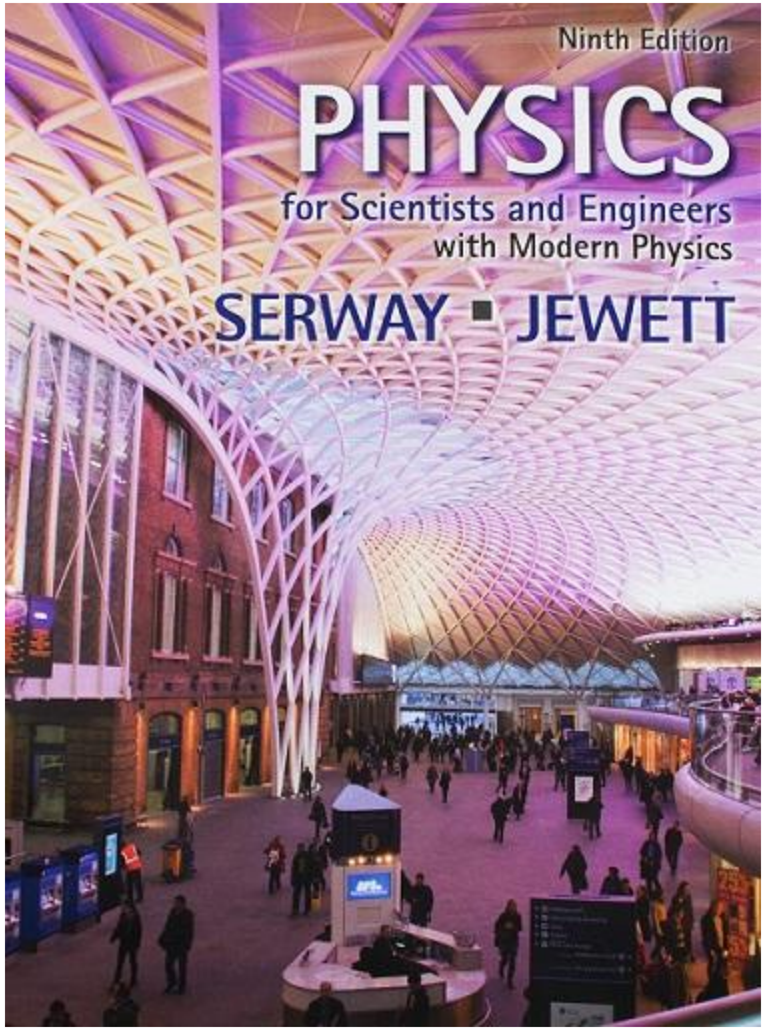
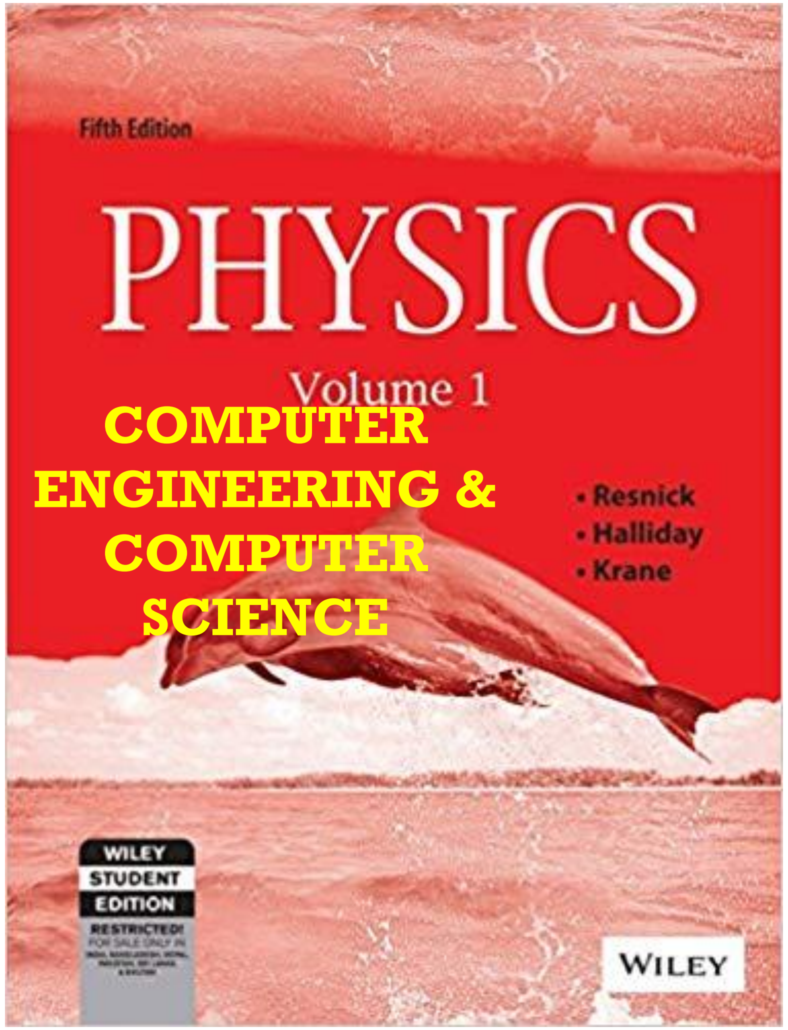
PHYSICS

