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CHAPTER 3: MASS TRANSFER

*) Law of Conservation of Mass:

Mass can neither be created nor ~~it~~ can be destroyed.

This law helps us to track pollutants.

Mass-Transfer Processes

In the bio-chemical or physical mass-transfer process, interphase diffusion occurs when a driving force is created.

In gas phase, the driving force is partial pressure gradient whereas in liquid or solid phase, the driving force is the concentration gradient.

Different ~~#~~ mass transfer processes depending upon the underlying ~~process~~ phases.

(i) Gas-liquid M.T: Supply of oxygen for decomposing wastes.
(ii) Liquid-Gas M.T: Methane released from anaerobic waste treatment.
(iii) Liquid-liquid M.T: Extracting organic solvents
(iv) Liquid-solid M.T: Adsorption of pollutants on activated carbon.
(v): Solid-gas M.T: Release of gases from solid wastes in landfill.

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x) Intensive Properties:

- They are bulk properties.
- They don't depend upon the amount of matter present.

Eg: density, colour, temperature, hardness, etc.

x) Extensive Properties:

- They depend upon amount of matter present.
- considered additive for ~~etc~~ subsystems.

Eg: volume, mass, size, length, etc.

Law of Conservation of Energy

Energy can neither be created nor be destroyed but can be converted to other forms of energy.

Using first law of thermodynamics, the flow of energy can be analyzed through energy balance equations. Mathematically,

$$\boxed{\text{change in energy of system} - \text{change in energy of surrounding} = 0}$$

Mass Balance

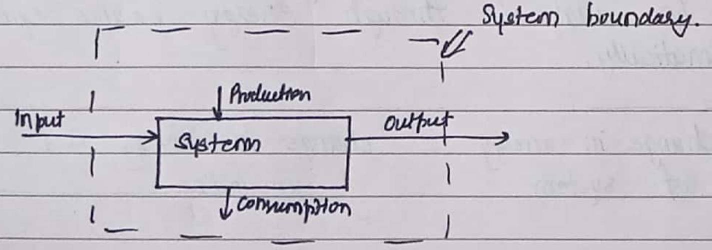
The concept of law of conservation of mass is useful to solve, analyze and describing environmental engineering problems. This concept is called mass balance.

x) Steps of Mass-Balance Analysis:

(i): Defining region of analysis. by use of 'Mass Balance Diagram.'

(ii) All input, output, production and consumption must be converted to same units.

(iii): After mentioning all sources, we draw a system boundary. to ease calculations and show no other effects.

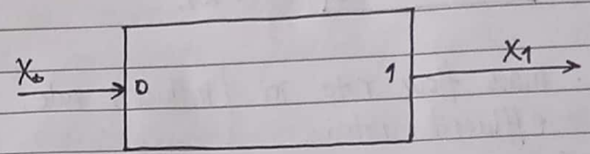


The mass balance equation is

Accumulation rate	=	Input rate	−	Output rate	+	Production rate	−	Consumption rate.

x) Material Balance with Single Material

Material flow is understood and analyzed by a black box.



All flows into the box is called influents.

X_0 = mass / time of X flowing into box.

All flows out of the box is called effluents.

X_1 = mass / time of X flowing out of the box.

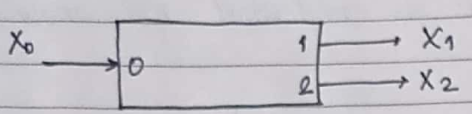
If no production or consumption in system and condition is steady,

Then,

$$X_0 = X_1.$$

We use generally volume ~~basics~~ balance for liquids and mass balance for solids.

x) Splitting Single-Material Flow streams:

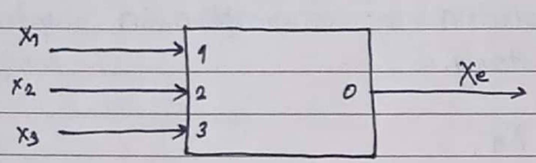


X_0 = ~~flow~~ . mass flow rate in / influent rate.
X_1 & X_2 = effluent rate.

