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KATHMANDU UNIVERSITY

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Subject: MATH 101

Assignment No: 4

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1) Decide the convergence and divergence of the following sequences. If it converges, then find the limit.

a) $a_n = \frac{n+3}{n^2+5n+6}$

Soln:

Given,

$$a_n = \frac{n+3}{n^2+5n+6}$$

$$= \frac{(n+3)}{n^2+(3+2)n+6}$$

$$= \frac{(n+3)}{n^2+3n+2n+6} = \frac{\cancel{(n+3)}}{\cancel{(n+3)}(n+2)} = \frac{1}{n+2}$$

$\therefore a_n = \frac{1}{n+2}$

Now, to check for convergence,

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{n+2} = 0 \text{ i.e, finite value.}$$

Here, limit exists.

This sequence is convergent to 0.

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(b) $a_n = \sqrt[n]{3^{2n+1}}$

Sol^n:

Given,

$a_n = \sqrt[n]{3^{2n+1}}$

$= (3^{2n+1})^{\frac{1}{n}}$

$= (3^{2n})^{\frac{1}{n}} \cdot (3^1)^{\frac{1}{n}}$

$= 3^{2\cancel{n} \times \frac{1}{\cancel{n}}} \cdot 3^{\frac{1}{n}}$

$= 3^2 \cdot 3^{\frac{1}{n}}$

So, $a_n = 9 \cdot 3^{\frac{1}{n}}$

To check for convergence,

$\lim_{n \to \infty} a_n = \lim_{n \to \infty} 9 \cdot 3^{\frac{1}{n}}$

$= 9 \cdot 3^{\frac{1}{\infty}}$

$= 9$

Here, limit exists.

So, this sequence is convergent to 9

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(c): $a_n = \left(1 - \frac{1}{n}\right)^n$

Sol^n:

Given,

$$a_n = \left(1 - \frac{1}{n}\right)^n$$

Let $z = -1/n$.

So,

If $n \to \infty$, $z \to 0$.

So,

$$a_n = (1+z)^{-1/n}$$

$$\therefore a_n = \frac{1}{(1+z)^{1/n}}$$

To check for ~~diverge~~ convergence.

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \left(1 - \frac{1}{n}\right)^n$$

$$= \lim_{z\to 0} \frac{1}{(1+z)^{1/z}}$$

$$= \cancel{\frac{1}{(1+0)^{1/0}}} = \frac{1}{e} \quad \left[\because \lim_{z\to 0} (1+z)^{-1/z} = e\right]$$

Here, limit exists.

This sequence is convergent to $1/e$.

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2) Find n^{th} partial sum and the sum of the series for the following.

a): $\frac{5}{1\cdot2} + \frac{5}{2\cdot3} + \frac{5}{3\cdot4} + \cdots\cdots$

Soln:

Given,

$$S_n = \frac{5}{1\cdot2} + \frac{5}{2\cdot3} + \frac{5}{3\cdot4} + \cdots + \frac{5}{(n-1)\cdot n} + \frac{5}{n\cdot(n+1)}$$

$$= 5\left[\frac{1}{1\cdot2} + \frac{1}{2\cdot3} + \frac{1}{3\cdot4} + \cdots + \frac{1}{(n-1)\cdot n} + \frac{1}{n(n+1)}\right]$$

$$= 5\left[\left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \cdots + \left(\frac{1}{n-1} - \frac{1}{n}\right) + \left(\frac{1}{n} - \frac{1}{n\right.$$

$$= 5\left[1 - \frac{1}{n+1}\right]$$

$$\therefore S_n = \frac{5n}{n+1}$$

So,

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} 5\left[1 - \frac{1}{n+1}\right]$$

$$= 5\left[1 - \frac{1}{\infty+1}\right]$$

$$= 5$$ Hence, this given series is convergent.

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(b): $\sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2}$

Sol^n:

Given,

$$S_n = \frac{2n+1}{n^2(n+1)^2} = \frac{1}{n^2} - \frac{1}{(n+1)^2}$$

$$= \left(\frac{1}{1} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{9}\right) + \left(\frac{1}{9} - \frac{1}{16}\right) + \cdots$$

$$\cdots + \left(\frac{1}{(n-1)^2} - \frac{1}{n^2}\right) + \left(\frac{1}{n^2} - \frac{1}{(n+1)^2}\right)$$

$$S_n = 1 - \frac{1}{(n+1)^2}$$

So,

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} 1 - \frac{1}{(n+1)^2}$$

$$= 1 - \frac{1}{(\infty+1)^2}$$

$$= 1$$

Hence, this series is convergent.

