Center of Gravity, Mass and Centroid

* Center of mass

The point where the mass of the body is supposed to be concentrated ~~betw~~ is called center of mass.

A line passing through the center of mass of body have equal moment on both side.

* Center of gravity:

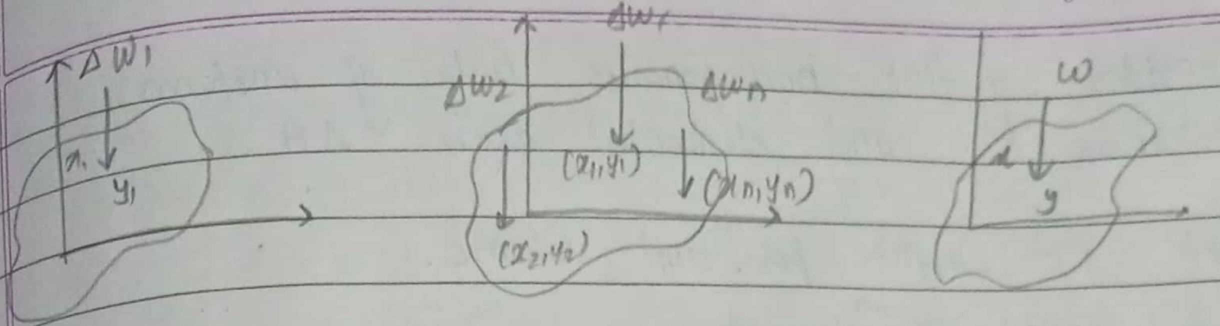
The point where the resultant gravitation force acts on the body is called center of gravity.

* Centroid:

The geometric center of the body is called centroid.

→ For a body having uniform density, center of mass and centroid coincide.

→ For a body having constant gravitational field and uniform density, center of mass, centroid and center of gravity coincide.



Let us consider a flat horizontal plate divided into 'n' small elements.

The force exerted by the earth on 'n' number of elements of the plate are denoted as $\Delta W_1, \Delta W_2, \ldots, \Delta W_n$. These all acts towards the center but are assumed parallel.

Let W be the total weight of the plate.

$$W = \Delta W_1 + \Delta W_2 + \ldots + \Delta W_n$$

To find $\bar{x}$ and $\bar{y}$ coordinates of point G (centre of gravity).

We know,

moments of W about y and x axes are equal to the sum of the corresponding moments of weight of elements.

$$\Sigma M_y = \bar{x} W = x_1 \Delta W_1 + x_2 \Delta W_2 + \ldots + x_n \Delta W_n.$$

$$\therefore \bar{x} = \frac{x_1 \Delta W_1 + x_2 \Delta W_2 + x_3 \Delta W_3 + \ldots + x_n \Delta W_n}{W}$$

$$\Sigma M_x = \bar{y} W = y_1 \Delta W_1 + y_2 \Delta W_2 + \ldots + y_n \Delta W_n$$

$$\therefore \bar{y} = \frac{y_1 \Delta W_1 + y_2 \Delta W_2 + y_3 \Delta W_n + \ldots + y_n \Delta W_n}{W}.$$

Date :
Page:

Consider a flat homogeneous body of uniform thickness 't'. and elemental area ΔA.

Let γ = weight per unit volume.

So, weight of an element

$$\Delta W = \gamma t \Delta A$$

Hence, the total weight of entire plate.

$$W = \gamma t A$$

Here,

A = total area of the plane.

