Managing Inventory In SC

Lect delivered by

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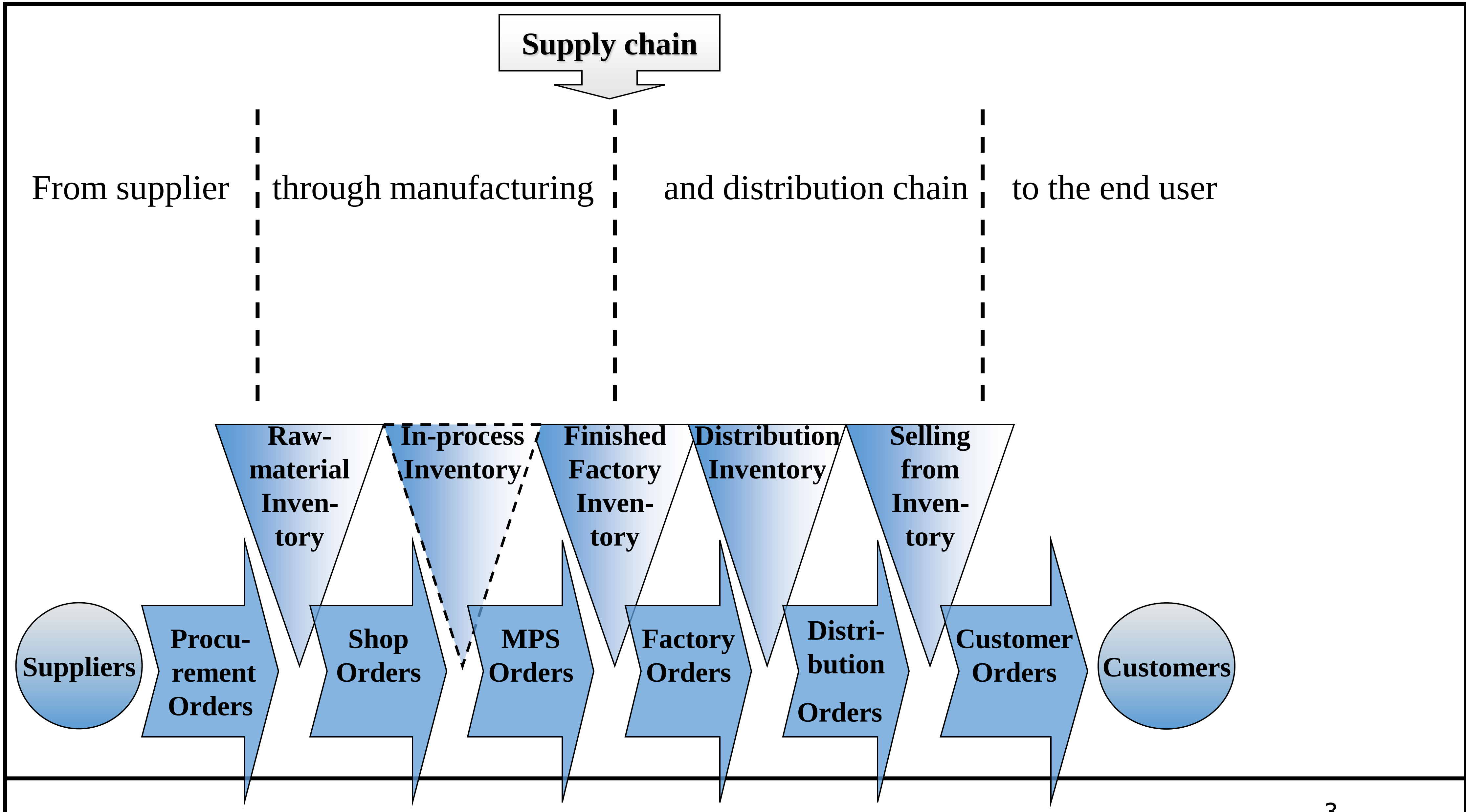
Introduction

Inventory is a material held in an idle or incomplete state awaiting future Sale or use.
Inventories are common to all organizations including:

- Farms
- Manufactures
- Retailers
- Hospitals
- Universities
- Families, etc

Inventory at different stages in Supply Chain



- Inventory may have significant impact on

    -Customer Service level &

    -Supply Chain system wide cost

TYPES OF INVENTORY MAINTAINED BY A MANUFACTURING ORGANIZATION

- **Supplies:** Inventory items consumed in the normal functioning that are not part of the final product.
- **Raw Materials:** Items purchased from suppliers to be used as inputs into the production process.
- **In-Process Goods:** Partially completed final products that are still in the production process.
- **Finished Goods:** Final product, available for sale, distribution, or storage.

