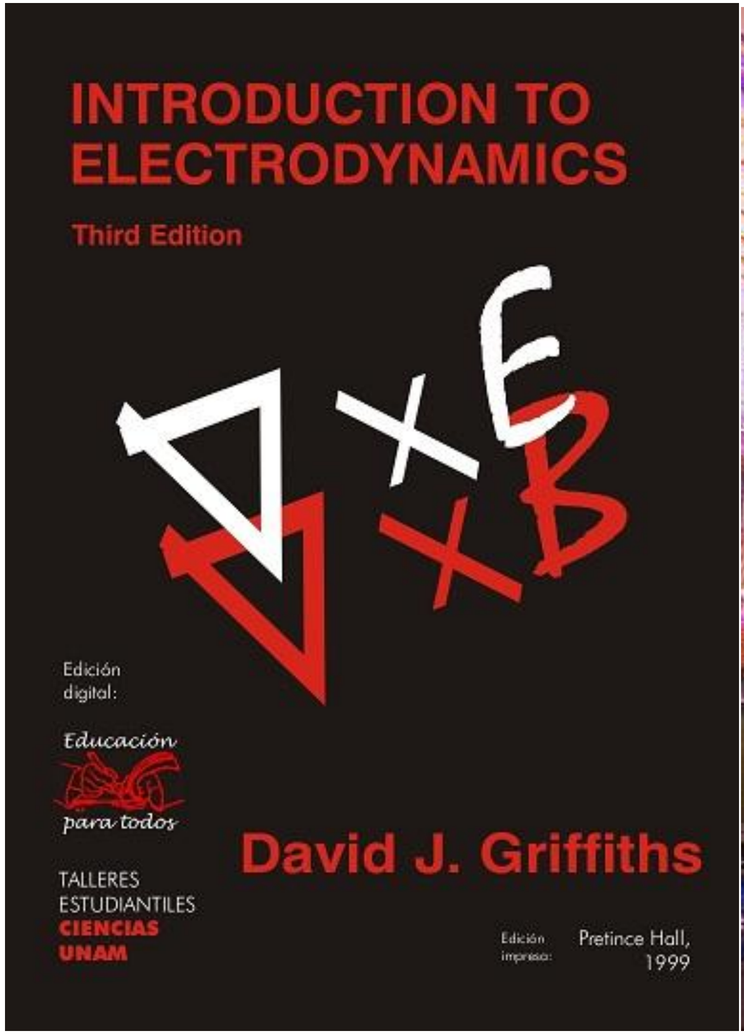
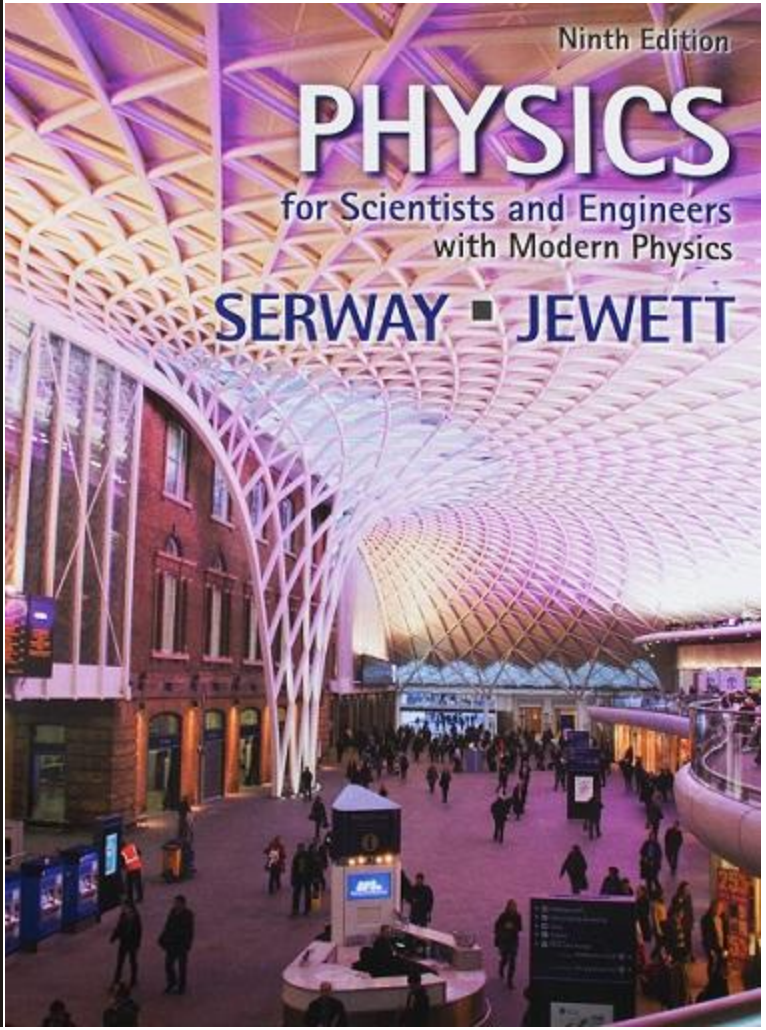
PHYSICS

General Physics II (PHYS 102)





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Course Outline

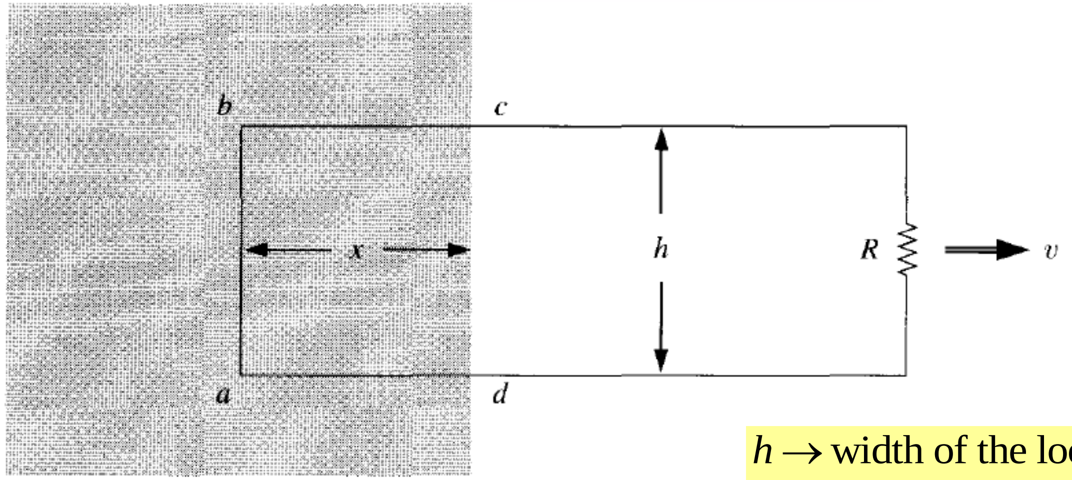
- Flux Rule for Motional emf
- Induction
- Faraday's Law of Induction
- Lenz's Law
- Self and Mutual Inductions
- Neumann Formula & Reciprocity Theorem
- Energy Stored in Magnetic Field

Flux Rule for Motional emf

ELECTROMAGNETIC INDUCTION

Flux Rule for Motional emf

- **Motional emf -** The emf induced in a conductor moving through a constant magnetic field
- Figure EI-1 shows a primitive model for a generator.



$h \rightarrow$ width of the loop

Figure EI-1

- In the **shaded region** there is a uniform magnetic field $\vec{B}$, pointing into the page.
- If the entire loop is pulled to the right with speed v, the charges in segment experience a magnetic force qvB whose vertical component drives current around the loop, in clockwise direction.

