Buffer or Suffer

Inventory Management in SC under Uncertainty

Lect. delivered by

S P Sarmah

The role of safety inventory

- Forecasts are rarely completely accurate
- If average demand is 1000 units per week, then half the time actual demand will be greater than 1000, and half the time actual demand will be less than 1000; what happens when actual demand is greater than 1000?
- If you kept only enough inventory in stock to satisfy average demand, half the time you would run out
- **Safety inventory:** Inventory carried for the purpose of satisfying demand that exceeds the amount forecasted in a given period.

The role of safety inventory (Contd.)

- Average inventory is therefore cycle inventory plus safety inventory
- There is a fundamental tradeoff:
  - Raising the level of safety inventory provides higher levels of product availability and customer service
  - Raising the level of safety inventory also raises the level of average inventory and therefore increases holding costs
    - Very important in high-tech or other industries where obsolescence is a significant risk (where the value of inventory, such as PCs, can drop in value)

Important Question to Answer in Planning Safety Inventory

- What is the appropriate level of safety inventory to carry?

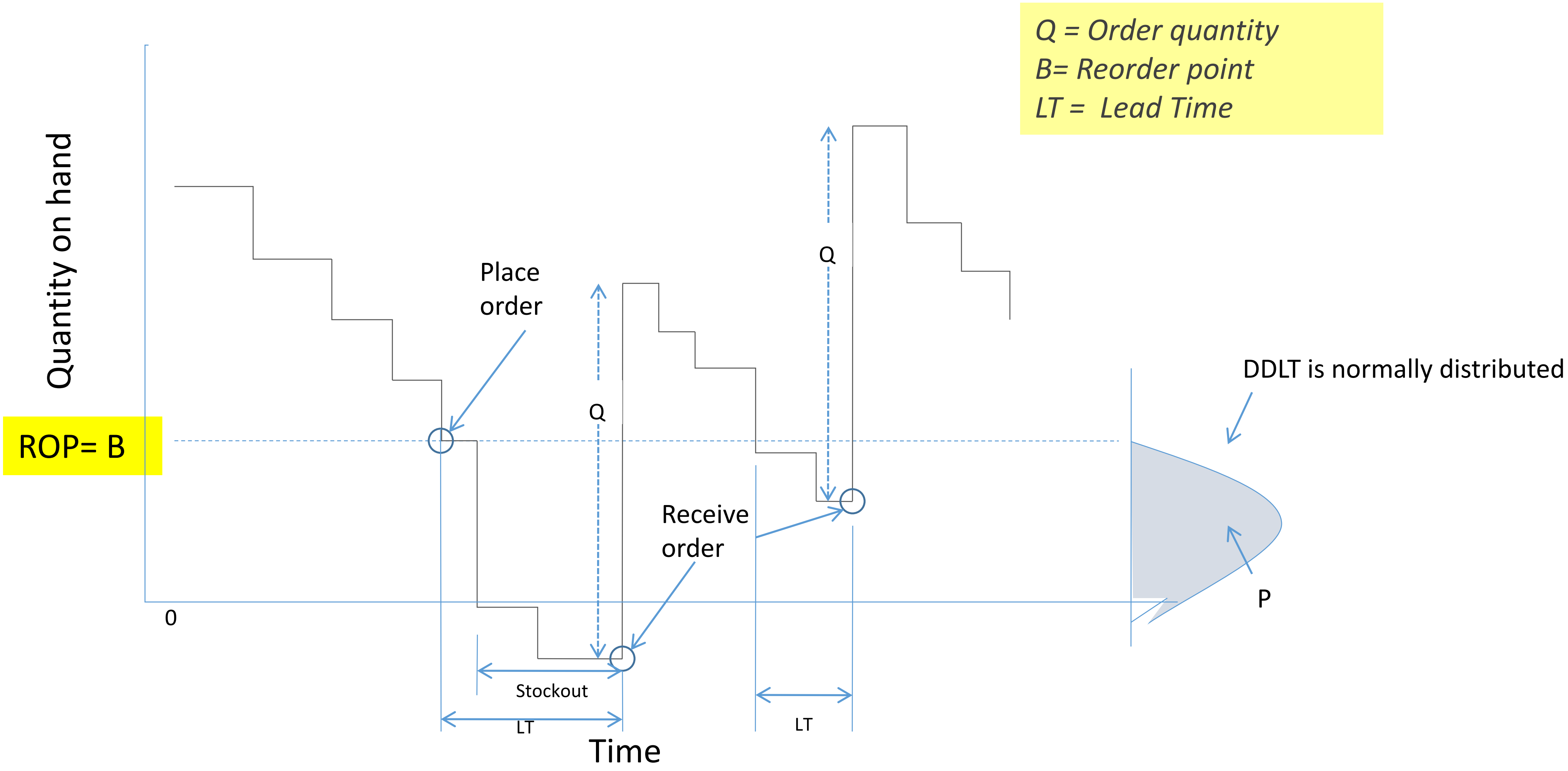
- Appropriate level of safety inventory is determined by:
  - supply or demand uncertainty
  - desired level of product availability
- Higher levels of uncertainty require higher levels of safety inventory given a particular desired level of product availability
- Higher levels of desired product availability require higher levels of safety inventory given a particular level of uncertainty

