



CMPUT 274

Recursion

Topics Covered:

- What is recursion?
- Conditions for termination
- Stack frames

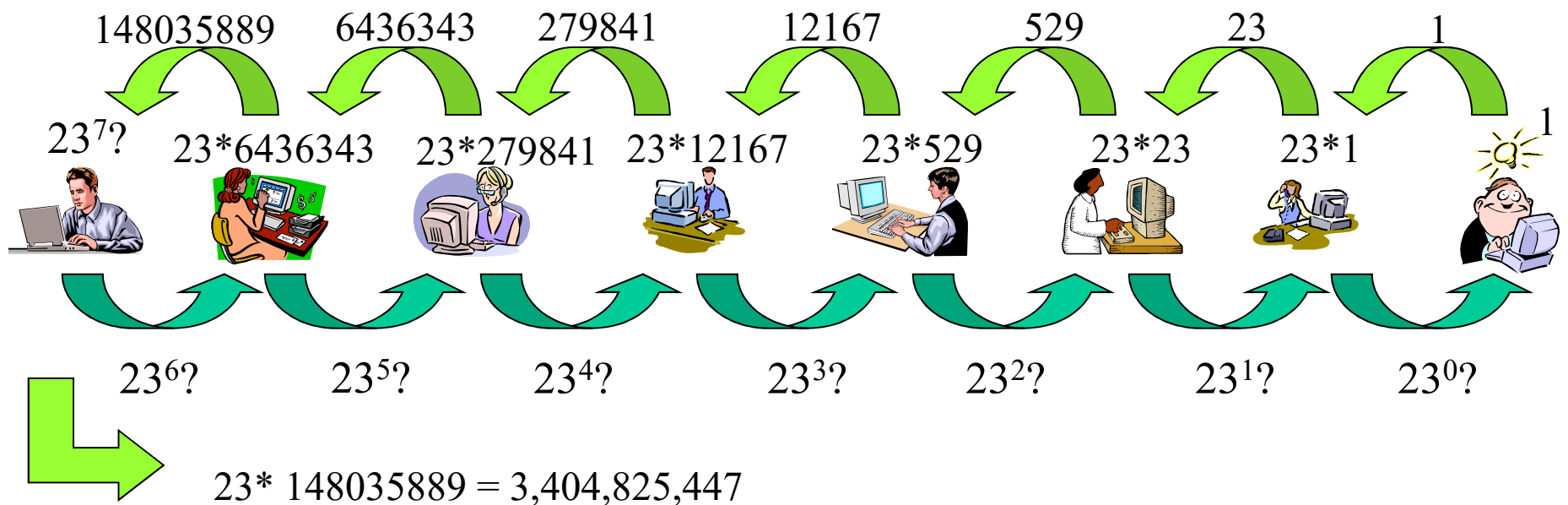
Recursion

- **Recursion** occurs when a function (or method) calls itself, either directly or indirectly.
- If a problem can be resolved by solving a simple part of it and resolving the rest of the big problem the same way, we can write a function that solves the simple part of the problem then calls itself to resolve the rest of the problem.
- This is called a **recursive function**.



Recursive Function Example

- Suppose we want to calculate 23^7 . We know that 23^7 is $23 * 23^6$. If we know the solution for 23^6 we would know the solution for 23^7 .



Recursive Methods

- For recursion to **terminate**, two conditions must be met:
 - there must be one or more simple cases that do **NOT** make recursive calls. (**base case**)
 - the recursive call must somehow be simpler than the original call. (Change the state to move towards the base case.)

Example: Factorial

- For example, how can we write a recursive function that computes the factorial of an Integer:

$$0! = 1$$

$$1! = 1$$

$$2! = 2 * 1 = 2$$

$$\rightarrow 2! = 2 * 1!$$

$$3! = 3 * 2 * 1 = 6$$

$$\rightarrow 3! = 3 * 2!$$

$$n! = n * (n-1) * \dots * 3 * 2 * 1$$

$$\rightarrow n! = n * (n-1)!$$

- The last observation, together with the simple cases, is the basis for a recursive function.

