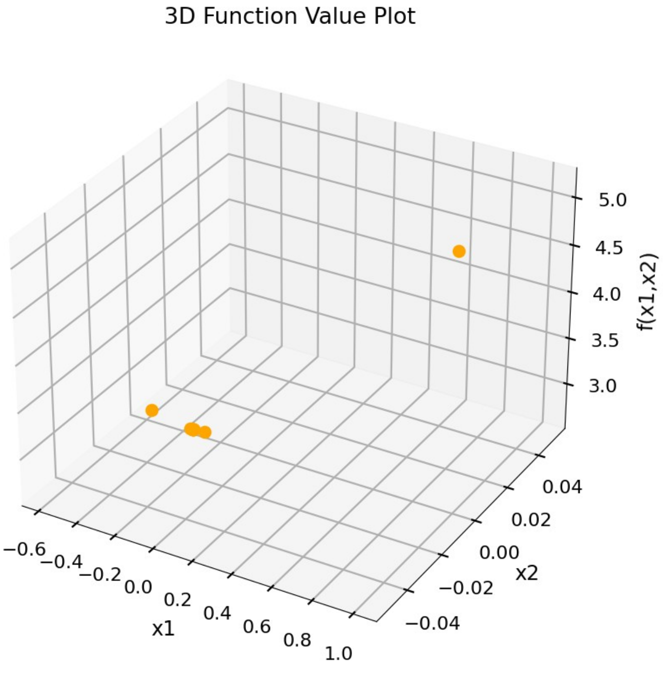
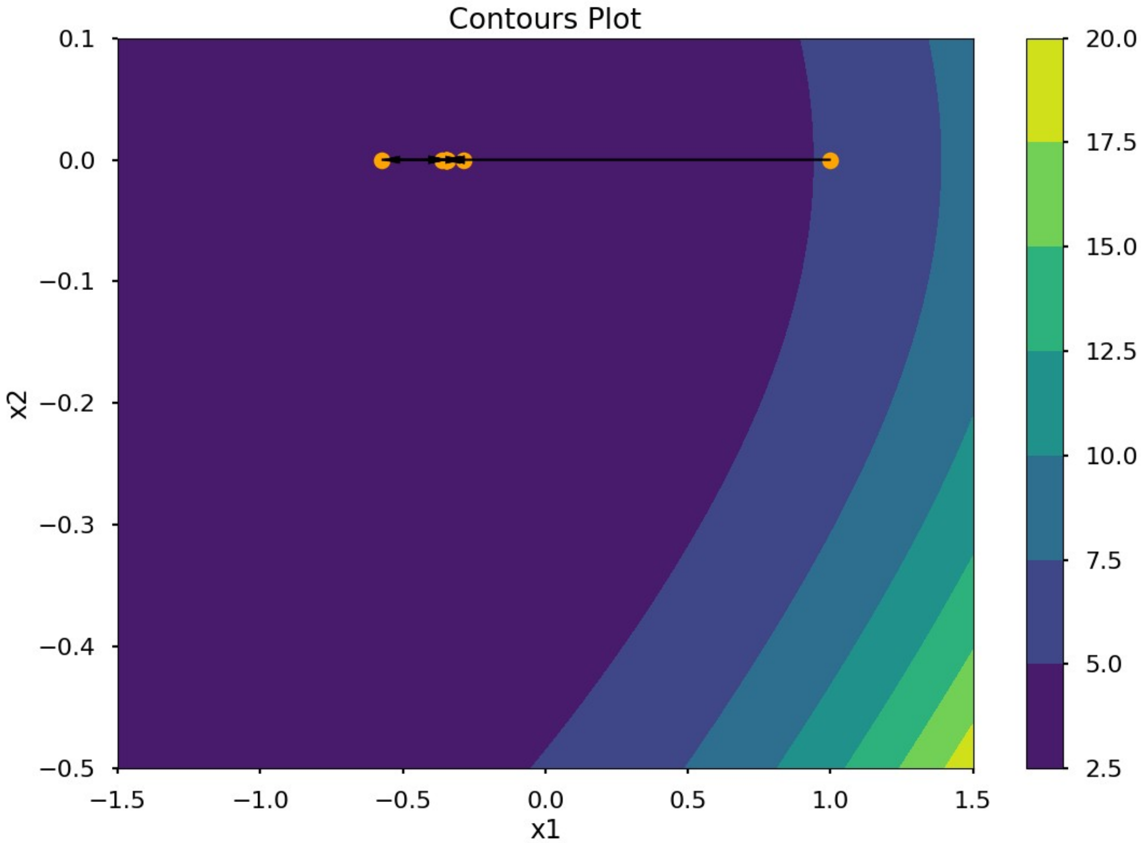
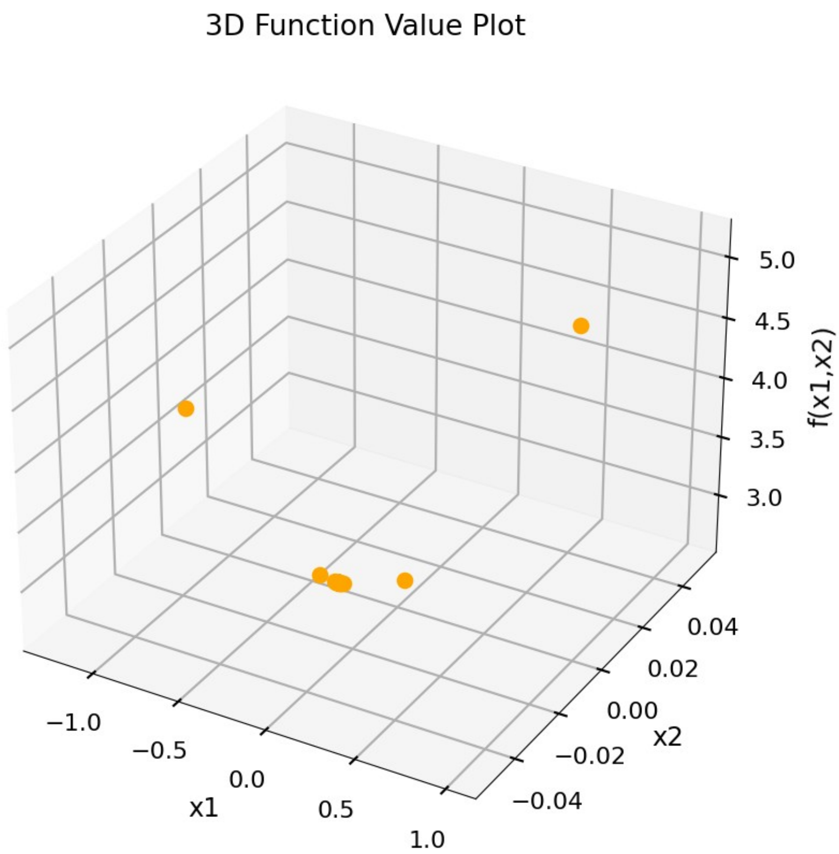
Q1)
Part-1 Armijo Goldstein Search