Putting W and ΔW in the centre of gravity equation, dividing through γt, we get

$$\bar{x} = \frac{x_1 \Delta A_1 \gamma t + x_2 \Delta A_2 \gamma t + \cdots + x_n \Delta A_n \gamma t}{A \gamma t}$$

$$\therefore \bar{x} = \frac{x_1 \Delta A_1 + x_2 \Delta A_2 + \cdots + x_n \Delta A_n}{A}$$

$$\bar{y} = \frac{y_1 \Delta A_1 \gamma t + y_2 \Delta A_2 \gamma t + \cdots + y_n \Delta A_n \gamma t}{A \gamma t}$$

$$\therefore \bar{y} = \frac{y_1 \Delta A_1 + y_2 \Delta A_2 + y_3 \Delta A_3 + \cdots + y_n \Delta A_n}{A}$$

Date :

Page:

First moment of area about y-axis $(Q_y) = \bar{x} A$

$= x_1 \Delta A_1 + x_2 \Delta A_2 + \cdots + x_n \Delta A_n.$

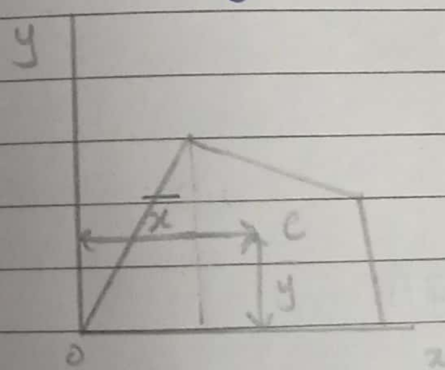
First moment of area about x-axis $(Q_x) = \bar{y} A$

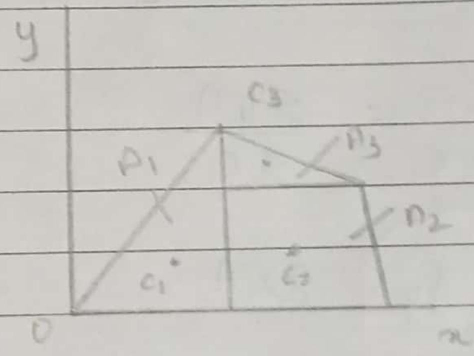
$= y_1 \Delta A_1 + y_2 \Delta A_2 + \cdots + y_n \Delta A_n.$

Here $(\bar{x}, \bar{y})$ gives centroid C for area A.

If the plate is not homogeneous, ~~the~~ G cannot be determined.

If plate is homogeneous with uniform thickness, the center of gravity coincides with centroid C of its area.





For composite area,

We can express the first moment Q_y of the composite area with respect to x-axis as:

i) the product of $\bar{X}$ and the total area.

ii) the sum of the first moments of the elementary areas with respect to the y-axis.

Date :
Page:

The first moment about y-axis.

$$Q_y = \bar{X}(A_1 + A_2 + \cdots + A_n)$$
$$= \bar{x}_1 A_1 + \bar{x}_2 A_2 + \cdots + \bar{x}_n A_n = \Sigma \bar{x} A$$

$$\therefore \bar{X} = \frac{\bar{x}_1 A_1 + \bar{x}_2 A_2 + \cdots + \bar{x}_n A_n \ \{\Sigma \bar{x} A\}}{(A_1 + A_2 + \cdots + A_n)}$$
$$= \frac{\Sigma \bar{x} A}{\Sigma A}$$

The first moment about x-axis.

$$Q_x = \bar{y}(A_1 + A_2 + \cdots + A_n)$$
$$= \bar{y}_1 A_1 + \bar{y}_2 A_2 + \cdots + \bar{y}_n A_n = \Sigma \bar{y} A.$$

$$\therefore \bar{y} = \frac{\bar{y}_1 A_1 + \bar{y}_2 A_2 + \cdots + \bar{y}_n A_n}{A_1 + A_2 + A_3 + \cdots + A_n}$$

$$= \frac{\Sigma \bar{y} A}{\Sigma A}$$

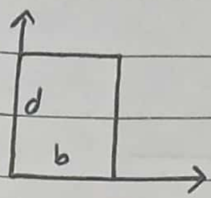
So, $Q_y = \Sigma \bar{x} A$ $\qquad$ $Q_x = \Sigma \bar{y} A$

$$\bar{X} = \frac{\Sigma \bar{x} A}{\Sigma A} \qquad \bar{y} = \frac{\Sigma \bar{y} A}{\Sigma A}.$$

