(c) $(\forall x\, P(x) \rightarrow \forall x\, q(x)) \rightarrow \forall x\, (P(x) \rightarrow q(x))$, False

(d) $(\forall x\, P(x) \rightleftarrows \forall x\, q(x)) \rightarrow \forall x\, (P(x) \rightleftarrows q(x))$ ⇒ False

---

*** NOTE**

① Conditional Proof rule.

$$(P \wedge q) \rightarrow (R \rightarrow S) = P \wedge q \wedge R \rightarrow S$$

② $(P \rightarrow q) \rightarrow (R \rightarrow S) = (P \rightarrow q) \wedge R \rightarrow S$

---

⑦ # PERMUTATIONS AND COMBINATIONS.

Combinatorics

9 Categories
- P & C ***
- Summation → Binomial / → Generating f^n **
- Recurrence Rel^n.

* Fundamental principal of counting

① Multiplicat^n principal

- We have an event E and to complete it we have to break down into multiple subevents in such a way that by completing each subevent individually the Entire Event will be completed.

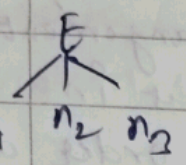
∴ E → E_1, E_2 --- E_x

No. of ways to complete ⇒ n_1, n_2, n_x

⇒ E will be completed in ⇒ $n_1 * n_2 * n_3 - - - n_x$

eg:- 3 Pants & 2 shirts ⇒ No. of ways to dress up $= 3 \times 2 = 6$ ways.

② Addition Principle ⇒ Here an Event is not broken into subevents rather if An event can be completed in n_1 ways also same can be completed in n_2 ways ... etc



⇒ ∴ Total no. of ways in which E can be completed $= n_1 + n_2 + n_3$

eg:- 5 persons, arrange atleast 3 persons in a row. how many ways ⇒ $\underline{5} \times \underline{4} \times \underline{3} + \underline{5} \times \underline{4} \times \underline{3} \times \underline{2} + \underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1}$

(1st way) (2nd way) (3rd way)

to arrange atleast 3 persons in a row.

---

* 10 letters, Forming a 4 letter word? How many ways? ~~except~~ repetition not allowed.

Sol^n $\underline{10} \times \underline{9} \times \underline{8} \times \underline{7} = \cancel{120}$. 5040 ways.

⇓

Generalizat^n ⇒ n letters, Forming a r letter word.

$n \times (n-1) \times (n-2) \times (n-3) \times \cdots (n-r+1)$

⇓

$$\frac{n \times (n-1) \times (n-2) \cdots (n-r+1) \times (n-r) \times (n-r-1) \times \cdots 3 \times 2 \times 1}{(n-r) \times (n-r-1) \times (n-r-2) \times \cdots 3 \times 2 \times 1}$$

$$= \frac{n!}{(n-r)!} = \boxed{{}^nP_r}$$

– Permutation (or) Arrangement

⇓

meaning, here order matters

abcd, bcad are 2 diff. things

* ${}^nP_r = \frac{n!}{(n-r)!} = {}^nC_r \times r!$

– order matters.

But in case when we <u>SELECT</u> persons then order doesnt matter.

* eg:- Picking 4 balls out of 10 balls from a bag

$B_1 \times B_2 \times B_3 \times B_4$

$B_1 \times B_3 \times B_2 \times B_4$

$\vdots$

$\times __ \times __ \times __$

4! arrangements → treated as single grp.

But in select^n order doesnt matter so treat 4! different arrag. as one (all ~~comb~~ arrangements are equal)

$\therefore \dfrac{{}^{10}P_4}{4!}$

* This is nothing but ${}^{10}C_4$

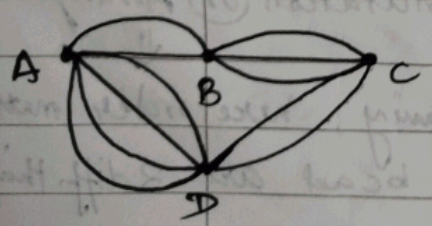
⇓

<u>In General</u>: Choosing r objects from n objects

$= \dfrac{{}^nP_r}{r!} = \boxed{{}^nC_r}$ - Combination (or) Selection

* ${}^nC_r = \dfrac{n!}{(n-r)!\,r!} = \dfrac{{}^nP_r}{r!}$ - order doesnt matter - choose, pick etc.

Que) How many way to go from A to C?



A to B AND B to C 2×3 $= 6$

(or) A to D AND D to C 4×2 $= +8$

14 ways

9 Categories of Problems

	w/o Repetition	Limited Repetition	(or) with repetn Unlimited Repetn
1. Permutation	^nP_r	$\frac{n!}{n_1! \times n_2! \times n_3! \cdots}$	n^r
6. Combination	^nC_r	Generating fn.	$^{n+r-1}C_r$

7. Distribution
8. Principle of Inclusion Exclusion ⇒ $A \cup B = A + B - A \cap B$ etc set theory
9. Pigeon hole Principle.

---

* **Note** :- First of all Differentiate Question ⇒ Is it Que of permutatn (order ✓ matters) or is it combinatn (order ✗)

Keywords :-

Permutatn	combinatn.
• Arrangement	select / collect.
• Sequence	• set (∵ set is unordered)
Que) How many ← • numbers/letters/word.	Subset → Que) How many subsets of a can be like:- given set have 3 etc.!
• Boxes, Chair	Team, committe, interview select^n

* Distribution ~~of~~ problems.

Keywords :- Distribute, Partition.

Que) How many ways to distribute 10 boys in 3 groups --

* Principle of Inclusion, Exclusion.

Keyword :- OR Que) How many no. b/w 1 & 1000 are divisible by 4 (or) 5

* Pigeon Hole Principle.

keyword :- Atleast how many.

Que) Atleast how many persons among 100 in a party have birthday on same date.

Practice Questions on Permutations.

without repetn.

Que) 5 letter word distinct from 26 letters of english

Permutatn

Soln

$^{26}P_5$ (or) $26 \times 25 \times 24 \times 23 \times 22$

Box method

26	25	24	23	22

with repetn

Que) 5 letter word from 26 letters (repetn allowed)

Soln

$n^r = 26^5$ (or)

26	26	26	26	26

Box method.

---

* Note :- By default, words are repeated.

Que) How many 5 letter password possible from 26 letters.

