Combinatorics :-

Type of Questions :-

① Permutation & combination.

② summation
- Binomial summation $\left(\sum_{r=0}^{n} {}^nC_r\right)$
- Generating F^n $\left(\sum_{r=0}^{\infty} x^r\right)$

③ Solution of Recurrence relation.

① Permutation & combinations

	No. repetation	unlimited repetation	Limited repetation
a) Permutation	~~No repetation~~ ${}^nP_r$	~~Unlimited Repet~~ n^r	$\frac{n!}{n_1!\, n_2! \ldots}$
b) combination	${}^nC_r$	${}^{n-1+r}C_r$	Generating F^n

Learn

c) Distribution

d) Inclusion & Exclusion

e) Pigeon-Hole principle.

Permutation & Combination Identities

Permutation	Combination
1) Arrange 2) sequence Ex Numbers words Boxes	1) Select Collection. 2) set, subset. Ex Set, subset, Commettee etc. Team

$$\boxed{{}^nP_r > {}^nC_r}$$

- **Distribution** :- Key is distribution.
  - ordered Partition
  - unordered Partition.

- **Inclusion - Exclusion problem** 

  ↳ select (3 king or 5 dec) like this type of Que.

*** $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ 

*** $n(A \oplus B) = n(A) + n(B) - 2n(A \cap B)$ 

*** $n(A - B) = n(A) - n(A \cap B)$ 

↑ A But not B

*** $n(A^c \cap B^c) = n(U) - n(A \cup B)$

↑ Neither A Nor B ✓

*** $n(A^c \cup B^c) = n(U) - n(A \cap B)$

↑ Either A or B

- **Pigeonhole Principle**:-

  ↳ (Min How many) is Key for identification.

Lecture -7 :-

Que from {a, b, c, d --- z} How many 5 letter words can you make. & No repetation allow.

Sol^n $\left({}^{26}C_5 * \lfloor 5\right)$ or $\left({}^{26}P_5\right)$

⇓ use

⇒ $\boxed{\text{Is tarah ke Question ko Box Bnakr solve kro}}$ ✓

26 × 25 × 24 × 23 × 22 ✓

when Repetation is allow

26 × 26 × 26 × 26 × 26 ✓

*** Que How many distinct 5 letter words from {a, b, c, d, -- z} can make. whose starting is a & ending is d ✓

⇒ $\boxed{\text{distinct means No Repetation allow}}$

Sol^n

a				d

24 × 23 × 22

If distinct is Not given then.

a				d

26 × 26 × 26

Que starting with vowel, ending with z & distict letter use

				z

5   24   23   22   1

Que How many ways to arrange ~~Select~~ 4 People from 10 peoples in which "A" Person selected all the times.

Sol^n $\left({}^{9}C_{3} * 4!\right)$ ✓

*** V. ImP ✓

Answer for —

① From 10 People & 4 chair

way to arrange these 10 People into 4 Chairs is equal to —

② If 4 people & 10 chair

ways these 4 people arrange into 10 Chairs. → Bcoz Here we firstly select the chair & then arrange people ✓

& In I^st we select People firstly & then arrange.

Que from 26 letter of Alphabets {A, B, C, —— Z} How many Password can you make of 5 letter.

Sol^n

$26 \times 26 \times 26 \times 26 \times 26$

{In Password repetation are allow} ✓

***

for repetation Allow ⇒ r can be — r>n, r=n, r<n

for repetation Not Allow ⇒ r must be → $(r \leq n)$ ✓

• Password Letters can be Repeted ∞ Times.

Que How many out comes from throughing a dice 5 times. If :-

a) first & Last out comes is 6

b) first two out comes is odd.

Sol^n a) $1 \times 6 \times 6 \times 6 \ 1 \rightarrow 6\times6\times6 \Rightarrow 6^3$

b) $3 \times 3 \times 6 \times 6 \times 6 \longrightarrow 3\times3 \Rightarrow 3^2 \times 6\times6\times6 \Rightarrow (3^2 \times 6^3)$

Que In GATE exam if their is 60 Que & each Ques has mcq —

a) if a candidate choose either one of these mcq or left the Ans

b) if a candidate must choose one of the mcq.

then How many possible way to find distinct Answer if 4 mcq present for each que.

Sol^n a) $\rightarrow 5\times5$ — 60 Times $\Rightarrow 5^{60}$

b) $\rightarrow 4\times4$ — 60 Times $\rightarrow 4^{60}$

$$\boxed{{}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + \text{---} \, {}^nC_n = 2^n}$$

Que In 60 Ques → 50 mcq, ↘ 10 msq } a candidate may fill or may not fill the answer So find Possible way To get distinct Ans

Sol^n 60 Que

50 mcq → $(5)^{50}$

10 msq → $({}^4C_0 + {}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4)^{10} \Rightarrow (16)^{10}$

So Total distinct possible Ans $\Rightarrow \{(5)^{50} \times (16)^{10}\}$

Que How many words can you make from
1) MISSISSIPPI
2) MATHEMATICS
3) ENGINEERING.

Sol^n 1) $\dfrac{11!}{4!\,4!\,2!}$ ✓ 2) $\dfrac{11!}{2!\,2!\,2!}$ ✓ 3) $\dfrac{11!}{3!\,3!\,2!\,2!}$ ✓

$$\boxed{{}^{n}P_{n} = n!\ ,\quad 0! = 1!\ ,\quad {}^{n}C_{n} = 1\ ,\quad {}^{n}C_{0} = 1\ ,\quad {}^{n}C_{1} = n}$$

Que How many jumble for —
1) EQUATION
2) ABACUS

Sol^n 1) $8!$

2) $\dfrac{6!}{2!}$

Que How many jumble for —
1) MISSISSIPPI starting with P. Also find the Probability.
2) MISSISSIPPI starting Not with P, also find the Probability

Sol^n 1) $\boxed{P}\boxed{\ }\boxed{\ }\boxed{\ }\boxed{\ }\boxed{\ }\boxed{\ }\boxed{\ }\boxed{\ }\boxed{\ }\boxed{\ }$

1 $\quad \underbrace{\qquad\qquad\qquad}$ $\to \dfrac{\lfloor 10}{\lfloor 4\ \lfloor 4}$ Ans & probability $= \dfrac{\dfrac{\lfloor 10}{\lfloor 4\ \lfloor 4}}{\dfrac{\lfloor 11}{\lfloor 4\ \lfloor 4\ \lfloor 2}}$

2) All − starting with P

$\dfrac{\lfloor 11}{\lfloor 4\ \lfloor 4\ \lfloor 2} - \dfrac{\lfloor 10}{\lfloor 4\ \lfloor 4}$ & probability $= \left(\dfrac{\dfrac{\lfloor 11}{\lfloor 4\lfloor 4\lfloor 2} - \dfrac{\lfloor 10}{\lfloor 4\ \lfloor 4}}{\dfrac{\lfloor 11}{\lfloor 4\ \lfloor 4\ \lfloor 2}}\right)$

Que If their is 5 identical red flag, 4 id. Green flag & 10 id Blue flags

Solⁿ find How many ways you can arrange.

PPPPP — — —

$$\Rightarrow \frac{\underline{19}}{\underline{5}\ \underline{4}\ \underline{10}}$$

Constrains (Mixed Type Que in Permutation)

1. starting with, Ending with, Not starting with.
2. All together, all Not together
3. No two object of set & type are together.
4. Alternating permutation
5. circular permutation.

*** By default Repetation is allowed

1. Ex. ① How many 4 digit number are even
   ② ,, ,, ,, ,, odd
   ③ ,, ,, ,, ,, even with No repeted digits
   ④ ,, ,, ,, ,, odd ,, ,, ,,

Solⁿ ①

9	10	10	5

→ 0,2,4,6,8

↳ 9×10×10×5 ⇒ 90×50 ⇒ 4500 ways

②

9	10	10	5

→ 1,3,5,7,9

③

8	8	7	4

↳ 2,4,6,8

+

9	8	7	0

⇒ (8×8×7×4)+(9×8×7)

④

8	8	7	5

→ 1,3,5,7,9

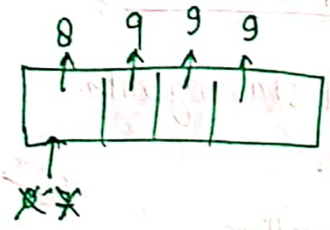
8 * 8 * 7 * 5

↳ Ans

Que4 Pick a Random No. from 1000 ——— 99999 so that it does not contain 7, find the Probability.

Sol^n $(1000-9999) \Rightarrow (9999-1000+1) \Rightarrow$ 9000 Numbers ✓

$$n(\text{Not Containing } 7) \Rightarrow \left(\frac{9\times9\times9\times8}{9000}\right) \Rightarrow \frac{81\times72}{9000} \Rightarrow \frac{5832}{9000}$$



Que Based on Together:-

Que) How many jumbles from "EQUATION." In which all the vowels are together.

Sol^n "EQUATION" ⇒ All vowels are together

↳ 5 vowels, 3 consonent ⇒ $(4! * 5!)$ ✓

b) all consonent are Together.

⇒ $6! * 3!$ ✓

c) All vowels are together & all consonent are Together.

⇒ $(2! * 5! * 3!)$

d) All Vowels are not Together

⇒ $\underbrace{8!}_{\text{all}} - \underbrace{(4! * 5!)}_{\text{all vowels are Together}}$ ✓

Que In "EEQUATION" How many Jumbles are Possible when all Vowels are Together

Soln :- "EEQUATION"

EEUAIO
QTN

$\rightarrow \left(4! * \frac{6!}{2!}\right)$

No Two object are Together type Que :-

** **Que** In "EQUATION" No Two Consonent are together then find No. of Jumble.

Soln "EQUATION" ⇒ EUAIO
QTN

⇒ All − {Consonent Together} ✓

⇒ ~~$\underline{|8} - \{\underline{|6} * \underline{|3}\}$~~

⇒ $\underline{|8} - \{\underline{|6} * \underline{|3}\}$

↑ E ↑ U ↑ A ↑ I ↑ O ↑

$\rightarrow \underline{|5} * {}^6P_3$

$\hookrightarrow \underline{|5} * {}^6P_3$

Que How many Jumble in "MISSISSIPPI" so that No two SS ~~Vowels~~ are together.

Soln ↑ M ↑ I ↑ I ↑ I ↑ P ↑ P ↑ I ↑

$$\boxed{\frac{\underline{|7}}{\underline{|4}\,\underline{|2}} * {}^8C_4}$$ ✓ Ans

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Que How many binary No. m 0's & n 1's such that No two 1's are together.

Sol^n

↑ 0 ↑ 0 ↑ 0 ↑ 0 — — ↑ 0 ↑

$\downarrow$

(m+1) Places

$\downarrow$

$$\frac{\lfloor m}{\lfloor m} * {}^{(m+1)}C_n \Rightarrow 1*\{{}^{(m+1)}C_n\}$$ ✓ Ans

ALTERNATING Arrangement Que :-

> If their is a group of Boys & girls & In Question it says No two girls are together & No two Boys are Together it means They are in Alternating Position

↓

Alternating Case is possible when

# Boys	# Girls
n	n
$n+1$	n
n	$n+1$

** good

Que How many arrangement are possible if their are 5G, 5B so that they are sit into alternate ways.

Sol^n

↑ G1 ↑ G2 ↑ G3 ↑ G4 ↑ G5 ↑ $\quad$ ~~^{5}~~ $^6P_5$

GBGBGBGBGB
or
BGBGBGBGBG

$\rightarrow (2 * \lfloor 5 \times \lfloor 5)$ ✓

Q2 → for 7 Boys 9 Girls arrange in the way so that they are in Alternate ways A s't.

Sol^n 0 ← Alternate ways

When objects are Identical

Que How many ~~Arro~~ Alternate arrangements are possible for—

1) 5 0's 5 1's
2) 6 0's 5 1's

Sol^n 1) 5(0's), 5(1's)

1 0 1 0 1 0 1 0 1 0

or

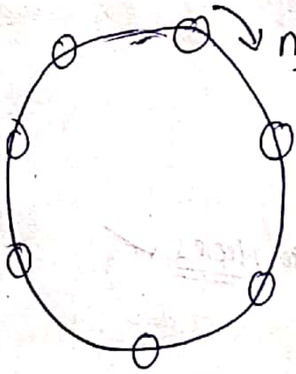
0 1 0 1 0 1 0 1 0 1

→ 2 ways.

2) 6(0's) – 5(1's)

0 1 0 1 0 1 0 1 0 1 0 → 1 ways

Circular Permutation :-



n distinct object

↳ $\frac{\lfloor n}{n}$ ways can arrange → views are same

↳ $\frac{n(n-1)!}{n} \rightarrow (n-1)!$

for Garland → if n flower are their

then # arrangement → $\left[\frac{(n-1)!}{2}\right]$

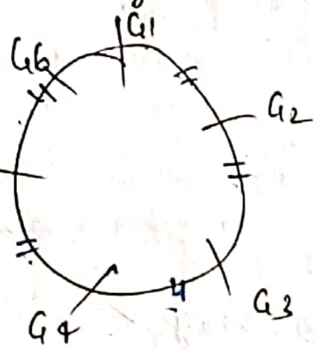
Q6 if 5B, 6G are their then —

(a) arrange in Circular Permutation so that all Boys are Together.

Sol^n :- $|\underline{6-1} * |\underline{5} \Rightarrow |\underline{5} * |\underline{6}$

↑ Circular Permutation ↑ Boys arrangement.

(b) No two Boys sit Together.



$\rightarrow \boxed{|\underline{6-1} * {}^6P_5}$

$\downarrow$

$\boxed{|\underline{5} * {}^6P_5}$

Q7 5 Identical Green, 6 Id Blue, 3 Id Yellow

(a) # arrangemt in straight line

(b) # arrangemt in Circular form

Sol^n (a)

$\frac{(6+5+3)!}{6!5!3!} \Rightarrow \frac{14!}{6!5!3!}$ Ans

(b) $\frac{|\underline{(6+5+3)-1}}{|\underline{5}\,|\underline{6}\,|\underline{3}} \Rightarrow \left(\frac{13!}{6!5!3!}\right)\frac{1}{2}$ Ans → due to Necles

:- Lecture - 8 :-

Combination

$$\boxed{{}^{n-1+r}C_r \equiv {}^{n-1+r}C_{n-1}}$$ ✓

Que: How may way to select ~~from~~ ~~10~~ 3 people from 10 People in which one person ~~is~~ (A) is Already Included.

Soln :- ${}^{10-1}C_{3-1} = \boxed{{}^{9}C_2}$ Ans

If a particular Person Always excluded.

Soln ${}^{10-1}C_3 \Rightarrow {}^{9}C_3$

Two person Always Included.

Soln ${}^{10-2}C_{3-2} = {}^{8}C_1$

Two person Always excluded.

Soln ${}^{8}C_3$ ✓

- from n people, select r from, if m people is already selected.

  $\Rightarrow {}^{n-m}C_{r-m}$

Que Set A = {1, 2, 3, - - - - 10}

a) find # subset of A

b) subset of element ≥ 1

c) " " ≥ 2

d) " " = 3

Sol^n a) # subset $= 2^{10} \longrightarrow \{{}^{10}C_0 + {}^{10}C_1 + {}^{10}C_2 \text{------} {}^{10}C_{10}\} \rightarrow 2^{10}$

b) # subset element $\geq 1 \Rightarrow 2^{10} - {}^{10}C_0 \Rightarrow 2^{10} - 1 = 1023$.

c) # subset $n \geq 2 \Rightarrow 2^{10} - ({}^{10}C_0 + {}^{10}C_1)$

d) # subset element $= 3 \rightarrow {}^{10}C_3$ ✓

e) # ,, ,, $\leq 3 \rightarrow {}^{10}C_0 + {}^{10}C_1 + {}^{10}C_2$

Que from set $A = \{1, 2, 3, 4, 5, \text{--} 10\}$.

Que
a) How many subset is present ??
b) Subset which Include 2 & 3 ??
c) subset which exclude 2 & 3 ??
d) subset which Include 2 & 3 & exclude 4 ??
e) 5 element subset ??
f) 5 element subset which Include 2 & 3 ??
g) ,, ,, ,, Exclude 2 & 3 ??
h) ,, ,, ,, Include 2 & 3 & exclude 4 ??

