14.1

18.) $$\int_1^4 \int_1^{\sqrt{x}} 2y e^{-x} \, dy \, dx = \int_1^4 y^2 e^{-x} \Big|_1^{\sqrt{x}} dx$$

y - first

$$= \int_1^4 xe^{-x} - e^{-x} \, dx = \int_1^4 (x-1) e^{-x} \, dx$$

Parts

We may need Int. by parts

+/−	u	dv
+	$x-1$	$\cancel{e^{-x}}$
−	1	$-e^{-x}$
+	0	e^{-x}

$$= -(x-1)e^{-x} - e^{-x} \Big|_1^4 = -xe^{-x} + \cancel{e^{-x}} - \cancel{e^{-x}} \Big|_1^{4}$$

$$= -xe^{-x} \Big|_1^4 = -4e^{-4} - {}^{-}1e^{-x} = \underline{\frac{-4}{e^4} + \frac{1}{e}}$$

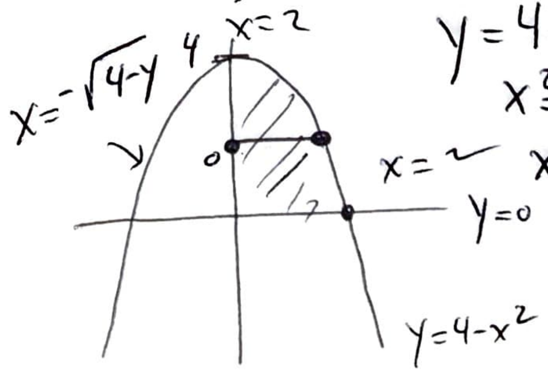
*

46.) $$\int_0^2 \int_0^{4-x^2} f(x,y) \, dy \, dx$$ To change order of integration