- Why hold inventory at all?
  - Due to mismatch between demand and supply.
  - Unexpected changes in customer demand. Uncertainty in customer demand has increased due to short life cycle of product.
  - Historical data about customer may not be available
  - Economies of scale offered by transportation also encourage firms to transport large quantities of items & therefore, hold large inventories
  - A significant uncertainty in quality of the supply, uncertainty with supplier cost and delivery lead time

INVENTORY COSTS

- **Purchase Cost (P):** It is the unit purchase price if it is obtained from an external source, or the unit production cost if it is produced internally.
- **Order/Setup Cost (C):** Expense incurred in issuing a purchase order to an outside supplier or from internal production setup costs.
- **Holding Cost (H):** The cost associated with investing in inventory and maintaining the physical investment in storage.

  It incorporates such items as cost of capital, taxes, insurance, handling, storage, shrinkage, obsolescence, and deterioration.

  On an annual basis, they most commonly range from 20% to 40% of the investment.
- **Stockout Costs:** It is the economic consequence of an external or internal shortage. External shortage can incur backorder costs, present profit loss and future profit loss. Internal shortages can result in lost production and a delay in a completion date.

Economies of Scale to Exploit Fixed Costs

- Lot sizing for a single product (EOQ)
- Aggregating multiple products in a single order
- Lot sizing with multiple products or customers
  - Lots are ordered and delivered independently for each product
  - Lots are ordered and delivered jointly for all products
  - Lots are ordered and delivered jointly for a subset of products

Lot sizing for a single product: Economic Order Quantity Model (EOQ)

