Fall 2017 - Mini project on sequence learning in a cerebellum-like model

Part 1

1a) The longest sequence the weighted network can learn.

In the command window below first pattern is A and the second pattern is B.

```
Command Window

n =

    10

     0     1     1     1     0     0     0     1     0     0     0     1     1     1     1     0     0     1     1     0

     0     0     0     0     0     0     0     1     1     1     1     1     1     1     1     0     1     0     1     0
```

W=

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	0	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
0	1	1	1	0	0	0	1	0	0	0	1	1	1	1	0	0	1	1	0
1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	0	0	1	1	1
1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1
1	0	1	0	1	1	1	1	1	0	1	0	0	1	1	1	1	1	1	0
1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	0	1	0	1	1	0	1	1	1	1	1	1	1	1	0	1	1	1	0
1	0	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	0	0	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	0	1	0	1	1	1	1	1	0	1	0	0	1	1	1	1	1	1	0

```
Longest sequence that could be learned : 7
```

1b)Example of a sequence of the same length that can't be learned:

```
Command Window

n =

    10

   0   1   1   0   0   1   1   0   0   1   1   0   0   1   0   0   1   1   0   0

     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1
```

```
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

Longest sequence that could be learned : 1
fx >>
```

1c)What is it that makes one sequence easier or harder to learn than another? I think it's the complexity of Patterns generated (i.e. depends on the number of 1s in the patterns while learning process is going on), and the length of the pattern generated.

In the command window below first pattern is A and the second pattern is B. Here we can make the pattern B more complicated by making it all 1s instead of initializing to 0 and randomly generating it to get 1's and 0's.

Part 2)

Consider a Range of values for *f* and to find out which value of f lets you learn the most.

2a) At f=0.1

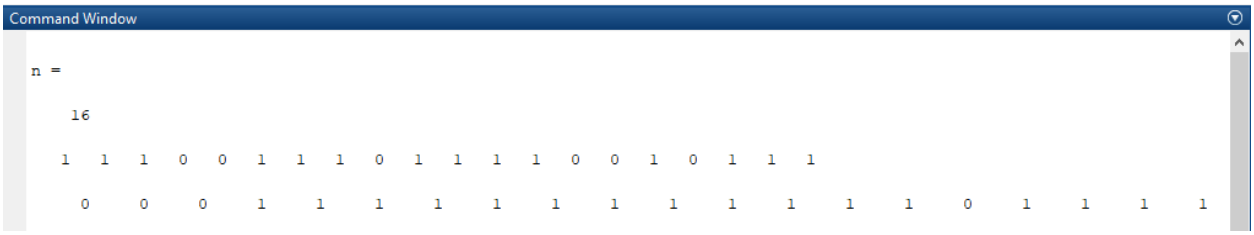
```
Command Window

n =

     2

     0     0     1     0     0     0     0     0     0     0     0     0     0     0     0     0     0     0     0     1

     0     0     0     0     0     0     0     0     0     0     0     0     1     0     0     0     0     0     0     1
```