sketch region

$0 \leq y \leq 4-x^2$, $0 \leq x \leq 2$

$$= \int_0^4 \int_0^{\sqrt{4-y}} f(x,y) \, dx \, dy$$



$y = 4 - x^2$

$x^2 = 4 - y$

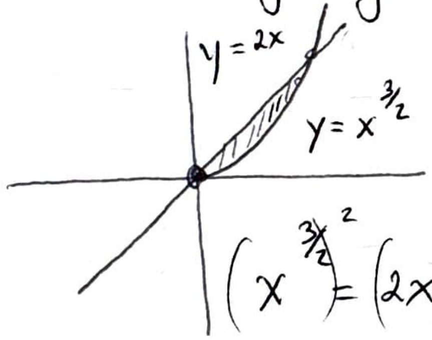
$x = \sqrt{4-y}$

Notes

$$\iint_R f(x,y)\,dA = \text{Volume of solid}$$

$$\iint_R 1\,dA = \text{Area of Region } R.$$

40.) Area of region bounded by $y = x^{3/2}$ and $y = 2x$



y 1st order Right →

$$\int_0^4 \int_{x^{3/2}}^{2x} 1\,dy\,dx$$

$2x$ ← Top of Region

0 ↑ Left

$x^{3/2}$ ↑ Bottom of Region

$(x^{3/2})^2 = (2x)^2$

$x^3 = 4x^2$

$x^3 - 4x^2 = 0$

$x^2(x-4) = 0$

$x = 0, 4$

If $y = 2x$ Then $x = \frac{1}{2}y$

$y = x^{3/2}$ Then $x = y^{2/3}$

$\left(\frac{1}{2}y\right)^3 = \left(y^{2/3}\right)^3$

$\frac{1}{8}y^3 = y^2$

$y^3 - 8y^2 = 0$

$y^2(y-8) = 0$ $y = 0, 8$

~~x 1st order $\int_0^8 \int_{\frac{1}{2}y}^{y^{2/3}} dx\,dy$~~

x 1st order Right

Top 8

$$\int_0^8 \int_{\frac{1}{2}y}^{y^{2/3}} 1\,dx\,dy$$

0 Bottom

$\frac{1}{2}y$ Left

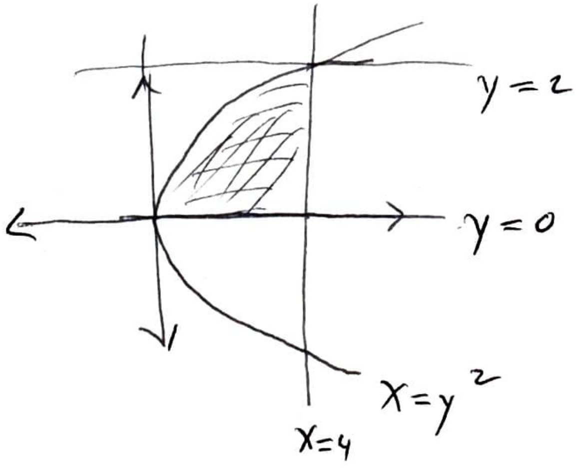
Set-up Limits based on $\int_{\text{Left}}^{\text{Right}} d\boxed{x}$

and $\int_{\text{Bottom}}^{\text{Top}} d\boxed{y}$

Note: MATH 9 Does <u>Not</u> work on Iterated Integrals.

66.) $\int_0^2 \int_{y^2}^4 \sqrt{x}\sin x \, dx\, dy$

We cannot evaluate This integral so let's Change The order.



Looking at This from bottom to Top

$y=0$ to $x=y^2 \rightarrow y=+\sqrt{x}$

Leftmost $x=0$ Rightmost $x=4$

$$\int_0^4 \int_0^{\sqrt{x}} \underbrace{\sqrt{x}\sin x}_{\text{constant}} \, dy\, dx = \int_0^4 (\sqrt{x}\sin x)\, y \Big|_0^{\sqrt{x}} dx$$

66.) continued

$$= \int_0^4 \left(\left(\sqrt{x}\ \sin x\right)\sqrt{x} - \left(\sqrt{x}\ \sin x\right)0 \right) dx$$

$$= \int_0^4 x \sin x \, dx \quad \text{by Parts}$$

+/-	u	dv
+	x	~~Sinx~~
-	1	- cos x
+	0	- Sinx

$$= \Big[-x\cos x + \sin x \Big]_0^4 = -4\cos(4) + \sin(4) - (0+0)$$

$$= \underline{\sin(4) - 4\cos(4)}$$

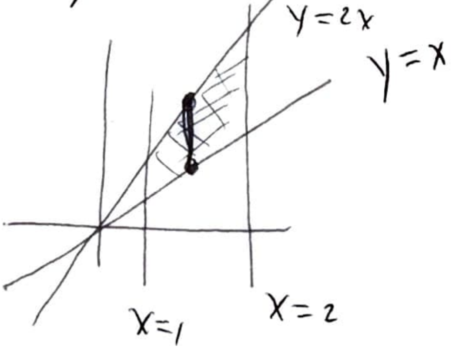
Check Last integral with MATH 9 ✓

14.2

15.) $\displaystyle\iint_R \frac{y}{x^2+y^2}\, dA$ ~~R is {(x,y)}~~ R is trapazoid bounded by $y=x$, $y=2x$, $x=1$, $x=2$

y 1st $\displaystyle\int_1^2 \int_x^{2x} \frac{y}{x^2+y^2}\, dy\, dx$

Bottom of Region is $y=x$, Top of Region is $y=2x$



$$\int_1^2 \int_x^{2x} \frac{y}{x^2+y^2}\, dy\, \cancel{dx} = \int_1^2 \frac{1}{2} \ln|x^2+y^2| \Big|_x^{2x} dx = \frac{1}{2}\int_1^2 \ln|5x^2| - \ln|2x^2|\, dx$$

