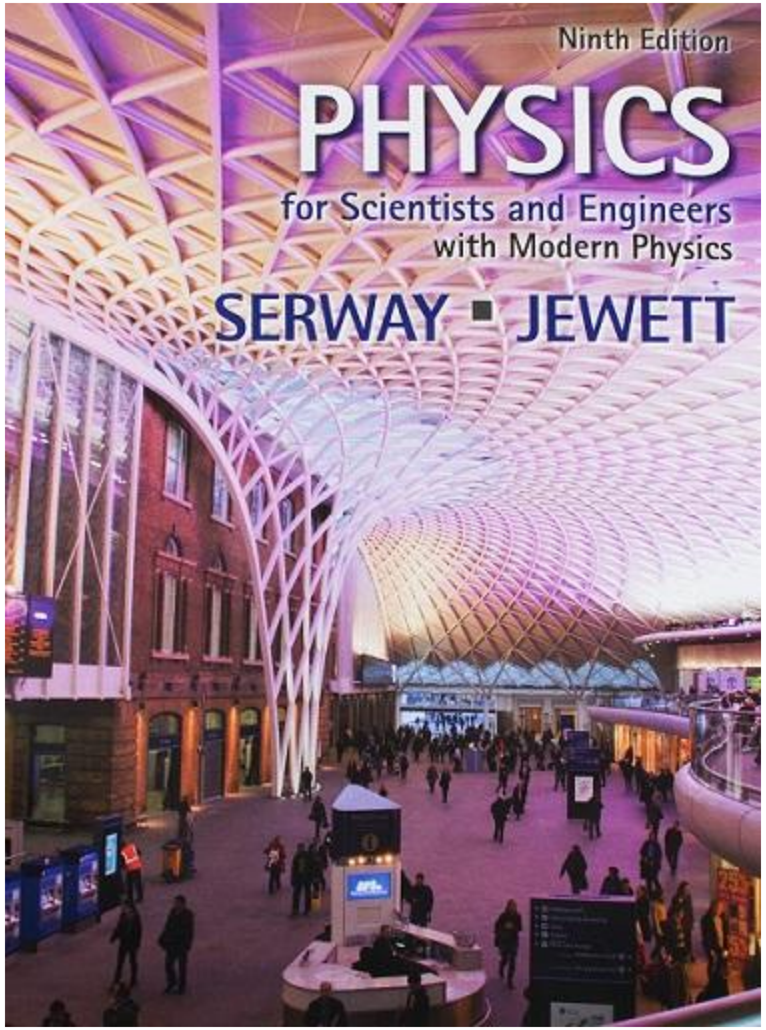
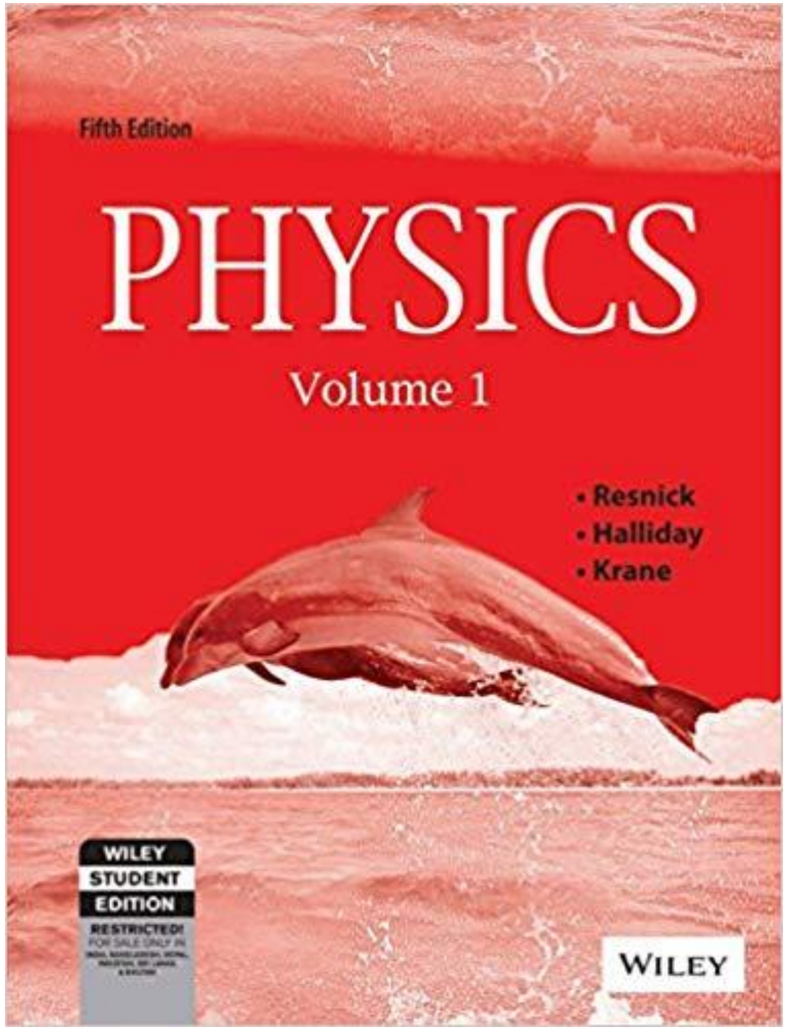
General Physics I (PHYS 101)





