**Course: Mathematical foundations of Computer Science (MA 714)**

Assignment - 1

Topics: Counting principles and Pigeonhole principle

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1. Let $S$ be a set with $n$ elements and $T$ be a set with $m$ elements. Calculate the total number of one-one functions from $S$ to $T$.
2. How many positive integers less than $10^6$ have the sum of their digits equal to 19?
3. Let $n$ objects be given and you need to choose $r$ objects from them with repetitions. Explain that the above process can be done in $\binom{n+r-1}{r}$ ways.
4. Prove that the coefficient of $X^aY^bZ^cW^d$ in the expansion of $(X+Y+Z+W)^n$ is $\frac{n!}{a!b!c!d!}$, where the numbers $a, b, c, d \in \mathbb{N}$ satisfies $a+b+c+d=n$.
5. Show that in any list of ten non-negative integers $\{a_0, a_1, \cdots, a_9\}$ there is a string of consecutive items of the list, i.e., $\{a_l, a_{l+1}, \cdots\}$ where $l \in 0, 1, \cdots, 9$, whose sum $a_l + a_{l+1} + \cdots$ is divisible by 10.
6. Let there be $n$ persons attending a gathering. Each of them knows certain number of persons (minimum is 0 and maximum is $n-1$). Prove that there exists atleast two people who knows the same number of persons.

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