Work done per unit charge

- The emf is $\mathcal{E} = \oint \vec{f}_{mag} \cdot d\vec{l} = vBh$ .... (1)

the magnetic force, per unit charge

[The horizontal segments bc and ad contribute nothing, since the force here is perpendicular to the wire]

- The magnetic flux through the rectangular loop $abcd$ in **Figure EI-1** is

$$\Phi = \int \vec{B} \cdot d\vec{a} = Bhx$$

As the loop moves, the flux decreases:

$$\frac{d\Phi}{dt} = Bh\frac{dx}{dt} = -Bhv \quad \text{..........} \quad (2)$$

- From Eq.(1) and Eq.(2), we get

$$\mathcal{E} = -\frac{d\Phi}{dt}$$

Flux rule for motional emf

Flux Rule for Motional emf

ELECTROMAGNET IC INDUCTION

Flux Rule for Motional emf in Arbitrary Loop

- **Figure EI-2** shows a loop of wire at time t and also a short time dt later.
- The change in the flux during the time dt is

$$d\Phi = \Phi(t+dt) - \Phi(t) = \Phi_{\text{ribbon}} = \int_{\text{ribbon}} \vec{B}\cdot d\vec{a}$$
$$= \oint \vec{B}\cdot(\vec{v}\times d\vec{l})dt$$

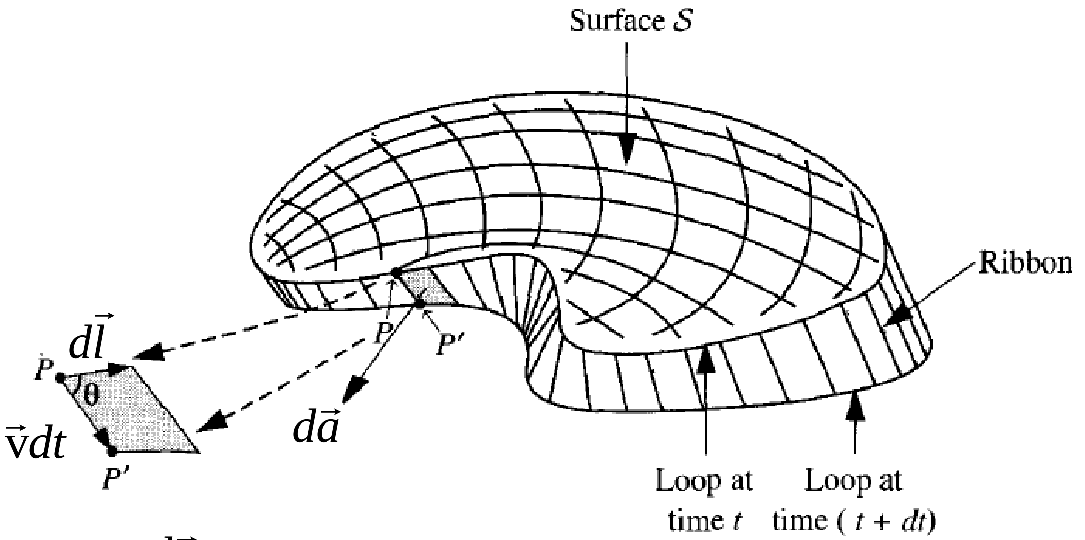


Figure EI-2

Therefore

$$\frac{d\Phi}{dt} = \oint \vec{B}\cdot(\vec{v}\times d\vec{l})$$

$$= \oint \vec{B}\cdot(\vec{w}\times d\vec{l}) \quad \left[\begin{array}{l} \because \text{Resultant velocity of a charge at } P,\ \vec{w} = \vec{u}+\vec{v} \\ \quad \text{where } \vec{v} = \text{velocity of the wire,} \\ \quad \vec{u} = \text{velocity of the charge down the wire} \\ \quad \text{and } \vec{u} \text{ is parallel to } d\vec{l}\,. \end{array}\right]$$

$$= -\oint (\vec{w}\times\vec{B})\cdot d\vec{l}$$

$$= -\oint \vec{f}_{mag}\cdot d\vec{l} \qquad \left[\because \text{the magnetic force per unit charge, } \vec{f}_{mag} = \vec{w}\times\vec{B}\right]$$

$$= -\mathcal{E}$$

$$\therefore\ \mathcal{E} = -\frac{d\Phi}{dt}$$

Flux rule for motional emf

Michael Faraday & Joseph Henry

ELECTROMAGNET IC INDUCTION

Michael Faraday

- **1791 – 1867**
- **British physicist and chemist**
- **Great experimental scientist**
- **Contributions to early electricity include:**
  - Invention of electric motor, electric generator, and transformer
  - Electromagnetic induction
  - Laws of electrolysis

Joseph Henry

- **1797 – 1878**
- **American physicist**
- **First director of the Smithsonian**
- **Improved design of electromagnet**
- **Constructed one of the first motors**
- **Discovered self-inductance**
- **Unit of inductance is named in his honor**

- Faraday's law of induction was discovered through experiments carried out by Michael Faraday in England in 1831 and by Joseph Henry in the United States at about the same time. Even though Faraday published his results first, which gives him priority of discovery.
- In addition to their independent simultaneous discovery of the law of induction, Faraday and Henry had several other similarities in their lives. Both were apprentices at an early age. Faraday, at age 14, was apprenticed to a London bookbinder. Henry, at age 13, was apprenticed to a watchmaker in Albany, New York. In later years Faraday was appointed director of the Royal Institution in London, whose founding was due in large part to an American, Benjamin Thompson (Count Rumford). Henry, on the other hand, became secretary of the Smithsonian Institution in Washington, DC, which was founded by an endowment from an Englishman, James Smithson.

Induction Experiments

Experiment - I

Figure FL - I shows a coil of wire as a part of a circuit containing an ammeter

- While the magnet is moving, the ammeter deflects, showing that a current has been set up in the coil.
- If we hold the magnet stationary with respect to the coil, the ammeter does not deflect.
- If we move the magnet away from the coil, the meter again deflects, but in the opposite direction, which means that the current in the coil is in the opposite direction.

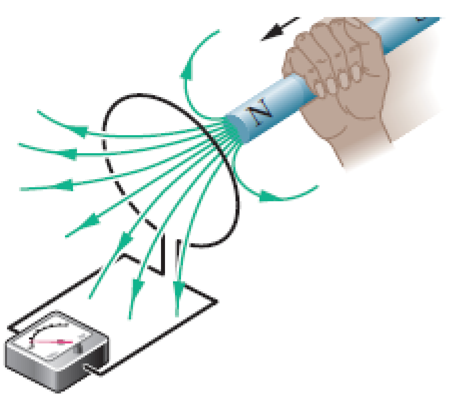


Figure FL – I

Experiment -2

Figure FL -2 shows two coils are placed close together but at rest with respect to each other.

- If we close switch S, to turn on a current in the right-hand loop, the meter suddenly and briefly registers a current – an induced current - in the left – hand loop.

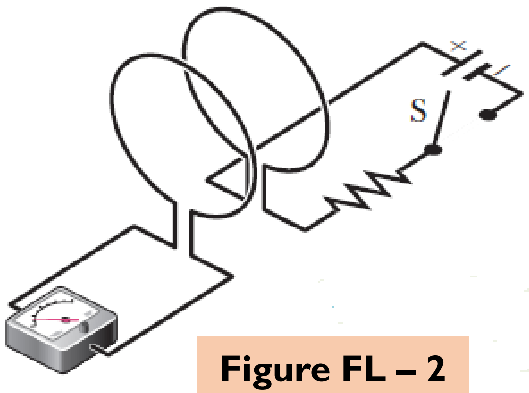


Figure FL – 2

- If we then open the switch, another sudden and brief induce current appears in the left-hand loop, but in opposite direction.

The common feature of these two experiments is motion or change. It is the moving magnet or the changing current that is responsible for the induced emfs.