Ganesh Kuwar Chhetri

Dept. of Physics, School of Science, Kathmandu University

HEAT TRANSFER

Course Outline

- **Heat Transfer**
  - ➢ **Conduction**
  - ➢ **Convection**
  - ➢ **Radiation**
- **Stefan's Boltzmann Law**
- **Wien's Displacement Law**
- **Rayleigh-Jeans Law**
- **Planck's Radiation Law**

Heat Tranfer

- The three mechanisms of heat transfer are conduction, convection, and radiation.
- Conduction occurs within a body or between two bodies in contact.
- Convection depends on motion of mass from one region of space to another.
- Radiation is heat transfer by electromagnetic radiation, such as sunshine, with no need for matter to be present in the space between bodies

Conduction

- Conduction is the transfer of heat within materials without bulk motion of the materials.
- It is the most significant means of heat transfer within a solid or between solid objects in thermal contact.
- Conduction can be viewed as an exchange of kinetic energy between colliding molecules or electrons.
- Heat transfer occurs only between regions that are at different temperatures, and the direction of heat flow is always from higher to lower temperature.
- The rate of thermal conduction depends on the properties of the substance being heated.
- Metals are good conductors of heat. Cork, paper, gases are poor conductors.
- In a good conductor, such as copper, conduction takes place both by means of the vibration of atoms and by means of the motion of free electrons.

Heat Transfer Through a Conducting Slab

Heat Transfer Through a Conducting Slab

- Consider a thin slab of homogeneous material of thickness Δx and cross-sectional area A. One face of the slab is at a temperature T_C , and the other face is at a temperature $T_h > T_C$ (Figure C-1).
- Let Q be the energy that is transferred as heat through the slab, from its face at temperature T_h to its face at temperature T_C in time t. Experiment shows that the conduction rate *H* (the amount of energy transferred per unit time) is

$$H = \frac{Q}{t} = kA\frac{T_h - T_C}{\Delta x} = kA\frac{\Delta T}{\Delta x}$$

For a slab of infinitesimal thickness *dx* and temperature difference *dT*, we can write the law of thermal conduction as

$$\boxed{H = kA\left|\frac{dT}{dx}\right|}$$

where k is the thermal conductivity of the material and $|dT/dx|$ is the temperature gradient (the rate at which temperature varies with position)

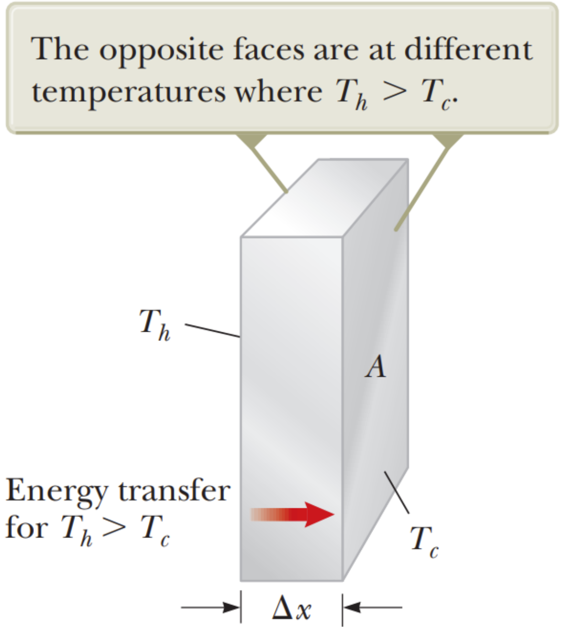


Figure C-1

Experimentlly

$$\frac{Q}{t} \propto A\frac{\Delta T}{\Delta x}$$

$$H \equiv P = kA\frac{\Delta T}{\Delta x}$$

Thermal Conductivity

Thermal Conductivity

- Thermal conductivity is the property of a material's ability to conduct heat.
- Heat transfer occurs at higher rate across materials of high thermal conductivity than across materials of low conductivity.
- Thermal Conductivity: $$k = \frac{H}{A\left|\frac{dT}{dx}\right|}$$ → Thermal conductivity of a material is defined as heat current per unit area per unit temperature gradient.
- The SI unit of thermal conductivity (k) is the watt per meter kelvin (W/m K).
- Thermal conductivity depends on the nature of the material.
- Substances that are good thermal conductors have large thermal conductivity values, whereas good thermal insulators have low thermal conductivity values.
- Table C lists thermal conductivities for various substances.