Centroid of Composite Area

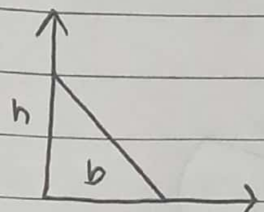
Shape | Figure | Centroid $G(\bar{x}, \bar{y})$

Rectangle



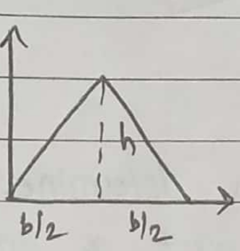
$\bar{x} = (b/2)$
$\bar{y} = (d/2)$

Triangle (⊿)



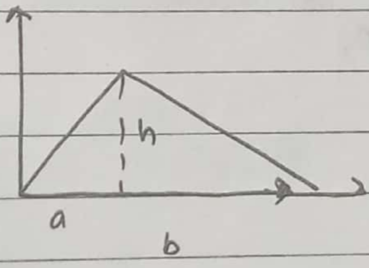
$\bar{x} = (b/3)$
$\bar{y} = (h/3)$

Triangle (△)



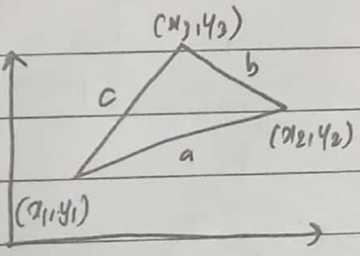
$\bar{x} = b/2$
$\bar{y} = h/3$

Triangle (unsym.)



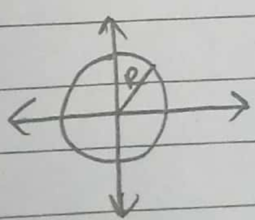
$\bar{x} = (a+b)/3$
$\bar{y} = (h/3)$

Triangle (general)



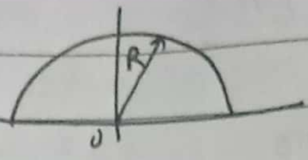
$\bar{x} = (x_1 + x_2 + x_3)/3$
$\bar{y} = (y_1 + y_2 + y_3)/3$

Circle



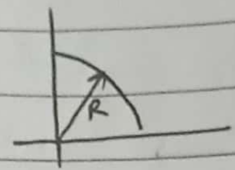
$\bar{x} = 0$
$\bar{y} = 0$

Semicircle.



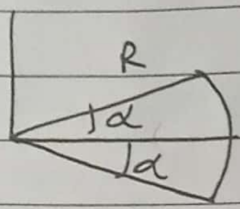
$\bar{x} = 0$
$\bar{y} = (4R/3\pi)$

Quadrant of circle.