Sol^n (a) $2 \times 2 \times 2 \times$ —— 10 Times $\rightarrow 2^{10}$

for 2 & 3 one choice only

(b) $2 \times 1 \times 1 \times 2$ —— 10 Times $\rightarrow 2^{8}$

(c) ,, ,, 10 Time $\rightarrow 2^{8}$

(d) ,, for 2, 3 & 4 only one choices $\rightarrow 2^{7}$

subset Not specified

(e) ${}^{10}C_5$ ✓

(f) ${}^{10-2}C_{5-2} \Rightarrow {}^{8}C_3$ ✓

(g) ${}^{10-2}C_5 \Rightarrow {}^{8}C_5$ ✓

(h) ${}^{10-2-1}C_{5-2} \Rightarrow {}^{7}C_3$ ✓

subset size specified

Que How many ways to make 3B & 2 girl committee from 5B & 6G

Sol^n:- $^5C_3 * {}^6C_2 \Rightarrow \frac{\lfloor 5}{\lfloor 3 \lfloor 2} * \frac{\lfloor 6}{\lfloor 2 \lfloor 4}$

$\Rightarrow \frac{5 \times 4}{2} * \frac{6 \times 5}{2} \Rightarrow 150$ ✓

at least 3 Boys

$\Rightarrow {}^5C_3 * {}^6C_2 + {}^5C_4 * {}^6C_1 + {}^5C_5 * {}^6C_0$

(or)

${}^{11}C_5 - \{ {}^5C_0 * {}^6C_5 + {}^5C_1 * {}^6C_4 + {}^5C_2 * {}^6C_3 \}$ ✓

at most 3 Boys

$\Rightarrow {}^5C_0\,{}^6C_5 + {}^5C_1\,{}^6C_4 + {}^5C_2\,{}^6C_3 + {}^5C_3\,{}^6C_4.$

at least 2B & at most 2G

B	G	Choose
2	3	✗
3	2	✓
4	1	✓
5	0	✓

$\boxed{{}^5C_3 * {}^6C_2 + {}^5C_4 * {}^6C_1 + {}^5C_5 * {}^6C_0}$

Que In an Exame two section A & B are their in which each having 25-25 Question

To done 10 Question it Needs atleast 6 Question from section A & at most 3 Question from section B ✓

Find # of combinations to do the Questions.

Sol^n:-

$$\begin{array}{ccl} & 10_Q & \\ A & & B \\ \geq 6 & & \leq 3 \\ \cancel{6} & \cancel{4} & ✗ \\ 7 & 3 & ✓ \\ 8 & 2 & ✓ \\ 9 & 1 & ✓ \\ 10 & 0 & ✓ \end{array}$$

} Choices

Que from a dec of 52 cards —

a) select 5 cards in which exactly 2 ~~are~~ are King.

* b) 5 card select exactly 2 are King & 3 are Queen.

c) select 5 cards in which exactly 2 are king & exactly 2 are Queen

Sol^n a) ${}^4C_2 * {}^{48}C_3$

b) ${}^4C_2 * {}^4C_3$

c) ${}^4C_2 * {}^4C_2 * {}^{44}C_1$

Que 11 People — (5 B & 6 G)

a) select 5 people in which 3 B & 2 G are there. in which Name "A" Boy must be Included.

b) " " " " " " " " also name "G" girl must be Included.

c) " " " " " " " " " " " Excluded

Sol^n :- a) ${}^4C_2 * {}^6C_2$

b) ${}^4C_2 * {}^5C_1$

c) ${}^4C_3 * {}^5C_2$

d) " " " " " " in which "A" & "G" Boys & girls can not work Together

Sol^n $\underbrace{({}^5C_3 * {}^6C_2)}_{\text{Total}} - \underbrace{({}^4C_2 * {}^5C_2)}_{\text{Working Together}}$

Que A = {1, 2, 3, — — 10}

a) How many ways to select 3 ~~People~~ Number & arrang them??

b) How many ways to select 3 Numbers & arrange them into ascending order??

Sol^n :- a) ${}^{10}P_3$ (b) In this type of que ${}^{10}C_3 = {}^{10}P_3$ {only one way to arrange it}

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Some time $\boxed{{}^nC_r \equiv {}^nP_r}$

- when Identical object are given
- when their is only one way to arrange it.

Que How many ways to arrange m zero's & n once's. So that No two onces are get Together.

Sol^n 0 0 0 0 - - 0

(m+1) Places $\longrightarrow$ $1 * {}^{(m+1)}C_n$

for placing 0 (zero)

Que How many 10 bit strings are their in which their are 3 ~~bit~~ No. of zeros are their.

Sol^n $\boxed{{}^{10}C_7 * {}^3C_3}$ ✓ (or) $\boxed{{}^{10}C_3 * {}^7C_7}$

$\boxed{{}^nC_r = {}^nC_{n-r}}$ ✓

★★ Que if their is 5 0's & 15 1's if • Every 0's followed at least 1 one then How many No. can you form.?

Sol^n:- (01 01 01 01 01) (1111111111)

$\Rightarrow \left(\frac{\lfloor 15}{\lfloor 5 \; \lfloor 10}\right)$ ✓

Every zero follows ~~exactly~~ at least 2 1's.

(011 011 011 011 011) (11111)

$\Rightarrow \left(\frac{\lfloor 10}{\lfloor 5 \; \lfloor 5}\right)$

Variations of $\left({}^{n-1+r}C_r\right)$ ✓

1) · In this type of combination

$\left.\begin{array}{l} n \leq r \\ n \geq r \\ n = r \end{array}\right\} \rightarrow$ anything possible

$n = \#$ group selected $\boxed{r = \text{RHS}}$ ✓

By Example it is clear ✓

Que If their are three type of chocklete R, G, B are given & we want to find out 10 out of them.

So, How many ways can we Taken 10 chocklete.

Soln $\cancel{x_1 + x_2 + x_3} = 10$

$\begin{pmatrix} r = 10 \\ n = 3 \end{pmatrix}$ $\quad {}^{3-1+10}C_{10} \Rightarrow {}^{12}C_{10} = {}^{12}C_2 \Rightarrow \frac{12 \times 11}{2} = 66$ ways ✓

Que a) If their is 4 type of CD's in the shop A, B, C, D

if a person want to choose 20 CD from these 4 type.

then How many ways to choose it.

b) If their is 5 type of coins are their C_1, C_2, C_3, C_4, C_5

If you want to choose 15 coin from it then No. of Possible ways to choose it.

Soln a) # CD's $\quad A + B + C + D = 20$

Here $\boxed{r = 20}$ & $\boxed{n = 4} \rightarrow$ 4 Type of CD's

$\uparrow$ Total No. of CD's.

${}^{4-1+20}C_r \Rightarrow \boxed{{}^{23}C_{20}} = {}^{23}C_3 \Rightarrow \frac{23 \times \cancel{22}^{11} \times \cancel{21}^{7}}{3 \times \cancel{2}}$

$\Rightarrow (77 \times 23)$ ways

b) Type of coins = 5

$c_1+c_2+c_3+c_4+c_5 = 15$

5 $\boxed{n=5 \quad r=15}$

$^{5-1+15}C_{15} \Rightarrow (^{19}C_{15}) = (^{19}C_4)$

2) Non-Negative Integral Solution Problem :-

• For Number of Non Negative solution. we will use –

$(^{n-1+r}C_r)$ formula

Ex a) How many ~~solu~~ Non-Negative solution for

$a+b+c+d = 10$.

b) How many solution for

$c_1+c_2+c_3+c_4+c_5 = 15$

where $\boxed{c_1, c_2, c_3, c_4, c_5 \geq 0}$

Soln :- (a) $a+b+c+d = 10$

$\boxed{n=4, \ r=10}$

Non Negative Solution $\Rightarrow {}^{n-1+r}C_r = {}^{4-1+10}C_{10} \Rightarrow {}^{13}C_{10} = {}^{13}C_3$

(b) Here $c_1, c_2, c_3, c_4, c_5 \geq 0$ given it means ~~i~~ a want to find Non Negative solution

So $\boxed{n=5, \ r=15}$

$^{5-1+15}C_{15} \Rightarrow (^{19}C_{15}) \Rightarrow \frac{\lfloor 19}{\lfloor 15 \ \lfloor 4} \Rightarrow \frac{19 \times 18 \times 17 \times 16}{4 \times 3 \times 2}$

$\Rightarrow (19 \times 16 \times 3 \times 4)$ Ans

~~Distrib~~

3) Identical Ball in Box (distribution problem):-

(or)

Ball in the Box ✓

Que if their is 10 Identical ball is given and if I ask to place these Identical ball into 4 Boxes then How many ways can you place these balls into Boxes??

Sol^n ~~x_1+x_2~~ $B_1+B_2+B_3+B_4=10$

Surely these are Non Negative in Nature so, we can apply.

$^{n-1+r}C_r \Rightarrow {}^{4-1+10}C_{10} \Rightarrow ({}^{13}C_{10}) \Rightarrow ({}^{13}C_3)$ ✓ Ans

4) Out comes of Identical coin & dice :-

Que a) If 10 identical dice are thrown then How many Possible out comes are Possible??

b) If 20 Identical coin are thrown then How many Possible out comes are ??

Sol^n (a) 10 Identical dice

$d_1+d_2+d_3+\cancel{d_4+d_5}+d_6 = 10$

$\left.\begin{matrix} n=6 \\ r=10 \end{matrix}\right\}$ ${}^{6-1+10}C_{10} \Rightarrow \boxed{{}^{15}C_{10}}$

(b) 20 Identical coin

$C_H+C_T=20$

$\left.\begin{matrix} n=2 \\ r=20 \end{matrix}\right\} \Rightarrow {}^{20+2-1}C_{20} \Rightarrow \boxed{{}^{21}C_{20}}$

→ also k/a Non-Negative solution

5) Dice sum & digit sum problems :-

Ex if we throw 3 dice & sum == 9

↳ $x_1 + x_2 + x_3 = 9$

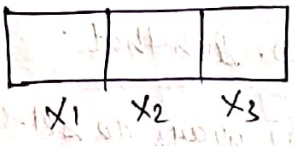
But Here $\boxed{n \neq 3}$ ⟸ we will see further in the Question

• In this way ~~8~~ digit sum problem are completed – ✓

Let's do Variation of Questions on $\left({}^{n-1+r}C_r\right)$ ✓ from Previous ⑤ Types

Que How many ways to put 10 Identical balls into ~~80~~ 3 boxes ✓

Sol^n



$\boxed{x_1 + x_2 + x_3 = 10}$ ✓

$n = 3,\ r = 10$

$^{3-1+10}C_{10} \Rightarrow \boxed{^{12}C_{10}}$

→ When each boxes have at least one balls. :-

In this type of problem put one ball in all the Boxes firstly & then apply the previous concept ☺

$\Rightarrow x_1 + x_2 + x_3 = 10 - 3$

$\Rightarrow \boxed{x_1 + x_2 + x_3 = 7}$ ✓

↳ $^{3-1+7}C_7 \Rightarrow \left({}^{9}C_7\right)$ ✓ $\Rightarrow \frac{9 \times 8}{2} \Rightarrow 36$ ✓

gate ✓

Que How many way we can put n balls in k boxes so, that ~~it is~~ the Boxes are non-Empty

Sol^n $x_1 + x_2 \text{ ------ } x_k = (n - k)$ ← NonEmpty ✓ // each Box have at least one Ball ✓

↳ $\left({}^{k-1+n-k}C_{n-k}\right) \Rightarrow \left({}^{n-1}C_{n-k}\right)$ ✓ $= \left({}^{n-1}C_{k-1}\right)$ ✓

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Que How many +ve Integral solution.

$$a+b+c+d=20$$

Sol^n a, b, c & d at least == 1

$$a+b+c+d=20-4$$

$$\boxed{a+b+c+d=16}$$ ✓

$$^{4-1+16}C_{16} \Rightarrow (^{19}C_{16}) = (^{19}C_3)$$ ✓

100 constraints with different-2 Values ✓

Que Three type of Chocklets Red, Green, Yellow. such that.

$\boxed{X_R+X_G+X_y=10}$ → find How many ways to select Chocklets.

where

$1\leq X_R$, $2\leq X_G$, $4\leq X_{yellow}$

Solutions:-

$$X_R+X_G+X_y=10-(4+2+1)$$

$$\boxed{X_R+X_G+X_y=3}$$

$$^{3-1+3}C_3 \Rightarrow (^5C_3) \Rightarrow \frac{5\times4}{2} = 10 \text{ Ways}$$

★★★ **Que** three dice are thrown in the way sum $(d_1+d_2+d_3=15)$

Find No. of way to find the combinations

Sol^n $d_1+d_2+d_3=15-(3)$ → we already know dice thrown will not be Blanle

$$\boxed{d_1+d_2+d_3=12}$$ ✓

So, take minimum constraints

$$^{3-1+12}C_{12} \Rightarrow (^{14}C_{12}) \Rightarrow (^{14}C_2)$$ ✓

Que How many Integral solution for —

$a+b+c=10$

constraints

$-2\leq a$, $-1\leq b$, $1\leq c$

Sol^n $\boxed{a+b+c=10}$

$a+b+c = 10-(-2-1+1)$

$\Rightarrow 10+2 \Rightarrow 12$

$\boxed{a+b+c=12}$

$\boxed{n=3, r=12}$ $\quad {}^{3-1+12}C_{12} \Rightarrow {}^{14}C_{12} = {}^{14}C_{2} \Rightarrow \frac{\cancel{14}^{7}\times 13}{\cancel{2}} \Rightarrow \boxed{91}$

Question Based on upper constraints :-

$\boxed{\text{upper constraints always less than ~~or gr~~ equal to right hand side}}$

Que $x_1+x_2+x_3=10$

$x_1\leq 8 \quad x_2\leq 5 \quad x_3\leq 7$

- ${}^{n-1+r}C_r$ Not for this for ~~the~~ multiple upper constraints

$\boxed{\text{Generating } F^n \text{ works for all type of constraints} \to \text{upper} \to \text{lower} \to \text{mixed}}$

- But it will consume time

- for one upper constraints we can go for $({}^{n-1+r}C_r)$

Ex $x_1+x_2+x_3=10$ & $x_1\leq 8$

Sol^n find complement of given constraints & then solve —

lower constraints

$x_1\leq 8 \xrightarrow{\text{comp}} \boxed{x_1\geq 9} \longrightarrow (x_1+x_2+x_3=1)$

${}^{3-1+10}C_{10} - \{{}^{3-1+1}C_1\} \Rightarrow {}^{12}C_{10} - \{{}^{3}C_{1}\} \Rightarrow \frac{\cancel{12}^{6}\times 11}{\cancel{2}} - 3$

$\Rightarrow 66-3 \Rightarrow \underline{63}$

In previous Question

↳ 3 sol^n removed $\left\{ \begin{array}{l} 9+0+1 \\ 9+1+0 \\ 10+0+0 \end{array} \right\}$

Que $\boxed{x_1+x_2+x_3 \leq 10}$

solve $x_1+x_2+x_3 = 1$
$= 2$
|
|
$= 10$

→ It will Takes Too much Time

So I Add one Virtual Box & Put Inequality into equality.

eg $\underbrace{x_1+x_2+x_3}_{\leq 10} + \underset{(x_4) \rightarrow \text{Remaining put Here}}{} = 10$

sol^n $({}^{4-1+10}C_{10}) \Rightarrow ({}^{13}C_{10})$

Que Solve for

(i) $x_1+x_2+x_3 \leq 13$

(ii) $x_1+x_2 \leq 12$

Sol^n (i) $x_1+x_2+x_3 \leq 13$

$x_1+x_2+x_3+x_4 = 13 \Rightarrow {}^{4-1+13}C_{13} \Rightarrow ({}^{16}C_{13})$

(ii) $x_1+x_2 \leq 12$

$x_1+x_2+x_3 = 12$

$\cdot {}^{3-1+12}C_{12} \Rightarrow ({}^{14}C_{12})$

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Que $x_1 + x_2 + x_3$ = 10 i'd Red

= 15 i'd yellow

= 12 i'd Green Balls.

