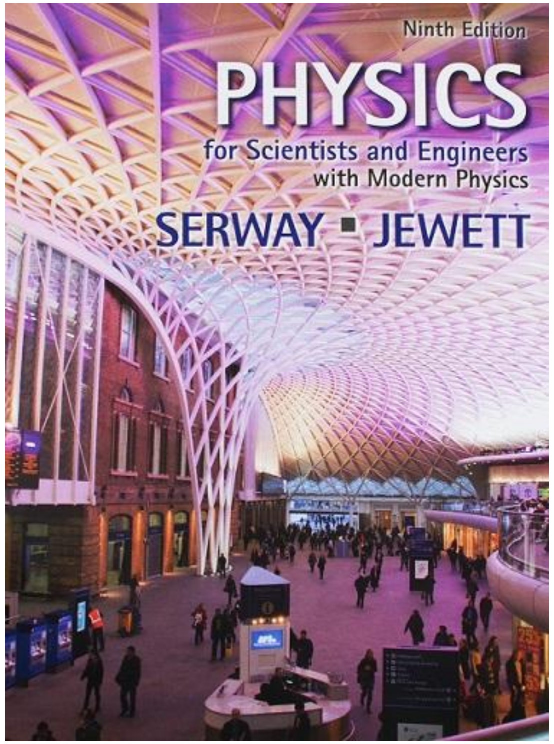
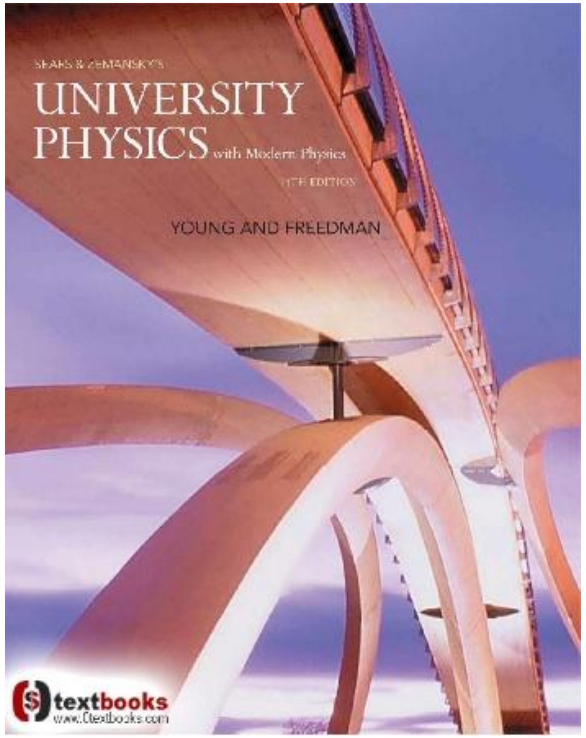
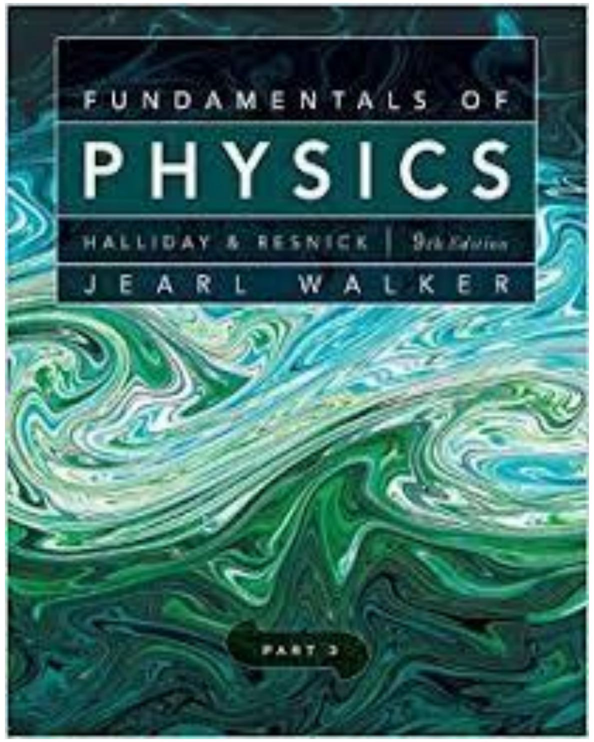
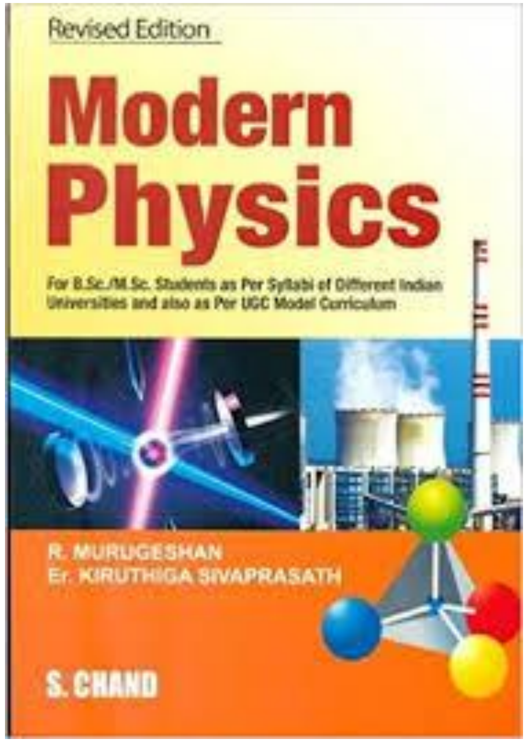
PHYSICS

General Physics II (PHYS 102)









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Course Outline

- Some Properties of Nuclei
- Nuclear Binding Energy
- Radioactivity
- Half Life
- The Decay Processes
- Nuclear Reactions
- Q-Value in Nuclear Reaction
- Nuclear Fission and Nuclear Fusion

Milestones in the Development of Nuclear Physics

NUCLEAR PHYSICS

The year **1896** marks the **birth of nuclear physics** when French physicist **Antoine-Henri Becquerel** (1852–1908) discovered radioactivity in uranium compounds. This discovery prompted scientists to investigate the details of radioactivity and, ultimately, the structure of the nucleus.