Soln

$26 \times 26 \times 26 \times 26 \times 26 = 26^5$

---

* Permutatn Ques with some constraints

Que) 5 letter word start with a end with z

a	26	26	26	z

$1 \times 26 \times 26 \times 26 \times 1 = 26^3$

w/o repetn $\Rightarrow$

a	24	23	22	z

$\Rightarrow 1 \times 24 \times 23 \times 22 \times 1$

Que) 5 letter password start with a or b & end with z.

permutat^n — By default repet^n allowed

$\Rightarrow$ | a/b | 26 | 26 | 26 | z |

$2 \times 26 \times 26 \times 26 \times 1$

Que) 5 letter password start with vowel & end with z?

Sol^n

| 5 | 26 | 26 | 26 | 1 |

$= 5 \times (26)^3$

Que) —— " —— . Repet^n not allowed

Sol^n

| a,e,i o,u | | | | z |

$5 \times 24 \times 23 \times 22 \times 1$

Que) 10 persons, Arrange 4 persons on 4 chairs. ways!

Sol^n persons cannot be repeated obviously $\therefore {}^{10}P_4$.

(or) First select 4 persons among 10 and then Arrange them on 4 chairs.

$\therefore {}^{10}C_4 \times 4!$

Que) 10 persons, Arrange 4 persons such that 'A' never get selected.

Sol^n remaining 9 persons $\Rightarrow \therefore {}^9C_4 \times 4!$ (or) ${}^9P_4$.

Que) 10 persons, Arrange 4 persons such that 'A' always get selected.

Sol^n A P_1 P_2 P_3 $\Rightarrow {}^9C_3 \times 4!$

Select 3 among remaining 9 people & arrange all 4 people

(M2) P_1 P_2 P_3 A

$= {}^9P_3 \times 4$

$= {}^9C_3 \times 3! \times 4$

$= {}^9C_3 \times 4!$

Que) —— " —— A and B always get selected.

(M1) ${}^8C_2 \times 4!$

(M2) P_1 P_2 A $\Rightarrow$ A P_1 P_2 B

${}^8P_2 \times 3 \Rightarrow ({}^8P_2 \times 3) \times 4$

$\Rightarrow {}^8P_2 \times 3 \times 4$

$= {}^8C_2 \times 2! \times 3 \times 4$

$= {}^8C_2 \times 4!$

Que) Metro → 4 seat vacant, 10 persons. ways?

Sol^n ① $^{10}P_4$

② $^{10}C_4 \times 4!$

③ $\boxed{10 \mid 9 \mid 8 \mid 7}$ $= 10 \times 9 \times 8 \times 7$

Seat 1 → Seat 4

In box method take that quantity which is less.

Que) Metro → 10 seats vacant, 4 persons ways?

① $^{10}P_4$

② $^{10}C_4 \times 4!$

③ $\boxed{10 \mid 9 \mid 8 \mid 7}$ $= 10 \times 9 \times 8 \times 7$.

Person 1 Person 4

* In Mathematics,

Both are equal { 10 Persons on 4 chair ⇒ Pick 4 Persons & arrange them
4 Persons on 10 chairs ⇒ Pick 4 chairs & arrange them

Que) 10 toys & Choose 4 toys and give them to 4 children.

Sol^n

Here toys arrangement matters → ∴ Permut^n w/o repet^n

∴ ① $^{10}P_4$, ② $^{10}C_4 \times 4!$

③ $\boxed{10 \mid 9 \mid 8 \mid 7}$

Child 1 Child 4

* Here we took Implicit Assumpt^n ⇒ Every kid get 1 toy,

Que) 10 toys ⟶ give them to 4 children

Sol^n Now, this is a Question of DISTRIBUTION problem
we will study this futher

* Games Question

Que) 2 teams A, B, They play a 10 games tournament
outcomes of each game can be ⇒ A win / B win / tie.
Total no. of outcomes of entire tournament.

Sol^n

$\boxed{3 \mid 3 \mid 3 \mid --- \mid 3} = 3^{10}$ ⇒ Permutation with repet^n (n^r)

A win / B win / tie ∴ 3 choices per game

order matters

$\boxed{\text{A win}}$ $\boxed{\text{B win}}$ and $\boxed{\text{B win}}$ $\boxed{\text{A win}}$
game 1 game 2 are both different.

* Dice Question

Que) 10 Dice are thrown. How many outcomes?

* By default, we take here distinct dice.
* ie $\boxed{5 \mid 6}$ and $\boxed{6 \mid 5}$ are different.
dice 1 dice 2 dice 1 dice 2

↓ distinct
∴ order matters
⇒ Permutat^n with unlimited repet^n.

∴ $\boxed{6 \mid 6 \mid --- \mid 6} = 6^{10}$ outcomes

each dice have 6 choice

* **Note:-** In DMS, By default we dont take any thing identical unless mentioned. Everything is distinct.
∴ Apply Permutat^n always unless mentioned

Que) 10 Dice are thrown. 1st & 2nd dice odd no. outcomes?

Sol^n $\underset{\downarrow}{\underline{1,3,5}} \times \underset{\downarrow}{\underline{1,3,5}} \times \underset{\downarrow}{\underline{\overset{1,2,3,4}{5,6}}} \times \underset{\downarrow}{\underline{\quad}} \; --- --$

3 choice × 3 choice × 6 choice × 6 choice × — —

⇒ $3^2 \times 6^8$.

Que) 10 Dice are thrown. 1st & 2nd Dice same no.

1no 1no 6 choices 6 6 -- 6

2 2no. 6 6 6 -- 6

⋮

6 6no 6 6 6 -- 6

no. on dice Possibilities.

$+ 1\times1\times6^8$

$+ 1\times1\times6^8$

$+ 1\times1\times6^8$

⋮

$+ 1\times1\times6^8$

$\Rightarrow 6\times6^8$

$= 6^9$

* Answer sheet

Que) A Question Paper has 65 Que & each Que is MCQ having 4 options each. one is correct. If student had solved all Que, How many answer sheets possible?

By default distinct answer sheets.

∴ Permutation

Q1
Q2
⋮
Q65

$\Rightarrow$ | 4 | 4 | 4 | - - - - | 4 | $= 4^{65}$ ways.

