UIT2201 Programming and Data Structures

Hashing Techniques

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Introduction

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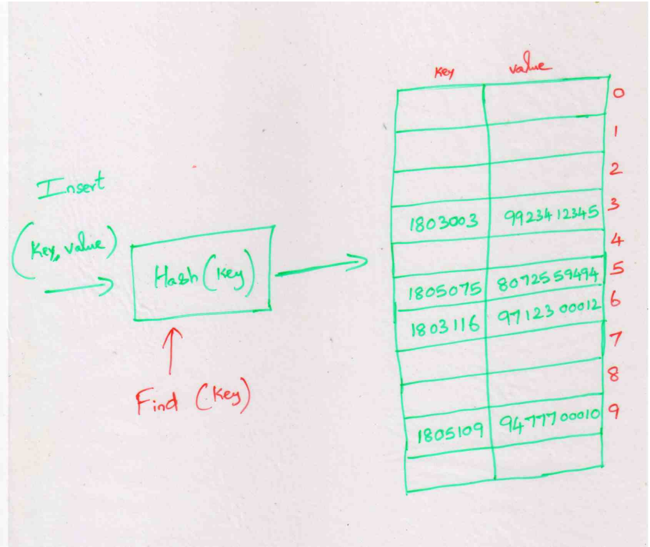
- Complexity: $O(\log n)$
- Is it possible to do this in constant time (on the average)!?

Basic Idea



Insert
(Key, value)
Hash (Key)
Find (Key)
Key
value
0
1
2
1803003
9923412345
3
4
1805075
8072559494
5
1803116
9712300012
6
7
8
1805109
9477700010
9

Issues

- Selection of hashing function is very crucial to achieve constant time on the average
- Hashing function should ideally handle any type of key (non-mutable)
- It should be easy to compute!
- Hashing function should ideally generate distinct indexes for distinct keys
- Generally, the "key density" is very high compared to the number of pairs we wish to store
- How do we choose appropriate TableSize?
- What to do when the hashing function maps more than one key to an index (collision handling)?

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- Hash code is to map a key to an integer (not necessarily in the range of the Table)
- Hash code may typically be a 32-bit integer
- Compression function should map a hash code to a legal index of the Table

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- If the key is an integer, then a simple hash function (key mod TableSize) may work
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- If the key is not an integer, then it should first be converted into a hash code before performing the modulus operation
- A simple analysis reveals that the keys are well distributed across the index space when TableSize is a prime number
- For example, consider the hash codes $[200, 205, 210, 220, \cdots, 600]$ with TableSize of 100 Vs TableSize of 101

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- Probability of collision is approximately $1/\mathit{TableSize}$

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- We can break the bit sequence into chunks of 32 bits, and either add them or take exclusive-or of those chunks

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- For example, “stop”, “tops”, “pots”, and “spot” will have same hash code!
- Another bad case is when TableSize is quite large and strings are small
- Example: TableSize $= 10007$ and strings are 8 characters at most
- Only indexes up to ($127 * 8 = 1016$) are used! (we have assumed ASCII representation here)

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- And the computation gets complex when the number of characters are increased

Polynomial Hash Codes with Horner's rule

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- Text book suggests $33, 37, 39, 41$ as good options for English character strings

Cyclic-Shift Hash Codes

- This is a variant of polynomial hash codes
- The multiplication is replaced by cyclic shift by a certain number of bits
- For example, $10110\cdots$ is replaced by $\cdots 10110$
- This operation has no mathematical meaning, but seems to generate better hash codes!