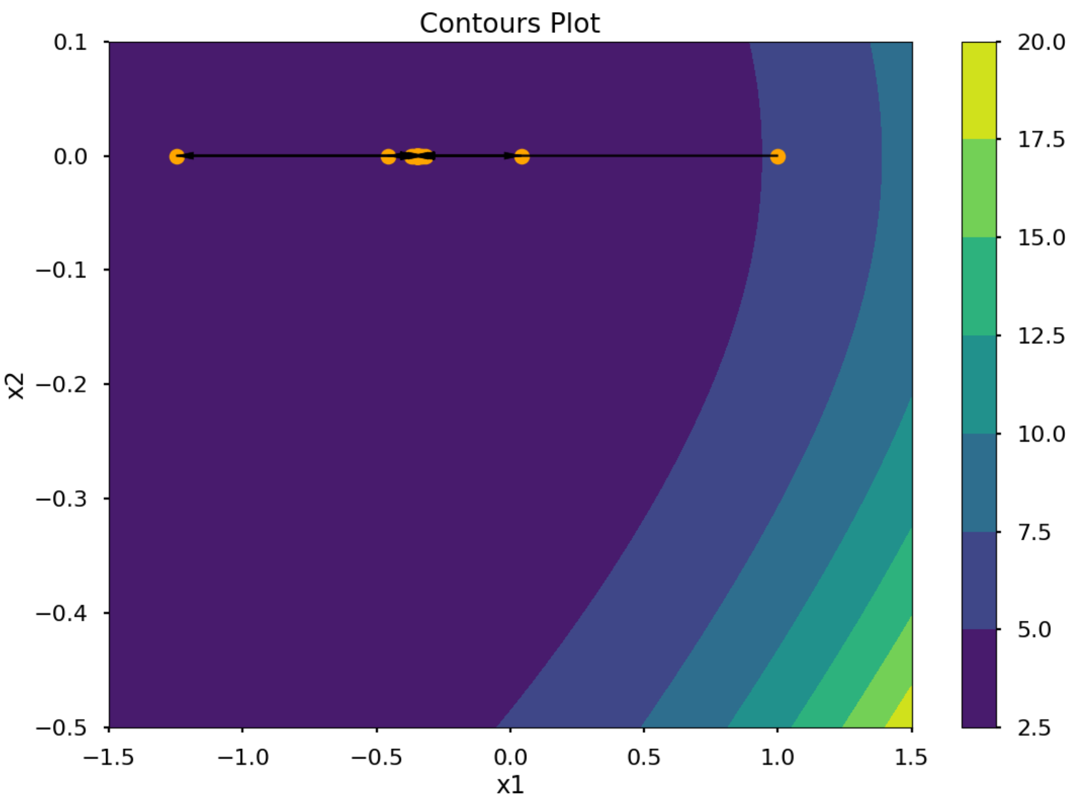
Algorithm converges in 12 iterations.
Initial Point = [1, 0]
Converges to [-0.34, 0] with function value 2.55



