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An Adaptive Pre-distortion Method Based on Orthogonal Polynomials

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Abstract: Wideband signals without pre-distortion are distorted by memory and nonlinear effect of high power amplifier. To compensate these distortions, we propose an adaptive pre-distorter based on orthogonal polynomials. Based on orthogonal criterion, we derive a series of orthogonal polynomials under circular symmetric complex Gaussian distribution. Indirect learning architecture is used for pre-distorter identification and recursive least square algorithm is introduced for parameters estimation. At last using two power amplifier models, we simulate the performance of the proposed pre-distorter in an OFDM system. Simulation results show that the proposed pre-distorter can effectively compensate the distortions produced by high power amplifier.

Key Words: Orthogonal Polynomials, Pre-distortion, Recursive Least Square, Nonlinear Distortion

1 Introduction

In broadband wireless communications, OFDM (Orthogonal Frequency Division Multiplexing) is adopted as a basis modulation technique for its high spectral efficiency and resistance to multi path fading. But OFDM also exists some disadvantages, for example, high PAPR (Peak-to-Average Power Ratio), sensitive to synchronization errors and so on [1]. The high PAPR requires good linearization of HPA (High Power Amplifier). Back off technique can provide the required linearization of HPA, but this can also lower the efficiency of HPA, resulting the increase of energy consumption [1][2]. In addition, nonlinear effect of HPA regenerates the spectrum, which boosts the ACPR (Adjacent Channel Power Ratio) and incurs adjacent channel interference [2].

DPD (Digital Pre-distortion) is a widely investigated technique for compensation the nonlinear effects of HPA. The principle is that a pre-distorter, which is an inverse module of HPA, is concatenated before the HPA. So the combinational system can be viewed as linear. Because of its realization is in the digital field, it is convenient to use different DPD algorithms and has the advantages of high digital precision, strong adaptive ability, performance stability and so on[2].

As the signal bandwidth in broadband OFDM is large, the memory effect of HPA can not be ignored. According to [4], DPD based on Volterra polynomials can compensate the distortion of HPA with memory effects. In [6] a pre-distortion method based on orthogonal polynomials is proposed. The closed form expression of the orthogonal polynomials is deduced under the uniform distribution in [7], and the Gaussian distribution case is done in [8]. In this paper, we will directly obtain the polynomials via orthogonalizing operation. For the DPD coefficients, we will use the adaptive training method with reference to [3]. At last, we simulate the performance of this DPD method and the results show that it is very effective to reduce the spectrum regeneration.

2 The Structure of Pre-distorter

2.1 Pre-distortion Model Based on Orthogonal Polynomials

DPD based on Volterra polynomials can compensate the nonlinear distortion of HPA. In [6] it is show that a pre-distortion method based on orthogonal polynomials also can resolve the nonlinear distortion problem and the numerical stability is better than those Volterra polynomials based. The model of orthogonal polynomials based DPD is expression in equation (1).

$$z(n)=\sum_{k=1}^{K}\sum_{q=0}^{Q}w_{kq}\phi_k(x(n-q)) \qquad (1)$$

Where K is the order of nonlinear, Q is the depth of memory, w_{kq} is the coefficient of the model and $\phi_k(x)$ is the orthogonal polynomials. $x(n)$ is the input baseband OFDM signal and $z(n)$ is the signal after DPD. In [7] the close form expression of $\phi_k(x)$ is deduced, while in [8] the Gaussian ones are given. Those two papers are sought to obtain the formula of orthogonal polynomials, but it is complex for deducing under a particular distribution. In addition, for simplicity, only the odd order polynomials are considered. Because the DPD only uses the first several order polynomials (often no more than 5), it is not need to deduce the general formulas. In this paper, we use the orthogonalizing procedure to directly obtain the required polynomials. The procedure is stated as followings.

Assume that the form of the orthogonal polynomial is $\phi_K(z)=\sum_{k=1}^{K}a_k\,|z|^{k-1}z$, where K is the order of nonlinear. Provided the input signal is circular symmetric complex Gaussian distribution $z\sim CN(0,1)$, the orthogonalizing procedure is as followings:

(1) Set $\phi_1(z)=z$.

