Date. No.

THERMODYNAMICS

Unit 1: HEAT TRANSFER

Thermodynamics:

Thermodynamics is the branch of physics which deals with the transformation of heat into mechanical work.

It involves the study of interaction of one body to another in terms of quantities of heat and work and gives interrelationship between heat and mechanical work.

Heat Flux (ϕ):

Heat flux is defined as the amount of heat transmitted per unit area per unit time from or to a surface from one plane to another.

Transmission of Heat:

Transmission of heat can be done in three ways. They are as follows:

a) Conduction b) Convection c) Radiation.

a) Conduction:

The process in which heat is transmitted from one point to another through the substance without the actual movement of particles is called conduction.

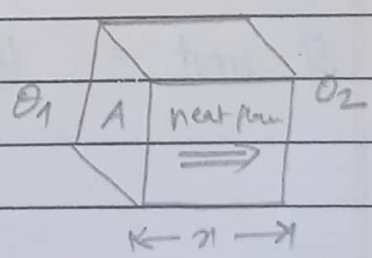
Date. No.

When one end of metal bar is heated, the molecules get heated, vibrate and then transmit heat to the cooler end. However, particles remain in their mean positions.
Eg: Heat transfer in metal bar.

b) Convection:
Convection is the process in which heat is transmitted from one place to another without actual movement of the heated particles.
It is mostly in liquid and gases. Here, the heated particles goes away from heat source and less heated particles come to fulfill the vacant space.
Eg: flow of blood in body, Hot air in heating system

c) Radiation:
Radiation is the process in which heat is transmitted from one place to another without necessity of an intervening medium.
Eg: Heat obtain from sun.

Thermal Conductivity



Let us suppose a cube of side x and area A.
Let θ_1 and θ_2 be temperature of any two opposite faces ($\theta_1 > \theta_2$).

Suppose 'Q' amount of heat flows through cube at time 't' from θ_1 to θ_2.

Hence,
the rate of flow of heat across the rod is given by,

$$\frac{Q}{t} \propto \frac{A(\theta_1 - \theta_2)}{x}$$

or,
$$\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{x} \quad \text{— (i)}$$

Here,
K = coefficient of thermal conductivity of material of cube.

$$\therefore K = \frac{Qx}{A(\theta_1 - \theta_2)t} \quad \text{—— (ii)}$$

Thus,
coefficient of thermal conductivity is defined as the amount of heat flowing in one second across the opposite faces of the cube of side one unit maintained at temperature different of $1^\circ C$.

SI unit = $Wm^{-1}K^{-1}$.

Kirchoff's Law:

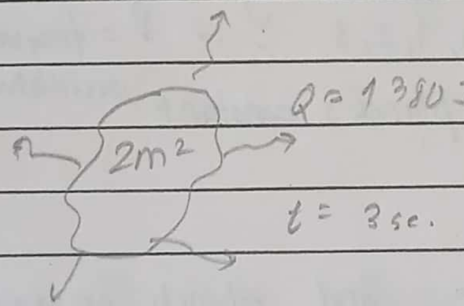
Kirchoff's law states that, "Good absorbes are good emitters."

Emissive Power (ϵ):

Emissive power of a material at a certain ~~group~~ temperature is defined as the rate of radiation emitted through a unit area of the material.

Mathematically,

$$\epsilon = \frac{Q}{A \cdot t}$$



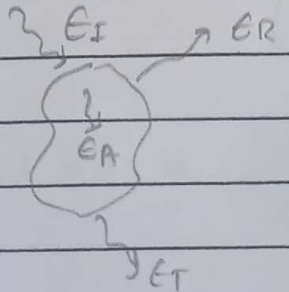
Here,

$$\text{Emissive power} = \frac{1380}{2 \times 3} = 230\ J/m^2 s$$

Black Body:

A body that absorbs all the radiation falling on it is called perfectly black body.

An ideal radiator with an emissivity of unity is also called ideal absorber.



Here,

$\epsilon_I = \epsilon_A + \epsilon_R + \epsilon_T$

if $\epsilon_R = \epsilon_T = 0$.

$\therefore \epsilon_I = \epsilon_A$. $\rightarrow$ condition for perfectly black body

An object for which emissivity (e) = 0 absorbs none of the energy incident on it. Such an object is called ideal reflector.

