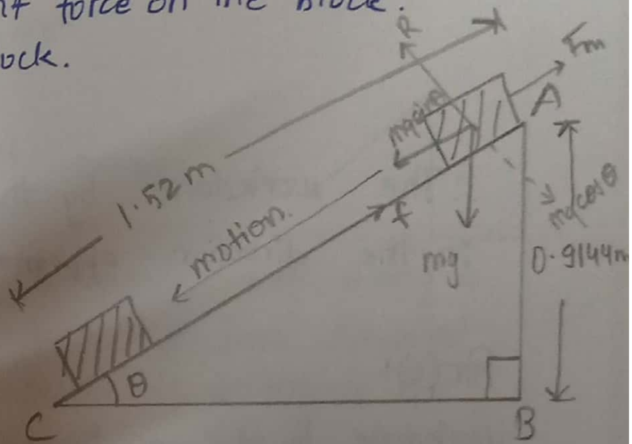
CHAPTER 1:

DYNAMICS OF SYSTEM OF PARTICLES

1: A ~~body~~ 45.36 kg block of ice slides down an incline 1.52 m long and 0.9144 m high. A man pushes up on the ice parallel to the incline so that it slides down at constant speed. The coefficient of friction between the ice and the incline is 0.1. Find

a) the force exerted by the man
b) the workdone by the man on the block.
c) the workdone by gravity on the block
d) the workdone by the surface of the incline on the block.
e) the workdone by the resultant force on the block.
f) the change in K.E. of the block.

Soln:



Given;
Mass of ice block (m) = 45.36 kg
length of incline (l) = 1.52 m
height of incline (h) = 0.9144 m
coefficient of friction (μ) = 0.1

For (a):

Base of $\Delta ABC = \sqrt{(1.52)^2 - (0.9144)^2}$
$= 1.2142$ m

Let Fm be force exerted by the man and f be frictional force.
According to Newton's second law;

$$F_m + f = mg\sin\theta \quad \text{—— (i)}$$

According to Newton's third law;

$$R = mg\cos\theta \quad \text{—— (ii)}$$

Using eqn (i),

$$F_m = mg\sin\theta - f$$
$$= mg\sin\theta - \mu R$$

$= mg\sin\theta - \cancel{\text{ær}}\ \mu\, mg\cos\theta$

$= mg\left[\sin\theta - \mu\cos\theta\right]$

$= 45.36 \times 9.8 \left[\frac{0.9144}{1.52} - \left(0.1 \times \frac{1.2142}{1.52}\right)\right]$

$\therefore F_m = \cancel{231.15\ N}\ \cancel{\{231.90\ N\}}\ 231.90\ N$

$\therefore$ The force exerted by the man is $231.5\ N$.

For (b):

Workdone by the man on the block $(W_m) = \vec{F_m}\cdot\vec{s}$

$= F.l.\cos 180°$

$= -1 \times 231.90 \times 1.52$

$= -352.48\ J$

$\therefore$ The workdone by the man is $352.48\ J$ but in the direction opposite to the ~~mot~~ direction of method.

For (c):

Workdone by the gravity on the block $(W_g) = \vec{F_g}\cdot\vec{s}$

$= F_g \times l \times \cos(90° - \theta)$

$= mg \times l \times \sin\theta$

$= 45.36 \times 9.8 \times \cancel{1.52} \times \frac{0.9144}{\cancel{1.52}}$

$= 406.47\ J$

$\therefore$ The workdone by gravity on the block is $406.47\ J$

For (d):

Workdone by the surface of the incline on the block

$(W_s) = \vec{f}\cdot\vec{s}$

$= f.l.\cos 180°$

$= \mu R.l.\cos 180°$

$= 0.1 \times (-1) \times 1.52 \times mg\cos\theta$

$= 0.1 \times (-1) \times 1.52 \times 45.36 \times 9.8 \times \frac{1.2142}{1.52}$

$= -53.97\ J$

$\therefore$ The workdone by surface on the block is $53.97\ J$ but in opposite direction of motion.

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For (e):

Workdone by resultant force on block $(W_r) = \vec{R}\cdot\vec{s}$

$= R\cdot s\cdot \cos 90^\circ$

$= 0$

∴ The resultant force does no work on the block.