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3) Decide whether the following series converges or diverges using

i): n^{th} term Test:

a): $\sum_{n=1}^{n} \frac{n}{n+1}$

Soln:

Here,

$a_n = \frac{n}{n+1}$

for n^{th} term test,

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n}{n+1}$$

$$= \lim_{n\to\infty} 1 - \frac{1}{n+1}$$

$$= 1-0$$

$$= 1 \neq 0$$

Hence, the series is divergent.

(b): $\sum_{n=1}^{\infty} n \sin\frac{1}{n}$

Sol^n.

Here,

$\cancel{\lim_{n\to\infty}} \quad a_n = n\sin\left(\frac{1}{n}\right)$

For n^{th} term test,

Let $n = \frac{1}{m}$

So, if $n \to \infty$, $m \to 0$.

$$\lim_{n\to\infty} n\sin\left(\frac{1}{n}\right) = \lim_{m\to 0} \frac{1}{m}\sin m$$

$$= \lim_{m\to 0} \frac{\sin m}{m} = 1 \neq 0$$

Hence, the series is divergent.

(c): $\sum_{n=1}^{\infty} \cancel{(-1)^{n+1}} \dfrac{3\sqrt{n+1}}{\sqrt{n}+1}$

Sol^n:

Here,

$$a_n = (-1)^{n+1} \frac{3\sqrt{n+1}}{\sqrt{n}+1}$$

For n^{th} term test,

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$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{3\sqrt{n+1}}{\sqrt{n}+1}$$

$$= \lim_{n\to\infty} \frac{3\cdot\cancel{\sqrt{n}}\cdot\sqrt{1+\frac{1}{n}}}{\cancel{\sqrt{n}}\left(1\cdot\cancel{\frac{\sqrt{n}}{\sqrt{n}}} + \frac{1}{\sqrt{n}}\right)}$$

$$= \frac{3\sqrt{1+1/\infty}}{(1+1/\infty)} = 3 \cancel{\neq 0} \neq 0$$

Hence, the series is divergent.

(d): $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{10^n}{n^{10}}$

Soln,

Given,

$$a_n = (-1)^{n+1} \frac{10^n}{n^{10}}$$

So,

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} (-1)^{n+1} \frac{10^n}{n^{10}}$$

Here, the limit doesn't exist.

So, this series is divergent.

(ii) Integral test:

a) $\sum_{n=2}^{\infty} \frac{\ln n}{\sqrt{n}}$

Soln:

Here,

$$a_n = \frac{\ln n}{\sqrt{n}}$$

Writing $\sum_{n=2}^{\infty} a_n$ in integral form,

$$\sum_{n=2}^{\infty} \frac{\ln n}{\sqrt{n}} = \int_2^{\infty} \frac{\ln n \, dn}{\sqrt{n}}$$

$$= \lim_{b\to\infty} \left[\ln n \int_2^b \frac{1}{\sqrt{n}} dn - \int_2^b \left(\frac{d \ln n}{dn} \int_2 \frac{1}{\sqrt{n}} dn \right) dn \right]_2^b$$

$$= \lim_{b\to\infty} \left[2\sqrt{n} \ln n - \int_2^b \frac{1}{n} \times 2\sqrt{n} \right]_2^b$$

$$= \lim_{b\to\infty} \left[2\sqrt{n} \ln n - 4\sqrt{n} \right]_2^b$$

$$= \lim_{b\to\infty} \left(2\sqrt{b} \ln b - 4\sqrt{b} - 2\times 2 \ln 2 - 4\sqrt{2} \right)$$

$$= \infty$$

The given series is divergent.