- **Ernest Rutherford** showed the radiation emitted from radioactive substances had three types:
  - alpha rays(He nuclei)
  - beta rays(electrons)
  - gamma rays(high-energy photons)
- In **1911**, **Rutherford, Hans Geiger, and Ernest Marsden** performed scattering experiments
  - Established that the nucleus of an atom can be modeled as a point mass and a point charge
  - Most of the atomic mass was contained in the nucleus
  - Nuclear force was a new type of force

The short-range force, which is predominant at particle separation distances less than approximately 10^{-14} m and is zero for large distances

- The observation of nuclear reactions in 1930 by Cockroft and Walton, using *artificially accelerated particles*
- The discovery of the neutron in 1932 by Chadwick and the conclusion that neutrons make up about half of the nucleus
- The discovery of artificial radioactivity in 1933 by Joliot and Irene Curie
- The discovery of nuclear fission in 1938 by Meitner, Hahn, and Strassmann
- The development of the first controlled fission reactor in 1942 by Fermi and his collaborators

During the past century, **applications of nuclear physics** have had enormous effects on humankind, some beneficial (reactors), some catastrophic (bombs)

Some Properties of Nuclei

Some Properties of Nuclei

- All nuclei are composed of two types of particles: **protons and neutrons**.
  - The only exception is the ordinary hydrogen nucleus, which is a single proton.
- The atomic nucleus is described by the number of protons and neutrons it contains, using the following quantities:
  - The **Atomic Number (Z)** equals the number of protons in the nucleus.
  - The **Neutron number (N)** is the number of neutrons in the nucleus.
  - The **Mass number (A=Z+N)** is the number of nucleons (neutrons plus protons) in the nucleus

Mass Number Is Not Atomic Mass

- A **nuclide** is a specific combination of atomic number and mass number that represents a nucleus.

Symbolism:

$$\text{Nuclide} \rightarrow {}^{A}_{Z}X$$

X $\downarrow$ the chemical symbol of the element

Example:

$${}^{27}_{13}Al$$

- Mass number is 27
- Atomic number is 13
- Contains 13 protons
- Contains 14 (27 – 13) neutrons

- The nuclei of all atoms of a particular element contain the same number of protons but often contain different numbers of neutrons. Nuclei related in this way are called **isotopes**.

The **isotopes** of an element have the same Z value but different N and A values.

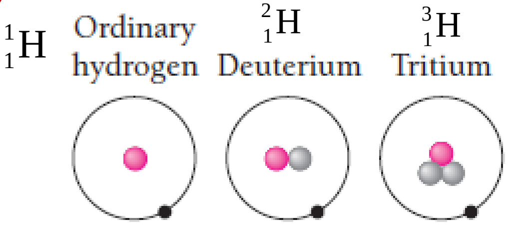


Figure M-1 The isotopes of Hydrogen

Some Properties of Nuclei

NUCLEAR PHYSICS

Charge and Mass

- The proton has a single positive charge e, equal in magnitude to the charge -e on the electron.

$$e = 1.6 \times 10^{-19}\ \text{C}$$

  The neutron has no charge

- The proton is approximately 1836 times as massive as the electron, and the masses of the proton and the neutron are almost equal.

- It is convenient to use **atomic mass units**, u, to express masses
  - 1 u = 1.660 539 x 10^{-27} kg
  - Based on definition that the mass of one atom of ^{12}C is exactly 12 u

Mass can also be expressed in MeV/c^2

- From E_R = mc^2
- 1 u = 931.494 MeV/c^2
  - Includes conversion 1 eV = 1.602 177 x 10^{-19} J

Table M-1 Masses of Selected Particles in Various Units

Particle	Mass kg	Mass u	Mass MeV/c^2
Proton	$1.672\,62 \times 10^{-27}$	1.007 276	938.27
Neutron	$1.674\,93 \times 10^{-27}$	1.008 665	939.57
Electron (β particle)	$9.109\,38 \times 10^{-31}$	$5.485\,79 \times 10^{-4}$	0.510 999
$^1_1$H atom	$1.673\,53 \times 10^{-27}$	1.007 825	938.783
$^4_2$He nucleus (α particle)	$6.644\,66 \times 10^{-27}$	4.001 506	3 727.38
$^4_2$He atom	$6.646\,48 \times 10^{-27}$	4.002 603	3 728.40
$^{12}_6$C atom	$1.992\,65 \times 10^{-27}$	12.000 000	11 177.9