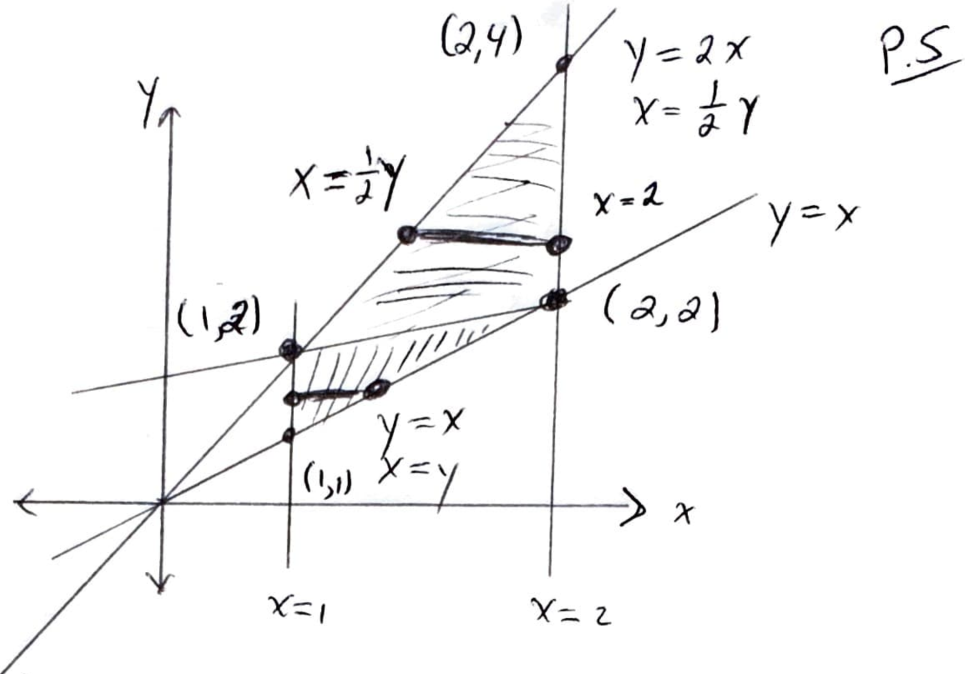
$u = x^2+y^2 \quad du = 2y\, dy$

$\frac{1}{2}$

Continued to *

P.S

$\#_{15}$ continued



Set-up

To use dx dy order

Need to divide The region into 2 Regions

Lower Region $1 \le x \le y$
$1 \le y \le 2$

upper Region $\frac{1}{2}y \le x \le 2$
$2 \le y \le 4$

$$\iint_R \frac{y}{x^2+y^2}\, dA = \int_1^2 \int_1^y \frac{y}{x^2+y^2}\, dx\, dy + \int_2^4 \int_{\frac{1}{2}y}^2 \frac{y}{x^2+y^2}\, dx\, dy$$

What a great example!

$$* \ \frac{1}{2} \int_1^2 \ln|5x^2| - \ln|2x^2|\, dx = \frac{1}{2} \int_1^2 \ln 5 + \cancel{2\ln x} - \ln 2 - \cancel{2\ln x}\, dx$$

$$= \frac{1}{2}\left[\ln 5 - \ln 2\right] x \Big|_1^2 = \frac{1}{2}(\ln 5 - \ln 2)(2-1) = \underline{\frac{1}{2}(\ln 5 - \ln 2)}$$

$$\int_1^2 \int_1^y \frac{y}{x^2+y^2}\, dx\, dy = \int_1^2 \cancel{y}\left(\frac{1}{\cancel{y}}\right) \operatorname{Tan}^{-1}\left(\frac{x}{y}\right)\Big|_1^{\cancel{y}} dy$$

$$= \int_1^2 \underset{\boxed{\pi/4}}{\operatorname{Tan}^{-1}(1)} - \operatorname{Tan}^{-1}\left(\frac{1}{\cancel{y}}\right) dy = \int_1^2 \frac{\pi}{4} - \operatorname{Tan}^{-1}\left(\frac{1}{y}\right) dy$$

$$= \int_1^2 \frac{\pi}{4}\, dy - \int_1^2 \operatorname{Tan}^{-1}\left(\frac{1}{y}\right) dy$$

$$\boxed{= \frac{\pi}{4}}$$

$$u = \operatorname{Tan}^{-1}\left(\frac{1}{y}\right) \qquad dv = dy$$

$$du = \frac{-1/y^2}{1+\frac{1}{y^2}}\, dy \qquad v = y$$

$$du = \frac{-1}{y^2+1}\, dy$$

$$= \frac{\pi}{4} - \left[y \operatorname{Tan}^{-1}\left(\frac{1}{y}\right) \overset{+}{\cancel{-}} \int_1^2 \frac{\cancel{-}y}{1+y^2}\, dy \right]$$