General Physics (PHYS 104)





COMPUTER ENGINEERING & COMPUTER SCIENCE

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Course Outline

- Work Done by a Constant Force
- Work Done by Variable Force
- Work Done by the Spring Force

Work Done by a Constant Force

Work

- **Work** is energy transferred to or from an object by means of force acting on the object.
- Work is a scalar quantity.
- The SI unit of work is the joule (J).

Work Done by a Constant Force

- When a constant force $\vec{F}$ acts on an object that undergoes a straight-line displacement $\vec{S}$, the work done by the force on the object is defined to be the scalar product of $\vec{F}$ and $\vec{s}$.
- Work done by a constant force :

$$\boxed{W = \vec{F}\cdot\vec{s} = Fs\cos\phi}$$ where ϕ is the angle between $\vec{F}$ and $\vec{s}$

  - If F = 0, then W = 0. For work to be done, a force must be exerted.
  - If s = 0, then W = 0. For work to be done by a force, there must be movement of the point of application of that force through some distance.
  - If $\phi = 90°$, then W = 0. For work to be done by a force, a component of the force must act in the direction of the displacement (or in the opposite direction). If a force is always perpendicular to the direction of motion, then the work done by that particular force is zero.
  - When $\phi = 0°$, then W = F s. If the force and the displacement are in the same direction.
  - When $\phi = 180°$, then W = –Fs. If the force acts opposite to the direction of the displacement, then that force does negative work.

Work Done by a Constant Force

Nature of Work

The work done will be **positive, zero or negative** depending upon the angle between

Positive Work

- If $0 \le \phi < 90^0$, the work done on an object by a force is positive.

  Example of positive work:

  When an object falls freely under gravity, the work done by gravitational force on the object is positive.

Negative Work

- If $90^0 < \phi \le 180^0$, the work done on an object by a force is negative .

  Example of negative work:

  When a body is made slide over a rough surface, the work done by the frictional force on the object is negative.

Zero Work

- If $\phi = 90^0$, then W= 0 , $F = 0$, then W= 0 , $s = 0$, then W= 0.

  If a force is always perpendicular to the direction of motion, then the work done by that particular force is zero.

  For work to be done, a force must be exerted.

  For work to be done by a force there must be movement of the point of application of that force through some distance.

Example of zero work:

Work done by the force of gravity on a body moving on the horizontal surface.

Work Done by a Variable Force

Variable Force

- A variable force is a force which changes in magnitude or direction as a body moves. e.g. The spring force

Work Done by a Variable Force, Straight – Line Motion

- Let a body moves along the x-axis from x_i to x_f as a variable force $F_x(x)$ is applied to it [Figure F_V-1].

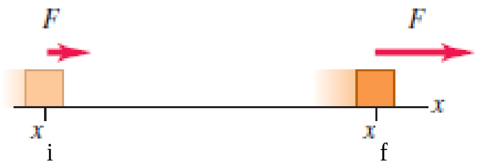


Figure Fv-1

- The smooth curve in Figure F_V-2 shows an arbitrary variable force that acts on a body that moves from x_i to x_f.