$\bar{x} = (4R/3\pi)$
$\bar{y} = (4R/3\pi)$

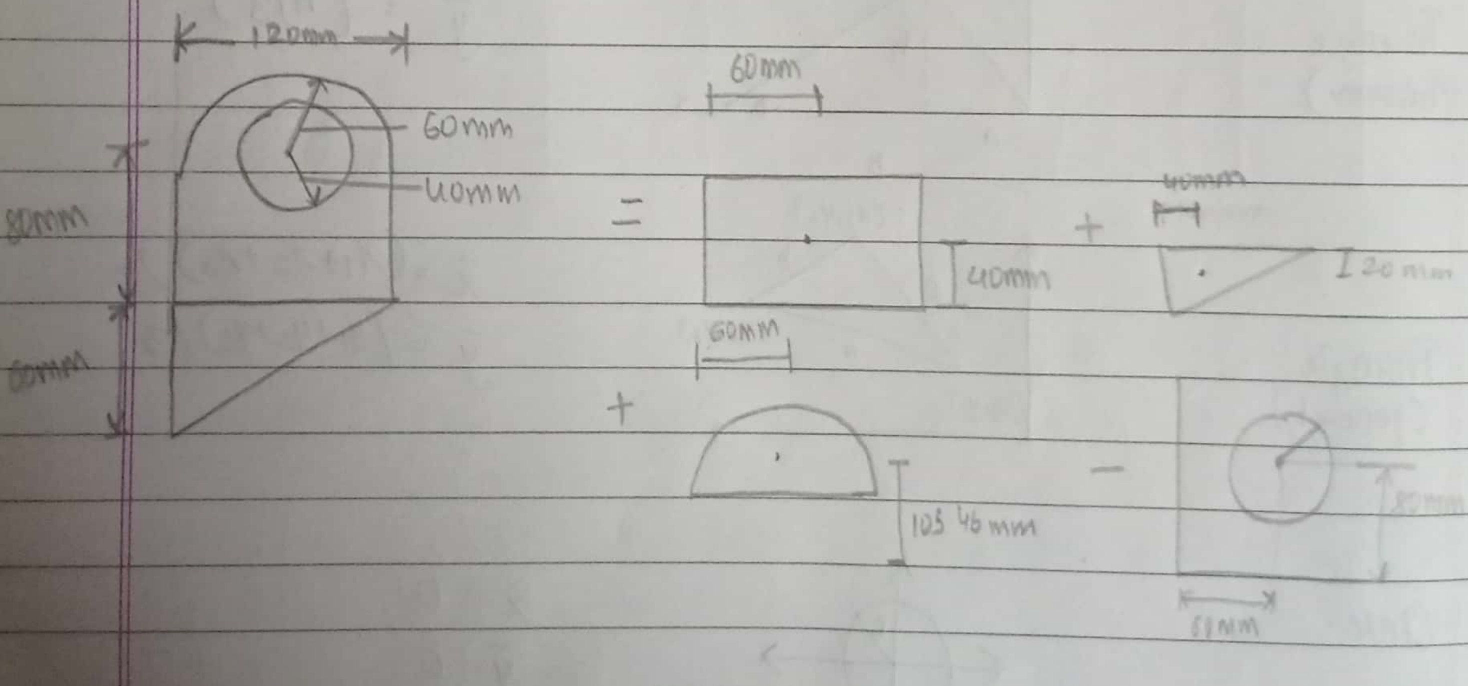
Circular sector.



$\bar{x} = (4R/3\pi)$
$\bar{y} = (4R/3\pi)$

<Q>: For the given plane, determine:
a) the first moments of w.r.t x and y axes
b) the location of the centroid.

Soln:



Date : ....
Page: ....

Component	Area (mm^2)	$\bar{x}$, mm	$\bar{y}$, mm	$x\bar{A}$	$y\bar{A}$
Rectangle	$120 \times 80 = 9.6 \times 10^3$	60	40	$+576 \times 10^3$	$+384 \times 10^3$
Triangle	$\frac{1}{2} \times 120 \times 60 = 3.6 \times 10^3$	40	-20	$+144 \times 10^3$	-72×10^3
Semicircle	$\frac{1}{2}\pi\, 60^2 = 5.655 \times 10^3$	60	105.46	$+339.3 \times 10^3$	$+596.4 \times 10^3$
Circle.	$-\pi(40)^2 = -5.027 \times 10^3$	60	80	-301.6×10^3	-402.2×10^3
	$\Sigma A = 13.828 \times 10^3$			$\Sigma \bar{x} A = +757.7 \times 10^3$	$\Sigma \bar{y} A = +506.2 \times 10^3$

So,
the first moment of area is
about y-axis $(\Sigma Q_y) = \Sigma \bar{x} A = 757.7 \times 10^3 \text{ mm}^3$.

about x-axis $(Q_x) = \Sigma \bar{y} A = 506.2 \times 10^3 \text{ mm}^3$

The location of centroid is :

$$\bar{X} = \frac{\Sigma \bar{x} A}{\Sigma A} = \frac{506.2 \times 10^3}{13.828 \times 10^3} = 54.8 \text{ mm}$$

$$\bar{Y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{757.7 \times 10^3}{13.828 \times 10^3} = 36.6 \text{ mm}$$