(2) Derivate $\phi_2(z)$. $\phi_1(z)$ and $\phi_2(z)$ are orthogonal, so there is $E\{\phi_1(z)\phi_2^*(z)\}=0$. Because of z is

circular symmetric complex Gaussian distribution, this has:

$$E\{a_2 \mid z\mid^3 + a_1 \mid z\mid^2\} = 1.3292 a_2 + a_1 = 0 \quad (2)$$

With the constraint of setting $a_1 = 1$, by solving equation (2) it has $a_2 = -0.7523$, then

$$\phi_2(z) = -0.7523 \mid z \mid z + z .$$

(3) Derivate $\phi_3(z)$. Just as the same as procedure (2), $\phi_1(z)$, $\phi_2(z)$ and $\phi_3(z)$ are orthogonal, so there are $E\{\phi_1(z)\phi_2^*(z)\} = 0$ and $E\{\phi_2(z)\phi_3^*(z)\} = 0$. With the condition of $z \sim CN(0,1)$, it has :

$$\begin{cases} a_1 + 1.3292 a_2 + 2a_3 = 0 \\ 0.1754 a_2 + 0.5 a_3 = 0 \end{cases} \quad (3)$$

With the constraint of setting $a_1 = 1$, by solving equation (3) it has $a_2 = -0.1730$, $a_3 = -0.5$, then

$$\phi_3(z) = -0.1730 \mid z\mid^2 z - 0.5 \mid z \mid z + z .$$

(4) Recursively $\phi_4(z)$ and $\phi_5(z)$ can be obtained:

$$\phi_4(z) = 0.1207 \mid z\mid^3 z + 0.1348 \mid z\mid^2 z - 1.2567 \mid z \mid z + z$$

$$\phi_5(z) = 0.1157 \mid z\mid^4 z - 1.0018 \mid z\mid^3 z + 2.7956 \mid z\mid^2 z - 2.9769 \mid z \mid z + z$$

By the above deducing, the first 5 order orthogonal polynomials under circular symmetric complex Gaussian distribution are listed in table 1.

Table 1 The First 5 Order Orthogonal Polynomials

$\phi_1(z) = z$
$\phi_2(z) = -0.7523 \mid z \mid z + z$
$\phi_3(z) = -0.1730 \mid z\mid^2 z - 0.5 \mid z \mid z + z$
$\phi_4(z) = 0.1207 \mid z\mid^3 z + 0.1348 \mid z\mid^2 z - 1.2567 \mid z \mid z + z$
$\phi_5(z) = 0.1157 \mid z\mid^4 z - 1.0018 \mid z\mid^3 z + 2.7956 \mid z\mid^2 z - 2.9769 \mid z \mid z + z$

The above recursive solution method is under the condition of circular symmetric complex Gaussian distribution, but this method can be applicable for any distribution such as uniform, exponential distribution and so on. It is a huge priority for its directly recursive solution for the orthogonal polynomials under different kinds of distribution.

2.2 Coefficients Training via Indirect Method

The coefficients of DPD model are obtained by two methods [5]: inverse model and indirect method. Inverse model need to know the model of HPA, and the coefficients are derived by solving an inverse problem. Indirect method estimates the coefficients via indirect training structure. This is not need to know the model of HPA, so it is very convenient for different kinds of DPD structure. The structure of indirect method is shown in figure 1.

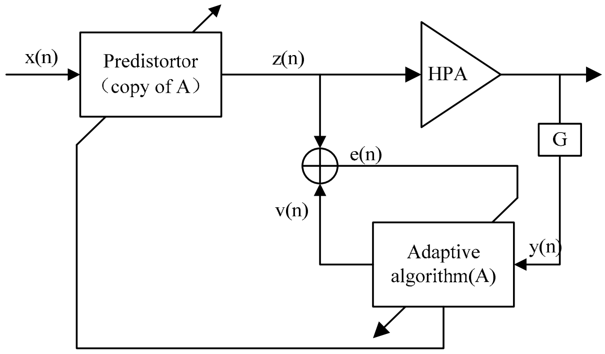

Figure 1 Indirect Learning Structure

From (1), $z(n)$ is the signal after DPD. The DPD linearly weights the polynomials by using the coefficient vector $\mathbf{w}$. The $\mathbf{w}$ can be estimated by LS algorithm. When it is converged, the LS leads to:

$$\mathbf{z} = \mathbf{U}\mathbf{w} \quad (4)$$

where $\mathbf{z} = [z(L+1); z(L+2);...; z(L+N)]$, $\mathbf{w} = [w_{10}; w_{30};...; w_{KQ}]$, $\mathbf{U} = [\mathbf{u}_1, \mathbf{u}_2, ..., \mathbf{u}_{kq}]$, $y(n) = y(n)/G$, $\mathbf{u}_{kq} = [\phi_k(y(L-q));...;\phi_k(y(L+N-q))]$.