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(b): $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}(\sqrt{n}+1)}$

Sol^n:

Let $a_n = \frac{1}{\sqrt{n}(\sqrt{n}+1)}$

Writing $\sum_{n=1}^{\infty}$ in integral form,

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}(\sqrt{n}+1)} = \cancel{\int_1^{\infty} \frac{1\, dx}{\sqrt{n}(\sqrt{n}+1)}} = \int_1^{\infty} \frac{dx}{\sqrt{x}(\sqrt{x}+1)}$$

Let $u = \sqrt{x}+1$ so, $du = \frac{1}{2\sqrt{x}} dx$

or $2 \cdot du = \frac{1}{\sqrt{x}} dx$

So,

$$= \int_1^{\infty} \frac{2\,du}{u}$$

$$= \lim_{b \to \infty} 2 \int_1^{b} \frac{du}{u}$$

$$= \lim_{b \to \infty} 2 \ln [u] \Big|_1^b$$

$$= 2 \cdot \lim_{b \to \infty} \ln(\sqrt{x}+1) \Big|_1^b$$

$$= 2 \lim_{b \to \infty} \left[\ln \sqrt{b}+1 - \ln \sqrt{1}+1 \right]$$

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$$= 2 \lim_{b\to\infty} \ln\left(\frac{\sqrt{b}+1}{2}\right)$$

$$= 2 \ln \frac{\sqrt{\infty}+1}{2}$$

$$= \infty$$

This series is divergent.

(c): $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$

Soln:

Here,

$$a_n = \frac{n}{n^2+1}$$

Writing $\sum_{n=1}^{\infty} n/n^2+1$ in integral form.

$$\sum_{n=1}^{\infty} n/(n^2+1) = \int_1^{\infty} \frac{x\,dx}{x^2+1}$$

$$= \frac{1}{2}\int_1^{\infty} \frac{2x\,dx}{x^2+1}$$

Let $u = x^2+1$ so, $\frac{du}{dx} = 2x$ or, $du = 2x \cdot dx$

$$= \frac{1}{2}\int_1^{\infty} \frac{du}{u}$$

$$= \frac{1}{2}\left[\lim_{b\to\infty} \ln u\right]_1^b$$

$$= \cancel{\frac{1}{2}\left[\lim_{b\to\infty} \ln b - \ln 1\right.}$$

$$= \frac{1}{2}\left[\lim_{b\to\infty} \ln (x^2+1)\right]_1^b$$

$$= \frac{1}{2}\left[\lim_{b\to\infty} \ln (b^2+1) - \ln (1^2+1)\right]$$

$$= \infty$$

$\therefore$ the series is divergent.

(iii) the ratio test:

(a): $\displaystyle\sum_{n=1}^{\infty} \frac{(n+3)!}{3!\,n!\,3^n}$

Soln:

Here,

$$a_n = \frac{(n+3)!}{3!\,n!\,3^n}$$

$$a_{n+1} = \frac{(n+1+3)!}{3!\,(n+1)!\,3^{n+1}}$$

Now,

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{(n+4)!}{3!\,(n+1)!\,3^{n+1}} \times \frac{3!\,n!\,3^n}{(n+3)!}$$

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$$= \lim_{n\to\infty} \frac{(n+4)\cancel{(n+3)!} \times \cancel{3!}\,\cancel{n!}\,\cancel{3^n}}{\cancel{3!}\,(n+1) \times \cancel{n!} \times \cancel{3^n} \times 3 \times \cancel{(n+3)!}}$$

$$= \lim_{n\to\infty} \frac{(n+4)}{(n+1) \times 3} = \lim_{n\to\infty} \frac{(n+4)}{3(n+1)}$$

$$= \lim_{n\to\infty} \frac{\cancel{n}(1+4/n)}{3\cancel{n}(1+1/n)}$$

$$= \frac{(1+0)}{3(1+0)} = \frac{1}{3} < 1$$

Through ratio test, the series converges.

(b): $\sum_{n=1}^{\infty} \frac{n\,2^n\,(n+1)!}{n!\,3^n}$

Sol^n:

Here,

$$a_n = \frac{n\,2^n\,(n+1)!}{n!\,3^n}$$

$$a_{n+1} = \frac{(n+1)\,2^{n+1}\,(n+1+1)!}{(n+1)!\,3^{n+1}}$$

Now,

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{(n+1)2^{n+1}(n+2)!}{(n+1)!\,3^{n+1}} \times \frac{n!\,3^n}{n\,2^n\,(n+1)!}$$

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$$= \lim_{n\to\infty} \frac{\cancel{(n+1)} \times \cancel{2^n} \times 2 \times (n+2) \times \cancel{(n+1)!} \times \cancel{n!}\ \cancel{3^n}}{\cancel{(n+1)} \times \cancel{n!} \times \cancel{3^n} \times 3 \times n \times \cancel{2^n} \times \cancel{(n+1)!}}$$

$$= \lim_{n\to\infty} \frac{2(n+2)}{3n}$$

$$= \lim_{n\to\infty} \frac{2\cancel{n}(1+2/n)}{3\cancel{n}}$$

$$= \frac{2}{3} < 1$$

Hence, this series converges.