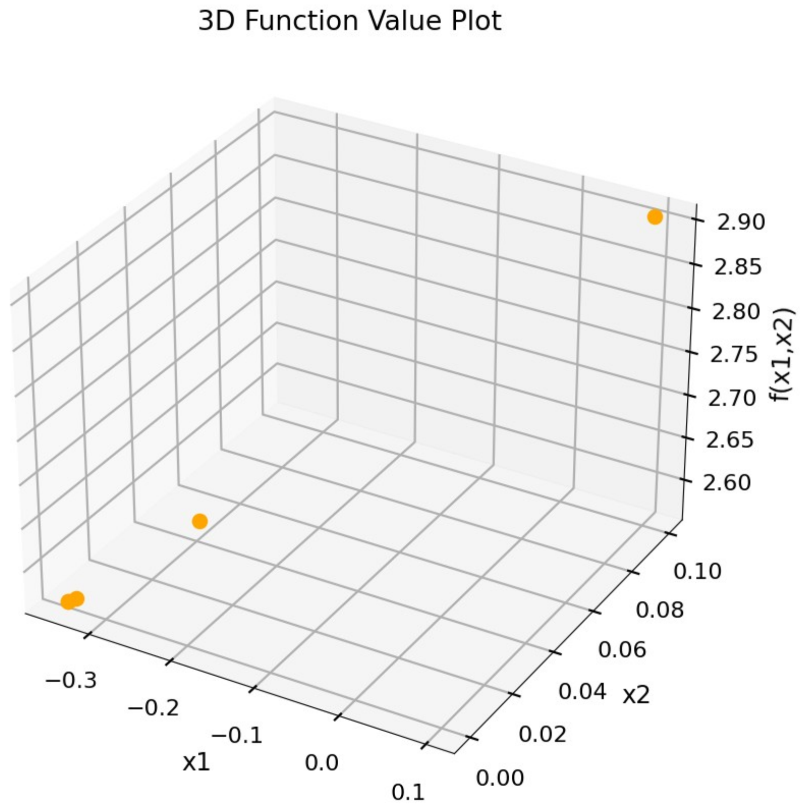
Part-2 Backtracking line search



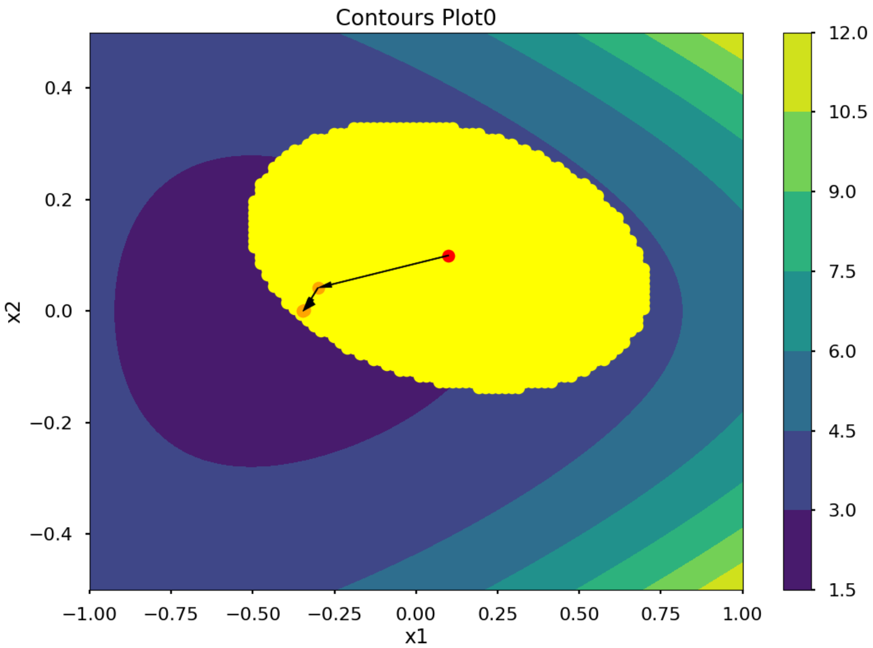
Converged in 53 iterations
Initial Point = [1, 0]
Converges to [-0.34, 0] with function value 2.55

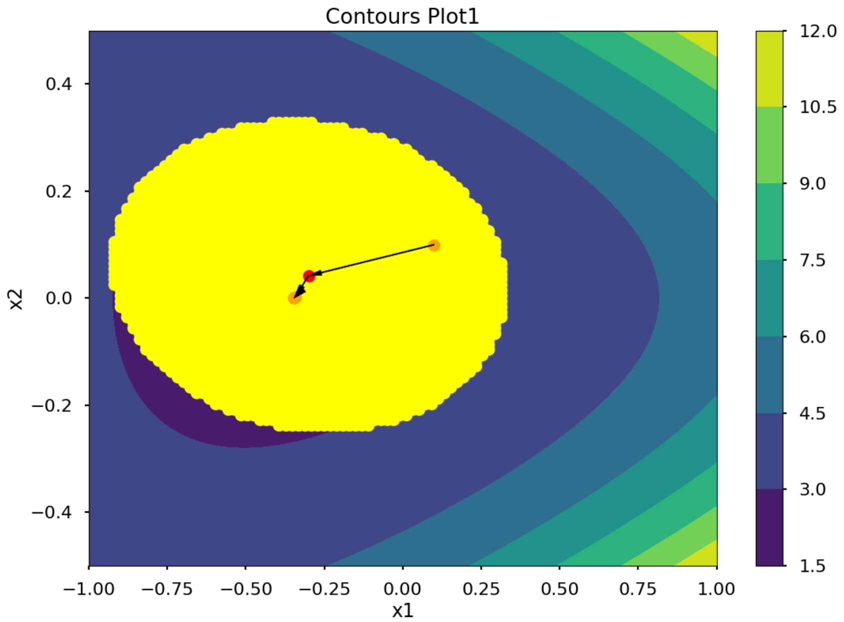


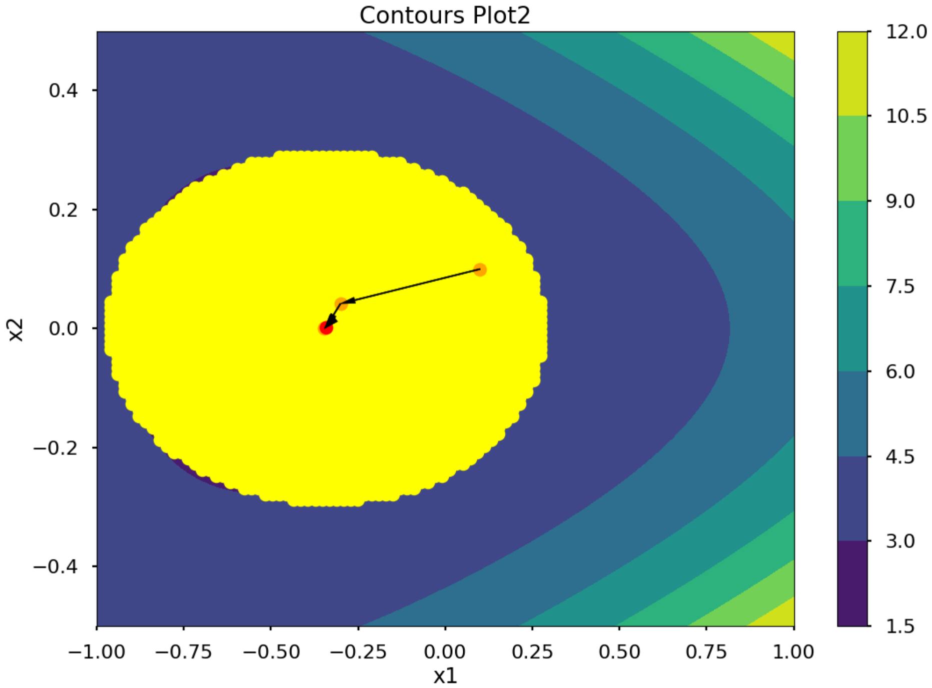
Q2)
Initial Point taken = [0.1, 0.1]
Converges to point [-0.346, 0] with function value 2.559.
The algorithm converges in 5 iterations.