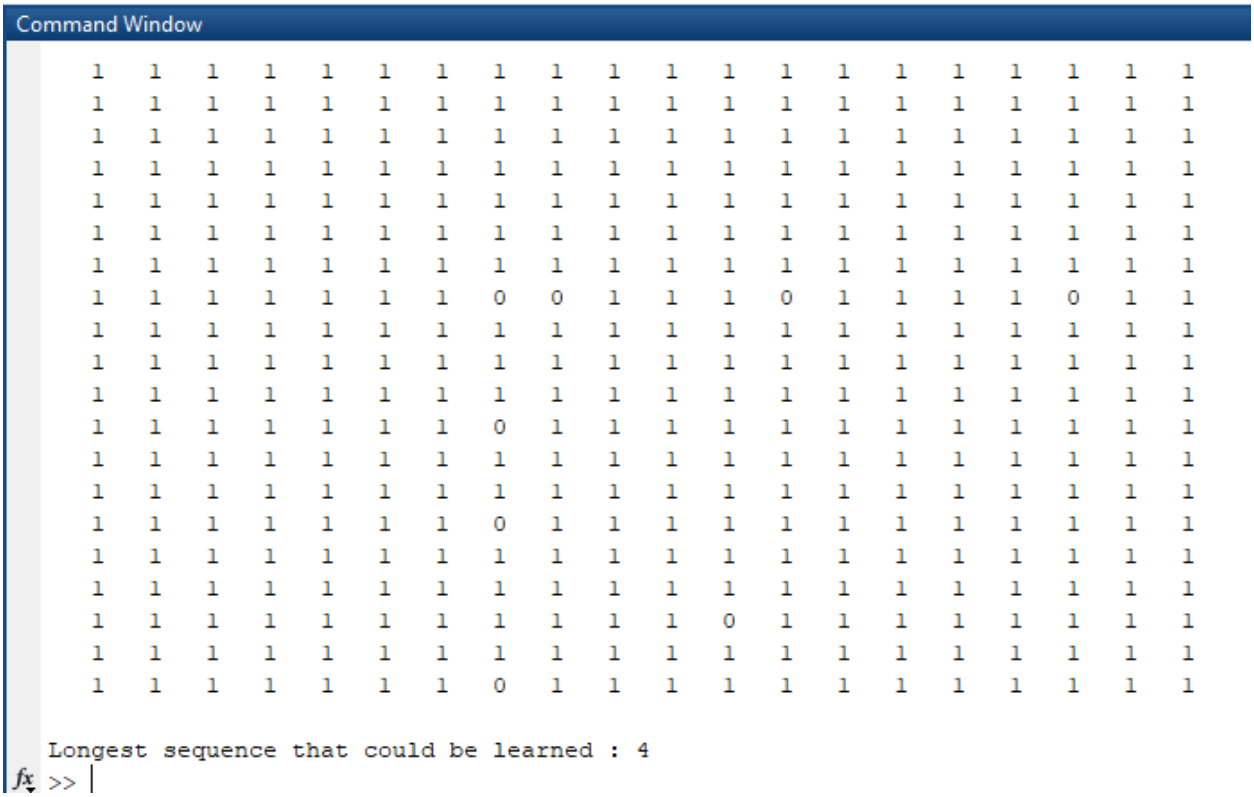
```
Command Window

    0    1    1    0    0    0    0    1    1    0    1    0    1    0    0    0    0    0    0    0
    1    0    1    0    1    0    1    0    0    0    0    0    1    0    0    1    0    0    0    0
    0    1    1    0    0    0    0    0    1    0    1    0    1    0    0    0    0    0    0    1
    0    0    0    0    0    1    0    0    0    0    0    1    0    0    0    0    0    0    0    0
    0    0    0    0    0    0    0    1    0    0    0    0    1    0    0    0    1    0    0    0
    1    0    0    0    1    0    0    1    0    1    0    0    1    0    0    1    0    0    1    0
    0    0    0    0    0    0    0    1    0    0    0    0    0    0    0    0    1    0    0    0
    0    1    1    0    0    1    0    0    0    1    0    0    0    0    0    0    0    1    0    1
    0    0    0    0    0    0    0    1    1    0    1    0    0    0    0    0    0    1    0    0
    1    1    0    0    1    0    0    0    0    0    1    0    0    0    0    0    0    0    0    0
    0    0    0    0    1    0    1    1    0    0    0    0    0    0    0    0    0    1    0    0
    0    1    0    0    0    1    0    1    0    0    0    1    0    0    0    0    0    0    1    1
    0    0    1    0    0    1    0    0    0    0    0    1    0    0    0    0    0    1    0    0
    0    0    0    0    0    1    0    0    0    0    0    1    0    0    0    0    0    0    0    0
    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0
    0    0    1    0    0    0    0    0    0    0    0    1    0    0    0    0    0    0    0    0
    0    0    1    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    0    1
    1    0    0    0    0    1    0    0    0    1    0    0    0    0    0    0    0    0    0    0
    0    1    0    0    0    1    0    0    0    0    0    0    0    0    0    0    0    0    0    0
    1    0    1    0    0    0    0    0    0    0    0    0    1    0    0    0    0    0    0    1

Longest sequence that could be learned : 20
fx >>
```