If the magnetic flux Φ_B through an area bounded by a closed conducting loop changes with time, a current is produced in the loop. The current produced in the loop is called an **induced current** ; the work done per unit charge to produce that current (to move the conduction electrons that constitute the current) is called an **induce emf**; and the process of producing the current and emf is called **Induction**.

Faraday’s Experiment

ELECTROMAGNET IC INDUCTION

Faraday’s Experiment

In 1831 **Michael Faraday** reported on a series of experiments, including three that can be characterized as follows:

Experiment 1. He pulled a loop of wire to the right through a magnetic field **[Figure FL-3 (a)]**. A current flowed in the loop.

Experiment 2. He moved the magnet to the left, holding the loop still **[Figure FL-3(b)].** Again, a current flowed in the loop.

Experiment 3. With both the loop and the magnet at rest **[Figure FL-3 (c)],** he changed the strength of the field (he used an electromagnet, and varied the current in the coil). Once again, current flowed in the loop.

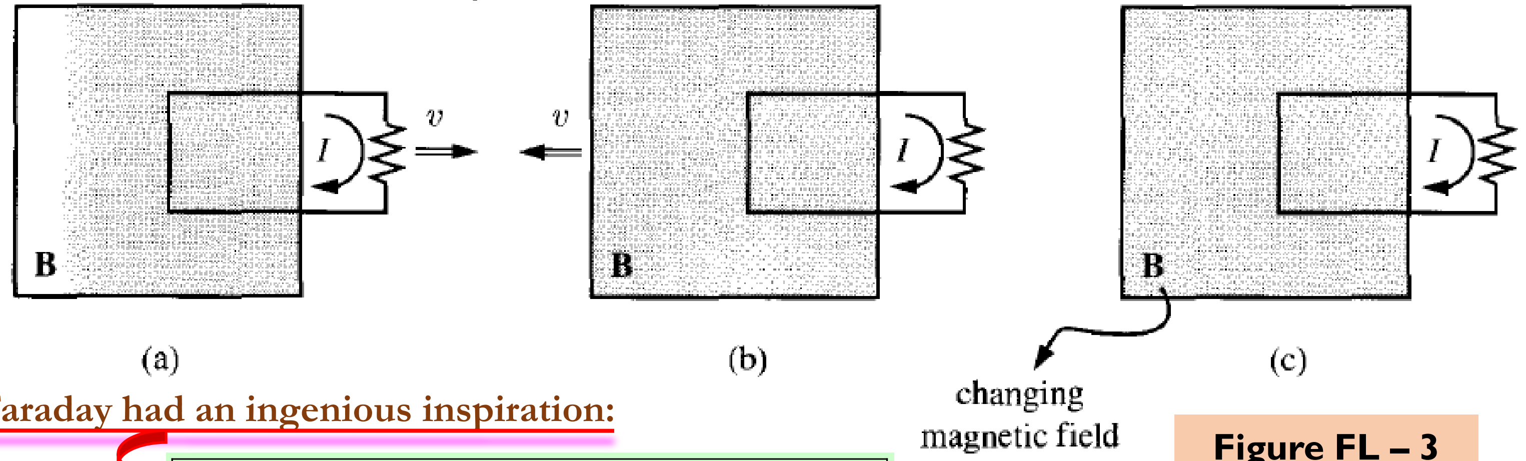


Figure FL – 3

Faraday had an ingenious inspiration:

A changing magnetic field induces an electric field.

Universal Flux rule

Whenever (and for whatever reason) the magnetic flux through a loop changes, an emf

$$\mathcal{E} = -\frac{d\Phi}{dt}$$

will appear in the loop.

In Faraday's first experiment it's the Lorentz force law at work; the emf is *magnetic. But in the other two it's an electric field (induced by* the changing magnetic field) that does the job. Viewed in this light, it is quite astonishing that all three processes yield the same formula for the emf.

In fact, it was precisely this "coincidence" that led Einstein to the special theory of relativity-he sought a deeper understanding of what is, in classical electrodynamics, a peculiar accident.

Faraday's Law of Induction

Faraday's Law of Induction

Experiments conducted by Michael Faraday in England in 1831 and independently by Joseph Henry in the United States that same year showed that an emf can be induced in a circuit by a changing magnetic field. The results of these experiments led to a very basic and important law of electromagnetism known as **Faraday's law of induction**.

Faraday's law of induction states:

The induced emf in a closed loop equals the negative of the time rate of change of magnetic flux through the loop.

Mathematically,

induced emf

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

Magnetic flux through the loop

$$\Phi_B = \int \vec{B}\cdot d\vec{a}$$

If the circuit consists of N loops, all of the same area, and if Φ_B is the flux through one loop, an emf is induced in every loop and Faraday's law becomes

total induced emf in the coil

$$\mathcal{E} = -N\frac{d\Phi_B}{dt}$$

- **Faraday's law**, in integral form:

$$\oint \vec{E}\cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t}\cdot d\vec{a}$$

$$\left[\because \mathcal{E} = -\frac{d\Phi_B}{dt} \Rightarrow \oint \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\left(\int \vec{B}\cdot d\vec{a}\right)\right]$$

- **Faraday's law**, in differential form:

$$\nabla\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla\times\vec{E} = -\frac{\partial}{\partial t}\left(\nabla\times\vec{A}\right) = \nabla\times\left(-\frac{\partial \vec{A}}{\partial t}\right)$$

$$\Rightarrow \vec{E} = -\frac{\partial \vec{A}}{\partial t}$$

A changing magnetic field induces an electric field.

Lenz’s Law

Lenz’s Law

ELECTROMAGNETIC INDUCTION

- The rule for determining the direction of the induced current was proposed in 1834 by Heinrich Friedrich Lenz, a Russian scientist (1804–1865) and is known as **Lenz' law**.

The induced current in a loop is in the direction that creates a magnetic field that opposes the change in magnetic flux through the area enclosed by the loop.

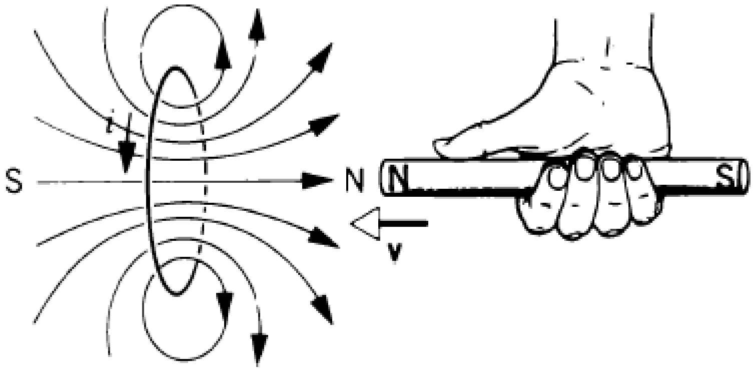


Figure LL-1

When the magnet is pushed toward the loop, the induced current i has the direction shown, setting up a magnetic field that opposes the motion of the magnet.

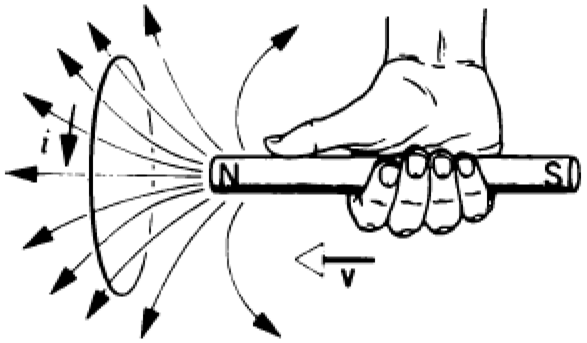


Figure LL-2

When the magnet is pushed toward the loop, the magnetic flux through the loop is increased. The induced current through the loop sets up a magnetic field that opposes the increase in flux.