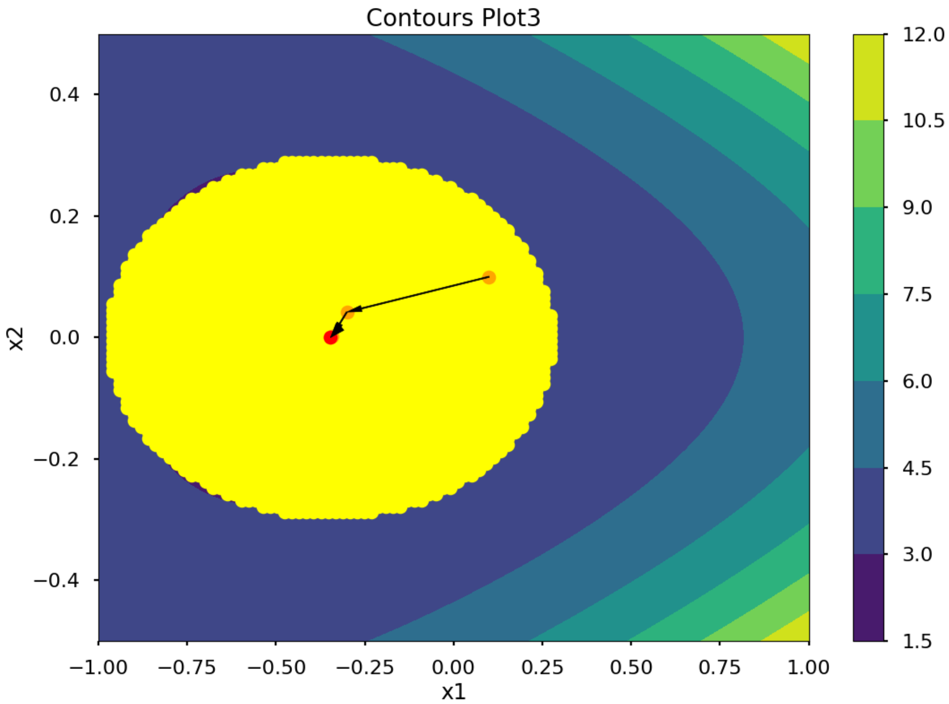


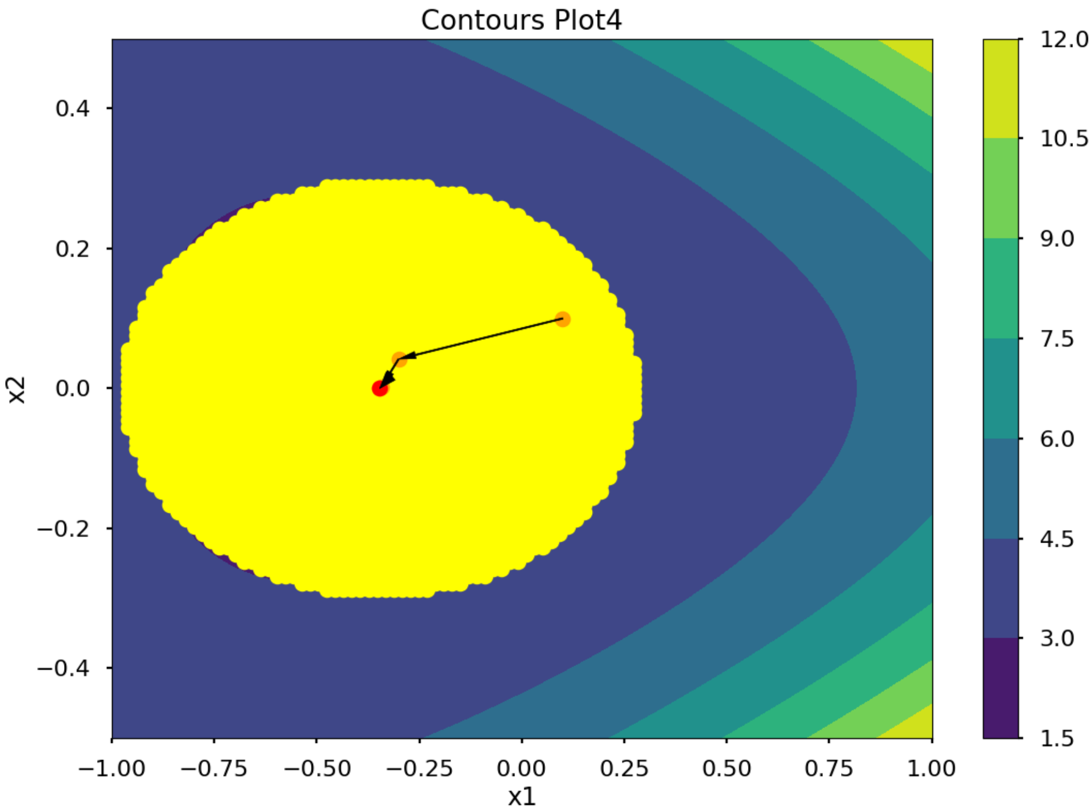
The contour plots below are color shaded values of the function with the color gradations indicating the value of the function for the values of x_1 and x_2. The ellipsoid is shown in yellow and its corresponding center in red. Arrows are marked in each contour plot.


Contours Plot0
x2
x1
0.4
0.2
0.0
−0.2
−0.4
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
12.0
10.5
9.0
7.5
6.0
4.5
3.0
1.5


Contours Plot1
x2
x1
0.4
0.2
0.0
−0.2
−0.4
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
12.0
10.5
9.0
7.5
6.0
4.5
3.0
1.5


Contours Plot2
x1
x2
12.0
10.5
9.0
7.5
6.0
4.5
3.0
1.5

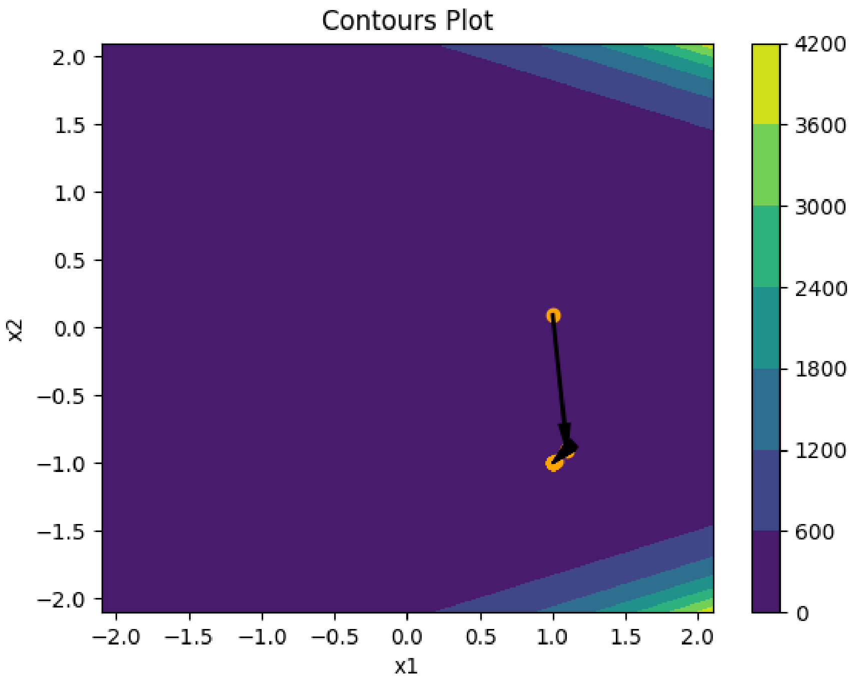

Contours Plot3
x1
x2
12.0
10.5
9.0
7.5
6.0
4.5
3.0
1.5



Contours Plot4
x2
x1
0.4
0.2
0.0
−0.2
−0.4
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
12.0
10.5
9.0
7.5
6.0
4.5
3.0
1.5