Types of Replenishment Policies

- Replenishment policy decisions regarding when to reorder and how much to reorder
- Continuous review: inventory is continuously monitored and an order of size Q is placed when the inventory level reaches the reorder point ROP
- Periodic review: inventory is checked at regular (periodic) intervals and an order is placed to raise the inventory to a specified threshold (the "order-up-to" level)
- Order upto level (S) is defined as demand over the lead time(D_L) plus demand over the review period (D_R).
- $S = D_L + D_R = D*L + D*R$ (For deterministic case)

Determination of Reorder Point (ROP) When demand is uncertain

Continuous Review Policy:



VARIABLE DEMAND AND CONSTANT LEAD TIME

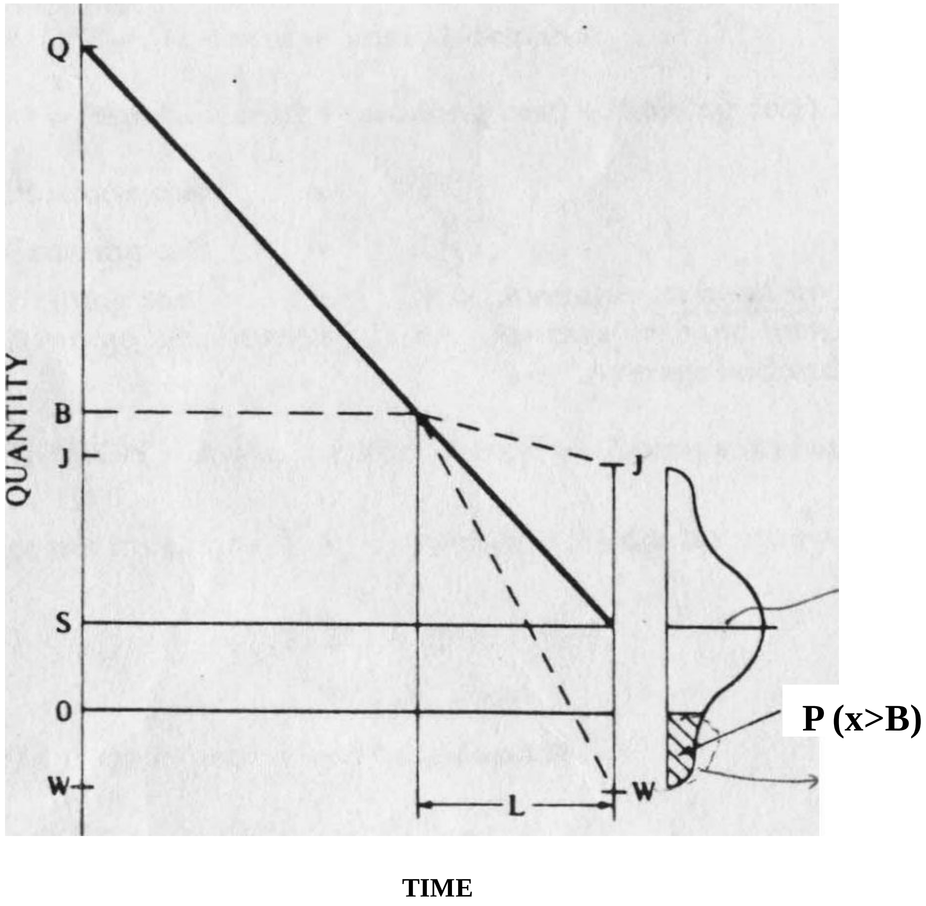
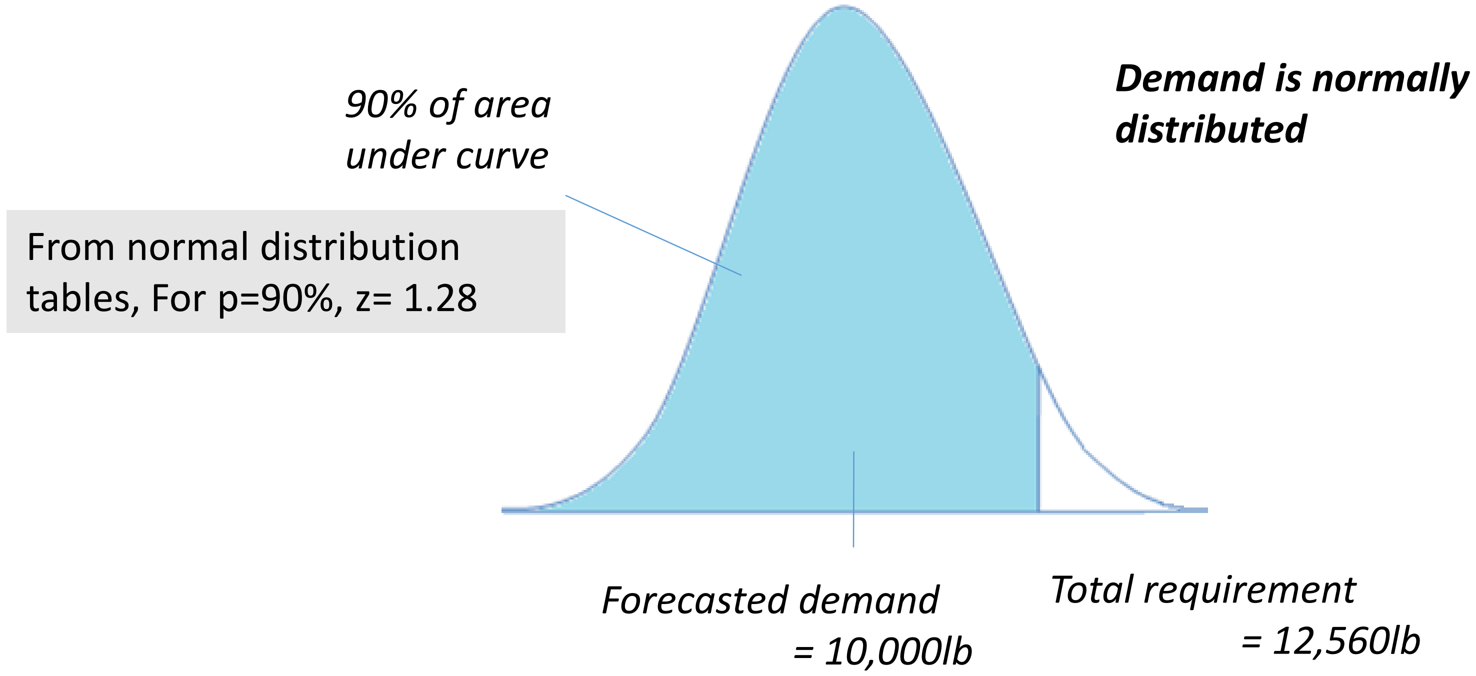


Figure Variable demand and constant lead time.

Key: B - Reorder point; *Q* - order quantity; *L* - constant lead time; O –S= safety stock; *B* - *S* - expected lead time demand; *B* - *J* - minimum lead time demand; *B* - *W* - maximum lead time demand; *P(x > B)* = probability of a stockout.

