Roll No.:	2	5	2	I	S	0	3	2

MACS

Department of Mathematical and Computational Sciences
National Institute of Technology Karnataka, Surathkal

Odd Semester (2025- 2026) **Examination: Mid Sem**
Course Code: MA714 **Course Name:** Mathematical Foundations of Computer Science
Date:07/10/2025 **Time:10.30 Hours to 12.00 Hours** **Maximum Marks: 50**

INSTRUCTIONS:

1. Answer ALL questions.
2. Use of only non-programmable Scientific Calculators permitted.
3. Rough work should NOT be done anywhere on the Question Paper.

Q.1. (A) We have a competition in which $n(>= 8)$ teams participate. Each team has exactly 5 participants. The rules of the competition are as follows: [05]

Rule 1: The competition has certain number of rounds (not fixed).

Rule 2: For each round every non-eliminated team must choose one representative to participate in the game. Every round has only one winner.

Rule 3: The losers of every rounds are totally eliminated from the competition and are not allowed to join back their teams. A team gets eliminated if it loses all of its players.

Rule 4: The competition ends when we have only one team of non-eliminated players remaining, and this team becomes the winner of the competition.

What is the total number of minimum and maximum rounds possible for the above competition?

(B) Justify or disprove: "An equilateral triangle can not be covered completely by two smaller equilateral triangles." [05]

Q.2. (A) You need to come up with a password that uses only the letters A, B, and C and which must use each letter at least once. How many such passwords of length 8 are there? [05]

(B) Let D_n denote the number of derangements on n objects. Show that D_n satisfies the recurrence relation $D_{n+1} = n(D_n + D_{n-1})$ along with the assumption $D_0 = 0, D_1 = 0$. [05]

Q.3. (A) Let $f(n)$ denote the number of words of length n in the alphabet $\{a, b, c\}$ in which a and b are never adjacent. So, $f(1) = 3$, and $f(2) = 7$ (the acceptable words of length 2 being aa, bb, cc, ac, bc, ca, and cb). Find a recurrence relation for $f(n)$ and calculate a formula for it. [05]

(B) Find the generating functions for the following sequences, with suitable explanation: [05]

i) $1, 1, 1, 1, 1, \cdots$

ii) $1, 3, 3, 1, 0, 0, 0, 0, \cdots$

$D_n = n! e^{-1}$

iii) $\binom{2025}{0}, \binom{2025}{1}, \binom{2025}{2}, \cdots, \binom{2025}{2025}, 0, 0, 0, \cdots$

Q.4. (A) What is the coefficient of x^{2005} in the generating function $G(x) = \frac{1}{(1-x)^2(1+x)^2}$? [05]

(B) Two persons A and B gamble Rupees on the toss of a fair coin. A has ₹70 and B has ₹30. In each play either A wins ₹1 from B or loss ₹1 to B. The game is played without stop until one wins all the money of the other or goes forever. Find the probability that A wins all the money of B. [05]

$1 + 3(2)^n$

Q.5. (A) Using the Generating Function Technique, solve $a_{n+2} - 5a_{n+1} + 6a_n = 2$, $n \geq 0$, $a_0 = 3$, $a_1 = 7$. [05]

(B) For the set of positive integers $\{1, 2, 3, \cdots, n-1, n\}$. we know that the first 6 digits appear in the first 6 positions. If there are 2385 derangement of this set, what is the value of n? [05]

*****

Department of Mathematical and Computational Sciences
National Institute of Technology Karnataka, Surathkal

Roll No.:	2	5	2	1	5	0	3	2

Odd Semester (2025- 2026) **Examination: End Sem**
Course Code: MA714 Course Name: Mathematical Foundations of Computer Science
Date: 26/11/2025 Time: 09.00 Hours to 12.00 Hours Maximum Marks: 100

INSTRUCTIONS:

1. Answer ALL questions.
2. Use of only non-programmable Scientific Calculators permitted.
3. Rough work should NOT be done anywhere on the Question Paper.

Q.1. (A) A company is issuing a 9-digit customer code number to all its clients. The pattern of the code number is set as each code number is odd and does not start with zero. Further, no digit is repeated. How many such code numbers can be generated? Explain. [06]

(B) (i) Given five points inside an equilateral triangle of side length 2, show, using the Pigeonhole Principle, that there are two points whose distance from each other is at most 1. [08]

(ii) Using Generating Functions technique, determine the number of ways of selecting, with repetitions allowed, r objects from n distinct objects.

(C) Suppose that 4 people are standing in line. How many ways are there to rearrange the line so that nobody is standing in their original place? Explain. [06]

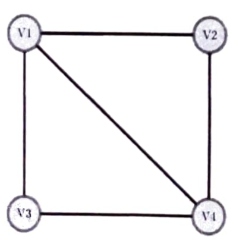
Q.2. (A) How many permutations of the n distinct elements $(1, 2, 3, \cdots, n)$ are there in which the element k is not in the k^{th} position $(k = 1, 2, 3, \cdots, n)$? Explain. [06]

(B) Using Generating Function Technique, solve $2a_n = a_{n-1} + 2^n$ where $a_0 = 1$. [08]

(C) If six people, designated as A, B, C, D, E, F are seated about a round table, how many different circular arrangements are possible, if arrangements are considered the same when one can be obtained from the other by rotation? Describe the solution. [06]

Q.3. (A) Prove or disprove: Every finite simple graph G with $n \geq 2$ vertices has at least two vertices of the same degree. [06]

(B) (i) Applying the Matrix Tree Theorem, determine the number of distinct labeled spanning trees of the following graph and also draw all the distinct labeled spanning trees: [08]



(ii) Which of the following partitions is/ are graphical? Describe with clear steps:

(I) $(5, 5, 3, 3, 2, 2, 2)$

(II) $(8, 7, 6, 5, 4, 3, 2, 2, 1)$

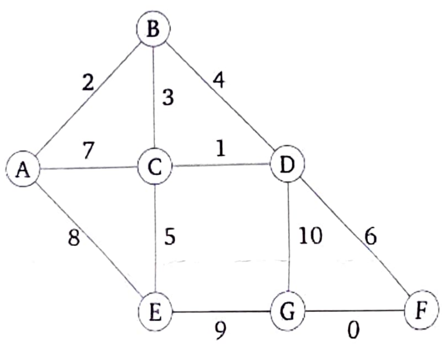
(C) Show that a connected graph G is Eulerian if and only if the degrees of all its vertices are even. [06]

Q.4. (A) If G is an Eulerian graph, show that its line graph $L(G)$ is also Eulerian. [06]

(B) (i) If the Chromatic Polynomial of a graph G on n vertices is given by $f(G, t) = t(t-1)(t-2)\cdots(t-n+1)$, derive the Chromatic Polynomial of $\overline{G}$, where $\overline{G}$ is the complement of the graph G. [08]

(ii) Prove or disprove: Either G or $\overline{G}$ is connected, where G is a simple graph.

(C) Apply Prim's Algorithm on the following graph and obtain the minimum spanning tree, starting from the vertex A. [06]



Explain the steps clearly describing the implementation of the algorithm pictorially.

Q.5. (A) Show that a group G can not be a union of two proper subgroups of G. [06]

(B) (i) Show that every subgroup of a cyclic group is cyclic. [08]

(ii) Show that every group of prime order is cyclic.

(C) Show that the intersection of two normal subgroups is also a normal subgroup. [06]

*****