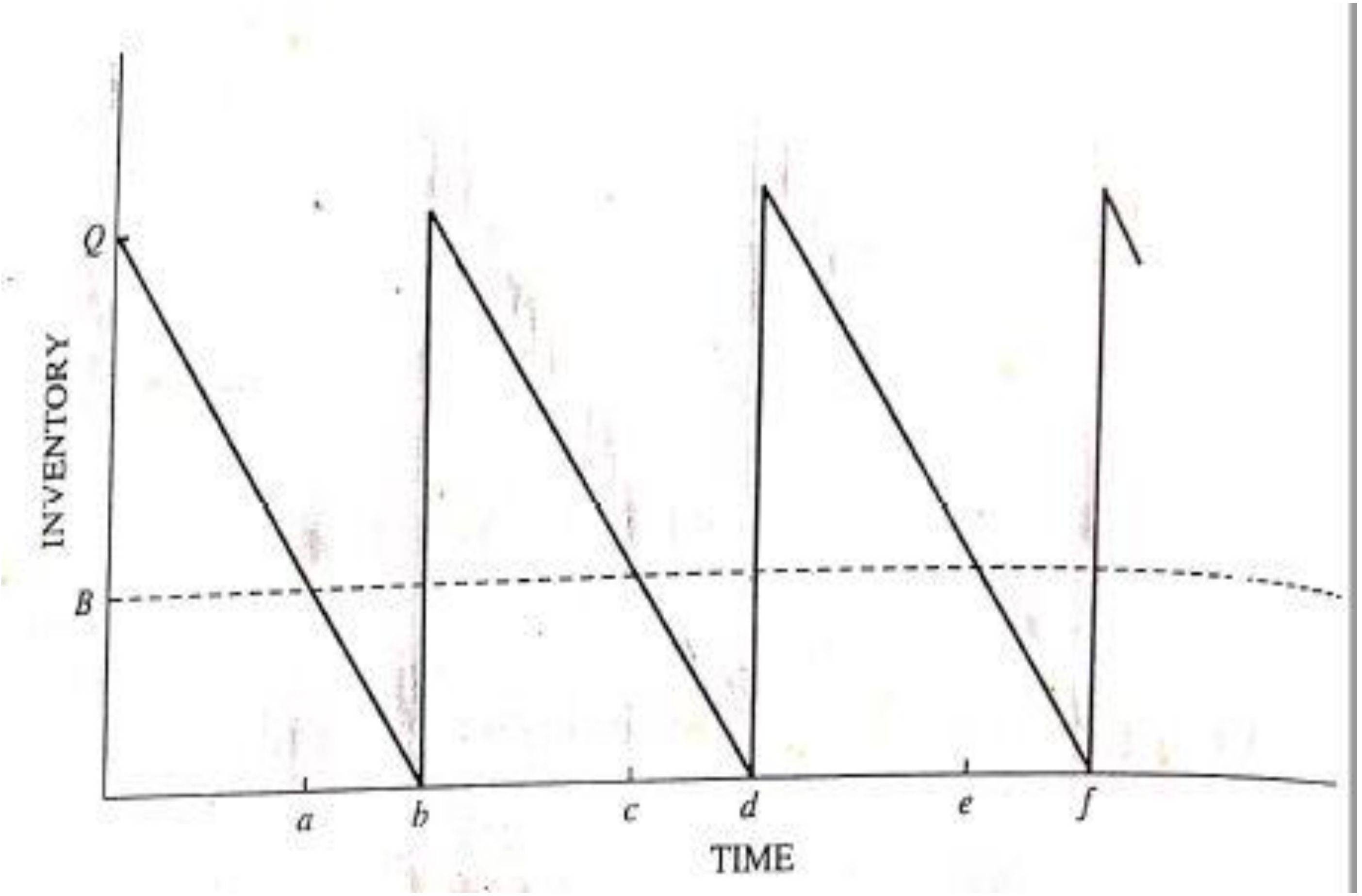


Fig. **Classical Inventory Model**

$$Q = lot\ size; Q/2 = average\ inventory; B = reorder\ point$$

$$ac = ce = interval\ between\ orders; ab = cd = elead\ time$$

Role of Cycle Inventory in a Supply Chain

- Lot, or batch size: quantity that a supply chain stage either produces or orders at a given time
- Cycle inventory: Average inventory that builds up in the supply chain because a SC stage either produces or purchases in lots that are larger than those demanded by the customer
  - Q = lot or batch size of an order
  - d = demand per unit time
- Inventory profile: Plot of the inventory level over time as shown in the fig.

- Cycle inventory = Q/2 (depends directly on lot size)
- Average flow time = Avg inventory / Avg flow rate
- Average flow time from cycle inventory = Q/(2d)

Role of Cycle Inventory in a Supply Chain

Suppose

Lot Size Q = 1000 units

Demand d = 100 units/day

Cycle inventory = Q/2 = 1000/2 = 500 = Avg inventory level from cycle inventory

Avg flow time = Q/2d = 1000/(2)(100) = 5 days

- Cycle inventory adds 5 days to the time a unit spends in the supply chain
- Lower cycle inventory is better because:
  - Average flow time is lower
  - Working capital requirements are lower
  - Lower inventory holding costs

Role of Cycle Inventory in a Supply Chain

- Cycle inventory is held primarily to take advantage of economies of scale in the SC
- SC costs influenced by lot size:
  - Purchase cost = P
  - Fixed ordering cost = A
  - Holding cost = H = PF (where F is the HC fraction expressed as percentage of Unit Purchase cost)
- Primary role of cycle inventory is to allow different stages to purchase product in lot sizes that minimize the sum of Purchase, Ordering, and Holding costs
- Ideally, cycle inventory decisions should consider costs across the entire supply chain, but in practice often, each stage generally makes its own supply chain decisions – increases total cycle inventory and total costs in the supply chain

Economies of Scale to Exploit Fixed Costs: Lot sizing for a single product (EOQ)

D = Annual demand (Unit per year)
A = Ordering cost (Rs per order) Independent of order size (Fixed Cost)
H = Inv. Carrying cost (Rs.per unit per year): Dependent on Order Size
P = Unit Purchase price (Rs per unit)
F = Fractional holding cost of unit purchase price
H = PF
Q = Lot size (Decision Variable)

