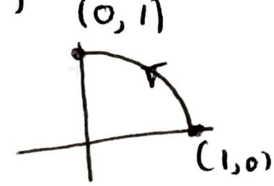
Section 15.2 Examples

Evaluate

1.) $\int_C (x^2+y^2)\, ds =$

where C is The counterclockwise path around The circle $x^2+y^2=1$ from $(1,0)$ to $(0,1)$



Solution

parameterize The curve.

recall The best way to parameterize a Circle is $x = \cos t \quad y = \sin t$

$\boxed{\text{or } x = r\cos t \quad y = r\sin t \quad \text{here } r = 1.}$

To start at $(1,0)$ $t=0$

To end at $(0,1)$ $t = \pi/2$

Now find $x' = -\sin t \quad y' = \cos t$

So $$\int_C (x^2+y^2)\, dx = \int_0^{\pi/2} (\cos^2 t + \sin^2 t)\sqrt{(-\sin t)^2 + (\cos t)^2}\, dt$$

$$= \int_0^{\frac{\pi}{2}} 1 \sqrt{1}\, dt = t \Big|_0^{\pi/2} = \frac{\pi}{2}.$$

2.) 15.2 #15 Evaluate $\int_C (2x + 3\sqrt{y})\, ds$ where

C is The Triangle with vertices at $(0,0)$, $(1,0)$ and $(0,1)$

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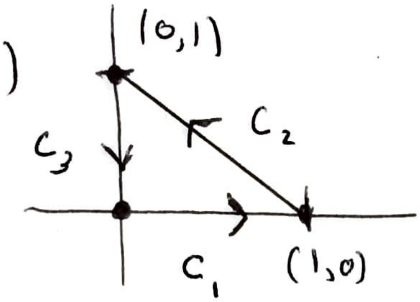
2) (continued)



Need 3 curves: C_1: from $(0,0) \to (1,0)$ $\quad x = t \quad 0 \le t \le 1$
$y = 0$

Practice forming Parametric Eq'n of Line using vectors.

C_2: from $(1,0) \to (0,1)$ $\quad x = 1 - t \quad 0 \le t \le 1$

$y = 1 - x \quad x = 1 - y \quad y = 0 + t$

Vector from (1,0) to (0,1) $= \langle -1, 1 \rangle$

(Recall ~~Eo~~ Parametric Eq'n)

C_3: from (0,1) to (0,0)

Vector $\langle 0, -1 \rangle$ $\quad x = 0 \quad 0 \le t \le 1$
$y = 1 - t$

C_1: $x = t \quad x' = 1$
$y = 0 \quad y' = 0$
$0 \le t \le 1$

$$\int_{C_1} (2x + 3\sqrt{y})\,ds = \int_0^1 2t\sqrt{1^2 + 0^2}\,dt = t^2\Big|_0^1 = 1$$

C_2: $x = 1 - t \quad x' = -1$
$y = t \quad y' = 1$
$0 \le t \le 1$

$$\int_{C_2} (2x + 3\sqrt{y})\,ds = \int_0^1 (2(1-t) + 3\sqrt{t})\sqrt{1+1}\,dt$$

$$= \int_0^1 (2 - 2t + 3t^{1/2})\sqrt{2}\,dt$$

$$= \sqrt{2}\left[2t - t^2 + 3t^{3/2} \cdot \tfrac{2}{3}\right]_0^1 = \sqrt{2}\left[(2 - 1 + 2) - 0\right] = 3\sqrt{2}$$

2 (continued)

C_3: $x = 0 \quad x' = 0$

$y = 1 - t \quad y' = -1$

$0 \le t \le 1$

$$\int_C (2x + 3\sqrt{y})\, ds = \int_0^1 3(1-t)^{1/2} \sqrt{0+1}\, dt$$

$$= \int_0^1 3(1-t)^{1/2}\, dt = \cancel{3}(1-t)^{3/2} \cdot \frac{2}{\cancel{3}}(-1) \Big|_0^1$$

$$= 0 - (-2) = 2$$

So $\int_C (2x + 3\sqrt{y})\, ds = 1 + 3\sqrt{2} + 2 = \underline{3 + 3\sqrt{2}}$

3.) $\int_C 2xyz\, ds \qquad C: \vec{r}(t) = 12t\,\hat{i} + 5t\,\hat{j} + 84t\,\hat{k}$

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$0 \le t \le 1$

$x = 12t \quad x' = 12$

$y = 5t \quad y' = 5$

$z = 84t \quad z' = 84$

$$\int_0^1 2(12t)(5t)(84t)\sqrt{12^2 + 5^2 + 84^2}\, dt$$

$$= \int_0^1 10080\, t^3 \underbrace{\sqrt{7225}}_{85}\, dt = \frac{856800\, t^4}{4} \Big|_0^1 = 214200$$

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4.) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = xy\hat{i} + xz\hat{j} + yz\hat{k}$

and $C: \vec{r}(t) = t\hat{i} + t^2\hat{j} + 2t\hat{k}$

$0 \le t \le 1$

$$x = t \qquad dx = dt$$
$$y = t^2 \qquad dy = 2t\,dt$$
$$z = 2t \qquad dz = 2\,dt \qquad 0 \le t \le 1$$

$$\int_C \vec{F} \cdot d\vec{r} = \int_C M\,dx + N\,dy + P\,dz$$

$$= \int_0^1 \overset{x}{(t)}\overset{y}{(t^2)}\,\overset{dx}{dt} + \overset{x}{(t)}\overset{z}{(2t)}\,\overset{dy}{2t\,dt} + \overset{y}{(t^2)}\overset{z}{(2t)}\,\overset{dz}{2\,dt}$$

$$= \int_0^1 \left(t^3 + 4t^3 + 4t^3\right) dt = \int_0^1 9t^3\,dt = \frac{9t^4}{4}\Big|_0^1 = \frac{9}{4}$$

5.) Evaluate $\int_C (x+y)\,dx + x\,dy + (x-z)\,dz$

Where C is The Line Segment from $(1,0,1)$ to $(3,1,2)$

Line Segment: $\vec{V} = \langle 3-1, 1-0, 2-1 \rangle = \langle 2, 1, 1 \rangle$

$x = 1 + 2t$, $y = 0 + t$, $z = 1 + t$ $\quad 0 \le t \le 1$

$dx' = 2\,dt \qquad dy = dt \qquad dz = dt$

$$\int_0^1 [(1+2t) + t]\,2\,dt + (1+2t)\,dt + [(1+2t) - (1+t)]\,dt$$

$$= \int_0^1 (2 + 6t + 1 + 2t + t)\,dt = \int_0^1 (3 + 9t)\,dt = 3t + \frac{9t^2}{2}\Big|_0^1 = \frac{15}{2}$$