Howmany ways we can put into Boxes??

Sol^n :- $x_1 + x_2 + x_3$ = 10 i'd Red —— n_1

= 15 i'd yellow —— n_2

= 12 i'd Green Ball —— n_3

Ans $\boxed{n_1 \times n_2 \times n_3}$

$\hookrightarrow \boxed{{}^{3-1+10}C_{10} * {}^{3-1+15}C_{15} * {}^{3-1+12}C_{12}}$

$\hookrightarrow \boxed{{}^{12}C_{10} * {}^{17}C_{15} * {}^{14}C_{12}}$ ✓

gate ✓

Que If their is 2 Girls & they picked 10 Lilly, 15 ~~Roses~~ deffidels, 12 sunflowers so, How many ways can these two girls ~~can~~ distribute these Flowers themselves ✓

Sol^n :- $G_1 + G_2$ = 10 Lilly

= 15 Roses

= 12 sunflowers

$= {}^{2-1+10}C_{10} + {}^{2-1+15}C_{15} + {}^{2-1+12}C_{12}$

$= \boxed{{}^{11}C_{10} + {}^{16}C_{15} + {}^{13}C_{12}}$ ✓ Ans ✓

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lecture - 9 :-

→ If their is limited constraints given then we can go for — Generating function.

(or)

- If we have limited No. of objects then we can use Generating Function

Ex :- for three dice rolling —

$$x_1 + x_2 + x_3 = 12 \quad // \; 1 \le x_1 \le 6,\; 1 \le x_2 \le 6,\; 1 \le x_3 \le 6$$

we know that.

dice have limit from 1 to 6 so Here we cannot use $^{n-1+r}C_r$ formola Because according to that formula

(12, 0, 0) is also possible which will not our soln.

So Here we use generating fn {Bcoz $x_1, x_2, \& x_3$ values are limited}.

- To use generating fn firstly form the generating function

ex :- form a generating fn for

(a) {a*5, b*3, c*6}

$$x_a + x_b + x_c = 10$$

$x_a \le 5$ $x_b \le 3$ $x_c \le 6$ ← Here upper Limit are given so use generating function

$$(1+x+x^2 - - x^5)\,(1+x+x^2+x^3)\,(1+x+x^2 - - + x^6)$$

~~Rough!- $x_1+x_2+x_3=12$ $1\le x_1\le 5$~~

- $1+x+x^2+x^3 - - - - - - - = \frac{1}{1-x}$
- $1+x+x^2+x^3 - - - - x^n = \frac{1-x^{n+1}}{1-x}$
- $\sum_{r=0}^{\infty} {}^{n-1+r}C_r\, x^r = \frac{1}{(1-x)^n}$

Formula ✓

Que $x_1+x_2+x_3=12$

$1\le x_1\le 6$ $\quad 1\le x_2\le 6$ $\quad 1\le x_3\le 6$

Sol^n To use Basic formula control Lower constraints firstly.

Here lower constraints start from 1 make it to zero ✓

$\overset{0}{x_1}+\overset{0}{x_2}+\overset{0}{x_3} = 12-(1+1+1) \Rightarrow 9$ ← our target is to getting coefficient of x^9.

$0\le x_1\le 5$ $\quad 0\le x_2\le 5$ $\quad 0\le x_3\le 5$

→ also 1 subtract Because you subtract one from Lower constraints

Then write Generating Function.

$$=(x^0+x^1+x^2 - - - x^5)^3$$

$$=\left[\frac{1(1-x^6)}{1-x}\right]^3 = \left(\frac{1-x^6}{1-x}\right)^3 = \frac{(1-x^6)^3}{(1-x)^3} = (1-x^6)^3\sum_{r=0}^{\infty} {}^{3-1+r}C_r\, x^r$$

$$=(1-x^6)^3\sum_{r=0}^{\infty} {}^{2+r}C_r\, x^r$$

$$=(1-x^{18}-3x^6(1-x^6))\sum_{r=0}^{\infty} {}^{2+r}C_2\, x^r$$

$$=(1+x^{18}-3x^6-3x^{12})\sum_{r=0}^{\infty} {}^{r+2}C_r\, x^r$$

But we are Interested in x^9

$$= (1-3x^6) \sum_{r\ge 0}^{\infty} {}^{r+2}C_2\, x^r$$

$$= (1-3x^6) \left[{}^{11}C_2 x^9 + {}^{5}C_2 x^3 \right]$$

$$\Rightarrow \quad {}^{11}C_2 x^9 - 3 * {}^{5}C_2 x^9$$

$$\Rightarrow \quad x^9 \left\{ {}^{11}C_2 - 3 * {}^{5}C_2 \right\} \Rightarrow \left\{ \frac{11*10}{2} - 3 * \frac{5\times 4}{2} \right\} x^9$$

$$= (55 - 30)\, x^9 \Rightarrow (25)\, x^9$$

So, 25 ways to get $\boxed{x_1+x_2+x_3=12}$

Ans 25

Que

5R	8B	7G

$\boxed{R+B+G=10}$ → selection with limited repetation.

$2 \le x_1 \le 5,\ 1 \le x_2 \le 8, \quad x \le 7$

Sol^n $\overset{00}{R} + \overset{0}{B} + G = 10$

$$\boxed{R+B+G = 10-3 = 7}$$

So, we will focus on x^7 coefficient

$$\boxed{0 \le x_1 \le 3,\ 0 \le x_2 \le 7,\ 0 \le x_3 \le 7}$$

$$(x^0+x^1+x^2+x^3)\,(x^0+x^1+x^2 - - x^7)\,(x^0+x^1 - - x^7)$$

$$\left(\frac{1-x^4}{1-x}\right)\left(\frac{1-x^8}{1-x}\right)\left(\frac{1-x^8}{1-x}\right)$$

$$\left(\frac{1-x^4}{1-x}\right) * \left(\frac{1-x^8}{1-x}\right)^2$$

$$= (1-x^4)\,(1-x^8)^2 \cdot \frac{1}{(1-x)^3}$$

$$= (1-x^4)\,(1-x^8)^2 \sum_{r\ge 0}^{\infty} {}^{3-1+r}C_r\, x^r$$

$$= (1-x^4)\,(1-x^8)^2 \sum_{r\ge 0}^{\infty} {}^{r+2}C_2\, x^r$$

$$= (1-x^4) \left\{ {}^{9}C_2 x^7 + {}^{5}C_2 x^3 \right\}$$

$$= x^7 \left\{ \frac{9\times 8}{2} - \frac{5\times 4}{2} \right\}$$

$$\Rightarrow (26)\, x^7$$

Ans 26

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Que How many ways can you distribute $(3n)$ identical balls into 2 Boxes. Such that each has atmost $(2n)$ balls.

Solⁿ

$$x_1 + x_2 = 3n$$

$$(0 \le x_1 \le 2n)(0 \le x_2 \le 2n)$$

$$= (x^0 + x^1 + x^2 - - - x^{2n})\,(x^0 + x^1 + x^2 - - - x^{2n})$$

$$= \left(\frac{1-x^{2n+1}}{1-x}\right)\left(\frac{1-x^{2n+1}}{1-x}\right)$$

directly we can write

$$\left(\frac{1-x^{2n+1}}{1-x}\right)^2$$

$$= \left(\frac{1-x^{2n+1}}{1-x}\right)^2$$

$$= (1-x^{2n+1})^2 \sum_{r=0}^{\infty} {}^{2-1+r}C_r\, x^r$$

$$= (1-x^{2n+1})^2 \sum_{r=0}^{\infty} {}^{r+1}C_r\, x^r$$

$$= (1-x^{2n+1})^2 \sum_{r=0}^{\infty} (r+1)\, x^r$$

$$= \{1 + \cancel{x^{4n+2}} - 2x^{2n+1}\} \sum_{r=0}^{\infty} (r+1)x^r$$

$$2n+1+P = 3n$$
$$P = 3n-2n-1$$
$$\boxed{P = (n-1)}$$

$$= (3n+1) - 2(n-1+1)$$

$$= 3n+1-2n \Rightarrow (n+1)$$

Ans

$\rightarrow (n+1)$ ways we put $3n$ balls into x_1 & x_2 Boxes

distribution problem (Partition Problem)

① ordered distribution → D to D {distinct}

② Unordered distribution → D to ID {Not distinct or Identical}

distribute (~~divide~~) → 10 toys — keyword

10 toys → C_1, C_2, C_3

distribute ~~divide~~ into 3 children

~~distribute~~ divide

10 Toys → P_1, P_2, P_3

divide 10 Toys into 3 Parts.

Ordered Partition (distribution)

1. distribution Fully specified ✓
2. " " Unspecified ✓
3. " " Partly specified. ✓

1. distribution Fully specified ✓

Ex → 10 Toys

A, B, C } distribute into 3 children A, B, & C ✓
2, 3, 5 find No. of ways to distribute it

$$\Rightarrow \frac{10!}{2!\,3!\,5!} \text{ Ans } ✓$$

(or)

$$\{{}^{10}C_2 * {}^{8}C_3 * {}^{5}C_5\} \Rightarrow \left(\frac{10!}{2!\,3!\,5!}\right) ✓$$

2. distribution unspecified :-

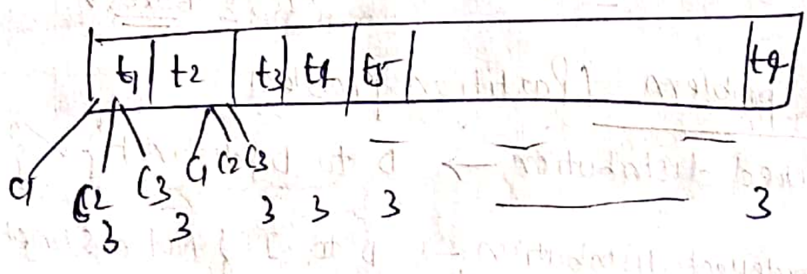
Ex:- How many way to distribute 10 Toys to 3 children.

Sol^n 10 ← upper (Because it will distribute) & distinct.

↓

3 ← lower

$(3)^{10} \rightarrow (3)^{10}$ Ans ✓



③ distribution Partilly specified

Que How many ways to distribute 4 Toys to 3 Children so that each child get 1 Toy

Sol^n

4 Toys

A B C

≥ 1 ≥ 1 ≥ 1

→ 1 1 1
+ (or) + (or) +
1 1 1

2	1	1
1	2	1
1	1	2

$\rightarrow \left(\frac{4!}{2!}\right) * 3$ Ans

→ If their is 5 Toys & 3 Children & each gets at least 1 ~~chi~~ Toys.

5

minimum satisfy → 1 1 1 → 2 Toys Remain distribute

3	1	1
1	3	1
1	1	3
2	2	1
2	1	2
1	2	2

$\rightarrow \left[\left(\frac{5!}{3!}\right)*3 + \left(\frac{5!}{2!\,2!}\right)*3\right]$ Ans

Que their is 20 Toys find How many Partition done if—

a) 20 Toys divide into three Boys & Boy A get 2 Toys exactly

b) 20 Toys divide into three Boys & Boy A get at least 1 Toy

c) Boy A get at least 2 Toys ~~& Boy ... at least ... Toy~~.

d) Boy A get at least 3 Toys

e) Boy A get at most 1 Toys.

f) Boy A get at most 2 Toys. (g) Boy A get at most 3 toys.

Sol^n :-

(a)

20 Toys

A B C

A ↓ 2 Toys

$\rightarrow \{ {}^{20}C_2 * 2^{18} \}$ ways

atleast

(b)

20 Toys

A B C

A ↓ ≤ 1 Toys

$\rightarrow \{ {}^{20}C_0 * 2^{20} + {}^{20}C_1 * 2^{19} \}$ ways

(c)

20 Toys

A B C

A ↓ ≤ 2

$\rightarrow ({}^{20}C_0\, 2^{20} + {}^{20}C_1\, 2^{19} + {}^{20}C_2\, 2^{18} \}$ ways

(d)

20 Toys

A B C

A ↓ ≤ 3

$\rightarrow ({}^{20}C_0\, 2^{20} + {}^{20}C_1\, 2^{19} + {}^{20}C_2\, 2^{18} + {}^{20}C_3\, 2^{17} \}$ ways

at most

(e)

20 Toys

A B C

A ↓ ≥ 1

$\rightarrow 3^{20} - \{ {}^{20}C_0 (2)^{20} \}$ ~~$+ {}^{20}C_1 (2)^{19}$~~ ways

↑ Total distribution

(f)

20 Toys

A B C

A ↓ ≥ 2

$\rightarrow 3^{20} - \{ {}^{20}C_0 (2)^{20} + {}^{20}C_1 (2)^{19} \}$ ways ~~$+ {}^{20}C_2 (2)^{18}$ ways~~

(g)

20 Toys

A B C

A ↓ ≥ 3

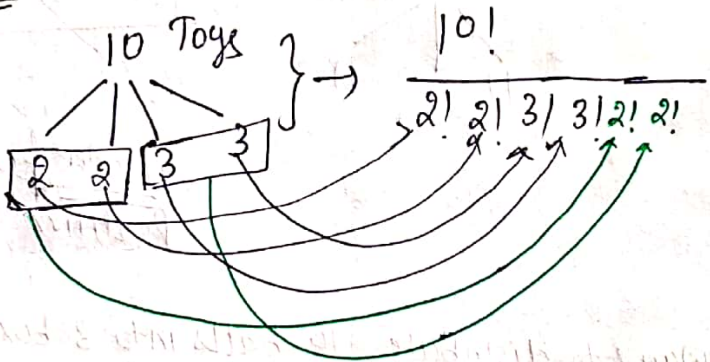
$\rightarrow 3^{20} - \{ {}^{20}C_0 (2)^{20} + {}^{20}C_1 (2)^{19} + {}^{20}C_2 (2)^{18} \}$ ways

Un-ordered Partition :- $\boxed{\text{Distinct} \longrightarrow \text{Indistinct}}$ ✓

↳ only fully specified ✓

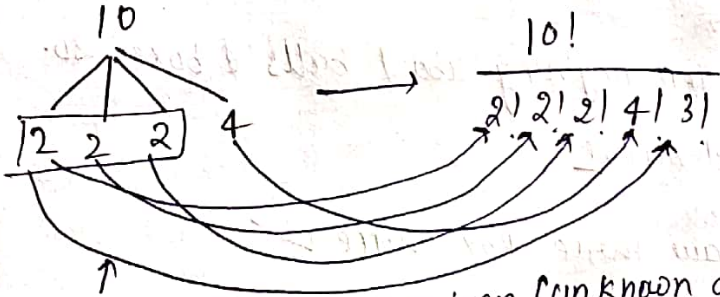
Ex 10 Toys distribute to 4 group such that they get 2, 2, 3 & 3 Toys. ✓

Sol^n



$$\frac{10!}{2!\,2!\,3!\,3!\,2!\,2!}$$

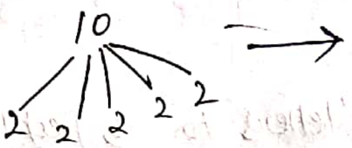
Ex



$$\frac{10!}{2!\,2!\,2!\,4!\,3!}$$

Bcoz groups Name Not given (unknown or unordered)

Ex



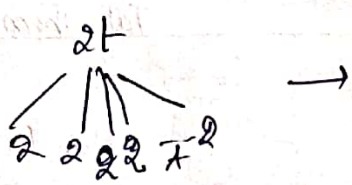
$$\frac{10!}{(2!)^5\, 5!}$$

Que How ~~Mays~~ Many ways divide 2t toys into ✓

(1) 2 groups "t" each

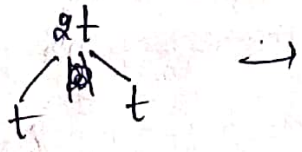
(2) t groups '2' each.

Sol^n ↳ (2)



$$\frac{(2t)!}{(2!)^t \times t!}$$ ✓

↳ (1)



$$\frac{(2t)!}{t!\,t!\,2!}$$ ✓

Que 8. Choose (11-11) people for two groups or teams from 30 people. How many ways are possible??