Lenz’s Law

Lenz’s Law

- Lenz’s law also helps us gain intuitive understanding of various induction effects and of the **role of energy conservation**.

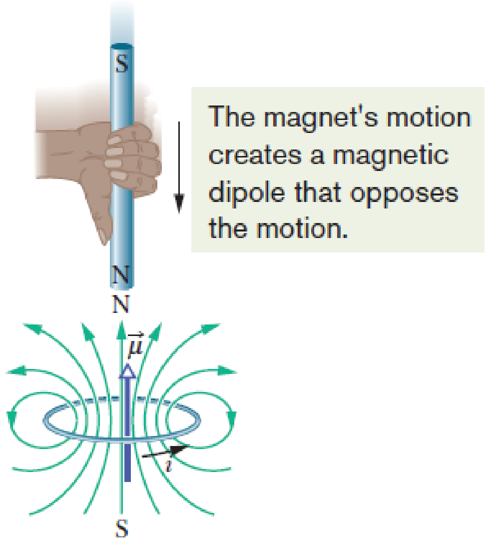
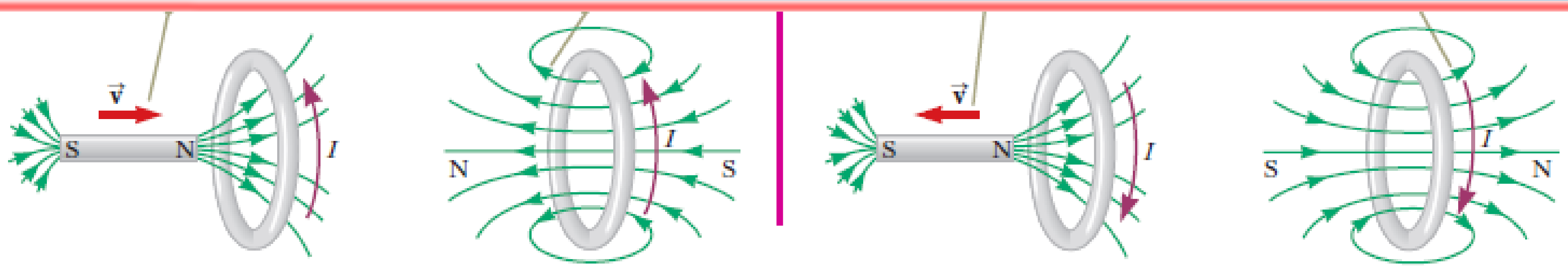


Figure LL- 3

- As the magnet is moved toward the loop, a current is induced in the loop. The current produces its own magnetic field, with magnetic dipole moment oriented so as to oppose the motion of the magnet. Thus, the induced current must be counterclockwise as shown in **Figure LL-3**
- The agent that causes the magnet to move, either toward the coil or away from it, always experiences a resisting force and is thus required to do work. From the conservation-of-energy principle this work done on the system must exactly equal the internal (Joule) energy produced in the coil, since these are the only two energy transfers that occur in the system. If the magnet is moved more rapidly, the agent does work at a greater rate and the rate of production of internal energy increases correspondingly.



The induced current tends to keep the original magnetic flux through the circuit from changing.

Multiple Choice Questions

ELECTROMAGNETIC INDUCTION

- **Figure EMI-1 shows a circular loop of wire falling toward a wire carrying a current to the left. What is the direction of the induced current in the loop of wire?**

  [a] clockwise

  [b] counterclockwise

  [c] zero

  [d] impossible to determine

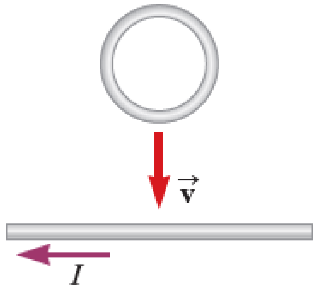


Figure EMI-1

Ans : [b]

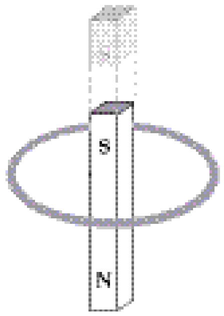
- **A bar magnet is dropped from above and falls through the loop of wire shown below. The north pole of the bar magnet points downward towards the page as it falls. Which statement is correct ?**

  [a] The current in the loop always flows in a clockwise direction.

  [b] The current in the loop always flows in a counterclockwise direction.

  [c] The current in the loop flows first in a clockwise, then in a counterclockwise direction.

  [d] The current in the loop flows first in a counterclockwise, then in a clockwise direction.



Ans : [d]

Figure EMI-2

Multiple Choice Questions

ELECTROMAGNETIC INDUCTION

- Which one of the following statements is equivalent to Faraday Law?

  [a] Time varying electric field induces a magnetic field.

  [b] Time Varying magnetic field induces an electric field.

  [c] Magnetic flux in a current loop is directly proportional to the current.

  [d] The direction of the induced current is such that it opposes the effect producing it.

  Ans : [b]

- Which one of the following pairs of equations concludes that $\vec{E}$ is purely Faraday's induce electric field?

  [a] $\nabla \cdot \vec{E} = \dfrac{\rho}{\varepsilon_0}; \quad \nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$

  [b] $\nabla \cdot \vec{E} = 0; \quad \nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$

  [c] $\nabla \cdot \vec{E} = 0; \quad \nabla \times \vec{E} = 0$

  [d] $\nabla \cdot \vec{E} = \dfrac{\rho}{\varepsilon_0}; \quad \nabla \times \vec{E} = 0$

  Ans : [b]

- Human brain activity produces weak variable electric currents. The way these are detected without surgery is by

  [a] measuring the force on a wire carrying a large electric current that is placed near the brain.

  [b] measuring the force on a solenoid carrying a large electric current that is placed near the brain.

  [c] measuring the magnetic fields they produce by means of small loops of wire of very low resistance placed near the brain.

  [d] measuring the potential difference between the leaves of an electroscope that is placed near the brain.

  Ans : [c]

ELECTROMAGNET IC INDUCTION

Inductor

- An **inductor** is a device that can be used to produce a known magnetic field in a specified region
- A circuit device that is designed to have a particular inductance is called an **inductor.**
- The circuit symbol for an inductor is  .
- A long solenoid is taken as our basic type of inductor.
- If a current i is established through each of the N windings of an inductor, a magnetic flux Φ_B links those windings.T

  The **inductance** L of the inductor is

$$L = \frac{N\Phi_B}{I}$$

$N \rightarrow$ number of turns

$N\Phi_B \rightarrow$ magnetic flux linkage

The inductance L is thus a measure of the flux linkage produced by the inductor per unit of current.

- The SI unit of inductance is the **henry** (H).

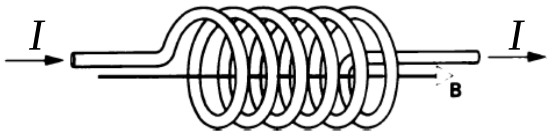
$$\boxed{1\mathrm{H} = 1\,\mathrm{T}\cdot\mathrm{m}^2/\mathrm{A}}$$

Inductance of a Solenoid

Consider a long solenoid of length l and cross-sectional area A.

