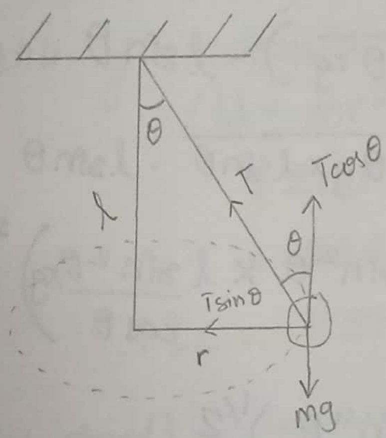
CHAPTER 2: ROTATIONAL DYNAMICS

<Q.1>: A conical pendulum consists of a bob of mass 'm' in motion in a circular path in a horizontal plane as shown in figure. During the motion, the supporting wire of length 'l' maintains the constant angle 'θ' with the vertical. Show that the magnitude of the angular momentum of the about circle's is

$$L = \left(\frac{m^2 g l^3 \sin^4\theta}{\cos\theta} \right)^{1/4}$$

Soln:



The bob is whirled in a circle and tension is acted on the string.

Here, the tension 'T' is resolved into two components: $T\sin\theta$ and $T\cos\theta$.

Here,

$T\cos\theta$ is balanced by weight of the body

$T\sin\theta$ is balanced by centripetal force.

So,

$$T\sin\theta = \frac{mv^2}{r} \quad \text{——— (i)}$$

$$T\cos\theta = mg \quad \text{——— (ii)}$$

Dividing (ii) from (i), we get

$$\tan\theta = \frac{v^2}{rg}$$

$$\text{or, } v = \sqrt{rg\tan\theta} \quad \text{——— (iii)}$$

From figure,

$$\sin\theta = \frac{r}{l} \qquad \text{or, } r = l\sin\theta \quad \text{——— (iv)}$$

We know,

Angular momentum $(\vec{L}) = \vec{r} \times \vec{p}$

$$= mvr$$

$$\therefore L = m \times (\sqrt{\tan\theta\, rg}) \cdot l\sin\theta$$

$$= m \times \sqrt{\tan\theta\, g \times l\sin\theta} \cdot l\sin\theta$$

$$= \left(m^2 \times l^2\sin^2\theta \times l\frac{\sin^2\theta}{\cos\theta} \times g\right)^{1/2}$$

$$= \left(\frac{m^2 g l^3 \sin^4\theta}{\cos\theta}\right)^{1/2}$$

$$\therefore L = \left(\frac{m^2 g\, l^3 \sin^4\theta}{\cos\theta}\right)^{1/2}$$

Hence, proved.

<Q.27>: Consider an oxygen molecule (O_2) rotating in the xy plane about z-axis. The axis passes through the center of the molecule, $\perp^r$ to its length. The mass of each oxygen atom is 2.66×10^{-26} kg and at room temperature, the average separation betn two atoms is $d = 1.21 \times 10^{-10}$ m (the atoms are treated as point masses)

(a) Calculate the moment of inertial of the molecule about the z-axis.

(b) If the angular speed of the molecule about the z-axis is 4.60×10^{-12} rad/s, what is R.K.E.?

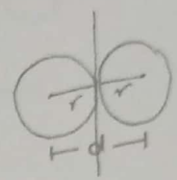
Soln:

Given,

mass of oxygen atom $(m) = 2.66 \times 10^{-26}$ kg

separation betn atoms $(d) = 1.21 \times 10^{-10}$ m

So radius of atom $(r) = \frac{d}{2} = \frac{1.21 \times 10^{-10} m}{2}$



$\therefore r = 6.05 \times 10^{-11}$ m.

Now, since the atoms are treated as point masses.

Moment of inertia of molecule about z-axis

$$(I_z) = mr^2 + mr^2$$
$$= 2mr^2$$
$$= 2 \times 2.66 \times 10^{-26} \times (6.05 \times 10^{-11})^2$$
$$\therefore I_z = 1.95 \times 10^{-46} \text{ kg.m}^2$$

Also, ω (Angular speed) $= 4.6 \times 10^{12}$ rad/s.