The Atomic Mass Unit

$$1\text{u} = \frac{1}{12}\left(\text{mass } m \text{ of one } ^{12}\text{C atom}\right)$$

$$= \frac{1}{12}\left(\frac{0.012\ \text{kg}}{6.02 \times 10^{23}\ \text{atoms}}\right)$$

$$= 1.66 \times 10^{-27}\ \text{kg}$$

Some Properties of Nuclei

NUCLEAR PHYSICS

Size and Structure of Nuclei

- First investigated by Rutherford in scattering experiments
- Since the time of Rutherford, many other experiments have concluded the following:
    - Most nuclei are approximately spherical
    - Average radius of Nuclei is

$$r = r_0 A^{\frac{1}{3}}$$

$$r_0 = 1.2 \text{ fm} = 1.2\times10^{-15}\text{m}$$

- A is the mass number

femtometer (sometimes called the fermi in honor of Enrico Fermi, a pioneer in Physics)

- The volume of the nucleus (assumed to be spherical) is directly proportional to the total number of nucleons

  This suggests that *all nuclei have nearly the same density*

- Nucleons combine to form a nucleus as though they were tightly packed spheres

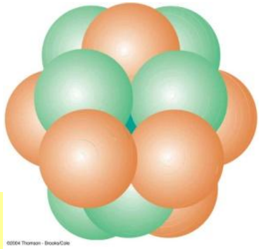


Figure M-2

A nucleus can be modeled as a cluster of tightly packed spheres, where each **is a nucleon.**

The Volume and Density of a Nucleus of mass number *A*:

- The volume of the nucleus : $V_{\text{nucleus}} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi a^3 A$

- The density of the nucleus : $\rho = \frac{m_{\text{nucleus}}}{V_{\text{nucleus}}} = \frac{mA}{\frac{4}{3}\pi r_0^{\ 3} A} = \frac{3m}{4\pi r_0^{\ 3}} = \frac{3\left(1.67\times10^{-27}\text{kg}\right)}{4\pi\left(1.2\times10^{-15}\text{m}\right)^3}$

$$= 2.3\times10^{17}\ \text{kg}/\text{m}^3$$

The nuclear density is approximately 2.3×10^{14} times as great as the density of water.

Some Properties of Nuclei

Nuclear Stability

- The nucleus consists of a closely packed collection of protons and neutrons.

Like charges (the protons) in proximity exert very large repulsive electrostatic forces on each other, which should cause the nucleus to fly apart. However, nuclei are stable because of the presence of the **nuclear force**.

Nuclear Force

- Nuclear Force is an attractive force, with a very short range (about 2 fm), that acts between *all nuclear* particles. The protons attract each other via the nuclear force, and at the same time they repel each other through the Coulomb force.
- The nuclear attractive force is stronger than the Coulomb repulsive force at the short ranges within the nucleus
- The attractive nuclear force also acts between pairs of neutrons and between neutrons and protons.

Features of the Nuclear Force

- Attractive force that acts between all nuclear particles
- Very short range
  - It falls to zero when the separation between particles exceeds about several fermis
- Independent of charge
  - The nuclear force on p-p, p-n, n-n are all the same
  - Does not affect electrons

Some Properties of Nuclei

Nuclear Stability

- The existence of the nuclear force results in approximately 270 stable nuclei; hundreds of other nuclei have been observed, but they are unstable.
- **Figure M-3** is a plot of N *versus* Z for a number of stable nuclei.
- The stable nuclei are represented by the black dots, which lie in a narrow range called the ***line of stability***.