Planck's Radiation Law:

Planck's law is obtained by using the following postulates:

→ A black body radiation chamber is filled up not only with radiation; but also with simple harmonic oscillators or resonators of the molecular dimension, which can only have energies given by

$$E = nh\nu \; ; \quad n = 0, 1, 2, 3 \ldots \quad \nu = \text{frequency of oscillator}$$

h = planck's constant

→ The oscillators cannot radiate and absorb energy continuously; but an oscillator of frequency ν can only radiate or absorb energy in units or quanta of magnitude $h\nu$.

→ Average energy of Planck's oscillator is given by

$$\bar{E} = \frac{h\nu}{e^{h\nu/kT} - 1} \quad \text{—— (i)}$$

Here

k = Boltzmann's constant

The number of resonators per unit volume in the frequency range ν and $\nu + d\nu$ is given by.

$$N_\nu \, d\nu = \frac{8\pi\nu^2}{c^3} d\nu \quad \text{—— (ii)}$$

The energy density belonging to $d\nu$ can be obtained by multiplying the average energy of Planck's oscillator by the number of resonators per unit volume in the ~~range~~ frequency range ν and $\nu + d\nu$ ie,

$$E_\nu \, d\nu = \bar{E} N_\nu \, d\nu = \frac{8\pi\nu^2}{c^3} \frac{h\nu}{e^{h\nu/kT} - 1}$$

$$= \frac{8\pi h\nu^3}{c^3} \frac{1}{e^{h\nu/kT} - 1} d\nu \quad \text{—— (iii)}$$

This is Planck's radiation law and $E_\nu d\nu$ is the energy density ie, total energy per unit volume.

The energy density $E_\lambda d\lambda$ belonging to range $d\lambda$ is obtained using relation $\nu = \frac{c}{\lambda}$ and hence,

$$|d\nu| = \left| -\frac{c}{\lambda^2} d\lambda \right| \quad \text{ie,}$$

$$E_\lambda \, d\lambda = \frac{8\pi h}{c^3} \left(\frac{c}{\lambda}\right)^3 \frac{1}{e^{hc/\lambda kT} - 1} \left| -\frac{c}{\lambda^2} d\lambda \right|$$

Date. No.

$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{hc/k\lambda T}-1} d\lambda \quad \text{— (iv)}$$

This is another form of Planck's radiation law.

According to this law, the energy density radiated from a radiation chamber at temperature T increases with increase in wavelength becomes maximum at a certain wavelength and then decreases with increases in wavelength.

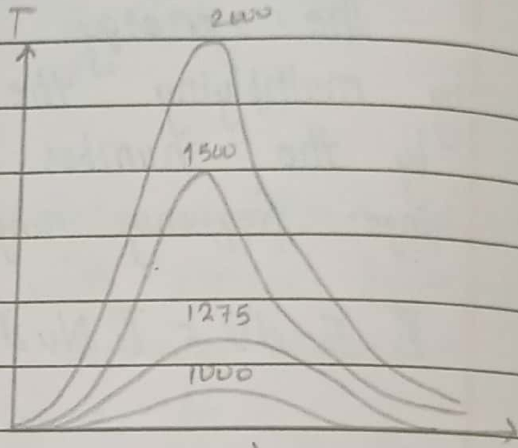


Fig: Planck's radiation curve.

(*) Rayleigh - Jeans law

From Planck's law,

$$E_\lambda dV = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{hc/\lambda kT}-1} d\lambda$$

For longer wavelengths,

$$e^{hc/\lambda kT} \approx 1 + \frac{hc}{\lambda kT}$$

and hence,

$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda} \left\{ \frac{1}{\left(1 + \frac{hc}{\lambda kT} - 1\right)} \right. d\lambda$$

$$= \frac{8\pi hc}{\lambda^5} \times \frac{\lambda kT}{hc} d\lambda$$

$$\therefore E_\lambda d\lambda = \frac{8\pi kT}{\lambda^4} d\lambda. \quad \text{——— (v)}$$

This is Rayleigh - Jeans law.

Wein's Displacement Law

Wein's displacement law states that, " the product of the wavelength corresponding to maximum energy 'λm' and the absolute temperature 'T' is constant."

Mathematically,

$$\lambda m \, T = \text{constant}$$

ie,

$$\lambda m \propto \frac{1}{T}$$

This constant is called Wien's displacement law and equals to 0·2896 cm. K.

Eg: Red ~~tr~~ light has higher wavelength, thus small refractive index and hence less temperature.

Date. No.

