INF 212
ANALYSIS OF PROG. LANGS
FUNCTION COMPOSITION

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Topics

- Recursion
- Higher-order functions
- Continuation-Passing Style
- Monads (take 1)
  - Identity Monad
  - Maybe Monad

Recursion

Prototypical Example

```
fact(n):
    if (n <= 1) then 1
    else n * fact(n-1)
```

Thinking Recursively

- Add numbers in a list
- Print a list of numbers
- Check if a number is in a list
- (Live coding)

Tail Recursion (first-order case)

- Function g makes a tail call to function f if return value of function f is return value of g
- Example

  tail call — not a tail call

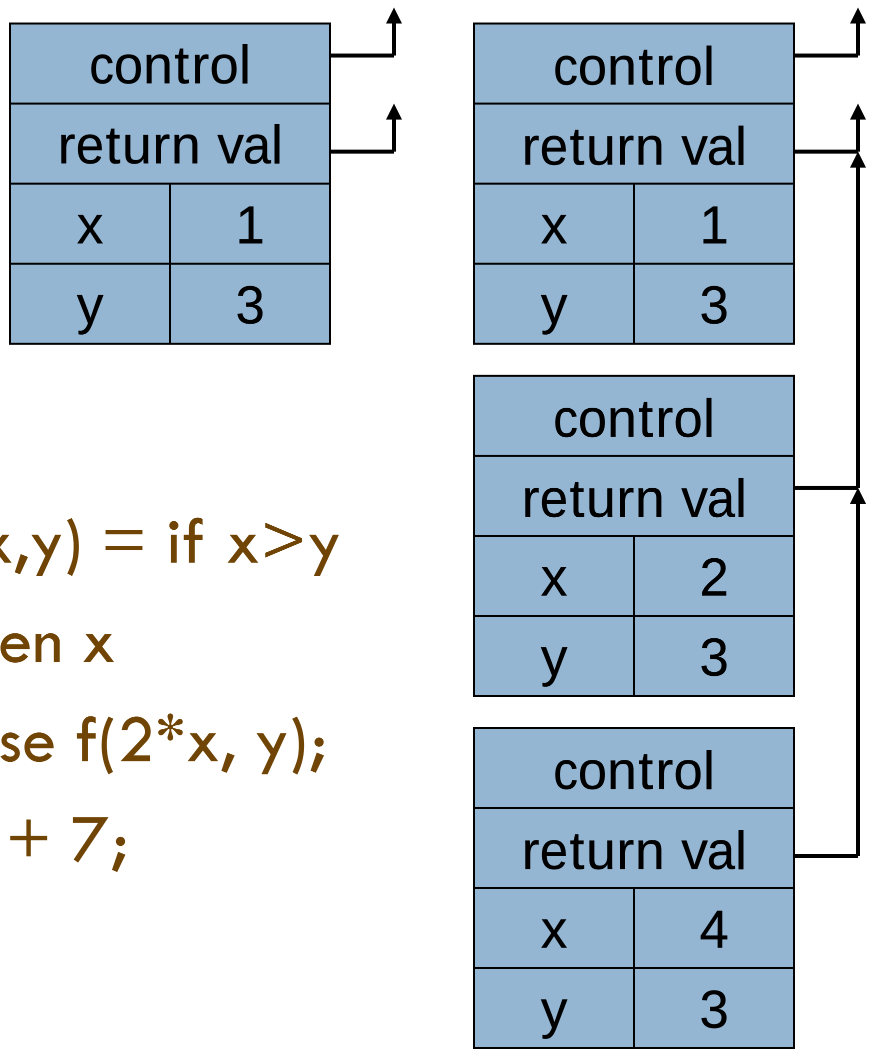
  fun g(x) = if x>0 then f(x) else f(x)*2

- Optimization: can pop current activation record on a tail call
  - Especially useful for recursive tail call because next activation record has exactly same form