The LS solution of equation (4) is [4]

$$\mathbf{w} = (\mathbf{U}^H \mathbf{U})^{-1} \mathbf{U}^H \mathbf{z} \quad (5)$$

Because the equation (5) involves the inversion of matrix, the computation is large. Matrix decomposition techniques can avoid the inversion operation and reduce the computation. There are many decomposition methods including QR (Orthogonal-triangular Decomposition) and SVD (Singular Value Decomposition). Besides, adaptive LS algorithm such as RLS also avoids the inversion of matrix. In this paper, we introduce RLS algorithm that is suitable for the structure of the DPD based on orthogonal polynomials with reference to [3].

According to the adaptive structure of figure 1, the coefficients are updated by the formula (6):

$$\mathbf{w}(n) = \mathbf{w}(n-1) + \mathbf{k}(n) e^*(n) \quad (6)$$

where $e(n)$ is the error signal. $\mathbf{k}(n)$ is the time-varying gain vector, expressed by formula (7)

$$\mathbf{k}(n) = \frac{\lambda^{-1}\mathbf{p}(n-1)\mathbf{y}^T(n)}{1 + \lambda^{-1}\mathbf{y}^*(n)\mathbf{p}(n-1)\mathbf{y}^T(n)} \quad (7)$$

where

$$\mathbf{y}(n) = [\phi_1(y(n))); \phi_1(y(n-1))...\phi_1(y(n-Q));...;\phi_K(y(n)); \phi_K(y(n-1))...\phi_K(y(n-Q))],$$

matrix $\mathbf{p}(n)$ is updated as formula (8).

$$\mathbf{p}(n) = \lambda^{-1}\mathbf{p}(n-1) - \lambda^{-1}\mathbf{k}(n)\mathbf{y}^*(n)\mathbf{p}(n-1) \quad (8)$$

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2.3 Power Amplifier Model

In the development of communication system, the simulation models must be constructed for emulating the PA's impaction to the system performance. There are several methods for modeling the PA with memory and nonlinear effect, such as Volterra model, memory polynomial model, Wiener-Hammerstein model, Wiener model, Hammerstein model and so on[4]. Here, memory polynomial and Wiener-Hammerstein models are introduced for the simulation of DPD.

(1) Memory Polynomial Model

This model characterizes the dynamic behavior of wideband PA very well and is commonly used in practice. The model is expressed in (9)

$$z(n)=\sum_{k=1}^{K}\sum_{q=0}^{Q} w_{kq}x(n-q)\,|\,x(n-q)\,|^{k-1} \quad (9)$$

where $x(n)$ is the input signal of DPD, $z(n)$ is the output signal of DPD, w_{kq} is the coefficient vector of DPD. K is the order of polynomial characterizing the nonlinear effect; Q is the depth of memory . In [4] w_{kq} extracted from a real PA is given as followings.

$w_{10}=1.0513+0.0904j$ $\quad w_{11}=-0.0680-0.0023j$

$w_{12}=-0.0289-0.0054j$ $\quad w_{30}=-0.0542-0.2900j$

$w_{31}=-0.2234+0.2317j$ $\quad w_{32}=-0.0621-0.0932j$

$w_{50}=-0.9657-0.7028j$ $\quad w_{51}=-0.2451-0.3735j$

$w_{52}=0.1229+0.1508j$

(2) W-H Model

It is a combination of Wiener and Hammerstein model, and can characterize the nonlinear and memory effect. The structure is depicted in figure 2.

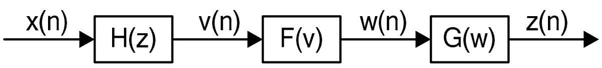

Figure 2 W-H PA Model

where H(z) and G(w) is the linear time-invariant filter, F(v) is the memoryless PA model. In [4] the parameters of this model from a real PA are given as followings.

$$H(z)=\frac{1+0.5z^{-2}}{1-0.2z^{-1}},\; G(w)=\frac{1-0.1w^{-2}}{1-0.4w^{-1}},$$

$$w(n)=\sum_{\substack{k=1\\ k\text{ odd}}}^{K} b_k v(n)\,|\,v(n)\,|^{k-1}$$

where

$b_1=1.0108+0.0858j;\; b_2=0.0879-0.1583j$

$b_3=-1.0992-0.8891j$.