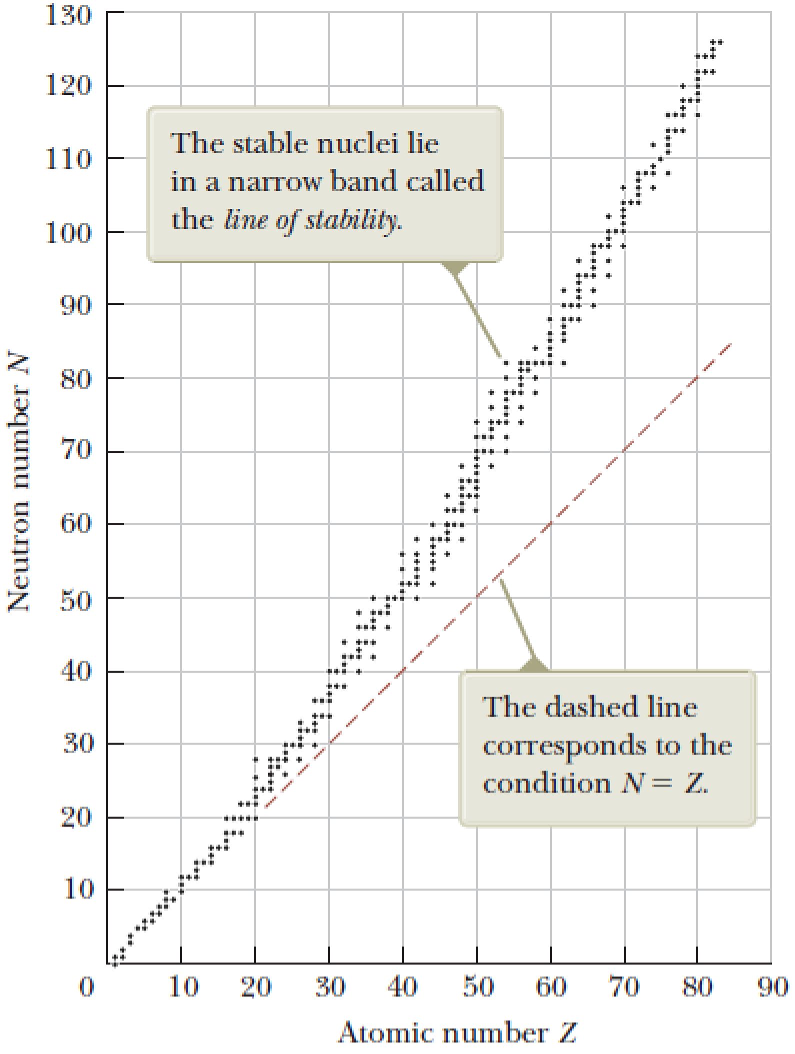


Figure M-3

- Light nuclei are most stable if $N = Z$
- Heavy nuclei are most stable when $N > Z$
  - Above about $Z = 20$
  - As the number of protons increases, the Coulomb force increases and so more neutrons are needed to keep the nucleus stable
- No nuclei are stable when $Z > 83$

Nuclear Binding Energy

Binding Energy

- The total mass of a nucleus is always less than the sum of the masses of its nucleons. Because mass is a measure of energy, the total energy of the bound system (the nucleus) is less than the combined energy of the separated nucleons. This difference in energy is called the **binding energy** of the nucleus and can be thought of as the energy that must be added to a nucleus to break it apart into its components.

Binding Energy -The missing energy that keeps a nucleus together.

B.E.>0 ➡ stable nucleus

- The binding energy can be calculated from conservation of energy and the Einstein mass-energy equivalence principle.

The binding energy E_B of the nucleus ${}^{A}_{Z}X$ is given by

$$E_B\,(\mathrm{MeV}) = \left[Zm\left({}^{1}_{1}\mathrm{H}\right) + Nm_n - m\left({}^{A}_{Z}\mathrm{X}\right) \right] \mathrm{u} \times 931.494\,\mathrm{MeV/u}$$

where $m\left({}^{1}_{1}\mathrm{H}\right)$ is the atomic mass of the neutral hydrogen atom,

m_n is the mass of the neutron

and, $m\left({}^{A}_{Z}\mathrm{X}\right)$ is the atomic mass of ${}^{A}_{Z}\mathrm{X}$.

Note:

$$E_B\,(\mathrm{J}) = \left[Zm\left({}^{1}_{1}\mathrm{H}\right) + Nm_n - m\left({}^{A}_{Z}\mathrm{X}\right) \right] \mathrm{kg} \times c^2$$

Calculate the binding energy of the deuteron, which consists of a proton and a neutron, given that the atomic mass of the deuteron is $\mathrm{M}_2 = 2.014102\ \mathrm{u}$.

Solution:

$$\begin{aligned} E_B\,(\mathrm{MeV}) &= \left[Zm\left({}^{1}_{1}\mathrm{H}\right) + Nm_n - M_2 \right] \mathrm{u} \times 931.494\,\mathrm{MeV/u} \\ &= [1.007825\ \mathrm{u} + 1.008665\ \mathrm{u} - 2.014\,102\ \mathrm{u}] \times 931.494\,\mathrm{MeV/u} \\ &= [0.002\,388\ \mathrm{u}] \times 931.494\,\mathrm{MeV/u} \\ &= 2.224\ \mathrm{MeV} \end{aligned}$$

Sample Problem

Calculate the binding energy per nucleon (MeV/nucleon) for tritium $\left({}_1^3\mathrm{H}\right)$, a radioactive isotope of hydrogen.

Assume:

$$m_p = 1.007825\ \mathrm{u}$$

$$m_n = 1.008665\ \mathrm{u}$$

$$m_t = 3.01605\ \mathrm{u}$$

$$\mathrm{u} = 1.66\times10^{-27}\mathrm{kg}$$

Solution:

Binding Energy:

$$E_B\left(\mathrm{MeV}\right) = \left[Zm_P + Nm_n - m\left({}_Z^A\mathrm{X}\right)\right]\mathrm{u}\times931.5\,\mathrm{MeV/u}$$

$$= \left[\left(1\times1.007825\mathrm{u} + 2\times1.008665\mathrm{u}\right) - 3.01605\mathrm{u}\right]\times931.5\,\mathrm{MeV/u}$$

$$= 8.48\ \mathrm{MeV}$$

Binding energy per nucleon

$$\frac{E_B}{A} = \frac{8.48\mathrm{MeV}}{3} = 2.8\mathrm{MeV/nucleon}$$

Radioactivity

- **Radioactivity** is the spontaneous emission of radiation
  - Discovered by Becquerel in 1896
  - Many experiments were conducted by Becquerel and the Curies
- Experiments suggested that radioactivity was the result of the decay, or disintegration, of unstable nuclei
- Three types of radioactive decay occur in radioactive substances:

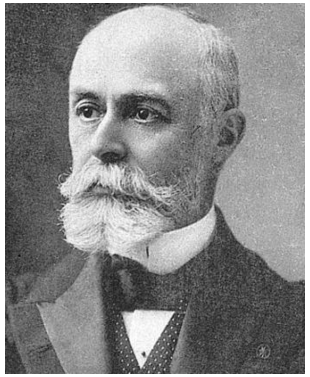
(i) alpha (α) decay: emitted particles – ${}^{4}_{2}\mathrm{He}$ nuclei

(ii) beta (β) decay: emitted particles – electrons or positrons

(iii) gamma (γ) decay: emitted particles – high-energy protons

Antoine-Henri Becquerel

- 1852 – 1908
- a French engineer, physicist, Nobel laureate, and the first person to discover evidence of radioactivity
- Shared Nobel Prize in 1903 for studies in radioactive substances
  - Prize in physics
  - Shared with Pierre Curie and Marie Curie



Alpha Particles

- The particles are ${}^{4}_{2}\mathrm{He}$ nuclei
- Barely penetrate a piece of paper

Beta Particles

- The particles are either electrons or positrons
- Can penetrate a few mm of aluminum

Gamma Rays

- The "rays" are high energy photons
- Can penetrate several cm of lead

Positron (e^{+}) : antiparticle of the electron

Positron is a particle like the electron in all respects except that the positron has a charge of $+e$.