4 choices for each Que

Que) — " — Student may leave Answer Blank — " —

| 5 | 5 | - - - - | 5 | $\Rightarrow 5^{65}$ ways.

Que) — " — 65 Que — " — 50 Que MCQ, 15 Que MSQ

Student may leave ans Blank — " — .

Sol^n

Q1
Q2
⋮
Q65

← 50 → ← 15 →

| 5 | 5 | - - - | 5 | | | | - - - | |

$1 + {}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4 = 2^4 = 16$

Blank

$= 5^{50} * 16^{15}$

* Que) 5 couples invited to a party. Condition is that every wife should be accompanied by her husband but a husband may come alone also. No. of different gatherings possible?

Sol^n

Couple 1 ... Couple 5

$= 3 \times 3 \times 3 \times 3 \times 3 = 3^5$

{ }, {H}, {W, H} = 3 possibilities for each couple

Permutat^n with unlimited repet^n.

Que) How many pairs of sets (A, B) are there that satisfy cond^n.
$A, B \subseteq \{1, 2, 3, 4, 5\}$ and $A \cap B = \{\ \}$

Sol^n we have to make two disjoint subsets A, B of a set $\{1, 2, 3, 4, 5\}$

1	2	3	4	5

A B None

⇒ element 1 can go in set A, or set B or None ∴ Each ele. has 3 choices. ⇒ 3^5

Its permutat^n because order matters here
$A = \{1\}$, $B = \{2\}$ and $A = \{2\}$, $B = \{1\}$ are 2 diff. things.

Que) Let x be a set of size n. How many pairs of sets (A, B) are there that satisfy the cond^n $A \subseteq B \subseteq x$?

Sol^n $x = \{1, 2, 3 \ldots n\}$

1	2	- - -	n

$= 3^n$.

Each ele has 3 choices. {A, B}, 'B' only, None.

If ele goes to A then it must go to B also.

---

* Category ③:- Permutat^n with limited Repetition (Mississippi Problem)

Multiset = {a, a, b} ⇒ How many 3 letter word can be formed

→ Here we cannot take [a|a|a] or [b|b|b] ∝ because its limited repet^n and not unlimited or w/o repet^n.

$\{a, a, b\} \Rightarrow$ Forming 3 letter word

If it was the case $\{a_1, a_2, b\}$ $\Longrightarrow$ But since a_1, a_2 are same $\Rightarrow \{a, a, b\}$

$\therefore$ a_1, a_2, b
a_1, b, a_2
a_2, a_1, b
a_2, b, a_1
b, a_1, a_2
b, a_2, a_1 $\Big\}$ $3!$

These are same

$a\ a\ b$
$a\ b\ a$
$a\ a\ b$
$a\ b\ a$
$b\ a\ a$
$b\ a\ a$ $\Big\}$ $\dfrac{3!}{2!}$ ← common elements.

Que) $\{a, a, a, b, b\}$ How many 5 letter word from this multiset?

$= \dfrac{5!}{3!\,2!}$ ← Arrangement of 5 people
← 3 a & 2b common.

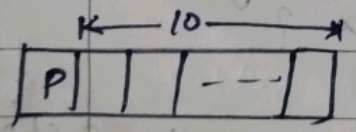
Que) How many jumbled words from word MISSISSIPPI.**

Sol^n

$\{M, I, I, I, I, S, S, S, S, P, P\} = \dfrac{11!}{4!\,4!\,2!}$

Que) —11— starts with P?

Sol^n



$M \to 1$
$I \to 4$
$S \to 4$
$P \to 1$

$= \dfrac{10!}{4!\,4!}$

Que) —11— not starts with P? $\Rightarrow$ (Complementary Problem)

$=$ Total $-$ starts with P

$= \dfrac{11!}{4!\,4!\,2!} - \dfrac{10!}{4!\,4!}$

Que) 4 identical math Books, 3 identical English Books, 2 identical Physics Books. Arrange on shelf. No. of ways?

Sol^n Permutation with limited repet^n (or) Mississippi problem

∵ identical.

$$\therefore \frac{9!}{4!\,3!\,2!}$$

Que) 5 identical Red Flags, 3 identical Blue Flags, 4 identical Green Flags. How many ways to Arrange them in a row.

Permutat^n.

limited repet^n ⇒ Its Mississippi Problem ⇒ $\frac{12!}{5!\,3!\,4!}$

* Note :- In Question if word 'identical' not given then by default we treat as distinct objects ⇒ In that case, answer will be ⇒ 12! (all flags distinct)

Que) From Jumbled words from MISSISSIPPI a word is picked at random what is probability that it starts with P.

Sol^n Probability = $\frac{\text{no. of outcomes favourable}}{\text{Total Outcomes}}$

$$= \frac{\frac{10!}{4!\times 4!}}{\frac{11!}{4!\,4!\,2!}} = \frac{2}{11}$$

Que) —11— it doesnt starts with P.

Sol^n P = 1 − Starts with P = $1 - \frac{2}{11} = \frac{9}{11}$

Que) Jumbled words from EQUATION. Probability that word picked at random starts with A or T

Sol^n A ---------- ⇒ 7!
T ---------- 7!
⟵ 7 ⟶ } 7! + 7!

∴ Prob. = $\frac{7!+7!}{8!}$ ← favourable / ← total.

(*) 6 categories of constraints on permutations.

1. Starting with, ending with, not starting with, not ending with, Starting & ending with --- digits

2. Containing :- 5 letter word containing atleast 1 a
   — " — exactly 1a
   — " — atmost 1a
   — " — not containing b.

3. Together :- MISSISSIPPI ⇒ All s together, All s not together
   EQUATION ⇒ All vowels together, Atleast 2 vowel together

4. No 2 objects together :- Mississippi :- No 2 's' together
   No 2 vowels together.

5. Alternating Arrangements :- B G B G
   (or) G B G B

6. Circular permutations

Category 01*. Starting & Ending with -- digits.

Que) How many 4 digit no. ⇒ [ ][ ][ ][ ]. = 9×10^3.

0 not allowed → 9 10 10 10

10 choices ⇒ 0 to 9.

Q) How many 4 digit even no?

⇒ [ ][ ][ ][ ]. = $9 \times 10^2 \times 5$ = 4500.

9 10 10 5 ← choices are 0, 2, 4, 6, 8.

Q) How many 4 digit even no are possible w/o repetn.

