SYSTEM OF LINEAR EQUATIONS

A set of linear equations is called system of linear equations.

Let the 'm' system of linear equations with 'n' unknown variables is $a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$ $a_{21}x_2 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$ $a_{m_1}x_1 + a_{m_2}x_2 + \dots + a_{m_m}x_n = b_m$ $a_{m_1}x_1 + a_{m_2}x_2 + \dots + a_{m_m}x_n = b_m$

Representing system of linear equations (i) in matrix form,

So, coefficient matrix

Augmented matrix

Augmented matrix

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Homogeneous and Non-homogeneous System of linear Equations.

A linear system of equations is said to be homogeneous if all the constant terms are zero.

Thomogeneous system of equations is always consistent $a_{11} M_1 + a_{12} M_2 + \cdots + a_{1n} M_n = 0$ $a_{21} m_1 + a_{22} m_2 + \cdots + a_{2n} m_n = 0$ \vdots $a_{n1} M_1 + a_{n2} m_2 + \cdots + a_{mn} m_n = 0$

If any of the constant terms in system of linear equations is to not equal to zero, the system of linear equations is said to be non-homogeneous.

 $a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = 0$ $a_{21}x_1 + a_{12}x_2 + \cdots + a_{2n}x_n = 1$ \vdots

ania, + anix+ - - - + amnan = 0

Sementary Row operations:

The row operations in matrices used to transform a system of linear equations into new systems having the same valutions as the original one.

Sementary Row operations are.

(i): Row swap: Rm \ Rn

(ii) Scalar multiplication: Rm -> KRm

(iii) Row sum/difference: Rm -> Rm + Rn

Rm -> & Rm + kRn

Solutions of System of linear Equations.

System of linear equations has three types of solutions.

i) Unique solution

ii) Infinitely many solutions

iii) No solution.

it is called consistent otherwise it is called inconsistent.

Let the system of equations: $a_{11} \times + a_{12} \times + a_{13} \times = b_{1}$ $a_{21} \times + a_{22} \times + a_{23} \times = b_{2}$ $a_{31} \times + a_{32} \times + a_{33} \times = b_{3}$

i) Writing in augmented form,

[a11 a12 a13 : b1

a21 a22 a23 : b2

[a31 a25 a33 : b3

ii) Dements azi, azi and azz are made zero wing row operations.

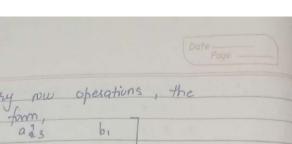
Here, $a_{11} x_1 + a_{12} y_1 + a_{13} z_2 = b_1$ $a_{12} x_1 + a_{23} x_1 + a_{23} z_2 = d_1$ $a_{23} x_2 + a_{23} z_3 = d_2$ $a_{23} x_3 z_4 z_2 = d_2$

Cases:

i) If $e_{33} \neq 0$ 4 $d_2 \neq 0$ or $e_{33} \neq 0$ 4 $d_2 = 0$, the system of linear equations has unique solution. ii) If $e_{33} = d_2 = 0$, the system of linear equations has

many sulutions

iii) If $e_{33} = 0$ but $d_2 \neq 0$, the system of linear equations has no solution.



if ofter elementary now operations, the matrix is in the form,

1	a12	a23	b1
0	1	c23	d2
0	0	1	d3

ie, upper triangular matrix with all main diagonals elements unity, the motrix is called echolen from d matrix.

If row operations are further done such that
the augmented matricest are in the is in the form.

0 1 0 : e2

0 0 1 : e3

This form is called now reduced echolen form.

Row Rank:

The number of non-zero nows in the motrix is known as its now rank.

If now rank of coefficient motrix is equal to the rank of augmented motrix, the system of linear equations is said to be consistent.

If now reduced echolen form 3.

 # Parametrically Represented Solutions

when the augmented matrix of the system is converted to now reduced echolen form and the number of non-zero nows is less than the number of variables, parametric solution method is used.

When this happens, there are hasic variables (corresponding to 1) and free variable (corresponding to 0). Let the free variable be k (any value) and find the solution in terms of k.