Radioactivity

- The rate at which a particular decay process occurs in a radioactive sample is proportional to the number of radioactive nuclei present .
- If N is the number of radioactive nuclei present at some instant, the rate of change of N is

$$-\frac{dN}{dt} \propto N$$

$$\therefore \boxed{\frac{dN}{dt} = -\lambda N} \quad .... \quad (1)$$

where λ is called the **decay constant** and determines the rate at which the material will decay.

- Eq.(1) can be written in the form

$$\frac{dN}{N} = -\lambda dt$$

Integrating both sides, we get

N_0 ↓ number of undecayed nuclei at t = 0

$$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_0^t dt$$

$$\ln\left(\frac{N}{N_0}\right) = -\lambda t + k \quad .... \quad (2)$$

Marie Curie

1867 -1934 (Polish Scientist)

- Shared Nobel Prize in Physics in 1903 for studies in radioactive substances with Pierre Curie and Becquerel
- Won Nobel Prize in Chemistry in 1911 for discovery of radium and polonium

At $t = 0,\ N = N_0,\ \text{So } k = \ln N_0$

$$\therefore \quad \ln N = -\lambda t + \ln N_0$$

$$\text{or,} \quad \ln\left(\frac{N}{N_0}\right) = -\lambda t$$

$$\text{or,} \quad \frac{N}{N_0} = e^{-\lambda t}$$

$$\therefore \quad \boxed{N = N_0 e^{-\lambda t}} \quad .... \quad (3)$$

Equation (3) shows that the number of radioactive nuclei in a sample decreases exponentially with time.

Figure M-5 illustrates the exponential decay law.

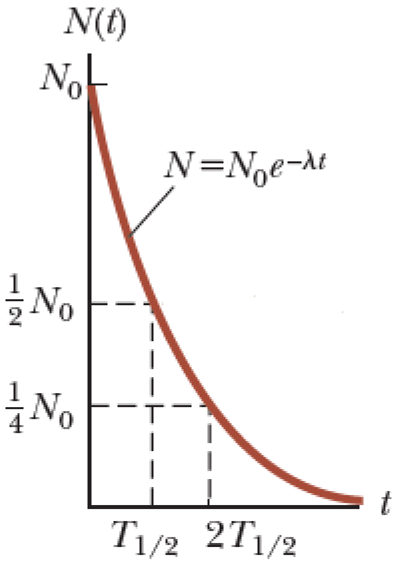


Figure M-5

Decay Rate

- The **decay rate** R of a sample is defined as the number of decays per second.

$$\boxed{R = \left|\frac{dN}{dt}\right| = \lambda N_0 e^{-\lambda t} = R_0 e^{-\lambda t}}$$

where $R_0 = \lambda N_0$ is the decay rate at t = 0 and $R = \lambda N$

- The decay rate of a sample is often referred to as its **activity**.
- A frequently used unit of activity is the **curie (Ci)**, defined as

$$1\ \text{Ci} \equiv 3.7 \times 10^{10}\ \text{decays/s}$$

The SI unit of activity is the **becquerel (Bq)**:

$$1\ \text{Bq} \equiv 1\ \text{decay/s}$$

Therefore

$$\boxed{1\ \text{Ci} \equiv 3.7 \times 10^{10}\ \text{Bq}}$$

Half-Life

- The half-life is defined as the time interval during which half of a given number of radioactive nuclei decay.
- The half-life is a useful parameter in characterizing the decay of a particular nucleus
- For $t = T_{1/2},\ N = \dfrac{N_0}{2}$

$$\text{So, N} = N_0 e^{-\lambda t} \Rightarrow \frac{N_0}{2} = N_0 e^{-\lambda t}$$

$$\Rightarrow e^{\lambda t} = 2$$

$$\Rightarrow \lambda t = \ln 2$$

$$\therefore \boxed{t = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}}$$

This is a convenient expression for relating half-life to the decay constant.

Sample Problem

NUCLEAR PHYSICS

How long does it take for 60% of a sample of radon to decay? Half-life of radon is 3.8 days.

Hint:

Number of undecayed nuclei:

$$N = N_0 - 60\% \text{ of } N_0$$

$$\Rightarrow N = \frac{40}{100} N_0$$

$$\therefore \frac{N_0}{N} = \frac{100}{40}$$

We have,

$$\boxed{N = N_0 e^{-\lambda t}}$$

$$\Rightarrow e^{\lambda t} = \frac{N_0}{N} \quad \Rightarrow \lambda t = \ln\left(\frac{N_0}{N}\right) \quad \Rightarrow t = \frac{\ln\left(\frac{N_0}{N}\right)}{\lambda} \quad \Rightarrow t = \frac{\ln\left(\frac{N_0}{N}\right)}{0.693} \times T_{1/2}$$

$$\therefore \quad t = \frac{\ln\left(\frac{100}{40}\right)}{0.693} \times 3.8 \text{ days} = 5.02 \text{days}$$

The half life of radon is 3.8 days. After how many days will only 1/20 of a radon sample be left over?