The Flux linkage is

$$N\Phi_B = (nl)(\mathrm{BA})$$



$n \rightarrow$ number of turns per unit length of the solenoid

$\mathrm{B} \rightarrow$ magnitude of the magnetic field within the solenoid

The inductance of the solenoid

$$L = \frac{N\Phi_B}{I} = \frac{(nl)(\mathrm{BA})}{I} = \frac{(nl)\{(\mu_0 nI)(\mathrm{A})\}}{I} = (\mu_0 n^2 \mathrm{A})l$$

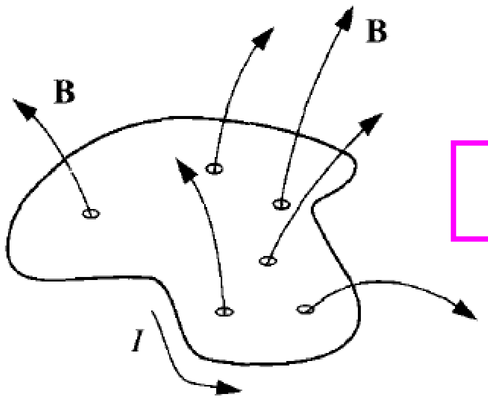
Thus, the inductance per unit length near the center of a long solenoid is

$$\frac{L}{l} = \mu_0 n^2 \mathrm{A}$$

Self-Inductance

Self -Inductance

- An **induced emf** appears in any coil in which the current is changing.



$$\Phi_B = LI$$

- The **self-induced emf**

$$\mathcal{E}_L = -L\frac{dI}{dt} \quad \left[\because \mathcal{E}_L = -\frac{d\Phi_B}{dt} = -\frac{d(LI)}{dt} = -L\frac{dI}{dt}\right]$$

- The **self-inductance** of the loop is $\boxed{L = -\frac{\mathcal{E}_L}{dI/dt}}$.

- The SI unit of inductance is the **henry** (H).

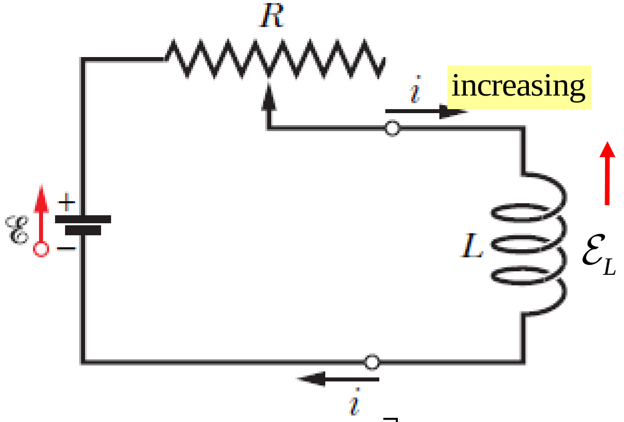
$$\boxed{1H = 1\frac{V\cdot s}{A}}$$

Inductance is a measure of how much opposition a loop offers to a change in the current in the loop.

Inductance

- An **self- induced emf** appears in any inductor (such as coil, solenoid) whenever the current changes with time.

$$\mathcal{E}_L = -L\frac{di}{dt}$$



$$\left[\because \mathcal{E}_L = -\frac{d(N\Phi_B)}{dt} = -\frac{d(Li)}{dt} = -L\frac{di}{dt}\right]$$

- The direction of the induced emf is given by lenz's law.

LR Circuit

ELECTROMAGNETIC INDUCTION

- **Figure IN-1** shows an LR Circuit.
- The total emf in this circuit is

$$\mathcal{E}_o + \left[-L\frac{dI}{dt}\right]$$

- Ohm's law:

$$\mathcal{E}_o - L\frac{dI}{dt} = IR$$

$$\Rightarrow \frac{dI}{\mathcal{E}_o - IR} = \frac{dt}{L}$$

$$\Rightarrow \frac{d(\mathcal{E}_o - IR)}{\mathcal{E}_o - IR} = -R\frac{dt}{L} \qquad \ldots\ldots\ldots\ldots \qquad (1)$$

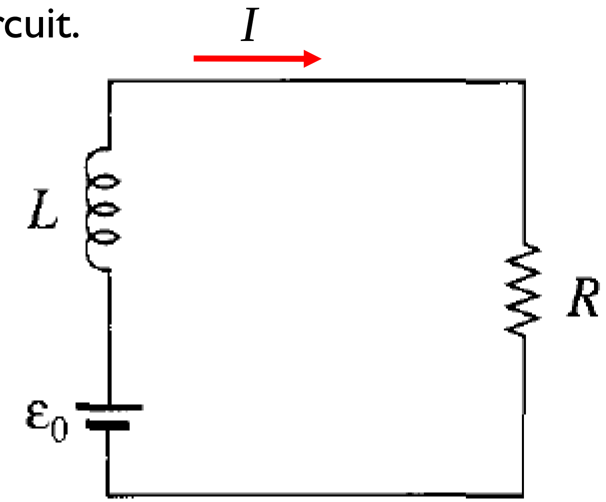


Figure IN-1

- Integrating both sides of Eq. (1), we get

$$\ln(\mathcal{E}_o - IR) = -\frac{R}{L}t + k$$

At $t = 0, I = 0,$ then $k = \ln\mathcal{E}_o$

$$\therefore \ln(\mathcal{E}_o - IR) = -\frac{R}{L}t + \ln\mathcal{E}_o$$

$$\Rightarrow \ln\left[\frac{\mathcal{E}_o - IR}{\mathcal{E}_o}\right] = -\frac{R}{L}t$$

$$\Rightarrow \frac{\mathcal{E}_o - IR}{\mathcal{E}_o} = e^{\left(-\frac{R}{L}\right)t}$$

$$\boxed{\therefore I(t) = \frac{\mathcal{E}_o}{R}[1 - e^{\left(-\frac{R}{L}\right)t}] = \frac{\mathcal{E}_o}{R}\left(1 - e^{-\frac{t}{\tau}}\right)}$$

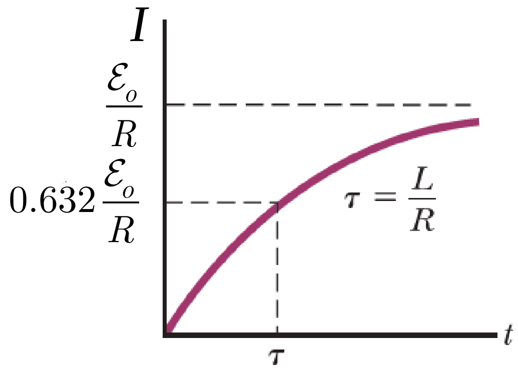


Figure IN-2 Plot of current Vs time

$\tau \rightarrow$ time constant of the LR-circuit

It tells you how long the current takes to reach a substantial fraction (roughly two-thirds) of its final value.

Sample Problems & Multiple Choice Questions

MAGNETIC FIELDS IN MATTER

- A solenoid has an inductance of 53 mH and a resistance of 0.37 Ω . If the solenoid is connected to a battery, how long will the current take to reach half its final equilibrium value? (This is a real solenoid because we are considering its small, but nonzero, internal resistance.)

Hint:

$$\because \; I(t) = \frac{\mathcal{E}_o}{R}\left[1 - e^{\left(-\frac{R}{L}\right)t}\right] = \frac{\mathcal{E}_o}{R}\left(1 - e^{-\frac{t}{\tau}}\right)$$

$$\Rightarrow \frac{1}{2}\frac{\mathcal{E}_o}{R} = \frac{\mathcal{E}_o}{R}\left[1 - e^{\left(-\frac{R}{L}\right)t}\right]$$

$$\Rightarrow e^{\left(-\frac{R}{L}\right)t} = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\Rightarrow \frac{R}{L}t = \ln(2)$$

$$\Rightarrow t = \frac{L}{R}\ln(2) = \frac{53\times10^{-3}\,H}{0.37\;\Omega}(0.693) = 0.1\text{ s}$$

- What is the inductance of a series *RL* circuit in which *R* = 1.0 KΩ if the current increases to one-third of its final value in 30 μs?

[a] 74 mH [b] 99 mH [c] 49 mH [d] 462 mH

Ans : [a]

- When a switch is closed, completing an *LR* series circuit, the time needed for the current to reach one half its maximum value is _____ time constants.