Example of Tail Recursion

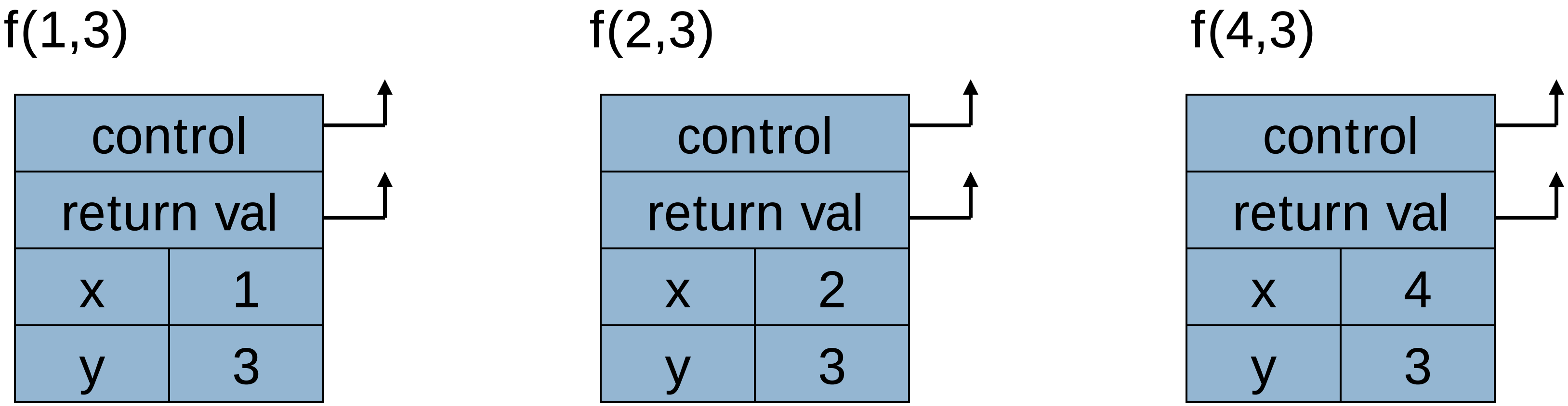
Calculate least power of 2 greater than y

f(1,3)




```
fun f(x,y) = if x>y
     then x
     else f(2*x, y);
f(1,3) + 7;
```

Tail Recursion Elimination

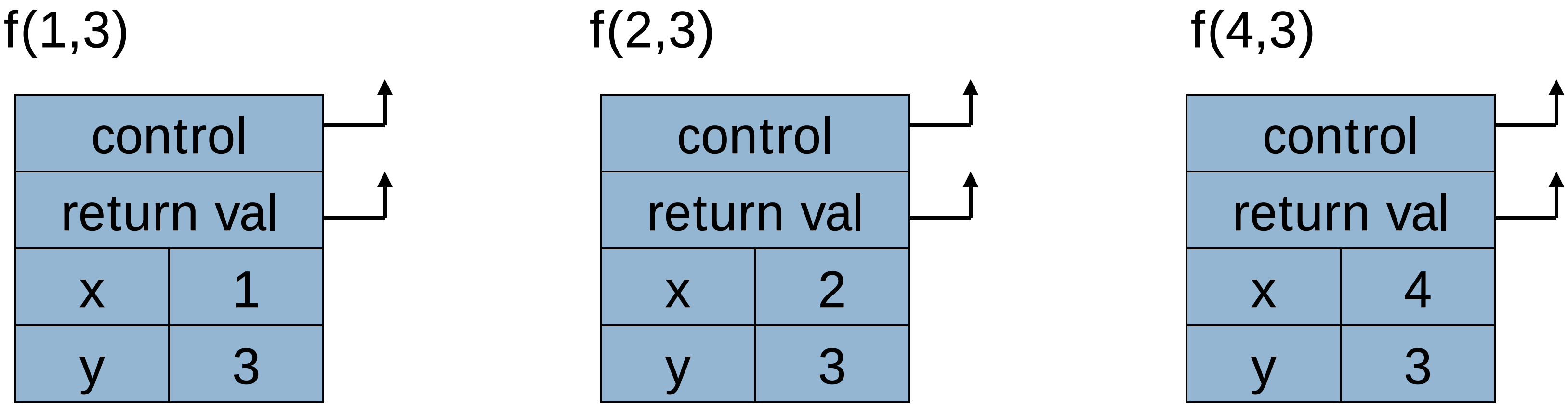



```
fun f(x,y) = if x>y
     then x
     else f(2*x, y);
f(1,3) + 7;
```

Optimization

- pop followed by push - reuse activation record in place
- Tail recursive function is equivalent to iterative loop

Tail Recursion and Iteration




```
fun f(x,y) = if x>y
      then x
      else f(2*x, y);
f(1,y);
```

test

loop body

initial value

```
function g(y) {
    var x = 1;
    while (!x>y)
        x = 2*x;
    return x;
}
```

Higher-order functions

Higher-Order Functions

- Function passed as argument
  - Need pointer to activation record "higher up" in stack
- Function returned as the result of function call
  - Need to keep activation record of the returning function
- Functions that take function(s) as input and return functions as output are known as <u>functionals</u>