Sol^n :- 30 People → 11, 11 ⟶ 30 $\xrightarrow{\text{choose}}$ 22 People → 11, 11 ⟹ $\boxed{{}^{30}C_{22} * \dfrac{22!}{11!\,11!\,2!}}$

Unordered Partition { Because Team have No Name

Que How many ways to distribute 10 balls into 3 boxes.

Sol^n Here it cann't Tell Anything about balls & boxes so assume these are Identical

So, each ball have three Box choice

Ans 3^{10} ← 10 → B1 B2 B3

Que How many ways can distribute 15 letters to 5 People

Sol^n 15 letters → P1 P2 P3 P4 P5 ⟶ 5^{15}

$\boxed{\text{· Jb Tk Identical Ke liye Na Kha jae Tb Tk Identical Na maano Take Them as distinct}}$

Inclusion- Exclusion Principle :-

$n(A\cup B) \rightarrow$ // (A or B) or (at least one from A or B)

$n(A\oplus B) \rightarrow$ // (A or B But not Both) or (Eactly one of A or B)

$n(A-B) \rightarrow$ // (A But (AND) not B) $\rightarrow (A\cap \bar{B})$ or $\boxed{A-(A\cap B)}$

$n(B-A) \rightarrow$ // (B But not A)

$n(A^c\cap B^c) \rightarrow$ // (Neither A Nor B) or (None of the given No)

$n(A^c\cup B^c) \rightarrow$ // (Either Not A or Not B)

↳ $n(A) - n(A\cap B)$

$$\boxed{n(A\cup B\cup C) = n(A)+n(B)+n(C) - n(A\cap B) - n(B\cap C) - n(C\cap A) + n(A\cap B\cap C)}$$

$$\boxed{n(A\oplus B\oplus C) = n(A)+n(B)+n(C) - 2n(A\cap B) - 2n(B\cap C) - 2n(C\cap A) + 3n(A\cap B\cap C)}$$

$$n(A^c\cap B^c\cap C^c) = n(U) - n(A\cup B\cup C)$$

$$n(A^c\cup B^c\cup C^c) = n(U) - n(A\cap B\cap C)$$

Que How many 8 bit Binary Numbers either start with (11) or end with (000)

Sol^n use $n(A\cup B)$ // Either A or B //

1	1				0	0	0

$n(A) = 2^6 \quad n(B) = 2^5$

$n(A\cap B) = 2^3 \quad n(U) = 2^8$

A: (1) Either start with (11) or end with (000) $\Rightarrow 2^6 + 2^5 - 2^3$

(2) Either start with (11) or end with (000) but Not both

$= 2^6 + 2^5 - 2\times 2^3$

Que How many No. are from 1 to 100

a) ~~+~~ divisible by 2 or 3

b) ,, ,, but Not both

c) ,, , 2 but Not 3

d) ,, ,, 3 but Not 2

e) Neither divisible by 2 Nor 3

f) either not divisible by 2 or 3.

Soln 1 —— 10

$$\left.\begin{aligned} n(\div 2) &= 50 \Rightarrow \left\lfloor \tfrac{100}{2} \right\rfloor \\ n(\div 3) &= 33 \Rightarrow \left\lfloor \tfrac{100}{3} \right\rfloor \\ n(2 \cap 3) &= 16 \Rightarrow \{LCM(2,3)\} \\ n(U) &= 100 \end{aligned}\right\} \checkmark$$

a) ~~50+~~ 50+33−16

b) 50+33−32

c) 50−16

d) 33−16

e) 100 − {50+33−16}

f) 100 −16

***

• If in the Question it ask divisible by 2 or 3 from 100 to 1000

↳ (1—1000) − (1−99)

do

Dearrangement Problem:-

Dearrangement is the way through which we can.
we can put each No. at wrong position.

~~Opposite~~ $\boxed{\text{It is not opposite of arrangement bcoz of each having wrong position}}$

Que from $A = \{1,2,3,4,5\}$
find the de-arrangement : {sequence in which No Number is at right position}.

Sol^n $n(\bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D} \cap \bar{E}) = \underbrace{n(U)}_{\text{all Combination}} - \underbrace{n(A \cup B \cup C \cup D \cup E)}_{\text{atleast one at right Position}}$

Very length so their is a formula

Shortcut

$$\boxed{D_n = \sum_{r=2}^{n} (-1)^r \frac{n!}{r!}}$$

D_n ↑ de-arrangement of n element

Que $D_5 = ?$

Sol^n $= \left\{ \frac{5!}{2!} - \frac{5!}{3!} + \frac{5!}{4!} - \frac{5!}{5!} \right\}$

$= \{60 - 20 + 5 - 1\}$

$= \{44\}$ way

:- Lecture - 10 :-

Que How many de arrangement of $\{1,2,3,4\}$

Sol^n $D4 \Rightarrow \sum_{r=2}^{4} (-1)^r \frac{n!}{r!}$

$\Rightarrow \frac{4!}{2!} + \frac{-4!}{3!} + \frac{4!}{4!}$

$\Rightarrow 4! \left\{ \frac{1}{2!} - \frac{1}{6} + \frac{1}{4!} \right\} \Rightarrow 4*3 - 4 + 1$

$\Rightarrow 13 - 4 = 9$ **Ans**

Que If there are 5 Box & 5 ball ~~Pat~~ Numbers 1, 2, 3, 4 & 5 on each Box and Ball. ~~Be~~ Putt ball into Boxes such a way that ith Number ball cannot palace at ith No. Box.

Sol^n It is De-arrangement Problem ✓

Ans $D5 \Rightarrow (-1)^2 \frac{5!}{2!} - \frac{5!}{3!} + \frac{5!}{4!} - \frac{5!}{5!}$

$\Rightarrow (5 \times 4 \times 3) - (5 \times 4) + (5) - 1$

$\Rightarrow 60 - 20 + 5 - 1$

$\Rightarrow (44)$ ways ✓

Que if their is ~~4~~ 5 Boxes & 5 balls

① All ball going to wrong position

② at least 1 ball go in correct box.

③ all go in correct Boxes

④ at least one ball go in Incorrect box.

Sol^n

1) D5 way

2) $(5! - D5)$ way

3) 1 way

(4) $(5! - 1)$ way

[all go in correct position.

Que If there is 5 ball & 5 boxes

a) How many ways to Put Exactly one ball into the correct box.

b) How many ways to Put Excty two ball into the correct box.

Solⁿ a) $({}^5C_1 * D_4)$ ways.
D_4 → Incorrect Boxes Putting Incorrect balls.
${}^5C_1$ → Correct Ball into correct box.

b) $({}^5C_2 * D_3)$
D_3 → Incorrect Boxes putting Incorrect balls.
${}^5C_2$ → Corred Ball into correct box.

- The above formula we studied Till now are Not using for Repetation. For repetation we will do by Normal way.

gate ★★★
Ex How many de-arrangement For "LILAC"

Solⁿ

L	I	L	A	C
~~L~~	~~I~~	~~L~~	~~A~~	~~C~~

$${}^3C_2 * 2 * 2 \rightarrow 3\times2\times2 \Rightarrow \underline{12 \text{ Ways}}$$
${}^3C_2$ → L, 2 → C, 2 → (A,I)

← other arrangement gets Less ways than this
So, Ans is 12 Ways

Pigeon hole principle :-

↳ Keyword "atleast How many" (or)(minimum) & "same"

If there is n pigeons occupy m pigeon holes.
whereas $\boxed{n>m}$

Then

★★ $\geq \left(\left\lfloor \frac{n-1}{m} \right\rfloor + 1\right)$ pigeons at same pigeon-hole

(or) ★★ $\left\lceil \frac{n}{m} \right\rceil$ pigeons at same pigeon hole.

Que If 50 Bicycle have Coloured with 7 colour then ~~How ma~~ "at least" Howmany bicycle have coloured "same" colour.

Soln at least $\left\{\left\lfloor \frac{50-1}{7} \right\rfloor + 1\right\}$ Bicycle have same colour.

$\hookrightarrow [7+1] \Rightarrow 8$ ✓

Que How many bicycle ~~of~~ are coloured with 7 Colour so that it Takes gurontee so at ~~at~~ least 4 bicycle have same Colour

Soln:-

$$\left\{4 = \left\lfloor \frac{n-1}{7} \right\rfloor + 1\right\}$$

$$3 = \left\lfloor \frac{n-1}{7} \right\rfloor$$

$$n-1 = 21$$

$\boxed{n = 22} \rightarrow$ bicycle ✓

(or)

$$\left\lceil \frac{n}{m} \right\rceil = 4$$

$$\left\lceil \frac{n}{7} \right\rceil = 4$$

$\boxed{n \Rightarrow \text{minimum} \Rightarrow 22}$ ✓

Que :- 30 people are invited at Birthday party at least Howmany of them you guranteed will be born on same day of the week?

Soln $\left\lceil \frac{30}{7} \right\rceil \Rightarrow 5$ people Born on same day of week

(or)

$\left\{\left\lfloor \frac{30-1}{7} \right\rfloor + 1\right\} \Rightarrow \{4+1\} \Rightarrow 5$ people ✓

Que If 30 people are Invited at Birthday at least How many of them ~~y~~ guranteed will be born on same month.

Soln $\left\lceil \frac{30}{12} \right\rceil \Rightarrow 3$ people (or) $\left\lfloor \frac{30-1}{12} \right\rfloor + 1 \Rightarrow 3$ people ✓

Que Minimum How many card must be dealt from a deck of 52 cards. So as to gurantee that at least 3 ~~cards~~ of the same suit.

Sol^n:- $\left\lfloor \frac{n-1}{m} \right\rfloor + 1 = 3$

$\frac{n-1}{4} + 1 = 3$

$\frac{n-1}{4} = 2$

$\boxed{n = 9}$ → at least 9 card dealt

for same kind of (3) card Guranteed. { means 2 comes 3 times, King comes 3 times etc }

Take $\boxed{m = 13}$

$\frac{n-1}{13} + 1 = 3$

$n = 26 + 1$

$\boxed{n = 27}$

Que ~~You~~ If you dealt 30 cards then How many cards guranteed to be dealt of same kind

Sol^n $\left.\begin{matrix} n = 30 \\ m = 13 \end{matrix}\right\}$ $\left\lfloor \frac{n-1}{m} \right\rfloor + 1$

$\left\lfloor \frac{30-1}{13} \right\rfloor + 1 \Rightarrow$ 3 card of same kind guranteed

Que If Every possible committee of 3B & 2G from 10B & 5G is written on chits of paper (1 committee / chits) & Put then distribute these chits in 4 heads then one of those hats guaranteed at least chits?

Sol^n # Committee $= {}^{10}C_3 * {}^5C_2$

$= \frac{10\times9\times8}{6} * \frac{5\times4}{2}$

$=$ 1200 committee $\rightarrow n$

So, Their is 1200 Chits are there ✓

then distribute it to 4 heads $\rightarrow m$

Chits / head $\Rightarrow$ max $\Rightarrow \lfloor \frac{n-1}{m} \rfloor + 1$

$\Rightarrow \lfloor \frac{1199}{4} \rfloor + 1$

$= \lfloor 299.\text{-} \rfloor + 1 \Rightarrow 300$ Chits / head ✓

★★★ Que If pair of distinct Number are Choosen from $\{1, 2, 3 \text{ --- } 10\}$ then at least how many How many Pairs have same sum??

Sol^n $m = ?$ $\rightarrow$ No. of distinct Pair to produce sum ✓

~~Take 1 as first No~~ ✓ Sum # $\left\langle \begin{array}{l} \text{min } (1+2) \Rightarrow 3 \\ \text{max } (9+10) \Rightarrow 19 \end{array} \right\}$

~~then (1+2), (1+3), --- 1+19~~

$m \Rightarrow (19-3+1) \Rightarrow 17$

holes ✓

choose pairs $\Rightarrow {}^{10}C_2$

$\Rightarrow \frac{10\times9}{2} \Rightarrow 45$ ways $\leftarrow$ Pigeon ✓

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~~Sum~~ same sum = $\left\lfloor \frac{n-1}{m} \right\rfloor + 1$

$\Rightarrow \left\lfloor \frac{45-1}{17} \right\rfloor + 1$

$\Rightarrow \left\lfloor \frac{44}{17} \right\rfloor + 1 \Rightarrow 3$ ✓

Sol' Que:- 10 pairs of Red socks
15 " " Green "
12 " " Blue "
minimom How many socks you must withdraw.
So that you get matching Pairs✓.

Soln:- 10 pairs of Red socks → 20 socks
15 Pairs of Green " → 30 socks
12 " " Blue " → 24 socks

To guranteed the socks is Pair min $\{(10+15+12)+1\}$ Taken out

$\Rightarrow (37+1)$ ~~38~~ $\Rightarrow$ 38 socks✓

minimum How many socks with draw you must guranteed Red socks Pair found.

Soln $\left(\underset{Red}{10} + \underset{Green}{30} + \underset{Blue}{24}\right) + 1 \Rightarrow$ 65 socks // Guranteed Red Pair of socks ✓ found. ✓

:SUMMATION :- ✓

① Binomial summation ✓ // Properties of Binomial coefficient $\boxed{\sum_{r=0}^{n} {}^nC_r}$

② Generating function ✓ // $\sum_{r=0}^{\infty} x^r$ ← power series expansion ✓

Properties of Binomial Coefficient :-

Binomial coefficient ⇒ $\boxed{{}^nC_r}$

for ${}^nC_r \rightarrow$ $\boxed{{}^nC_0,\ {}^nC_1 \text{ ——— } {}^nC_n}$ are Binomial Coefficient & these are $(n+1)$ in numbers ✓ for $({}^nC_r)$ ✓

Comes when we do expansion of Binomial ✓

Binomial Expansion :-

$$(x+y)^n = \boxed{\sum_{r=0}^{n} {}^nC_r\, x^r y^{n-r}} \checkmark$$

Ex

$$(x+y)^2 = {}^2C_0 (x)^0 (y)^2 + {}^2C_1 (x)^1 (y)^1 + {}^2C_2 (x)^2 y^0$$

$$= 1y^2 + 2xy + 1x^2 \Rightarrow x^2 + y^2 + 2xy \checkmark$$

Que :- How many Term are their If you Expand $(x+y)^{50}$; ✓

Sol^n $(x+y)^{50} \rightarrow$ their are $(50+1)$ Term in the expansion.

from $\boxed{{}^{50}C_0 \longrightarrow {}^{50}C_{50}}$

Que If You Expand $(x+y)^{10}$ then what is the coefficient of x^2y^8 = ?

Sol^n ${}^{10}C_2\, x^2 y^8 \Rightarrow$ coefficient of $x^2y^8 \Rightarrow {}^{10}C_2 \Rightarrow \frac{\cancel{10}^{5} \times 9}{\cancel{2}} \Rightarrow 45$

Que for $\left(x^2+\frac{1}{x^3}\right)^{10}$ Find coefficient of x^5 ??

