14.2 # 31) Volume of The region bounded by

$z = x + y, \quad x^2 + y^2 = 4, \quad 1^{st}$ octant.

$V = \iint_R (z_{Top} - z_{Bottom})\, dA$ where R is The region in The xy-plane That bounds The sides of The solid.

1^{st} Octant $\Rightarrow x \geq 0,\ y \geq 0,\ z \geq 0$ so $z_{Top} = x + y$

$z_{Bottom} = 0$

$V = \int_{\_}^{\_} \int_{\_}^{\_} x + y \; d\_\, d\_$ still have 6 blanks to fill based on The order desired.

Lateral bounds are $x^2 + y^2 = 4$ so solve for x or y.

$y = \pm\sqrt{4 - x^2}, \quad -2 \leq x \leq 2$ but 1^{st} oct. so

$0 \leq y \leq \sqrt{4 - x^2}, \quad 0 \leq x \leq 2$

$V = \int_0^2 \int_0^{\sqrt{4-x^2}} x + y \; dy\, dx$ Here ends The concepts and begins The grunt work.

$$\int_0^2 \int_0^{\sqrt{4-x^2}} x+y \; dy\, dx = \int_0^2 xy + \frac{1}{2}y^2 \Big|_0^{\sqrt{4-x^2}} dx$$

$$= \int_0^2 x\sqrt{4-x^2} + \frac{1}{2}(4-x^2)\,dx = \int_0^2 \underbrace{\boxed{x\sqrt{4-x^2}}}_{\text{u-sub}} + 2 - \frac{1}{2}x^2\, dx$$

$u = 4-x^2$

$du = -2x\, dx$

$-\frac{1}{2}$

$$\int -\frac{1}{2}u^{1/2}\,du = \frac{-1}{\cancel{2}}u^{3/2}\frac{\cancel{2}}{3} = -\frac{1}{3}(4-x^2)^{3/2}$$

$$= \left[-\frac{1}{3}(4-x^2)^{3/2} + 2x - \frac{1}{6}x^3\right]_0^2 = \left(0 + 4 - \frac{8}{6}\right) - \left(-\frac{1}{3}(4)^{3/2} + 0 - 0\right)$$

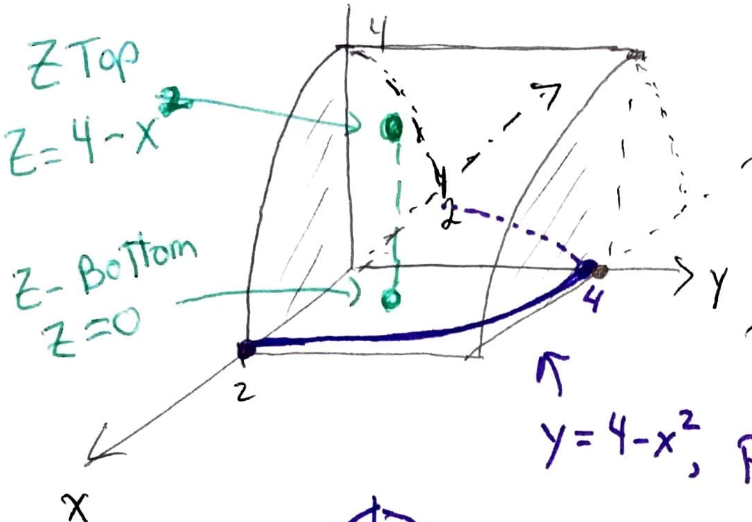
$$= 4 - \frac{4}{3} + \frac{8}{3} = 4 + \frac{4}{3} = \underline{16/3}$$

33.) Find Volume of solid bound by $\underbrace{z = 4 - x^2}$, $y = 4 - x^2$ in **1st Octant.** $z \geq 0,\ x \geq 0,\ y \geq 0$

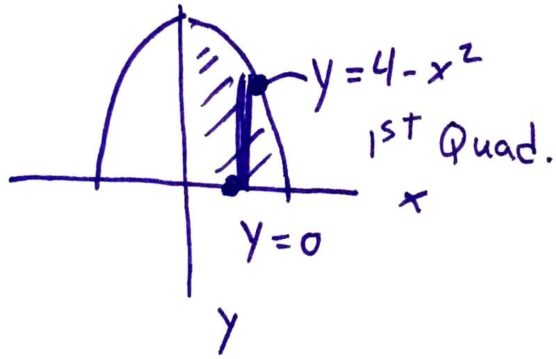
parabola in xz-plane

Since y is Not incuded y can be anything.

parabolic cylinder.



$y = 4 - x^2$, parabola in x,y plane



$$V = \int_0^2 \int_0^{4-x^2} 4 - x^2 \, dy \, dx = \int_0^2 (4-x^2)y \Big|_0^{4-x^2} dx$$

$$= \int_0^2 (4-x^2)^2 \, dx = \int_0^2 16 - 8x^2 + x^4 \, dx$$

$$= 16x - \frac{8x^3}{3} + \frac{x^5}{5} \Big|_0^2 = 32 - \frac{64}{3} + \frac{32}{5} - 0 = \frac{256}{15}$$