Then,
If no production or consumption and steddy state conditions then,

$$X_0 = X_1 + X_2$$

x) Combining Single-Material Flow Streams:



In this case,

$$X_1 + X_2 + X_3 = X_e.$$

$$\left[\sum_{i=1}^{m} X_i\right] = X_e.$$

Conservative and Non-conservative Substances

Conservative substances are substances that dont convert physically or chemically to other substances in normal conditions. They tend to be stable, long-lived compounds persisting within the environment.

Non-conservative substances are the substances that are transformed to other substances through physical, ~~oter~~ chemical and biological, process ~~conditions occuring in the environment.~~ ~~and it depends rate of transformation~~ process in the environment and the rate of transformation depends upon the physical, chemical and biological conditions occuring in environment.

(x) Complex Processes with Single Material

If a system is considered, we use the assumption of steady state and conservative material.

We know the full material balance equation is,

Material accumulated per unit time = Material input per unit time − Material output per unit time + material produced per unit time − material consumed per unit time.

So, for mass balance equation,

Mass of A accumulated per unit time	=	Mass of A input per unit time	−	mass of A output per unit time	+	mass of A produced per unit time	−	Mass of A consumed per unit time.

If the density of the material doesn't change, the volume balance equation.

volume of A accumulated per unit time	=	volume of A input per unit time	−	volume of A output per unit time	+	volume of A produced per unit time	−	volume of A consumed per unit time.

In simple case, we can write the equation to be.

Accumulation rate = Input rate − Output rate + Production rate − Consumption rate.

Steady State Condition:

The characteristic of a ~~..~~ condition where its value changes negligibly over long period of time.
In this case,

$$\frac{dM}{dt} = 0.$$

ie. there is no accumulation in system.
So
rate of accumulation = 0.

Steady State System with Conservative Pollutant:

Steady state system means accumulation rate = 0 and.
for conservative pollutant, the transformation rate = 0.
Thus,

Input rate = Output rate.

Steady State Condition with Non-conservative Pollutant

In this case, the accumulate rate is 0.
~~&~~ Also, the production rate = 0 but the consumption/ decay rate exists.
Thus, the equation becomes.

Input rate = Output rate + Decay rate.

*) Calculation of decay rate:

The decay of non-conservative pollutant is frequently modelled as first-order reaction.
The rate of loss of substance is proportional to the amount of substance present at any given time.

$$\frac{dC}{dt} = -kC \quad \text{—(i)}$$

Here,
k = reaction rate coefficient
C = pollutant concentration.

or, $\frac{dC}{C} = -k \cdot dt$

Integrating,

$\ln \frac{C}{C_0} = -kt$

or $C = C_0 e^{-kt}$ —— (ii)

For a uniformly distributed system,

Mass = Concentration x Volume.

Then,

Decay rate $= \frac{dM}{dt} = \frac{d(CV)}{dt} = V \frac{dC}{dt}$

∴ decay rate = kCV.

C = concentration (uniform) throughout volume (V).

Thus, for ~~steda~~ steady state condition with non-conservative pollutant.

Input rate = Output rate + KCV.

The condition of uniform concentration is complete mixing assumption and such idealization is Continuously Stirred Tank Reactor (CSTR) model.

or,

Complete Mix Box Model.

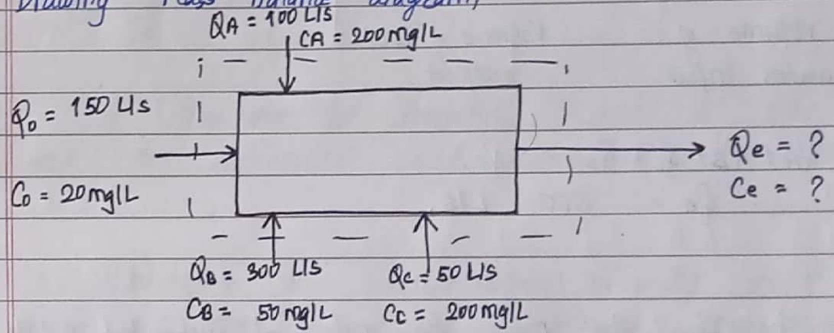
<Q>: A stream flowing at 150 L/s and 20 mg/L suspended solids, receives wastewater from three different sources:

Source	Quantity	Solid Conc:
A	100	200
B	300	50
C	50	200

What is the flow rate and suspended solid concentration at sampling point?