Return Function as Result

- Language feature (e.g., Python, ML, …)
- Functions that return "new" functions
  - Example: `fun compose(f,g) = (fn x => g(f x));`
  - Function is "created" dynamically
    - Expression with free variables; values determined at run-time
  - Function value is closure = ⟨env, code⟩
  - Code <u>not</u> compiled dynamically (in most languages)
  - Need to maintain environment of the creating function

Closures

- Function value is pair closure = ⟨env, code ⟩
  - Statically scoped function must carry a link to its static environment with it
  - Only needed if function is defined in a nested block
- When a function represented by a closure is called…
  - Allocate activation record for call (as always)
  - Set the access link in the activation record using the environment pointer from the closure

Closures

- Function with free variables that are bound to values in the enclosing environment

(lambda (x)
(lambda (y)
x+y))

closure

```
def makeInc(x):
    def inc(y):
        # x is "closed" in the definition of inc
        return y + x
    return inc
```

Note to self: illustrate closures in Python and C# (my examples)

What are closures good for?

- For changing your mind later!
  - Replaces constants and variables with functions
  - Replaces conditionals
  - …

Return Function with Private State


```
fun mk_counter (init : int) =
    let  val count = ref init
         fun counter(inc:int) =
             (count := !count + inc; !count)
    in
         counter
    end;
val c = mk_counter(1);
c(2) + c(2);
```

- Function to “make counter” returns a closure
- How is correct value of count determined in c(2) ?

Implementing Closures

- Closures as used to maintain static environment of functions as they are passed around
- May need to keep activation records after function returns
  - Stack (last-in-first-out) order fails! (why?)
- Possible "stack" implementation:
  - Put activation records on heap
  - Instead of explicit deallocation, invoke garbage collector as needed

Continuations

Continuations

- Representation of the control state of a program
  - Data structure available to the programmer instead of hidden
  - Contains the current stack and point in the computation
- Can be later used to return to that point

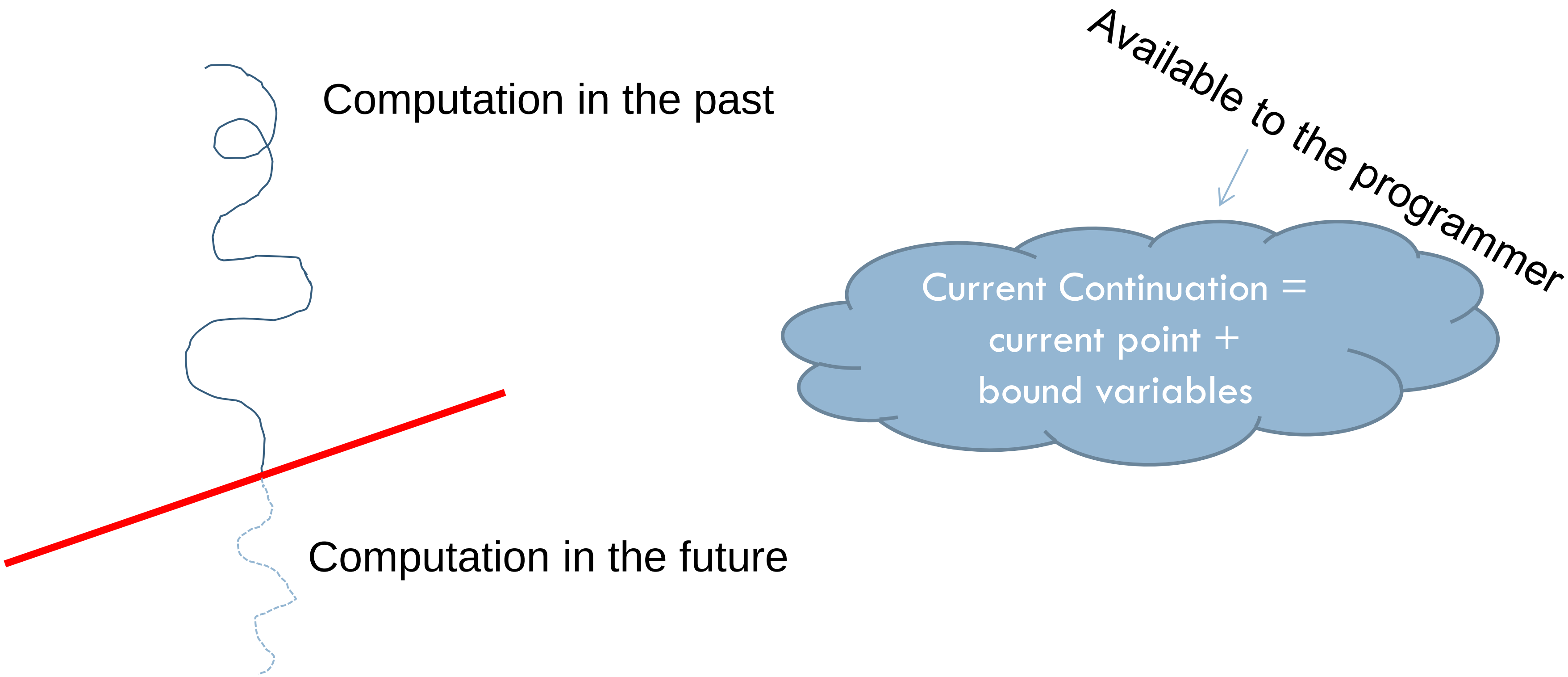
Remember Goto

```
A: blah
   blah
   if something GOTO A else GOTO B
B: ...
```

