1. Back Savers is a company that produces backpacks primarily for students. They are considering offering some combination of two different models—the Collegiate and the Mini. Both are made out of the same rip-resistant nylon fabric. Back Savers has a long-term contract with a supplier of nylon and receives a 5000-square-foot shipment of the material each week. Each Collegiate requires 3 square feet while each Mini requires 2 square feet. The sales forecasts indicate that at most 1000 Collegiates and 1200 Minis can be sold per week. Each Collegiate requires 45 minutes of labor to produce and generates a unit profit of $32. Each Mini requires 40 minutes of labor and generates a unit profit of $24. Back Savers has 35 laborers that each provide 40 hours of labor per week. Management wishes to know what quantity of each type of backpack to produce per week.

   a. Clearly define the decision variables

   b. What is the objective function?

   c. What are the constraints?

   d. Write down the full mathematical formulation for this LP problem.

ANSWER: -

a. Decision Variables:

   Decision Variables are used to represent the quantities or levels of different activities or decisions that need to be determined.

   Let’s define these variables:

   C = The number of Collegiate backpacks to produce per week.

   M = The number of Mini backpacks to produce per week.

a. Objective Function:

   It represents what you are trying to optimize or maximize (or minimize) in the context of the problem. In the Back Savers problem, the objective function is to maximize profit, which is typically represented as follows:

   Maximise z = 32C + 24M

   Here:

   Z represents the total profit that the company aims to maximize.

   C represents the number of Collegiate backpacks produced per week.

   M represents the number of Mini backpacks produced per week.

   The coefficients 32 and 24 represent the unit profit for each Collegiate and Mini backpack, respectively.

b. Constraints:

- Back Savers can only use the available rip-resistant nylon fabric, which is limited to 5,000 square feet per week:

$$3C + 2M <= 5000$$

- Back Savers has 35 laborers, each providing 40 hours of labor per week, and the labor required for each type of backpack:

$$45C + 40M <=35 \times 40 \times 60(mins)$$
$$45C + 40M <= 84,000$$

- The sales forecasts indicate that at most 1000 Collegiates and 1200 Minis can be sold per week:

$$C <= 1000, M <=1200$$

- The number of backpacks produced cannot be negative:

$$C >= 0, M >= 0$$

c. Full mathematical formulation: -
Maximize Z = 32C + 24M
Subject to: -
3C +2M <= 5000
45C + 40M <= 84,000
C <= 1000, M <= 1200
C >= 0, M >= 0

2. The Weigelt Corporation has three branch plants with excess production capacity. Fortunately, the corporation has a new product ready to begin production, and all three plants have this capability, so some of the excess capacity can be used in this way. This the product can be made in three sizes--large, medium, and small--that yields a net unit profit of $420, $360, and $300, respectively. Plants 1, 2, and 3 have the excess capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved.
The amount of available in-process storage space also imposes a limitation on the production rates of the new product. Plants 1, 2, and 3 have 13,000, 12,000, and 5,000 square feet, respectively, of in-process storage space available for a day's production of this product. Each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively.
Sales forecasts indicate that if available, 900, 1,200, and 750 units of the large, medium, and small sizes, respectively, would be sold per day.
At each plant, some employees will need to be laid off unless most of the plant’s excess production capacity can be used to produce the new product. To avoid layoffs if possible, management has decided that the plants should use the same percentage of their

excess capacity to produce the new product. Management wishes to know how much of each of the sizes should be produced
by each of the plants to maximize profit.
a. Define the decision variables
b. Formulate a linear programming model for this problem.

ANSWER: -

b. Decision Variables:
Decision Variables are used to represent the quantities or levels of different activities or decisions that need to be determined.

Let's define these variables:

L = Number of units of the large size produced per day.
M = Number of units of the medium size produced per day.
S = Number of units of the small size produced per day.

c. Formulation of the Linear Programming Model:
Objective Function:
The objective is to maximize the total profit, which is the sum of the profit from each size produced at each plant:

Maximize Z = 420X1 + 360X2 + 300X3

Subject to the following constraints:

- Plants 1, 2, and 3 produce up to 750, 900, and 450 units per day respectively

X1 <= 750
X2 <= 900
X3 <= 450

- The in-process storage area requirements for large, medium, and small units, respectively, are 20,15, and 12 square feet. Plants 1,2 and 3 have 13000, 12000, and 5000 square feet available respectively.

20X1 + 15X2 + 12X3 <= 13,000
20X1 + 15X2 + 12X3 <= 12,000
20X1 + 15X2 + 12X3 <= 5,000

- Sales forecasts indicate that if available, 900, 1,200 and 750 units of large, medium, and small sizes, respectively, would be sold per day.

X1 <=900; X2<=1,200; X3 <= 750

- Since the number of units cannot be negative.

$$X1 >= 0; X2 >= 0; X3 >=0$$

- Management wants each plant to use the same percentage of their excess capacity to produce the new product.

$$X1/750 = X2/900 = X3/450$$