At f=0.5

```
Command Window

n =

    10

     0     1     1     1     0     0     0     1     0     0     0     1     1     1     1     0     0     1     1     0

     0     0     0     0     0     0     0     1     1     1     1     1     1     1     1     0     1     0     1     0

    1     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1
    1     0     1     0     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1     0
    1     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1
    1     0     1     0     0     1     1     1     1     1     1     1     1     1     1     1     1     1     1     0
    0     1     1     1     0     0     0     1     0     0     0     1     1     1     1     0     0     1     1     0
    1     1     1     1     1     1     0     1     1     1     1     1     1     1     1     0     0     1     1     1
    1     1     1     1     0     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1
    1     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1
    1     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1     0     1     1     1
    1     0     1     0     1     1     1     1     1     0     1     0     0     1     1     1     1     1     1     0
    1     1     1     1     0     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1
    1     0     1     0     1     1     0     1     1     1     1     1     1     1     1     0     1     1     1     0
    1     0     1     0     0     1     1     1     1     1     1     1     1     1     1     1     1     1     1     0
    1     1     1     1     0     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1
    1     1     1     1     0     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1
    1     1     1     1     0     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1
    1     1     1     0     0     1     1     1     1     1     1     1     0     1     1     1     0     1     1     1
    1     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1
    1     1     1     1     0     1     1     1     1     1     1     1     1     1     1     1     1     1     1     1
    1     0     1     0     1     1     1     1     1     0     1     0     0     1     1     1     1     1     1     0

Longest sequence that could be learned : 7
```

At f=0.8





We can see from the results obtained above that as the frequency increases the ability of the network to learn decreases. That means as the number of 1's increases in the patterns it gets complicated and the system seizes to learn after some particular frequency.

Part 3

Alternate way of forming patterns: f is the exact fraction of bits set to 1. In each pattern rather than having f be the probability that each bit is set.

Command Window

```
    1   0   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1
    1   0   1   1   1   1   0   0   1   1   1   0   1   1   1   0   1   1   0   1
    1   0   1   1   1   1   1   1   1   1   1   0   1   1   1   1   1   1   1   1
    1   0   1   1   1   1   1   0   1   1   1   1   1   1   1   0   1   1   1   1
    1   1   1   1   1   1   1   1   1   1   1   0   1   1   1   1   0   1   1   1   1
    1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1
    1   1   1   1   1   1   0   1   1   1   0   0   0   1   1   1   1   1   1   0
    1   1   1   1   1   1   0   1   1   1   1   0   0   1   1   0   1   1   1   1
    1   1   1   1   1   1   1   1   1   1   1   1   0   1   1   1   1   1   1   1
    1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1
    1   1   1   1   1   1   1   1   1   1   0   1   1   1   1   1   1   1   1   0
    1   1   1   1   1   1   0   0   1   0   0   0   0   0   0   1   1   1   0   0
    0   1   1   1   1   0   1   1   1   1   1   0   1   1   1   1   1   1   1   0
    1   0   0   1   1   1   1   0   0   1   1   1   1   1   1   0   1   1   1   1
    1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1
    1   1   1   1   1   1   0   1   1   1   0   0   1   1   1   1   1   1   1   1
    1   1   1   1   1   1   1   1   1   1   1   0   1   1   1   1   1   1   1   1
    1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1
    1   1   1   1   1   1   0   1   1   1   1   0   0   1   1   1   1   1   1   1
    1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   0

Longest sequence that could be learned : 6
```

Previously :

Command Window

```
    0   0   1   1   0   1   1   1   0   0   1   0   0   0   1   1   0   0   1   1
    1   1   1   1   0   1   1   1   0   1   1   1   0   1   1   1   0   0   1   1
    0   0   1   1   0   1   1   1   1   1   1   1   1   1   1   1   0   1   1   1
    1   1   1   0   0   1   1   0   1   1   0   1   1   1   1   1   0   1   0   1
    0   1   0   1   0   1   0   1   1   0   1   0   1   0   0   0   1   1   1   0
    1   1   1   1   0   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1
    0   1   1   1   0   1   1   1   1   1   1   1   1   1   1   1   1   1   1   0
    1   1   1   1   0   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1
    1   1   1   1   0   1   0   1   1   1   1   1   1   1   1   1   1   1   1   1
    0   1   1   1   0   1   1   1   1   0   1   0   1   0   1   1   1   1   1   1
    1   1   1   0   0   1   1   0   1   1   0   1   1   1   1   1   0   1   0   1
    0   1   1   1   0   1   1   1   1   0   1   0   1   0   1   1   1   1   1   1
    1   1   1   0   0   1   0   0   0   1   0   1   0   1   1   1   0   0   0   1
    0   0   1   1   0   1   1   1   0   0   1   0   0   0   1   1   0   0   1   1
    0   1   1   1   0   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1
    0   1   1   1   0   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1
    1   1   1   1   0   1   0   1   1   1   1   1   1   1   1   1   1   1   1   1
    1   1   1   0   0   1   0   0   0   1   0   1   0   1   1   1   0   0   0   1
    1   1   1   1   0   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1
    0   0   1   1   0   1   1   1   1   1   1   1   1   1   1   1   0   1   1   1

Longest sequence that could be learned : 4
fx >>
```

From the above examples we can see that the longest sequence that could be learned is greater when f is the exact fraction of bits set to 1. Neural circuit that could provide this type of normalization is the one which has a low frequency. It helps provide a better learning efficiency.