Sol^n $^{10}C_r (x^2)^r \left(\frac{1}{x^3}\right)^{10-r}$

$\Rightarrow {}^{10}C_r (x)^{2r} (x)^{3r-30}$

$\Rightarrow {}^{10}C_r (x)^{2r+3r-30}$

$\Rightarrow {}^{10}C_r (x)^{5r-30}$

$5r-30=5$

$\boxed{r=7}$ $\longrightarrow$ Ans $({}^{10}C_7)$ ✓

$P(x^{-5}) \rightarrow$ coefficient

$\hookrightarrow 5r-30=-5$

$\boxed{r=5}$ ✓

$\hookrightarrow ({}^{10}C_5)$ Ans

Multinomial Expansion:-

① Trinomial Expansion

$$\boxed{(x+y+z)^n = \sum \frac{n!}{n_1!\, n_2!\, n_3!} x^{n_1} y^{n_2} z^{n_3} \qquad \boxed{n = n_1+n_2+n_3}}$$

Here if anyone ask for x^{10} ~~How~~ find coefficient ??

then 10 divide in (n_1, n_2, n_3) & # of (n_1, n_2, n_3)

$10,0,0$
$1,2,7$
$3,2,5$
$\vdots$

\# Time n_1, n_2, n_3 finding How many ways we can divide it ✓

Solution = # of time coefficient comes

for $\boxed{n_1+n_2+n_3=n}$

$\hookrightarrow$ # Solution finding use generating Function ✓

$\boxed{{}^{(n-1+r)}C_r}$

Que In the Expansion of $(x+y+z)^{10}$ what is the coefficient of $x^2y^3z^5$

Solⁿ $\frac{10!}{2!\,3!\,5!} \Rightarrow \frac{10\times 9\times \cancel{8}^{4}\times \cancel{7\times 6}}{\cancel{3\times 2\times 2}} \Rightarrow 90 * 28$
$\Rightarrow 2520$ ✓

$\boxed{\frac{n!}{n_1!\,n_2!\,n_3!} = P(n, n_1, n_2, n_3)}$ ✓
repeated ✓

$\boxed{{}^nC_r = C(n,r)}$ ✓

$\boxed{{}^nC_r = \binom{n}{r}}$

Que Is it true or false
$$\binom{n}{r} = \binom{n}{n-r}$$

Ans True → $\boxed{{}^nC_r = {}^nC_{n-r}}$ ✓

Que Find coefficient of $x^2y^3z^5$ in $(3x+4y+5z)^{10}$

Solⁿ :- $\frac{10!}{2!\,3!\,5!}(3)^2 * (3)^3 (5)^5 \Rightarrow \frac{10\times 9\times 8\times 7\times 6}{\cancel{2\times 3\times 2}} * 4\times 3\times 3^2\times 5^5$

$\Rightarrow (420 * 48 * 9\times 5^5)$ Ans

Properties of Binomial coefficient:-

1. Symmetric property ⇒ ${}^nC_r = {}^nC_{n-r}$
2. Pascals formula : $\boxed{{}^nC_r = {}^{n-1}C_r + {}^{n-1}C_{r-1}}$
3. ★★ Newtons formula : $\boxed{{}^nC_R * {}^RC_K = {}^nC_K * {}^{n-K}C_{r-K}}$ ✓
4. Row sum $\boxed{\sum_{r=0}^{n} {}^nC_r = 2^n}$

It means $\boxed{{}^nC_0 + {}^nC_1 \text{ --- } {}^nC_n = 2^n}$

(5) Even & odd summation $\boxed{{}^nC_0 + {}^nC_2 + {}^nC_4 \text{ --- } = {}^nC_1 + {}^nC_3 + {}^nC_5 \text{ --- } = 2^{n-1} = \left(\frac{2^n}{2}\right)}$

(6) Alternating sign row sum

$$\boxed{\sum_{r=0}^{n} (-1)^r \, {}^nC_r = 0}$$

$\hookrightarrow \left({}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 \text{ --- } = 0\right)$

(7) row square summation

$$\boxed{\sum_{r=0}^{n} {}^nC_r^2 = {}^{2n}C_n} \rightarrow \boxed{{}^nC_0^2 + {}^nC_1^2 + {}^nC_2^2 \text{ --- } {}^nC_n^2 = {}^{2n}C_n}$$

(8) Column summation :-

$$\boxed{\sum_{k=0}^{N} \overset{\swarrow \text{variable}}{{}^kC_r} = {}^{N+1}C_{r+1}}$$

$\uparrow$ fix

(9)

$$\boxed{\sum_{r=0}^{n} r\,({}^nC_r) = n \cdot 2^{n-1}}$$

$$\boxed{\sum_{r=0}^{n} r(r-1)\,({}^nC_r) = n(n-1)\, 2^{n-2}}$$

$$\boxed{\sum_{r=0}^{n} r(r-1)(r-2)\,({}^nC_r) = n(n-1)(n-2)\, 2^{n-3}}$$

$r(r-1)(r-2)$

(10) Vandermonde's Identity :-

$$\boxed{\sum_{r=0}^{k} {}^{m}C_{r} \cdot {}^{n}C_{k-r} = {}^{m+n}C_{k}}$$

Boys Girls

$(m+n)$

m Boy n Girls

k Choose

0 B k Girls

1 B (k-1) Girls

k Boys 0 Girls.

Lecture - 11 :-

1) Symmetry

$$\boxed{{}^nC_r = {}^nC_{n-r}} \equiv \binom{n}{r} = \binom{n}{n-r}$$

$$\boxed{{}^{n-1+r}C_r \equiv {}^{n-1+r}C_{n-1}}$$

② Pascal's formula

$$\boxed{{}^nC_r = {}^{n-1}C_r + {}^{n-1}C_{r-1}}$$

↳ It tells if we select r people from n people then for a person x
- → It may be Included
- → It may be Excluded

for x Included ⇒ ${}^{n-1}C_{r-1}$
for x Excluded ⇒ ${}^{n-1}C_r$

$$\boxed{{}^nC_r = {}^{n-1}C_r + {}^{n-1}C_{r-1}}$$

[↳ more deep]

if (x, y) two are in (r) then four possibility are :-

① Both Excluded $(x\uparrow, y\uparrow) \longrightarrow {}^{(n-2)}C_r$
② Both Included $(x\downarrow, y\downarrow) \longrightarrow {}^{(n-2)}C_{(r-2)}$
③ x Included, y exclude $(x\downarrow, y\uparrow) \longrightarrow {}^{n-2}C_{r-1}$ ← x Include, y Exclude
④ x Exclude, y Include $(x\uparrow, y\downarrow) \longrightarrow {}^{n-2}C_{r-1}$ ← y Include

$$\boxed{{}^nC_r = \underset{x\uparrow\, y\uparrow}{{}^{n-2}C_r} + 2\cdot \underset{x\downarrow\, y\uparrow \,+\, x\uparrow\, y\downarrow}{{}^{n-2}C_{r-1}} + \underset{x\downarrow\, y\downarrow}{{}^{n-2}C_{r-2}}}$$

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Ques $\;{}^{89}C_{50} + {}^{89}C_{49} + {}^{90}C_{51} = ?$

Sol^n $\;\underbrace{{}^{89}C_{50} + {}^{89}C_{49}} + {}^{90}C_{51}$

$\underbrace{{}^{90}C_{50} + {}^{90}C_{51}}$

$\downarrow$

$({}^{91}C_{51})$ Ans

$= ({}^{91}C_{40})$ Ans $\quad \boxed{{}^nC_r = {}^nC_{n-r}}$

Ques $\;{}^{50}C_{30} + 2\left({}^{50}C_{29}\right) + {}^{50}C_{28} = ?$

Sol^n $\;\underbrace{{}^{50}C_{30} + {}^{50}C_{29}} + \underbrace{{}^{50}C_{29} + {}^{50}C_{28}}$

$= {}^{51}C_{30} + {}^{51}C_{29}$

$= ({}^{52}C_{30})$ Ans

3) Newton's formula :-

$$\boxed{{}^nC_r * {}^rC_k = {}^nC_k * {}^{n-k}C_{r-k}}$$

n
$\downarrow$
r committeee
$\downarrow$
K Leaders

Ex $\;\boxed{\left({}^{50}C_{20} * {}^{20}C_3\right) = \left({}^{50}C_3 \; {}^{47}C_{17}\right)}$

(1) (2) (3)

Choose leader

then make committee

Que How many way to place 8 white pawns & 8 black pawns on 8×8 chessBoard.

Sol^n :- Firstly choose 16 square in which choose 8 square for white then others (remain) will be for black.

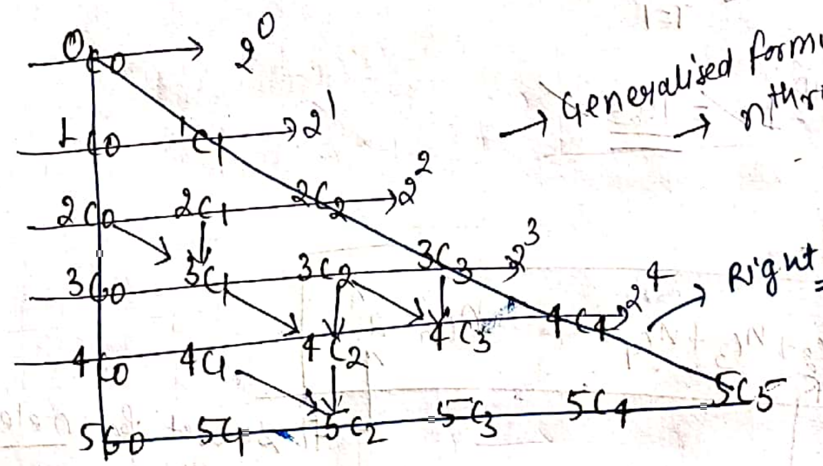
$$\left\{ {}^{64}C_{16} \times {}^{16}C_8 \right\} \equiv {}^{64}C_8 \times {}^{56}C_8$$

4) **Row sum :-**

$$\boxed{\sum_{r=0}^{n} {}^nC_r = 2^n}$$

$$\boxed{{}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + \cdots {}^nC_n = 2^n}$$

Pascal Trangle



→ Generalised formula → nth row addition → (2^n)

→ Right ∠ Δ

⇓

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

↓ ↓ ↓

Proof

$$\sum_{r=0}^{n} {}^nC_r\, x^r y^{n-r} = (x+y)^r$$

put $x=1$ & $y=1$

$$\sum_{r=0}^{n} {}^nC_r (1)^r (1)^{n-r} = (1+1)^r \Rightarrow (2)^r$$

Que :- What is the value of $\sum_{r=0}^{10} 2^4 3^6\, {}^{10}C_4$?

Sol^n $(2+3)^{10} \Rightarrow (5)^{10}$ Ans

Eg $\boxed{{}^6C_0 + {}^6C_1 + {}^6C_2 + {}^6C_3 + \dots {}^6C_6 = 2^6}$

Que What is the value of $\sum_{r=1}^{10} {}^{10}C_r$

Sol^n $\sum_{r=1}^{10} {}^{10}C_r = 2^{10} - 1 = \underline{1023}$

Another proof :-

$$\boxed{{}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + {}^nC_4 \dots {}^nC_n = 2^n}$$

zero element subset

one element subset

two element subset

subset for n elements

Combination of all $\Rightarrow (2^n)$

total No. of subsets

gate Que How many No. of total Possible labled graphs are from n vertices {1, 2, 3, - - n}

Soln for n vertices # Edges = $\left\{\frac{n(n-1)}{2}\right\}$ ✓

possible graphs ⇒ $^{\frac{n(n-1)}{2}}C_0 + {}^{\frac{n(n-1)}{2}}C_1 + \text{———} \; {}^{\frac{n(n-1)}{2}}C_{\frac{n(n-1)}{2}}$

$\Rightarrow \left[2^{\frac{n(n-1)}{2}}\right]$ Ans

↑ Total Possible No. of graph ✓

for Binomial

$$\sum_{r=0}^{n} {}^nC_r = 2^n$$

Trinomial

$$\sum_{n_1+n_2+n_3=n} P(n, n_1, n_2, n_3) = 3^n$$

$$\boxed{\sum_{n_1+n_2+ \text{——} n_r = n} P(n, n_1, n_2, \text{——} n_r) = r^n}$$ ← multinomial ✓

$$\boxed{\sum \frac{n!}{n_1!\,n_2!\,n_3!} = (3)^n}$$ ✓

5) Alternating sign row sum

$$\boxed{\sum_{r=0}^{n} (-1)^r \, {}^nC_r = 0}$$ ✓ $\xrightarrow{\text{Proof}}$ $\left(\sum_{r=0}^{n} (-1)^r \cdot (1)^{n-r} \, {}^nC_r\right)$

↓

$(-1+1)^n \Rightarrow (0)^n \Rightarrow 0$ ✓

Que What is the Value of -

$$\sum_{r=0}^{n} (-2)^r \, {}^nC_r$$

Sol^n $(-2+1)^n \Rightarrow (-1)^n$ Ans ✓

6) Even & odd summation

We know that :-

$${}^nC_0 + {}^nC_1 + {}^nC_2 + \cdots {}^nC_n = 2^n$$

& $${}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 \cdots {}^nC_n = 0$$

$$\boxed{{}^nC_0 + {}^nC_2 + {}^nC_4 \cdots = {}^nC_1 + {}^nC_3 + {}^nC_5 \cdots = \frac{2^n}{2} \Rightarrow 2^{n-1}}$$

Ex

$$\sum_{K=0}^{15} {}^{30}C_{2K} = \sum_{K=1}^{15} {}^{30}C_{2K-1} = 2^{29}$$ ✓

Que $A = \{1, 2, 3, \cdots 50\}$

a) How many subsets of odd No. of sum

b) How many subsets of even No. of sum.

Sol^n a) # odd No. of sum $\Rightarrow {}^{50}C_1 + {}^{50}C_3 + {}^{50}C_5 + \cdots = 2^{50-1}$

$\Rightarrow 2^{49}$ ✓

b) # even No. of sum $\Rightarrow 2^{49}$ ✓

Ques $\frac{1}{0!\,10!}+\frac{1}{2!\,8!}+\frac{1}{4!\,6!}+\frac{1}{}$ ---- $\frac{1}{10!\,0!}=?$

This could be written as $\rightarrow$ $\frac{2^m}{n!}$ then find $m, n = ?$

Soln $\frac{10!}{10!}\left\{\frac{1}{0!\,10!}+\frac{1}{2!\,8!}+\frac{1}{4!\,6!}+ ---- \frac{1}{10!\,0!}\right\}$

$\Rightarrow \frac{1}{10!}\left\{2^{10-1}\right\}$

$\Rightarrow \left(\frac{2^9}{10!}\right) \rightarrow \boxed{m=9,\ n=10}$

7) **Row square summation :-**

$$\boxed{\sum_{r=0}^{n} {}^nC_r^2 = {}^{2n}C_n}$$ ✓

$\left({}^nC_0^2+{}^nC_1^2+{}^nC_2^2+{}^nC_3^2 ---- {}^nC_n^2 = {}^{2n}C_n\right)$

2n People

nB nG

n People choose

0B nG
1B (n-1)G
2B (n-2)G
|
nB 0G

$\rightarrow \underbrace{{}^nC_0\,{}^nC_n}_{\text{same}} + \underbrace{{}^nC_1\,{}^nC_{n-1}}_{\text{same}} + {}^nC_2\,{}^nC_{n-2}$ ---- $+\,{}^nC_n\,{}^nC_0 = \left({}^{2n}C_n\right)$

*** **Ques** from 2n People (n Boys, n Girls) make a committee in which equal No. of Boys & girls are their ✓

Soln $\boxed{{}^nC_0 * {}^nC_0 + {}^nC_1\,{}^nC_1 + {}^nC_2\,{}^nC_2 + ---- {}^nC_n\,{}^nC_n \Rightarrow {}^{2n}C_n}$ ✓

8) Column sum

$$\boxed{\sum_{k=0}^{N} {}^{k}C_{r} = {}^{N+1}C_{r+1}}$$

fix

$$\boxed{\underbrace{{}^{0}C_{r} + {}^{1}C_{r} + {}^{2}C_{r}}_{0 \text{ provide}} + \dots {}^{r}C_{r} \dots {}^{N}C_{r} = {}^{N+1}C_{r+1}}$$

So

$$\boxed{\sum_{k=r}^{N} {}^{k}C_{r} = {}^{N+1}C_{r+1}}$$

Ex $\sum_{k=10}^{50} {}^{k}C_{10} = ?$

Sol^n ${}^{10}C_{10} + {}^{11}C_{10} + {}^{12}C_{10} + \dots \dots {}^{50}C_{10} = {}^{50+1}C_{10+1} = ({}^{51}C_{11})$

Proof → ${}^{0}C_{0}$ column

${}^{1}C_{0}$ ${}^{1}C_{1}$ $\boxed{{}^{N+1}C_{r+1}}$

${}^{2}C_{0}$ ${}^{2}C_{1}$ ${}^{2}C_{2}$

${}^{3}C_{0}$ ${}^{3}C_{1}$ ${}^{3}C_{2}$ ${}^{3}C_{3}$

${}^{4}C_{0}$ ${}^{4}C_{1}$ ${}^{4}C_{2}$ ${}^{4}C_{3}$ ${}^{4}C_{4}$

9) Variation of row sum.

$$\sum_{r=0}^{n} {}^{n}C_{r} = 2^{n} \rightarrow \boxed{\sum_{r=0}^{n} r^{0}\, {}^{n}C_{r} = n^{0} 2^{n-0}} \rightarrow 2^{n}$$

$$\boxed{\sum_{r=0}^{n} r \cdot {}^{n}C_{r} = n^{1} 2^{n-1}} \rightarrow n2^{n-1}$$