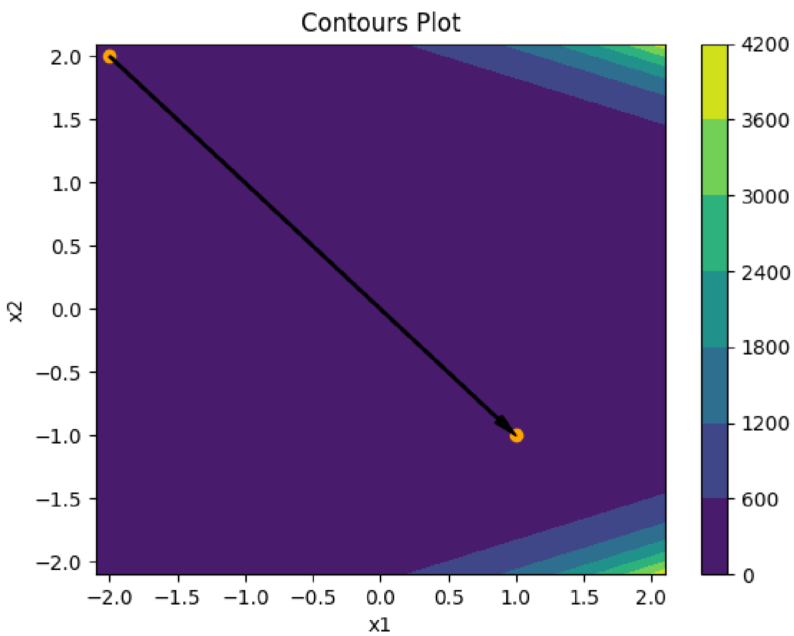
Q3)
1.
Eigenvalues = [9, 11]
Max of eigenvalues = 11
Initial Points = [1, 0.1], [-2, 2], [2, -2], [2, 2], [-2, -2]
Chosen Alpha value = 1 / 11

Initial Point 1:



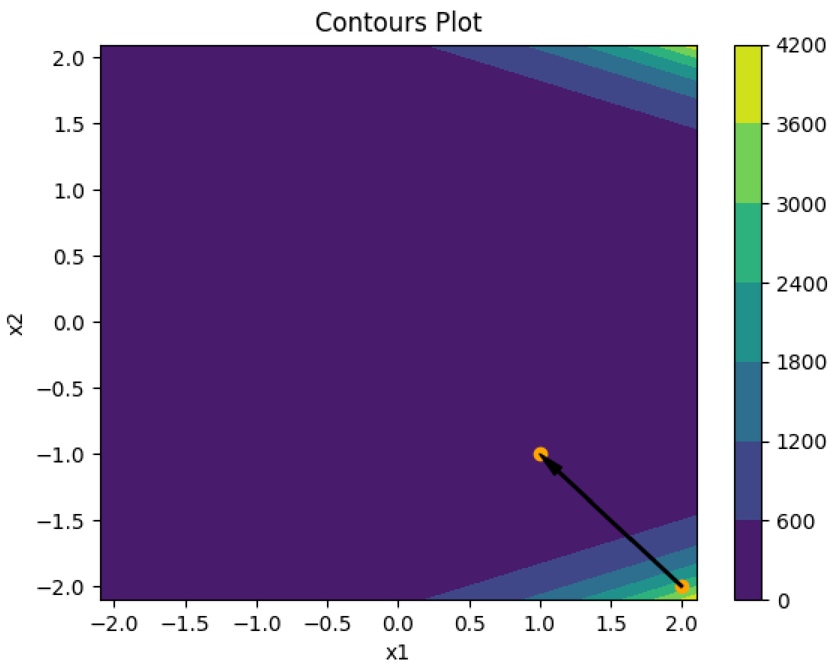
Number of iterations to converge = 11

Initial Point 2:



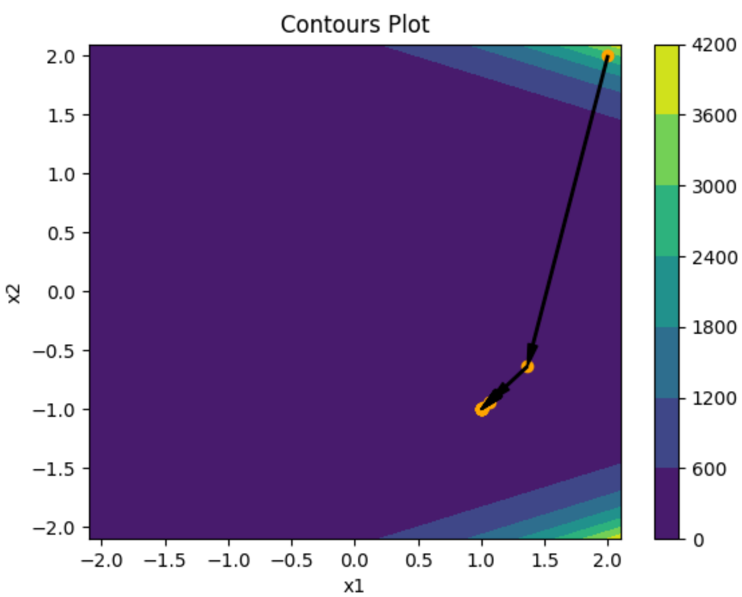
Number of iterations to converge = 2

Initial Point 3:



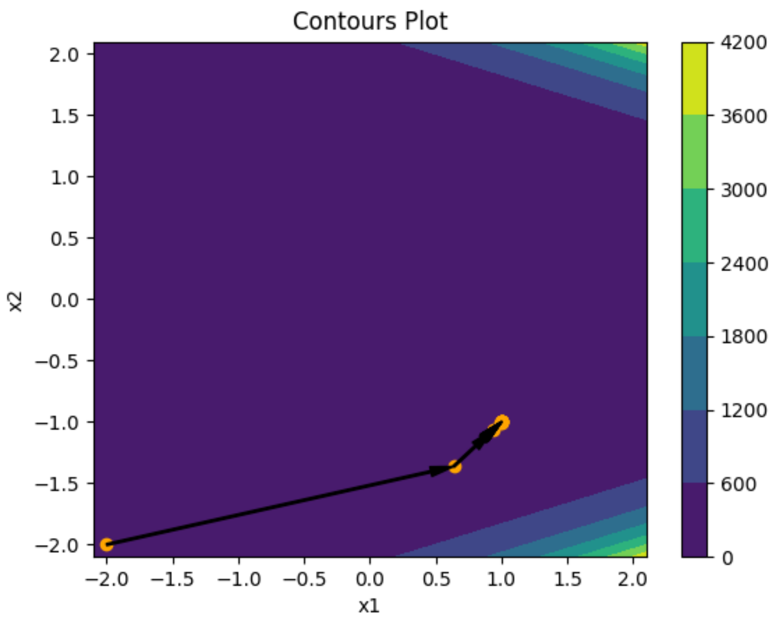
Number of iterations to converge = 2

Initial Point 4:



Number of iterations to converge = 12

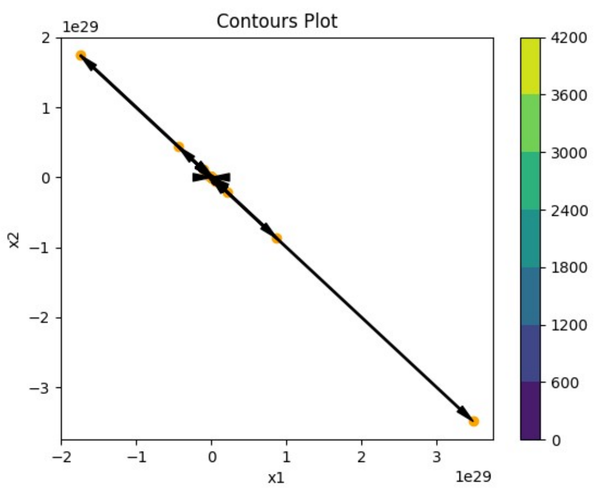
Initial Point 5:



Number of iterations to converge = 12

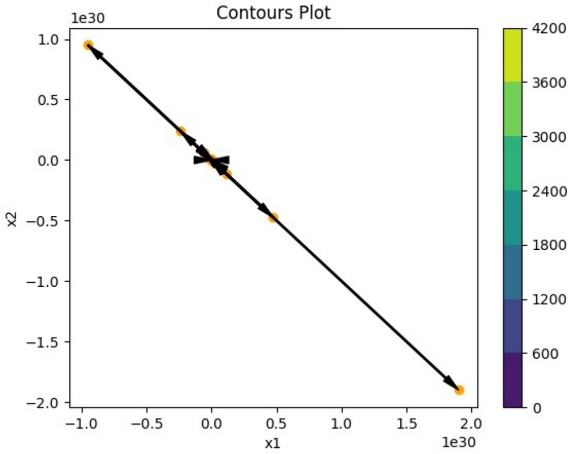
2.
Eigenvalues = [9, 11]
Max of eigenvalues = 11
Initial Points = [1, 0.1], [-2, 2], [2, -2], [2, 2], [-2, -2]
Chosen Alpha value = 3 / 11

Initial Point 1:



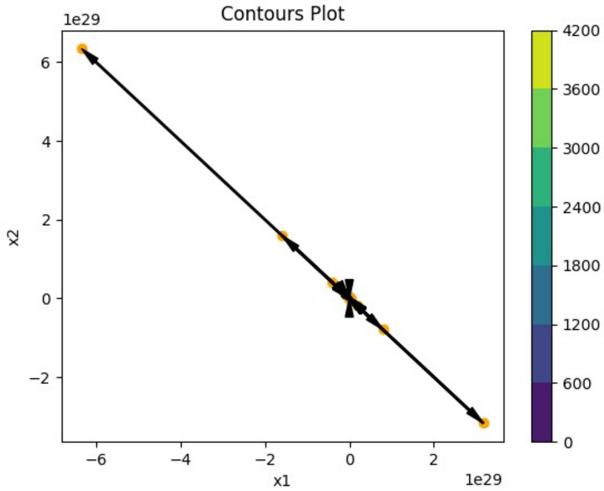
Number of iterations to converge = Didn’t converge in 100 iterations

Initial Point 2:



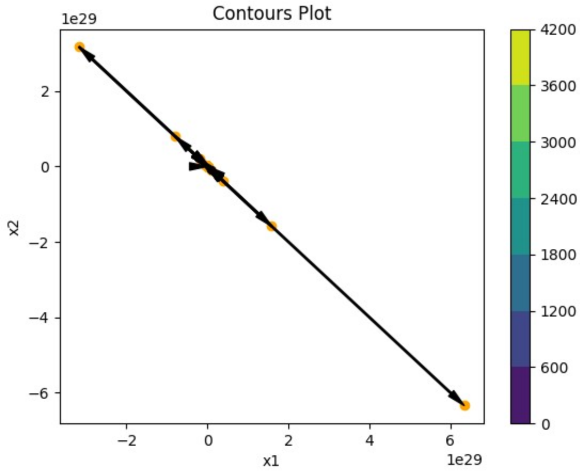
Number of iterations to converge = Didn’t converge in 100 iterations

Initial Point 3:



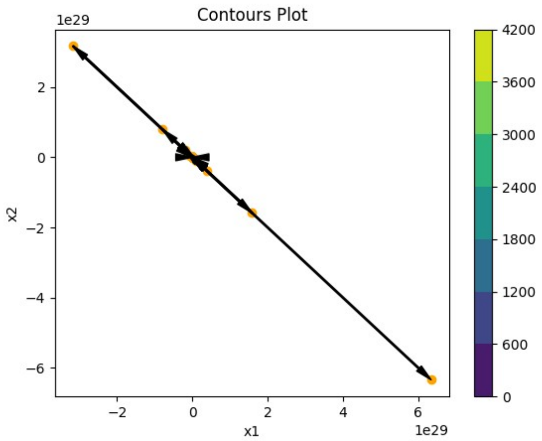
Number of iterations to converge = Didn’t converge in 100 iterations

Initial Point 4:



Number of iterations to converge = Didn’t converge in 100 iterations

Initial Point 5:



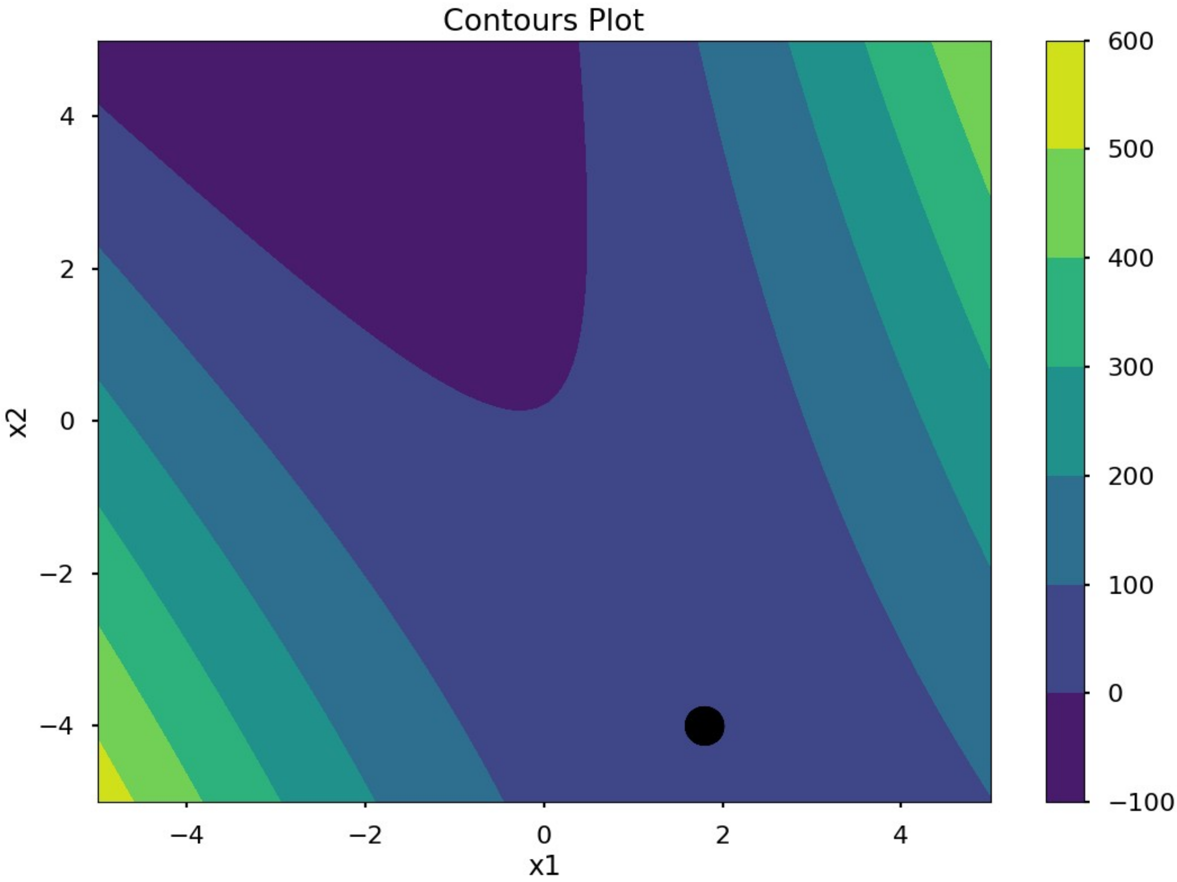
Number of iterations to converge = Didn’t converge in 100 iterations

Conclusions:
In case-1 when alpha value is between 0 and 2/(max. Eigenvalue), we obtain convergence. In case-2 when alpha value is greater than 2/(max. Eigenvalue), the function doesn’t converge. We verified this for 5 different starting points. So, we can conclude that for fixed step size gradient algorithm we get convergence **if and only if** alpha lies between 0 and 2/(maximum eigenvalue of hessian).

Q4

(a) For the function f (x) = 10 x_1^2 + 10x_1 x_2 + x_2^2 + 4x_1 − 10x_2 + 2, for different values of theta and alpha = 0.01, function value is computed and compared with the initial point [1.8, -4].

From the contour plot, it is visible that in two directions, the value of the function is decreasing while in other two directions, the value of the function is increasing.



The difference between the value of [f(x* + alpha * d_theta) – f(x*)] is positive as well as negative for certain values of theta. In two directions, the difference is negative and in other orthogonal two directions, the difference is positive.

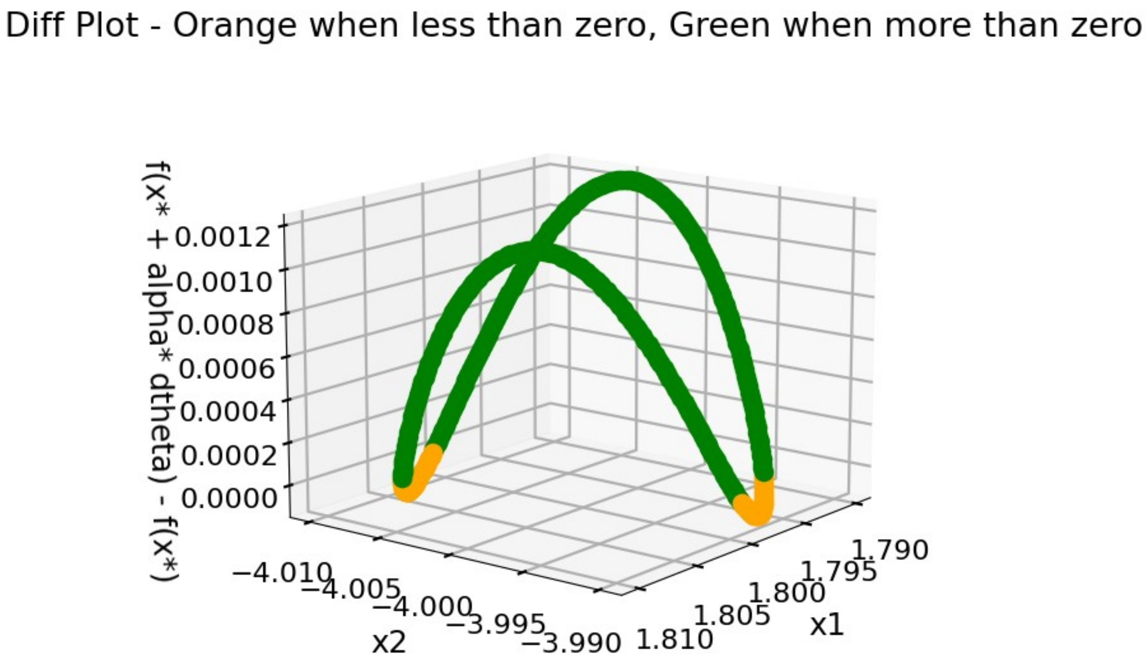
del_fx = [0. 0.]