Matlab Code:

```
%Neural Sequence Learning
clear
clc
close all

%the frequency of 1's in a single pattern
f = 0.8;

% no. of elements in a pattern
%the number of 1's in a given pattern
N = 20;
n = N*f

%sequence consists of a collection of binary patterns
Q=0;

%  Weighted Matrix
W = zeros(N); %initialise to Zero

% Input pattern

A=rand(1,20)<f;

%Output pattern
B = zeros(1,N);%initialise to Zero
%B = ones(1,N);
B(randperm(N,n))=1; % Permuted variable

W_T = zeros(N);
%Pattern C
C = zeros(1,N);%initialise to zero
```

```
%GIVEN: 100 ITERATIONS
for t = 1:100
Q = Q + 1; % to generate a sequence of Q+1 patterns
for i=1:N
W(i,:) = A(i).*B';
end
W_T = W_T + W;
W = W_T >= 1;

% plug the values of A and B in the sequence of patterns and compare it
patterns(t,:) = A;
patterns(t+1,:) = B;


for j = 1:N
C(j) = sum(A'.*W(:,j)) >= N*f;
end

comapare(t,:) = C;
% Now Check if both B and C are euqal
check = isequal(B,C);
if check == 0
break % Loop breaks if there is an error in the sequence
end

A = B;
B = zeros(1,N);
B(randperm(N,N*f)) = 1;

end
disp([A])
disp([B])
disp([W])

disp(['Longest sequence that could be learned : ' num2str(Q)])
```

Extra credit

To explore the advantage of a high-dimensional recoding of the patterns analogous to that performed by the granule cells in the cerebellum

Command Window

```
n =

    14

ans =

     1
```

Command Window

```
z =

  20×20 logical array

   0   1   1   1   1   1   0   1   1   0   0   1   1   0   1   1   1   1   1   1
   1   0   0   0   0   0   1   0   0   1   1   0   0   1   0   0   0   0   0   0
   1   0   0   0   0   0   1   0   0   1   1   0   0   1   0   0   0   0   0   0
   1   0   0   0   0   0   1   0   0   1   1   0   0   1   0   0   0   0   0   0
   1   0   0   0   0   0   1   0   0   1   1   0   0   1   0   0   0   0   0   0
   1   0   0   0   0   0   1   0   0   1   1   0   0   1   0   0   0   0   0   0
   0   1   1   1   1   1   0   1   1   0   0   1   1   0   1   1   1   1   1   1
   1   0   0   0   0   0   1   0   0   1   1   0   0   1   0   0   0   0   0   0
   1   0   0   0   0   0   1   0   0   1   1   0   0   1   0   0   0   0   0   0
   0   1   1   1   1   1   0   1   1   0   0   1   1   0   1   1   1   1   1   1
   0   1   1   1   1   1   0   1   1   0   0   1   1   0   1   1   1   1   1   1
   1   0   0   0   0   0   1   0   0   1   1   0   0   1   0   0   0   0   0   0
   1   0   0   0   0   0   1   0   0   1   1   0   0   1   0   0   0   0   0   0
   0   1   1   1   1   1   0   1   1   0   0   1   1   0   1   1   1   1   1   1
   1   0   0   0   0   0   1   0   0   1   1   0   0   1   0   0   0   0   0   0
   1   0   0   0   0   0   1   0   0   1   1   0   0   1   0   0   0   0   0   0
   1   0   0   0   0   0   1   0   0   1   1   0   0   1   0   0   0   0   0   0
   1   0   0   0   0   0   1   0   0   1   1   0   0   1   0   0   0   0   0   0
   1   0   0   0   0   0   1   0   0   1   1   0   0   1   0   0   0   0   0   0
   1   0   0   0   0   0   1   0   0   1   1   0   0   1   0   0   0   0   0   0
```