For (f);

Here, the block moves in constant speed. So,

initial velocity (u) = final velocity (v).

∴ Change in KE (ΔKE) = $KE_f - KE_i$

$= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$= \frac{1}{2}mv^2 - \frac{1}{2}mv^2$

$= 0$.

∴ The K.E of the block doesn't change.

2. A man pushes a 27.215 kg block 9.144 m along a level floor at constant speed with the force directed 45° below the horizontal. If coefficient of KE is 0.2, how much work does the man do on the block?

Soln:

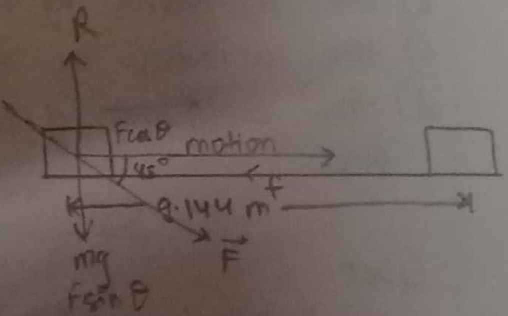
Given,

mass of block (m) = 27.215 kg

distance (s) = 9.144 m

coefficient of KE (μ) = 0.2.

angle of force to horizontal (θ) = 45°



According to Newton's third law of motion,

$R = mg + F\sin\theta$ — (i)

According to Newton's second law of motion;

$F\cos\theta = f$ — (ii)

or, $F\cos\theta = \mu R$ — (iii)

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Using eq^(i) in eq^ (iii)

$$F\cos\theta = \mu(mg + F\sin\theta)$$

or, $F\cos 45° = 0.2\,(27.215 \times 9.8 + F.\sin 45°)$

or, $F\cos 45° = 0.2 \times 27.215 \times 9.8 + F\sin 45° \times 0.2$

or, $F(\cos 45° - \sin 45° \times 0.2) = 0.2 \times 27.215 \times 9.8$

$$\therefore F = \frac{0.2 \times 27.215 \times 9.8}{\cos 45° - \sin 45° \times 2} = 94.29 \text{ N}$$

Now, the workdone by the man on the block $(W_m) = \vec{F}\cdot\vec{s}$

$$= F.s.\cos 45°$$

$$= 94.29 \times 9.144 \times \frac{1}{\sqrt{2}}$$

$$= 609.65 \text{ J}$$

$\therefore$ The workdone by the man on the bloch is 609.65 J

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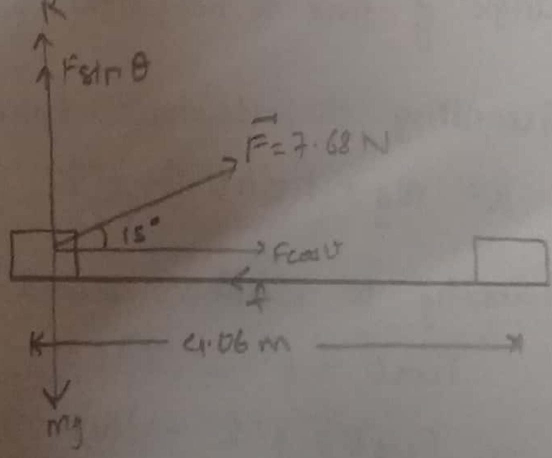
3. A block of mass m=3.57 kg is drawn at a constant speed a distance d=4.06 m along a horizontal floor by rope exerting a constant force of magnitude F=7.68 N making the angle 15° with the horizontal. Compute.
a) the total workdone on the block
b) the workdone by the rope on the block
c) the workdone by the friction on the block
d) the coefficient of ~~fri~~ KE friction between the block and floor.