Ex $\boxed{0\cdot {}^nC_0 + 1\cdot {}^nC_1 + 2\cdot {}^nC_2 + 3\cdot {}^nC_3 + \cdots\cdots n\cdot {}^nC_n \Rightarrow n\cdot 2^{n-1}}$

derivation

$$\boxed{\sum_{r=0}^{n} {}^nC_r x^r y^{n-r} = (x+y)^n} \rightarrow \text{We know}$$

$\downarrow$

Put $(y=1)$

$$\sum_{r=0}^{n} {}^nC_r x^r = (1+x)^n$$

$\downarrow$, differentiate

$$\boxed{\sum_{r=0}^{n} {}^nC_r\, r\cdot x^{r-1} = n(1+x)^{n-1}} \rightarrow \text{Put } (x=1) \quad \sum_{r=0}^{n} r\cdot {}^nC_r = n(1+1)^{n-1}$$

$$\downarrow \quad \boxed{n\cdot(2)^{n-1}}$$

So,

1) $\sum_{r=0}^{n} r\cdot {}^nC_r = n\cdot 2^{n-1}$

2) $\sum_{r=0}^{n} {}^nC_r = 2^n$

3) $\sum_{r=0}^{n} r(r-1)\, {}^nC_r = n(n-1)\, 2^{(n-2)}$

~~3) $\sum_{r=0}^{n} r^2[{}^nC_r] = n(n-1)\,2$~~

~~4) $\sum_{r=0}^{n} r^3[{}^nC_r] =$~~

$\hookrightarrow$ from this you can find $\boxed{\sum_{r=0}^{n} r^2\, {}^nC_r = n(n-1)2^{(n-2)} + \sum_{r=0}^{n} r\, {}^nC_r}$

$$= n(n-1)2^{n-2} + n\,2^{(n-1)}$$

$$= n2^{n-2}\{(n-1) + 2\}$$

$$= \{n(n+1)\,2^{n-2}\}$$

Que If $\sum_{r=0}^{n} (3r^2+4r+5)\, {}^{10}C_r = ?$

Sol^n $\boxed{\sum_{r=0}^{n} 3r^2\times {}^{10}C_r + \sum_{r=0}^{n} 4r\times {}^{10}C_r + \sum_{r=0}^{n} 5\times {}^{10}C_r}$

One more Variation

$$\sum_{r=0}^{n} {}^{n}C_r\, x^r = (1+x)^n$$

Integrate on bothside w.r.t x.

$$\sum_{r=0}^{n} {}^{n}C_r \frac{x^{r+1}}{(r+1)} = \frac{(1+x)^{n+1}}{(n+1)}$$

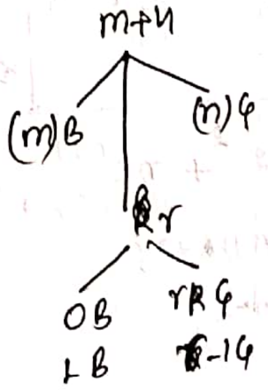
put $\boxed{x=1}$

$$\boxed{\sum_{r=0}^{n} {}^{n}C_r \frac{1}{(r+1)} = \frac{(2)^{n+1}}{(n+1)}}$$

Onetime more Integration :-

$$\boxed{\sum_{r=0}^{n} {}^{n}C_r \frac{1}{(r+1)(r+2)} = \frac{(2)^{n+2}}{(n+1)(n+2)}}$$

9) Vandermonde's Identity.



$${}^{m}C_0\, {}^{n}C_r + {}^{m}C_1\, {}^{n}C_{r-1} \text{ ------ } {}^{m}C_r\, {}^{n}C_0 = {}^{m+n}C_r$$

$$\boxed{\sum_{K=0}^{r} {}^{m}C_K\, {}^{n}C_{r-K} = {}^{m+n}C_r}$$

Que $\sum_{r=0}^{10} {}^{30}C_k * {}^{40}C_{r-k} = ?$

Sol^n ${}^{(30+40)}C_r = ({}^{70}C_{10})$ ✓ **Ans** ✓

Que what is the coefficient of x^8 in $(1+x)^{10}$.

Sol^n $(1+x)^{10} = {}^{10}C_r (x)^r \leftarrow$ Put $\boxed{r=8}$

$\hookrightarrow ({}^{10}C_8) = {}^{10}C_2$ **Ans** ✓

Que what is the coefficient of x^8 in $(x^2+3x^3-5x^4)(1+x)^{10}$

Sol^n: $(x^2+3x^3-5x^4)(1+x)^{10}$

$\hookrightarrow$ find (x^6, x^5, x^4). To get x^8 coefficient

$\hookrightarrow {}^{10}C_6 + {}^{10}C_5 + {}^{10}C_4$

So $\left\{({}^{10}C_6 * 1) + ({}^{10}C_5 * 3) + ({}^{10}C_4 * 5)\right\} x^8$

$\hookrightarrow \left\{\left(\frac{10\times 9\times 8\times 7}{4\times 3\times 2}\right)+1\right\} + \left(\frac{10\times 9\times 8\times 7\times 6}{5\times 4\times 3\times 2}+3\right) + \left(\frac{10\times 9\times 8\times 7}{4\times 3\times 2}\right)5$

$= (210+1) + (36\times 7+3) + (210-5)$

$= 211 + (252+3) + 205$

$= 211 + 255 + 205$

$= 211 + 460 \Rightarrow \underline{671}$ **Ans**

$\Rightarrow \boxed{{}^{10}C_6 + 3\,{}^{10}C_5 - 5\,{}^{10}C_4}$ ✓

Generating Function:-

When ever summation ——

$$\boxed{\begin{array}{l} r \text{ is from } \rightarrow (0 - n) \leftarrow \text{Binomial coefficient} \\ r \text{ is from } \rightarrow (0 - \infty) \leftarrow \text{Generating } f^n \end{array}}$$

:- Like Integration for any ~~Valu~~ Numeric Value you get Integration In this way If we use generating Function then we get formula Like Integration.

But Integration work on continious Value & Generating F^n work on discrete Value

Table of Generating Function :-

Sequence	Numeric f^n $\boxed{[a_r]_{r=0}^{\infty}}$	Generating Function $\boxed{A(x) = \sum_{r=0}^{\infty} a_r x^r}$
1, 1, 1, 1, - - -	1	$\frac{1}{1-x} \Rightarrow \sum_{r=0}^{\infty} 1 \cdot x^r = \sum_{r=0}^{\infty} x^r$ $1+x+x^2 —— \infty$
5, 5, 5, 5, ——	5	$\sum_{r=0}^{\infty} 5 \cdot x^r \Rightarrow 5 \cdot \sum_{r=0}^{\infty} x^r \Rightarrow 5\{1+x+x^2 —— \infty\}$ $\left(\frac{5}{1-x}\right)$
$\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$ ——	1/3	$\left\{\frac{1}{3(1-x)}\right\}$
0.1, 0.1, 0.1 ——	0.1	$\left\{\frac{0.1}{1-x}\right\}$
−3, −3, −3, ——	−3	$\left\{\frac{-3}{1-x}\right\}$

Lecture-12

Generating Function Table :- Application is to solve Ball & Box Problem with upper constraints

Table-I

Sequence a_0, a_1, a_2 ---	Numeric Function $[a_r]_0^\infty$	Generating f^n. $A(x) = \sum_{r=0}^{\infty} a_r x^r$
$1,1,1,1,$ ---	1	$\sum_{r=0}^{\infty} 1 \cdot x^r = \left(\frac{1}{1-x}\right)$ $x^0 + x^1 + x^2 + x^3 +$ ---
$1,-1,+1,-1,+1,-1$ ---	$(-1)^r$	$\left(\frac{1}{1+x}\right) \to \sum_{r=0}^{\infty} (-1)^r x^r$
k^0, k^1, k^2, k^3 --- ex $5^0, 5^1, 25, 125,$ $\Rightarrow (5)^r \to \left(\frac{1}{1-5x}\right)$ ✓	$(K)^r$	$\sum_{r=0}^{\infty} (kx)^r \Rightarrow \sum_{r=0}^{\infty} K^r (x)^r$ $\Rightarrow \left(\frac{1}{1-kx}\right)$
$1, -k, k^2, -k^3,$ ---	$(-1)^r (k)^r = (-K)^r$	$\frac{1}{1+Kx} = \sum_{r=0}^{\infty} (-1)^r (kx)^r$
Table-II $1,0,1,0,1,0,$ ---	$a_r = 1$; $r = 2x$ $= 0$; $r = 2x-1$ or $r \neq 2x$	$\frac{1}{1-x^2} = \sum_{r=0}^{\infty} (x^2)^r$ $\Rightarrow (x^0 + x^2 + x^4 + x^6 + x^8$ --- $)$
$1,0,0,1,0,0,1$ ---	$a_r = 1$; $r = 3x$ $= 0$; $r \neq 3x$	$\frac{1}{1-x^3} = \sum_{r=0}^{\infty} (x^3)^r$ $\Rightarrow (x^0 + x^3 + x^6 +$ --- --- $)$
$1,\underbrace{0,0 \text{---}}_{k-1},1,0,0$ ---	$a_r = 1$; $r = Kx$ $= 0$; $r \neq Kx$	$\frac{1}{1-x^k}$
${}^{n-1}c_0, {}^{n}c_1, {}^{n+1}c_2, {}^{n+2}c_3$ ---	${}^{n-1+r}c_r$	$\frac{1}{(1-x)^n}$ $\to$ use in Ball & Box Problem ✓
${}^nc_0, {}^nc_1, {}^nc_2$ --- ${}^nc_n, 0, 0,$ ---	${}^nc_r$	$(1+x)^n \Rightarrow \sum_{r=0}^{n} {}^nc_r x^r = (1+x)^n$
$1, 1, \frac{1}{2!}, \frac{1}{3!}, \frac{1}{4!}$ ---	$\frac{1}{r!}$	e^x $(r=\infty)$ ✓

Que :- If $a_r = 1$ then find $A(x) = ?$ **Ans** :- $A(x) = \sum_{r\geq 0}^{\infty} 1 \cdot x^r \Rightarrow \left(\frac{1}{1-x}\right)$

Que :- If $A(x) = \frac{1}{1-x}$ then find $a_r = ?$ **Ans** :- $A(x) = \frac{1}{1-x}$ $A \Rightarrow a_r \Rightarrow 1$ ✓

Que :- if $\sum_{r\geq 0}^{\infty} x^r = \frac{1}{1-x}$ then find Numeric f^n. **Ans** :- Numeric $f^n = 1$.

:- Properties of Generating Function :-

1) $a_r \rightarrow A(x)$

then

$K a_r \rightarrow K \cdot A(x)$

Ex $\Rightarrow \sum_{r\geq 0} a_r x^r = A(x)$ ✓

$K\left(\sum_{r=0}^{\infty} a_r x^r\right) = K \cdot A(x)$ ✓

2) $A(x) \rightarrow a_r$

$A(Kx) \rightarrow a_r (K^r)$

Ex $\Rightarrow A(x) = \sum_{r=0}^{\infty} a_r x^r$

$A(Kx) = \sum_{r\geq 0}^{\infty} a_r (Kx)^r \Rightarrow \sum_{r=0}^{\infty} K^r a_r (x)^r$

3) Linearity property

$\left.\begin{array}{l} a_r \rightarrow A(x) \\ b_r \rightarrow B(x) \end{array}\right\} \rightarrow a_r + b_r \rightarrow A(x) + b(x)$ ✓

$\rightarrow a_r - b_r \rightarrow A(x) - b(x)$ ✓

So, Generally

$$\boxed{K_1 a_r + K_2 b_r \rightarrow K_1 A(x) + K_2 B(x)}$$ ✓

$$\boxed{\cancel{a_r \cdot b_r = A(x) \cdot B(x)}}$$

↳ multiplication & division rule cannot follow this ✓

④ shifting property of generating F^n ✓

In the given generating f^n if we multiply it by x then all the series will shift Right, if we multiply it by x^2 then all the series will shift two time right ✓.

$a_0, a_1, a_2, a_3, \text{-----} \rightarrow A(x)$

$0, a_0, a_1, a_2 \text{-------} \rightarrow x A(x)$

$0, 0, a_0, a_1, a_2 \text{------} \rightarrow x^2 A(x)$

Que What is the Numeric Value of $\frac{1}{1-3x}$??

Sol^n $\frac{1}{1-3x} \Rightarrow (3)^r$

$\frac{1}{1-x} \Rightarrow (1)^r = 1$

$\frac{5}{1-x} \Rightarrow 5*(1)^r = 5$

Que what is the Generating F^n of $1, -\frac{1}{3}, \frac{1}{9}, -\frac{1}{27}$ --- ??

Sol^n:- $1, -\frac{1}{3}, \frac{1}{9}, -\frac{1}{27}$, ---

$\Rightarrow (-1)^r \left(\frac{1}{3}\right)^r \Rightarrow \frac{1}{1-\left(\frac{-1}{3}\right)x} \Rightarrow \frac{1}{1+\frac{1}{3}x} \Rightarrow \left(\frac{3}{3+x}\right)$ Ans

Que what is the generating F^n of $\frac{1}{1-x^5}$??

Sol^n:- $a_r = 1$; $r = 5K$
$a_r = 0$; $r \neq 5K$ } $\rightarrow$ sequence $1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1,$ ---

Que what is the generating F^n of $1, 0, 0, 1, 0, 0, 1,$ ---

Sol^n $1, 0, 0, 1, 0, 0, 1$ ---

$k-1 = 2 \longrightarrow \frac{1}{1-x^3}$ Ans

$(k=3)$

Que what is the generating F^n of $5, 0, 0, 5, 0, 0,$ ---

Sol^n $5, 0, 0, 5, 0, 0,$ --- Take 5 as common

$5\{1, 0, 0, 1, 0, 0, 1,$ --- $\}$

$\Rightarrow 5\left(\frac{1}{1-x^3}\right) \Rightarrow \frac{5}{1-x^3}$

Que what is the sequence of $\frac{1}{1-(3x)^5}$

Sol^n:- $\frac{1}{1-(3x)^5} \Rightarrow a_r \begin{cases} (3)^r ; & \text{when } r = 5K \\ 0 & \text{when } r \neq 5K \end{cases}$

sequence

$1, 0, 0, 0, 0, (3)^5, 0, 0, 0, 0, (3)^{10}, 0, 0, 0, 0$

Que what is the Numeric Value of $\frac{1}{(1-x)^5}$

Solⁿ $5-1+r\,C_r \Rightarrow {}^{4+r}C_r \Rightarrow ({}^{r+4}C_r) = ({}^{r+4}C_4)$ Ans

& coefficient of (x^7)

put $r=7 \Rightarrow ({}^{11}C_4)$ ✓

***Que If $A(x) = \frac{7}{(1+3x)^5}$ then what is the Numeric Function correspond to this generating Function??

Solⁿ:- $A(x) = \frac{7}{(1+3x)^5} \Rightarrow \frac{7}{(1-(-3x))^5}$

$\Rightarrow 7 * (-1)^r (3)^r * {}^{5-1+r}C_r$

$\Rightarrow (7) * (-1)^r (3)^r * {}^{4+r}C_r$ ✓ Ans

Que what is the generating Function of. $\sum_{r=0}^{\infty} 3.5^r \, {}^{r+3}C_3 \, x^r$

Solⁿ $A \sum_{r=0}^{\infty} 3.5^r \, {}^{r+3}C_3 \, x^r$

$3\left\{\frac{1}{(1-5x)^4}\right\} \Rightarrow \frac{3}{(1-5x)^4}$ ✓

Que what is the generating function of ${}^5C_0, {}^6C_1, {}^7C_2, ---$

Solⁿ ${}^5C_0 \Rightarrow {}^{6-1+0}C_0 \Rightarrow (n=6) \Rightarrow \left\{\frac{1}{(1-x)^6}\right\}$ ✓

When in ${}^xC_y$; x & y both Increase by 1 the $\frac{1}{\not{p}}$ ${}^{n-1+r}C_r$ generating fⁿ used.