[a] 0.250 [b] 0.500 [c] 0.693 [d] 1.00

Ans : [c]

Mutual-Inductance

ELECTROMAGNETIC INDUCTION

Mutual -Inductance

- If loops 1 and 2 are near each other, a changing current in either loop can induce an emf in the other.

  This mutual induction is described by

$$\mathcal{E}_2 = M\frac{dI_1}{dt}$$

&

$$\mathcal{E}_1 = M\frac{dI_2}{dt}$$

M

Mutual inductance of the two loops

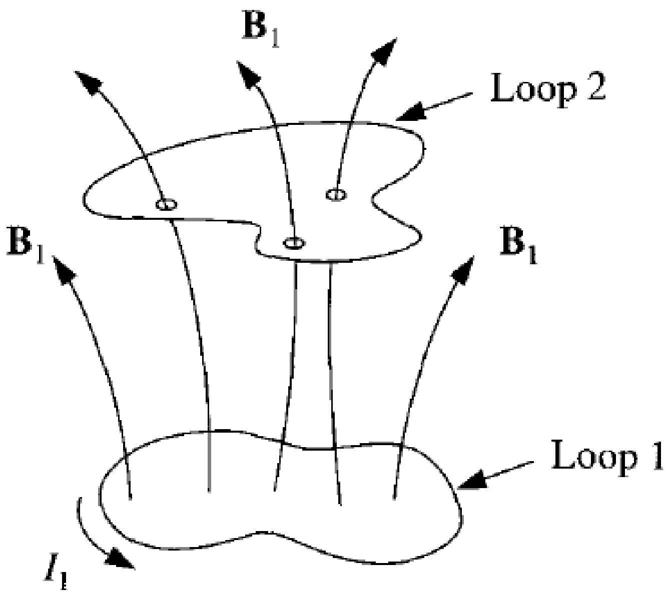


Figure MI-1

- **Mutual inductance** depends on the geometry of both circuits and on their orientation with respect to each other.
- The SI unit of mutual inductance is the **henry** (H).

Mutual Induction → to suggest the mutual interaction of the two coils

Suppose two coils are placed near each other, as shown in **Figure MI-2**.

If a current I_1 in a coil 1 change with time, the flux through loop 2 will vary accordingly, and Faraday's law says this changing flux will induce an emf in loop 2:

$$\mathcal{E}_2 = -\frac{d\left(N_2\Phi_{21}\right)}{dt} = -M_{21}\frac{dI_1}{dt}$$

$\left[\because N_2\Phi_{21} = M_{21}I_1\right.$

Φ_{21} = the magnetic flux through one turn of coil 2 due to $\left.I_1\right]$

$$M_{21} = \frac{N_2\Phi_{21}}{I_1}$$

the mutual inductance M_{21} of coil 2 with respect to coil 1

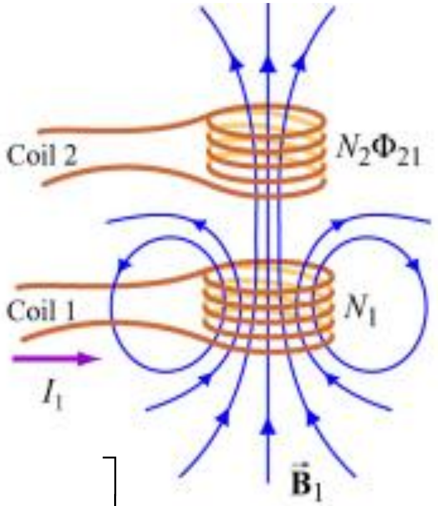


Figure MI-2
Changing current in coil 1 produces changing magnetic flux in coil 2

Neumann Formula & Reciprocity Theorem

Neumann Formula

- Suppose we have two loops of wire at rest. If we run a steady current I_1 around loop 1, it produces a magnetic field $\vec{B}_1$.

  Some of the field lines pass through loop 2 **[Figure NR-1]**.

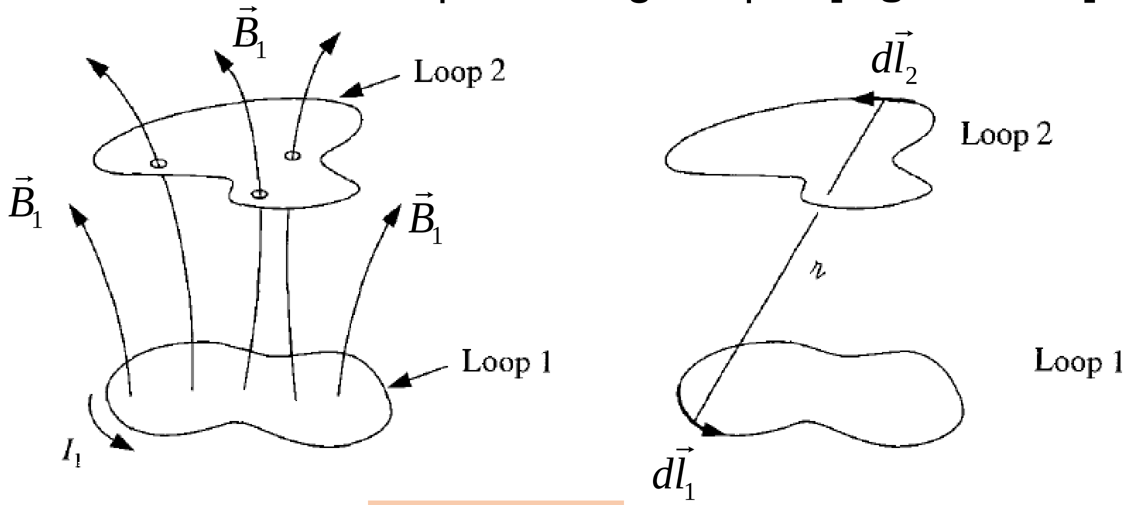


Figure NR-1

- The Biot-Savart law,

$$\vec{B}_1 = \frac{\mu_0 I_1}{4\pi} \oint \frac{d\vec{l}_1 \times \hat{\mathscr{r}}}{\mathscr{r}^2}$$

reveals one significant fact about this field : ***It is proportional to the current*** I_1 .

The flux of $\vec{B}_1$ through loop 2: $\Phi_2 = \int \vec{B}_1 \cdot d\vec{a}_2$

Thus,

$$\Phi_2 = M_{21} I_1 \quad \text{..........} \quad (1)$$

where M_{21} is the constant of proportionality; it is known as **mutual inductance** of the two loops.

Again, The flux of $\vec{B}_1$ through loop 2:

$$\Phi_2 = \int \vec{B}_1 \cdot d\vec{a}_2 = \int \left(\nabla \times \vec{A}_1\right) \cdot d\vec{a}_2$$

$$= \oint_{C_2} \vec{A}_1 \cdot d\vec{l}_2 \qquad \text{[using Stoke's theorem]}$$

$$= \oint_{C_2} \left[\frac{\mu_0 I_1}{4\pi} \oint_{C_1} \frac{d\vec{l}_1}{\mathscr{r}} \right] \cdot d\vec{l}_2$$

$$\boxed{\therefore \Phi_2 = \left[\frac{\mu_0}{4\pi} \oint_{C_2} \oint_{C_1} \frac{d\vec{l}_1 \cdot d\vec{l}_2}{\mathscr{r}} \right] I_1} \quad \text{..........} \quad (2)$$

Comparing Eq.(1) and Eq.(2), we get

$$\boxed{M_{21} = \frac{\mu_0}{4\pi} \oint_{C_2} \oint_{C_1} \frac{d\vec{l}_1 \cdot d\vec{l}_2}{\mathscr{r}}}$$

Neumann Formula

Neumann Formula & Reciprocity Theorem

Reciprocity Theorem

Neumann Formula:

$$M_{21} = \frac{\mu_0}{4\pi} \oint_{C_2} \oint_{C_1} \frac{d\vec{l}_1 \cdot d\vec{l}_2}{\mathscr{r}} \qquad \ldots\ldots\ldots\ldots (3)$$

It's not very useful for practical purposes, but it does reveal two important things about mutual inductance:

(1) M_{21} is purely geometrical quantity, having to do with the sizes, shapes, and relative positions of the two loops.