Sol^n Starting with odd no. = [ ] [ ] [ ] [ ]
(1, 3, 5, 7, 9)

$5 \times 8 \times 7 \times 5$ ← (choices are 0, 2, 4, 6, 8)

Starting with even no. = [ ] [ ] [ ] [ ]
(choices are 2, 4, 6, 8)

$4 \times 8 \times 7 \times 4$ choices (∵ if we are selecting a no. at 1st place we cannot repeat it)

$\therefore \; 8 \times 7 \, [5 \times 5 + 4 \times 4]$

$= \underline{\underline{56 \times 41}}$.

Method 2

For a no. to be 4 digit and even, it should end with 0, 2, 4, 6, 8.

∴ Ending with 0

[ ] [ ] [ ] [0]

$9 \times 8 \times 7 \times 1$ choice

Ending with 2, 4, 6, 8

0 and the no. chosen at end not allowed → [ ] [ ] [ ] [ ]

$8 \times 8 \times 7 \times 4$

0 allowed.

$\therefore \; 9 \times 8 \times 7 \times 1 + 8 \times 8 \times 7 \times 4 = 56 \times 41$

Q2 * Containing - - -
category

Q1) How many 5 letter words containing exactly 1 a. (among 26 letters)

Sol^n

[a] [ ] [ ] [ ] [ ]

$1 \times 25 \times 25 \times 25 \times 25$ ⇒ ${}^5C_1 \times (25)^4$

$+$

$= 5 \times (25)^4$

[ ] [a] [ ] [ ] [ ]

$25 \times 1 \times 25 \times 25 \times 25$

$+$

⋮ and so on.

Q) How many 5 letter words contains

- exactly 2 a $= {}^5C_2 \times (25)^3$
- exactly 3 a $= {}^5C_3 \times (25)^2$

Q) How many 5 ~~digit~~ letter words contain atleast 1 a.

Sol^n exactly 1 a, + 2a + 3a + 4a + 5a.

$= {}^5C_1 \times (25)^4 + {}^5C_2 \times 25^3 + {}^5C_3 \times 25^2 + {}^5C_4 \times 25^1 + {}^5C_5$

M2 (complementary method) Atleast 1 a $\Rightarrow$ Total − No a.

$(26)^5 - (25)^5$

Q) How many 5 ~~digit~~ ~~two~~ letter words contain atmost 1a

Sol^n exactly 0 'a' + exactly 1 'a'

$(25)^5 + {}^5C_1 \times (25)^4$.

Que) How many 5 digit no contain exactly 1 '7'

Sol^n 7 at 1st place — 7 at remaining 4 places

| 7 | | | | | + | | 7 | | | |

$1 \times 9 \times 9 \times 9 \times 9$ — $8 \times 1 \times 9 \times 9 \times 9$

$\therefore (9)^4 + {}^4C_1 \times 8 \times (9)^3$.

Que) How many 5 digit no contain exactly two '7'.

Sol^n One 7 and 1st place, other 7 at any other place from remaining 4 places. — Two 7 at every other place but not at 1st place

| 7 | | | | | + | | 7 | 7 | | |

$^4C_1 \times 9^3$ — 8 1 1 9 9 — $^4C_2 \times 9^2 \times 8$

Category 03 * Together with - - -

Que 1) Word is EQUATION

How many words are there with All vowels ~~together~~ together

Sol^n EQUATION ⇒ $\boxed{EUAIO}$ $\boxed{QTN}$

vowels consonants.

∴ $\boxed{EUAIO}$ Q T N ⇒ $5! \times 4!$

① ② ③ ④

arrang. all vowels in 5! ways. arrange ①② ③,④ in 4! ways.

Q2) All consonants together.

Sol^n $\boxed{QTN}$ E U A I O ⇒ $3! \times 6!$

① ② ③ ④ ⑤ ⑥

Q3) All vowels & All consonants together.

Sol^n $\boxed{EUAIO}$ $\boxed{QTN}$ ⇒ $5! \times 3! \times 2!$

① ②

Q4) All vowels Not together (Complementary problem)

⇒ Not Together = Total − All together

= 8! total words − $5! \times 4!$

Q5) All consonants Not together.

= $8! - (3! \times 6!)$

Q6) All vowels not together & All consonants not together.

⇒ Solved by Principle of Inclusion Exclusion.

Que) MISSISSIPPI

① ② ③ ④ ⑤ ⑥ ⑦ ⑧

All S together ⇒ $\boxed{SSSS}$ M I I I I P P ⇒ $\frac{8!}{4! \times 2!}$

I repetⁿ → 4! ; P repetⁿ → 2!

Q2) All S Not together.

Total Jumbled words − All S together.

$$\left(\frac{11!}{4! \times 4! \times 2!}\right) - \left(\frac{8!}{4! \times 2!}\right)$$

Q3) All characters of same type together.

⇒ $\boxed{SSSS}$ M $\boxed{IIII}$ $\boxed{PP}$ ⇒ 4!

① ② ③ ④

Category 04 ✱ No two objects of certain type together.

Q1) EQUATION ⇒ No 2 Consonants together

Solⁿ Consonants = Q, T, N.

Vowels = A E I O U ⇒ _A_E_I_O_U_

~~_A_E_I_O_~~ $5! \times ({}^6C_3 \times 3!)$

Q2) No 2 vowels together

_Q_T_N_ ⇒ $3! \times {}^4C_5$

↑ Not possible.

∴ No. of ways = 0.

Que) MISSISSIPPI ⇒ No 2 'S' together

_M_I_I_I_P_P_I_

$$\frac{7!}{4! \times 2!} \times {}^8C_4 \times 1$$

← all S arrangement in $\frac{4!}{4!}$ ways ← repetⁿ

Category 05 * Alternating Arrangements.

Q1) 5 Boys & 4 Girls.

. No 2 Boys together

$_ G_1 _ G_2 _ G_3 _ G_4 _$

$= 4! \times 5!$

. No 2 Girls together

$_ B_1 _ B_2 _ B_3 _ B_4 _ B_5 _$

$= 5! \times {}^6C_4 \times 4!$

Que) Alternating Array. ⇒ No 2 Boys together & No 2 girls together.