Que what is the generating fⁿ of $\sum_{r=0}^{\infty} {}^nC_r (3)^r x^r$ & $\sum_{r=0}^{\infty} {}^nC_r (-1)^r (3)^r x^r$.

Solⁿ $\Rightarrow (1+3x)^n$ & $(1-3x)^n$

Que $\sum_{r=0}^{\infty} \frac{(-1)^r 5^r}{r!} x^r = ?$ $\rightarrow$ e^{-5x}

Solⁿ $\frac{(-1)^r 5^r}{r!}$ $(-1)^r 5^r \rightarrow e^{5x}$, $r! \rightarrow e^x$

Ans e^{-5x} ✓

Que $\sum_{r=0}^{\infty} 7 \cdot \frac{(-1)^r 7^r}{r!} x^r = ? \rightarrow 7 \cdot (e^{-7x})$ Ans

Que coefficient of a_{10} in e^{-2x} ??

Solⁿ – $\frac{(-1)^{10} (2)^{10}}{10!} \Rightarrow \frac{(2)^{10}}{10!}$ ✓ Ans

Que What is the value of $\sum_{r=0}^{\infty} (5 \cdot 3^r + 7(-1)^r) x^r$

Solⁿ $\sum_{r=0}^{\infty} (5 \cdot 3^r + 7(-1)^r) x^r$

$A(x) = 5 \cdot \frac{1}{1-3x} + 7 \cdot \frac{1}{1+x} \Rightarrow \left(\frac{5}{1-3x} + \frac{7}{1+x}\right) \rightarrow$ solve according to option

Table-3 of Generating Function :-

Sequence	Numeric Function (a_r)	Generating Fⁿ $A(x)$
1, 1, 1, 1, ---	1	$\frac{1}{(1-x)}$
0, 1, 2, 3, 4, ---	r	$\frac{x}{(1-x)^2}$
0, 1, 4, 9, 16, ---	r^2	$\frac{x(x+1)}{(1-x)^3}$

$\boxed{\sum_{r=0}^{\infty} {}^{n-1+r}C_r x^r = \frac{1}{(1-x)^n}}$ ↑ Prove by this

Que What is the Value of $A(x)$ for $\sum_{r=0}^{\infty}(3r^2-4r+5)x^r$

Sol^n:- $\sum_{r=0}^{\infty}\left(3r^2-4r+5\right)x^r$

$$= 3 \left\{\frac{x(x+1)}{(1-x)^3}\right\} - 4\left\{\frac{x}{(1-x)^2}\right\} + \left\{\frac{5}{1-x}\right\}$$

Que what is the sequence of (1) $\frac{1}{1-3x}$ (2) $\frac{x}{1-3x}$ (3) $\frac{x^2}{1-3x}$

Sol^n (1) $1, 3, 3^2, 3^3, \ldots = 3^r$ ✓

(2) $0, 1, 3, 3^2, 3^3, \ldots = 3^{r-1}$ ✓

(3) $0, 0, 1, 3, 3^2, 3^3 \ldots = 3^{r-2}$ ✓

Que what is the Value of $a_8 = ?$ in $\left(\frac{x^2}{1-7x}\right)$ ✓

Sol^n $\left(\frac{x^2}{1-7x}\right) \Rightarrow (7)^{r-2} \Rightarrow$ for $a_8 \rightarrow (7)^6$ Ans.

Que What is a_n of $\frac{1}{x^2-5x+6}$??

Sol^n $\frac{1}{x^2-5x+6}$

$\frac{1}{(x-2)(x-3)} \Rightarrow \frac{A}{x-2} + \frac{B}{x-3}$ ✓

$\Rightarrow \left(\frac{-1}{x-2}\right) + \left(\frac{1}{x-3}\right)$

$\Rightarrow \frac{1}{2-x} - \frac{1}{3-x} \Rightarrow \frac{1}{2}\left(\frac{1}{2}\right)^r - \frac{1}{3}\left(\frac{1}{3}\right)^r$

$\Rightarrow \left(\frac{1}{2}\right)^{r+1} - \left(\frac{1}{3}\right)^{r+1}$ ✓

Conversion

① $\frac{1}{3x-5} \rightarrow \frac{-1}{5-3x}$

$\downarrow$

$\left(\frac{-1/5}{1-3/5\,x}\right)$ And then solve

② If Quadratic f^n is at denominator then use Partial fraction ✓

③ $\frac{1}{2x+6} \Rightarrow \frac{1}{6+2x} \Rightarrow \frac{1}{6}\left(\frac{1}{1+\frac{1}{3}x}\right)$ & then solve

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Gate Questions ~~for~~ from generating Function :-

gate 2005

Que-1 $G(x) = \frac{1}{(1-x)^2} = \sum_{i=0}^{\infty} g(i)\, x^i$ when $|x|<1$

what is $g(i)$

(a) i (b) $i+1$ (c) $2i$ (d) 2^i

Sol^n $G(x) = \frac{1}{(1-x)^2} \Rightarrow \left({}^{n-1+r}C_r\right)$, $n = 2$, $r = i$ $\Rightarrow {}^{2-1+i}C_i \Rightarrow {}^{i+1}C_1 \Rightarrow (i+1)$ ✓ Ans (b) ✓

Gate 2016

The coefficient of x^{12} in $(x^3+x^4+x^5+x^6+ \text{---})^3$ is ___

Sol^n :- $x^9 \{1+x+x^2+ \text{---}\}^3$

$= \frac{x^9}{(1-x)^3} \Rightarrow x^9 * \sum_{r=0}^{\infty} {}^{3-1+r}C_r\, x^r$

$\Rightarrow$ for $r=3$ ✓

${}^{3-1+3}C_3 \Rightarrow {}^5C_3 \Rightarrow \frac{5\times 4}{2} \Rightarrow 10$ ✓

★★★ **Gate 2017** If the coefficient of ordinary generating Function ✓ of a sequence $\{a_n\}_{n=0}^{\infty}$ is $\frac{1+z}{(1-z)^2}$ then $a_3 - a_0 = ?$

Sol^n $\frac{1+z}{(1-z)^2} \Rightarrow \frac{1+x}{(1-x)^2} \Rightarrow \frac{1}{(1-x)^2} + \frac{x}{(1-x)^2}$

$\Rightarrow {}^{2-1+r}C_r + {}^{1+(r-1)}C_{r-1}$

$\Rightarrow \boxed{{}^{(1+r)}C_1 + \boxed{{}^{r}C_{r-1}}}$ → $r \geq 1$ } shifting ; $r=0$ → 0

$\Rightarrow$ ~~$(r+1)+r \Rightarrow (2r+1)$~~

$(r+1) + (r)$

$a_3 \rightarrow (3+1)+(3) \Rightarrow 7$ } $7-1 = 6$ Ans

$a_0 \Rightarrow (0+1)+0 \Rightarrow 1$

***Gate 2018 :-**

Which one of the following is the closed form of sequence (a_n) where as $(a_n = 2n+3)$ $(n = 0, 1, 2, 3, \text{---})$

a) $\frac{3}{(1-x)^2}$ b) $\frac{3x}{(1-x)^2}$ (c) $\frac{2-x}{(1-x)^2}$ (d) $\frac{3-x}{(1-x)^2}$

Sol^n $a_n = 2n+3$

$\downarrow$

$a_r = 2r+3$

$\downarrow$

$2 \cdot \frac{x}{(1-x)^2} + \frac{3}{(1-x)} = \frac{3-x}{(1-x)^2}$ Ans

***Que** what is the generating function for:

a) $(3, -1, 6, 10, 0, 1, 0, 0, 0)$ b) $3, -5, 0, 0, \text{---}$

Sol^n (a) $(3 \times x^0) + (-1)x^1 + 6 \times x^2 + 10 \times x^3 + 1 \times x^5$

$= 3 - x + 6x^2 + 10x^3 + x^5$

(b) $3 \times x^0 + (-5) \times x^1$

$\Rightarrow 3 - 5x$

Lecture - 13

Recurrence relation

$\boxed{a_n = 2n}$ → This is solved form of recurrence relation.

$a_0 = 0$
$a_1 = 2$
$a_2 = 4$
$a_3 = 6$

Some time, Solved recurrence relation Not given so firstly we will solved it & then give Answer

Ex $\boxed{a_n = 2a_{n-1}}$ → order of recurrence rel^n ⇒ Heigher Index − lower Index = order

Function

recurring

Index → Always Integer

If Integer come.

Solve this & Find a_n → in the Term of n

& Every Recurrence Rel^n have some Termination Cond^n

We can solve recurrence rel^n but that Takes Too much Time, so this is not a correct way to solve it.

So, we use some other method which Takes less time & gives correct Answers.

Recurrence Rel^n → determinate order → $\boxed{a_n = 4a_{n-1} + 5a_{n-2}}$

Recurrence Rel^n → $\boxed{\text{Indeterminate order}}$

determinate order → Linear, Non Linear

All the Power of $F^n = 1$ → $a_n = a_{n-1} + 5a_{n-2}$ → coefficient are constant

Linear → LRCC (constant coefficient) → LRCC Homogenious, LRCC Non Homogenious

Linear → LRVC → coefficient of F^n are variable. Ex $\boxed{a_n = n a_{n-1}}$

Non Linear → power of $F^n \geq 2$

Ex. $a_n = a_{n-1}^2 + n^2$

• $a_n = a_{n-1} a_{n-2} + n$ ← non Linear

• $a_n = a_{n-1} + n^2$ ← Linear (F^n Power ⇒ 1)

• $a_n = 2\sqrt{a_{n-1}}$ ← Non linear

Que Find the order of $\boxed{a_n = 4a_{n-1} + 5a_{n-2}}$

→ $n - (n-2)$
⇒ $n - n + 2$ ⇒ 2 ← order

- A recurrence relation have minimum order = 1 ← determinate order ✓
$\uparrow$ constant

- For Indeterminate recurrence relⁿ → $a_n = k\, a_{n/2} + b$
↳ order ⇒ $n - n/2 = n/2$ ← variable ✓✓

LRCC Homomogenious :- (constant coefficient)

↳ If we shift all Fⁿ Term {an Term} at left, Then at right hand-side only zero will left.

Ex $a_n = 2a_{n-1} + 5a_{n-2} \Rightarrow \boxed{a_n - 2a_{n-1} - 5a_{n-2} = 0}$ ✓✓

LRCC Non Homogenous :-

After shifting all fⁿ all left hand side something Remain at right hand side. K/a LRCC Non Homogenous.

Ex $a_n = 2a_{n-1} + 5a_{n-2} + 5 \Rightarrow \boxed{a_n - 2a_{n-1} - 5a_{n-2} = 5}$ ✓

(or)

$\boxed{a_n - 2a_{n-1} - 5a_{n-2} = n^2}$

etc ✓

We have solution for recurrence relation for —

1. LRCCH
2. LRCCH → If RHS have
   - Polynomial → $n^2 + n + 5$
   - Power → $2^n, 3^n, 5\cdot 2^n$
   - Polynomial * Power → $n^2 2^n$

25.00

LRCCH { Linear recurrence relation with constant coefficient Homogenous. }

Characteristic roots method :-

$\boxed{\text{No. of Constant given} = \text{No. of orders}}$

Ex:- $a_n = 5a_{n-1} - 6a_{n-2}$ $\boxed{a_0 = 1,\ a_1 = 5}$ two constant given

order $\Rightarrow n-(n-2) \Rightarrow 2$

Que solve the recurrence relation—

$$a_n = 5a_{n-1} - 6a_{n-2} \quad \boxed{a_0 = 1,\ a_1 = 5}$$

Sol^n:

$$a_n = 5a_{n-1} - 6a_{n-2}$$

$$\boxed{a_n - 5a_{n-1} + 6a_{n-2} = 0}$$

Put lowest coefficient as 1 &

$$t^2 - 5t + 6 \times 1 = 0 \quad // \text{ k/a characteristic eq}^n$$

$$\Rightarrow \boxed{t^2 - 5t + 6 = 0}$$

$$\boxed{t = 3, 2}$$

Sol^n → $\boxed{a_n = c_1 2^n + c_2 3^n}$

*****

for one root

$a_n = c_1 \alpha^n$

for two root

$a_n = c_1 \alpha^n + c_2 \beta^n$

$$n=0 \Rightarrow a_0 = c_1 2^0 + c_2 3^0 = 1 \quad \text{—} (1)$$

$$n=1 \Rightarrow a_1 = c_1 2^1 + c_2 3^1 = 5 \quad \text{—} (2)$$

$\boxed{c_1 = -2,\ c_2 = 3}$ after solving

directly satisfy option by a_0 & a_1 value Taking

So, Final Ans $\Rightarrow$ $\boxed{a_n = -2\, 2^n + 3 \cdot 3^n \Rightarrow 3^{n+1} - 2^{n+1}}$ Ans when root is repeated :-

Que $a_n = 4a_{n-1} - 4a_{n-2}$ $a_0 = 4$, $a_1 = 6$

Sol^n:- $a_n - 4a_{n-1} + 4a_{n-2} = 0$

$\boxed{t = 2, 2}$ ✓

$\boxed{a_n = C_1(2)^n + C_2 2^n}$ → It tells → $(C_1 + C_2) 2^n = \boxed{C \cdot 2^n}$

↑ But Question want two C value

So, this Homogenous equation rule Breaks when roots are repeated ✓

So for Repeated Roots in Homogenous eq^n –

we use → $\boxed{a_n = C_1 \alpha^n + C_2 n \alpha^n}$ ✓

↑ for repeated Root ✓

Que If characteristic eq^n for 5 roots are. —

$(t-\alpha)^2 (t-\beta)^3 = 0$

then find the formate in an form of this characteristic equation.

Sol^n $(t-\alpha)^2 (t-\beta)^3 = 0$

$$\boxed{a_n = C_1\alpha^n + C_2 n \alpha^n + C_3 \beta^n + C_4 n \beta^n + C_5 n^2 \beta^n}$$ ✓

Sol^n for Homogenous equation

① $\boxed{a_n = C_1\alpha^n + C_2 \beta^n + C_3 r^n \text{ —}}$ ✓

when roots are repeated

② $\boxed{a_n = C_1 \alpha^n + C_2 n \alpha^n}$ ✓

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Right Brain develop using Practice only

LRCCIH {InHomogenous}

Ex $a_n = 5a_{n-1} - 6a_{n-2} + 5$; $\boxed{a_0 = 1, a_1 = 5}$

Soln) $a_n - 5a_{n-1} + 6a_{n-2} = 5$ → for this type of Question Two type of solution we need to find
① Homogenous soln (a_n^h)
② Particular soln (a_n^p)

$$\boxed{a_n = a_n^h + a_n^p}$$

Homogenous soln — Particular solution

soln for In Homogenous :- (Table)

	RHS	Particular soln (Trial)
Polynomial	c	d
	$c_1 n + c_2$	$d_1 n + d_2$
	$c_1 n^2 + c_2 n + c_3$	$d_1 n^2 + d_2 n + d_3$
Power	$c.a^n$	$d.a^n$
Polynomial * Power	$(c_1 n + c_2) a^n$	$(d_1 n + d_2) a^n$
	$n.a^n$	

→ Then solve this → $a_n - 5a_{n-1} + 6a_{n-2} = 5$

$a_n = a_n^h + a_n^p$ ← Particular solution

Homo genous solution,

for, $a_n^h \Rightarrow \boxed{a_n - 5a_{n-1} + 6a_{n-2} = 0}$

$\boxed{a_n^h = c_1 2^n + c_2 3^n}$

$\boxed{a_n^p = d}$ ← Bcoz RHS ⇒ constant

Put & Find the value d

$d - 5d + 6d = 5$

$2d = 5 \Rightarrow \boxed{d = 5/2} \rightarrow \boxed{a_n^p = 5/2}$

So → $\boxed{a_n = c_1 2^n + c_2 3^n + 5/2}$

$\boxed{a_0 = 1 \ \& \ a_1 = 5}$ Put & find $c_1 \& c_2$

(or) do it by option

· If right hand side is Constant then we can stuck ~~at the~~ in finding one solution like —

$$\boxed{a_n = \underbrace{c_1(1)^n + c_2(2)^n + d}}$$ Here (Constant + Constant) got collepse.

So, Before getting the value (d) Check is their any collepse occure of Not.