Total cost = Purchase cost + Ordering cost + Holding cost

$$TC(Q) = DP + \frac{D}{Q}A + \frac{Q}{2}H$$

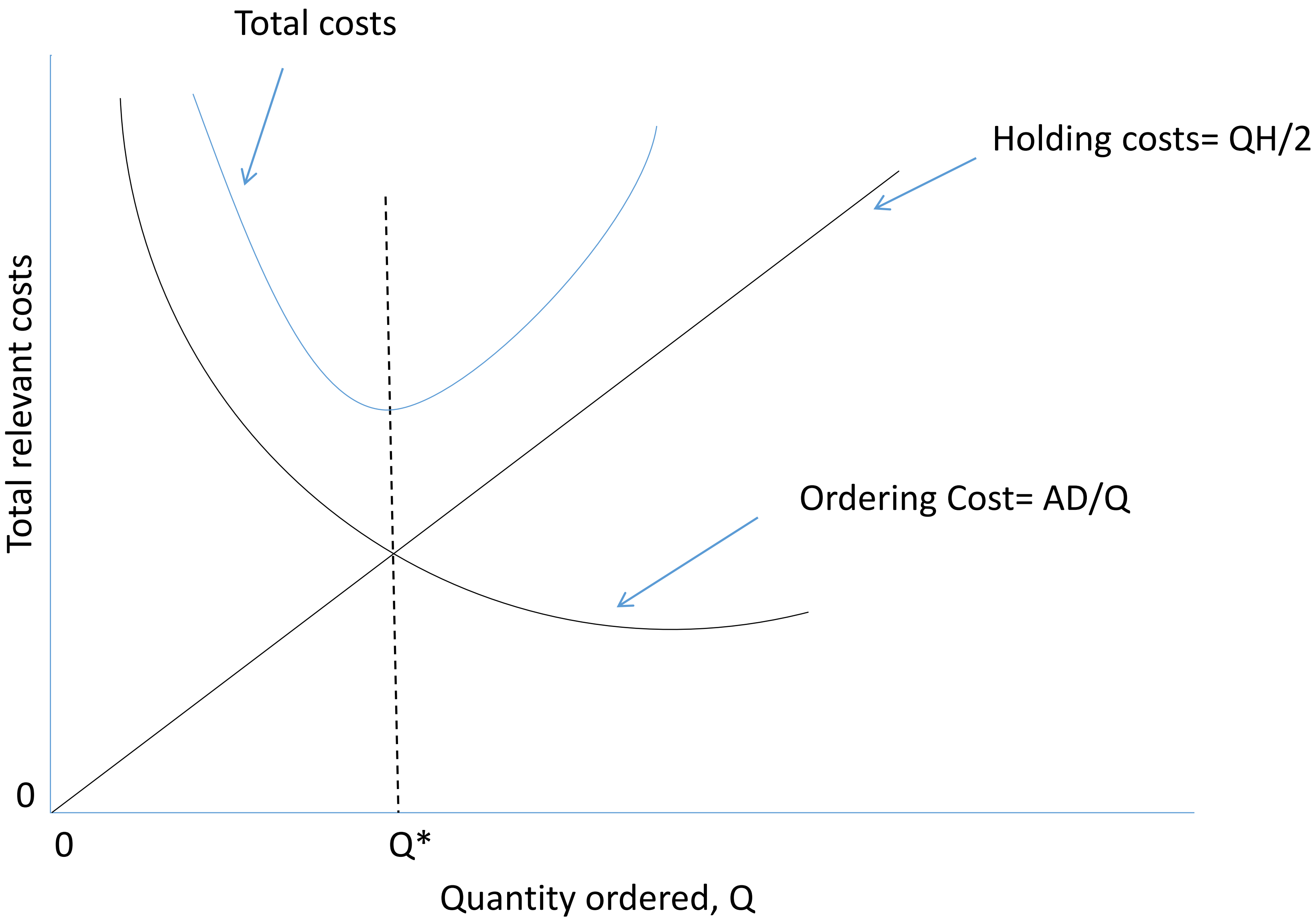
$$\frac{d(TC)}{dQ} = 0, we get$$

$$Q^* = \sqrt{\frac{2AD}{H}} = \sqrt{\frac{2AD}{PF}}$$

$$Q^* = \sqrt{\frac{2 \times Ordering\ \text{Cost x Annual demand}}{\text{Inventory holding Cost}}}$$

Note : As unit price of the item increases, lot size decreases. It means order frequently in smaller lot size. This is applicable for high valued items.