Solⁿ

$\underline{B_1}\ \underline{G_1}\ \underline{B_2}\ \underline{G_2}\ \underline{B_3}\ \underline{G_3}\ \underline{B_4}\ \underline{G_4}\ \underline{B_5}$ ⇒ $5! \times 4!$

Que) 4B & 4 Girls in alternate arrangement

$\underline{B}\ \underline{G}\ \underline{B}\ \underline{G}\ \underline{B}\ \underline{G}\ \underline{B}\ \underline{G}$

and $\underline{G}\ \underline{B}\ \underline{G}\ \underline{B}\ \underline{G}\ \underline{B}\ \underline{G}\ \underline{B}$ } ⇒ $4! \times 4! \times 2$.

Que) 5 0's & 5 1's No 2 0's & No 2 1's together

and. $\underline{0}\ \underline{1}\ \underline{0}\ \underline{1}\ \underline{0}\ \underline{1}\ \underline{0}\ \underline{1}\ \underline{0}\ \underline{1}$

$\underline{1}\ \underline{0}\ \underline{1}\ \underline{0}\ \underline{1}\ \underline{0}\ \underline{1}\ \underline{0}\ \underline{1}\ \underline{0}$ } 2 ways. (or) $\frac{5!}{5!} \times \frac{5!}{5!} \times 2$

Category 06 * Circular Permutations → Distinct / Identical.

Distinct

Arrange 3 Boys on a Circular table

Linear ⇒ 3! ways.

$b_1\ b_2\ b_3$

$b_1\ b_3\ b_2$

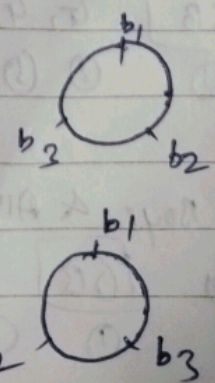
$b_2\ b_3\ b_1$

$b_2\ b_1\ b_3$

$b_3\ b_1\ b_2$

$b_3\ b_2\ b_1$

Circular ⇒



⇒ $\frac{3!}{3}$ ways.

(∵ 3 boys)

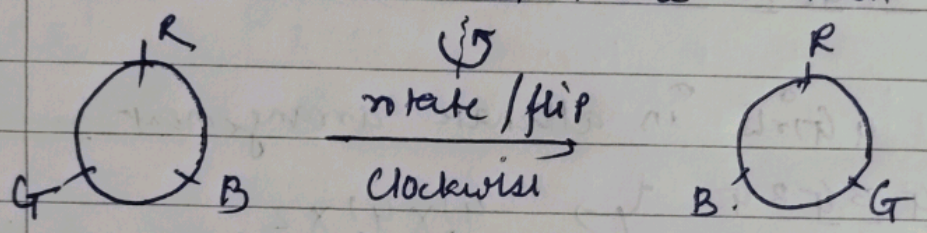
$\therefore$ To Arrange n persons on a Circular Table

$$= \frac{n!}{n} = (n-1)! \text{ ways.}$$

* Garland / Neclace problems.

Here in Garland or neclace, we can flip it clockwise and we observe that same thing is obtained

eg:- A Garland with Red, Blue & Green flower.



$\therefore$ These Both are Same

$\therefore$ Red, Blue, Green & Red, Green, Blue are same

$\therefore$ To Arrange n distinct flowers in a Circular Garland

$$= \frac{(n-1)!}{2} \text{ ways.}$$

Que) 6 Boys & 5 Girls arrange in circular. All Boys together

$\boxed{GB}$ $G_1\ G_2\ G_3\ G_4\ G_5$

① ② ③ ④ ⑤ ⑥ $\Rightarrow 5! \times 6!$

Circular arrangement of 6 people

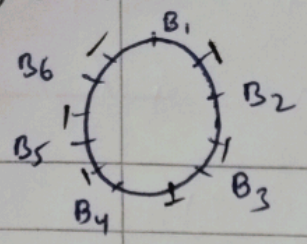
6 Boys Arrangement internally

Que) All Boys & All girls together $\Rightarrow$ $\boxed{6B}$ $\boxed{5G}$

① ② $\Rightarrow \frac{2!}{2} \times 6! \times 5!$

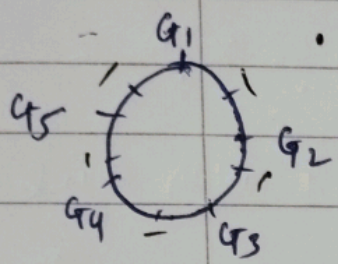
Que) 6B , 5G . No 2 girls together.

Soln



$\Rightarrow \frac{6!}{6} \times {}^6C_5 \times 5! = 5! \times 6 \times 5!$ ways.

Boys arrang in circular.

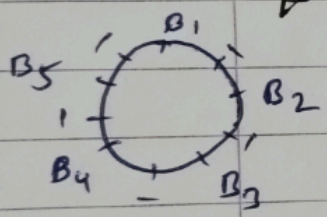


. No 2 Boys together.

$\Rightarrow \frac{5!}{5} \times {}^5C_6$ ← not possible $\Rightarrow$ 0 ways.

Que) 5 Boys , 5 Girls.

. No 2 Boys & No 2 Girls together



$\Rightarrow \frac{5!}{5} \times 5! \Rightarrow 4! \times 5!$ ← Girls arrangement in 5 places.

Boys arrang. in circular.

Que) 6 Boys & 4 Girls arrange such that exactly 2 girls together

Soln . Linear $\Rightarrow$ $\boxed{G\ G}$ G, G, B_1 B_2 B_3 -- B_6

$\Rightarrow {}^4C_2 \times 2! \times 9!$ ways

← pick 2 girls & arrange them.

. Circular $\Rightarrow \frac{9!}{9} \times {}^4C_2 \times 2!$

arrange 9 people in circular way. → $\frac{9!}{9}$

← pick 2 girls & arrange them

Identicle

4 identicle Red , 3 identicle blue , 6 identicle yellow flowers. How many arrangement in Garland?

$\Rightarrow \frac{13!}{13 \times 2} \times \frac{1}{4! \times 3! \times 6!}$ ← identicle.

Circular → 13 × 2 ← Garland

:- Lecture - 8 :-

Combination

$$\boxed{{}^{n-1+r}C_r \equiv {}^{n-1+r}C_{n-1}}$$

Que: How many way to select ~~from~~ 3 people from 10 people in which one person ~~is~~ (A) is Already Included.