Questions:

1:
$$\alpha - 2y + z = 0$$

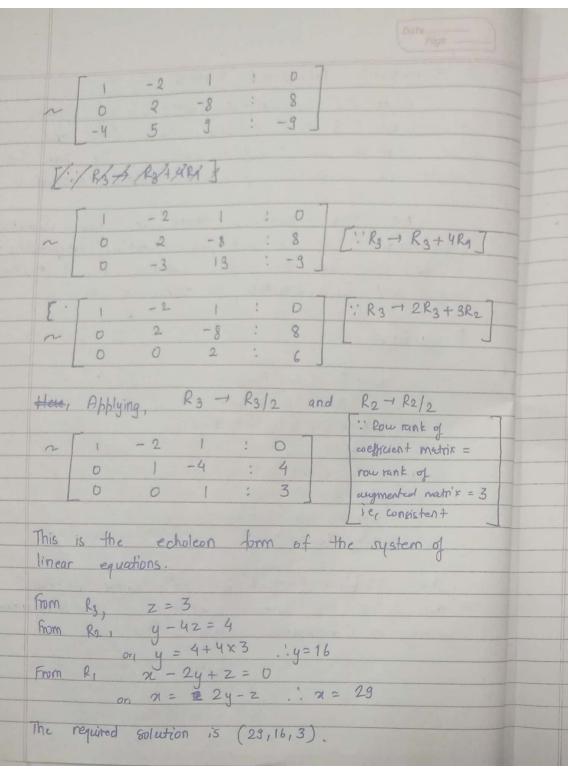
 $2y - 8z = 8$
 $-4\alpha + 5y + 9z = -9$.
Solp:

aiven,

$$2y - 2y + z = 0$$

 $2y - 8z = 8$
 $-49 + 5y + 9z = -9$

Since, all the constant is not equal to zero, the system of linear equations is non-homogeneous. Writing in augmented matrix form,



2) x + 2y - 3z = 3 -2x - 5y + 4z = 5 -5x - 13y + 9z = 180x + 2y - 3z = 3-2x - 5y + 4z = 5-5a - 13y + 92 = 18 Since, all the constant terms is not equal to zero. the system of linear equations is non-homogeneous. Witting in augmented form. Applying R2 = R2 + 2R1 and R3 -> R3 + 5R1 11 Applying R2 - 12 - 1XR2 R3 - 1 R3 - 3R2 -3: -1 -2: 11 0 0 :

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Applying R2 -1 XR2
                                                            (3): x + 2y 4-z = 4
                                                             3x + 7y - 2z = 1

-2x + 3y + 3z = -1
   2 -3 : 3
0 1 2 : -11
0 0 0 0 : 0
                                                             aiver,
                                                             x + 2y - z = 4

3x + 7y - 2z = 1
  This is the echelon form.
                                                               -2x+34 +3z=-1
  From Rg, DXZ=0
  fo, I is a free variable.
                                                            Since, all the constant terms is not equal to zero,
 from R_2,

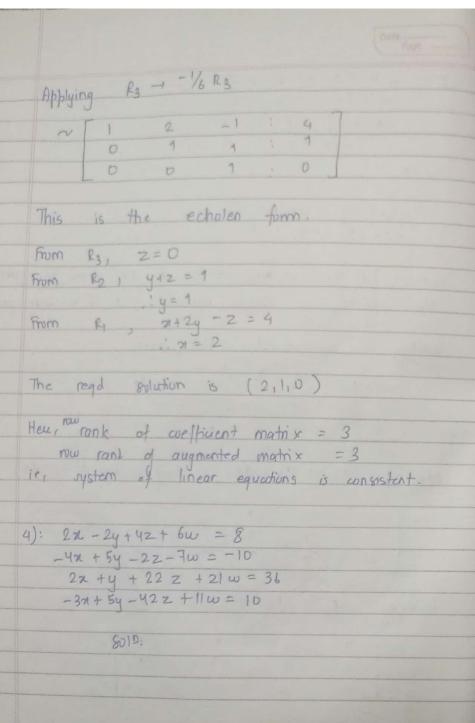
y + 2z = -11

y = -11 - 2z
                                                            the system of linear equations is non-homogeneous.
                                                            Writing in augmented matrix form,
from R_1, x + 2y - 3z = 3

x + 2y - 3z = 3

x + 2y - 3z = 3
                                                               ~ [ 1 2 -1 : 4 ] 3 7 -2 : 1 ] -2 3 3 : -1
The parametric form of solution is,
 Putting Z=r.
                                                           Applying R2 - 12-3R, and R3-1 R3+2R,
 y = -11 - 2r
y = 7r + 25
When r=0, the required solp is (25,-11,0)
When r=1, the required solp is (32,-13,1)
                                                           Applying R3 -> R3 - 7R2
  and so on.
Alex, now rank of coefficient matrix = 2.

Now rank of augmented matrix = 3
                                                               n 1 2 -1: 4
0 1 1: 1
  ie, inconsistent system of linear equations.
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Given.

2x - 2y + 4z + 6w = 8 -4x + 5y - 2z - 7w = -10 2x + 4y + 21z + 21w = 36-3x + 5y - 42z + 11w = 10

Here, all the constant ferms are not zero, so the system is non-homogeneous.

