Department of Mathematical and Computational Sciences
National Institute of Technology Karnataka, Surathkal

Mid-Semester Examination
Mathematical Foundations of Computer Science (MA714)
Odd Semester (2024-25)

Date: 9-10-2024   Time: 1.30 PM to 3 PM   Maximum Marks: 50

INSTRUCTIONS

1. Do not write anything on the question paper except your Roll Number and Name.
2. Mobile phones, calculators and other electronic gadgets are not allowed.

Q.1. (a) Is the following equality true for all positive integers n? Justify your answer.   [3]

$$\binom{-1/2}{n} = \binom{2n}{n} \Big/ (-4)^n$$

(b) Does the coefficient of x^n in the expansion of $(1 - 4x)^{-1/2}$ equal $\binom{2n}{n}$ for all positive integers n? Justify your answer.   [3]

(c) Let $(1 - \sqrt{1 - 4x})/(2x) = \sum_{n=0}^{\infty} C_n x^n$. Is the following equality true for all positive integers n? Justify your answer.   [6]

$$C_n = \frac{1}{n+1}\binom{2n}{n}$$

Q.2. Use the principle of inclusion–exclusion to find the number of primes not exceeding 110.   [8]

Q.3. A chess tournament has 15 participants, and any two players play one game against each other. Then is it true that at any given point of time, there are at least two players who have finished the same number of games? Justify your answer.   [6]

Q.4. How many solutions does   [4]

$$x_1 + x_2 + x_3 = 6$$

have, where x_1, x_2 and x_3 are nonnegative integers?

Q.5. A bowl consists of 8 red balls, 8 blue balls and 8 green balls, (i.e., a total of 24 balls). In how many different ways can 8 balls be selected from the bowl such that all the following three conditions are satisfied: (1) at least 3 balls selected are red, (2) an even number of blue balls is selected and (3) an odd number of green balls is selected?   [8]

Q.6. Solve the following recurrence relation: [5]

$$a_n = 5a_{n-1} - 6a_{n-2} \text{ for all } n \geq 2; \quad a_0 = 1 \quad a_1 = 0.$$

Q.7. A cat has a staircase of 100 stairs to climb. Each step it takes can cover either one stair or two stairs. Find the number of different ways for the cat to ascend the staircase.   [7]

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End-Semester Examination (Part B)
Mathematical Foundations of Computer Science (MA714)
Odd Semester (2024-25)

Date: 9-12-2024   Time: 9 AM to 12 PM   Maximum Marks: 60

INSTRUCTIONS
1. Do not write anything on the question paper except your Roll Number and Name.
2. Mobile phones, calculators and other electronic gadgets are not allowed.
3. Do the page numbering of your answer booklet and do the indexing.

Q.1. Solve the following recurrence relation: [5]

$$a_n = 4a_{n-1} - 4a_{n-2} \text{ for all } n \geq 2; \quad a_0 = 6, \quad a_1 = 8.$$

Q.2. Find the generating function for the sequence $\{a_n\}$, where $\{a_n\}$ satisfies the following recurrence   [9]
relation:

$$a_n = a_{n-1} + n^2 \text{ for all } n \geq 1; \quad a_0 = 0.$$

Q.3. Let (G, o) and $(H, *)$ be groups with respective identities e_G and e_H. And $f : G \to H$ is a   [8]
homomorphism. Either prove or disprove the following statements:
- (i) $f(e_G) = e_H$.
- (ii) $f(a^{-1}) = [f(a)]^{-1}$.

Q.4. Let G be a group. Let e be the identity element of G. Let $a \in G$ be such that $a \neq e$. Let   [10]
$<a> = \{a^m | m \in \mathbb{Z}\}$. Suppose that $<a>$ is finite. Prove or disprove the following statement:
There exists a positive integer n such that $\{a, a^2, a^3, \ldots, a^n\} = <a>$.

Q.5. Suppose H is a subgroup of a finite group G. Then prove or disprove the following statements: [8]
- (i) For all $a \in G$, $|aH| = |H|$.
- (ii) For all $a, b \in G$, either $aH = bH$ or $aH \cap bH = \emptyset$.

Q.6. If G is a group, let $H = \{a \in G : ag = ga \text{ for all } g \in G\}$. Prove that H is a subgroup of G.   [8]

Q.7. How many different strings can be made by reordering the letters of the word MISSISSIPPI ?   [6]

Q.8. Let G be a simple graph with 1200 vertices and 2500 edges. Also, G does not contain any cycle   [6]
of length 3. Can we deduce whether G is planar or non-planar? Justify your answer.

Odd Semester (2022-2023)
Course Code: MA714   Course Name: Math. Foundations of Computer Science   **Examination: Mid Sem**
Date: 13/10/2022   Time: 08:30 AM to 10.00 AM   **Maximum Marks: 50**

INSTRUCTIONS:
1. Answer ALL questions.
2. Rough work should NOT be done anywhere on the question paper.

Q.1. What is the generating function for $\{b_k\}$ where b_k is the number of solutions of   [2]

$$x_1 + x_2 + x_3 = k,$$

where x_1, x_2 and x_3 are integers with $x_1 \geq 2$, $0 \leq x_2 \leq 3$ and $2 \leq x_3 \leq 5$?

Q.2. There are 38 different time periods during which classes at a university can be scheduled. If   [2]
there are 677 different classes, how many different rooms will be needed?

