23/8/25

MA714
Assignment - 1

1) If $m \le n$

Total no of one-one functions from set S to T = $n(n-1)(n-2) ---- (n-m+1)$

$$= \frac{n!}{(n-m)!} = P(n,m)$$

2) $d_1 + d_2 + d_3 + d_4 + d_5 + d_6 = 19$ $\quad 0 \le d_i \le 9$

$\therefore$ No of ways to select 19 objects from 6 different boxes = $\binom{6+19-1}{19} = C(24,19) = 42504$

Now voilating conditions $\Rightarrow$

If we want sum of digits exactly = 19, then only ~~and~~ one digit can be >10 because $\underline{10+10>19}$

$\therefore$ No of solutions of $d_1'' + d_2 + \cdots d_6 = 19-10 = 9$

$$= \binom{6+9-1}{9} = C(14,5) = 2002$$

Now there are 6 ways to choose a digit that will exceed 9.

$\therefore$ No of voilations = $2002 \times 6 = 12012$

$\therefore$ $\boxed{\text{Ans} = 42504 - 12012 = 30{,}492}$

3) The proof uses stars and bars method.
Its equivalent to arranging r items from $n-1$ dividers (divisions for n categories), creating $n-1$ positions which are empty between n items.

- Total Positions :-
  The total no. of positions available is the sum of r items and $(n-1)$ positions
  $\therefore (n-1) + r$ total positions
  $= n+r-1$

- Choosing Positions
  No of ways to choose r positions for stars out of total $n+r-1$ positions $= C(n+r-1, r)$
  $= \binom{n+r-1}{r}$ ways.

4) Coeff. of $x^a y^b z^c w^d$ in $(x+y+z+w)^n$ is determined by considering no. of ways to select x, y, z, w from n factors

- No. of ways to choose x :
  No of ways to choose a factors from n total
  $= \binom{n}{a}$

- No of ways to choose y :
  After choosing a factors for x, there are $n-a$ factors remaining. $\therefore$ No of ways to choose b factors $= \binom{n-a}{b}$

- Similarly for z : $= \binom{n-a-b}{c}$

- For $w = \binom{n-a-b-c}{d}$

Now, since $a+b+c+d = n$
$\therefore d = n-a-b-c$
$\therefore$ For $w = \binom{d}{d}$

Total no of ways $= \binom{n}{a}\binom{n-a}{b}\binom{n-a-b}{c}\binom{d}{d}$

$$= \frac{n!}{a!(n-a)!} \times \frac{(n-a)!}{b!(n-a-b)!} \times \frac{(n-a-b)!}{c!\,d!} \times \frac{d!}{d! \times 0!}$$

$$= \frac{n!}{a!\,b!\,c!\,d!}$$ is the coeff of $x^a y^b z^c w^d$.

5) Let the sum of k consecutive items be denoted by S_k, $k = 1, 2 \ldots 10$

$\therefore S_0 = 0$

$S_1 = a_0$

$S_2 = a_0 + a_1$

$S_3 = a_0 + a_1 + a_2$ --- $S_k = a_0 + a_1 + \ldots a_{k-1}$

For proving any one of these is divisible by 10 Is an example of Pigeon hole Principle

- There are 11 cumulative sums possible $S_0, S_1, \ldots S_{10}$
- There are only 10 possible remainders when dividing by 10 (i.e 0, 1, 2, -- 9)
- By Pigeon hole principle, if you have 11 items (sums) to place into 10 pigeonholes (the remainders), then atleast one pigeon hole must contain more than one item.
- This means atleast two sums must have same remainder (Say S_i and S_j)
- This implies $S_j - S_i$ must be divisible by 10

$\therefore S_j - S_i = (a_0 + a_1 + \ldots a_{j-1}) - (a_0 + a_1 + \ldots a_{i-1})$

$= a_i + a_{i+1} + \ldots a_{j-1}$

is divisible by 10.

6) There are n persons in the gathering,
Max^m no. of persons one can know is $n-1$.
So every single person must be assigned to one of these $(n-1)$ people.

But since there are n persons, one of these $(n-1)$ people must be assigned twice. It is an example of pigeon hole principle

Therefore, There will be atleast 2 people who knows the same number of people