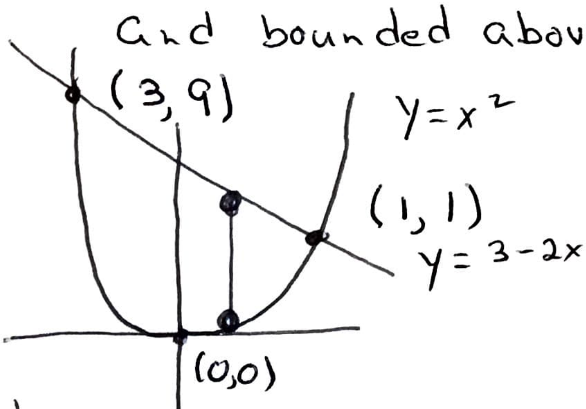
Math 2153 Additional Examples.

1.) Change The order of Integration.

$$\int_{-3}^{1}\int_{x^2}^{3-2x} f(x,y)\,dy\,dx$$

since we are Not evaluating the integrand is Not important.

We <u>must</u> understand The xy region being described. Region is bounded below by $y=x^2$ and bounded above by $y=3-2x$ for $-3 \le x \le 1$



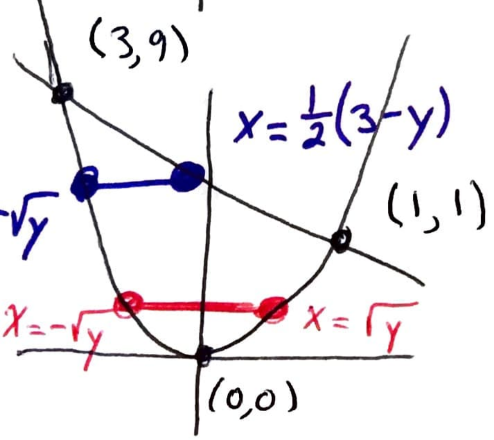
$x^2 = 3-2x$

$x^2+2x-3=0$

$(x+3)(x-1)=0$

$x=-3 \quad x=1$

Good News that The x-Limits are The intersections.



To change order we need to solve for x. $y=x^2 \rightarrow x=-\sqrt{y},\ x=\sqrt{y}$

$y=3-2x \rightarrow x=\frac{1}{2}(3-y)$

And identify The left & Right bounds for any Horizontal slice.

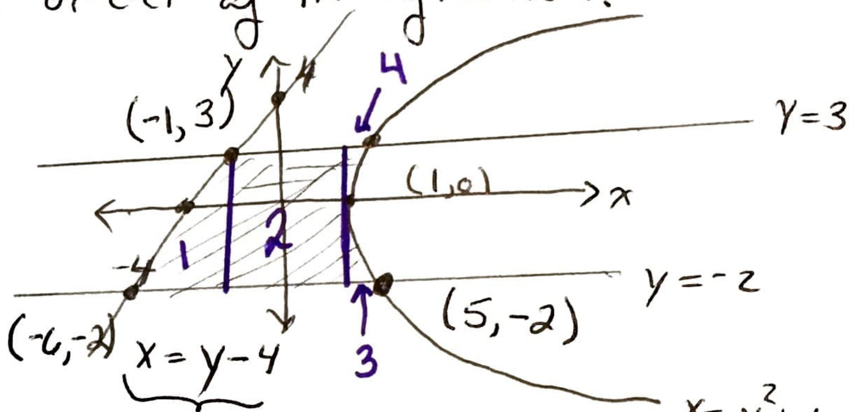
Notice we will Need 2 double integrals.

so $\displaystyle\int_{-3}^{1}\int_{x^2}^{3-2x} f(x,y)\,dy\,dx$

$$= \int_{y=0}^{y=1}\int_{-\sqrt{y}}^{\sqrt{y}} f(x,y)\,dx\,dy + \int_{y=1}^{y=9}\int_{-\sqrt{y}}^{\frac{1}{2}(3-y)} f(x,y)\,dx\,dy$$

<u>one more</u> Change the order of integration.

$$\int_{-2}^{3}\int_{y-4}^{y^2+1} xe^y\,dx\,dy$$



$Y = x+4$ $y = \pm\sqrt{x-1}$

Notice changing the order will require 4 double integrals.

