<Q-1>: A wire of length L, Young's modulus Y and cross-sectional area A is stretched elastically by an amount ΔL. By Hooke's law, the restoring force is $-k\Delta L$. Show that

a) $K = \frac{YA}{L}$

b) the workdone in stretching wire ΔL is
$$W = \frac{1}{2} YA \frac{\Delta L^2}{L}$$

Soln:

For (a):
We know,
$$Y = \frac{FL}{A\Delta L}$$

or, $Y = \frac{K\cancel{\Delta L} \times L}{A \times \cancel{\Delta L}}$ $\therefore K = \frac{YA}{L}$ Hence, proved.

For (b):
$$W = \int_0^{\Delta L} F\,dx$$ Let $\Delta L = x$.

So,
$$W = \int_0^{x} \frac{YAx}{L}\,dx$$

$$= \frac{YA}{L}\int_0^{x} x\,dx = \frac{YA}{L} \times \frac{x^2}{2}$$

$$W = \frac{1}{2}\frac{YAx^2}{L} = \frac{1}{2} YA \frac{\Delta L^2}{L}$$

Hence, proved.

<Q2>: A 200 kg laid is hung on a wire of length 4.00 m, cross-sectional area $0.200 \times 10^{-4} m^2$, and Young's Modulus $= 8.00 \times 10^{10} N/m^2$. What is the increase in length?

Soln:

Given,

mass $(m) = 200\,kg$

Length $(L) = 4.00\,m$

Area$(A) = 0.200 \times 10^{-4}\, m^2$

Young's modulus $(Y) = 8 \times 10^{10}\, N/m^2$

$\Delta L = ?$

We know,

$$Y = \frac{F \times L}{A \times \Delta L}$$

or $$\Delta L = \cancel{\frac{F\ Y \times A}{F \times}} \ \frac{F \times L}{A \times Y} = \frac{m \times g \times L}{A \times Y}$$

$$= \frac{200 \times 9.81 \times 4}{0.200 \times 10^{-4} \times 8 \times 10^{10}}$$

$$\therefore \Delta L = 0.0049\ m$$

<Q.3> Assume Young's Modulus of bone is $1.50 \times 10^{10}\ N/m^2$. The bone breaks if stress greater than $1.5 \times 10^8\ N/m^2$ is applied on it.

a) What is the maximum force that can be exerted on the femur bone in the leg if it has minimum effective diameter of 2.50 cm?

b) If this much force is applied compressively, by how much does the 25.0 cm ~~bone~~ long bone shorten.

<u>Soln</u>

Given,

Young's modulus of bone (Y) $= 1.50 \times 10^{10}\ N/m^2$

Breaking stress $= 1.5 \times 10^8\ N/m^2$

for (a):

diameter $= 2.50\ cm = 2.50 \times 10^{-2}\ m$

$F_{max} = ?$

So,

$Stress = \frac{F}{A}$ or, $Breaking\ stress = \frac{F_{max}}{A}$

or $B.S \times A = F_{max}$

or $F_{max} = 1.5 \times 10^8 \times \frac{\pi}{4} \times (\cancel{1.5 \times 10}\ 2.5 \times 10^{-2})^2$

$\therefore F_{max} = 73.6 \times 10^3\ N$

$\therefore F_{max} = 73.6\ kN$

~~Sol~~: For <b>:

$L = 25\,cm = 25 \times 10^{-2}\,m$

$\Delta L = ?$

~~Stress~~ $F = 73.6 \times 10^3\,N$

$A = \frac{\pi}{4} \times d^2.$

Now,

$Y = \frac{B.S}{strain}$ or, $Y = \frac{BS}{\Delta L} \times L$.

or $Y = \frac{B.S. \times L}{Y}$ or $\Delta L = \frac{1.5 \times 10^8 \times 25 \times 10^{-2}}{1.5 \times 10^{10}}$

$\therefore \Delta L = 25 \times 10^{-4}\,m$

$= 2.5\,mm.$

<Q.4>: A solid brass sphere is initially surrounded by air, and the air is exerted on it is $1 \times 10^5\,N/m^2$. The sphere is lowered to an ocean to a depth where pressure is $2 \times 10^7\,N/m^2$. The volume of the sphere is $0.50\,m^3$. By how much does the volume change once it is submerged? [Bulk Modulus of brass $= 6.1 \times 10^{10}\,N/m^2$].

Soln:

Given,

Atmospheric pressure $= 1.0 \times 10^5\,N/m^2$

sea pressure $= 2 \times 10^7\,N/m^2$

volume of sphere in air $(V) = 0.50\,m^3$

Bulk modulus $(B) = 6.1 \times 10^{-10}\,N/m^2$

$\Delta V = ?$

We know,

$$B = \frac{-P}{\frac{\Delta V}{V}}$$

or, $$\Delta V = \frac{-PV}{B}$$

$$= \frac{-(2.0\times10^{7} - 1\times10^{5})\times 0.5}{6.1\times10^{10}}$$

$$\therefore \Delta V = -1.63\times10^{-4}\ m^3$$

Here, the negative sign denotes decrease in volume.

This decreases by $1.63\times10^{-4}\ m^3$.