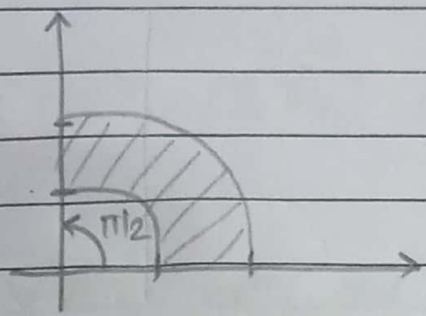
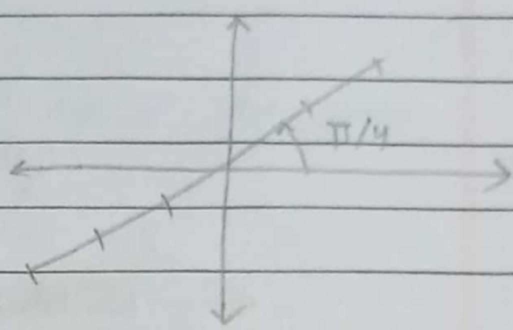
Plot the following sets:

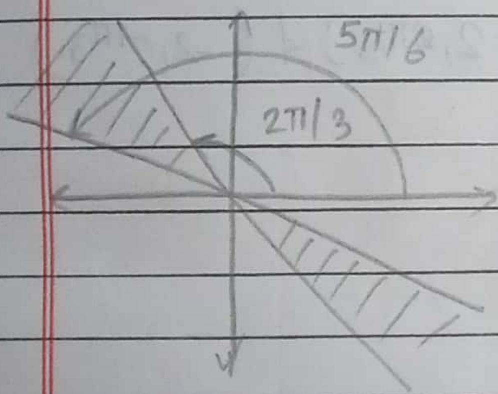
(i): $1 \le r \le 2, \quad 0 \le \theta \le \pi/2$



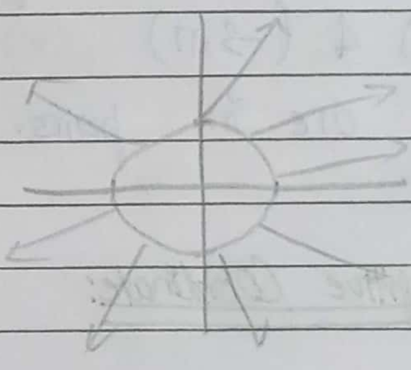
ii) $-3 \le r \le 2, \quad \theta = \pi/4$



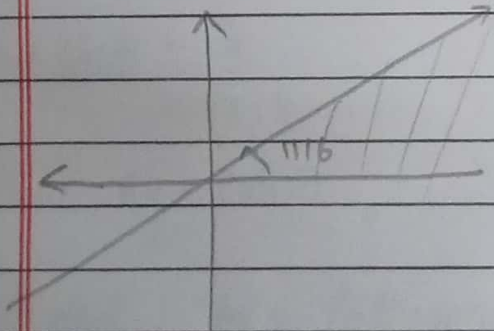
(iii) $2\pi/3 \le \theta \le 5\pi/6$



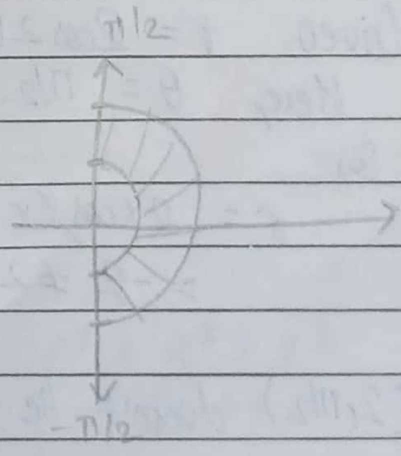
iv) $r \ge 1$



v) $0 \le \theta \le \pi/6, \; r \ge 0$



vi) $-\pi/2 \le \theta \le \pi/2, \quad 1 \le r \le 2$.



Date. No.

and

$\tan\theta = \frac{y}{x}$ $\therefore \theta = \tan^{-1}\left(\frac{y}{x}\right)$

To know the quadrant, we check the sign of x and y.

→ for 1st, $x = +ve$ & $y = +ve$. $\theta = \theta^\circ$
→ for 2nd, $x = -ve$ & $y = +ve$ $\theta = \pi - \theta$
→ for 3rd, $x = -ve$ & $y = -ve$ $\theta = \pi + \theta$
→ for 4th, $x = +ve$ & $y = -ve$ $\theta = 2\pi - \theta$.