3 Simulation and Analysis

For the performance evaluation of DPD, the simulation system is constructed as figure 3. The bandwidth of OFDM signal is 10MHz, the number of sub carriers is 1024, and the modulation mode is QPSK. The nonlinear order of DPD is K=5, and the memory depth is Q=4. OFDM baseband signal is up sampled by 5 fold, then goes through the DPD. The signal from DPD pass through the HPA model, and the output signal of HPA is $y(n)$. $y(n)$ is then used as the feedback to the adaptive training module. After the estimation of delay and gain, the data is aligned, and the RLS algorithm is applied to the estimation of coefficients. The computation is carried out for every data sample, then the coefficients is used to update the DPD.

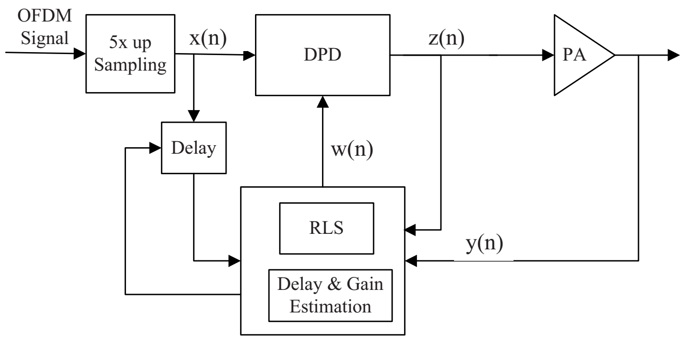

Figure 3 Simulation Flow

Comparing the PSD and ACPR before and after the pre-distortion, the performance of pre-distorter can be evaluated. The simulation results of memory polynomials and W-H PA models are shown in figure 4 and figure 5 respectively.

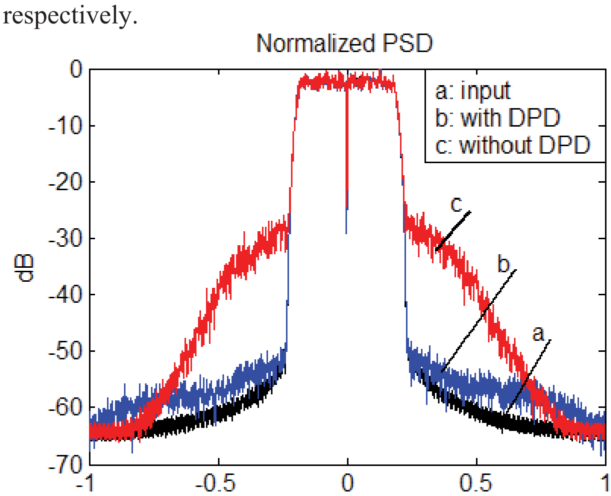

Figure 4 DPD Performance under Memory Polynomial Model

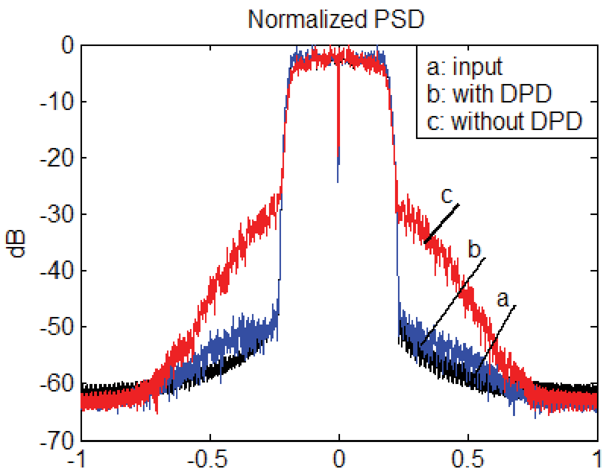

Figure 5 Performance under W-H Model

From figure 4 and figure 5, the signal without pre-distortion regenerates out band spectrum heavily, while the pre-distortion can effectively suppress the regeneration. It is shown that the PSD with pre-distortion is very close to the original signal. In figure 4, with the memory polynomial model, the ACPR with pre-distortion is about -50dB, which is 23dB smaller than the no pre-distortion case. In figure 5, with the W-H model, the improvement of ACPR is about 21dB. The two simulation results show that the proposed pre-distortion method is very effective to compensate the HPA nonlinear distortion.

4 Conclusion

In this paper, an orthogonal polynomials based pre-distorter is discussed. Using the direct orthogonalizing method, the first 5 order orthogonal polynomials under the circular symmetric complex Gaussian distribution are obtained recursively. LS and RLS algorithm is introduced for the coefficients estimation. The proposed pre-distortion method is the combination of RLS and indirect learning structure. The simulation results in two PA model show that it is very effective to suppress the nonlinear distortion and memory effect of HPA in terms of the improvement of ACPR. The discussion of proposed pre-distortion can be a reference to the practical design of pre-distorter.

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