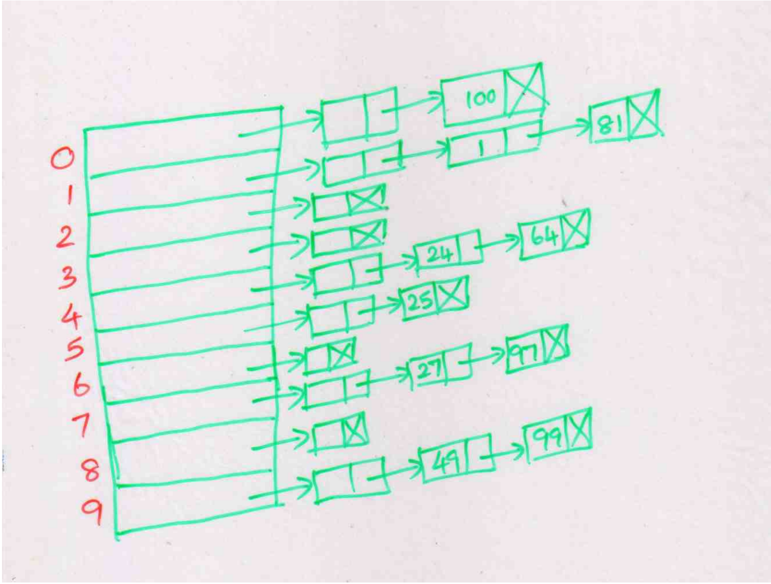
Collision Handling

- Separate Chaining — use additional data structures to store the pairs having the same home index
- Open Addressing — use only the array and systematically look for an alternate index
    - Linear Probing
    - Quadratic Probing
    - Double Hashing
- Rehashing to increase or reduce the TableSize
- Extendible Hashing

Separate Chaining

- Idea here is to use another collection data structure to keep all the pairs that hash to the same index
- A simple linked list will be an ideal choice!

Separate Chaining



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- Average length of a linked list will be λ
- We may achieve constant average time, if we can keep the load factor λ close to 1 and the hash function distributes the keys well
- Some other data structure, such as BST or another hash table, may be tried instead of linked lists, but may not be worth the effort

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- The function F is referred to as a collision resolution probing strategy

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- And, we should add status field (Valid, Empty, Deleted) to each entry in the table

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- A new object can be inserted as long as there are free cells in the array

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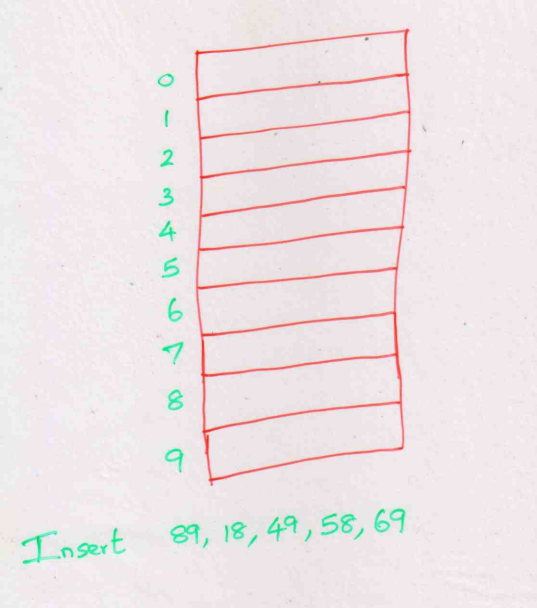

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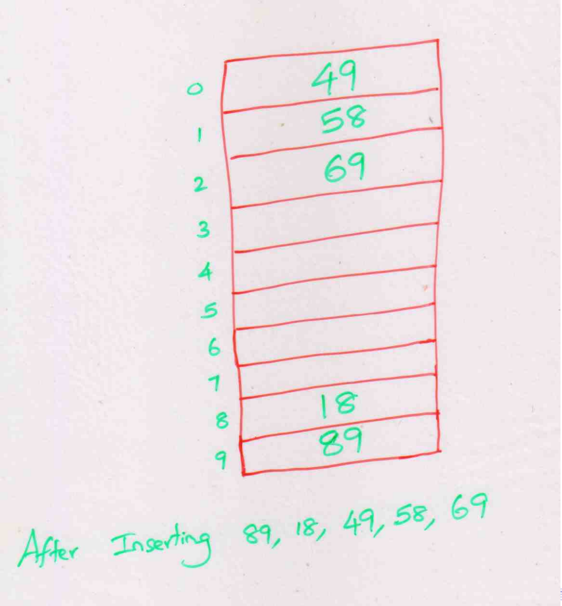