Solⁿ :- ${}^{10-1}C_{3-1} = \boxed{{}^{9}C_2}$ Ans

If a particular Person Always excluded.

Solⁿ ${}^{10-1}C_3 \Rightarrow {}^{9}C_3$

Two person Always Included.

Solⁿ ${}^{10-2}C_{3-2} = {}^{8}C_1$

Two person Always excluded.

Solⁿ ${}^{8}C_3$

- from n people, if m people is already selected. select r, ~~from~~.

  $\Rightarrow {}^{n-m}C_{r-m}$

Que Set $A = \{1, 2, 3, \ldots \ldots 10\}$

a) find # subset of A

b) subset of element ≥ 1

c) " " ≥ 2

d) " " $= 3$

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Sol^n a) # Subset $= 2^{10} \rightarrow \{{}^{10}C_0 + {}^{10}C_1 + {}^{10}C_2 \text{ ---- } {}^{10}C_{10}\} \rightarrow 2^{10}$

b) # subset element $\geq 1 \Rightarrow 2^{10} - {}^{10}C_0 \Rightarrow 2^{10} - 1 = 1023$.

c) # subset " $\geq 2 \Rightarrow 2^{10} - ({}^{10}C_0 + {}^{10}C_1)$

d) # subset element $= 3 \rightarrow {}^{10}C_3$ ✓

e) # " " $\leq 3 \rightarrow {}^{10}C_0 + {}^{10}C_1 + {}^{10}C_2$

Qu from set $A = \{1, 2, 3, 4, 5, \text{---} 10\}$.

a) How many subset is present ??
b) Subset which Include 2 & 3 ??
c) subset which exclude 2 & 3 ??
d) subset which Include 2 & 3 & exclude 4 ??
e) 5 element subset ??
f) 5 element subset which Include 2 & 3 ??
g) " " " Exclude 2 & 3 ??
h) " " " " Include 2 & 3 & exclude 4 ??

Sol^n (a) $2 \times 2 \times 2 \times$ ---- 10 Times $\rightarrow 2^{10}$

(b) $2 \times \underline{1} \times 1 \times 2$ ---- 10 times $\rightarrow 2^8$ (for 2 & 3 one choice only)

(c) " " " 10 Time $\rightarrow 2^8$

(d) " for 2, 3 & 4 only one choices $\rightarrow 2^7$

Subset Not specified

(e) ${}^{10}C_5$ ✓

(f) ${}^{10-2}C_{5-2} \Rightarrow {}^8C_3$ ✓

(g) ${}^{10-2}C_5 \Rightarrow {}^8C_5$ ✓

(h) ${}^{10-2-1}C_{5-2} \Rightarrow {}^7C_3$ ✓

Subset size specified

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Qu How many ways to make 3B & 2G in committee from 5 B & 6 G

Sol^n :- $5C_3 * 6C_2 \Rightarrow \frac{\lfloor 5}{\lfloor 3 \lfloor 2} * \frac{\lfloor 6}{\lfloor 2 \lfloor 4}$

$\Rightarrow \frac{5 \times 4}{2} * \frac{6 \times 5}{2} \Rightarrow 150$ ✓

at least 3 Boys

$\Rightarrow 5C_3 * 6C_2 + 5C_4 * 6C_1 + 5C_5 * 6C_0$

(or)

$11C_5 - \{5C_0 * 6C_5 + 5C_1 * 6C_4 + 5C_2 * 6C_3\}$ ✓

at most 3 Boys

$\Rightarrow 5C_0\, 6C_5 + 5C_1\, 6C_4 + 5C_2\, 6C_3 + 5C_3\, 6C_4$.

at least 2B & at most 2G

B	G	Choose
2	3	✗
3	2	✓
4	1	✓
5	0	✓

$\boxed{5C_3 * 6C_2 + 5C_4 * 6C_1 + 5C_5 * 6C_0}$

Qu In an Exam two section A & B are their in which each having 25-25 Question

To done 10 Question it Needs atleast 6 Question from section A & at most 3 Question from section B ✓

Find # of combinations to do the Questions.

Sol^n :-

10

A ↙ B ↘

A: ≥6	B: ≤3
~~6~~	~~4~~ ✗
7	3 ✓
8	2 ✓
9	1 ✓
10	0 ✓

} Choices ✓

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Qu from a dec of 52 cards —

a) select 5 cards in which exactly 2 ~~are~~ are King.

* b) 5 card select exacty 2 are King & 3 are Queen.

c) select 5 cards in which exacty 2 are King & exacty 2 are Queen

Sol^n a) $^4C_2 * {}^{48}C_3$

b) $^4C_2 * {}^4C_3$

c) $^4C_2 * {}^4C_2 * {}^{44}C_1$

Qu 11 People — (5 B & 6 G)

a) select 5 people in which 3 B & 2 G are there. in which Name "A" Boy must be Included.

b) " " " " " " " also name "G" girl must be Included.

c) " " " " " " " " " " Excluded

Sol^n:- a) $^4C_2 * {}^6C_2$

b) $^4C_2 * {}^5C_1$

c) $^4C_3 * {}^5C_2$

d) " " " " " in which "A" & "G" Boys & girls can not work Together.

Sol^n $\underbrace{(^5C_3 * {}^6C_2)}_{\text{Total}\checkmark} - \underbrace{(^4C_2 * {}^5C_2)}_{\text{Working Together}\checkmark}$

Qu A = { 1, 2, 3, — — — 10 }

a) How many ways to select 3 ~~People~~ Number & arrang them??

b) How many ways to select 3 Numbers & arrange them into ascending order??

Sol^n:- a) $^{10}P_3$ (b) In this type of qu $^{10}C_3 = {}^{10}P_3$ // {only one way to arrange it}

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Sometime $\boxed{{}^nC_r \equiv {}^nP_r}$

- when Identical object are given
- when their is only one way to arrange it.

Que How many ways to arrange m zero's & n once's. So that No two onces are get Together.

Sol^n $\underbrace{0\ 0\ 0\ 0\ \cdots\ 0}_{(m+1)\text{ Places}} \longrightarrow 1 * {}^{(m+1)}C_n$

for placing 0(zero)

Que How many 10 bit strings are their in which their are 3 ~~bit~~ No. of zeros are their.