Table C

Thermal Conductivities

Substance	Thermal Conductivity (W/m · °C)
Metals (at 25°C)	
Aluminum	238
Copper	397
Gold	314
Iron	79.5
Lead	34.7
Silver	427
Gases (at 20°C)	
Air	0.023 4
Helium	0.138
Hydrogen	0.172
Nitrogen	0.023 4
Oxygen	0.023 8

Substance	Thermal Conductivity (W/m · °C)
Nonmetals (approximate values)	
Asbestos	0.08
Concrete	0.8
Diamond	2 300
Glass	0.8
Ice	2
Rubber	0.2
Water	0.6
Wood	0.08

Energy Transfer Through Two Slabs

Energy Transfer Through Two Slabs

Two slabs of thickness L_1 and L_2 and thermal conductivities k_1 and k_2 are in thermal contact with each other as shown in Figure ET-1. The temperatures of their outer surfaces are T_C and T_H, respectively, $T_H > T_C$. Determine the temperature at the interface and the rate of energy transfer by conduction through an area A of the slabs in the steady-state condition.

Hint:

✓ At steady state, the rate of energy transfer through slab 1 equals the rate of energy transfer through slab 2.

$$H_2 = H_1$$

$$k_2 A\left[\frac{T_h - T}{L_2}\right] = k_1 A\left[\frac{T - T_C}{L_1}\right]$$

$$\Rightarrow T = \frac{k_1 L_2 T_C + k_2 L_1 T_h}{k_1 L_2 + k_2 L_1}$$

Rate of heat transfer,

$$H = H_1 = k_1 A\left[\frac{T - T_C}{L_1}\right] = k_1 A\left[\frac{\left(\frac{k_1 L_2 T_C + k_2 L_1 T_h}{k_1 L_2 + k_2 L_1}\right) - T_C}{L_1}\right] = \frac{A\left(T_h - T_C\right)}{\frac{L_1}{k_1} + \frac{L_2}{k_2}}$$

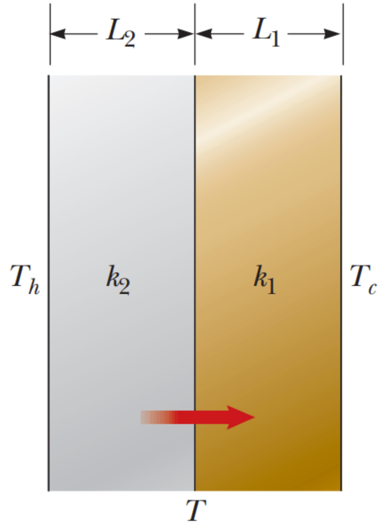


Figure ET-1
Energy transfer by conduction through two slabs in thermal contact with each other.

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Convection

- Convection is the transfer of heat by mass motion of a fluid from one region of space to another.
- Convection is usually the dominant form of heat transfer in liquids and gases.
- **Convective heat transfer is a very complex process, and there is no simple equation to describe it.**

Examples:

Hot-air and hot-water home heating system

The flow of blood in the body

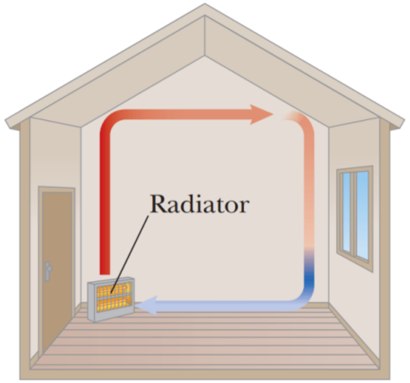

Figure CN-1
Convection currents are set up in a room warmed by a radiator.

Forced Convection

If the fluid is circulated by a blower or pump, the process is called forced convection.

The most important mechanism for heat transfer within the human body (needed to maintain nearly constant temperature in various environments) is forced convection of blood, with the heart serving as the pump.

Free Convection

If the flow is caused by differences in density due to thermal expansion, such as hot air rising, the process is called natural convection or free convection.

Free convection in the atmosphere plays a dominant role in determining the daily weather, and convection in the oceans is an important global heat-transfer mechanism.