(2) The integral in Eq. (3) is unchanged if we switch the roles of loops 1and 2; if follows that

$$M_{21} = M_{12} \qquad \left[\because \quad \Phi_1 = \left[\frac{\mu_0}{4\pi} \oint_{C_1} \oint_{C_2} \frac{d\vec{l}_2 \cdot d\vec{l}_1}{\mathscr{r}} \right] I_2 \right]$$

This is the **reciprocity theorem**.

Whatever the shapes and positions of the loops, the flux through 2 when we run a current around 1 is identical to the flux through 1 when we send the same current around 2.

$$\Phi_2 = \left[\frac{\mu_0}{4\pi} \oint_{C_2} \oint_{C_1} \frac{d\vec{l}_1 \cdot d\vec{l}_2}{\mathscr{r}} \right] I = \left[\frac{\mu_0}{4\pi} \oint_{C_1} \oint_{C_2} \frac{d\vec{l}_2 \cdot d\vec{l}_1}{\mathscr{r}} \right] I = \Phi_1$$

Sample Problem

- **A short solenoid** (length l and radius a, with n_1 turns per unit length) **lies on the axis of a very long solenoid** (radius b, with n_2 turns per unit length) **as shown in (Figure SP-1). Current I flows in the short solenoid. What is the flux through the long solenoid? What is the mutual inductance?**

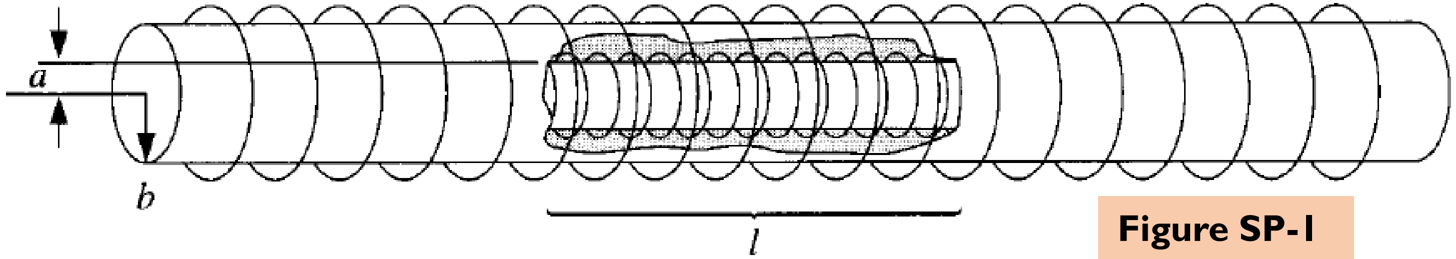


Figure SP-1

Solution:

Let the current I flows in the long solenoid.

The field inside the long solenoid is constant:

$$B = \mu_0 n_2 I$$

So the total flux through the short solenoid is

$$\Phi_S = (n_1 l)\ [(\mu_0 n_2 I)(\pi a^2)]$$

$$\left[\begin{aligned} \because \Phi_S &= \text{No. of turns in short solenoid} \\ &\times \text{ the flux through the single loop of the short solenoid} \end{aligned}\right]$$

According to the reciprocity theorem,

This is also the flux through the long solenoid when the current I flows in the short solenoid.

$$\Phi_L = \Phi_S = (n_1 l)\ [(\mu_0 n_2 I)(\pi a^2)] = \left[\mu_0 \pi a^2 n_1 n_2 l\right] I$$

Mutual Inductance

$$M = \mu_0 \pi a^2 n_1 n_2 l \qquad [\because \Phi = MI]$$

Multiple Choice Questions

ELECTROMAGNET IC INDUCTION

- Coil 1, connected to a 100 Ω resistor, sits inside coil 2. Coil 1 is connected to a source of 60 cycle per second AC current. Which statement about coil 2 is correct?

  [a] No current will be induced in coil 2.

  [b] DC current (current flow in only one direction) will be induced in coil 2.

  [c] AC current (current flow in alternating directions) will be induced in coil 2.

  [d] Both AC and DC current will be induced in coil 2.

  Ans: [c]

- An emf of 16V is induced in a coil of inductance 4H. The rate of change of current must be

  [a] 64 A/s [b] 32 A/s [c] 16 A/s [d] 4 A/s

  Ans: [d]

- The magnetic flux through a circular loop is given by $0.02\, t^3$ Wb. The magnitude of the induced emf in the loop at t = 1 millisecond is

  [a] 6×10^{-8} V [b] 6×10^{-2} V [c] 6×10^{-10} V [d] 6×10^{-6} V

  Ans: [a]

Multiple Choice Questions

ELECTROMAGNET IC INDUCTION

- The mutual inductance of a loop with respect to the other

  [a] depends on the current flowing along the loops.

  [b] does not depend on the relative positions of the loops.

  [c] does not depend on the current flowing along the loops.

  [d] does not depend on the shapes and sizes of the two loops.

  Ans: [c]

- The self-inductance of a long solenoid

  [a] does not depend on the radius of the solenoid.

  [b] depends on the current flowing along the solenoid.

  [c] does not depend on the current flowing along the solenoid.

  [d] does not depend on the number of the turns per unit length.

  Ans: [c]

- In an inductor of self-inductance L=2 mH, current changes with time according to the relation $I = t^2 e^{-t}$. At what time, emf is zero?

  [a] 1 s [b] 2 s [c] 3 s [d] 4 s

  Ans: [b]

Energy in a Magnetic Field

ELECTROMAGNET IC INDUCTION

- **Figure MF-1** shows a circuit in which a coil is connected is series with a battery.

  As soon as the switch S is closed, the current starts rising in the circuit. The back *emf* produced in the coil opposes this rise in current. So, work must be done against the back *emf* to get the current going.

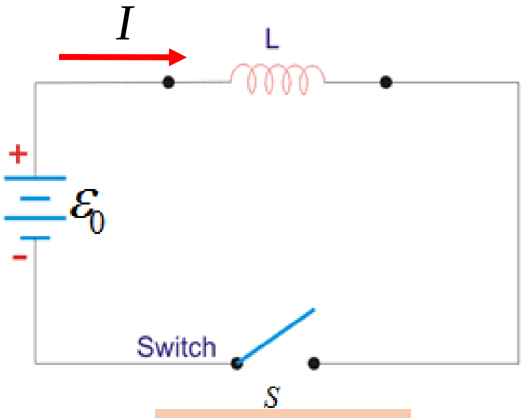


Figure MF-1

- The work done on a unit charge, against the back emf, in one trip around the circuit is $-\mathcal{E}$.

  The amount of charge per unit time passing down the wire is I.