Soln:

Given,
mass of block (m) = 3.57 kg
distance (s) = 4.06 m
force (F) = 7.68 N
Angle (θ) = 15°



For (a):

Total workdone on the block (W_t) = $W_r + W_f + W_g + W_R$

[∵ Sum of workdone by rope, friction, gravity and reaction]

$$= F \cdot s \cdot \cos 15^\circ + Ff \cdot s \cdot \cos 180^\circ + m \cdot g \cdot \cos 90^\circ + R \cdot s \cdot \cos 90^\circ \quad \text{— (i)}$$

Here,

according to Newton's third law of motion,

$$R + F\sin\theta = mg \quad \text{— (ii)}$$

According to Newton's second law of motion,

$$F\cos\theta = f \quad \text{— (iii)}$$

or $7.68 \times \cos 15 = f$ $\quad$ ∴ $f = 7.418$ N

~~Applying the value of f in 1,~~

~~$W_t = 7.68 \times 4.06 \times \cos 15^\circ + 7.418 \times 4.06 \times \cos 180^\circ + 0 + 0$~~

~~∴ $W_t =$~~

Using eqn (iii) in eqn (i),

$$W_t = F \cdot s \cdot \cos\theta + F\cos\theta \cdot s \cdot \cos 180^\circ + 0 + 0$$

$$= Fs\cos\theta + Fs\cos\theta \times (-1)$$

$$\therefore W_t = 0$$

~~Here,~~ Thus, the total workdone on the block is 0.

For (b):

Workdone by rope on the block (W_r) = $\vec{F} \cdot \vec{s}$

$$= F \cdot \cos 15 \times 4.06$$

$$= 7.68 \times \cos 15^\circ \times 4.06$$

$$\therefore W_r = 30.11 \text{ J}$$

The total workdone by the rope is 30.11 J for (c):

Workdone by friction on the block $(W_f) = \vec{f} \cdot \vec{s}$

$= f \cos 180° \times 4.06$

$= F \cos 15° \times \cos 180° \times 4.06$

[∵ from eqⁿ iii]

$= 7.68 \times \cos 15° \times \cos 180° \times 4.06$

$\therefore W_f = -30.11\ J$

The workdone by friction on the block is 30.11 J but in opposite direction of its motion.

for (d):

from eqⁿ (ii); $R = mg - F\sin\theta$

and

from eqⁿ (iii): $f = F\cos\theta$

We know,

Coefficient of Kinetic friction $(\mu) = \frac{f}{R} = \cancel{\frac{mg - F\sin\theta}{F\cos\theta}} = \frac{F\cos\theta}{mg - F\sin\theta}$

$= 1 \Big/ \left(\frac{3.57 \times 9.8 - 7.68 \times \sin 15°}{7.68 \times \cos 15°}\right)$

$\therefore \mu = 0.224$

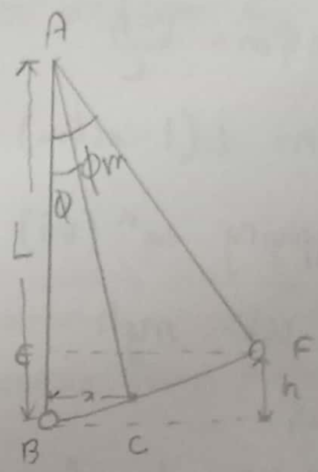
The coefficient of Kinetic friction is 0.224.

4: A small object of mass 'm' is suspended from a string of length 'L'. The object is pulled sideways by a force 'F' that is always horizontal until string makes angle ϕm with vertical. The displacement accomplished a small constant speed. Find the workdone by all forces that act on the object.

Soln

From free body diagram,

$$T\cos\phi = mg \quad \text{— (i)}$$
$$T\sin\phi = F \quad \text{— (ii)}$$

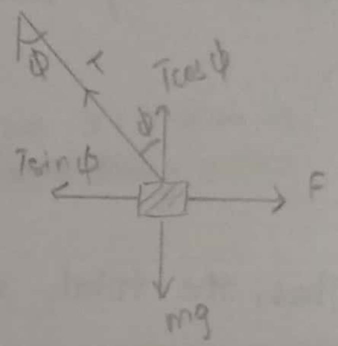


Dividing (i) from (ii);

$$\frac{T\sin\phi}{T\cos\phi} = \frac{F}{mg}$$

or, $\tan\phi = \frac{F}{mg}$

or, $\vec{F} = mg\tan\phi$ — (iii)