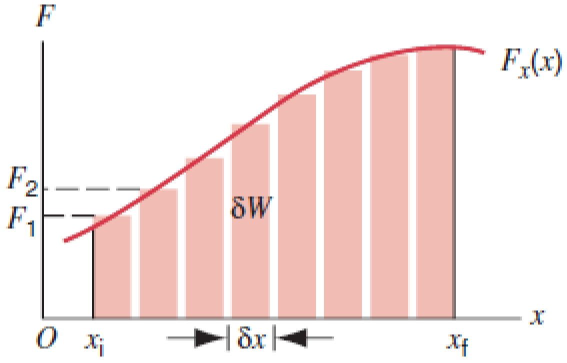


Figure Fv-2

$F_x(x) \rightarrow$ force that varies in magnitude only

$F_1,\ F_2 \rightarrow$ nearly constant forces in the first and second interval respectively

Let us divide the total displacement into a large number of very small intervals of equal width , such that in each interval force can be considered to be constant.

DYNAMICS OF SYSTEM OF PARTICLES

Work Done by a Variable Force

Work Done by a Variable Force, Straight – Line Motion

- Consider the first interval in which there is a small displacement δx from x_i to $x_i + \delta x$.

  The work done by the force F_1 in that interval is $\delta W_1 = F_1 \ \delta x$.

  Similarly, in the second interval, in which the body moves from $x_i + \delta x$ to $x_i + 2\delta x$.

  The work done by the force F_2 in that interval is $\delta W_2 = F_2 \ \delta x$.

Therefore, the total work done by the force in the total displacement from x_i to x_f is approximately

$$W = \delta W_1 + \delta W_2 + \ldots$$

$$= F_1 \ \delta x + F_2 \ \delta x + \ldots$$

$$\text{or} \quad W = \sum_{n=1}^{N} F_n \ \delta x \qquad \ldots\ldots\ldots\ldots \ (1)$$

When the number of intervals becomes infinite and the width of each interval tends to zero, we get the exact value of work done.

Hence,
$$\boxed{W = \lim_{\delta x \to 0} \sum_{n=1}^{N} F_n \ \delta x \qquad \ldots\ldots\ldots\ldots \ (2)}$$

- The relation $\lim_{\delta x \to 0} \sum_{n=1}^{N} F_n \ \delta x = \int_{x_i}^{x_f} F_x(x) \, dx$ defines the integral $F_x(x)$ of with respect to x from x_i to x_f .

Numerically, this quantity is exactly equal to the area between the force curve and the axis between the limits x_i to x_f .

Work Done by a Variable Force, Straight – Line Motion

- Hence, the total work done by $F_x(x)$ in displacing a body from x_i to x_f is

$$W = \int_{x_i}^{x_f} F_x(x)\, dx$$

The shaded area between the force curve and the x-axis between the limits x_i to x_f [Figure F_V-3] gives the total work done by $F_x(x)$ in displacing a body from x_i to x_f .

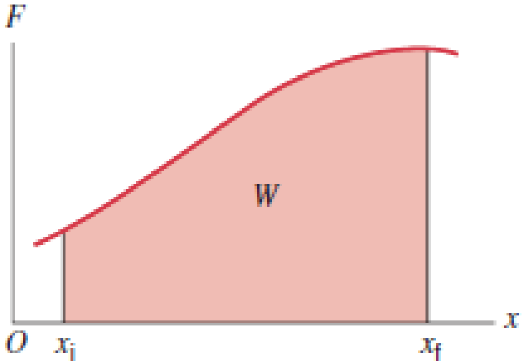


Figure Fv-3

Three Dimensional Analyses

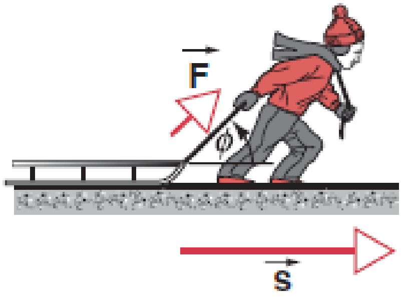
- The work done by a variable force $\vec{F}$ while the particle moves from an initial position $\vec{r}_i$ having coordinates (x_i, y_i, z_i) to a final position $\vec{r}_f$ having coordinates (x_f, y_f, z_f) is

$$W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{r} = \int_{\vec{r}_i}^{\vec{r}_f} \left(F_x dx + F_y dy + F_z dz\right)$$

$$\therefore \quad W = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \int_{z_i}^{z_f} F_z dz$$

