Quantum Teleportation and Reality

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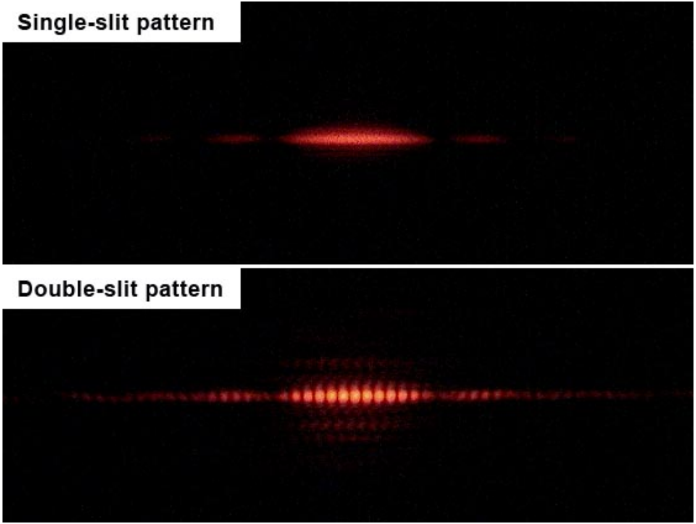
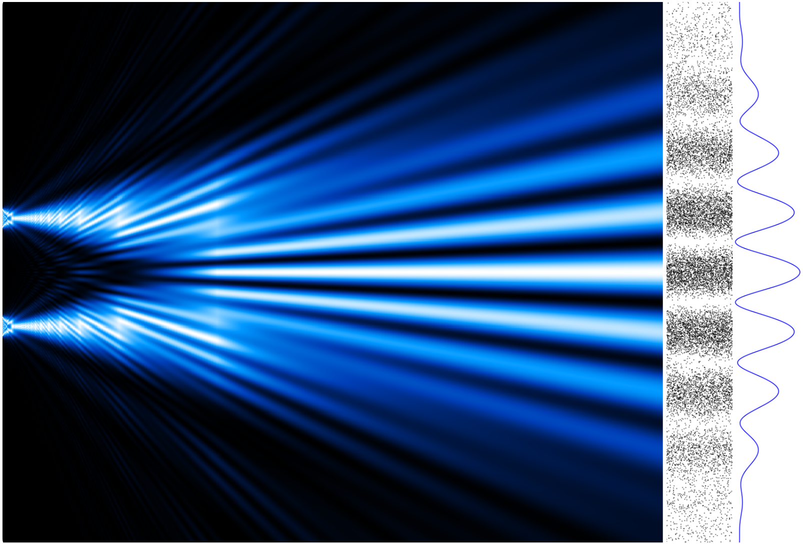
Double Slit Experiment



Double Slit Experiment (continued)



Quantum Teleportation

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● (qenv) devcbhatt@islab-HP:~/Desktop/Sem 2/Quantam Cryptography/Assignment$ python3 teleport.py

Quantum Teleportation Circuit:
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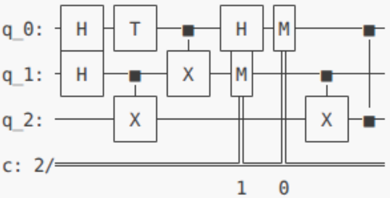


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Measurement Results:
{'01': 240, '11': 243, '10': 272, '00': 269}

