3)$ (d) $(-3, \pi)$ (e) $(-3, 2\pi)$
   (f) $(-2, -\pi/3)$.

5. How is the idea of symmetry of a polar curve $r = f(\theta)$ helpful to sketch the graph of the curve? Write the conditions when the curves $r = f(\theta)$ is symmetric about x-axis, and y-axis. Check the symmetry, sketch the graph of the curve $r = \sin 2\theta$, and find the area covered by the curve.

6. Sketch the graph of the following equations:
   (a) $r = -2\sin\theta$ (b) $r = 2\cos\theta$ (c) $r = 1 - \cos\theta$ (d) $r = -1 + \sin\theta$
   (e) $r = 5 - 5\sin\theta$ (f) $r = 12 + \cos\theta$ (g) $r = 2 + \cos\theta$ (h) $r = 32 + \cos\theta$
   (i) $r = 2\cos 2\theta$ (j) $r = \sin 4\theta$ (k) $r^2 = 4\sin 2\theta$.

7. Find the areas of the following regions:
   (a) Inside the lemniscate $r^2 = 4\sin 2\theta$.
   (b) Inside one leaf of the four leaved rose $r = \cos 2\theta$.
   (c) Inside the limacon $r = 4 + 2\cos\theta$.
   (d) Inside the cardiod $r = 2(1 + \cos\theta)$.
   (e) Inside the circle $r = 1$ and outside the cardiod $r = 1 - \cos\theta$.
   (f) The inner loop of the limacon $r = 12 + 24\cos\theta$.
   (g) Shared by the circle $r = 2$ and cardiod $r = 2 - 2\cos\theta$.
   (h) Inside the circle $r = 4\cos\theta$ and to the right of the vertical line $r = \sec\theta$.
   (i) Shared by the circles $r = 2a\cos\theta$ and $r = 2a\sin\theta$.

8. (a) Find the polar equation of the line passing through $P_0(2, \pi/3)$.
   (b) Find the Cartesian equation of the polar line $r\cos(\theta - 2\pi/3) = 1$.