Sample Problem

- **A child pulls a 5.6 kg sled a distance of s =12 m along a horizontal surface at a constant speed. What work does the child do on the sled if the coefficient of kinetic friction μ_k is 0.20 and the rope makes an angle $\phi = 45^0$ with the horizontal?**



- According to Newton's First Law

$$\sum F_x = 0 \qquad \text{and} \qquad \sum F_y = 0$$

or, $F\cos\phi = f$ — or, $N + F\sin\phi = mg$

$\therefore \boxed{F\cos\phi = \mu_k N}$ .... (1) $\quad \therefore \boxed{N = mg - F\sin\phi}$ .... (2)

- From equations (1) and (2), we get

$$F\cos\phi = \mu_k\left(mg - F\sin\phi\right)$$

$$\therefore \boxed{F = \frac{\mu_k mg}{\cos\phi + \mu_k \sin\phi}} \quad \text{.......... (3)}$$

- Therefore, work done by the child on the sled,

$$W = \vec{F}\cdot\vec{s} = Fs\cos\phi = \left[\frac{\mu_k mg}{\cos\phi + \mu_k\sin\phi}\right] s\ \cos\phi$$

$$= \left[\frac{0.20\times 5.6\times 9.8}{\cos45^0 + 0.20\sin45^0}\right]\times 12\times\ \cos45^0$$

$$= 110\ \text{J}$$

Hint:

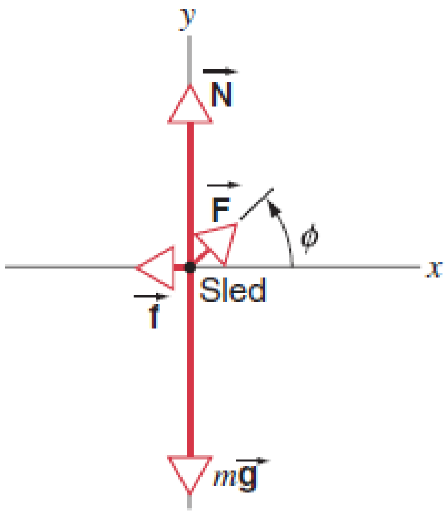


Figure SP-1
A Free-body diagram for the sled

Sample Problem

A 45.36-kg block of ice slides down an incline 1.52 m long and 0.9144 m high. A man pushes up on the ice parallel to the incline so that it slides down at constant speed. The coefficient of friction between the ice and the incline is 0.1.

Find (a) the force exerted by the man,
(b) the work done by the man on the block,
(c) the work done by the gravity on the block,
(d) the work done by the surface of the incline on the block,
(e) the work done by the resultant force on the block,
and (f) the change in kinetic energy of the block.

Hint:

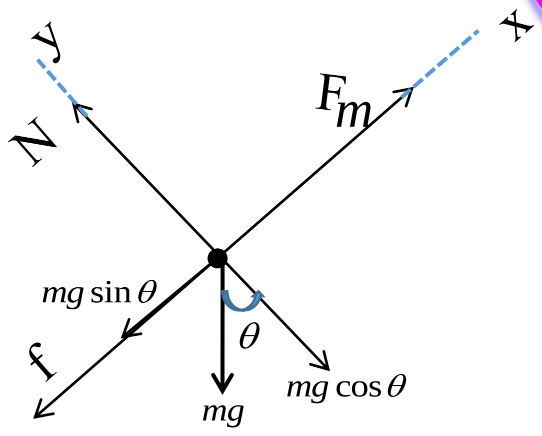


Figure SP-1
A Free-body diagram for the block of ice

According to Newton's First Law

$$\sum F_x = 0 \qquad \text{and} \qquad \sum F_y = 0$$

or, $F_m + f = mg\sin\theta$

$\therefore \boxed{F_m = mg\sin\theta - f}$ .... (1)

$\therefore \boxed{N = mg\cos\theta}$ .... (2)

(a) The force exerted by the man,

$$F_m = mg\sin\theta - f$$
$$= mg\sin\theta - \mu_k(mg\cos\theta)$$
$$= \ldots$$
$$= 231.15\ \text{N}$$

(b) The work done by the man on the block,

$$W_m = \vec{F}_m \cdot \vec{s}$$
$$= -F_m s$$
$$= \ldots$$
$$= -352.56\ \text{J}$$

DYNAMICS OF SYSTEM OF PARTICLES

Sample Problem

A 45.36-kg block of ice slides down an incline 1.52 m long and 0.9144 m high. A man pushes up on the ice parallel to the incline so that it slides down at constant speed. The coefficient of friction between the ice and the incline is 0.1.