Soln:

Drawing Mass-Balance diagram,



Let suspended solid = SS.

We know,

~~Accumulatation~~

Accumulation rate of SS = Input rate of SS - Output rate of SS + Production rate of SS - Consumption rate of SS

Let us assume: i) Steady state condition

ii) Conservative Pollutant.

Then, the above eq^n ~~becomes~~ in volumetric rate condition,
i.e., balancing volumetric rate,

~~Volume rate~~

Volume of water accumulated = volume of water in − volume of water out + volume of water produced − volume of water consumed.

So, using the assumptions,

0 = Volume of water input − Volume of water output + 0 − 0.

or Volume of water input = Volume of water output.

or, $Q_A + Q_B + Q_C + Q_0 = Q_e$.

$\therefore Q_e = 600$ L/s.

Now, balancing the mass flow rate of suspended solids,

Rate of SS accumulated = Rate of SS input − Rate of SS output + Rate of SS produced − Rate of SS consumed.

Let us assume: i) steady state condition
ii) conservative pollutant.

Then,

0 = rate of SS input − rate of SS output + 0 − 0

or rate of SS input = rate of SS output.

or, $Q_0C_0 + Q_AC_A + Q_BC_B + Q_CC_C = Q_sC_s$

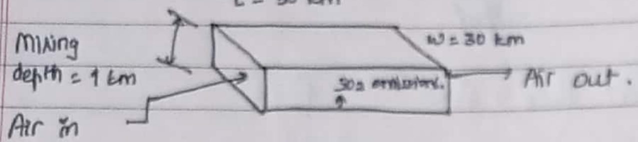
or $C_s = \dfrac{Q_0C_0 + Q_AC_A + Q_BC_B + Q_CC_C}{Q_0}$

$= \dfrac{(150 \times 20 + 100 \times 200 + 300 \times 50 + 50 \times 200)\ L/s \times mg/L}{600\ L/s}$

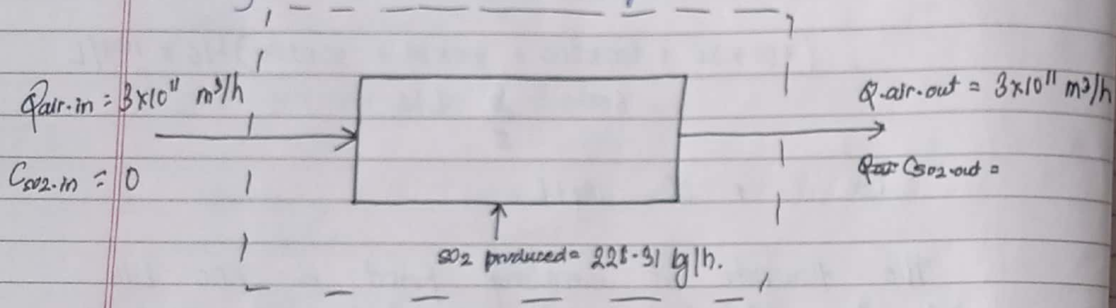
$\therefore C_s = 80$ mg/L

The flowrate at sampling point is 600 L/s and the suspended solid concentration is 80 mg/L.

Q7: Estimate the concentration of SO_2 in the urban air ~~the~~ above KU, Dhulikhel. The mixing height above KU is 1 km. The length and width of the box is 50 km and 30 km. The average annual wind speed is 10000 m/h. There are 200 brick kilns in the area. SO_2 release rate is 0.2 kg/brick produced and annual brick production rate is 10 million bricks/year.



Drawing mass balance diagram:



Given,

Mixing depth (h) = 1 km = ~~100~~

Width of box (w) = 30 km $\quad v = 10000$ m/h

Length of box (L) = 50 km.

$\therefore$ The volume of air entering the box

$$(Q_{air.in}) = w \times h \times v$$

$$= 1\,km \times \cancel{10000\,m}\ \frac{10^3 m}{1\,km} \times 30 km \times \frac{10^3 m}{1 km} \times 10^4 \frac{m}{h}$$

$$\therefore Q_{air.in} = 3\times10^{11}\ m^3/h$$

Concentration of SO_2 produced = ~~0.2~~

$$= 0.2 \frac{kg}{brick} \times 10^7 \frac{bricks}{year} \times \frac{1\ year}{365\ days} \times \frac{1\ day}{24\ hr}$$

$$= 228.31\ kg/h$$

Let us assume 0 kg/h of SO_2 entering into the box.