Command Window

```
y =

  20×20 logical array

   1   0   0   0   0   0   1   0   0   1   1   0   0   1   0   0   0   0   0   0
   0   1   1   1   1   1   0   1   1   0   0   1   1   0   1   1   1   1   1   1
   0   1   1   1   1   1   0   1   1   0   0   1   1   0   1   1   1   1   1   1
   0   1   1   1   1   1   0   1   1   0   0   1   1   0   1   1   1   1   1   1
   0   1   1   1   1   1   0   1   1   0   0   1   1   0   1   1   1   1   1   1
   0   1   1   1   1   1   0   1   1   0   0   1   1   0   1   1   1   1   1   1
   1   0   0   0   0   0   1   0   0   1   1   0   0   1   0   0   0   0   0   0
   0   1   1   1   1   1   0   1   1   0   0   1   1   0   1   1   1   1   1   1
   0   1   1   1   1   1   0   1   1   0   0   1   1   0   1   1   1   1   1   1
   1   0   0   0   0   0   1   0   0   1   1   0   0   1   0   0   0   0   0   0
   1   0   0   0   0   0   1   0   0   1   1   0   0   1   0   0   0   0   0   0
   0   1   1   1   1   1   0   1   1   0   0   1   1   0   1   1   1   1   1   1
   0   1   1   1   1   1   0   1   1   0   0   1   1   0   1   1   1   1   1   1
   1   0   0   0   0   0   1   0   0   1   1   0   0   1   0   0   0   0   0   0
   0   1   1   1   1   1   0   1   1   0   0   1   1   0   1   1   1   1   1   1
   0   1   1   1   1   1   0   1   1   0   0   1   1   0   1   1   1   1   1   1
   0   1   1   1   1   1   0   1   1   0   0   1   1   0   1   1   1   1   1   1
   0   1   1   1   1   1   0   1   1   0   0   1   1   0   1   1   1   1   1   1
   0   1   1   1   1   1   0   1   1   0   0   1   1   0   1   1   1   1   1   1
```

fx

```
G =

   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
   0   1   1   1   1   1   0   1   1   0   0   1   1   0   1   1   1   1   1   1
   0   1   1   1   1   1   0   1   1   0   0   1   1   0   1   1   1   1   1   1
   0   1   1   1   1   1   0   1   1   0   0   1   1   0   1   1   1   1   1   1
   0   1   1   1   1   1   0   1   1   0   0   1   1   0   1   1   1   1   1   1
   0   1   1   1   1   1   0   1   1   0   0   1   1   0   1   1   1   1   1   1
   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
   0   1   1   1   1   1   0   1   1   0   0   1   1   0   1   1   1   1   1   1
   0   1   1   1   1   1   0   1   1   0   0   1   1   0   1   1   1   1   1   1
   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
   0   1   1   1   1   1   0   1   1   0   0   1   1   0   1   1   1   1   1   1
   0   1   1   1   1   1   0   1   1   0   0   1   1   0   1   1   1   1   1   1
   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
   0   1   1   1   1   1   0   1   1   0   0   1   1   0   1   1   1   1   1   1
   0   1   1   1   1   1   0   1   1   0   0   1   1   0   1   1   1   1   1   1
   0   1   1   1   1   1   0   1   1   0   0   1   1   0   1   1   1   1   1   1
   0   1   1   1   1   1   0   1   1   0   0   1   1   0   1   1   1   1   1   1
   0   1   1   1   1   1   0   1   1   0   0   1   1   0   1   1   1   1   1   1
```

Matlab Code:

```
clear
clc
close all

%the frequency of 1's in a single pattern
f = 0.8;

% no. of elements in a pattern
%the number of 1's in a given pattern
N = 20;
n = N*f

%sequence consists of a collection of binary patterns
Q=0;

%  Weighted Matrix
W = zeros(N); %initialise to Zero

A=rand(1,20)<f; % Random input variable
z = xor(A,A')
y = ~z
G = A.*y'

B = zeros(1,N);
B(randperm(N,n))=1;
W_T = zeros(N);
C = zeros(1,N);


for t = 1:100
Q = Q + 1;

for i=1:N
W(i,:) = G(i).*B';
end
W_T = W_T + W;
W = W_T >= 1;


for j = 1:N
C(j) = sum(A'.*G(:,j)) >= N*f;
```

```
end
compare(t,:) = C;
tf = isequal(B,C);
if tf == 0
break
end
G = B;
B = zeros(1,N);
B(randperm(N,N*f)) = 1;
end
disp([W])

disp(['The longest sequence that can be learned is' num2str(Q)])
```