(*) Derivation from Wien's Displacement law:

We know,

$$E_\lambda = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1} \quad \text{——— (i)}$$

So, to calculate maximum energy radiated by chamber,

$$\frac{dE_\lambda}{d\lambda} = 0$$

$$\text{or, } \frac{d}{d\lambda}\left[\frac{8\pi hc}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}\right] = 0$$

$$\text{or } \frac{d}{d\lambda}\left[\frac{1}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}\right] = 0$$

$$\text{or } \frac{-5}{\lambda^6} \frac{1}{e^{hc/\lambda kT} - 1} + \frac{1}{\lambda^5}\left(\frac{-1}{(e^{hc/k\lambda T} - 1)^2}\right)\left(e^{hc/\lambda kT}\right)\left(\frac{hc}{\lambda kT}\right)\left(\frac{-1}{\lambda^2}\right) = 0$$

$$\text{or, } -5 - \frac{hc}{\lambda kT} \frac{e^{hc/\lambda kT}}{e^{hc/\lambda kT} - 1} = 0$$

Let $x = \frac{hc}{\lambda kT}$, we get.

पाठशाला

Date. No.

$$5 - \frac{x e^x}{e^x - 1} = 0$$

Solving,

$x \approx 4.98$

or, $\lambda T = \frac{hc}{kx} \approx 0.2898$ cm K.

Hence, the product of wavelength corresponds to maximum radiation energy and the absolute temperature is constant

$$\lambda_m T = \text{constant} = 0.2898 \text{ cm K}.$$

Stefan's Boltzmann's law

Stefan's law states that,

" the rate of emission of radiant energy by unit area of perfectly black-body is directly proportional to the fourth power of its absolute temperature."

ie, $I \propto T^4$

or, $I = \sigma T^4$

Here, σ = stefan's constant.

A black body at absolute temperature T surrounded by another black body at absolute temperature T_0 not only loses an amount of energy σT^4 but also gains σT_0^4.

Date. No.

Thus, the amount of heat lost by the former per unit time is given by

$$I = \sigma(T^4 - T_0^4)$$

This is called Stefan-Boltzmann's law.

*) <u>Derivation of Stefan's Law from Planck's law:</u>

We know,

$$E_v = \frac{8\pi h v^3}{c^3} \frac{1}{e^{hv/kt} - 1} \quad \text{--- (i)}$$

The total radiation energy per unit volume emitted by the black body radiation chamber ~~all~~ over all range of frequency or wavelength can be calcula~~ting~~ted by integrating eqⁿ (i) from $v \to 0$ to $v \to \infty$ and.

$\lambda \to 0$ to $\lambda \to \infty$ ie,

So,

$$E = \int_0^\infty E_v \, dv$$

$$= \int_0^\infty \frac{8\pi h v^3}{c^3} \frac{1}{e^{hv/kt} - 1} \, dv$$

पाठशाला

Date. No.

Let $\frac{h\nu}{kT} = x$

or, $\nu = \frac{kT}{h} x$. Hence,

$$E = \frac{8\pi h}{c^3} \int_0^\infty \left(\frac{kTx}{h}\right)^3 \frac{1}{e^x - 1} \left(\frac{kT}{h}\right) dx$$

$$= \frac{8\pi k^4 T^4}{c^3 h^3} \int_0^\infty \frac{x^3}{e^x - 1} dx.$$

Here, $\int_0^\infty \frac{x^3}{e^x - 1} dx$ is standard integral equating to $\frac{\pi^4}{15}$

$$\cancel{\therefore E = \frac{1}{4} \frac{8\pi^5 k^4 T^4}{15 c^3 h^3} c = \frac{2\pi^5 k^4}{15 c^2 h^3} T^4}$$

$$\therefore E = \frac{8}{15} \frac{\pi^5 k^4 T^4}{c^3 h^3}$$

The energy per second radiated by a unit area of the black body chamber is effectively equal to $I = \frac{1}{4} E c$. Therefore, rate of energy radiated per unit area

Date. No.

$$I = \frac{1}{4} \frac{8\pi^5 k^4 T^4}{15 c^3 h^3} c$$

$$= \frac{2\pi^5 k^4 T^4}{15 c^2 h^3} T^4$$

$$\therefore I = \sigma T^4$$

Here,

$$\sigma = \frac{2\pi^5 k^4}{15 c^2 h^3}$$

$$= \frac{2\pi^5 \times (1.38 \times 10^{-23})^4}{15 \times (3 \times 10^8)^2 \times (6.62 \times 10^{-34})^3}$$

$$\therefore \sigma = 5.67 \times 10^{-8} \ Wm^{-2} K^{-4}.$$