Hessian inverse = [[-0.03333333 0.16666667]

[0.16666667 -0.33333333]]

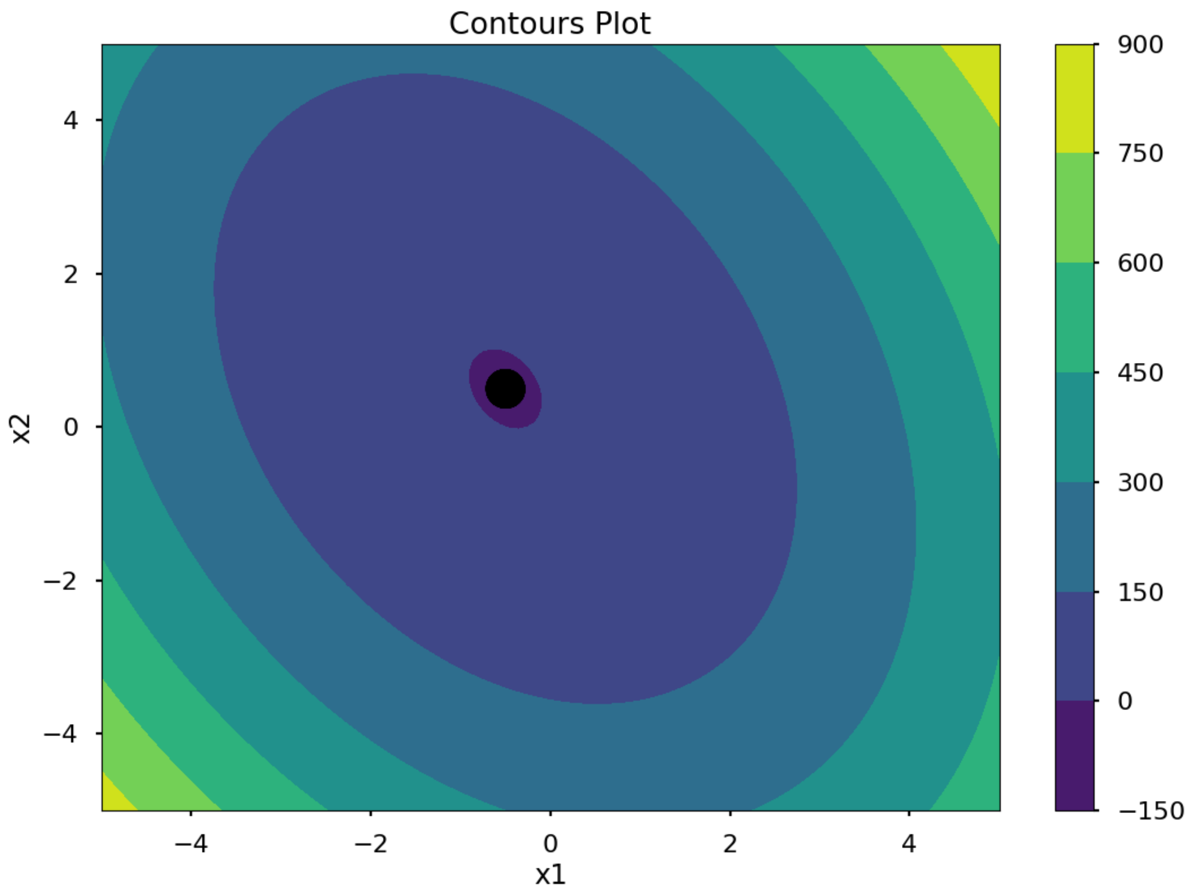
eigen values of Hessian = (array([24.45362405, -2.45362405]))

max of 2 eigen values 24.45362404707371

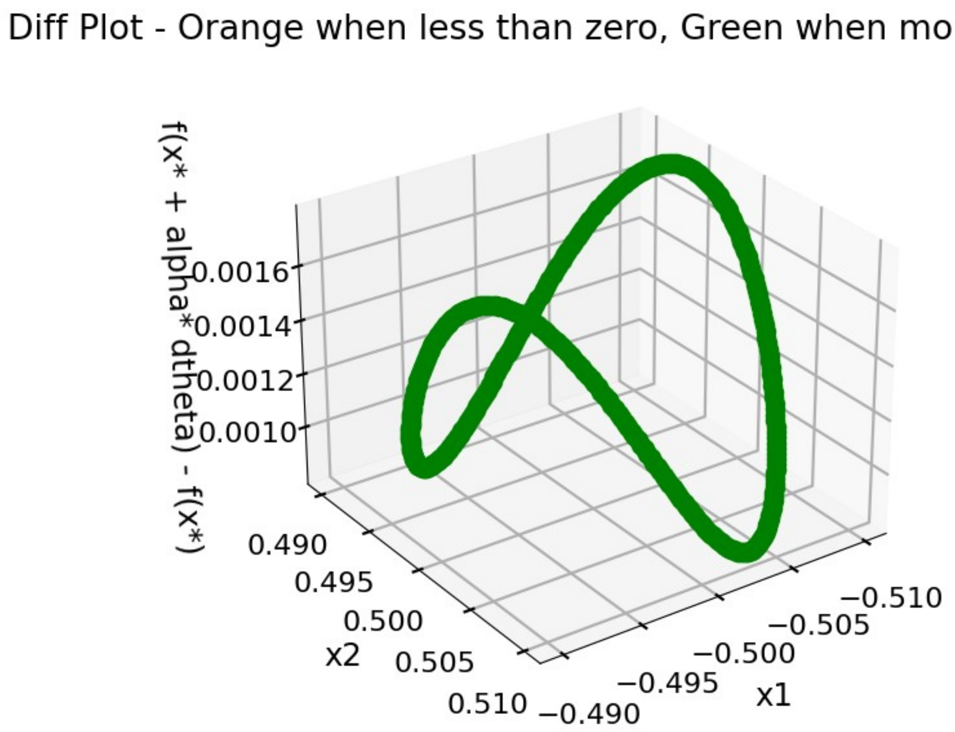


(b) For the second function, f (x) = 16x_1^2 + 8x_1 x_2 + 10x_2^2 + 12x^1 − 6x^2 + 2

From the contour plot of the function, it is visible that function is increasing in almost all directions and the minima is at the point [-0.5, 0.5].



The same thing is also visible from the 3D plot. The difference in the function value is positive for all values of theta around the point [-0.5, 0.5].



del_fx =[0. 0.]

Hessian inverse = [[0.03472222 -0.01388889]
[-0.01388889 0.05555556]]

eigen values of Hessian = (array([36., 16.]))

max of 2 eigen values 36.0

	$\nabla f(x*)$	eigen values of ∇_2 f(x*)	at x * local maxima/minima/saddle point?
Part 1	[0, 0]	24.45, -2.45	Saddle Point
Part 2	[0, 0]	36, 16	Local minima