Writing matrix in augmented form.

	2	-2	4	6		8	
~	-4	5	-2	-7	1	-10	
	2	1	+22	21		34	1
	-3	5	-42	- 11	:	10	1

Applying RI - 1/2 x RI

~	1	-1	2	3	:	4
	-4	5	-2	-7	1	-10
	2	1	22	21	:	36
	-3	5	42	11		10

Applying $R_2 \rightarrow R_2 + 4R_1$, $R_3 \rightarrow R_3 - 2R_1$, $R_4 + 3R_1$

		-1	2	3	-	4
n	0	1	6	5	i	6
	0	3	18	15	3	28
	0	2	48	20	;	22
-						

Applying	R3	- P3	-3R2	and	Ry→ fy-2R
T	- 1	-1	2	3	: 4
n	0	1	6	5	: 6
	D	0	0	D	: 16
	0	0	36	10	: 10

phlying		Ry -	Ky - 6	1-2		
113		-1	2	3	:	4
n	0	1	6	5	:	6
	0	0	0	0		16
	0	. 0	0	-20	•	-26

Du - Tonx Ru

Theyn	9	79		7 .		
	1	-1	2	3	1	4
~	0	1	6	5	:	6
	0	0	0	0	:	16
	0	6	0	1	,	1.3

From Ry, W=1.3From R3, 0.2=16 ir, not possible. Thus, Z has no solution.

do, the system in whole has no solution.

Hence, it is inconsistent,

(5): -5x + 3y - 2z + 4w = 8 6x + 8y + 5z - 3w = 4 x + 2y - 7z - 5w = -1-3x - 7y + 6z + 2w = 2

Given, -5x + 3y - 2z + 4w = 8 6x + 8y + 5z - 3w = 4 9x + 2y - 7z - 5w = -1-3x - 7y + 6z + 2w = 2

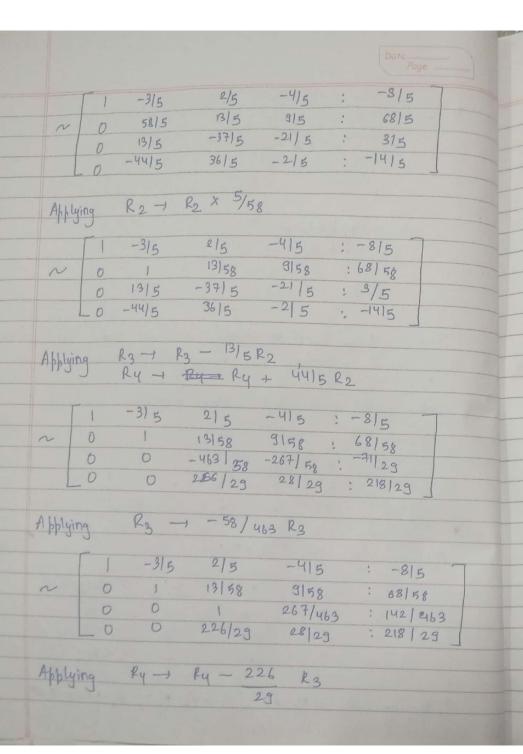
Since, all the constant forms are not zero, the system of linear equations is non-homogeneous.

U	7					
- 5	3	-2	4	:	8	Ī
6	8	5	-3	:	4	1
1	2	-7	-5	*	-1	1
-3	-7	6	2	?	2	1

Writing into augmented matrix form,

Applying -315 -315 -415 -815 -6 8 5 -3 4 1 2 -7 -5 1 1 2 -7 6 2 2

Applying R2 -> R2-6R1, R3 + R3-R1, R4-> R4+ 73R,



~ [1 -315 215	-415 : -815
0 1 13158	9/58 : 69/58
0 0 1	267/468 : 142/463
0 0 0	-3-52 : 5.12
Applying: Ry -> -1/3.52	Ry.
~ 1 -315 215	-415 '8/5
0 1 13/5	8 9158 : 68/58
0.01	
0 0 0	: -16/112
From Ry	
$\mu = -16/11 = -1.45$	1
From R3, Z+ 267 W=	142
968	463
1. 2 = 1.136	
from R2, y + 13 Z +	9 w = 68
	58 58
.'.y = 1.143	
From P1, 21 to -3 y +	$\frac{2}{5}$ $\frac{2}{5}$ $\frac{2}{5}$
2.520	
The required solution is (-2-528 1-143, 1-136 -1-454)
And And	
now rank of welficient ma	trix = 4
row rank of augmented	matrix = 4
The rystem of linear	equations is consistent.