Radiation

- Radiation is the transfer of heat by electromagnetic waves such as visible light, infrared, and ultraviolet radiation.
- Everyone has felt the warmth of the sun's radiation and the intense heat from a charcoal grill or the glowing coals in a fireplace. Most of the heat from these very hot bodies reaches you not by conduction or convection in the intervening air but by radiation. This heat transfer would occur even if there were nothing but vacuum between you and the source of heat.
- No medium is required for heat transfer via radiation — the radiation can travel through vacuum from, say, the Sun to you.

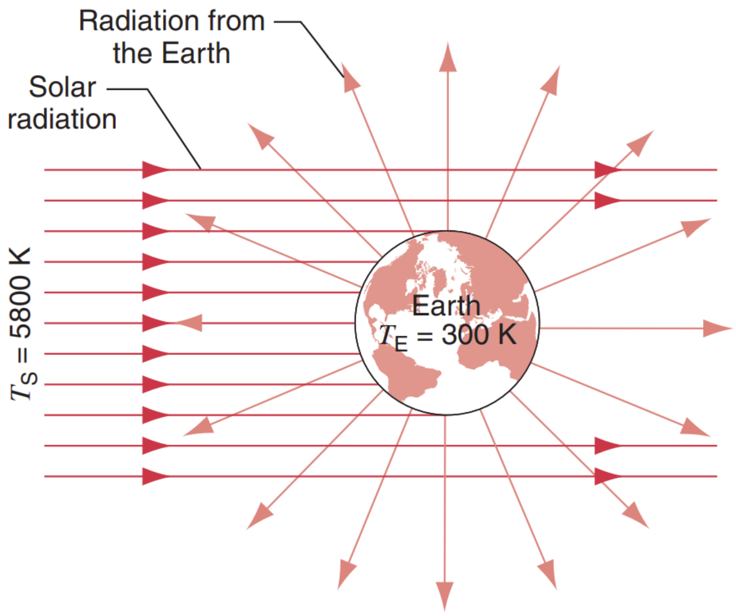

Figure BB-1

Solar radiation is intercepted by the Earth and is (mostly) absorbed. The temperature T_E of the Earth adjusts itself to a value at which the Earth's heat loss by radiation is just equal to the solar heat that it absorbs.

Stefan-Boltzmann Law

Stefan-Boltzmann Law

- The rate P_{rad} at which an object emits energy via electromagnetic radiation depends on the object's surface area A and the temperature T of that area in kelvins and is given by

$$\boxed{P_{rad} = \sigma \varepsilon A T^4} \quad \ldots\ldots\ldots\ldots\ldots \quad \text{(SB-1)}$$

Here σ = 5.6704 ×10^{-8} W/m^2·K^4 is called the Stefan–Boltzmann constant after Josef Stefan (who discovered Eq. SB-1experimentally in 1879) and Ludwig Boltzmann (who derived it theoretically soon after). The symbol ε represents the emissivity of the object's surface, which has a value between 0 and 1, depending on the composition of the surface.

- The rate P_{abs} at which an object absorbs energy via thermal radiation from its environment, which we take to be at uniform temperature T_{env} (in kelvins), is

$$\boxed{P_{abs} = \sigma \varepsilon A T_{env}^4} \quad \ldots\ldots\ldots\ldots\ldots \quad \text{(SB-2)}$$

- If an object is at a temperature T and its surroundings are at an average temperature T_{env} , the net rate of energy gained or lost by the object as a result of radiation is

$$\boxed{P_{net} = \sigma \varepsilon A \left(T^4 - T_{env}^4\right)} \quad \ldots\ldots\ldots \quad \text{(SB-3)}$$

- When an object is in equilibrium with its surroundings, it radiates and absorbs energy at the same rate and its temperature remains constant. When an object is hotter than its surroundings, it radiates more energy than it absorbs and its temperature decreases.

Emissivity

- A dimensionless number between 0 and 1
- The ratio of the rate of radiation from a particular surface to the rate of radiation from an equal area of an ideal radiating surface at the same temperature.
- A surface with the maximum emissivity of 1.0 is said to be a blackbody radiator, but such a surface is an ideal limit and does not occur in nature.

Black Body

- An object for which emissivity is unity is often referred to as a black body (ideal absorber). It absorbs all the energy incident on it.
- A body that is good absorber must also be a good emitter.
- A good approximation of a black body is a small hole leading to the inside of a hollow object as shown in Figure BB-1.
- Any radiation incident on the hole from outside the cavity enters the hole and is reflected a number of times on the interior walls of the cavity; hence, the hole acts as a perfect absorber.