- So, the total work done per unit time is

$$\frac{dW}{dt} = -\mathcal{E}\, I = LI\frac{dI}{dt}.$$

If we start with zero current and build it up to a final value I, the total work done is

$$W = \int_0^I LI\frac{dI}{dt}\,dt = L\int_0^I I\,dI$$

$$\therefore\ \boxed{W = \frac{1}{2}LI^2} \qquad \text{.............} \ (1)$$

$$= \frac{1}{2}I(LI)$$

$$= \frac{1}{2}I\Phi \qquad [\because \text{ the flux through the loop, } \Phi = LI]$$

$$= \frac{1}{2}I\int_S \vec{B}\cdot d\vec{a}$$

$$= \frac{1}{2}I\int_S (\nabla\times\vec{A})\cdot d\vec{a}$$

$$= \frac{1}{2}I\oint_c \vec{A}\cdot d\vec{l}$$

$$\therefore\ \boxed{W = \frac{1}{2}\oint_c\left(\vec{A}\cdot\vec{I}\right)dl} \qquad \text{...........} \ (2)$$

Energy in a Magnetic Field

- In this form, the generalization to volume currents is obvious:

$$W=\frac{1}{2}\int_V\left(\vec{A}\cdot\vec{J}\right)d\tau$$

$$=\frac{1}{2\mu_0}\int_V\vec{A}\cdot(\nabla\times\vec{B})\,d\tau \qquad \left[\because \text{Ampere's law}, \nabla\times\vec{B}=\mu_0\vec{J}\right]$$

$$=\frac{1}{2\mu_0}\int_V\left[\vec{B}\cdot(\nabla\times\vec{A})-\nabla\cdot(\vec{A}\times\vec{B})\right]d\tau \quad \left[\because \nabla\cdot(\vec{A}\times\vec{B})=\vec{B}\cdot(\nabla\times\vec{A})-\vec{A}\cdot(\nabla\times\vec{B})\right]$$

$$=\frac{1}{2\mu_0}\left[\int_V B^2 d\tau-\int_V\nabla\cdot(\vec{A}\times\vec{B})d\tau\right]$$

$$=\frac{1}{2\mu_0}\left[\int_V B^2 d\tau-\oint_S(\vec{A}\times\vec{B})\cdot d\vec{a}\right] \qquad \text{..................} \ (3)$$

where S is the surface bounding the volume V .

In Eq. (3) the larger the region we pick the greater is the contribution from the volume integral, and therefore the smaller is that of the surface integral.

In particular, if we integrate over all space, then the surface integral goes to zero, and we are left with

$$\therefore \boxed{W=\frac{1}{2\mu_0}\int_{\text{all space}}B^2\,d\tau}$$

Thus, the energy stored per unit volume (i.e. energy density) is equal to

$$U_B=\frac{B^2}{2\mu_0}$$

Sample Problem

- **Find the energy stored in a section of a length l of a long solenoid** (radius R and current I, n turns per unit length) **and hence obtain the expression for energy density.**

Solution:

The energy stored in a section of length l of a long solenoid is

$$W = \frac{1}{2\mu_0}\int_{\text{all space}} B^2\, d\tau$$

$$= \frac{1}{2\mu_0} B^2 \int d\tau$$

$$= \frac{1}{2\mu_0}(\mu_0 n I)^2 (\pi R^2 l)$$

$$\therefore \boxed{W = \frac{1}{2}\mu_0 n^2 \pi R^2 l I^2}$$

The energy density is

$$U_B = \frac{B^2}{2\mu_0}$$

$$= \frac{(\mu_0 n I)^2}{2\mu_0}$$

$$\therefore \boxed{U_B = \frac{1}{2}\mu_0 n^2 I^2}$$

Sample Problem

- **A long coaxial cable carries current I (the current flows down the surface of the inner cylinder, radius a, and back along the outer cylinder, radius b) as shown in Figure SP-2. Find the magnetic energy stored in a section of length . Also find the self-inductance.**

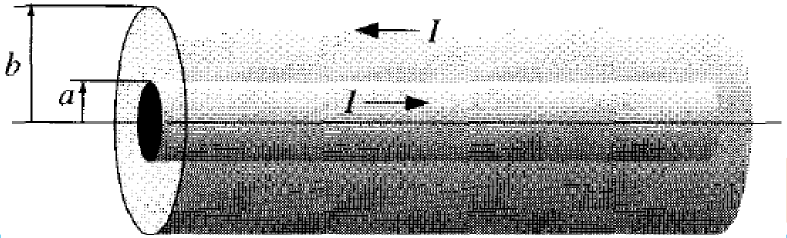


Figure SP-2

Ampere's law: $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$

$$\Rightarrow B(2\pi r) = \mu_0 I_{enc}$$

$$\Rightarrow B = \frac{\mu_0}{2\pi r} I_{enc}$$

Solution

According to Ampere's Law, the field between the cylinders is

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

Elsewhere, the field is zero.

Thus, the energy per unit volume is

$$\boxed{U_B = \frac{B^2}{2\mu_0} = \frac{1}{2\mu_0}\left(\frac{\mu_0 I}{2\pi r}\right)^2}$$

The energy in a cylindrical shell of length l, radius r, and thickness dr, then is

$$dW = \left[\frac{1}{2\mu_0}\left(\frac{\mu_0 I}{2\pi r}\right)^2\right](2\pi r dr\ l)$$

Therefore, the magnetic energy stored in a section of length l is

$$W = \frac{1}{2\mu_0}\left(\frac{\mu_0 I}{2\pi}\right)^2 (2\pi l)\int_a^b \frac{dr}{r}$$

$$\therefore \boxed{W = \frac{\mu_0 I^2 l}{4\pi}\ln\left(\frac{b}{a}\right) == \frac{1}{2}\left[\frac{\mu_0 l}{2\pi}\ln\left(\frac{b}{a}\right)\right] I^2}$$

The self-inductance is

$$L = \frac{\mu_0 l}{2\pi}\ln\left(\frac{b}{a}\right) \qquad \left[\because\ W = \frac{1}{2} L I^2\right]$$

Sample Problem & Multiple Choice Questions

ELECTROMAGNET IC INDUCTION

- A coil has an inductance of 53 mH and a resistance of 0.35 Ω .

  (a) If a 12 V emf is applied across the coil, how much energy is stored in the magnetic field after the current has built up to its equilibrium value?

  (b) After how many time constants will half this equilibrium energy be stored in the magnetic field?

Hint:

(a) $U_{B\infty} = \frac{1}{2} L I_\infty^{\ 2} = \frac{1}{2} L \left(\frac{\mathcal{E}}{R}\right)^2 = \frac{1}{2}\left(53\times10^{-3} H\right)\left(\frac{12\ V}{0.35\ \Omega}\right) = 31\ \mathrm{J}$

(b) $\because U_B = \frac{1}{2} U_{B\infty} \Rightarrow \frac{1}{2} L I^2 = \frac{1}{2}\left(\frac{1}{2} L I_\infty^{\ 2}\right) \Rightarrow I = \frac{1}{\sqrt{2}} I_\infty \Rightarrow \frac{\mathcal{E}_o}{R}\left(1 - e^{-\frac{t}{\tau}}\right) = \frac{1}{\sqrt{2}} \frac{\mathcal{E}_o}{R}$

$\Rightarrow \left(1 - e^{-\frac{t}{\tau}}\right) = \frac{1}{\sqrt{2}} \Rightarrow t \approx 1.2\tau$

- A 100 mH coil carries a current of 1A. Energy stored in the form of magnetic field is

  [a] 0.5 J [b] 0.1 J [c] 0.05 J [d] 1 J

  Ans : [c]

- The unit of $\boxed{\left(\frac{B^2}{2\mu_0}\right)}$ is

  [a] $\mathrm{J\ m^{-3}}$ [b] $\mathrm{J\ m^{-2}}$ [c] $\mathrm{W\ m^{-3}}$ [d] $\mathrm{W\ m^{-2}}$

  Ans : [a]

Text Books & References

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2. **R.A. Serway and J.W. Jewett**, Physics for Scientist and Engineers with Modern Physics
3. **Halliday and Resnick**, Fundamental of Physics
4. **D. Halliday, R. Resnick, and K. Krane** , Physics, Volume 2, Fourth Edition
5. **Hugh D.Young, Roger A. Freedman,** University Physics with Modern Physics, 13TH Edition


Thank you