Factorial Function

$$n! = n * (n-1)!$$

```
def factorial(number):  
    '''  
    Return the factorial of number.  
    '''  
    if (number == 0 or number == 1): # base case  
        answer = 1  
    else:  
        answer = number * factorial(number-1)  
  
    return answer
```

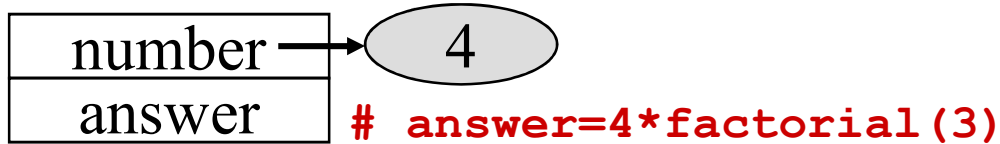
Function Activations and Frames

- When a function is invoked, a **frame** or **stack frame** corresponding to that function is created and pushed onto the call stack.
- The frame stores all of the **local variables** associated with that function call.
- The frame is created when the function is invoked, and destroyed when the function finishes.
- If a function is invoked again, a new frame is created for it with all its local variables.

Multiple Activations of a Function

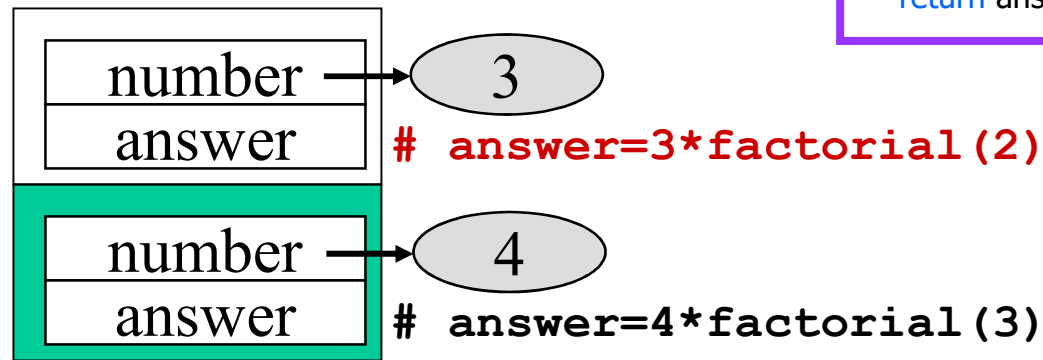
- When we invoke a recursive function, the function becomes active.
- Before it is finished, it makes a recursive call to the same function.
- This means that when recursion is used, there is more than one copy of the same function active at once.
- Therefore, each active function has its own frame which contains independent copies of its local variables.
- These frames are stored on the call stack.

Calling factorial(4)

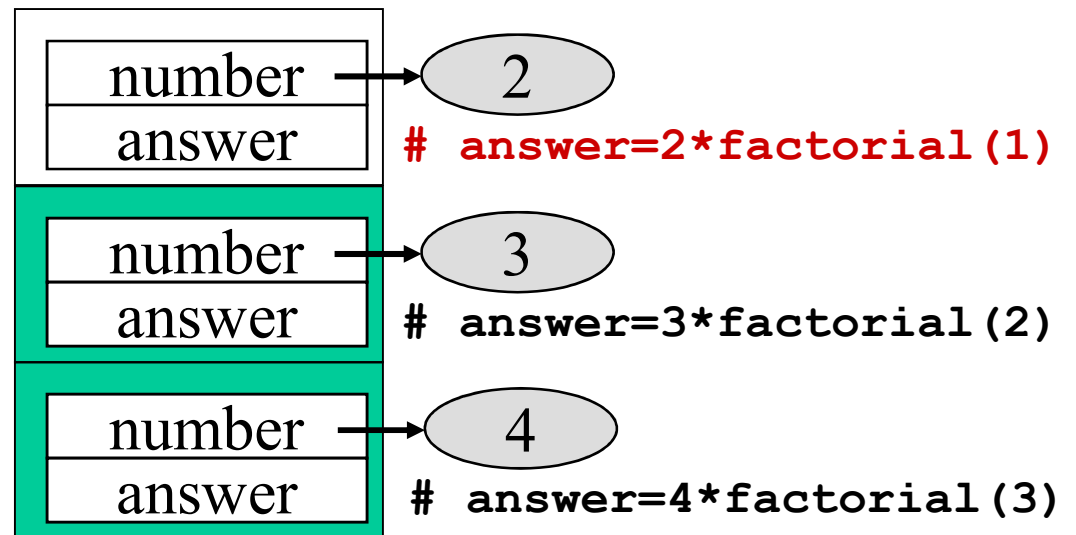


Call Stack

```
def factorial(number):  
    if (number == 0 or number == 1):  
        answer = 1  
    else:  
        answer = number * factorial(number-1)  
    return answer
```