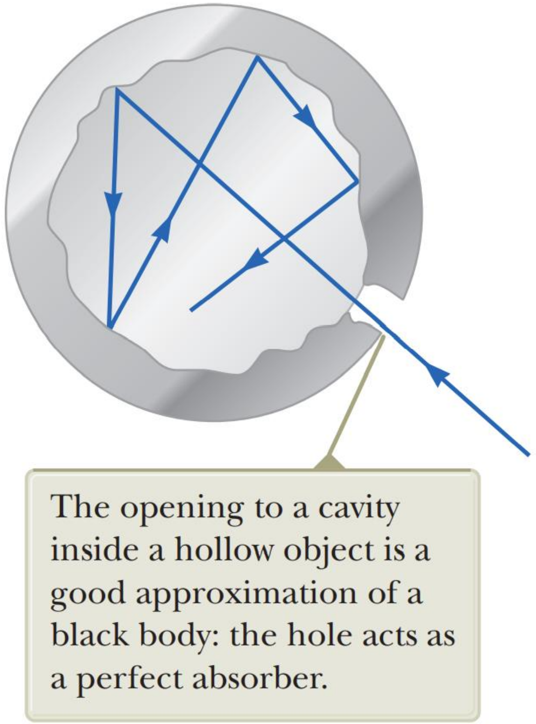

Figure 40.1
A physical model of a black body.

Black Body Radiation

- An object at any temperature emits electromagnetic waves in the form of thermal radiation from its surface. The characteristics of this radiation depend on the temperature and properties of the object's surface. Careful study shows that the radiation consists of a continuous distribution of wavelengths from all portions of the electromagnetic spectrum. If the object is at room temperature, the wavelengths of thermal radiation are mainly in the infrared region and hence the radiation is not detected by the human eye. As the surface temperature of the object increases, the object eventually begins to glow visibly red, like the coils of a toaster. At sufficiently high temperatures, the glowing object appears white, as in the hot tungsten filament of an incandescent lightbulb.
- A black body is an ideal system that absorbs all radiation incident on it. The electromagnetic radiation emitted by the black body is called blackbody radiation.
- The continuous-spectrum radiation that a blackbody emits is called blackbody radiation
- The spaces between lumps of hot charcoal (Figure BB-2) emit light that is very much like blackbody radiation.

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Figure BB-2
The glow emanating from the spaces between these hot charcoal briquettes is, to a close approximation, blackbody radiation. The color of the light depends only on the temperature of the briquettes.

Black Body Radiation

Black Body Radiation

- Figure BBR-1 shows how the intensity of blackbody radiation varies with temperature and wavelength
- The following two consistent experimental findings were seen especially significant..

1. The total power of the emitted radiation increases with temperature.

$$P_{rad} = \sigma \varepsilon A T^4$$

Stefan's Boltzmann Law

where P is the power in watts radiated at all wavelengths from the surface of an object, σ = 5.6704 $\times 10^{-8}$ W/m^2 $\cdot$K^4 is the Stefan–Boltzmann constant, A is the surface area of the object in square meters, e is the emissivity of the surface, and T is the surface temperature in kelvins.

2. The peak of the wavelength distribution shifts to shorter wavelengths as the temperature increases. This behaviour is described by the following relationship, called **Wien's displacement law**:

$$\lambda_{\max} T = 2.898 \times 10^{-3} \text{ m} \cdot \text{K}$$

where λ_{max} is the wavelength at which the curve peaks and T is the absolute temperature of the surface of the object emitting the radiation. The wavelength at the curve's peak is inversely proportional to the absolute temperature; that is, as the temperature increases, the peak is "displaced" to shorter wavelengths (Figure BBR-1)

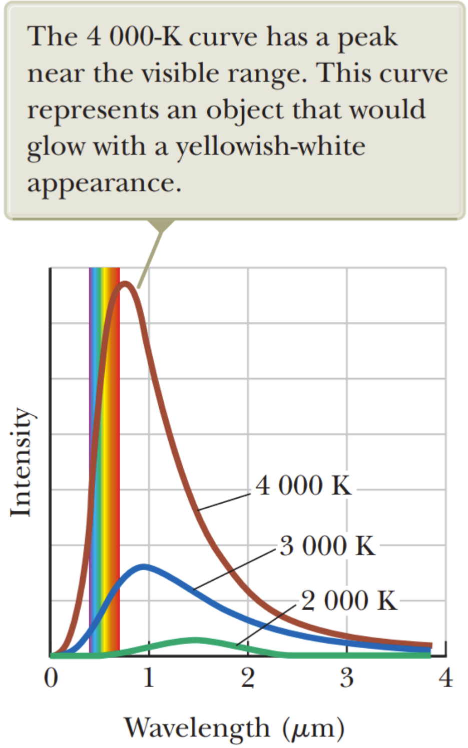

Figure BBR-1
Intensity of blackbody radiation versus wavelength at three temperatures.