Optimum Order Lot Size: Trade off Between OC &HC



Reorder point $B = \frac{DL}{12}$, when lead time is in month (as both units should be same)

No. of orders = $\frac{D}{Q*}$

Order interval time T = $\frac{Q*}{D}$

Total Min. cost $= DP + \frac{AD}{Q*} + \frac{Q*}{2}H$

$$TC^* = \sqrt{2ADH} = \sqrt{2ADPF}$$

Example:

Demand, D = 12,000 computers per year

d = 1000 computers/month

Unit cost, P = $500

Holding cost fraction, F = 0.2

Fixed cost, A = $4,000/order

Q* = Sqrt[(2)(12000)(4000)/(0.2)(500)] = 980 computers

Cycle inventory = Q/2 = 490

Flow time = Q/2d = 980/(2)(1000) = 0.49 month

Reorder interval, =Q/d= T = 0.98 month

Example (continued)

Annual ordering and holding cost =TC=

= (12000/980)(4000) + (980/2)(0.2)(500) = $97,980

Suppose lot size is reduced to Q=200, which would reduce flow time:

Annual ordering and holding cost =

= (12000/200)(4000) + (200/2)(0.2)(500) = $250,000

To make it economically feasible to reduce lot size, the fixed cost associated with each lot would have to be reduced

Example

If desired lot size = Q^* = 200 units, what would 'A' have to be?

D = 12000 units

P = $500

F = 0.2

Use EOQ equation and solve for S:

$A = [PF(Q^*)^2]/2D = [(0.2)(500)(200)^2]/(2)(12000)$

$= \$166.67$

To reduce optimal lot size by a factor of k, the fixed order cost must be reduced by a factor of k^2

Key Points from EOQ Model

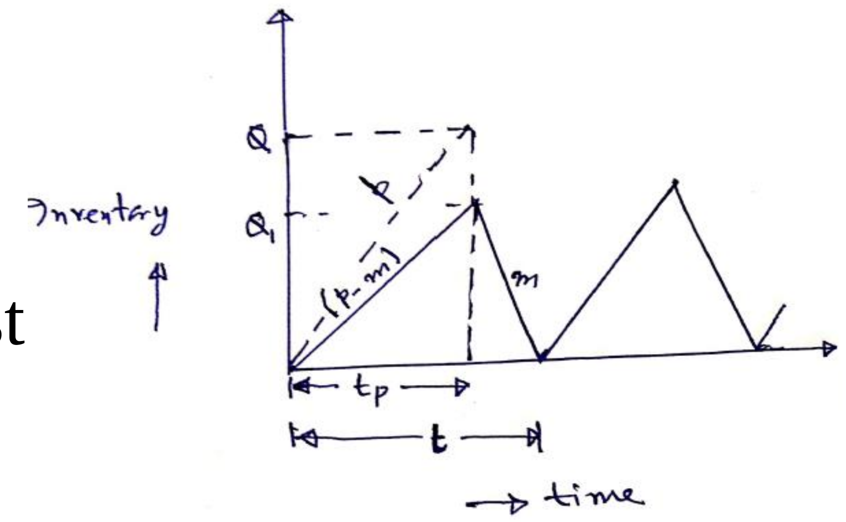
- In deciding the optimal lot size, the tradeoff is between setup (order) cost and holding cost.

- If demand increases by a factor of 4, it is optimal to increase batch size by a factor of 2 and produce (order) twice as often. *Cycle inventory (in days of demand) should decrease as demand increases*.

- If lot size is to be reduced, one has to reduce fixed order cost. To reduce lot size by a factor of 2, order cost has to be reduced by a factor of 4.

Economic Production Quantity Model (EPQ)

- D- Annual demand
- p – production rate
- m – demand rate
- P- unit variable cost
- H- Inventory holding cost



Objective – What is the optimum production quantity to minimize the total cost in the system?

