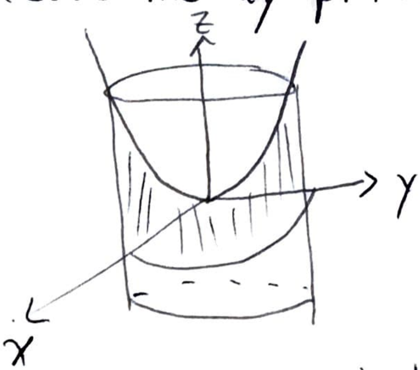
2153 additional examples for weekly Assign. #3

A.) Use a triple integral to find The volume of The solid inside The cylinder $x^2+y^2=4$, ~~above~~ below The paraboloid $z=x^2+y^2$ and above the xy-plane,

In Rectangular Form

$$V=\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{0}^{x^2+y^2} 1\,dz\,dy\,dx$$

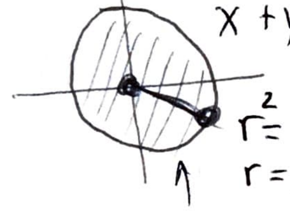


$0 \le z \le$ paraboloid

$0 \le z \le x^2+y^2$

↓ Project onto xy-plane.

$x^2+y^2=4$



$r^2=4$

$r=2$

In Cylindrical Form.

$$V=\int_{0}^{2\pi}\int_{0}^{2}\int_{0}^{r^2} 1\,r\,dz\,dr\,d\theta$$

For practice only: in spherical form

Intersection of $z=x^2+y^2$ and $x^2+y^2=4$

$y=\pm\sqrt{4-x^2}$

Worked out at The end

will help with #7.

Intersection of paraboloid & cylinder

$$\frac{\cos\phi}{\sin^2\phi}=\frac{2}{\sin\phi}$$

$\cot\phi=2$

$\frac{1}{2}=\tan\phi$

$\phi=\tan^{-1}\left(\frac{1}{2}\right)$

$z=r^2$ paraboloid

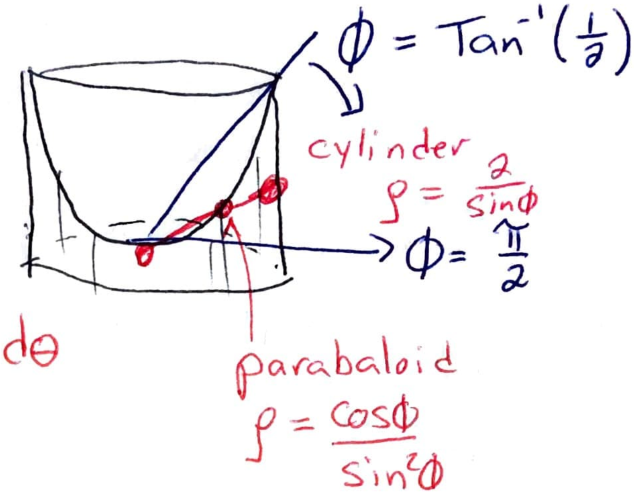
$\rho\cos\phi=\rho^2\sin^2\phi$

$\rho=\frac{\cos\phi}{\sin^2\phi}$

Cylinder $r=2$ $\rho=\frac{2}{\sin\phi}$

1) Spherical Nightmare.

$$V = \int_0^{2\pi} \int_{\tan^{-1}(\frac{1}{2})}^{\frac{\pi}{2}} \int_{\frac{\cos\phi}{\sin^2\phi}}^{\frac{2}{\sin\phi}} 1\, \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$



Best: Cylindrical

$$V = \int_0^{2\pi} \int_0^{2} \int_0^{r^2} r \, dz \, dr \, d\theta = \int_0^{2\pi} \int_0^{2} r z \Big|_0^{r^2} dr \, d\theta$$

$$= \int_0^{2\pi} \int_0^{2} r^3 \, dr \, d\theta = \int_0^{2\pi} \frac{r^4}{4} \Big|_0^{2} d\theta = \int_0^{2\pi} 4 \, d\theta$$

$$= 4\theta \Big|_0^{2\pi} = \underline{8\pi}$$

$$\int_0^{2\pi}\int_{\text{Tan}^{-1}\left(\frac{1}{2}\right)}^{\frac{\pi}{2}}\int_{\frac{\cos\phi}{\sin^2\phi}}^{\frac{2}{\sin\phi}} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

call me crazy but let's try spherical.

$$= \int_0^{2\pi}\int_{\text{Tan}^{-1}\left(\frac{1}{2}\right)}^{\frac{\pi}{2}} \frac{\rho^3}{3}\sin\phi \Big|_{\frac{\cos\phi}{\sin^2\phi}}^{\frac{2}{\sin\phi}} d\phi \, d\theta = \frac{1}{3}\int_0^{2\pi}\int_{\text{Tan}^{-1}\left(\frac{1}{2}\right)}^{\frac{\pi}{2}} \frac{8\sin\phi}{\sin^3\phi} - \frac{\cos^3\phi\sin\phi}{\sin^6\phi} \, d\phi \, d\theta$$

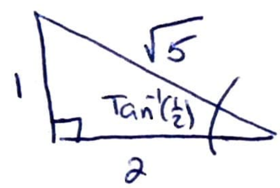
$$= \frac{1}{3}\int_0^{2\pi}\int_{\text{Tan}^{-1}\left(\frac{1}{2}\right)}^{\frac{\pi}{2}} \frac{8}{\sin^2\phi} - \frac{\cos^3\phi}{\sin^5\phi} \, d\phi \, d\theta = \frac{1}{3}\int_0^{2\pi}\int_{\text{Tan}^{-1}\left(\frac{1}{2}\right)}^{\pi/2} \underbrace{8\csc^2\phi}_{-8\cot\phi} - \underbrace{\cot^3\phi\csc^2\phi} \, d\phi \, d\theta$$

$u = \cot\phi$

$du = -\csc^2\phi \, d\phi$

$\int u^3 \, du = \frac{u^4}{4}$

$$= \frac{1}{3}\int_0^{2\pi} -8\cot\phi + \frac{1}{4}\cot^4\phi \Big|_{\text{Tan}^{-1}\left(\frac{1}{2}\right)}^{\frac{\pi}{2}} d\theta$$



oops!

$$= \frac{1}{3}\int_0^{2\pi}\left(-8(2) + \frac{1}{4}(2^4)\right) - (0+0) \, d\theta$$

$\cot(\text{Tan}^{-1}(\frac{1}{2})) = \frac{2}{1}$

$$= \frac{1}{3}\int_0^{2\pi} 0 - (-16+4) \, d\theta = \int_0^{2\pi} 4 \, d\theta = \underline{8\pi}$$