(c): $\sum_{n=1}^{\infty} \frac{(3n)!}{n!\,(n+1)!\,(n+2)!}$

Sol^n

Here,

$$a_n = \frac{(3n)!}{n!\,(n+1)!\,(n+2)!}$$

$$a_{n+1} = \frac{(3(n+1))!}{(n+1)!\,(n+2)!\,(n+3)!} = \frac{(3n+3)!}{(n+1)!\,(n+2)!\,(n+3)!}$$

Now,

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{(3n+3)!}{(n+1)!(n+2)!(n+3)!} \times \frac{n!\,(n+1)!\,(n+2)!}{(3n)!}$$

$$= \lim_{n\to\infty} \frac{(3n+3)(3n+2)(3n+1)\cancel{(3n)!} \times \cancel{n!} \times \cancel{(n+1)!} \times \cancel{(n+2)!}}{\cancel{(n+1)!\,(n+2)!\,(n}\ \cancel{(3n)!} \times (n+1)\cancel{n!}(n+2)\cancel{(n+1)!} \times (n+3) \times \cancel{(n+2)!}}$$

$$= \lim_{n\to\infty} \frac{(3n+3)(3n+2)(3n+1)}{(n+1)(n+2)(n+3)}$$

$$= \lim_{n\to\infty} \frac{3(3n+2)(3n+1)}{(n+2)(n+3)}$$

$$= \lim_{n\to\infty} \frac{3\cancel{n^2}(3+2/n)(3+1/n)}{\cancel{n^2}(1+2/n)(3+1/n)}$$

$$= \frac{3 \times 3 \times \cancel{3}}{1 \times \cancel{3}} = 9 > 1$$

This series is divergent.

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(iv) the n^{th} root test.

(a) $\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n^2}\right)^n$

Sol^n:

Here,

$$a_n = \left(\frac{1}{n} - \frac{1}{n^2}\right)^n$$

Now,

$$\lim_{n\to\infty} \sqrt[n]{a_n}$$

$$= \lim_{n\to\infty} \left(\frac{1}{n} - \frac{1}{n^2}\right)^{\cancel{n} \times 1/\cancel{n}}$$

$$= \lim_{n\to\infty} \frac{1}{n} - \lim_{n\to\infty} \frac{1}{n^2}$$

$$= 0 < 1$$

This series converges.

(b): $\sum_{n=2}^{\infty} \frac{n}{(\ln n)^{n/2}}$

Sol^n:

Here,

$$a_n = \frac{n}{(\ln n)^{n/2}}$$

Now,

$$\lim_{n\to\infty} \sqrt[n]{a_n} = (a_n)^{1/n}$$

$$= \lim_{n\to\infty} \left(\frac{n}{(\ln n)^{n/2}}\right)^{1/n}$$

$$= \lim_{n\to\infty} \frac{n^{1/n}}{(\ln n)^{\frac{\cancel{n}}{2}\times\frac{1}{\cancel{n}}}}$$

$$= \lim_{n\to\infty} \frac{1}{n}$$

$$= \frac{1}{\infty}$$

$$= 0 < 1$$

This series converges.

(c) $\sum_{n=1}^{\infty} \frac{(n!)^n}{n^{n^2}}$

Soln,

Given,

$$a_n = \frac{(n!)^n}{n^{n^2}}$$

Now,

$$\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \left(\frac{n!^n}{n^{n^2}}\right)^{\frac{1}{n}}$$

$$= \lim_{n\to\infty} \frac{n!}{n^n}$$

Here, we have,

$$0 \leq \frac{n!}{n^n}$$

and.

$$\frac{n!}{n} = \frac{n}{n} \cdot \frac{(n-1)}{n} \cdot \frac{(n-2)}{n} \cdots \frac{2}{n} \cdot \frac{1}{n}$$

$$\leq 1 \cdot 1 \cdot 1 \cdots 1 \cdot \frac{1}{n}$$

So, we can say,

$$0 \leq \frac{n!}{n^n} \leq \frac{1}{n}$$

or, $\lim_{n\to\infty} 0 \leq \lim_{n\to\infty} \frac{n!}{n^n} \leq \lim_{n\to\infty} 0$