Time between production run = t_p

Production run quantity = $Q = t_p \times p$

$$t_p = \frac{Q}{p}$$

Max inventory = $Q_1 = t_p(p - m)$

Average inventory $= \frac{Q_1}{2} = \frac{1}{2} t_p (p - m)$

$$= \frac{1}{2} \frac{Q}{P} (p - m)$$

$$= \frac{1}{2} Q\left(1 - \frac{m}{p}\right)$$

Total annual cost (TAC) = $DP + \frac{AD}{Q} + \frac{1}{2}\frac{Q}{p}(p-m)H$

$$\frac{d(TAC)}{dQ} = 0$$

$$Q^* = \sqrt{\frac{2AD}{H\left(1 - \frac{m}{p}\right)}}$$

Note – When production rate is infinite i.e $p \to \infty$

$Q = \sqrt{\frac{2AD}{H}}$ (same as or EOQ model)

Total minimum cost

$$(TC)_{Q*} = DP + \frac{AD}{Q*} + \frac{Q*}{2}\left(1 - \frac{m}{p}\right)H$$

$$(TC)_{Q*} = DP + \sqrt{2ADH\left(1 - \frac{m}{p}\right)}$$

Exercise: An item may be purchased for Rs 25 per unit or manufactured at a rate of 10,000 units per year for Rs. 23. if purchased, the order cost will be Rs. 5 compared to a Rs. 50 set up cost for manufacturing

Annual demand D = 2500 units per year; Holding cost fraction F = 10%

Should the item be purchased externally or produced internally ?

When the item is purchased from outside

$$EOQ = Q^* = \sqrt{\frac{2AD}{PF}} = \sqrt{\frac{2\times 5\times 2500}{0.1\times 25}}$$

$$Q^* = 100$$

$$(TAC)_{Q^*} = DP + \sqrt{2ADPF}$$

$$= 2500\times 25 + \sqrt{2\times 5\times 2500\times 25\times 0.1}$$

$$(TAC)_{Q^*} = 62750 \ \ Rs.$$

When the item is produced internally

$$EPQ \Rightarrow Q^* = \sqrt{\frac{2AD}{PF\left(1-\frac{m}{p}\right)}}$$

$$= \sqrt{\frac{2\times 50\times 2500}{23\times 0.1\times\left(1-\frac{2500}{10000}\right)}}$$

$$Q^* = 380$$

$$(TAC)_{Q^*} = DP + \sqrt{2ADPF\left(1-\frac{m}{p}\right)}$$

$$= 58156.7 \ \ Rs.$$

$$(TAC)_{\Pr oduction} < (TAC)_{Purchase}$$

Any Question?

Aggregating Multiple Products in a Single Order

- Transportation is a significant contributor to the fixed cost per order
- One can possibly combine shipments of different products from the same supplier
  - same overall fixed cost
  - shared over more than one product
  - effective fixed cost is reduced for each product
  - lot size for each product can be reduced
- One can also have a single delivery coming from multiple suppliers or a single truck delivering to multiple retailers
- Aggregating across products, retailers, or suppliers in a single order allows for a reduction in lot size for individual products because fixed ordering and transportation costs are now spread across multiple products, retailers, or suppliers

Example: Multiple products

Company is dealing four types of computer model and demand of each one is given below

Demand, D_i = 12,000 of each type of computers per year

d_i = 1000 computers/month

Unit cost, P = $500

Holding cost fraction, F = 0.2

Fixed cost, A = $4,000/order

Q* = Sqrt[(2)(12000)(4000)/(0.2)(500)] = 980 computers

Example: Aggregating Multiple Products in a Single Order

- If each product is ordered separately:
  - Q* = 980 units for each product
  - Total cycle inventory = 4(Q/2) = (4)(980)/2 = 1960 units
- Aggregate orders of all four products:
  - Combined Q* = 1960 units
  - For each product: Q* = 1960/4 = 490
  - Cycle inventory for each product is reduced to 490/2 = 245
  - Total cycle inventory = 1960/2 = 980 units
  - Average flow time, inventory holding costs will be reduced

Lot Sizing with Multiple Products or Customers

- In practice, the fixed ordering cost is dependent at least in part on the variety associated with an order of multiple models
  - A portion of the cost is related to transportation (independent of variety)
  - A portion of the cost is related to loading and receiving (**<u>not</u>** independent of variety)
- Three scenarios:
  - Lots are ordered and delivered independently for each product
  - Lots are ordered and delivered jointly for all three models
  - Lots are ordered and delivered jointly for a selected subset of models

Lot Sizing with Multiple Products

- Demand per year
  - D_L = 12,000; D_M = 1,200; D_H = 120
- Common transportation cost, A = \$4,000
- Product specific order cost
  - A_L = \$1,000; A_M = \$1,000; A_H = \$1,000
- Holding cost fraction, F= h = 0.2
- Holding cost H = FP=hP
- Unit purchase cost
  - P_L = \$500; P_M = \$500; P_H = \$500