Hint:

$$N = N_0 e^{-\lambda t}$$

$$\Rightarrow t = \frac{\ln\left(\frac{N_0}{N}\right)}{\lambda} = \frac{\ln\left(\frac{N_0}{N}\right)}{0.693} \times T_{1/2} = \frac{\ln(20)}{0.693} \times 3.8 \text{ days} = 16.43 \text{ days}$$

- $N = \frac{1}{20} N_0$
- $\lambda = \frac{0.693}{T_{1/2}}$

Decay Processes

- A radioactive nucleus spontaneously decays by means of one of three processes: alpha decay, beta decay, or gamma decay.

Alpha Decay

- A nucleus emitting an alpha particle , loses two protons and two neutrons.
- The atomic number Z *decreases by 2, the mass number* A *decreases by 4*, and the neutron number decreases by 2.
- The decay can be written as

$$\boxed{{}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}X + {}^{4}_{2}He}$$

where X is called the parent nucleus and Y the daughter nucleus.

- Example:

$${}^{238}_{92}\mathrm{U} \rightarrow {}^{234}_{90}\mathrm{Th} + {}^{4}_{2}\mathrm{He}$$

$${}^{226}_{88}\mathrm{Ra} \rightarrow {}^{222}_{86}\mathrm{Rn} + {}^{4}_{2}\mathrm{He}$$

Beta Decay

When a radioactive nucleus undergoes beta decay, the daughter nucleus has the same number of nucleons as the parent nucleus, but the atomic number is changed by 1:

$$\boxed{\begin{aligned} {}^{A}_{Z}X &\rightarrow {}_{Z+1}^{A}X + e^{-} + \overline{\nu} \\ {}^{A}_{Z}X &\rightarrow {}_{Z-1}^{A}X + e^{+} + \nu \end{aligned}}$$

Example:

$${}^{12}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N} + e^{-} + \overline{\nu}$$

$${}^{12}_{7}\mathrm{N} \rightarrow {}^{12}_{6}\mathrm{C} + e^{+} + \nu$$

neutrino

Gamma Decay

Very often, a nucleus that undergoes radioactive decay is left in an excited energy state. The nucleus can then undergo a second decay to a lower energy state, perhaps to the ground state, by emitting a high-energy photon:

$$\boxed{{}^{A}_{Z}X* \rightarrow {}^{A}_{Z}X + \gamma}$$

where $X*$ indicates a nucleus in an excited state.

Photons emitted in such a deexcitation process are called gamma rays. Such photons have very high energy (in the range of 1 MeV to 1 GeV) relative to the energy of visible light (about 1 eV).

Example of a decay sequence:

$${}^{12}_{5}\mathrm{B} \rightarrow {}^{12}_{6}\mathrm{C}^{*} + e^{-} + \overline{\nu}$$

$${}^{12}_{6}\mathrm{C}^{*} \rightarrow {}^{12}_{6}\mathrm{C} + \gamma$$

Nuclear Reactions

Nuclear Reaction

- The structure of nuclei can be changed by bombarding them with energetic particles
  - The changes are called **nuclear reactions.**
- Rutherford was the first to observe them in 1919, using naturally occurring radioactive sources for the bombarding particles.
- As with nuclear decays, the atomic numbers and mass numbers must balance on both sides of the equation.
- **Nuclear reactions** can occur when a target nucleus X is bombarded by a particle a, resulting in a daughter nucleus Y and an outgoing particle b:

$$a + \mathrm{X} \rightarrow \mathrm{Y} + b \xrightarrow{\text{in more compact form}} \mathrm{X}(a,b)\mathrm{Y}$$

Example:

$${}^{1}_{1}\mathrm{H} + {}^{7}_{3}\mathrm{Li} \rightarrow {}^{4}_{2}\mathrm{He} + {}^{4}_{2}\mathrm{He}$$

- The mass–energy conversion in such a reaction, called the **reaction energy Q,** is

$$\boxed{\mathrm{Q} = \left[(M_a + M_X) - (M_Y + M_b)\right] \times \mathrm{c}^2}$$

Difference between the initial and final rest energies resulting from the reaction

- The **conservation laws** for nuclear reactions:
  - Conservation of mass number
  - Conservation of charge
  - Conservation of energy, linear momentum and angular momentum

NUCLEAR PHYSICS

Nuclear Reactions

Some main types of Nuclear Reactions

- Elastic Scattering
  - In this case incident particle strikes the target nucleus and leaves without loss of energy but its direction may change.
  - Example:

    Scattering of α-particles from a thin gold foil

$$\boxed{{}_{79}Au^{197} + {}_{2}He^{4} \rightarrow {}_{79}Au^{197} + {}_{2}He^{4}}$$

- Inelastic Scattering
  - In this case the incident particle loses a part of its energy in exciting the target nucleus to a higher allowed energy level. The excited nucleus later decays radiating a γ- ray photon.
  - Example:

$$\boxed{\begin{array}{l} {}_{3}Li^{7} + {}_{1}H^{1} \rightarrow \left({}_{3}Li^{7}\right)^{*} + {}_{1}H^{1} \\ \left({}_{3}Li^{7}\right)^{*} \rightarrow {}_{3}Li^{7} + \gamma \end{array}}$$

