CHAPTER: 1:

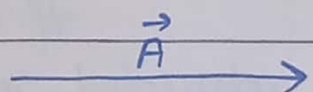
VECTOR ANALYSIS

Scalars:

Scalars have only magnitude and obey the rules of arithmetic and ordinary algebra.
Eg: distance, mass, charge, electric potential, etc.

Vectors:

Vectors have both magnitude and direction and obeys the rules of vector algebra.
Eg: displacement, torque, velocity, etc.



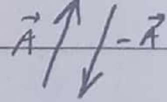
→ Vector notation: $\vec{A}$ or $\vec{a}$

→ Magnitude notation: $|\vec{A}|$ or A

$|\vec{a}|$ or a

→ Equal vectors: Two vectors having equal magnitude and same direction.

→ Negative vector: The vector of same magnitude with opposite direction and when added to ~~opposite~~ original vector gives resultant of zero.



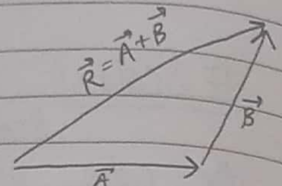
Mathematically,

$$\vec{A} + (-\vec{A}) = 0$$

Four Vector Operations:

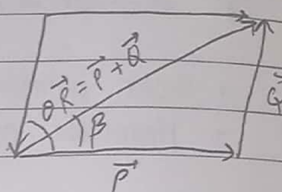
a) Addition of Two Vectors:

*) Triangle law of Vector Addition:



If two sides of a triangle taken in the same order represents the two vectors in magnitude and direction, then the third side in the opposite order represents the resultant of two vectors.

*) Parallelogram law of Vector Addition:



If two vectors are represented in magnitude and direction by the two sides of a parallelogram drawn from a point, then their resultant is given by the diagonal of the parallelogram passing through that point.

Mathematically,

$$R = |\vec{P} + \vec{Q}| = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$\tan\beta = \frac{Q\sin\theta}{P + Q\cos\theta}$$

*) Properties:

i) Addition is commutative: $\vec{A} + \vec{B} = \vec{B} + \vec{A}$

ii) Addition is associative: $(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$

b) Multiplying by a Scalar:

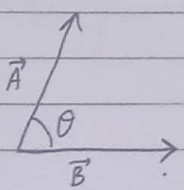
→ When multiplied with positive scalar, the magnitude changes, direction is unchanged.

→ When multiplied with negative scalar, the magnitude changes and the direction is reversed.

Scalar ~~product~~ multiplication is distributive.

$$a(\vec{A} + \vec{B}) = a\vec{A} + a\vec{B}$$

c) Dot product of Two Vectors:



The dot product of two vectors is defined as:

$$\vec{A} \cdot \vec{B} = AB\cos\theta$$

Since the dot product gives scalar, it is also called scalar product.

*) Properties:

i) Dot product is commutative: $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$

ii) Dot product is distributive: $\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$

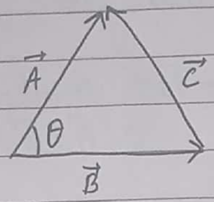
iii) If $\theta = 0^\circ$ ie, parallel $\Rightarrow \vec{A}\cdot\vec{B} = AB$

iv) If $\theta = 90^\circ$ ie, perpendicular $\Rightarrow \vec{A}\cdot\vec{B} = 0$.

To find magnitude of any vector $\vec{E}$,

$$E = \sqrt{\vec{E}\cdot\vec{E}}$$

*) Example:



If $\vec{C} = \vec{A} - \vec{B}$ then,

$$\begin{aligned}\vec{C}\cdot\vec{C} &= C^2 \\ &= (\vec{A}-\vec{B})\cdot(\vec{A}-\vec{B}) \\ &= (\vec{A}-\vec{B})^2 \\ &= A^2 - 2\vec{A}\cdot\vec{B} + B^2 \\ &= A^2 - 2AB\cos\theta + B^2\end{aligned}$$

$$\therefore\ C^2 = A^2 + B^2 - 2AB\cos\theta$$

This is the law of cosines.

(d): Cross Product of Two Vectors:

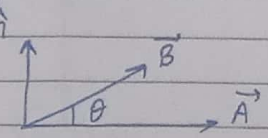
The cross product of two vectors is defined as

$$\vec{A}\times\vec{B} = AB\sin\theta\ \hat{n}$$

where,

$\hat{n}$ is unit vector containing A and B in perpendicular direction.

$$|\vec{A}\times\vec{B}| = AB\sin\theta.$$



Geometrically the magnitude of $\vec{A}\times\vec{B}$ gives the area of parallelogram generated by $\vec{A}$ and $\vec{B}$.