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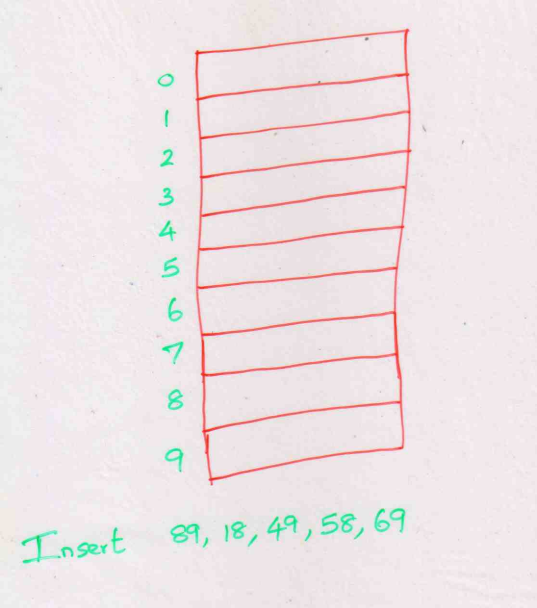
- Ideally, λ should be kept below 0.5

Quadratic probing

- Quadratic probing may be tried to eliminate the primary clustering problem
- $F(i) = i^2$ is a popular choice!
- The number of alternative locations may be severely reduced

Quadratic Probing



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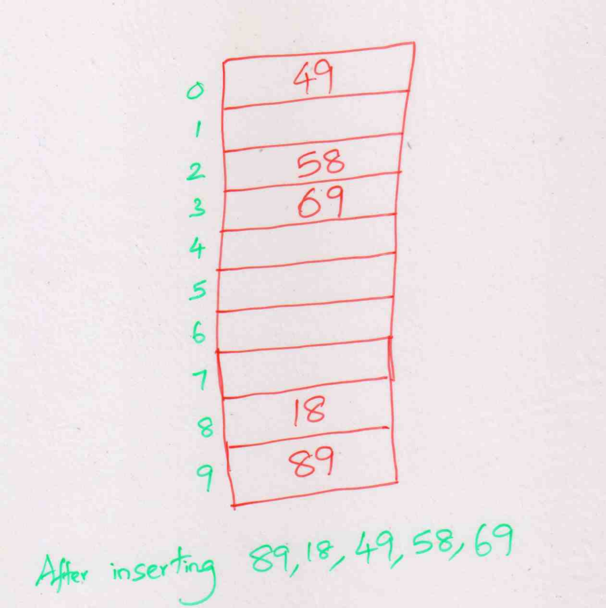

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- However, it has been proved that insertion is always possible if the table is at least half empty and TableSize is prime
- For all the keys that are mapped to the same index, same alternate locations are tried resulting in "secondary clustering"

Double hashing

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- Note that the step size is different for different keys!
- Also note that this second hash function should not result in 0 for any key! (Why?)
- Typically, $Hash_2(Key) = R - (Key \% R)$, where R is a prime number smaller that TableSize

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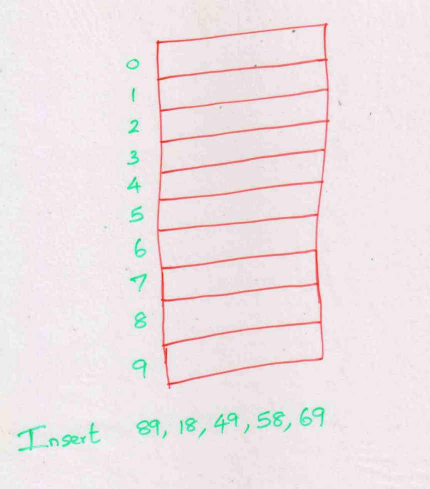

- Let us choose $R = 7$

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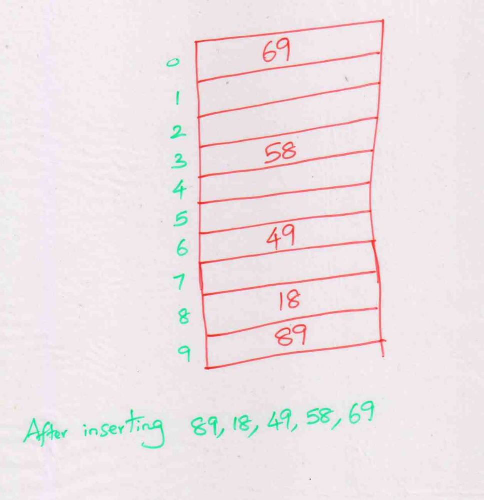