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(b) the work done by the man on the block,
(c) the work done by the gravity on the block,
(d) the work done by the surface of the incline on the block,
(e) the work done by the resultant force on the block,
and (f) the change in kinetic energy of the block.

Hint:

(c) The work done by the gravity on the block

$$\begin{aligned} W_m &= \vec{F}_g \cdot \vec{s} \\ &= F_g s \cos(90^0 - \theta) \\ &= mg\, s \sin\theta \\ &= \ldots \\ &= 406.8 \text{ J} \end{aligned}$$

(d) The work done by the surface of the incline on the block

$$\begin{aligned} W_m &= \vec{f} \cdot \vec{s} \\ &= -f\, s \\ &= \mu_k N s \\ &= \mu_k (mg \cos\theta) s \\ &= -54.24 \text{ J} \end{aligned}$$

(e) The work done by the resultant force on the block

$$W_{F_R} = \vec{F}_R \cdot \vec{s} = 0$$

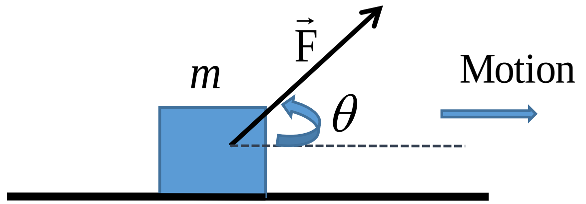
(f) The change in kinetic energy of the block,

$$W_{F_R} = \Delta K = 0$$

Sample Problem

A block of mass m = 3.57 kg is drawn at a constant speed a distance d = 4.06 meters along a horizontal floor by rope exerting a constant force of magnitude F = 7.68 N making an angle θ = 15° with the horizontal.

Compute (a) the total work done on the block,
(b) the work done by the rope on the block,
(c) the work done by the friction on the block,
(d) the coefficient of kinetic friction between the block and floor.



Hint:

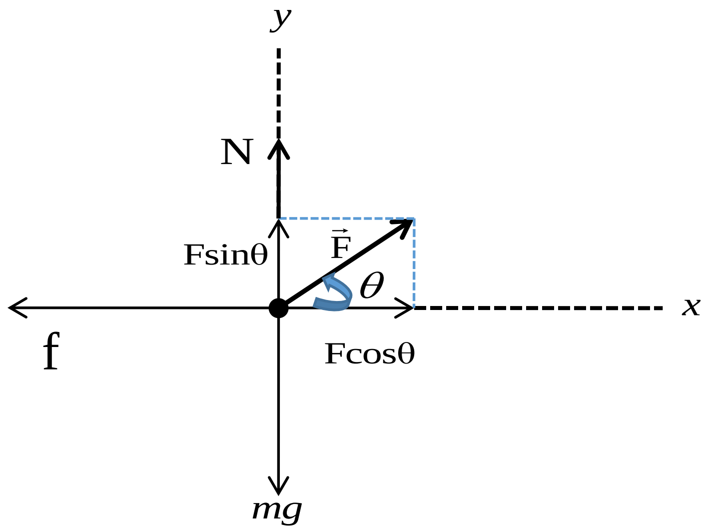


Figure SP-1
A Free-body diagram for the block

According to Newton's First Law

$$\sum F_x = 0 \quad \text{and} \quad \sum F_y = 0$$

$$\therefore \boxed{F\cos\phi = f} \quad \ldots\ldots\ldots\ (1) \qquad \therefore \boxed{N + F\sin\phi = mg} \quad \ldots\ldots\ldots\ (2)$$

(a) $$W_T = W_T + W_N + W_g + W_f = F\cos\phi\, d + 0 + 0 + \left(-F\cos\phi\, d\right) = 0$$