Q.3. Simplify $\binom{n+2}{r+1} - 2\binom{n+1}{r+1} + \binom{n}{r+1}$.   [2]

Q.4. Suppose that the increasing function g satisfies the recurrence relation $g(n) = 2g(\sqrt{n}) + 1$ [2]
whenever n is a perfect square greater than 1 and $g(2) = 1$. What is the big-$\mathcal{O}$ estimate for
$g(n)$ where n is the power of 2?

Q.5. Write a recurrence relation such that both $b_n = 4^n$ and $b_n = 2n3^n$ are its solutions.   [2]

Q.6. Use generating functions to solve the recurrence relation   [8]

$$a_n = 5a_{n-1} - 6a_{n-2}$$

with initial conditions $a_0 = 6$ and $a_1 = 30$.

Q.7. Find all the solutions of the recurrence relation   [8]

$$b_n = 4b_{n-1} - 4b_{n-2} + (n+1)2^n.$$

Q.8. State and prove the generalized principle of inclusion and exclusion.   [8]

Q.9. Prove the identity   [8]

$$\sum_{k=0}^{r} \binom{n+k}{k} = \binom{n+r+1}{r}$$

whenever n and r are positive integers.

Q.10. Let f be an increasing function that satisfies the recurrence relation   [8]

$$f(n) = af(n/b) + c$$

whenever n is divisible by b where $a > 1$, $b \in \mathbb{N}$ and $c > 0$. Then show that $f(n) = O(n^{\log_b a})$.

Odd Semester (2022-2023)   Examination: End Sem
Course Code: MA714   Course Name: Math. Foundations of Computer Science
Date: 17/12/2022   Time: 09:00 AM to 12.00 NOON   Maximum Marks: 100

INSTRUCTIONS:
1. Answer ALL questions.
2. Rough work should NOT be done anywhere on the question paper.

Q.1. An arm wrestler is the champion for a period of 75 hours. The arm wrestler had at least [4]
one match an hour but no more than 125 total matches. Show that there is a period of
consecutive hours during which the arm wrestler had exactly 24 matches.

Q.2. State and prove Vandermonde's identity. [4]

Q.3. What is the general form of the particular solution (no need to find the coefficients, if any) [4]
guarenteed to exist for the recurrence relation $a_n = 8a_{n-2} - 16a_{n-4} + F(n)$ if

(a) $F(n) = n^4 2^n$

(b) $F(n) = (n^2 - 2)(-2)^n$.

Q.4. Find $f(n)$ when $n = 3^k$ where f satisfies the recurrence relation $f(n) = 2f(n/3) + 4$ with [4]
$f(1) = 1$. Give a big-O estimate for the function f if it is an increasing function.

Q.5. Let G be a cycle on n vertices. Find the possible values of n if G is self complementary. [4]

Q.6. Find the length of the longest path in the hypercube Q_8. [4]

Q.7. Is $K_{3 \times 3}$ a planar graph? Justify. [4]

Q.8. Define cyclic group. Prove that a homomorphic image of a cyclic group is cyclic. [4]

Q.9. How many elements in the group $\mathbb{Z}_{225}$ are generating the group? [4]

Q.10. Consider $\mathcal{P}(\Omega)$ with the addition and multiplication defined by [4]

$$A + B = A \cup B \quad \text{and} \quad A.B = A \cap B, \text{ for } A, B \subseteq \Omega.$$

Is $(\mathcal{P}(\Omega), \cup, \cap)$ a ring? If it is not a ring, write down the axiom(s) for a ring which is/are
failing to hold.

Q.11. On the set $\mathbb{Q}$ of rational numbers, we define addition and multiplication by [4]

$$a \oplus b = a + b + 11 \quad \text{and} \quad a \odot b = a + b + \frac{ab}{11}, \text{ for } a, b \in \mathbb{Q}.$$

What is the additive inverse? Is $\mathbb{Q}$ an integral domain? Prove or disprove it.

Q.12. Let $c_1, c_2 \in \mathbb{R}$. Let r_1 and r_2 be two distinct real solutions of $r^2 = c_1 r + c_2$. Let a_n be a [8]
solution of the recurrence relation $a_n = c_1 a_{n-1} + c_2 a_{n-2}$, for $n = 2, 3, \ldots$. Then show that
$a_n = \alpha_1 r_1^n + \alpha_2 r_2^n$ for $n = 0, 1, 2, \ldots$ for some constants α_1, α_2.
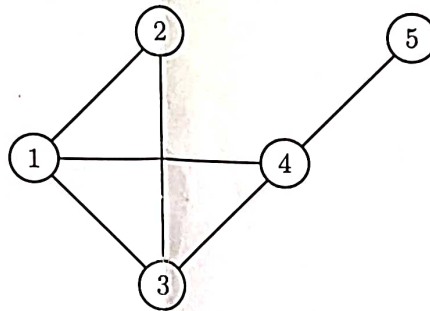
Quiz

Part - B (10 marks)

Answer ALL questions

1. Show that every loop-free connected planar graph has a vertex v with $deg(v) < 6$. [2M]

2. Find two non-isomorphic spanning trees for $K_{2,3}$. [2M]

3. By finding the chromatic polynomial of the graph G given below, calculate its chromatic number. [2M]

4. Let $G = (V, E)$ be a loop-free graph with $|V| = n \geq 2$. If for each $x, y \in V$, $x \neq y$,

$$deg(x) + deg(y) \geq n - 1,$$

then show that G has a Hamilton path. [4M]

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