Now, the workdone by the force 'F' is $W_F = \int F \cdot dx$

$$W_F = \int mg\tan\phi\, dx \quad \text{— (iv)}$$

From ΔABC,

$\sin\phi = \frac{x}{L}$

or, $x = L\sin\phi$

Differentiating both sides w.r.t. ϕ,

$$dx = L\cos\phi\, d\phi \quad \text{— (v)}$$

Using eq^n(v) in eq^n(iv);

$$W_f = \int mg\tan\phi\, L\cos\phi\, d\phi$$

Since the work is done from 0 to ϕm,

$$W_f = \int_0^{\phi m} mgL\sin\phi\, d\phi$$
$$= Lmg\left[-\cos\phi m + \cos 0°\right]$$
$$= Lmg\left[1 - \cos\phi m\right] \quad \text{— (vi)}$$

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In ΔAEF,

$\cos\phi_m = \frac{L-h}{L}$

or, $h = L(1-\cos\phi_m)$ — (vii)

Applying eq^n (vii) in eq^n (vi), we get

$W_F = mgh$.

Now,

workdone by force of gravity $(W_g) = -(W_F + W_T)$

[$\because W_g + W_F + W_T = 0$ i.e, conservative force]

$= -(mgh + 0)$

$\therefore W_g = -mgh$. [$\because \vec{T} \perp \vec{ds}$; $W_t = 0$]

Thus, the total workdone by all forces is mgh.

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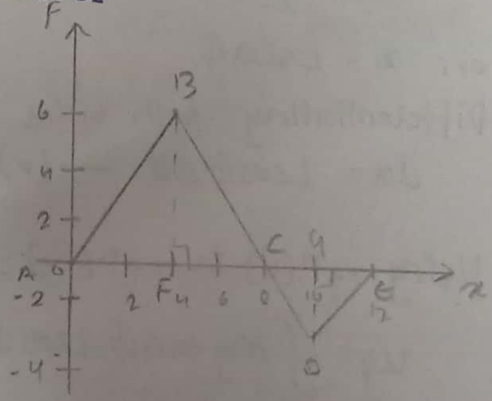
5. The force acting on the particle varies as shown in the figure. Find the workdone by the force on the particle as it moves

(a) from $x=0$ to $x=8.00m$,

(b) from $x=8.00$ to $x=10.00\,m$,

(c) from $x=0$ to $x=10.0\,m$.



Sol^n:

For (a):

We know,

Workdone ~~as particle~~ = Area under the graph.

So,

workdone as particle moves from $x=0$ to $x=8.00m$

(W_a) = Area covered by ΔABC

$= \cancel{\frac{1}{2}\times}$ Area of ΔABF + Area of ΔBFC.

$= \frac{1}{2}\times 4\times 6 + \frac{1}{2}\times 4\times 6 = 24\,J$

$\therefore$ Workdone in moving to 8.00 m is 24 J

For (b):

Workdone by force when particle moves from $x = 8{\cdot}00m$ to $x = 10{\cdot}00m$ (W_B) = Area of Δ CDG

= Area of ΔCGD ~~+ Δ GDE~~

$= \frac{1}{2} \times 2 \times (-3)$

$= -3$ J [$\because$ indicating the work is done in opposite direction]

Thus, 3 J work is done in moving particle from 8·00m to 10·00 m but in opposite direction.

For (c):

Workdone by force when particle moves from $x = 0{\cdot}00m$ to $x = 10{\cdot}00m$ (W_c) = Area of ΔABC + Area of ΔCDG

$= W_A + W_B$

$= 24J - 3J = 21J$

$\therefore$ The total workdone when particle moves from 0m to 10·00 m is 21 J.

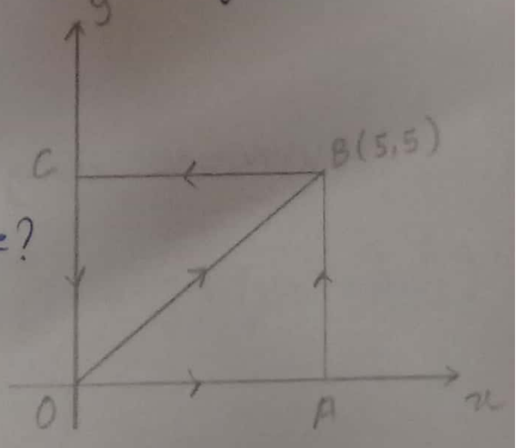
6) A force acting on a particle moving in xy plane is given by $\vec{F} = (2y\hat{i} + x^2\hat{j})$ N, where x and y are in meters. The particle moves from origin to the final position with coordinates $x = 5{\cdot}00$ m and $y = 5{\cdot}00$m as shown in figure. Calculate the workdone by $\vec{F}$ on the particle as it moves along

a) OAC

b) OBC

c) OC

d) Is $\vec{F}$ conservative or non-conservative?