By Sandwich theorem, $\lim_{n\to\infty} \frac{n!}{n^n}$

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$$= \lim_{n\to\infty} \sqrt[n]{a^n} = \lim_{n\to\infty} \left(\frac{n!}{n^n}\right)^{1/n}$$

$$= \lim_{n\to\infty} \frac{n!^{\frac{1}{n}}}{n}$$

$$= \cancel{\lim_{n\to\infty}} \; 0 < 1$$

This series is convergent.

<4>: (Multi-system and Row rank)

a): Solve the following system of linear equations:

i): $x_1 + 2x_2 - 3x_3 = -1$
$3x_1 + 5x_2 + 3x_3 = 2$
$-5x_1 - 4x_2 + 8x_3 = 51$

Sol^n:

The augmented matrix is

$$\sim \left[\begin{array}{ccc:c} 1 & 2 & -3 & -1 \\ 3 & 5 & 3 & 2 \\ -5 & -4 & 8 & 51 \end{array}\right]$$

$[\because R_2 \to R_2 - 3R_1 \quad , \quad R_3 \to R_3 + 5R_1]$

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$$\sim \left[\begin{array}{ccc:c} 1 & 2 & -3 & -1 \\ 0 & -1 & 12 & 5 \\ 0 & 6 & -7 & 46 \end{array}\right]$$

$\because R_3 \to R_3 + 6R_2$.

$$\sim \left[\begin{array}{ccc:c} 1 & 2 & -3 & -1 \\ 0 & -1 & 12 & 5 \\ 0 & 0 & 65 & 76 \end{array}\right]$$

Here,

$$x_1 + 2x_2 - 3x_3 = -1 \quad \text{—— (i)}$$
$$-x_2 + 12x_3 = 5 \quad \text{—— (ii)}$$
$$65x_3 = 76 \quad \text{—— (iii)}.$$

From eq^n (iii);

$$x_3 = \frac{76}{65}$$

From eq^n (ii).

$$-x_2 + 12 \times \frac{76}{65} = 5$$

$$\therefore x_2 = 587/65$$

From eq^n (i);

$$x_1 + 2 \times \frac{587}{65} - 3 \times \frac{76}{65} = -1$$

$$\therefore x_1 = \frac{-1011}{65}$$

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(ii) $x_1 + 2x_2 - 3x_3 = 2$
$3x_1 + 5x_2 + 3x_3 = -3$
$-5x_1 - 4x_2 + 8x_3 = -16$

Soln:

The augmented matrix is:

$$\left[\begin{array}{ccc:c} 1 & 2 & -3 & 2 \\ 3 & 5 & 3 & -3 \\ -5 & -4 & 8 & -16 \end{array}\right]$$

$\because R_2 \to R_2 - 3R_1$ and $R_3 \to R_3 + 5R_1$.

$$\sim \left[\begin{array}{ccc:c} 1 & 2 & -3 & 2 \\ 0 & -1 & 12 & -9 \\ 0 & 6 & -7 & -6 \end{array}\right]$$

$\because R_3 \to R_3 + 6R_2$

$$\sim \left[\begin{array}{ccc:c} 1 & 2 & -3 & 2 \\ 0 & -1 & 12 & -9 \\ 0 & 0 & 65 & -60 \end{array}\right]$$

Here,

$x_1 + 2x_2 - 3x_3 = 2$ — (i)
$-x_2 + 12x_3 = -9$ — (ii)
$65x_3 = -60$ — (iii)

From eqn (iii), $x_3 = \dfrac{-60}{65} = \dfrac{-12}{13}$

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$$-x_2 + 12x \times \frac{-12}{13} = -9$$

$$\therefore x_2 = -27/13$$

from eq^n (i),

$$x_1 + 2 \times \frac{(-27)}{13} - 3 \times \left(\frac{-12}{13}\right) = 2$$

$$\therefore x_1 = \frac{49}{13}$$

b) Define row rank of matrix. Find the rank of coefficient and augmented matrix and check consistency of the given system of equations.

i):
$$2x - 6y + 8z = 2$$
$$-4x + 13y + 3z = 6$$
$$-6x + 20y + 14z = -2$$

Sol^n:

{∵ The maximum number of linearly independent rows of a matrix is called row rank of a matrix.}

The augmented matrix is.

$$\sim \left[\begin{array}{ccc:c} 2 & -6 & 8 & 2 \\ -4 & 13 & 3 & 6 \\ -6 & 20 & 14 & -2 \end{array}\right] \underset{R_1 \to \frac{1}{2}R_1}{\sim} \left[\begin{array}{ccc:c} 1 & -3 & 4 & 1 \\ -4 & 13 & 3 & 6 \\ -6 & 20 & 14 & -2 \end{array}\right]$$

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$\because R_2 \to R_2 + 2R_1 \qquad R_3 \to R_3 + 63 R_1$

$$\sim \begin{bmatrix} 1 & -3 & 4 & : & 1 \\ 0 & 1 & 19 & : & 10 \\ 0 & 2 & 38 & : & 4 \end{bmatrix}$$

$\because R_3 \to \frac{1}{2} R_3.$

$$\sim \begin{bmatrix} 1 & -3 & 4 & : & 1 \\ 0 & 1 & 19 & : & 10 \\ 0 & 1 & 19 & : & 2 \end{bmatrix}$$

$\because R_3 \to R_3 - R_2$

$$\sim \begin{bmatrix} 1 & -3 & 4 & : & 1 \\ 0 & 1 & 19 & : & 10 \\ 0 & 0 & 0 & : & -8 \end{bmatrix}$$

Here, rank of coefficient matrix $\neq$ rank of augmented matrix.
This is inconsistency.

Rank of coefficient matrix = 2
Rank of augmented matrix = 3

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ii): $2x - 2y + 4z + 6w = 8$

$-4x + 5y - 2z - 7w = -10$

$2x + y + 22z + 21w = 36$

$-3x + 5y - 4z + 11w = 10$

Sol^n:

The augmented matrix is.

$$\sim \left[\begin{array}{cccc:c} 2 & -2 & 4 & 6 & 8 \\ -4 & 5 & -2 & -7 & -10 \\ 2 & 1 & 22 & 21 & 36 \\ -3 & 5 & -4 & 11 & 10 \end{array}\right]$$

$\because R_1 \to \frac{1}{2} R_1$

$$\sim \left[\begin{array}{cccc:c} 1 & -1 & 2 & 3 & 4 \\ -4 & 5 & -2 & -7 & -10 \\ 2 & 1 & 22 & 21 & 36 \\ -3 & 5 & -4 & 11 & 10 \end{array}\right]$$

$R_2 \to R_2 + 4R_1$ $\quad$ $R_3 \to R_3 - 2R_1$ $\quad$ $R_4 \to R_4 + 3R_1$

$$\sim \left[\begin{array}{cccc:c} 1 & -1 & 2 & 3 & 4 \\ 0 & 1 & 6 & 5 & 6 \\ 0 & 3 & 18 & 15 & 28 \\ 0 & 2 & 2 & 20 & 22 \end{array}\right]$$

$R_3 \to R_3 - 3R_2$ $\quad$ $R_4 \to R_4 - 2R_2$

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$$\sim \left[\begin{array}{cccc:c} 1 & -1 & 2 & 3 & 4 \\ 0 & 1 & 6 & 5 & 6 \\ 0 & 0 & 0 & 0 & 10 \\ 0 & 0 & -10 & -10 & 10 \end{array}\right]$$

Here, rank of coefficient matrix $\neq$ rank of augmented matrix.

$$3 \neq 4$$

$\therefore$ This is not consistent.

Q (c). Obtain row rank and parametrically represented solution of the system. Write down two specific solⁿ:

(i):
$$-4x + 12y - 7z = 8$$
$$x - 3y + 2z = -1$$

Solⁿ:

The augmented matrix is.

$$\left[\begin{array}{ccc:c} -4 & 12 & -7 & 8 \\ 1 & -3 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$[\because R_1 = -\frac{1}{4} R_1]$

$$\sim \left[\begin{array}{ccc:c} 1 & -3 & 7/4 & -2 \\ 1 & -3 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$R_2 \to R_2 - R_1$

$$\sim \left[\begin{array}{cccc} \cancel{1} & \cancel{-3} & \cancel{7/4} & \vdots\ \cancel{-2} \\ \cancel{0} & \cancel{0} & \cancel{1/4} & \cancel{1} \\ \cancel{0} & \cancel{0} & \cancel{0} & \vdots\ \cancel{0} \end{array}\right]$$