① $\displaystyle\int_{-6}^{-1}\int_{-2}^{x+4} xe^y\,dy\,dx$ ② $\displaystyle\int_{-1}^{1}\int_{-2}^{3} xe^y\,dy\,dx$

③ $\displaystyle\int_{1}^{5}\int_{-2}^{-\sqrt{x-1}} xe^y\,dy\,dx$

$-\sqrt{x-1} = -2$ $\sqrt{x-1} = 2$

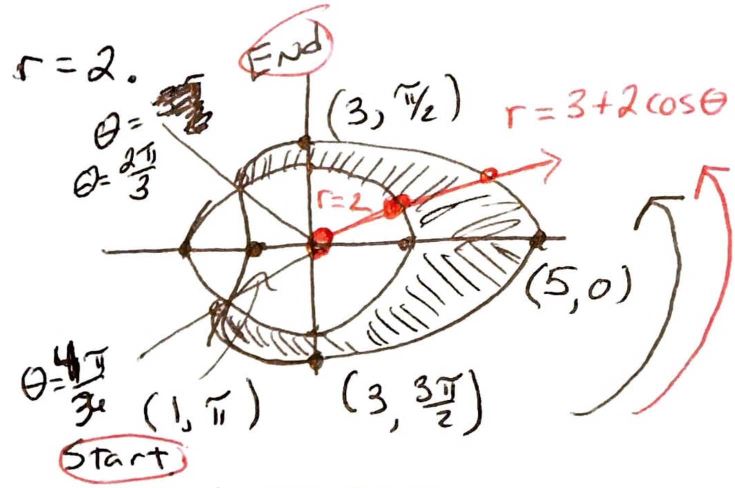
$x = 4 + 1 = 5$

$\sqrt{x-1} = 3$ $x = 10$

④ $\displaystyle\int_{1}^{10}\int_{\sqrt{x-1}}^{3} xe^y\,dx$

Wow!

3.) Use a double integral to find the area inside the dimpled Limaçon $r = 3 + 2\cos\theta$ and outside the circle $r = 2$.



Find pts of intersection.

$$3 + 2\cos\theta = 2$$

$$2\cos\theta = -1$$

$$\cos\theta = -1/2$$

$\theta_r = \pi/3$ reference angle

So in Quad II and III.

$\theta = \frac{2\pi}{3}$ and $\theta = \frac{4\pi}{3}$ (No Degrees)

Sorry I have No art skill but graph Them in polar mode on your calculator.

We Need θ Limits That are in The proper order for a counterclockwise rotation. Need r to go from and to on a ray from The origin.

$$A = \int_{-\frac{2\pi}{3}}^{\frac{2\pi}{3}} \int_{2}^{3+2\cos\theta} 1 \underbrace{r\,dr\,d\theta}_{\text{polar}}$$

Area

$\frac{4\pi}{3}$ coterminal with $-\frac{2\pi}{3}$

or could use

$\frac{4\pi}{3}$ to $\frac{8\pi}{3}$

Or we can use symmetry and double the result.

$$2\int_0^{\frac{2\pi}{3}}\int_2^{3+2\cos\theta} 1 \; r\,dr\,d\theta = \cancel{2}\int_0^{\frac{2\pi}{3}} \cancel{\tfrac{1}{2}}\, r^2 \Big|_2^{3+2\cos\theta} d\theta$$

$$= \int_0^{\frac{2\pi}{3}} (3+2\cos\theta)^2 - 4 \; d\theta = \int_0^{\frac{2\pi}{3}} 9 + 12\cos\theta + 4\cos^2\theta - 4 \; d\theta$$

$$= \int_0^{\frac{2\pi}{3}} 5 + 12\cos\theta + 4\cos^2\theta \; d\theta = \left[5\theta + 12\sin\theta + 4\left(\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right)\right]_0^{\frac{2\pi}{3}}$$

$$= \Big[7\theta + 12\sin\theta + \sin(2\theta)\Big]_0^{\frac{2\pi}{3}}$$

$$= \cancel{\left(\frac{35\pi}{6} + 12\sin\left(\frac{5\pi}{6}\right) + \sin\left(\frac{10\pi}{6}\right)\right) - \left(0 + 12\sin 0 + \sin 0\right)}$$

$$= \cancel{\frac{35\pi}{6} + 12\left(\frac{\sqrt{3}}{2}\right) + \left(-\frac{1}{2}\right) = \frac{35\pi}{6} + 6\sqrt{3} - \frac{1}{2}}$$

$$= \left(\frac{14\pi}{3} + 12\sin\left(\frac{2\pi}{3}\right) + \sin\left(\frac{4\pi}{3}\right)\right) - \left(0 + 12\sin(0) + \sin(0)\right)$$

$$= \frac{14\pi}{3} + 12\left(\frac{\sqrt{3}}{2}\right) + -\frac{\sqrt{3}}{2} - 0 = \frac{14\pi}{3} + 6\sqrt{3} - \frac{\sqrt{3}}{2}$$

$$= \frac{14\pi}{3} + \left(\frac{11\sqrt{3}}{2}\right)$$

Yes I did make a mistake solving my Trig equation