(b) $$W_F = \vec{F}\cdot\vec{d} = Fd\cos\phi = \ldots = 30.1\text{ J}$$

(c) $$W_F = \vec{F}\cdot\vec{d} = -Fd\cos\phi = \ldots = -30.1\text{ J}$$

(d) $$\mu_k = \frac{f}{N} = \frac{F\cos\phi}{mg - F\sin\phi} = \ldots = 0.225$$

Calculating Total Work Done From a Fraph

- **A force acting on a particle varies with x, as shown in Figure W-1. Calculate the work done by the force as the particle as it moves from** $x_i = 0$ to $x_f = 6.0$ m **.**

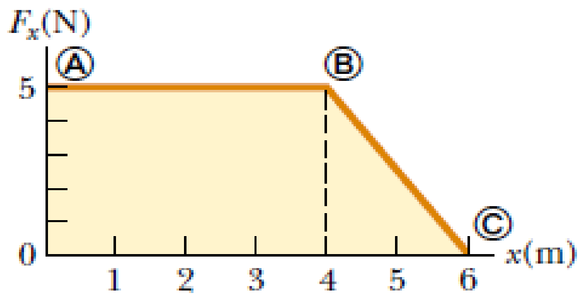


Figure W-1

Solution:

- The work done by the force is equal to the area under the curve from $x_i = 0$ to $x_f = 6.0$ m.

$$\therefore \quad W = (5.0N)(4.0\text{ m}) + \frac{1}{2}(5.0N)(2.0\text{ m})$$

$$= 20J + 5.0J$$

$$= 25J$$

- **The force acting on a particle varies as shown in Figure W-2. Find the work done by the force as the particle moves from** $x_i = 0$ to $x_f = 10.0$ m

- **Solution:**

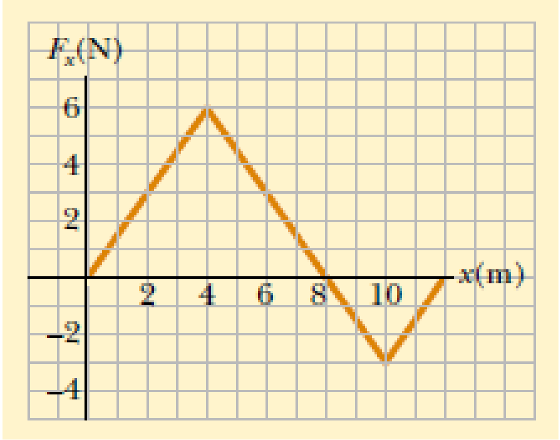


Figure W-2

- The work done by the force as the particle moves from $x_i = 0$ to $x_f = 10.0$ m is equal to the area under the curve from $x_i = 0$ to $x_f = 10.0$ m

$$\because \quad W = \frac{1}{2}(4.0N)(6.0\text{ m}) + \frac{1}{2}(4.0N)(6.0\text{ m}) - \frac{1}{2}(2.0N)(3.0\text{ m})$$

$$= 12J + 12J - 3J$$

$$= 21J$$

Spring Force

Spring Force

- The force exerted by a spring when it is stretched or compressed is called ***a spring force***.
- A spring force is an example of a one-dimensional variable force.
- The spring force is sometimes called *a restoring force*, because it acts to restore the relaxed state.

Figure S-1 shows a block attached to a spring.

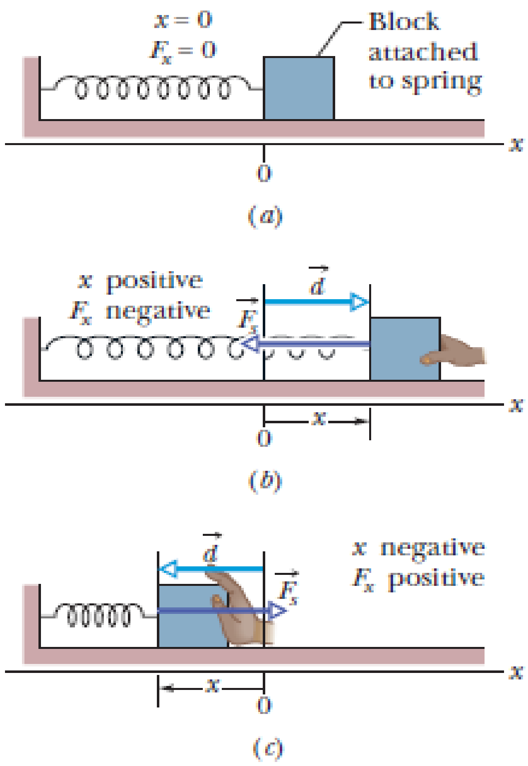


Figure S-1

- Figure S-1a *shows* a spring in its **relaxed state.**
- If we stretch the spring by pulling the block to the right as in Figure S-1b, the spring pulls on the block toward the left.
- If we compress the spring by pushing the block to the left as in Figure S-1c, the spring now pushes on the block toward the right.