Example: Suppose the demand forecast of an item in a warehouse is 10000 units and the Forecast error i.e. Standard deviation of demand (s_D) is 2000 units. The manager wants to maintain 90% product availability. What is the requirement of the warehouse?



Area under forecasted distribution for warehouse

Total requirements = Forecast + (z* Forecast error)

Total requirements of warehouse = 10,000 + (1.28*2,000) = 12,560 units

Need of Safety Stock

Safety Stocks are needed as a hedge against

(i) Demand Uncertainty

(demand higher than forecasted)

(ii) Lead Time Uncertainty

(late delivery of goods)

Demand during lead time (DDLT)

Demand = D = 100 units/week

Standard deviation of demand = s_D *= 10 units (variance =* $s_D^2 = 10^2$*)*

Lead time = LT = 3 weeks

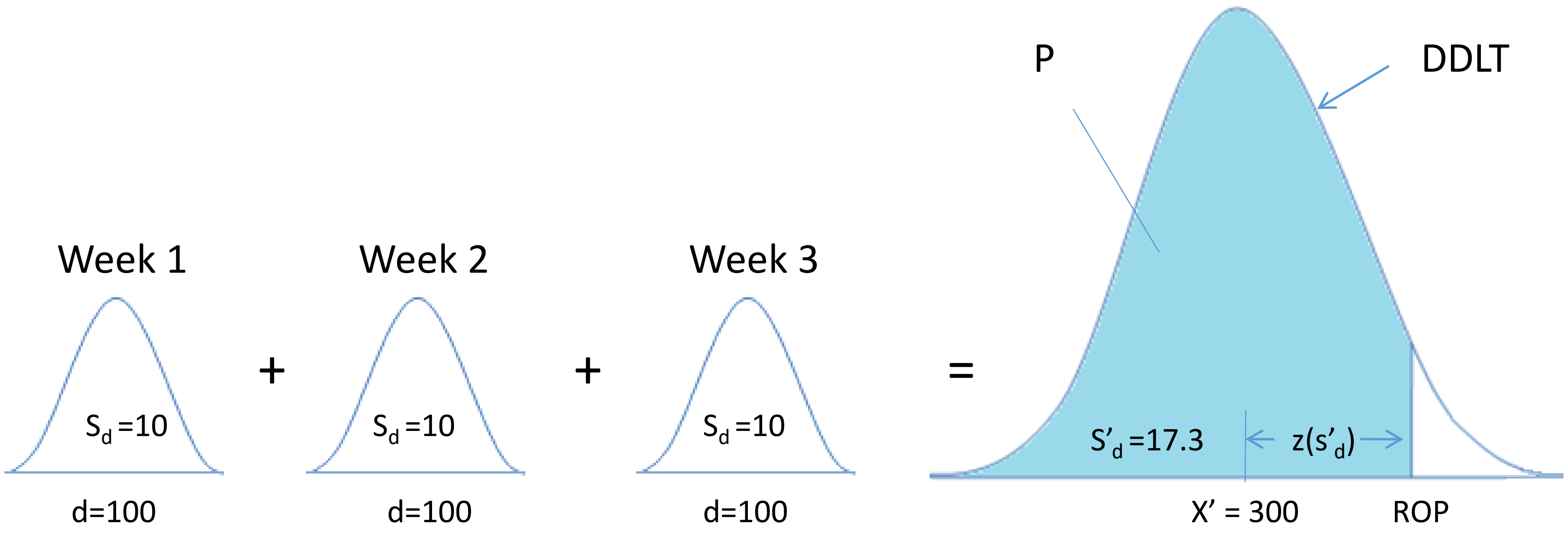
So,

*Mean demand during lead time (DDLT) = D*LT = 100 *3 = 300*

Variance of DDLT = $LT * s_D^2 = 3 * 10^2 = 300$

Standard deviation of DDLT = $s'_D = s_D\sqrt{LT} = \sqrt{300} = 10\sqrt{3} = 17.3$

Demand During Lead Time (DDLT) cond…



Methods for determination of Safety Stock

- Safety stock for the random demand can be determined based on customer service
- Again Customer service can be divided mainly into the following two categories
  - (a) Service per order cycle or Cycle service level
  - (b) Service per unit demanded or Item fill rate

Service per order cycle or Cycle service level

- This approach is not concerned with how large the shortage is but with how often it can occur.
- Cycle service level is defined as specified probability P_1 of no stock out per replenishment cycle.