Let us assume: i) steady state condition
ii) conservative pollutant.

Balancing the volume flow rate:

~~Rate~~ volume of air ~~SO_2~~ accumulated = volume of air ~~SO_2~~ input − volume of air ~~SO_2~~ output + volume of air ~~SO_2~~ produced − volume of air consumed

$$\therefore Q_{air.in} = Q_{air.out} = 3\times10^{11}\ m^3/h.$$

Balancing the mass flow rate of SO_2.

Rate of SO_2 accumulated = Rate of SO_2 input − Rate of SO_2 output + Rate of SO_2 produced − rate of SO_2 consumed.

From given conditions and above assumptions,

0 = rate of SO_2 ~~accumulat~~ input − rate of SO_2 output + rate of SO_2 produced − 0.

or $0 = Q_{air\text{-}in} \times C_{SO_2\text{-}in} - Q_{air\text{-}out} \times C_{SO_2\text{-}out} + 228.31 \text{ kg/h}$

or, $Q_{air\text{-}out} \times C_{SO_2\text{-}out} = 228.31 \text{ kg/h}$

or $C_{SO_2\text{-}out} = \dfrac{228.31 \text{ kg/h}}{3 \times 10^{11} \text{ m}^3/\text{h}}$

$= 228.31 \text{ kg} \times \dfrac{10^9 \mu g}{1 \text{ kg}} \times \dfrac{1}{3 \times 10^{11} \text{ m}^3}$

$\therefore C_{SO_2\text{-}out} = 0.761 \ \mu g/m^3$

The SO_2 concentration in KV is $0.761 \ \mu g/m^3$.

<Q7: A lake has a surface area of $2.6 \times 10^6 m^2$. The average depth is 12 m. The lake is fed by a stream having flow rate of $1.2 m^3/s$ and phosphorus concentration of 0.045 mg/L. Runoff from homes along lake adds 2.6 g/s. The degradation rate of the lake is 0.36 /day. A river flow from the lake at a flowrate of $1.2 m^3/s$. What is the steady state concentration of phosphorus in lake.

Soln:

Given,

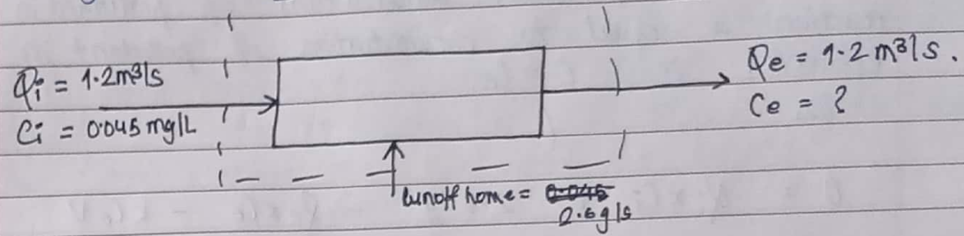
surface area (A) = $2.6 \times 10^6 m^2$

Depth (h) = 12 m

$\therefore$ Volume of water = $A \times h = 2.6 \times 10^6 \times 12$

$= 3.12 \times 10^7 m^3$

Drawing mass body diagram,



According to question, the river fed into lake has flown out of the lake maintaining constant flow rate.

and degradation rate of phosphorus (k) = 0.36 /day.

For mass flow rate,

rate of P accumulated = rate of P input − rate of P output + rate of P produced − rate of P consumed.

Given is steady state condition.

Let us assume i) no production of phosphorus. Then,

0 = rate of P input − rate of P output + 0 − rate of P consumed.

or, $0 = Q_i \times C_i - Q_e \times C_e - KCV + 2.6 \frac{g}{s}$

Let us assumed the lake is completely mixed. In such condition of CSTR, concentration of pollutant in reaction is equal to concentration of pollutant in effluent. ie, $C = C_e$.