Thus, the relationship while converting from

a) Cartesian to Polar:

$r = \sqrt{x^2+y^2}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

b): Polar to Cartesian

$x = r\cos\theta$
$y = r\sin\theta$

$r = \sqrt{x^2+y^2}$

Change the following equations into cartesian form.

i) $r=2$.

Soln.

$r = 2$ or, $\sqrt{x^2+y^2} = 2$

$\therefore$ ~~x^y~~ or, $x^2+y^2 = 4$.

Date. No.

(ii): $r\sin\theta = 2$

Soln.

We know,

$\sin\theta = \frac{y}{r}$.

So,

$r \cdot \frac{y}{r} = 2$ $\therefore y = 2$.

(iii) $r = 1 + 2r\cos\theta$

Soln:

Given,

$r = 1 + 2r\cos\theta$

We know, $r = \sqrt{x^2+y^2}$ and $r\cos\theta = x$.

So,

$\sqrt{x^2+y^2} = 1 + 2x$

Squaring both sides, we get.

$x^2+y^2 = (1+2x)^2$

or, $x^2+y^2 = 1 + 4x + 4x^2$

or, $4x^2+4x - y^2 + 1 = 0$

(iv): $r = 1 - \cos\theta$

Soln:

Given,

$r = 1 - \cos\theta$

Multiply both sides by r, we get.

Date. No.

$r^2 = r - r\cos\theta$

or, $x^2+y^2 = \sqrt{x^2+y^2} - x$

or, $x + x^2+y^2 = \sqrt{x^2+y^2}$

Squaring both sides, we get

$\cancel{x^2 + 2x^3y^2 + x^4+y^4 +}$

$\cancel{x^2}+x^4+y^4+2x^3+2xy^2+2x^2y^2 = \cancel{x^2}+y^2$

or, $x^4+y^4+2x^3+2xy^2+2x^2y^2-y^2=0.$

v) $r = 4\operatorname{cosec}\theta$

Soln,

Given,

$r = 4 \times \frac{1}{\sin\theta}$

or, $r = 4 \times \frac{r}{y}$ $\therefore y = 4.$

(vi) $r^2\sin 2\theta = 2$

Soln

Given,

$r^2 \cdot 2\sin\theta\cos\theta = 2$

or, $r^2 \cdot 2 \cdot \frac{y}{r} \cdot \frac{x}{r} = 2$

Or, $xy = 1$

Date. No.

(vii) $r = \frac{5}{\sin\theta - 2\cos\theta}$

Soln:

Given,

$r = \frac{5}{\sin\theta - 2\cos\theta}$

or, $r\sin\theta - 2r\cos\theta = 5$

or $y - 2x = 5$

(viii) ~~rsin~~ $r\sin\theta = \ln r + \ln\cos\theta$

Soln.

Given,

$r\sin\theta = \ln r + \ln\cos\theta$

or, $r\sin\theta = \ln(r\cos\theta)$

or, $y = \ln x$

$\therefore x = e^y$

(ix) $r\sin(\theta + \pi/6) = 2$

Soln:

$r\left\{\sin\theta\cdot\cos\frac{\pi}{6} + \cos\theta\cdot\sin\frac{\pi}{6}\right\} = 2$

or, $\frac{\sqrt3}{2}\cdot r\cdot\sin\theta + r\cdot\cos\theta\cdot\frac{1}{2} = 2$

or, $x + \sqrt3 y = 4$

Date. No.

Change the following polar points into cartesian points:

i) $(\sqrt{2}, \pi/4)$

Sol^n:

Given,

$r = \sqrt{2}$

$\theta = \pi/4$

So,

$x = r\cos\theta = \sqrt{2}\cdot\cos\frac{\pi}{4} = \sqrt{2}\times\frac{1}{\sqrt{2}} = 1$

$y = r\sin\theta = \sqrt{2}\cdot\sin\frac{\pi}{4} = \sqrt{2}\times\frac{1}{\sqrt{2}} = 1$

$\therefore (\sqrt{2}, \pi/4) = (1, 1)$

(ii): $(5, \tan^{-1}(4/3))$

Sol^n:

Given,

$r = 5$

$\theta = \tan^{-1}\left(\frac{4}{3}\right)$

$\therefore \tan\theta = \frac{4}{3}$

Since, $p = 4$, $b = 3$, $h = \sqrt{4^2+3^2} = 5 = r$.

Thus, $(5, \tan^{-1}(4/3)) = (3, 4)$.

Date. No.

Change the following cartesian equations to polar form.

i) $x = 2$.