Then,

Rotational Kinetic Energy $= \frac{1}{2} I\omega^2$

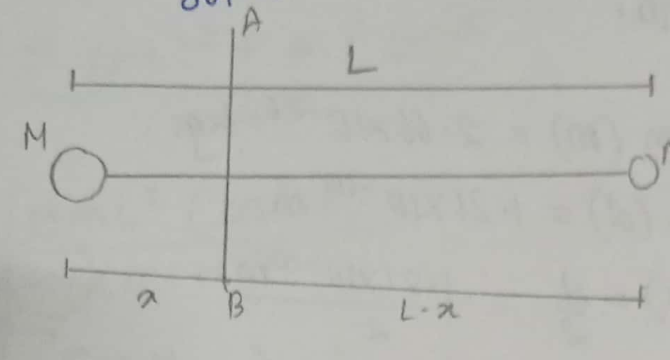
$$= \frac{1}{2} \times 1.95 \times 10^{-46} \times (4.6 \times 10^{12})^2$$
$$\therefore RKE = 2.06 \times 10^{-21} \text{ J}$$

<Q.3>: Two masses 'M' and 'm' are connected by a rigid rod of length 'L' and of negligible mass, as in figure. For an axis passes $\perp^r$ to the rod, show that the system has the minimum moment of inertia when the axis passes through the center of mass. show that

moment of inertia is

$$I = \left(\frac{mM}{m+M}\right)L^2$$

Soln:



Here, the moment of inertia of system of two masses M and m about the axis AB is.

$$I_{AB} = Mx^2 + m(L-x)^2 \quad \text{— (i)}$$

For minimum value of I,

$$\frac{dI_{AB}}{dx} = 0$$

or, $2Mx - 2m(L-x) = 0$

or, $Mx - mL + mx = 0$ $\quad \therefore x = \left(\frac{m}{m+M}\right)L \quad \text{— (ii)}$

Also,

$$x_{cm} = \frac{M.0 + m.L}{m+M} \qquad \therefore x_{cm} = \left(\frac{m}{m+M}\right)L$$

This shows that eqn (ii) is identical of moment of inertia of center of mass of object and thus, moment of inertia does reach minimum value at center of mass

From eqn (i),

$$I = M\left(\frac{mL}{m+M}\right)^2 + m\left(L - \frac{mL}{m+M}\right)^2$$

$$= \frac{M(mL)^2 + m(LM)^2}{(m+M)^2}$$

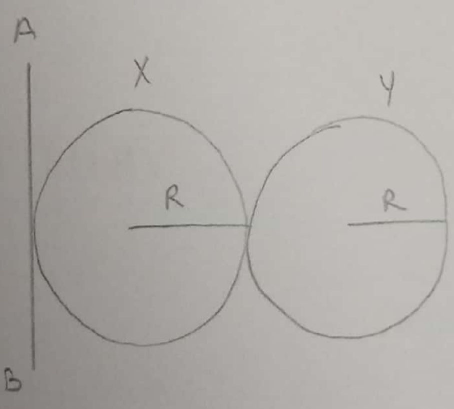
$$= \frac{MmL^2 \cancel{(m+M)}}{(m+M)^{\cancel{2}}}$$

$$\therefore I = \left(\frac{mM}{m+M}\right)L^2$$

Hence, proved.

<Q.67: Two identical solid spheres of mass M and radius R are joined together, and the combination is rotated about an axis tangent to one sphere and ⊥r to the line connecting them. What is the rotational inertia of the combination?

Soln:



Here,

moment of inertia of body X about AB axis (I_1)

$= \frac{7}{5}MR^2$ {∵ solid sphere}

Moment of inertia of body Y about axis AB (I_2)

$$= \frac{2}{5}MR^2 + M(3R)^2 \quad \{\because \text{parallel axis theorem}\} \quad \{\because I_{AB} = I_{cm} + Mh^2\}$$

$$= \frac{2}{5}MR^2 + 9MR^2 = \frac{47}{5}MR^2$$

So, the moment of inertia of combination (I)

$$= I_1 + I_2$$

$$= \frac{7}{5}MR^2 + \frac{47}{5}MR^2$$

$$= \frac{47+7}{5}MR^2$$

$$= \frac{54}{5}MR^2$$

$$\therefore I = 10.8\ MR^2$$