Soln:

Given;

$\cancel{\vec{F} = 2y\hat{\imath} + 2x}$ $\vec{F} = 2y\hat{\imath} + x^2\hat{\jmath}$

For (a):

Workdone along the path OAC $(W_{OAC}) = W_{OA} + W_{AC}$

Now,

$$W_{OA} = \int_{x_i}^{x_f} \vec{F} \cdot dx \cdot \hat{\imath}$$ [∵ Work is done along x-axis]

$$= \int_0^5 (2y\hat{\imath} + x^2\hat{\jmath})\, dx \cdot \hat{\imath}$$

$$= \int_0^5 (2y\hat{\imath})\, dx$$

$\therefore W_{OA} = 0$ [y = 0 along x-axis] [∵ constant path]

Workdone along AC $(W_{AC}) = \int_{y_i}^{y_f} \vec{F} \cdot dy \cdot \hat{\jmath}$ [∵ Work along y-axis]

$$= \int_0^5 (2y\hat{\imath} + x^2\hat{\jmath})\, dy\, \hat{\jmath}$$

$$= \int_0^5 x^2\, dy$$

$$= 5^2 \times [y]_0^5$$ [∵ x = 5 at A]

$$= 25 \times 5 = 125 \text{ J}$$

$\therefore$ Workdone along OAC $(W_{OAC}) = 125 + 0$

$= 125$ J.

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For (b):

Workdone along path OBC $(W_{OBC}) = W_{OB} + W_{BC}$

Here,

$$W_{OB} = \int_{y_i}^{y_f} \vec{F} \cdot dy \cdot \hat{j} \quad [\text{Workdone along } y\text{-axis}]$$

$$= \int_0^5 (2y\hat{i} + x^2\hat{j})\, dy\, \hat{j}$$

$$= \int_0^5 x^2\, dy \quad [\text{Here, } x = 0 \text{ and } y = 5]$$

$$\therefore W_{OB} = 0$$

$$W_{BC} = \int_{x_i}^{x_f} \vec{F} \cdot dx \cdot \hat{i} \quad [\because \text{Workdone along } x\text{-axis}]$$

$$= \int_0^5 (2y\hat{i} + x^2\hat{j})\, dx\, \hat{i}$$

$$= \int_0^5 2y\, dx \quad [\because \text{Here, } y = 5 \text{ and } x = 5]$$

$$= 2 \times 5 \times [x]_0^5$$

$$= 2 \times 5 \times 5 = 50\ J$$

$\therefore$ Workdone along path OBC $(W_{OBC}) = 50\ J$

For (c):

Workdone along path OC $(W_{OC}) = \int_0^5 \vec{F} \cdot \vec{dr}$

$$= \int_0^5 (2y\hat{i} + x^2\hat{j})(dx\hat{i} + dy\hat{j})$$

$$= \int_0^5 2y\, dx + \int_0^5 x^2\, dy$$

Here, $x = y$ [$\because$ particle moves along $x = y$]

Diffrentiating both sides by dx or dy, we get

$$dx = dy$$

So,

$$W_{OC} = \int_0^5 2y\,dx + \int_0^5 x^2\,dx$$

Here, $y = 5$ and $x = 5$

~~$W_{OC} = 2\left[\frac{5^2}{2}\right]_0^5 \quad 2$~~

$$W_{OC} = \cancel{2}\left[\frac{y^2}{\cancel{2}}\right]_0^5 + \left[\frac{x^3}{3}\right]_0^5$$

$$= 5^2 + \frac{125}{3} = 66.67\ J$$

$\therefore$ Work done along W_{OC} path $= 66.67\ J$.

For (d)

We check,

$$= W_{OA} + W_{AC} + W_{CB} + W_{BO}$$

$$= 0 + 125 + (-50) + 0$$

$$= 75\ J \neq 0$$

Hence, the force $\vec{F} = 2y\hat{i} + x^2\hat{j}$ acting on the particle is non-conservative.