So,

$$0 = Q_i \times C_i + 2.6\ \frac{g}{s} - Q_e \times C_e - k C_e V$$

or, $$Q_e C_e + K C_e V = Q_i C_i + 2.6\ \frac{g}{s}$$

or, $$1.2\ \frac{m^3}{s} \times 0.045\ \frac{mg}{L} + 2.6\ \frac{g}{s} = \left(1.2\ \frac{m^3}{s} + \frac{0.36}{day} \times 3.12\times10^7\ m^3\right) C_e$$

or, $$1.2\ \frac{m^3}{s} \times 0.045\ \frac{mg}{L} \times \frac{10^3\ L}{1\ m^3} + 2.6\ \frac{g}{s} \times \frac{10^3\ mg}{1\ g} = \left(1.2\ \frac{m^3}{s} + \frac{0.36}{day} \times \frac{1\ day}{86400\ s} \times 3.12\times10^7\ m^3\right) C_e$$

or, $$54\ \frac{mg}{s} + 2600\ \frac{mg}{s} = C_e \left(1.2\ \frac{m^3}{s} + 130\ \frac{m^3}{s}\right)$$

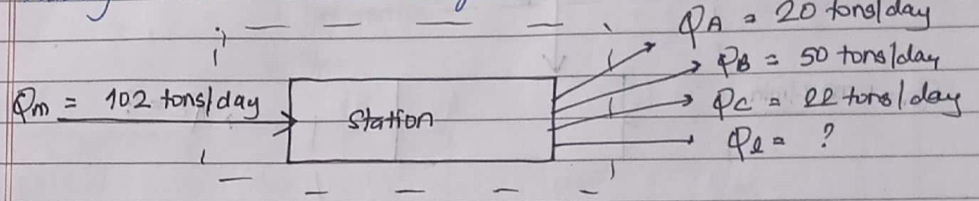
$$C_e = \frac{2654\ \frac{mg}{s}}{131.2\ \frac{m^3}{s} \times \frac{10^3\ L}{1\ m^3}}$$

$$\therefore C_e = \cancel{0.02}\ 0.020\ mg/L$$

<Q7: A city generates 102 tons/day of refuse, all of which goes to the transfer station. At the transfer station, the refuse is split up into four flow streams headed for three Incinerators and one landfill. If the capacity of the incinerators is 20, 50, and 22 tons/day, how much refuse must go into the landfill?

Sol:

Drawing mass balance diagram,



Balancing the mass flow rate,

refuse accumulated rate = refuse input rate − refuse output rate + refuse production rate − refuse consumption rate.

Let us assume (i): steady state condition.

(ii) conservative refuse.

Then, the equation becomes,

0 = refuse input rate − refuse output rate + 0 + 0

or refuse input rate = refuse output rate.

$$Q_m = Q_A + Q_B + Q_C + Q_\ell$$

$$\therefore Q_\ell = 10\ \text{tons/day}$$

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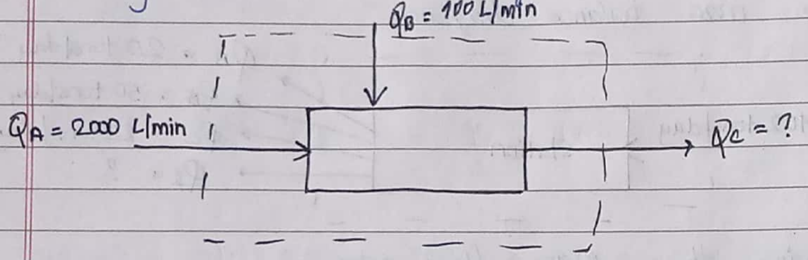
Q7: A sewer carrying stormwater to manhole 1 has constant flow rate of 2000 L/min (QA). At manhole 1, it receives constant lateral flow of 100 L/min. (Q_B). What is the flow to manhole 2. (Q_C)?

Soln:

Drawing mass balance diagram.



Writing mass-balancing equation for flowrate:

rate of water accumulated = rate of water input − rate of water output + rate of water produced − rate of water consumed.

Let us assume i) steady state condition
ii) conservative pollutant.

Then,

0 = rate of water input − rate of water output + 0 + 0

or, rate of water input = rate of water output

or, $Q_C = Q_A + Q_B$ ∴ Q_C = 2100 L/min.