Sol^n $\boxed{{}^{10}C_7 * {}^3C_6}$ ✓ (or) $\boxed{{}^{10}C_3 * {}^7C_7}$

$\boxed{{}^nC_r = {}^nC_{n-r}}$ ✓

** Que if their is 5 0's & 15 1's if Every 0's followed at least 1one then How many No. can you form.?

Sol^n:- (0 1 0 1 0 1 0 1 0 1) (1 1 1 1 1 1 1 1 1 1)

$\Rightarrow \left(\frac{\lfloor 15}{\lfloor 5 \ \lfloor 10}\right)$ ✓

Every zero follows ~~exactly~~ at least 2 1's.

(0 1 1 0 1 1 0 1 1 0 1 1 0 1 1) (1 1 1 1)

$\Rightarrow \left(\frac{\lfloor 10}{\lfloor 5 \ \lfloor 5}\right)$

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Variations of $({}^{n-1+r}C_r)$

1) · In this type of combination

$n < r$
$n \geq r$ } → anything possible
$n = r$

n = # group selected $\boxed{r = \text{RHS}}$

By Example it is clear

Que If their are three type of chocklete R, G, B are given & we want to find out 10 out of them.
So, How many ways can we Taken 10 chocklete.

Sol^n $x_1 + x_2 + x_3 = 10$

$\begin{pmatrix} r = 10 \\ n = 3 \end{pmatrix}$ $\quad {}^{3-1+10}C_{10} \Rightarrow {}^{12}C_{10} = {}^{12}C_2 \Rightarrow \frac{12 \times 11}{2} = 66$ ways

Que a) If their is 4 type of CD's in the shop A, B, C, D
if a person want to choose 20 CD from these 4 type.
then How many ways to choose it.

b) If their is 5 type of coins are their C_1, C_2, C_3, C_4, C_5
If you want to choose 15 coin from it then No. of Possible ways to choose it.

Sol^n a) # CD's $A + B + C + D = 20$

Here $\boxed{r = 20}$ & $\boxed{n = 4}$ → 4 Type of CD's
↑ Total No. of CD's.

${}^{4-1+20}C_r \Rightarrow \boxed{{}^{23}C_{20}} = {}^{23}C_3 \Rightarrow \frac{23 \times \cancel{22}^{11} \times \cancel{21}^{7}}{\cancel{3} \times \cancel{2}}$

$\Rightarrow (77 \times 23)$ ways

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b) Type of coins = 5

$c_1+c_2+c_3+c_4+c_5=15$

$\boxed{\begin{matrix} n=5 \\ r=15 \end{matrix}}$

$^{5-1+15}C_{15} \Rightarrow (^{19}C_{15}) = (^{19}C_4)$

2) <u>Non-Negative Integral Solution Problem:-</u>

- For Number of Non Negative solution. we will use –

$(^{n-1+r}C_r)$ formula

<u>Ex</u> a) How many ~~solu~~ Non-Negative solution for

$a+b+c+d=10$.

b) How many solution for

$c_1+c_2+c_3+c_4+c_5=15$

where $\boxed{c_1, c_2, c_3, c_4, c_5 \geq 0}$

<u>Sol^n:-</u> (a) $a+b+c+d=10$

$\boxed{n=4,\ r=10}$

Non Negative solution $\Rightarrow {}^{n-1+r}C_r \Rightarrow {}^{4-1+10}C_{10} \Rightarrow {}^{13}C_{10} = {}^{13}C_3$

(b) Here $c_1, c_2, c_3, c_4, c_5 \geq 0$ given it means I want to find Non Negative solution

<u>So</u> $\boxed{n=5,\ r=15}$

$^{5-1+15}C_{15} \Rightarrow (^{19}C_{15}) \Rightarrow \frac{\lfloor 19}{\lfloor 15 \ \lfloor 4} \Rightarrow \frac{19 \times \overset{6}{\cancel{18}} \times 17 \times \overset{2}{\cancel{16}}}{\cancel{4} \times \cancel{3} \times \cancel{2}}$

$\Rightarrow (19 \times 16 \times 34)$ Ans

~~Distrib~~

3) Identical Ball in Box (distribution problem) :-

(or)

Ball in the Box ✓

Que if their is 10 Identical ball is given and if I ask to place these Identical ball into 4 Boxes then How many ways can you place these balls into Boxes??

Sol^n ~~x_1+x_2+~~ $B_1+B_2+B_3+B_4=10$

Surely these are Non Negative in Nature so, we can apply.

$n-1+r C_r \Rightarrow 4-1+10 C_{10} \Rightarrow ({}^{13}C_{10}) \Rightarrow ({}^{13}C_3)$ ✓ Ans

4) Out comes of Identical coin & dice :-

Que a) If 10 identical dice are thrown then How many Possible out comes are Possible??

b) If 20 Identical coin are thrown then How many Possible out comes are ??

Sol^n (a) 10 Identical dice

$d_1+d_2+d_3+d_4+d_5+d_6=10$

$n=6$, $r=10$ $\Rightarrow 6-1+10 C_{10} \Rightarrow \boxed{{}^{15}C_{10}}$

(b) 20 Identical coin

$C_H+C_T=20$

$n=2$, $r=20$ $\Rightarrow 20+2-1 C_{20} \Rightarrow \boxed{{}^{21}C_{20}}$

→ also k/a Non-Negative solution

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5) Dice sum & digit sum problem :-

Ex if we throw 3 dice & sum = = 9

↳ $x_1 + x_2 + x_3 = 9$

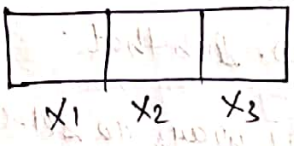
But Here $\boxed{n \neq 3}$ ← we will see further In the question

- In this way digit sum problem are completed - ✓

Let's do Variation of Questions on $({}^{n-1+r}C_r)$ ✓ from Previous ⑤ Types

Que How many ways to put 10 Identical balls into 3 boxes ✓

Sol^n



$\boxed{x_1 + x_2 + x_3 = 10}$ ✓

$n = 3, \; r = 10$

$\Rightarrow {}^{3-1+10}C_{10} \Rightarrow \boxed{{}^{12}C_{10}}$

-: When each boxes have at least one balls. :-

In this type of problem put one ball in all the Boxes firstly & then apply the previous concept ☺