Delivery Options

- No Aggregation: Each product ordered separately
- Complete Aggregation: All products delivered on each truck
- Tailored Aggregation: Selected subsets of products on each truck

No Aggregation: Order Each Product Independently

	Litepro	*Medpro*	*Heavypro*
Demand per year	**12,000**	**1,200**	**120**
Fixed cost / order	**$5,000**	**$5,000**	**$5,000**
Optimal order size	**1,095**	**346**	**110**
Order frequency	**11.0 / year**	**3.5 / year**	**1.1 / year**
Annual cost	**$109,544**	**$34,642**	**$10,954**

Total cost = $155,140

Aggregation: Order All Products Jointly

All three models are included each time an order is placed

The combined fixed order cost per order is given by

$A^* = A + A_L + A_M + A_H = 4000+1000+1000+1000 = \$7000$

Suppose n be the number of orders placed per year.

Total annual Ordering Cost = n A*

Total annual HC = $D_L F P_L/2n + D_M F P_M/2n + D_H F P_H/2n$

TC = Ordering Cost + Holding Cost

$n^* = \text{Sqrt}[(D_L F P_L + D_M F P_M + D_H F P_H)/2A^*]$

= 9.75

$Q_L = D_L/n^* = 12000/9.75 = 1230$

$Q_M = D_M/n^* = 1200/9.75 = 123$

$Q_H = D_H/n^* = 120/9.75 = 12.3$

Cycle inventory = Q/2

Average flow time = (Q/2)/(weekly demand)

Complete Aggregation: Order All Products Jointly

	Litepro	*Medpro*	*Heavypro*
Demand per year	**12,000**	**1,200**	**120**
Order frequency	**9.75/year**	**9.75/year**	**9.75/year**
Optimal order size	**1,230**	**123**	**12.3**
Annual holding cost	**$61,512**	**$6,151**	**$615**

Annual order cost = 9.75 × $7,000 = $68,250

Annual total cost = $136,528

Lessons from Aggregation

- Aggregation allows firm to lower lot size without increasing cost
- Complete aggregation is effective if product specific fixed cost is a small fraction of joint fixed cost
- Tailored aggregation is effective if product specific fixed cost is a large fraction of joint fixed cost

Any Question?

Little’s Law for measurement of performance of a system

Introduction

- For any process, three fundamental performance measures are Inventory, flow time and flow rate.
- Suppose patients are entering in a radiology department for check up. The arrival and departure time is as follows.

Patient Seriel No.	Arrival Time	Departure Time	Flow Time
1	7:35	8:50	1:15
2	7:45	10:05	2:20
3	8:10	10:10	2:00
4	9:30	11:15	1:45
5	10:15	10:30	0:15
6	10:30	13:35	3:05
7	11:05	13:15	2:10
8	12:35	15:05	2:30
9	14:30	18:10	3:40
10	14:35	15:45	1:10
11	14:40	17:20	2:40