Histogram saved as teleportation_result.png
```

Initial State

Unknown qubit (Alice):

$$|\psi\rangle_0 = \alpha |0\rangle + \beta |1\rangle$$

Shared Bell pair:

$$|\Phi^+\rangle_{12} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

Total state:

$$|\Psi\rangle = |\psi\rangle_0 \otimes |\Phi^+\rangle_{12}$$

Expand the Combined State

$$
\begin{aligned}
|\Psi\rangle &= (\alpha|0\rangle + \beta|1\rangle) \otimes \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \\
&= \frac{1}{\sqrt{2}}(\alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle + \beta|111\rangle)
\end{aligned}
$$

After CNOT (q0 control, q1 target)

$$|100\rangle \rightarrow |110\rangle, \quad |111\rangle \rightarrow |101\rangle$$

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(\alpha\,|000\rangle + \alpha\,|011\rangle + \beta\,|110\rangle + \beta\,|101\rangle)$$

After Hadamard on q0

Using:

$$H|0\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}, \quad H|1\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

Final decomposed state:

$$\begin{aligned}|\Psi\rangle = \frac{1}{2}[\,|00\rangle(\alpha|0\rangle + \beta|1\rangle) + |01\rangle(\alpha|1\rangle + \beta|0\rangle) \\ + |10\rangle(\alpha|0\rangle - \beta|1\rangle) + |11\rangle(\alpha|1\rangle - \beta|0\rangle)]\end{aligned}$$

Measurement Outcomes

Alice measures first two qubits:

Result	Bob's State
00	$\lvert\psi\rangle$
01	$X\lvert\psi\rangle$
10	$Z\lvert\psi\rangle$
11	$XZ\lvert\psi\rangle$

Bob's Correction

Bob applies conditional gates:

Classical Bits	Operation
00	I
01	X
10	Z
11	XZ

After correction:

$$|\psi\rangle_{Bob} = \alpha |0\rangle + \beta |1\rangle$$

Conclusion

- Original state destroyed at Alice
- Two classical bits transmitted
- State reconstructed at Bob
- No faster-than-light communication

Entanglement (EPR)

Bell State:

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

- Cannot be factorized:
$$|\Phi^+\rangle \neq |\psi\rangle_A \otimes |\phi\rangle_B$$
- Measurement outcomes are perfectly correlated.
- Individually: maximally mixed states.

References: Nielsen & Chuang (Ch. 2), Bernhardt (Ch. 4)

EPR Paradox: The Setup (1935)

Entangled State (Spin Version):

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

- Alice and Bob share an entangled pair.
- If Alice measures spin in z:

$$\uparrow_A \Rightarrow \downarrow_B$$

- Perfect anti-correlation.

Key Question:
How does Bob's particle "know" what Alice measured?
Reference: Nielsen & Chuang (Sec. 2.6)

EPR Argument: Is Quantum Mechanics Complete?

EPR introduce **Elements of Reality**:

> *If we can predict a value with certainty without disturbing the system, then it corresponds to a physical reality.*

- Alice can measure spin in z or x.
- Either choice lets her predict Bob's result.
- Therefore Bob must have definite z *and* x values.

But:

$$[\sigma_x, \sigma_z] \neq 0$$

Quantum mechanics forbids simultaneous definite values.
EPR Conclusion: Quantum mechanics is incomplete.

Bell's Theorem: Classical Bound (CHSH)

Assume hidden variables λ determine outcomes:

$$A, A', B, B' \in \{\pm 1\}$$

For fixed λ, define:

$$X = AB + AB' + A'B - A'B'$$

Factor:

$$X = A(B + B') + A'(B - B')$$

Since $B, B' = \pm 1$:

- Either $B + B' = \pm 2$ and $B - B' = 0$
- Or $B - B' = \pm 2$ and $B + B' = 0$

Therefore:

$$|X| = 2$$

Bell's Theorem: Classical vs Quantum

Classical (Local Hidden Variables)
For predetermined outcomes:

$$S = \langle AB \rangle + \langle AB' \rangle + \langle A'B \rangle - \langle A'B' \rangle$$

Since $|X| = 2$ for each hidden variable λ:

$$|S| \leq 2$$

Quantum Mechanics
Choosing optimal angles:

$$|S| = 2\sqrt{2}$$

Experimentally:

$$|S| > 2$$

Conclusion: Nature violates local realism

Quantum Prediction: Correlations

Singlet Bell state:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

Spin measurement along direction $\vec{a}$:

$$A = \vec{a} \cdot \vec{\sigma}$$

Quantum result:

$$\langle A \otimes B \rangle = -\vec{a} \cdot \vec{b} = -\cos\theta$$

Key Idea:
Correlation depends on angle between measurement directions.

Quantum Violation of CHSH

Choose measurement angles:

$$a = 0^\circ, \quad a' = 90^\circ$$

$$b = 45^\circ, \quad b' = -45^\circ$$

Using $\langle AB \rangle = -\cos\theta$:

$$\langle AB \rangle = \langle AB' \rangle = \langle A'B \rangle = -\frac{1}{\sqrt{2}}$$

$$\langle A'B' \rangle = +\frac{1}{\sqrt{2}}$$

CHSH (Clauser-Horne-Shimony-Holt) expression:

$$S = -\frac{4}{\sqrt{2}} = -2\sqrt{2}$$

$$|S| = 2\sqrt{2}$$

Quantum Teleportation

Unknown state:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Shared entangled pair:

$$|\Phi^+\rangle$$

Protocol:

1. Bell measurement by Alice
2. Send 2 classical bits
3. Bob applies correction

Result:

$$|\psi\rangle_B \text{ reconstructed exactly}$$

Reference: Nielsen & Chuang (Sec. 1.3)

Einstein–Rosen Bridge (1935)

Einstein & Rosen studied black hole solutions of General Relativity.
They found:

- Spacetime can form a "bridge" between two distant regions.
- This structure is called a wormhole.
- It connects two regions of spacetime.
- It is non-traversable (no faster-than-light travel).

Key Idea:
Spacetime geometry can create connectivity between distant locations.

ER = EPR Hypothesis (2013)

Maldacena & Susskind proposed:

$$\text{ER} = \text{EPR}$$

- ER: Einstein–Rosen bridge (wormhole)
- EPR: Quantum entanglement

Main Claim:

Entangled systems may be connected by microscopic wormholes.

- Entanglement = geometric connectivity
- Spacetime may emerge from quantum correlations
- No violation of relativity

Teleportation = Traversing a Wormhole?

In holographic duality:

- A gravitational spacetime can be equivalent to a quantum system.
- Two entangled black holes form a wormhole.
- The same system, viewed quantum mechanically, is just entanglement.

In this framework:

- Quantum teleportation corresponds to information moving through the wormhole.
- Classical communication is still required.
- No faster-than-light signaling occurs.

Key Idea:

Entanglement may be spacetime connectivity.

Physical Interpretation

ER $=$ EPR suggests:

- Geometry is not fundamental.
- Entanglement builds spacetime.
- Quantum information is fundamental structure.

Connections to:

- Black hole information paradox
- Holographic principle
- Emergent spacetime

Thank You

References

Textbooks

- M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information*, Cambridge University Press (2010).
- C. Bernhardt, *Quantum Computing for Everyone*, MIT Press (2019).

Research Papers

- J. Maldacena and L. Susskind, “Cool Horizons for Entangled Black Holes,” Fortschritte der Physik 61, 781–811 (2013). arXiv:1306.0533.
- C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W. K. Wootters, “Teleporting an Unknown Quantum State via Dual Classical and Einstein-Podolsky-Rosen Channels,” 70, 1895–1899 (1993).

Images and Resources

- Wikimedia Commons – Double slit images
- Qiskit Documentation: https://docs.quantum.ibm.com/