F0 – Decimals and Scientific Notation

Introduction

Scientific notation provides us with a different way to write numbers, particularly numbers that are extremely large or very small, to be able to express the magnitude of the size of the numbers while providing a certain amount of precision to their value. In many fields of science, engineering, computer technology, and mathematics, the size of the numbers that are used can be extremely large (such as vast distances between stars in astronomy, or the number of particles in a unit of a substance in chemistry) or extremely small (such as the time it takes a computer to make a calculation, or the mass of an electron). In particular for Statistics, there will be times when probabilities that we calculate end up being very small - close to 0, and our technology (Excel, calculator, etc.) will display the very small numbers with scientific notation, so we will need to know how to read and interpret them.

For example, in chemistry, Avogadro's number is the number of particles (atoms, molecules, etc.) in one mole of a substance, and that number is usually written as 6.022×10^{23}. This is scientific notation, and it's easier to write and work with than the standard decimal form of this number: $602{,}214{,}076{,}000{,}000{,}000{,}000{,}000$. The mass of an electron is $0.00000000000000000000000000000091$ kg, but again this is much easier to write and work with in scientific notation as: 9.1×10^{-31}. In this section we'll practice converting back and forth between scientific notation and standard decimal form.

$$6.022 \times 10^{23} \quad = 602{,}200{,}000{,}000{,}000{,}000{,}000{,}000$$

significant digits x power of ten

$$9.1 \times 10^{-31} = 0.00000000000000000000000000000091$$

Powers of 10

Since the number system we use is a base 10 system, powers of 10 are very important to be able to describe place values, as we worked on previously. Because of that, we use powers of 10 in scientific notation. When 10 is multiplied by itself several times (raised to a positive power), we can write the result easily as a 1 followed by as many 0's as we had in the exponent: $10^2 = 10 \cdot 10 = 100$, $10^3 = 10 \cdot 10 \cdot 10 = 1000$, and so on.

When we divide by a power of 10, we can also treat it as multiplication by 10 raised to a negative power, which can also be written in decimal form without calculation by writing a number of 0's between the decimal point and 1. The number of 0's is 1 less than the exponent: $10^{-2} = \frac{1}{10^2} = 0.01$, $10^{-3} = \frac{1}{10^3} = 0.001$, and so on.

Power of 10	Standard decimal form
10^4	$10{,}000$
10^3	$1{,}000$
10^2	100
10^1	10
10^0	1
10^{-1}	0.1
10^{-2}	0.01
10^{-3}	0.001
10^{-4}	0.0001

An easy way to think of this is to start with 1, and for a positive exponent on a power of 10, beginning with the decimal point just after the 1, move the decimal point to the right that many places, and fill in the missing places with 0's. For a negative power of 10, beginning with the decimal point just after the 1, move the decimal point to the left that many places, and again fill in the missing places with 0's.

Because powers of 10 can be written in this way, multiplying a number by a power of 10 can be done easily by shifting the decimal point. We've done this before when we convert between percentages and decimal values by moving the decimal point to the right or left two places to multiply or divide by 100. This is what allows scientific notation to be a convenient way to write extremely large and extremely small numbers – the digits do not change, it's just the location of the decimal point that changes.

Standard Decimal Form → Scientific Notation

To convert a number written in standard decimal form into scientific notation, shift the decimal point to move just after the first non-zero digit. The number of places you moved the decimal point will become the power of 10 we write – a positive power if you shifted the decimal point to the left because the number was very large, or a negative power if you shifted it to the right because the number was a very small decimal value. Write the number with the shifted decimal point (it should always be a number between 1 and 10, rounded to a specified number of significant digits, if necessary, then write " $\times 10^{power}$ " after it.

Examples: Write each number in scientific notation.

$64{,}200 = 6.42 \times 10^4$

$0.00243 = 2.43 \times 10^{-3}$

$1{,}000{,}000{,}000 =$

$0.00001 =$

$0.00000992 =$

$54{,}000{,}000 =$

Scientific Notation → Standard Decimal Form

To go the other way and write a number that is given in scientific notation back into standard decimal form, we again just move the decimal point. For a number in scientific notation with a *positive power* of 10, shift the decimal point *to the right* that many places, and fill in 0's at the end, if needed. For a number in scientific notation with a *negative power* of 10, shift the decimal point *to the left* that many places, and fill in 0's between the new decimal point and your other digits, as needed.

Examples: Write each number in standard decimal form

$8.33 \times 10^{-6} = .00000833$

$4.44 \times 10^3 = 4{,}440$

$1 \times 10^{-10} =$

$3.36 \times 10^{-4} =$

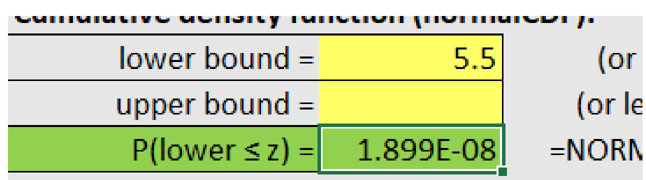
$7.2 \times 10^{11} =$

Technology Displays

The most common way that we will encounter scientific notation in our Statistics course is through probabilities calculated with technology (calculator, Excel, etc.) that end up being very small and need to be displayed with scientific notation. Most technology displays scientific notation a little bit differently than the way we've practiced with it so far – they will use a letter 'E' or 'e' to indicate the power of 10 in place of the " $\times 10^{power}$ ". See the examples below, and the live examples in the videos.

Examples: Write the calculated number shown in each technology display in scientific notation with 3 significant digits. Then write it in standard decimal form.

$4.68E\text{-}6 = 4.68 \times 10^{-6} = 0.00000468$



$1.899E\text{-}8 =$