General Physics I (PHYS 101)

Lecture 02

Dynamics of system of particles

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Outline

Work-Energy Theorem

Statement: Work done by the resultant force is equal to change in kinetic energy

$$W_{net} = \Delta K = K_F - K_I$$

For constant force: Let a constant net force $\vec{F}_{net}$ acts on a body of mass m. As the body moves through displacement $\vec{s}$, this net force causes its velocity to change from $\vec{v}_i$ to $\vec{v}_f$.

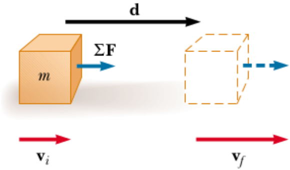

Figure 1: A particle undergoing a displacement $\vec{d}$ and a change in velocity under the action of a constant net force $\sum \vec{F}$.

According to Newton's second law,

$$\vec{F}_{net} = m\vec{a} = m\left[\frac{\vec{v}_f - \vec{v}_i}{\Delta t}\right]$$

where Δt is the time interval for the body to move through the displacement $\vec{s}$. The net work done by the constant net force is

$$\begin{aligned}
W_{net} &= \vec{F}_{net} \cdot \vec{s} \\
&= m \left[\frac{\vec{v}_f - \vec{v}_i}{\Delta t} \right] \cdot \left[\frac{\vec{v}_f + \vec{v}_i}{2} \right] \Delta t \\
&\left[\because \vec{v}_{av} = \frac{\vec{s}}{\Delta t} = \frac{\vec{v}_f + \vec{v}_i}{2} \right] \\
&= \frac{1}{2} m \left[\left(\vec{v}_f - \vec{v}_i \right) \cdot \left(\vec{v}_f + \vec{v}_i \right) \right] \\
&= \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = K_f - K_i = \Delta K
\end{aligned}$$

This equation states that the net work done on a particle by the constant resultant force is equal to the change in kinetic energy of the particle.

For Variable force: Let us consider an object moving under the action of a variable force $\vec{F}$ in the direction of force.

$$W = \int F ds = \int m a ds = \int m \frac{dv}{dt} ds = \int_{v_i}^{v_f} m dv \frac{ds}{dt}$$
$$= \int_{v_i}^{v_f} m v dv = \frac{1}{2} m {v_f}^2 - \frac{1}{2} m {v_i}^2 = \Delta K$$

Therefore work done on the particle by the resultant force is equal to the change in kinetic energy of the particle. This is known as work energy theorem.

Work-Energy Theorem (contd.)

Physical Significance: From work energy theorem, we have easily defined work and kinetic energy and derive the relation between them directly from Newton,s second law. It is useful for finding the work done by the resultant force and finding the speed at certain position.

Limitation: This work energy theorem directly defined from Newton's second law, and hence it is applicable to the particles. Objects that are like particles are considered for this rule. So, if all the object particles behave like particles, we can consider the whole object as a particle. Also, it doesn't define the direction of velocity.

Work-Energy Theorem (contd.)

Problem: A body of mass $m = 4.5g$ is dropped from rest at a height $h = 10.5m$ above the Earth's surface. Neglecting air resistance, what will its speed be just before it strikes the ground?

Hints: According to the work-energy theorem,

$$W_{net} = K_f - K_i \implies mgh = \frac{1}{2}mv_f^2 - \frac{1}{2}m\cancelto{0}{v_i^2}$$

$$\implies mgh = \frac{1}{2}mv_f^2$$

$$\implies v_f = \sqrt{2gh}$$

$$\implies v_f = 14.3m/s$$

Work-Energy Theorem (contd.)

Problem: A block of mass $m = 3.63kg$ slides on a horizontal friction less table with a speed of $v = 1.22m/s$. It is brought to rest in compressing a spring in its path. By how much is the spring compressed if its force constant k is $135N/m$?

Hints: According to the work-energy theorem,

$$W_{net} = K_f - K_i \implies -\frac{1}{2}kx^2 = \frac{1}{2}m\cancelto{0}{v_f^2} - \frac{1}{2}mv_i^2$$

$$\implies x = v_i\sqrt{\frac{m}{k}} = 0.20m$$

Conservative Force

Conservative Force If the work done by a force during a round trip of a system is always zero, the force is said to be conservative.

OR

If the work done by a force in moving a body from an initial position to a final position is independent of the path taken between two points then the force is conservative otherwise it is non-conservative.

Conservative Force (contd.)

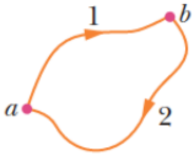

Figure 2: Conservative force

Suppose a particle goes from 'a' to 'b' along the path 1 and back from 'b' to 'a' along the path 2. If the force acting on the particle is conservative then the work done on the particle by the force for the round trip is zero.

Conservative Force (contd.)

$$i.e. \quad W_{ab,1} + W_{ba,2} = 0 \implies W_{ab,1} = -W_{ba,2} \qquad (1)$$

Now if the particle goes from 'a' to 'b' along the path 2 and back from 'b' to 'a' along the same path 2. Then

$$i.e. \quad W_{ab,2} + W_{ba,2} = 0 \implies W_{ab,2} = -W_{ba,2} \qquad (2)$$

From Equation(1) and Equation(2)

$$\boxed{W_{ab,1} = W_{ab,2}}$$

Conservative Force (contd.)

i.e. the work done on the particle by a conservative force in going from 'a' to 'b' is same for either path. Hence if the work done by a force on a particle that moves between two points depends only on the initial and final points and not on the path taken then the force is said to be conservative otherwise the force is said to be non-conservative.

Examples of Conservative forces are Gravitational force, Electrostatic force, spring force etc. and non conservative forces are frictional force, viscous force etc.

Conservative Force (contd.)

Gravitational force upward $= -mgh$

downward $= mgh$

total $= 0$

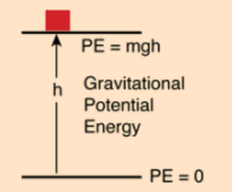

Figure 3: Gravitational force

i.e. gravitational force is conservative.

Conservative Force (contd.)

Elastic (Spring) force The work done by elastic force $W = -\frac{1}{2}kx^2$

Work done from initial to final position is

$W_{if} = -\frac{1}{2}(x_f^2 - x_i^2)$

Work done from final to initial position is

$W_{fi} = -\frac{1}{2}(x_i^2 - x_f^2)$

Total $W_{net} = W_{if} + W_{fi} = 0$

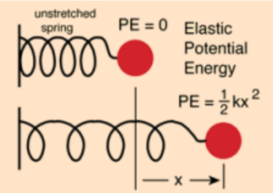

Figure 4: Elastic force

i.e. elastic force is conservative.

Frictional force The work done by frictional force is always negative

$W = -fd$

Work done from 'a' to 'b' is

$W_{ab} = Fdcos\theta = fdcos180 = -fd$

Work done from 'b' to 'a' is

$W_{ab} = Fdcos\theta = fdcos180 = -fd$

Total $W_{net} = W_{ab} + W_{ba} = -fd - fd = -2fd$

Conservative Force (contd.)

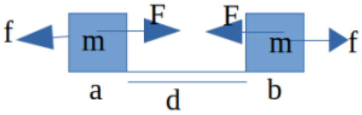

Figure 5: Frictional force

i.e. Frictional force is non-conservative.

Note: The work done by conservative force is capturable it is not in the case of non-conservative. That is why non-conservative forces are known as dissipative force.