CHAPTER: 4

MAGNETOSTATICS

Magnetic fields

A stationary charge produces only an electric field $\vec{E}$ in the space around it, whereas a moving charge generates in addition a magnetic field $\vec{B}$.

Hand Rules

(a) Right hand Thumb Rule:
For straight conductor
Thumb: current direction
Curled fingers: direction of magnetic field

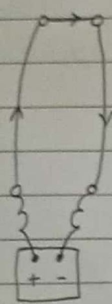
(b) Right hand Fist Rule.
For circular current carrying conductor
Thumb: magnetic field direction
Curled fingers: current direction

(c) Fleming's Right Hand Rule:
For direction of forces in electric motor.
Thumb: motion of conductor
Forefinger: direction of magnetic field
Middle finger: direction of induced current.

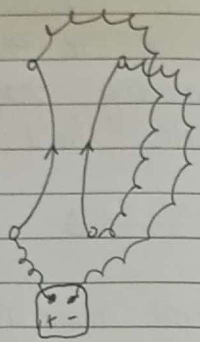
(d) Fleming's Left Hand Rule:

Index: direction of magnetic field
Middle finger: direction of motion of charge particle
Thumb: direction of Lorentz force.

Magnetic Fields



currents in opposite direction repel.

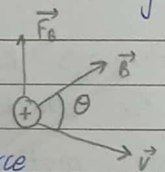


currents in same direction attract

Magnetic Force

The magnetic force in a charge Q, moving with velocity $\vec{v}$ in magnetic field $\vec{B}$ is given by

$$\vec{F_B} = Q(\vec{v} \times \vec{B})$$



the magnitude of the magnetic force on a charged particle moving in the magnetic field is.

$$F_B = QvB\sin\theta$$

Here,
θ = angle between $\vec{v}$ and $\vec{B}$.

When $\theta = 90^\circ$ i.e, $\vec{v} \perp \vec{B}$ $(F_B)_{max} = QvB$

when $\theta = 0^\circ/180^\circ$ ie, $\vec{v} \uparrow\uparrow \vec{B}$ $(F_B) = 0$

o) Note:

(i) Magnetic force vector is perpendicular to the magnetic field.
(ii) Magnetic force acts on charged particles when particle is in motion.
(iii) Magnetic force associated with a steady magnetic field does no work when a particle is displaced because force is $\perp$ to displacement.
$\therefore W_{mag} = 0$.

Kinetic energy of a charged particle moving through magnetic field can't be altered by magnetic field alone.

Now,
$$\text{Magnetic field } (B) = \frac{F_B}{Qv\sin\theta}$$

SI unit = Tesla (T) = $N A^{-1} m^{-1}$
Another unit = Gauss (G)

$$\therefore 1\,T = 10^4\,G$$

Also, Wb/m^2 = Weber per square Meter.

Lorentz Force

The force experienced by a charge particle moving in magnetic field and electric field is called Lorentz Force.

$$\vec{F_L} = (Q\vec{E}) + Q(\vec{v}\times\vec{B})$$
$$\therefore \vec{F_L} = Q[\vec{E} + \vec{v}\times\vec{B}]$$

Here,
$\vec{F_L}$ = Lorentz Force.
Q = charge
$\vec{E}$ = electric field
$\vec{B}$ = magnetic field
$\vec{v}$ = velocity.

Magnetic ~~Field~~ Flux

Magnetic flux is a measurement of the total magnetic field which passes through a given area.

Magnetic ~~field~~ flux through a surface.
$$\Phi_B = \int_S \vec{B}\cdot d\vec{a}$$

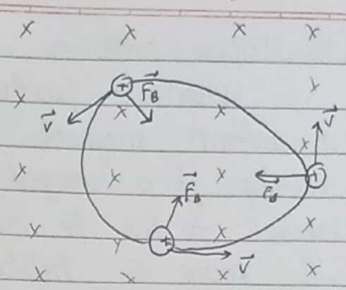
Unit : Weber (Wb).

Motion of Charged Particles in Uniform Magnetic Field

(a): Cyclotron Motion:

When a positively charged particle enters a magnetic field in ~~the~~ a direction perpendicular to the field, the particle moves in a circle because the magnetic force $\vec{F_B}$ is perpendicular $\vec{v}$ and $\vec{B}$ and has constant magnitude qvB.

From Newton's 2nd law for particle,



$$F_B = ma$$
or, $$qvB = \frac{mv^2}{r}$$

or, $$p = qBr \quad \text{—(i)}.$$
Here,
m = particle's mass
p = momentum

Equation (1) is known as cyclotron formula.