Rayleigh-Jeans Law

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Rayleigh-Jeans Law

- A successful theory for blackbody radiation must predict the shape of the curves in Figure BBR-1, the temperature dependence expressed in Stefan's law, and the shift of the peak with temperature described by Wien's displacement law. Early attempts to use classical ideas to explain the shapes of the curves in Figure BBR-1 failed.

One of the early attempt

- To describe the distribution of energy from a black body, we define $I(\lambda, T)\, d\lambda$ to be the intensity, or power per unit area, emitted in the wavelength interval $d\lambda$.

  The result of a calculation based on a classical theory of blackbody radiation known as the Rayleigh–Jeans law is

$$I(\lambda, T) = \frac{2\pi c k_B T}{\lambda^4}$$

 where k_B is Boltzmann's constant.

- An experimental plot of the blackbody radiation spectrum, together with the theoretical prediction of the Rayleigh–Jeans law, is shown in Figure BBR-2. At long wavelengths, the Rayleigh–Jeans law is in reasonable agreement with experimental data, but at short wavelengths, major disagreement is apparent.

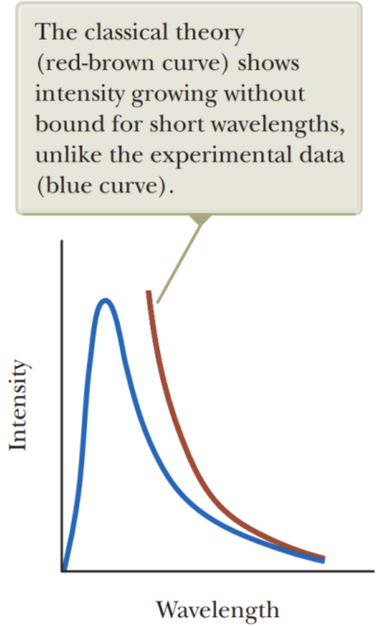

Figure BBR-2
Comparison of experimental results and the curve predicted by the Rayleigh– Jeans law for the distribution of blackbody radiation

Planck and the Quantum Hypothesis

- Finally, in 1900, the German physicist Max Planck succeeded in deriving a function, now called the Planck radiation law, that agreed very well with experimental intensity distribution curves.
- In his derivation he made what seemed at the time to be a crazy assumption. Planck assumed that electromagnetic oscillators (electrons) in the walls of Rayleigh's box vibrating at a frequency f could have only certain values of energy equal to nhf , where n = 0,1,2,3…. and h is the Planck's constant.
- These oscillators were in equilibrium with the electromagnetic waves in the box, so they both emitted and absorbed light. His assumption gave quantized energy levels and said that the energy in each normal mode was also a multiple of This was in sharp contrast to Rayleigh's point of view that each normal mode could have any amount of energy.

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Max Planck German Physicist (1858–1947) Planck introduced the concept of "quantum of action" (Planck's constant, h) in an attempt to explain the spectral distribution of blackbody radiation, which laid the foundations for quantum theory. In 1918, he was awarded the Nobel Prize in Physics for this discovery of the quantized nature of energy

Planck Radiation Law

Planck Radiation Law

- A mathematical relationship formulated in 1900 by German physicist Max Planck to explain the spectral-energy distribution of radiation emitted by a black body.
- A theoretical expression for the wavelength distribution that agreed remarkably well with the experimental curves in Figure BBR-1:

$$I(\lambda,T)=\frac{2\pi hc^2}{\lambda^5\left(e^{\frac{hc}{\lambda k_B T}}-1\right)} \qquad \text{..........} \quad \text{(PR)}$$

where $h = 6.626\times10^{-34}\ \mathrm{J\cdot s}$ is the Planck's constant, a fundamental constant of nature.

At long wavelengths, Equation PR reduces to the Rayleigh–Jeans expression, and at short wavelengths, it predicts an exponential decrease in with decreasing wavelength, in agreement with experimental results.