Flow control via textual labels mixes computation (beta-reductions) and representation (the text of the program)

Continuations continued

- Elegant concept for arbitrary flow control



Note to self: illustrate continuations in Scheme (Wikipedia)

What are continuations good for?

- Everything control flow!
  - Co-routines
  - Exceptions
  - Preserving flow
    in non-blocking I/O
      - (rhymes!)

The continuation nature of exceptions

```
function fact (n) {
  if (n < 0)
    throw "n < 0" ;
  else if (n == 0)
    return 1 ;
  else
    return n * fact(n-1) ;
}

function total_fact (n) {
  try {
    return fact(n) ;
  } catch (ex) {
    return false ;
  }
}

document.write("total_fact(10): " + total_fact(10)) ;
document.write("total_fact(-1): " + total_fact(-1)) ;
```

The continuation nature of exceptions – desugaring the previous slide

```
function fact (n,r,t) {
 if (n < 0)
   t ("n < 0")
 else if (n == 0)
   r(1)
 else
   fact(n-1,
        function (t0) {
          r (n*t0) ;
        },
        t)
}

function total_fact (n,ret) {
  fact (n,ret,
    function (ex) {
      ret(false) ;
    }) ;
}
```

```
total_fact(10, function (res) {
  document.write("total_fact(10): " + res)
}) ;

total_fact(-1, function (res) {
  document.write("total_fact(-1): " + res)
}) ;
```

I/O and continuations

Blocking (I/O in most systems)

```
contents = fs.ReadFile(path);
with contents do
    blah
```

Blocks here until we have the result

Non-blocking

```
contents = fs.ReadFileAsync(path);
with contents do
    blah
```

Uh-oh, we still don't have it

How to solve this?

I/O and continuations

Non-blocking

```
fs.ReadFileAsync(path, lambda(contents)
  {
        with contents do
          blah
  });
```

It's a callback!

It's the "current continuation" of the blocking form

JavaScript is FULL of this, so are jquery and node.js

Monads

Monads – what is the problem?

- The problem: how to affect the world
- Problem is more prevalent in pure functional programming style
  - No side-effects
  - That's right: no side-effects!
- But you've all seen it too!

No side effects?! Why?

- Easier to test: idempotent functions
- Easier to parallelize

- But the world is ALL about side-effects, right?
  - Storage, network, UI, …
  - Programs affect and control objects and activities in the real world

Example – a Tracing monad

```
def hypotenuse(x, y):
      return math.sqrt(math.pow(x, 2) + math.pow(y, 2))
```

Now we want to trace it, or affect the world in it:

```
def hypotenuse(x, y):
      h = math.sqrt(math.pow(x, 2) + math.pow(y, 2))
      print “In hypotenuse ” + h
      return h
```

Example – a Tracing monad

```
def hypotenuse(x, y):
        h = math.sqrt(math.pow(x, 2) + math.pow(y, 2))
        return h, “In hypotenuse” + h
```

Signature was	float, float -> float
Signature now is	float, float -> float, string

```
> math.pow(hypotenuse(6, 16), 4);
```

What is a monad?

- It’s a container
- An active container… it has behavior to:
  - Wrap itself around a [typed] value
  - Bind *functions* together

What is a monad?

- [A type constructor, m]
- A function that builds values of that type
  a -> m a (what you'd normally call a constructor in OOP)
- A function (bind) that combines values [of that type] with computations that produce values [of that type]
  m a -> (a -> m b) -> m b
- An unwrap function that shows "what's inside"