- Radiative capture
  - In this case the incident particle is captured by the target nucleus and a new nucleus is formed. The new nucleus then decays with the emission of one or more γ rays photon.
  - Example:

$$\boxed{{}_{6}C^{12} + {}_{1}H^{1} \rightarrow \left({}_{7}N^{13}\right)^{*} \rightarrow {}_{7}N^{13} + \gamma}$$

- Disintegration
  - The incident particle is absorbed by the target nucleus and the ejected particle is a different one. The composition of product nucleus is different from parent nucleus.
  - Example:

$$\boxed{{}_{4}Be^{9} + {}_{2}He^{4} \rightarrow {}_{6}C^{12} + {}_{0}n^{1}}$$

- Photodisintegration
  - When target materials are bombarded with radiations, the compound nucleus decays with formation of neutrons.
  - Example:

$$\boxed{{}_{1}H^{2} + \gamma \rightarrow {}_{1}H^{1} + {}_{0}n^{1}}$$

Nuclear Reactions

Q- Values for Reactions

- Nuclear Reaction: $a + \mathrm{X} \rightarrow \mathrm{Y} + b$
- Q-Value of Nuclear Reaction:

$$\mathrm{Q\,(J)} = \left[\left(M_a + M_X\right) - \left(M_Y + M_b\right)\right]\mathrm{kg} \times \mathrm{c}^2$$

$$\mathrm{Q\,(MeV)} = \left[\left(M_a + M_X\right) - \left(M_Y + M_b\right)\right]\mathrm{u} \times 931.5\,\mathrm{MeV/u}$$

- The Q value determines the type of reaction
  - An **exothermic** reaction
    - There is a mass "loss" in the reaction
    - There is a release of energy
    - Q is positive
  - An **endothermic** reaction
    - There is a "gain" of mass in the reaction
    - Energy is needed, in the form of kinetic energy of the incoming particles
    - Q is negative
    - The minimum energy necessary for the reaction to occur is called the **threshold energy**

Threshold Energy

$$E_{th} = (-Q)\left(1 + \frac{m_a}{M_X}\right)$$

Q- Values for Reactions

- Nuclear Reaction: $a + \mathrm{X} \rightarrow \mathrm{Y} + b$
- Suppose a target nucleus X (which is at rest), is bombarded by an incident particle a, resulting in a product nucleus Y and outgoing particle b and the product nucleus and product particle scattered at angles θ and ϕ.

  Let m_a M_X, M_Y, m_b denote the masses of a, X, Y, b and E_a, E_Y, E_b be their respective kinetic energies.

- From the conservation of energy

$$(E_a + m_a c^2) + M_X c^2 = (E_Y + M_Y c^2) + (E_b + m_b c^2)$$

$$\Rightarrow (M_X + m_a - M_Y - m_b)c^2 = E_Y + E_b - E_a$$

- The Q-value of nuclear reaction:

$$\mathrm{Q} = (M_X + m_a - M_Y - m_b)c^2 = E_Y + E_b - E_a \quad \text{..........} \quad (1)$$

- From the law of conservation of momentum:

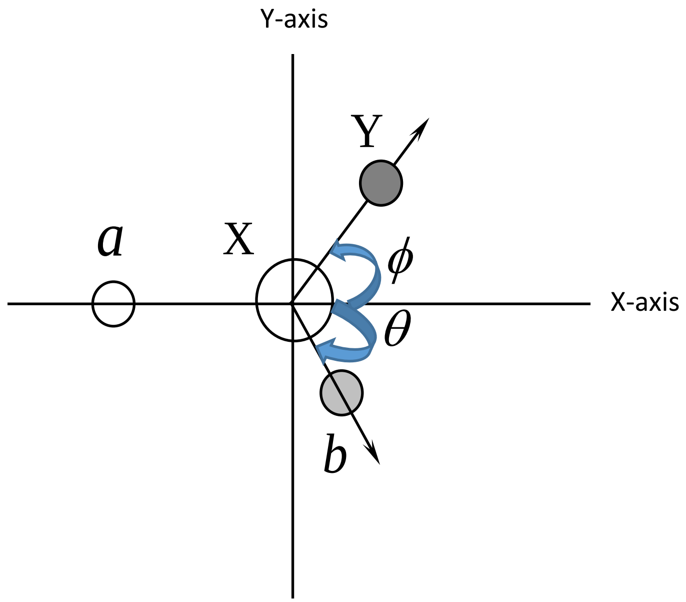
Along X-direction

$$m_a v_a = M_Y V_Y \cos\phi + m_b v_b \cos\theta$$

$$M_Y V_Y \cos\phi = m_a v_a - m_b v_b \cos\theta \quad \text{..........} \quad (2)$$

Along Y-direction

$$M_Y V_Y \sin\phi = m_b v_b \sin\theta \quad \text{..........} \quad (3)$$



Q- Values for Reactions

Squaring and adding Eq. (2) and Eq. (3), we get

$$(M_Y V_Y)^2 = (m_a v_a)^2 + (m_b v_b)^2 - 2 m_a m_b v_a v_b \cos\theta$$

$$\Rightarrow 2 M_Y E_Y = 2 m_a E_a + 2 m_b E_b - 2 m_a m_b \sqrt{\frac{2E_a}{m_a}} \sqrt{\frac{2E_b}{m_b}} \cos\theta$$

$$\therefore \boxed{E_Y = \frac{m_a}{M_Y} E_a + \frac{m_b}{M_Y} E_b - \frac{2}{M_Y} \sqrt{E_a E_b m_a m_b} \cos\theta} \quad \text{..........} \quad (4)$$