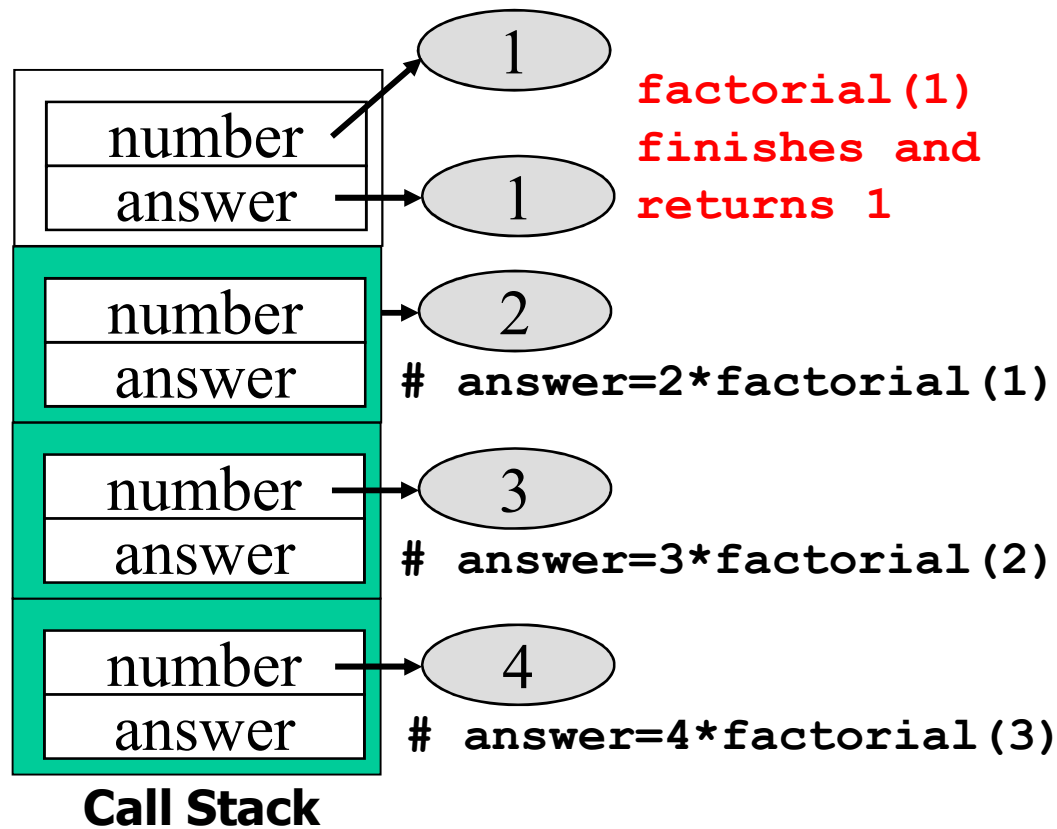
Call Stack



Call Stack

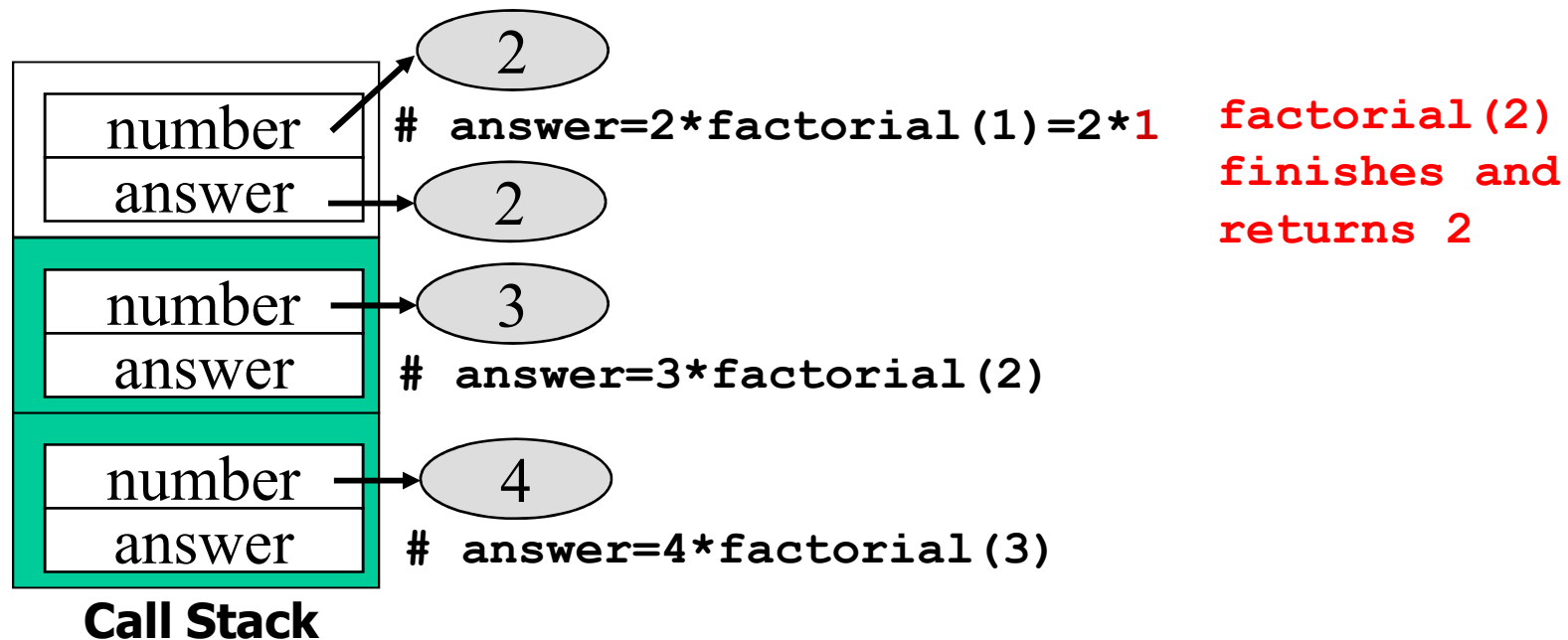
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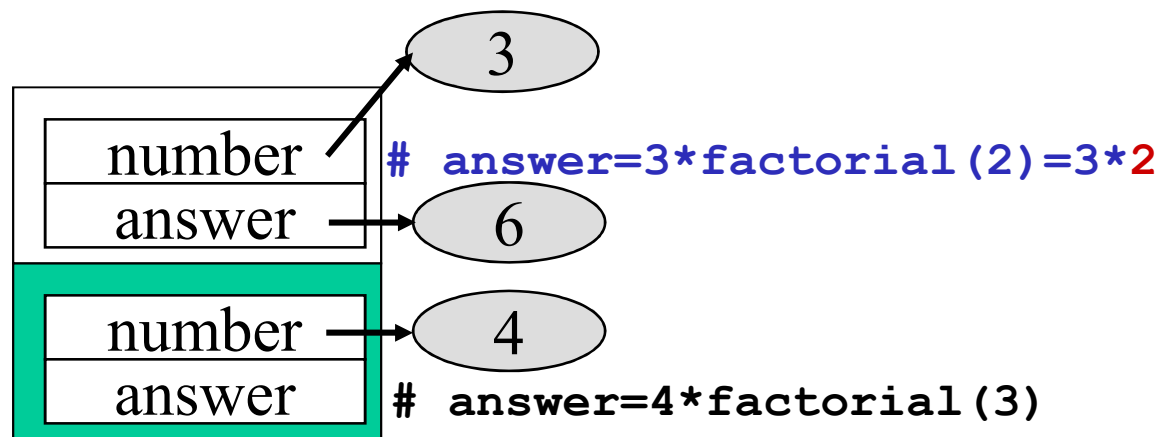
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Calling factorial(4)

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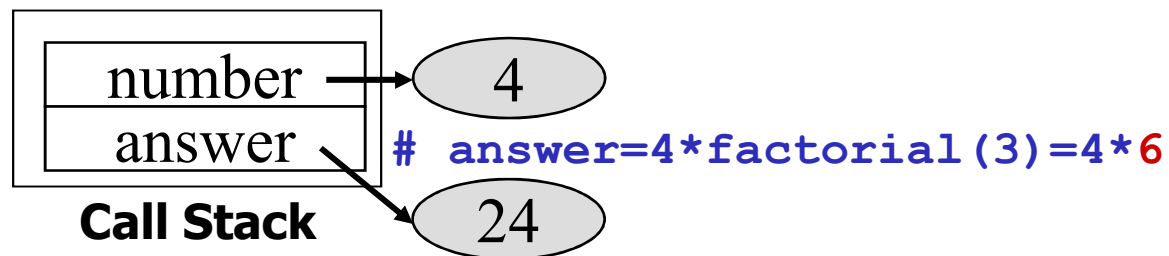


Call Stack

**factorial(3)
finishes and
returns 6**

Calling factorial(4)

```
def factorial(number):  
    if (number == 0 or number == 1):  
        answer = 1  
    else:  
        answer = number * factorial(number-1)  
    return answer
```



**factorial(4)
finishes and
returns 24**