Radius of circular path $(r) = \frac{mv}{qB}$

Cyclotron angular frequency $(\omega) = \frac{v}{r} = \frac{qB}{m}$

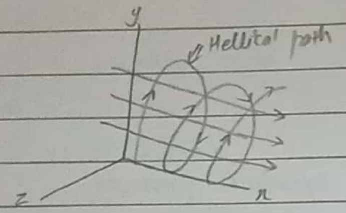
Time period $(T) = \frac{2\pi m}{qB}$

Frequency $(f) = \frac{qB}{2\pi m}$

Here, T and f is not depended on ~~velocity~~ speed of the particle or radius of the orbit.

(b) Hellical path:

When a positively charged particle enters a uniform magnetic field obliquely, if the velocity of a charged particle has a component parallel to the ~~to~~ magnetic field, the particle will move in hellical path about the direction of the field vector.



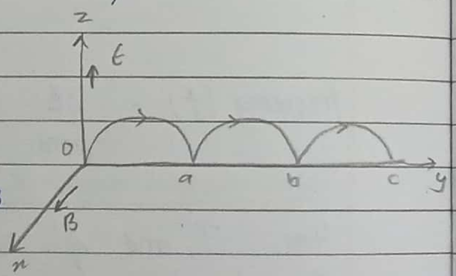
velocity component $\perp^r$ to $\vec{B}$ magnetic field causes circular motion whereas the component of velocity parallel to field moves the particle along st. line.

Thus, hellical path.

(c): Cycloid motion

<Q>: Suppose that $\vec{B}$ points in the x-direction and $\vec{E}$ in the z-direction. A particle at rest is released from origin, what path will it follow?

Ans:



When particle is at rest, magnetic force is zero, thus electric field accelerates the charge in z-direction.

With increase in speed, the magnetic force pulls charge towards the ~~y-axis~~ right and with increase in velocity, it curves particle back towards y-axis.

At this point, the charge, moving against electric field, it ~~become~~ begins slowing down decreases the magnetic field bringing to rest at a.

Similarly, the process carries the charge to point b.

The charged particle follows cycloid path.

Magnetic Force on System of Moving Particles

Consider a number of point charges $q_1, q_2, q_3, \dots, q_n$ are moving with velocities $\vec{v_1}, \vec{v_2}, \dots, \vec{v_n}$ respectively in magnetic field.

The net magnetic field is

$$\vec{F_m} = q_1(\vec{v_1} \times \vec{B}) + q_2(\vec{v_2} \times \vec{B}) + \dots + q_n(\vec{v_n} \times \vec{B})$$

$$= \sum_{n=1}^{n} q_i(\vec{v_i} \times \vec{B}) \quad \text{------} \quad (i)$$

For continuous system of moving charges, eq^n (i)

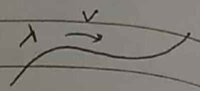
$$\vec{F_m} = \int dq\, (\vec{v} \times \vec{B})$$

where,

$\vec{v}$ = velocity of elemental charge dq in magnetic field $\vec{B}$

(x) Line Current:

The magnetic force on the line current



$$\vec{F}_{mag} = I\int(\vec{dl} \times \vec{B})$$

A line charge λ travelling down a wire at speed v constitutes a current

$$I = \lambda v.$$

(x) Surface Current:

$$\vec{F}_{mag} = \int(\vec{K} \times \vec{B})\,da$$

Surface current density $(\vec{K}) = \frac{d\vec{I}}{dl_\perp}$

$\therefore K = \sigma\vec{v}$ {current per unit width perpendicular to flow.}

(x) Volume Current

$$\vec{F}_{mag} = \int(\vec{J} \times \vec{B})\,d\tau$$

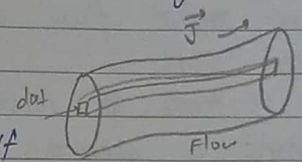
Volume current density $(\vec{J}) = \frac{d\vec{I}}{da_\perp}$ {current per unit area $\perp^r$ to flow}

$\therefore J = \rho\vec{v}$

Continuity Equation

When the flow of charge is distributed throughout a three dimensional region, we describe it by volume current density $(\vec{J})$

Consider a tube of infinitesimal cross section $da_\perp$ running to the flow, if the current in this tube is dI, the volume current density is.



$$\vec{J} = \frac{d\vec{I}}{da_\perp} \quad \text{—— (i)}$$

According to eq^n(i), the current crossing a surface S can be written as

$$I = \int_S J \cdot da_\perp = \int_S \vec{J} \cdot \vec{da} \quad \text{—— (ii)}$$

In particular, the total charge per unit time leaving a volume V is

$$I = \oint_S \vec{J} \cdot \vec{da} = \int_V (\nabla \cdot \vec{J})\,d\tau \quad \text{—— (iii)}$$

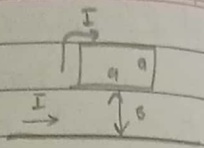
Since charge is conserved, whatever flows through the surface must come at expense of that remaining inside.

$$\int_V (\nabla \cdot \vec{J})\,d\tau = -\frac{d}{dt}\int_V \rho\,d\tau = -\int_V \left(\frac{\partial \rho}{\partial t}\right) d\tau \quad \text{—— (4)}$$

[$\because$ Minus sign indicates outward flow decreasing the charge left in V]. Since, it applies to any volume,

$\therefore \nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$ —— This is continuity equation

Questions:



(i) A square loop is placed near an infinite wire as shown in figure.
When current flows as shown in figure, the square loop:
⇒ tends to move away because ~~current~~ direction of current is opposite.