Equivalently

P_1 is the fraction of cycles in which no stock out takes place. Thus Cycle service level

$P_1 = 1 - \text{Probability of Stock out}$

Assuming demand distribution normal, one can determine the value of safety factor from normal distribution table

Example: Cycle Service level

- Suppose forecasted system generate following values.

  Mean demand during lead time = 58.3 units

  Standard Deviation of demand during lead time = s_D = 13.1 units

  Management desired a cycle service level P_1 = 0.9

  Determine the safety stock and reorder point.

We know, P_1 = 1 – Probability of Stock out

Probability of Stock out = 0.1

From normal distribution table, z = 1.30

Thus safety Stock = z s_D = 1.30*13.1 = 17.03

Reorder Point = Mean demand during lead time + Safety Stock

ROP = 58.3 + 17.03 = 75.33 units

Service per unit demanded or Item Fill rate

- Specified fraction (P_2) of customer of demand to be satisfied directly from the shelf is called Item fill rate.
- This term of service has considerable appeal for practitioners.
- Again for determination of fill rate, following two cases can be considered
  - (a) Complete Backordered
  - (b) Complete lost sales case

Item Fill Rate : complete backordered

Let us consider for the complete back order case.

Each replenishment is of size Q

Fraction back ordered = (Expected Shortage per replenishment cycle *Total no of replenishment Cycle)/ Total unit Demanded.

Thus fraction of demand satisfied directly from the shelf

P_2 = 1 – Fraction backordered

$$P_2 = 1 - \frac{\left(\frac{D}{Q}\right) s'_D (E_{(z)})}{D}$$

$$P_2 = 1 - \frac{s'_D (E_{(z)})}{Q}$$

E (z) = Unit normal loss integral or Standardized stock out quantity for a standard normal distribution

S'_D E (z) = Expected no of units out of stock during an order cycle

$$E(z) = \frac{Q}{s_D^{'}}\left[1 - P_2\right]$$

- If the Item fill rate P_2 is specified, management can find the value of safety factor z. The value of E(z) can be obtained from table.