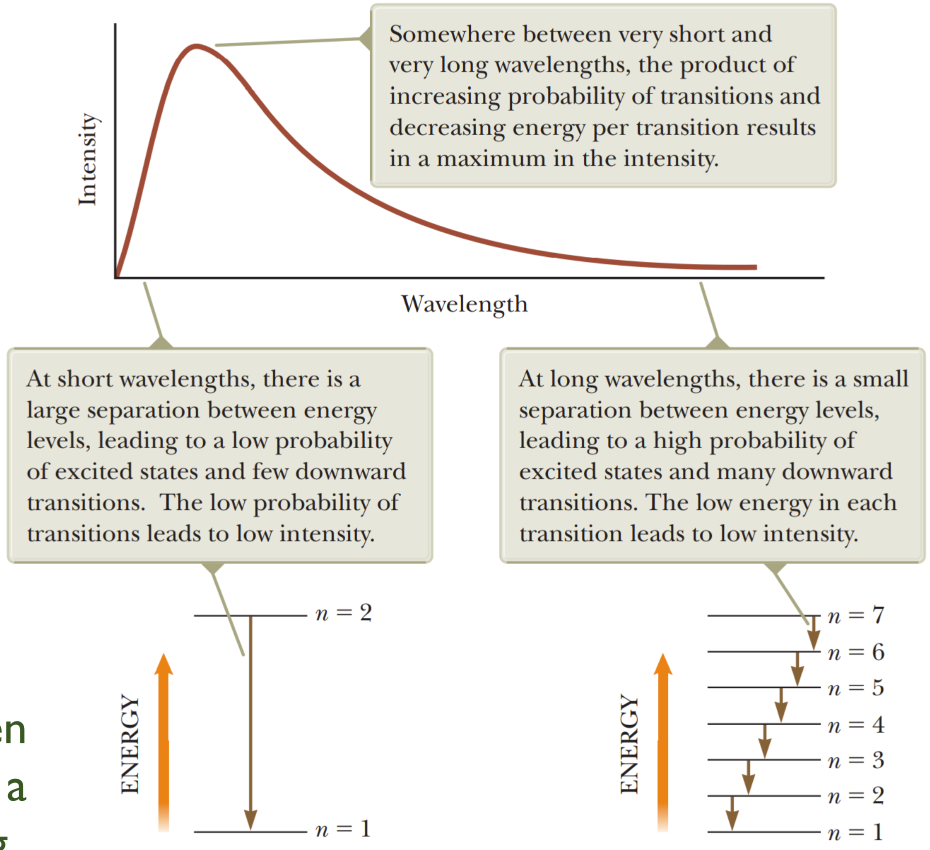

Figure BBR-3

In Planck's model, the average energy associated with a given wavelength is the product of the energy of a transition and a factor related to the probability of the transition occurring.

Planck Radiation Law

Planck Radiation Law

Planck's radiation law is obtained by using the following postulates:

1. A black body radiation chamber is filled up not only with radiation, but also with simple harmonic oscillators or resonators, of the molecular dimensions; which can not have any value of energy, but only energies given by $E = nh\upsilon$. where n=0,1,2,3,…

   h = Planck's constant

   υ = frequency of the oscillator

2. The oscillators can not radiate or absorb energy continuously; but an oscillator of frequency υ can only radiate or absorb energy in units or quanta of magnitude $h\upsilon$.

- The **average energy** of a Planck's oscillator is given by

$$\bar{E} = \frac{h\upsilon}{e^{\frac{h\upsilon}{kT}} - 1} \qquad \text{..........} \qquad (1)$$

- The **number of resonators per unit volume** in the frequency range υ and $\upsilon + d\upsilon$ is given by

$$N_\upsilon d\upsilon = \frac{8\pi\ \upsilon^2 d\upsilon}{c^3} \qquad \text{..........} \qquad (2)$$

Planck Radiation Law

Planck Radiation Law

- Energy density belonging to the range $d\upsilon$:

$$E_\upsilon d\upsilon = N_\upsilon d\upsilon \times \bar{E}$$

$$= \frac{8\pi\ \upsilon^2 d\upsilon}{c^3} \times \frac{h\upsilon}{e^{\frac{h\upsilon}{kT}} - 1}$$

$$E_\upsilon d\upsilon = \frac{8\pi\ h\upsilon^3}{c^3} \frac{1}{e^{\frac{h\upsilon}{kT}} - 1} d\upsilon \qquad \text{..........} \qquad (3)$$

- Energy density belonging to the range $d\lambda$:

$$E_\lambda d\lambda = \frac{8\pi\ h}{c^3}\left(\frac{c}{\lambda}\right)^3 \frac{1}{e^{\frac{hc}{\lambda kT}} - 1}\left|-\frac{c}{\lambda^2} d\lambda\right|$$

Using, $\upsilon = \frac{c}{\lambda}$

$$E_\lambda d\lambda = \frac{8\pi\ hc}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} d\lambda \qquad \text{..........} \qquad (4)$$

Rayleigh-Jean's Law

- For high temperature and larger wavelength,

$$e^{\frac{hc}{\lambda kT}} \approx 1 + \frac{hc}{\lambda kT}$$

- So, $E_\lambda d\lambda = \frac{8\pi\ hc}{\lambda^5} \frac{1}{\frac{hc}{\lambda kT}} d\lambda$

$$\therefore\ E_\lambda d\lambda = \frac{8\pi kT}{\lambda^4} d\lambda$$

This is Rayleigh-Jeans law.