Here,

$$\sim \left[\begin{array}{cccc} 1 & -3 & 7/4 & \vdots\ -2 \\ 0 & 0 & 1/4 & \vdots\ 1 \\ 0 & 0 & 0 & \vdots\ 0 \end{array}\right]$$

Here,

$$x - 3y + 7/4\,z = -2 \quad \text{—(i)}$$

$$\frac{1}{4} z = 1 \quad \text{(ii)} \qquad \therefore z = 1/4$$

$$0 \cdot y = 0$$

Here,

y is free variable.

Let $y = r$.

So, $z = 1/4$

$x - 3r + 7 = -2$

or, $x - 3r = -9$

If $r = 0$,

$y = 0$.

$x = -9$

$(-9, 0, 1/4)$

If $r = 1$,

$y = 1$

$x - 3 \times 1 = -9$

$\therefore x = -6$

$(-6, 1, 1/4)$

Here, row rank = 2

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ii) $3x_1 - x_2 + 3x_4 = 5$

$x_1 + 2x_2 - 3x_4 = -1$

$2x_1 + 5x_2 + 4x_3 + 2x_4 = 10$

Sol^n.

The augmented matrix is.

$$\begin{bmatrix} 3 & -1 & 0 & 3 & : & 5 \\ 1 & 2 & 0 & -3 & : & -1 \\ 2 & 5 & 4 & 2 & : & 10 \\ 0 & 0 & 0 & 0 & : & 0 \end{bmatrix}$$

$R_1 \rightarrow 1/3 R_1$

$$\sim \begin{bmatrix} 1 & -1/3 & 0 & 1 & : & 5/3 \\ 1 & 2 & 0 & -3 & : & -1 \\ 2 & 5 & 4 & 2 & : & 10 \\ 0 & 0 & 0 & 0 & : & 0 \end{bmatrix}$$

$R_2 \rightarrow R_2 - R_1$ $\quad R_3 \rightarrow R_3 - 2R_1$

$$\sim \begin{bmatrix} 1 & -1/3 & 0 & 1 & : & 5/3 \\ 0 & 7/3 & 0 & -4 & : & -8/3 \\ 0 & 17/3 & 4 & 0 & : & 20/3 \\ 0 & 0 & 0 & 0 & : & 0 \end{bmatrix}$$

$R_3 \rightarrow 1/4 R_3$

$$\sim \begin{bmatrix} 1 & -1/3 & 0 & 1 & : & 5/3 \\ 0 & 7/3 & 0 & -4 & : & -8/3 \\ 0 & 17/12 & 1 & 0 & : & 5/3 \\ 0 & 0 & 0 & 0 & : & 0 \end{bmatrix}$$

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$R_2 \to \frac{3}{7}R_2$

$$\sim \begin{bmatrix} 1 & -1/3 & 0 & 1 & : & 5/3 \\ 0 & 1 & 0 & -12/7 & : & -8/7 \\ 0 & 17/12 & 1 & 0 & : & 5/3 \\ 0 & 0 & 0 & 0 & : & 0 \end{bmatrix} \quad (RR = 3)$$

$R_3 \to R_3 - 17/12\, R_2$

$$\sim \begin{bmatrix} 1 & -1/3 & 0 & 1 & : & 5/3 \\ 0 & 1 & 0 & -12/7 & : & -8/7 \\ 0 & 0 & 1 & 17/7 & : & 5/3 \\ 0 & 0 & 0 & 0 & : & 0 \end{bmatrix}$$

Here,

x_4 is a free variable.

So,

$x_3 + 17/7\, x_4 = 5/3$

$x_2 - 12/7\, x_4 = -8/7$

$x_1 - 1/3\, x_2 + x_4 = 5/3$

The parametric form of solution is.

Let $x_4 = r$.

So,

$x_3 + \frac{17}{7} r = \frac{5}{3}$ $\therefore r = \frac{5}{3} - \frac{17}{7} r$

$x_2 = -\frac{8}{7} + \frac{12}{7} r$

$x_1 = \frac{5}{3} - r + \frac{1}{3}\left(-\frac{8}{7} + \frac{12}{7}\right) r$