So, from Eq. (1), we get

$$Q = \frac{m_a}{M_Y} E_a + \frac{m_b}{M_Y} E_b - \frac{2}{M_Y} \sqrt{E_a E_b m_a m_b} \cos\theta + E_b - E_a$$

$$\Rightarrow Q = \left(\frac{m_a}{M_Y} - 1\right) E_a + \left(\frac{m_b}{M_Y} + 1\right) E_b - \frac{2}{M_Y} \sqrt{E_a E_b m_a m_b} \cos\theta$$

$$\therefore \boxed{Q = E_b \left(1 + \frac{m_b}{M_Y}\right) - E_a \left(1 - \frac{m_a}{M_Y}\right) - \frac{2}{M_Y} \sqrt{E_a E_b m_a m_b} \cos\theta}$$

For $\theta = 90^0$

$$\boxed{Q = E_b \left(1 + \frac{m_b}{M_Y}\right) - E_a \left(1 - \frac{m_a}{M_Y}\right)}$$

Nuclear Fission & Nuclear Fusion

Nuclear Fission

- **Nuclear fission** is the process that occurs in present-day nuclear reactors and ultimately results in energy supplied to a community by electrical transmission.
- **Nuclear fission** occurs when a very heavy nucleus, such as ^{235}U, splits into two smaller **fission fragments.** Thermal neutrons can create fission in ^{235}U:

$$^{1}_{0}\mathrm{n} + ^{235}_{92}\mathrm{U} \rightarrow ^{236}_{92}\mathrm{U}^{*} \rightarrow \mathrm{X} + \mathrm{Y} + \text{neutrons}$$

where ^{236}U* is an intermediate excited state and X and Y are the fission fragments.

- A typical fission reaction for uranium is

$$^{1}_{0}\mathrm{n} + ^{235}_{92}\mathrm{U} \rightarrow ^{141}_{56}\mathrm{Ba} + ^{92}_{36}\mathrm{Kr} + 3(^{1}_{0}\mathrm{n})$$

- Nuclear fission was first observed in 1938 by Otto Hahn (1879–1968) and FritzStrassmann (1902–1980) following some basic studies by Fermi.
- In 1951 the first electricity from a nuclear plant was generated in Idaho. Today over 400 reactors in 26 countries produce about 200,000 MW of electric power—the equivalent of nearly 10 million barrels of oil per day.
- The hydrogen (fusion) bomb, which was first exploded in 1952, is an example of an uncontrolled thermonuclear fusion reaction.

Nuclear Fusion

Nuclear fusion is an area of active research, but it has not yet been commercially developed for the supply of energy.

In **nuclear fusion**, two light nuclei fuse to form a heavier nucleus and release energy.

Two examples of energy-liberating fusion reactions are as follows:

$$^{1}_{1}\mathrm{H} + ^{1}_{1}\mathrm{H} \rightarrow ^{2}_{1}\mathrm{H} + \mathrm{e}^{+} + \nu$$

$$^{1}_{1}\mathrm{H} + ^{2}_{1}\mathrm{H} \rightarrow ^{3}_{2}\mathrm{He} + \gamma$$

These reactions occur in the core of a star and are responsible for the outpouring of energy from the star. The second reaction is followed by either hydrogen–helium fusion or helium–helium fusion:

$$^{1}_{1}\mathrm{H} + ^{3}_{2}\mathrm{He} \rightarrow ^{4}_{2}\mathrm{He} + \mathrm{e}^{+} + \nu$$

$$^{3}_{2}\mathrm{He} + ^{3}_{2}\mathrm{He} \rightarrow ^{4}_{2}\mathrm{He} + ^{1}_{1}\mathrm{H} + ^{1}_{1}\mathrm{H}$$

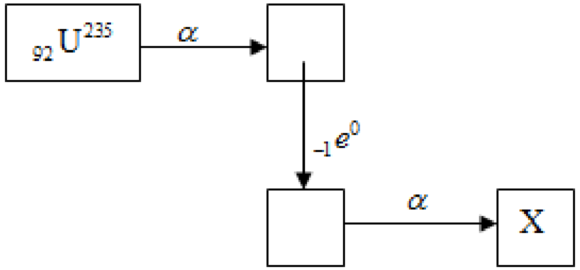
These fusion reactions are the basic reactions in the **proton–proton cycle,** believed to be one of the basic cycles by which energy is generated in the Sun and other stars that contain an abundance of hydrogen.

Some Notes

- The number of nuclei present for a certain radioactive material at time and are and respectively. The half-life for the material is
$$T_{1/2} = \frac{0.693(t_2 - t_1)}{\ln N_1 - \ln N_2} .$$

- In case of an artificial radioactive transformation as given by $_{15}P^{30} \rightarrow {}_{14}Si^{30} + X,$ the emitted particle X is ***positron***.
- The unknown atomic number and mass number respectively, for the following reaction
$${}_0^1n + {}_{92}^{235}U \rightarrow {}_Z^AX + {}_{38}^{94}Sr + 3\,{}_0^1n$$ are 54, 140 .

- The chart below shows part of the radioactive series beginning with the isotope ${}_{92}^{235}U$.



The isotope marked with an X is $_{89}Ac^{227}$.

Text Books & References

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Thank you