$$= \frac{\pi}{4} - \left[y \operatorname{Tan}^{-1}\left(\frac{1}{y}\right) + \frac{1}{2} \operatorname{Ln}|1+y^2| \right]_1^2$$

$$= \frac{\pi}{4} - \left[2 \operatorname{Tan}^{-1}(1/2) + \frac{1}{2} \operatorname{Ln} 5 - \left(\underset{\left(\frac{\pi}{4}\right)}{\operatorname{Tan}^{-1}(1)} + \frac{1}{2} \operatorname{Ln} 2 \right) \right]$$

$$\int_2^4 \int_{\frac{1}{2}y}^{2} \frac{y}{x^2+y^2}\,dx\,dy = \int_2^4 \underset{\nearrow}{\mathrm{Tan}^{-1}\left(\frac{x}{y}\right)}\Big|_{\frac{1}{2}y}^{2}\,dy$$

From 1st integral.

$$\int_2^4 \mathrm{Tan}^{-1}\left(\frac{2}{y}\right) - \mathrm{Tan}^{-1}(1/2)\,dy = \int_2^4 \mathrm{Tan}^{-1}\left(\frac{2}{y}\right)dy - \mathrm{Tan}^{-1}\left(\frac{1}{2}\right)y\Big|_2^4$$

$=$

$$u = \mathrm{Tan}^{-1}\left(\frac{2}{y}\right) \quad dv = dy$$

$$du = \frac{-2/y^2}{1+\frac{4}{y^2}}\,dy \quad v = y$$

$$du = \frac{-2}{y^2+4}\,dy \quad v = y$$

$$= y\,\mathrm{Tan}^{-1}\left(\frac{2}{y}\right)\Big|_2^4 - \int_2^4 \frac{-2y}{y^2+4}\,dy - \mathrm{Tan}^{-1}\left(\frac{1}{2}\right)(4-2)$$

$$= \underline{4\,\mathrm{Tan}}^{-1}\left(\frac{1}{2}\right) - 2\,\mathrm{Tan}^{-1}(1) + \mathrm{Ln}|y^2+4|\Big|_2^4 - 2\,\underline{\mathrm{Tan}}^{-1}(1/2)$$

$$= 2\,\mathrm{Tan}^{-1}(1/2) - 2(\pi/4) + \underbrace{\mathrm{Ln}\,20}_{2\mathrm{Ln}2+\mathrm{Ln}5} \underbrace{-\ \mathrm{Ln}\,8}_{-3\mathrm{Ln}2}$$

$$= 2\,\mathrm{Tan}^{-1}\left(\frac{1}{2}\right) - 2\left(\frac{\pi}{4}\right) + \mathrm{Ln}5 - \mathrm{Ln}2$$

So Together

$$\int_1^2 \int_1^y \frac{y}{x^2+y^2}\,dx\,dy + \int_2^4 \int_{\frac{1}{2}y}^{2} \frac{y}{x^2+y^2}\,dx\,dy$$

$$= \left[\frac{\pi}{4} - 2\operatorname{Tan}^{-1}\left(\frac{1}{2}\right) - \frac{1}{2}\operatorname{Ln} 5 + \frac{\pi}{4} + \frac{1}{2}\operatorname{Ln} 2\right] + \left[2\operatorname{Tan}^{-1}\left(\frac{1}{2}\right) - 2\left(\frac{\pi}{4}\right) + \operatorname{Ln} 5 - \operatorname{Ln} 2\right]$$

$$-\frac{1}{2}\operatorname{Ln} 5 + \frac{1}{2}\operatorname{Ln} 5 + \frac{1}{2}\operatorname{Ln} 2 - \operatorname{Ln} 2$$

$$= \cancel{2\left(\frac{\pi}{4}\right)} - \cancel{2\left(\frac{\pi}{4}\right)} - \cancel{2\operatorname{Tan}^{-1}\left(\frac{1}{2}\right)} + \cancel{2\operatorname{Tan}^{-1}\left(\frac{1}{2}\right)} + \frac{1}{2}\operatorname{Ln} 5 - \frac{1}{2}\operatorname{Ln} 2 \Big]$$

$$= \underline{\frac{1}{2}\operatorname{Ln} 5 - \frac{1}{2}\operatorname{Ln} 2}$$

same result but way Too much work to get here.