Spring Force and Spring Constant

Spring Force

- To a good approximation for many springs, the force from a spring $\vec{F}_s$ is proportional to the displacement $\vec{d}$ of the free end from its position when the spring is in the relaxed state.
- The ***spring force*** is given by

$$\vec{F}_s = -k\vec{d} \qquad (\text{Hooke's law}) \qquad \ldots\ldots\ldots. \qquad (1)$$

$$F_s = -kx$$

which is known as **Hooke's law** after Robert Hooke, an English scientist of the late 1600s.

- The minus sign in Eq. (1) indicates that the direction of the spring force is always opposite the direction of the displacement of the spring's free end

Spring Constant (k)

- It is a measure of the stiffness of the spring. The larger k is, the stiffer the spring; that is, the larger k is, the stronger the spring's pull or push for a given displacement.
- The SI unit for k is the newton per meter.

The Work Done by the Spring Force

The Work Done by the Spring Force

- A common physical system for which the force varies with position is shown in Figure S-2.

  A block on a horizontal, frictionless surface is connected to a spring.

- If the spring is stretched a small distance from its unstretched (equilibrium) configuration by the applied force $\vec{F}_{app}$, it exerts on the block a force of magnitude

$$F_s = -kx \qquad \text{(Hooke's law)}$$

where x is the displacement of the block from its unstretched (x=0) position and k is a positive constant called the force constant of the spring.

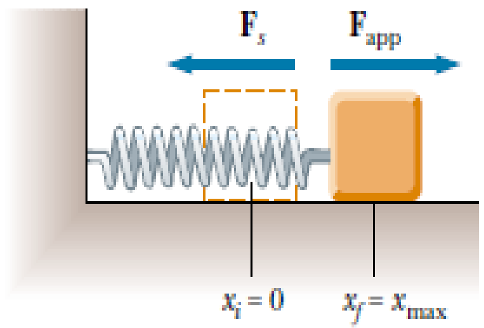


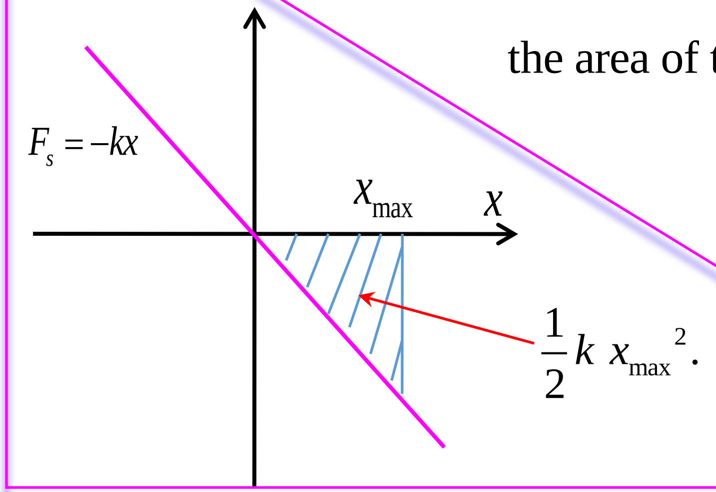
Figure S-2

The spring force F_s always acts in a direction to restore the block to its location at $x = 0$.

The work done by the spring force as the block moves from $x_i = 0$ to $x_f = x_{max}$ is

$$W_s = \int_0^{x_{max}} F_s dx = \int_0^{x_{max}} (-kx)\, dx = -\frac{1}{2} k\, x_{max}{}^2$$

The work done by the spring force as the block moves from $x_i = 0$ to $x_f = x_{max}$ is the area of the shaded triangle, $\frac{1}{2} k\, x_{max}{}^2$.



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Thank you