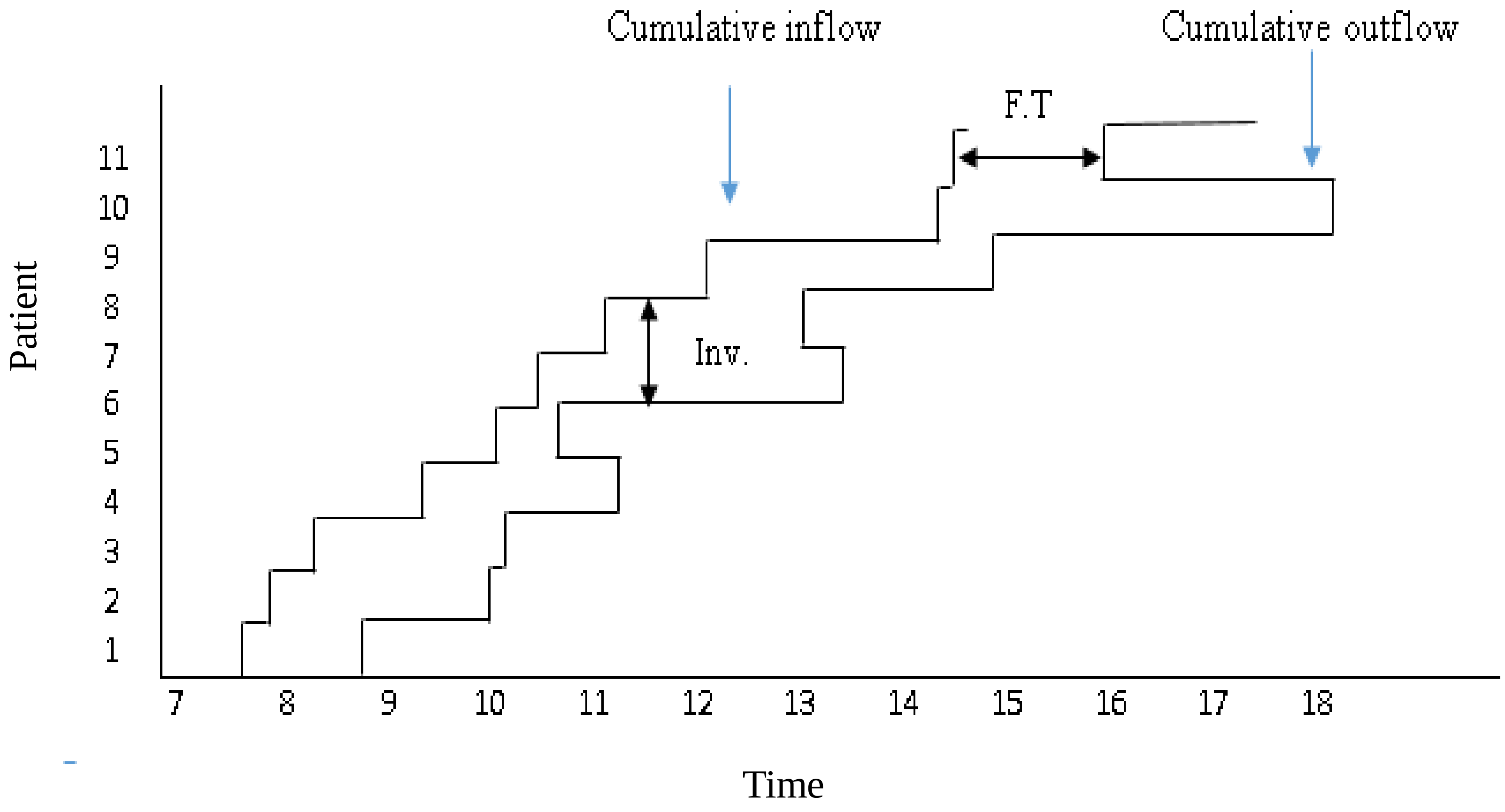
Total hours=22 hrs 50 min

Average flow time= 2 hrs 4 min 33 sec

Little's Law states that

Average Inventory=Average flow rate * Average flow time

The clinic is open for 11 hours



A brute force to compile average inventory is to count inventory at every moment in time through out the day say every five minutes and then take the average.

Little's law is useful in finding the third performance measure that is average flow time when two others are known.

For example, if you want to find out how long the patient in the radiological centre will spend waiting for their x ray. Follow the steps as below.

Step 1:
Observe the inventory of patients at a couple of random points during the day.
This gives an average inventory.
Step 2:
From the record, check how many patients were treated that day. This is the day's output.
From day's output find average output per hour.

Step 3:

Now use Little's law to compute

Average Inventory=Average flow rate * Average flow time

Flow time= Inventory/Flow rate

This gives an average a patient has to wait in the system.

Little's law hold always.

It does not depend on the sequence in which the flow units are reserved (But the sequence could influence the flow time of a particular flow unit)

Furthermore, Little's law does not depend on randomness.

It does not matter if there is variability in the number of patients or how long treatment takes for patient.

All that matter is the average flow rate and average flow time.

- Inventory turn & Inventory cost

Sales from the organization within a certain time can be considered as flow rate.

Flow rate= Sales within a certain time = cost of goods sold per year

= Rs. 26,258 per year (say)

Inventory = Rs. 4825 (In terms of value)

From Little's law,

Flow time=Inventory/Flow rate

= 0.18 years

= 67 day

Thus an average, the organization needs 67 days to translate a rupee investment into a rupee of profitable revenue.

Inventory turn=1/Flow time

THANKS