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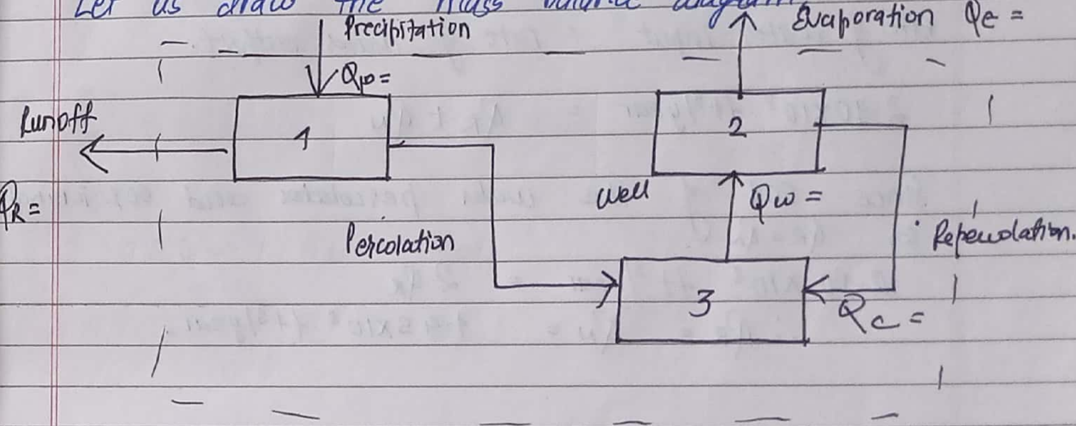
Q7: Suppose the rainfall is 40 inch/year of which 50% percolates into the ground. The farmer irrigates crops using well water. Of extracted well water, 80% is lost by evaporation, the remainder percolates back into the ground. How much groundwater could a farmer on a 2000 acre farm extract from the ground per year without depleting the groundwater reservoir volume?

Soln:

Let us draw the mass balance diagram.



The rainfall = 40 $\frac{\text{inch}}{\text{year}}$ = 40 $\frac{\text{inch}}{\text{year}}$ × $\frac{1\ \text{ft}}{12\ \text{inch}}$

= 3.33 ft/year.

Land area = 2000 acre = 2000 acre × $\frac{43650\ \text{ft}^2}{1\ \text{acre}}$

= 8.712×10^7 ft^2

Applying mass-balance equation in black box 1,

rate of water accumulated = rate of water input - rate of water output + rate of water produced - rate of water consumed.

Let us assume (i) : steady state condition.
(ii) : conservative source.

Then, the eqn becomes,

rate of water input = rate of water output.

$$2.90 \times 10^8 \ ft^3/year = Q_R + Q_N$$

Since 50% of the water percolates and 50% is runoff.

so, $Q_R = Q_N$.

$$2.90 \times 10^8 \ ft^3/year = 2 Q_R$$

$$\therefore Q_R = Q_N = 1.45 \times 10^8 \ ft^3/year.$$

Now, in black box 2,
Assuming steady state condition and conservative material,

rate of input rate = rate of output rate.

or, $Q_W = Q_C + Q_E$

According to question, ~~80~~ 80% of the water goes evapotranspiration and 20% of the water goes to rep~~x~~erlocation. so,

$$Q_E = 0.8 \ Q_W.$$

or, $Q_C = Q_W - 0.8 Q_W$

$$\therefore Q_C = 0.2 \ Q_W.$$

Now, in black box 3,
Assuming steady state condition and conservative source,

rate of water input = rate of water output.

$$Q_N + Q_C = Q_W.$$

or,

we know, $Q_C = 0.2 \ Q_W$

So,

$$0.2 Q_W + Q_N = Q_W.$$

$$\frac{\cancel{2.90}\ 1.45 \times 10^8}{0.8} = Q_W.$$

$$\therefore Q_W = 1.81 \times 10^8 \ ft^3/year.$$

So,

$$Q_C = 0.2 \times 1.81 \times 10^8 \ ft^3/year = 3.62 \times 10^7 \ ft^3/year$$

$$Q_E = 0.8 \times 1.81 \times 10^8 \ ft^3/year = 1.448 \times 10^8 \ ft^3/year.$$