(*): Properties:

i) Cross product is not commutative: $\vec{A}\times\vec{B} \neq \vec{B}\times\vec{A}$

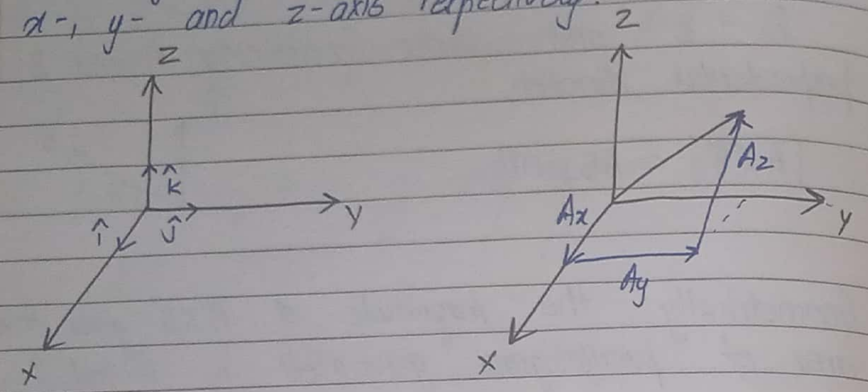
ii) Cross product is distributive: $\vec{A}\times(\vec{B}+\vec{C}) = \vec{A}\times\vec{B} + \vec{A}\times\vec{C}$

iii) If $\theta = 0^\circ$ ie, parallel, $|\vec{A}\times\vec{B}| = 0$

iv) If $\theta = 90^\circ$ ie, perpendicular $|\vec{A}\times\vec{B}| = AB$

Vector Algebra in Component Form:

Let $\vec{i}, \vec{j}, \vec{k}$ be unit vectors parallel to x-, y- and z-axis respectively.



Now,

$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$ $\qquad$ $\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$

$a\vec{A} = aA_x \hat{i} + bA_y \hat{j} + cA_z \hat{k}$

$\vec{A} + \vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k}$

$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$ $\quad \{\hat{i}\cdot\hat{i} = 1,\ \hat{i}\cdot\hat{j} = 0\}$

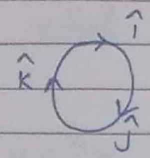
$|\vec{A}| = \sqrt{A_x^2 + \cancel{B_x^2} + A_y^2 + A_z^2}$

$\vec{A} \times \vec{B} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \times (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$

$= (A_y B_z - A_z B_y)\hat{i} + (A_z B_x - A_x B_z)\hat{j} + \cancel{(A_x B_y - A_y B_x)}\hat{k}$

$(A_x B_y - A_y B_x)\hat{k}$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$



Triple ~~Co~~ Product:

a) Scalar Triple Product:

$\vec{A} \cdot (\vec{B} \times \vec{C}) = \vec{B} \cdot (\vec{C} \times \vec{A}) = \vec{C} \cdot (\vec{A} \times \vec{B})$

$= |\vec{B} \times \vec{C}| \cdot (A\cos\theta)$

= Volume of parallelepiped.

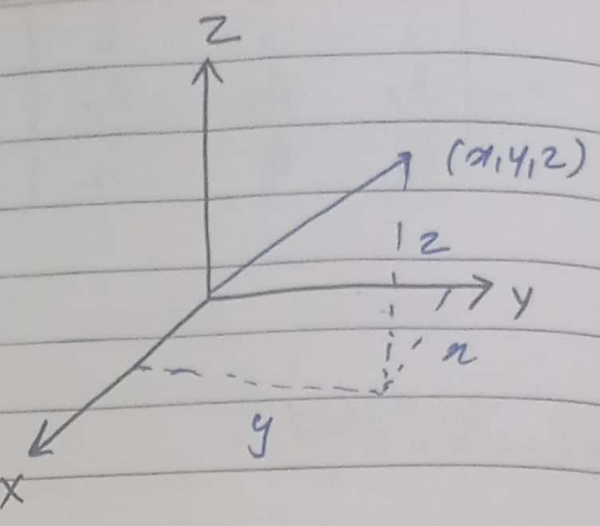
= Area of base x altitude of parallelepiped

$\vec{A} \cdot (\vec{B} \times \vec{C}) = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$

b) Vector Triple Product:

$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$.

x) Position vector:



$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$r = \sqrt{x^2+y^2+z^2}$$

$$= (x^2+y^2+z^2)^{1/2}$$

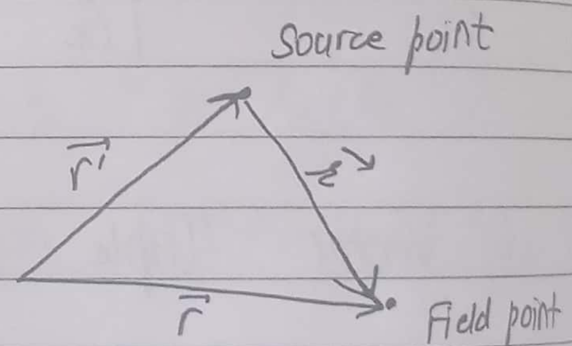
$$\hat{r} = \frac{\vec{r}}{r} = \frac{x\hat{i} + y\hat{j} + z\hat{k}}{\sqrt{x^2+y^2+z^2}}$$

x) Infinitesimal Displacement Vector:

$$d\vec{r} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$$

x) Seperation Vector:

The separation vector from the source point to the field point is.



$$\vec{\mathscr{r}} = \vec{r} - \vec{r'}$$

$$= (x-x')\hat{i} + (y-y')\hat{j} + (z-z')\hat{k}$$