If their is collision then — Put

$$\boxed{a_n^p = n^2 d}$$

n^2 because In ~~Ro Homogi~~ root repeated Term of Homogenious we use (n) ✓

Solution when RHS have one degree polynomial :—

Ex $$\boxed{a_n = 5a_{n-1} - 6a_{n-2} + 3n + 5}$$

Sol^n → $a_n = a_n^h + a_n^p$

(Normally solve)

$$\boxed{a_n^p = d_1 n + d_2}$$ → Write Particular Solution in this form

If collision occure with Homogenouse eq^n

then $$\boxed{a_n^p = (d_1 n + d_2) n}$$ ✓

multiply "n" to whole ✓

Que:- Solve

$$a_n = 5a_{n-1} - 6a_{n-2} + 3n + 5$$

Sol^n $\boxed{a_n - 5a_{n-1} + 6a_{n-2} = 3n+5}$

$$a_n - 5a_{n-1} + 6a_{n-2} = 0$$

$$t^2 - 5t + 6 = 0$$

$$t^2 - (3+2)t + 6 = 0$$

$$t^2 - 3t - 2t + 6 = 0$$

$$\boxed{t = 3, 2}$$

$$a_n = c_1(2)^n + c_2(3)^n + a_n^p$$

for a_n^p ← particular

$$\cancel{\boxed{a_n = 3d+5}}$$

$$\cancel{\boxed{a_n = d_1 n + d_2}}$$

$$\cancel{d_1 n + d_2 + 5\{d_1(n-1) + d_2\} + 6\{d_1(n-2) + d_2\} = 3n+5}$$

$$\cancel{dn + 5 + 5\{dn - d + 5\} + 6\{dn - 2d + 5\}}$$

$$\cancel{5 - 5d + 25 - 12d + 30 = 5}$$

$$\cancel{-17d + 60 = 5}$$

$$\cancel{d = \frac{-55}{-17} \Rightarrow \boxed{d = \frac{55}{17}}}$$

$$\cancel{d_1 n + d_2 + 5d_1 n - 5d_1 + 5d_2 + 6d_1 n - 12d_1 + 6d_2 = 3n+5}$$

$$\cancel{d_1 + 5d_1 + 6d_1 = 3}$$

$$d_2$$

Sol^n for a_n^p ✓

$$\boxed{a_n^p = d_0 + d_1 n}$$

Put the value into real eq^n ✓

$$(d_0 + d_1 n) - 5(d_0 + d_1(n-1)) + 6(d_0 + d_1(n-2)) = 3n+5$$

$$d_0 - 5d_0 + 5d_1 + 6d_0 + 6d_1(-2) = 5$$

$$-4d_0 + 6d_0 + 5d_1 - 12d_1 = 5$$

$$\boxed{2d_0 - 7d_1 = 5} \quad (1)$$

$$d_1 - 5d_1 + 6d_1 = 3$$

$$2d_1 = 3$$

$$\boxed{d_1 = 3/2} \quad (2)$$

$$2d_0 - \frac{7\times 3}{2} = 5$$

$$d_0 = \left(5 + \frac{21}{2}\right)\frac{1}{2}$$

$$\left(d_0 = \frac{31}{4}\right) ✓$$

$$a_n = c_1(2)^n + c_2(3)^n + \frac{31}{4} + \frac{3}{2}n$$

$$\boxed{a_n = c_1(2)^n + c_2(3)^n + \frac{3}{2}n + \frac{31}{4}}$$ ✓

gate 2010
Que

$a_1 = 8, \quad a_n = 6n^2 + 2n + a_{n-1}$

Let $a_{99} = K * 10^4$

The value of K is??

Sol^n $\boxed{a_n - a_{n-1} = \underline{6n^2+2n}}$ → If RHS is Polynomial then use substitution method

You can use formula but that will be time taken ← this will be faster than previous formula method ✓

$\boxed{a_n = a_{n-1} + 6n^2 + 2n}$ ✓

for $n=2$ $a_2 = a_1 + 6 \cdot 2^2 + 2 \cdot 2$

$= 8 + 6 \cdot 2^2 + 2 \cdot 2$

$a_3 = a_2 + 6 \cdot 3^2 + 2 \cdot 3$

$= 8 + 6 \cdot 2^2 + 2 \cdot 2 + 6 \cdot 3^2 + 2 \cdot 3$

$a_4 = a_3 + 6 \cdot 4^2 + 2 \cdot 4$

$= 8 \cdot 6 \cdot 2^2 + 2 \cdot 2 + 6 \cdot 3^2 + 2 \cdot 3 + 6 \cdot 4^2 + 2 \cdot 4$

$a_{99} = 8 + 6 \cdot 2^2 + 2 \cdot 2 + 6 \cdot 3^2 + 2 \cdot 3 + 6 \cdot 4^2 + 2 \cdot 4 + \dots 6 \cdot 99^2 + 2 \cdot 99$

$= 8 + 6\{2^2 + 3^2 + 4^2 \dots 99^2\} + 2\{2+3+4+ \dots 99\}$

$= \cancel{8} \; 6\{1^2 + 2^2 + 3^2 + 4^2 \dots 99^2\} + 2\{1+2+3+ \dots 99\}$ $\cancel{-6-2}$

$= 6\left\{\frac{99(99+1)(198+1)}{6}\right\} + 2\left(\frac{99(99+1)}{2}\right)$

$\Rightarrow (100 \times 199 \times 99) + 100 \times 99 \Rightarrow \frac{100 \times 99\{200\}}{10^4}$ $\Rightarrow \underline{198 \text{ Ans}}$ ✓

Recurrence relation for Power function :-

$a_n = 5a_{n-1} - 6a_{n-2} + 3.2^n$

$$\boxed{a_n - 5a_{n-1} + 6a_{n-2} = 3.2^n}$$

Homogenious

$a_n - 5a_{n-1} + 6a_{n-2} = 0$

$$\boxed{a_n = c_1(2)^n + c_2(3)^n}$$

$a_n^P = d.2^n \rightarrow$ Change

$$\boxed{a_n^P = dn.2^n}$$

multiply with n when repetation comes.

Collide with this

Put into eq^n

$d.n.2^n - 5\{d(n-1)2^{(n-1)}\} + 6\{d(n-2)2^{n-2}\} = 3.2^n$

divide by $(2)^{(n-2)}$

$d.n(4) - 5\{d(n-1)2\} + 6\{d(n-2)\} = 3*4$

$4dn - 10d(n-1) + 6d(n-2) = 12$

$4dn - 10dn + 10d + 6dn - 12d = 12$

$10d - 12d = 12$

$-2d = 12 \Rightarrow \boxed{d = -6}$

So, our solution $\boxed{a_n = c_1(2)^n + c_2(3)^n - 6.n2^n}$

Concept for Power * Polynomial

So Trick → for $\boxed{a_n^p = (d_0+d_1 n)2^n}$ ← simply

↓ when repetation comes

$$\boxed{a_n^p = (d_0+d_1 n)\, n\, 2^n}$$

Converting from LRVC → LRCC

(LRVC: variable; LRCC: constant coefficient.)

Que. $n.a_n = 5n a_{n-1} - 5a_{n-1} + 3$, $a_1 = 2$

Sol^n $n a_n = (5n-5)\, a_{n-1} + 3$

$n a_n = 5(n-1)a_{n-1} + 3$

Put $\boxed{n.a_n = b_n}$

$\boxed{b_n = 5 b_{n-1} + 3}$ → LRCC

$b_n - 5b_{n-1} = 0$

↓

$b_n = c.5^n + b_n^p$

$\boxed{b_n = d}$

$d - 5d = 3 \Rightarrow \boxed{d \Rightarrow -3/4}$

$b_n = c.5^n + (-3/4)$

$n.a_n = \left(c.5^n - 3/4\right)$, $a_n = \dfrac{c.5^n - 3/4}{n}$ By $\boxed{a_1 = 2}$

$2 = \dfrac{c.5 - 3/4}{1} \Rightarrow 20c - 3 = 8 \quad \boxed{c = 11/20} \rightarrow \boxed{a_n = \dfrac{\frac{11}{20}5^n - 3/4}{n}}$ Ans

Converting from Non-Homogenous To Homogenous :-

Example :-

$$\boxed{a_n^2 = 5a_{n-1}^2 + 6}$$

Put $\boxed{b_n = a_n^2}$

$\boxed{b_n = 5b_{n-1} + 6}$

↓ then further solve it

& Finally put $\boxed{b_n = a_n^2}$

↓

$a_n = \pm\sqrt{c.5^n - 3/2}$

$a_n = \sqrt{5.c + (-3/2)} \Rightarrow 5$

$5c - 3/2 = 25$

$5c = 25 + 3/2$

$\boxed{c \Rightarrow \frac{53}{10}}$

Put & get sol^n

-ve sign discard bcoz $a_1 = +5$

Lecture - 14

$$\cancel{a^{\log_b c} = c^{\log_b a}}$$

$$\boxed{c^{\log_b a} = a^{\log_b c}}$$

When power is not same

Ex:- $a_n^2 = 8\, a_{n-1}^1$

Take log on Both side

$$\boxed{2\log a_n = \log 8 + \log a_{n-1}}$$

Say $\boxed{b_n = \log a_n}$

$$\boxed{2(b_n) = 3 + b_{n-1}} \rightarrow \text{solve it}$$

Sol^n for Indeterminate order Recurrence rel^n :-

Que $\cancel{a_n = \frac{a_n}{2} + 5}$ $\left(a_n = a_{\frac{n}{2}} + 5\right)$, $\boxed{a_1 = 6}$

Let $\boxed{n = 2^k}$

$$\boxed{a_{2^k} = a_{2^{k-1}} + 5}$$

$$\boxed{b_k = a_{2^k}}$$

$$\boxed{b_k = b_{k-1} + 5} \leftarrow \text{Put}$$

$b_k - b_{k-1} = 5$

$\boxed{b_k^p = \cancel{5}\, dk}$ ← Because of collision

$dk - d(k-1) = 5$

$\boxed{d = 5}$

$b_k = C(1)^k$

$b_k = C + b_k^p$

$$\boxed{b_k = C + 5k}$$

$a_{2^k} = C + 5\log_2 n$

$$\boxed{a_n = C + 5\log_2 n} \rightarrow a_1 = 6 \quad \boxed{C \Rightarrow 6}$$

$$\boxed{a_n = 6 + 5\log_2 n}$$

Que - How many factors are there n { # Positive Integral divisions}

$\hookrightarrow n = p_1^{n_1} p_2^{n_2} p_3^{n_3} \cdots p_r^{n_r} \longrightarrow \#factor = (n_1+1)(n_2+1)(n_3+1) \cdots (n_r+1)$

Prime No

gate 2014

Que # +ve Integral of 2014 ; # factors = ?

2	2014
19	1007
	53

→ try uptill $\sqrt{1007} \approx 33$ ✓

↳ ~~3~~, ~~5~~, ~~7~~, ~~11~~, ~~13~~, ~~17~~, 19 ✓

Prime no

$2^1 \times 19^1 \times 53^1 \Rightarrow (1+1)(1+1)(1+1) \Rightarrow 8$ factors ✓

gate 2005

divisors → 2100

Sol^n

2	2100
2	1050
3	525
5	175
5	35
	7

$\Rightarrow 2^2 \times 3^1 \times 5^2 \times 7^1 \Rightarrow (3) * (2) * (3) * (2)$

$\Rightarrow$ 36 factors ✓

Que How many No. from $(1 - n)$ are relatively prime to n

Sol^n relatively prime of x & n → $\boxed{gcd(x, n) = 1}$ ✓

Ans $\phi(n)$

↑

Euler Totient Function.

procedure to find $\phi(n)$ → Firstly Break down the Number into prime factor.

$n = p_1^{n_1} p_2^{n_2} p_3^{n_3} \cdots p_r^{n_r}$

$\phi(n) = \phi(p_1^{n_1} p_2^{n_2} \cdots p_r^{n_r})$

$= \phi(P_1^n) \times \phi(P_2^n) \ldots\ldots \times \phi(P_n^n)$

use formula:

$$\boxed{\phi(p^n) = p^n - p^{n-1}}$$

Ex $\phi(10) = \phi(2*5)$
$\Rightarrow \phi(2^1) * \phi(5) = (2^1 - 2^0) * (5^1 - 5^0)$
$= (1) * (4) \Rightarrow 4$

Que (2005 gate)
if $\boxed{n = p^2 q}$ p, & q are prime No.

Sol^n $\phi(p^2 q) \Rightarrow \phi(p^2)\,\phi(q)$
$\Rightarrow (p^2 - p^1)(q^1 - q^0)$

Que How many ~~relatively~~ prime No. from (1–60) which is relatively prime to 60.

Sol^n $\phi\ 60 \Rightarrow 2^2 * 3^1 \times 5^1$
$\Rightarrow \phi(2^2 * 3^1 * 5^1)$
$\Rightarrow \phi(2^2) * \phi(3^1) * \phi(5^1)$
$\Rightarrow (2^2 - 2^1) * (3^1 - 3^0) * (5^1 - 5^0)$
$\Rightarrow 2 * 2 * 4 \Rightarrow \underline{16}$

Que If from all the factors of 2^{100} ~~if~~ what is the probability to pick one factor which is the ~~multiple~~ divisor of 2^{90}.

Sol^n $\dfrac{n(2^{90})}{n(2^{100})} \Rightarrow \left(\dfrac{91}{101}\right)$

for $6^{100} \xrightarrow{\phi} 6^{90}$

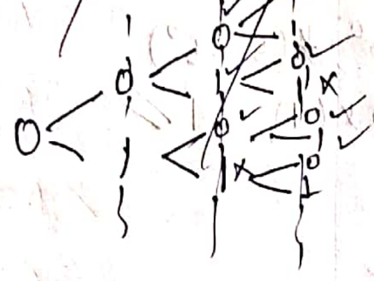
$\dfrac{(6)^{90}}{(6)^{100}} \Rightarrow \dfrac{2^{90} \times 3^{90}}{2^{100} \times 3^{100}} \Rightarrow \left\{\dfrac{91 \times 91}{101 \times 101}\right\}$

gate Que $a_n \Rightarrow$ ~~No. of bit string of n with consecutive 1~~
~~find recurrence relation of a_n?~~

a) ~~$a_{n-2} + a_{n-1} + 2^{n-2}$~~ b) ~~$a_{n-2} + 2a_{n-1} + 2^{n-2}$~~

c) ~~$2a_{n-2} + a_{n-1} + 2^{n-2}$~~ d) $2a_{n-2} + 2a_{n-1} + 2^{n-2}$

Sol^n



~~$a_4 = 2$~~ ~~$a_2 = 3$~~ $a_3 = 4$

$a_1 = 0$
$a_2 = 1$
$a_3 = 3$

Substitute into option & get Answer ✓

:- Question Based on setting of Recurrence rel^n :-

Gate 2008

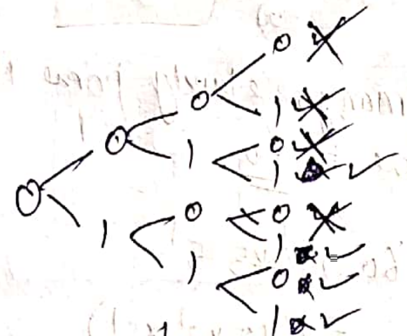
No. consecutive zeros

a) $x_n = x_{n-1} + x_{n-2}$

b) $x_n = 2x_{n-1}$

c) $x_n = x_{n/2} + 1$

d) $x_n = x_{n/2} + n$



satisfy by option ✓ Ans (a)

all setting problem do like this.

gate Que Let a_n be the numbers of bit strings of length "n" containing consecutive 1's is ✓

What is the Recurrence rel^n for a_n??

a) $a_{n-2} + a_{n-1} + 2^{n-2}$

b) $a_{n-2} + 2a_{n-1} + 2^{n-2}$ ✓

c) $2a_{n-2} + a_{n-1} + 2^{n-2}$

d) $2a_{n-2} + 2a_{n-1} + 2^{n-2}$

$a_1 \to 0$
$a_2 \to 1$
$a_3 \to 2$
$a_4 \to 4$

Put & find Ans ✓ Ans (a)