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Derivation of Wien's Displacement Law from Planck Radiation Law

Planck Radiation Law

- Energy density belonging to the range $d\lambda$: $E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda kT}}-1}d\lambda$

- To calculate the maximum energy radiated by the chamber, we have

$$\frac{dE_\lambda}{d\lambda}=0$$

$$\Rightarrow \frac{d}{d\lambda}\left[\frac{8\pi hc}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda kT}}-1}\right]=0$$

$$\Rightarrow \frac{d}{d\lambda}\left[\frac{1}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda kT}}-1}\right]=0$$

$$\Rightarrow \left(-\frac{5}{\lambda^6}\right)\frac{1}{e^{\frac{hc}{\lambda kT}}-1}+\frac{1}{\lambda^5}\left[\frac{-1}{\left(e^{\frac{hc}{\lambda kT}}-1\right)^2}\left(e^{\frac{hc}{\lambda kT}}\right)\left(\frac{hc}{kT}\right)\left(-\frac{1}{\lambda^2}\right)\right]=0$$

$$\Rightarrow -5-\frac{hc}{\lambda kT}\frac{e^{\frac{hc}{\lambda kT}}}{e^{\frac{hc}{\lambda kT}}-1}=0$$

- Letting $\frac{hc}{\lambda kT}=x$ we get

$$5-\frac{xe^x}{e^x-1}=0$$

$$\Rightarrow x\approx 4.98$$

$$\Rightarrow \lambda T=\frac{hc}{kx}=0.002898 \text{ m K}=0.2898 \text{ cm K}$$

Hence the product of wavelength corresponds to maximum radiation energy and the absolute temperature is constant i.e.

$$\boxed{\lambda_m T=\text{constant}=0.2898 \text{ cm K}}$$

This is **Wien's Displacement Law**.

Derivation of Stefan-Boltzmann Law from Planck Radiation Law

Planck Radiation Law

- Energy density belonging to the range $d\upsilon$: $E_\upsilon d\upsilon = \frac{8\pi h\upsilon^3}{c^3}\frac{1}{e^{\frac{h\upsilon}{kT}}-1}d\upsilon$ ....(PRL-1)

- The total radiation energy per unit volume emitted by the black body radiation chamber over all range of frequency or wavelength can be calculated by integrating equation (PRL-1) from the limit $\nu \to 0$ to $\nu \to \infty$ i.e

$$E = \int_0^\infty E_\upsilon d\upsilon$$

$$= \int_0^\infty \frac{8\pi h\upsilon^3}{c^3}\frac{1}{e^{\frac{h\upsilon}{kT}}-1}d\upsilon$$

- Let $\frac{h\upsilon}{kT} = x \Rightarrow \upsilon = \frac{kT}{h}x$

Hence $$E = \frac{8\pi h}{c^3}\int_0^\infty \left(\frac{kTx}{h}\right)^3 \frac{1}{e^x-1}\left(\frac{kT}{h}\right)d\upsilon$$

$$= \frac{8\pi k^4T^4}{c^3h^3}\int_0^\infty \frac{x^3}{e^x-1}dx$$

But the integral is the standard integral and has the value

$$\int_0^\infty \frac{x^3}{e^x-1}dx = \frac{\pi^4}{15}$$

Therefore

$$E = \frac{8\pi k^4T^4}{c^3h^3}\left(\frac{\pi^4}{15}\right) = \frac{8\pi^5 k^4T^4}{15c^3h^3}$$

$$\sigma = \frac{2\pi^5 k^4}{15c^2h^3} = 5.67\times10^{-8}\,\mathrm{W/m^2{\cdot}K^4}$$

The energy per second radiated by a unit area of the black body chamber is effectively equal to $P_{rad} = \frac{c}{4}E$

Therefore, the rate of energy radiated per unit area is

$$P_{rad} = \frac{c}{4}\left[\frac{8\pi k^4T^4}{c^3h^3}\left(\frac{\pi^4}{15}\right)\right] = \frac{2\pi^5 k^4}{15c^2h^3}T^4 = \sigma T^4$$

$\therefore \boxed{P_{rad} = \sigma T^4}$ This is the **Stefan-Boltzmann law**.

References

1. **R.A. Serway and J.W. Jewett**, Physics for Scientist and Engineers with Modern Physics

2. **Halliday and Resnick**, Fundamental of Physics

3. **D. Halliday, R. Resnick, and K. Krane** , Physics, Volume 1, Fifth Edition

4. **Hugh D.Young, Roger A. Freedman,** University Physics with Modern Physics, 13TH Edition

Thank you