11.4 Examples

Let $\vec{u} = \langle 2, 1, 7 \rangle$ and $\vec{v} = \langle -4, 2, 3 \rangle$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 7 \\ -4 & 2 & 3 \end{vmatrix} = (1(3) - 2(7))\hat{i} - (2(3) - -4(7))\hat{j} + (2(2) - -4(1))\hat{k}$$

$$= (3-14)\hat{i} - (6+28)\hat{j} + (4+4)\hat{k}$$

$$= \langle -11, -34, 8 \rangle$$

Show $\vec{u} \times \vec{v}$ is orthogonal to $\vec{u}$ and $\vec{v}$.

$$(\vec{u} \times \vec{v}) \cdot \vec{u} = \langle -11, -34, 8 \rangle \cdot \langle 2, 1, 7 \rangle$$

$$= -22 + -34 + 56 = 0 \Rightarrow \text{orthogonal}$$

$$(\vec{u} \times \vec{v}) \cdot \vec{v} = \langle -11, -34, 8 \rangle \cdot \langle -4, 2, 3 \rangle$$

$$= 44 - 68 + 24 = 0 \Rightarrow \text{orthogonal}$$

Find $\text{Proj}_{\vec{v}} \vec{u} = \dfrac{\vec{u} \cdot \vec{v}}{\|\vec{v}\|^2} \vec{v}$

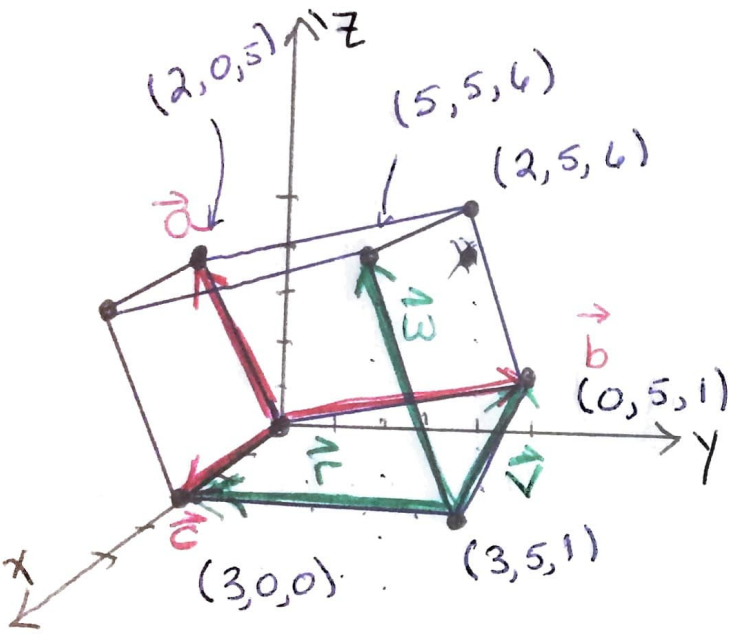
$$= \frac{2(-4) + 1(2) + 7(3)}{(-4)^2 + (2)^2 + (3)^2} \langle -4, 2, 3 \rangle$$

$$= \frac{15}{29} \langle -4, 2, 3 \rangle$$

$$\| \text{Proj}_{\vec{u}} \vec{v} \| = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\|} = \frac{15}{\sqrt{4+1+49}} = \frac{15}{\sqrt{54}} = \frac{15}{3\sqrt{6}} = \frac{5}{\sqrt{6}}$$

11.4 Examples (2)

Find Volume of parallelepiped with vertices at $(0,0,0)$, $(3,0,0)$, $(0,5,1)$, $(2,0,5)$, $(3,5,1)$, $(5,0,5)$, $(2,5,6)$, $(5,5,6)$



$$\text{Volume} = \left| \vec{a} \cdot (\vec{c} \times \vec{b}) \right|$$

$\vec{a} = \langle 2,0,5 \rangle$
$\vec{b} = \langle 0,5,1 \rangle$
$\vec{c} = \langle 3,0,0 \rangle$

$$\vec{c} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & 0 \\ 0 & 5 & 1 \end{vmatrix} = \langle 0, -3, 15 \rangle$$

$$\vec{a} \cdot \vec{c} \times \vec{b} = 0 - 0 + 75 = 75 = \text{Vol.}$$ cu. units

Or $\vec{v} = \langle 0-3, 5-5, 1-1 \rangle = \langle -3, 0, 0 \rangle$
$\vec{\omega} = \langle 5-3, 5-5, 6-1 \rangle = \langle 2, 0, 5 \rangle$
$\vec{r} = \langle 3-3, 0-5, 0-1 \rangle = \langle 0, -5, -1 \rangle$

$$\vec{v} \times \vec{\omega} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 0 & 0 \\ 2 & 0 & 5 \end{vmatrix} = \langle 0, 15, 0 \rangle \qquad \vec{r} \cdot (\vec{v} \times \vec{\omega}) = 0 - 75 + 0 = -75$$

$$\text{Vol} = |-75| = \underline{75 \text{ cu units}}$$