Reorder point method with Both Demand and Lead time Uncertainty

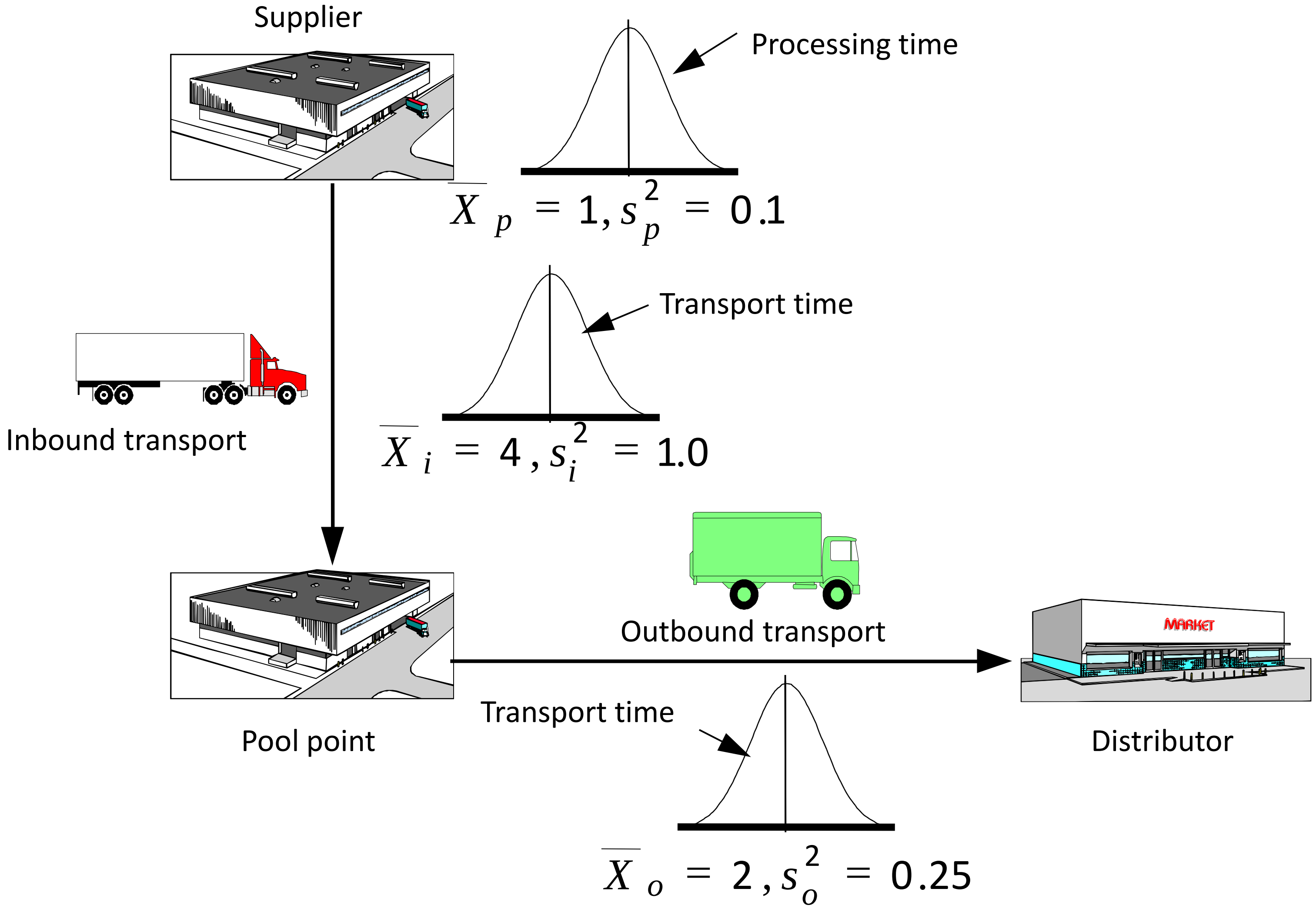
Standard deviation of demand due to variation in demand and lead time is given by

(s_D') = $\sqrt{LT(s_D^2) + D^2(s_{LT}^2)}$

S_{LT} - Lead time Standard deviation

Important Inference: *The impact on Standard deviation of demand during lead time is more due to the second component as both are squared. Normally, we donot have control on variability associated with demand. But we can control the variability in lead time by selecting nearby supplier and thereby improve the performance*

Example: Suppose inventory is to be maintained on a distributor's self for an item whose demand forecast d = 100 units per day and s_d = 10 units per day. Continuous review (ROP) is the method of inventory control. There are multiple points through out the supply channel where time is incurred in the product flow between source point and customer as shown in the **fig.** No significant inventory is maintained at pool point or in trucks. I = 10%; S = $ 10/order; C = $5/unit; P = 0.99; Determine avg. inventory at distributor.



Example cond…

Solution The reorder point inventory theory applies. However, determining the statistics of the demand-during-lead-time distribution requires taking the lead-time for the entire channel into account.

Recall,

$$s' = \sqrt{LT(s_d^2) + d^2(s_{LT}^2)}$$

where

$$\begin{aligned} s_{LT}^2 &= s_p^2 + s_i^2 + s_o^2 \\ &= 0.1 + 1.0 + 0.25 \\ &= 1.35 \text{ days} \end{aligned}$$

Example cond…

Average lead time

$$LT = \overline{X}p + \overline{X}i + \overline{X}o = 1 + 4 + 2 = 7 \text{ days}$$

Now

$$s' = \sqrt{7x10^2 + 100^2 x1.35} = \sqrt{14{,}200} = 119.16 \text{ days}$$

and

$$Q^* = \sqrt{\frac{2(100)(10)}{0.1(5)}} = 63 \text{ units}$$

$$AIL = \frac{Q^*}{2} + z(s') = \frac{63}{2} + 2.33(199.16) = 309 \text{ units}$$

Advantages and Disadvantages of Continuous review Model:

Advantage:

- Lowest total relevant cost

Disadvantage:

- Constant monitoring
- Missing joint
  - Order production
  - Transportation
  - Buying Economies

Can be overcome using Periodic Review Model

THANKS