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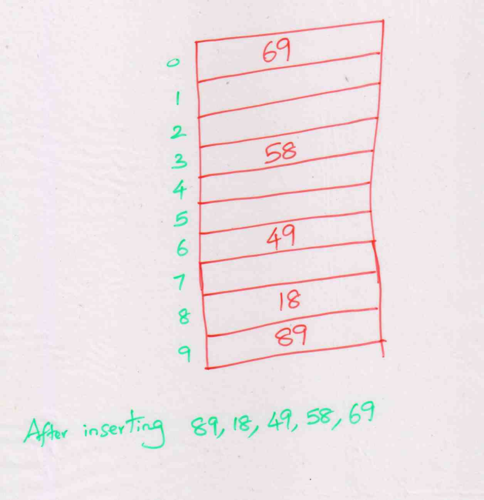

- Try inserting 60 and 23!

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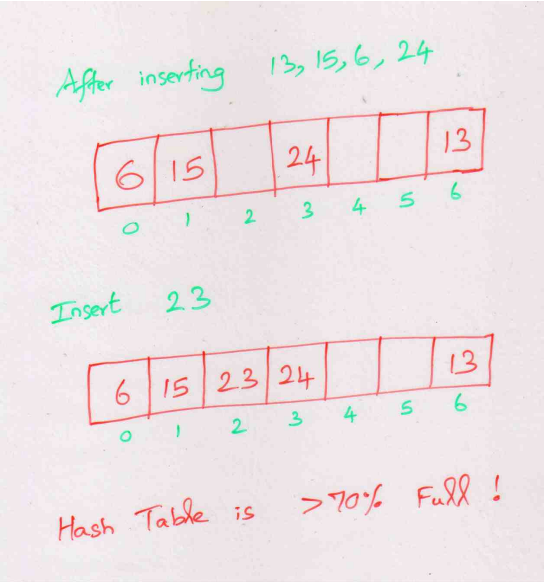
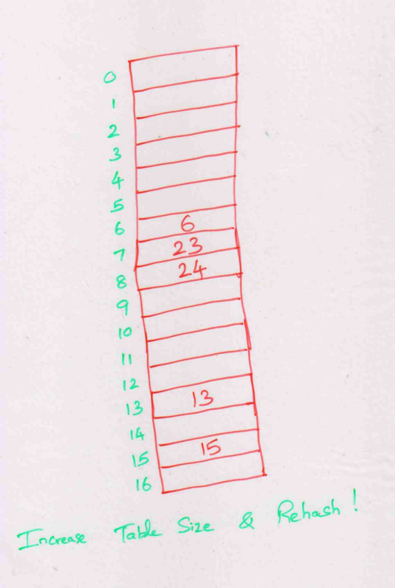
- Rehashing is a good opportunity to purge all the deleted objects!

Rehashing


After inserting 13, 15, 6, 24
6
15
24
13
0 1 2 3 4 5 6
Insert 23
6
15
23
24
13
0 1 2 3 4 5 6
Hash Table is >70% Full !

Rehashing


0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
6
23
24
13
15
Increase Table Size & Rehash !

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- Hence the complexity is $O(n)$
- However, next rehashing is likely to occur only after n more insertions
- So, the amortized complexity can be considered as constant

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- Examples: Password hashing, checksum of data for integrity check, message digest
- Popular choices: SHA-2 and SHA-3 (Keccak) families of algorithms, MD5 (MD6 has also been introduced), RIPEMD, BLAKE, etc.

Summary

- Several applications need collections that support only insertion, deletion, search
- Hashing is an ideal solution that can achieve constant average time
- We have discussed some simple hash functions and the issues involved
- Collision is a major issue in implementing hashing technique
- Separate chaining is one of the solutions to handle collision — use a secondary data structure, such as linked lists, to store all the objects hashing to the same index
- Load factor λ needs to be close to 1 for effective separate chaining

Summary

- We have also explored the open addressing hashing (also known as closed hashing) techniques in this lecture
- In particular, we discussed three probing techniques, namely linear, quadratic, and double hashing
- Analysis reveals that the load factor should be kept below 0.5 for effective closed hashing
- Rehashing may be necessary to control the load factor
- Hash codes are also useful in several cryptographic applications