Sol^n:

Given,

$x = 2$

or, $r\cos\theta = 2$

(ii) $\frac{x^2}{4} + (y-2)^2 = 1$

Sol^n:

Given,

$\frac{x^2}{4} + (y-2)^2 = 1$

or, $x^2 + 4(y-2)^2 = 4$

or, $x^2 + 4(y^2 - 4y + 4) = 4$

or, $x^2 + 4y^2 - 16y + 16 = 4$

or, $r^2\cos^2\theta + r^2\sin^2\theta + 3r^2\sin^2\theta - 16r\sin\theta + 16 = 4$

or, $r^2 + 3r^2\sin^2\theta - 16r\sin\theta + 12 = 0$

Change into polar points:

(i) (3,4)

Sol^n:

Given,

$x = 3 \qquad y = 4$

$\therefore r = \sqrt{4^2+3^2} = 5$

and $\tan\theta = \frac{y}{x} = \frac{4}{3}$

$\therefore \theta = \tan^{-1}\left(\frac{4}{3}\right)$

$\therefore (3,4) = \left(5, \tan^{-1}(4/3)\right)$

(ii) (-2,0)

Sol^n:

Given,

$x = -2 \qquad y = 0$

$\therefore r = \sqrt{(-2)^2+(0)^2} = 2$

$\theta = \tan^{-1}\left(\frac{0}{-2}\right) \qquad \theta = \tan^{-1}(-0)$

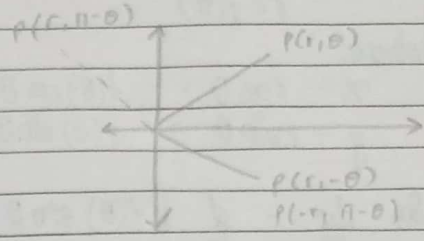
Since the point lies on negative x-axis, $\theta = \pi$

$\therefore (-2,0) \approx (2,\pi)$

Symmetry:

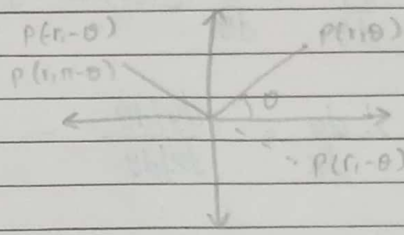
a) Symmetry about x-axis (initial ray):

If a polar curve is symmetrical about x-axis, both the points (r,θ) and $(r,-\theta)$ or $(-r, \pi-\theta)$ lie on the curve.



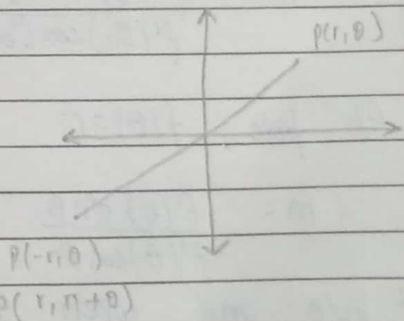
b) Symmetry about y-axis:

If a polar ~~coordinate~~ curve is symmetrical about y-axis, both the points (r,θ) and $(-r,-\theta)$ or $(r,\pi-\theta)$ lie on the curve.



(c): Symmetry about origin:

If a polar curve is symmetrical about origin, both the points (r,θ) and $(-r,\theta)$ or $(r, \pi+\theta)$ lie on the curve.



Slope of a polar curve:

The equation of polar curve is given by.

$$r = f(\theta)$$

where,

$$x = r\cos\theta = f(\theta)\cos\theta$$
$$y = r\sin\theta = f(\theta)\sin\theta$$

So, $\dfrac{dy}{d\theta} = \dfrac{d}{d\theta}[f(\theta)\sin\theta] = f'(\theta)\sin\theta + \cos\theta \cdot f(\theta)$

$\dfrac{dx}{d\theta} = \dfrac{d}{d\theta}[f(\theta).\cos\theta] = f'(\theta)\cos\theta - \sin\theta \cdot f(\theta)$

$\therefore \dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta} = \dfrac{f'(\theta)\sin\theta + \cos\theta \cdot f(\theta)}{f'(\theta)\cos\theta - \sin\theta\, f(\theta)}$

$\therefore$ Slope at $\theta = \theta_0$

$$\therefore m = \frac{f'(\theta_0)\sin\theta_0 + \cos\theta_0\, f(\theta_0)}{f'(\theta_0)\cos\theta_0 - \sin\theta_0\, f(\theta_0)}$$

At pole, $f(\theta) = 0$

$$\therefore m = \frac{f'(\theta)\sin\theta}{f'(\theta)\cos\theta} = \tan\theta$$

At pole and when $\theta = \theta_0$ $\quad m = \tan\theta_0$