$\Rightarrow x_1 + x_2 + x_3 = 10 - 3$

$\Rightarrow \boxed{x_1 + x_2 + x_3 = 7}$ ✓

↳ ${}^{3-1+7}C_7 \Rightarrow ({}^{9}C_7)$ ✓ $\Rightarrow \frac{9 \times 8}{2} \Rightarrow 36$ ✓

gate ✓

Que How many way we can put n balls in k boxes so, that ~~it is~~ the Boxes are non-Empty ← NonEmpty ✓ // each Box have at least one Ball ✓

Sol^n $x_1 + x_2 \text{ ---- } x_k = (n - k)$

↳ $({}^{k-1+n-k}C_{n-k}) \Rightarrow ({}^{n-1}C_{n-k})$ ✓ $= ({}^{n-1}C_{k-1})$ ✓

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JNU

Que How many +ve Integral solution.

$$a+b+c+d=20$$

Sol^n a, b, c & d at least == 1

$$a+b+c+d=20-4$$

$$\boxed{a+b+c+d=16}$$

$$4-1+16C_{16} \Rightarrow (19C_{16}) = (19C_3)$$

How constraints with different-2 Values

Que Three type of Chocklets Red, Green, Yellow. such that

$\boxed{X_R+X_G+X_y=10}$ → find How many ways to select chocklets.

where

$1 \le X_R$, $2 \le X_G$, $4 \le X_{yellow}$

Solutions:-

$$X_R+X_G+X_y=10-(4+2+1)$$

$$\boxed{X_R+X_G+X_y=3}$$

$$3-1+3C_3 \Rightarrow (5C_3) \Rightarrow \frac{5\times4}{2}=10 \text{ ways}$$

***

Que three dice are thrown in the way sum $(d_1+d_2+d_3=15)$

find No. of way to find the combinations

Sol^n $d_1+d_2+d_3=15-(3)$ → we already know dice thrown will not be blank

$$\boxed{d_1+d_2+d_3=12}$$

$$3-1+12C_{12} \Rightarrow (14C_{12}) \Rightarrow (14C_2)$$

So, take minimum constraints

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Que How many Integral solution for —

$a+b+c=10$

constraints

$-2 \le a$, $-1 \le b$, $1 \le c$

Solⁿ $\boxed{a+b+c=10}$

$a+b+c = 10-(-2-1+1)$

$\Rightarrow 10+2 \Rightarrow 12$

$\boxed{a+b+c=12}$

$\boxed{n=3, r=12}$ $\quad 3-1+{}^{12}C_{12} \Rightarrow {}^{14}C_{12} = {}^{14}C_2 \Rightarrow \frac{14\times13}{2} \Rightarrow \boxed{91}$

Question Based on upper constraints :-

$\boxed{\text{upper constraints always less than or equal to right hand side}}$

Que $x_1+x_2+x_3=10$

$x_1 \le 8 \quad x_2 \le 5 \quad x_3 \le 7$

• $n-1+{}^{r}C_{r}$ Not for this for multiple upper constraints

$\boxed{\text{Generating } F^n \text{ works for all type of constraints} \rightarrow \text{upper} \rightarrow \text{lower} \rightarrow \text{mixed}}$

↳ • But it will consume time

• for one upper constraints we can go for $({}^{n-1+r}C_{r})$

Ex $x_1+x_2+x_3=10$ & $x_1 \le 8$

Solⁿ find complement of given constraints & then solve —

$x_1 \le 8 \xrightarrow{\text{comp}} \boxed{x_1 \ge 9}$ → lower constraints $\longrightarrow (x_1+x_2+x_3=1)$

$3-1+{}^{10}C_{10} - \{3-1+{}^{1}C_{1}\} \Rightarrow {}^{12}C_{10} - \{{}^{3}C_{1}\} \Rightarrow \frac{12\times11}{2} - 3$

$\Rightarrow 66-3 \Rightarrow 63$

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In Previous Question

↳ 3 sol^n removed $\left\{ \begin{array}{l} 9+0+1 \\ 9+1+0 \\ 10+0+0 \end{array} \right\}$

Que $\boxed{x_1+x_2+x_3 \leq 10}$

solve $x_1+x_2+x_3 = 1$
$= 2$
$\vdots$
$= 10$ → It will Takes Too much Time

So, Add one virtual Box & Put Inequality into equality

$(x_4) \rightarrow$ Remaining put Here

Or $x_1+x_2+x_3+x_4 = 10$
≤ 10

Sol^n $\left(^{4-1+10}C_{10}\right) \Rightarrow \left(^{13}C_{10}\right)$

Que Solve for

(i) $x_1+x_2+x_3 \leq 13$

(ii) $x_1+x_2 \leq 12$

Sol^n (1) $x_1+x_2+x_3 \leq 13$

$x_1+x_2+x_3+x_4 = 13 \Rightarrow {}^{4-1+13}C_{13} \Rightarrow \left(^{16}C_{13}\right)$

(2) $x_1+x_2 \leq 12$

$x_1+x_2+x_3 = 12$

$^{3-1+12}C_{12} \Rightarrow \left(^{14}C_{12}\right)$

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Que $x_1 + x_2 + x_3$ = 10 i'd Red
= 15 i'd yellow
= 12 i'd Green Balls.

How many ways we can put into Boxes??

Solⁿ :- $x_1 + x_2 + x_3$ = 10 i'd Red —— n_1
= 15 i'd yellow —— n_2
= 12 i'd Green Ball —— n_3

Ans $\boxed{n_1 \times n_2 \times n_3}$

$\boxed{{}^{3-1+10}C_{10} * {}^{3-1+15}C_{15} * {}^{3-1+12}C_{12}}$

$\boxed{{}^{12}C_{10} * {}^{17}C_{15} * {}^{14}C_{12}}$

gate

Que If their is 2 Girls & they picked 10 Lilly, 15 ~~Roses~~ deffides, 12 sunflowers

So, How many ways can these two girls ~~can~~ distribute these flowers themselves

Solⁿ :- $G_1 + G_2$ = 10 Lilly
= 15 Roses
= 12 sunflowers

$= {}^{2-1+10}C_{10} + {}^{2-1+15}C_{15} + {}^{2-1+12}C_{12}$

$= \boxed{{}^{11}C_{10} + {}^{16}C_{15} + {}^{13}C_{12}}$ Ans

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