ASSIGNMENT-III (2023)

MATH 104

1. Find all the local maxima, local minima, and saddle points of the functions

   a. $f(x,y) = x^2 + xy + y^2 + 3x - 3y + 4$ [Ans. local min. - 5]

   b. $f(x,y) = x^2 + xy + 3x + 2y + 5$ [Ans. Saddle point (-2,1)]

2. Find the absolute maxima and minima of the functions on the given domains.

   a. $f(x,y) = 2x^2 - 4x + y^2 - 4y + 1$ on the closed triangular plate bounded by the lines $x = 0, \quad y = 2, \quad y = 2x$ in the first quadrant. [Ans. 1 at (0,0) and - 5 at (1,2)]

   b. $f(x,y) = 48xy - 32x^3 - 24y^2$ on the rectangular plate $0 \le x \le 1$, $0 \le y \le 1$. [Ans. 2 at $(1/2, 1/2)$ and -32 at $(1,0)$]

3. Maximize the function $f(x,y,z) = x^2 + 2y - z^2$ subject to the constraints $2x - y = 0$. and $y + z = 0$ [Ans. $f\left(\frac{2}{3}, \frac{4}{3}, -\frac{4}{3}\right) = \frac{4}{3}$]

4. Find the extreme values of $f(x,y,z) = x - y + z$ on the unit sphere $x^2 + y^2 + z^2 = 1$. [Ans. $\sqrt{-3}$, $\sqrt{3}$]

5. Define the double integral of a function $f(x,y)$ over a rectangular region in xy-plane. State first form and stronger from of Fubini's theorem in plane. Sketch the region of integration of the function $f(x,y) = x/y$ over the region in the first quadrant bounded by the line $y = x, y = 2x, x = 1, x = 2$ and then integrate it. [Ans. $3/2 \ln 2$]

6. Evaluate the followings

   a. $\displaystyle\int_0^3 \int_0^2 (4 - y^2)\,dydx$ [Ans. 16] *b.* $\displaystyle\int_{-1}^0 \int_{-1}^1 (x + y + 1)\,dxdy$ [Ans. 1]

   c. $\displaystyle\int_0^\pi \int_0^x (x \sin y)\,dydx$ [Ans. $\frac{\pi}{2} + 2$] *d.* $\displaystyle\int_0^{\ln 8} \int_0^{\ln y} e^{(x+y)}\,dydx$ [Ans. $\ln 8 - 16 + e$]

   e. The integral of $f(x,y) = x^2 + y^2$ over the triangular region with vertices $(0,0), (1,0), (0,1)$. [Ans. 1/6]

7. Reverse the order of integration and evaluate: (See Ex.13.1 answer key)

   a. $\displaystyle\int_0^1 \int_2^{4-2x} dydx$ *b.* $\displaystyle\int_0^1 \int_y^{\sqrt{y}} dxdy$ *c.* $\displaystyle\int_0^1 \int_1^{e^x} dydx.$

8. Find the volume of the region that lies under the paraboloid $z = x^2 + y^2$ and above the triangle enclosed by the lines $y = x, x = 0$ and $x + y = 2$ in the xy-plane. [Ans.4/3]

9. Find the volume of the prism whose base is the triangle in the $xy-$plane bounded by the $x-$axis and the line $y = x$ and $x = 1$ whose top face is the plane $z = f(x,y) = 3 - x - y$. [Ans. 1]

10. Change the Cartesian integrals into equivalent polar integrals and evaluate this polar integral:

$a.\ \int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} dydx$ [Ans. $\pi/2$] $\qquad b.\ \int_{-1}^{1}\int_{0}^{\sqrt{1-y^2}} (x^2+y^2)dxdy$ [Ans. $\pi/8$]

$c.\ \int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} dydx$ [Ans πa^2] $\qquad d.\ \int_{0}^{6}\int_{2}^{y} xdxdy$ [Ans. 36]

$e.\ \int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \ln(x^2+y^2+1)dxdy$ [Ans. $\pi/2+1$]

11. Find the average value of $f(x,y) = x\cos xy$ over the rectangle $R: 0 \le x \le \pi$ and $0 \le y \le 1$. [Ans. $2/\pi$]

12. Evaluate the integrals:

$a.\ \int_{0}^{1}\int_{0}^{1}\int_{0}^{1} (x^2+y^2+z^2)dzdxdy$ [Ans. 1]

$b.\ \int_{0}^{1}\int_{0}^{\pi}\int_{0}^{\pi} (y\sin z)dxdydz$ [Ans. $\pi^3(1-\cos 1)/3$]

13. Find the volume of the sphere $x^2+y^2+z^2 = a^2$ by using triple integral. [Ans. $4\pi a^3/3$]

14. Find the average value of $F(x,y,z)$ over the given region:

   a. $F(x,y,z) = x^2+9$ over the cube in the first octant bounded by the coordinates plane and the planes $x = 2, y = 2$ and $z = 2$. [Ans. 31/2]

   b. $F(x,y,z) = x^2+y^2+z^2$ over the curve in the first octant bounded by the coordinates planes and the planes $x = 2, y = 2$ and $z = 2$. [Ans. 1]

15. Evaluate the cylindrical coordinate's integral $\int_{0}^{2\pi}\int_{0}^{1}\int_{r}^{\sqrt{2-r^2}} (y\sin z)dzrdrd\theta$ $\left[\text{Ans. } \frac{4\pi\sqrt{2}-1}{3}\right]$

16. Evaluate the spherical coordinate's integral $\int_{0}^{2\pi}\int_{0}^{\pi}\int_{r}^{(1-\cos\phi)/2} \rho^2\sin\phi d\rho d\phi d\theta$ [Ans. $\pi/3$]

17. (i) Define the Jacobian determinant or Jacobian of the coordinate transformation $x = g(u,v)$, $y = h(u,v)$. Find the Jacobian $\dfrac{\partial(x,y)}{\partial(u,v)}$ for the transformation:

   $a.\ x = u\cos v, y = u\sin v$ $\qquad b.\ x = u\sin v, y = u\cos v$

   (ii) Find the Jacobian $\dfrac{\partial(x,y,z)}{\partial(u,v,w)}$ of the transformation:

   $a.\ x = u\cos v, y = u\sin v, z = w$ $\qquad b.\ x = 2u-1, y = 3v-4, z = \frac{1}{2}(w-4).$

18. Evaluate the following integrals

   i. $\int_{0}^{3}\int_{0}^{4}\int_{x=\frac{y}{2}}^{x=\frac{y}{2}+1} \left(\frac{2x-y}{2}+\frac{z}{3}\right) dx\,dy\,dz$ by applying the transformation

$$u = \frac{2x-y}{2}, \quad v = \frac{y}{2}, \quad w = \frac{w}{3}$$

and integrating over an appropriate region in uvw - space. Ans: 12.

ii. Use the transformation $u = x + 2y$, $v = x - y$ to evaluate the integral

$$\int_0^{2/3} \int_y^{2-2y} (x+2y)e^{y-x}\,dxdy$$

by writing it as an integral over the region G in the uv - plane.

Ans: $\frac{1}{3}\left(1 + \frac{3}{e^2}\right) \approx 0.4687$.

iii. Evaluate

$$\int_0^1 \int_0^{1-x} (y-2x)^2 \sqrt{x+y}\,dydx$$

(Hints: Integrand suggests the transformation $u = x + y$, $v = y - 2x$) Ans: $\dfrac{2}{9}$.

** The End **