(ii) A charge of 3C is moving with velocity $\vec{v} = (4\hat{i}+3\hat{j})$ m/s in a magnetic field $\vec{B} = (4\hat{i}+3\hat{j})$ Wb m^{-2}. The force acting on the test charge is 0 (zero)

$$\vec{F_B} = q(\vec{v}\times\vec{B}) = 3\times\begin{vmatrix}\hat{i} & \hat{j} & 0\\ 4 & 3 & 0\\ 4 & 3 & 0\end{vmatrix}$$

$$= 0 \text{ N.}$$

(iii) A proton is moving in a circular orbit of radius 14 cm in a uniform 0.35 T magnetic field $\perp^r$ to the velocity of the proton. Find the speed of the proton.
If the proton is removed by electron with the same speed, what will happen to the radius?

Soln:

Given,

$r = 14$ cm

$B = 0.35$ T

$v_p = ?$

$m = 1.67\times10^{-27}$ kg

$q = 1.6\times10^{-19}$ C

Now,

$$v_p = \frac{qBr}{m_p} = \frac{1.6\times10^{-19}\times0.35\times14\times10^{-2}}{1.67\times10^{-27}}$$

$$= 4.7\times10^{6} \text{ m/s.}$$

Now,

mass of electron $(m_e) = 9.1\times10^{-31}$ kg

$$r = \frac{v_p m_e}{qB} = \frac{4.7\times10^{6}\times9.1\times10^{-31}}{1.6\times10^{-19}\times0.35}$$

$$= 76\times10^{-6} \text{ m}$$

∴ The radius of electron will be smaller.

(iv) A charge particle is circling in a magnetic field with cyclotron frequency 1.5×10^{8} rad/s. If the speed of charge is doubled, the new cyclotron frequency is 1.5×10^{8} rad/s because the frequency is independent of velocity.

(v) In 1897 J.J. Thompson discovered the electron by measuring the charge to mass ratio of cathode ray as follows:
(a) First he passed beam through uniform crossed electric and magnetic fields (mutually perpendicular) and adjusted electric field until he got zero deflection. What was the speed of the particle.

Soln:

We know,

$$qE = qvB \qquad \therefore v = E/B$$

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Q: Then, he turned off the electric field and measure radius of curvature of beam as deflected by magnetic field alone. In terms of E, B, R, what is charge to mass ratio of the particles.

Sol^n

We know,

$$mv = qBR$$

or $$\frac{q}{m} = \frac{v}{BR} \qquad \therefore \frac{q}{m} = \frac{E}{B^2 R}$$

(vi) You set out to reproduce Thomson's e/m experiment with an accelerating potential of 150 V and a deflecting electric of magnitude 6.0×10^6 N/C.

(a): At what fraction of speed of light do electrons move.
(b) What magn~~itude~~etic-field magnitude will yield zero beam deflection?

Sol^n:

(a): $$\frac{1}{2} mv^2 = eV$$

or $$v = \sqrt{2\left(\frac{e}{m}\right)V} = \sqrt{2 \times 1.75 \times 10^{11} \times 150}$$

$$\therefore v = 7.245 \times 10^6 \text{ m/s}.$$

fraction of speed of light $= 2.415 \times 10^{-8}$.

(b): $$B = \frac{E}{v} = \frac{6 \times 10^6}{7.245 \times 10^6} = 0.827 \text{ T}$$

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(vii) A current I is uniformly distributed over a wire of circular cross-section with radius 'a'.
(a) Find the volume current density. If $J = ks$. (s = distance from the axis).
(b) Find the total current in wire.

Sol^n.

(a): Sol^n:

$$J = \frac{I}{\pi a^2}$$

(b): $$I = \int_s J \, da_\perp = \int_0^a (ks)(2\pi s \, ds)$$

$$= \int_0^a 2\pi k \, s^2 \, ds$$

$$\therefore I = \frac{2\pi k a^3}{3}$$

(*) Magnetostatics:

Steady current produces magnetic fields that are constant in time; the theory of steady current is called magnetostatics.

In magnetostatics, the continuity equation.

$$\nabla \cdot \vec{J} = 0$$

When a steady current flows in a wire, its magnitude I must be the same all along the line. Thus.

$$\frac{\partial \rho}{\partial t} = 0 \quad \text{in magnetostatics.}$$