

# Setup

this section loads and installs all the packages. You should be setup already from assignment 1, b if not please read and follow the `instructions.md` for further details.

```
• begin
•     using CSV , DataFrames , Random
•     using PlutoUI
•     using PlotlyJS
•     import Colors : Colors, @colorant_str
•     using LinearAlgebra : dot, norm, norm1, norm2, I
•     using Distributions : Distributions, Uniform
•     using Statistics
•     using MultivariateStats : MultivariateStats, PCA
•     using StatsBase : StatsBase
```

## !!!IMPORTANT!!!

Insert your details below. You should see a green checkmark.

Welcome Sanad Masannat! 

```
• let
•     def_student = (name="NAME as in eclass", email="UofA Email", ccid="CCID",
idnumber=0)
•     if length(keys(def_student) ∩ keys(student)) != length(keys(def_student))
•         md"You don't have all the right entries! Make sure you have 'name', 'email',
'ccid', 'idnumber'. ✘"
•     elseif any(getfield(def_student, k) == getfield(student, k) for k in
keys(def_student))
•         md"You haven't filled in all your details! ✘"
•     elseif !all(typeof(getfield(def_student, k)) === typeof(getfield(student, k))
in keys(def_student))
•         md"Your types seem to be off: 'name::String', 'email::String', 'ccid::String',
'idnumber::Int' ✘"
•     else
•         md"Welcome $(student.name)! ✌"
•     end
```



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```
student =  
(name = "Sanad Masannat", email = "sanad@ualberta.ca", ccid = "sanad", idnumber = 162622  
• student = (  
•     name="Sanad Masannat",  
•     email="sanad@ualberta.ca",  
•     ccid="sanad",  
•     idnumber=1626221
```

Important Note: You should only write code in the cells that has:

- ##### BEGIN SOLUTION
- 
- 

# Preamble

In this assignment, we will implement:

- Q2(a) Mini-batch Gradient Descent ✓
- Q2(b,c) Loss functions MSE ✓ , and gradient of MSE ✓
- Q2(d-f) Optimizers: Constant LR ✓ , Heuristic Stepsize ✓ , and AdaGrad ✓
- Q2(g) Polynomial Features ✓

check\_elements (generic function with 1 method)

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# Q2: Multi-variate Regression

In the last assignment you learned the weight for a simplistic univariate setting, to predict  $y$  from  $x$ . Now we get to move to the multivariate setting! This means more than one input, which is a much more realistic problem setting.

Unlike before, instead of having a struct be all the properties of an ML systems we will break our systems into smaller pieces. This will allow us to more easily take advantage of code we've already written, and will be more useful as we expand the number of algorithms we consider. We make several assumptions to simplify the code, but the general type hierarchy can be used much more broadly.

We split each system into:

- Model
- Gradient Descent Procedure
- Loss Function
- Optimization Strategy

## Baselines

### Mean Model

train! (generic function with 1 method)

```
begin
    """
    MeanModel()
    Predicts the mean value of the regression targets passed in through `epoch!`.
    """
    mutable struct MeanModel <: AbstractModel
        μ::Float64
    end
    MeanModel() = MeanModel(0.0)
    predict(reg::MeanModel, X::AbstractVector) = reg.μ
    predict(reg::MeanModel, X::AbstractMatrix) = fill(reg.μ, size(X,1))
    Base.copy(reg::MeanModel) = MeanModel(reg.μ)
    function train!(::MiniBatchGD, model::MeanModel, lossfunc, opt, X, Y, num_epochs)
        model.μ = mean(Y)
    end
end
```

### RandomModel



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```
train! (generic function with 2 methods)
```

```
• begin
  • """
  •     RandomModel
  • 
  •     Predicts 'b*x' where 'b' is sampled from a normal distribution.
  • """
  • struct RandomModel <: AbstractModel # random weights
  •     w::Matrix{Float64}
  • end
  • RandomModel(in, out) = RandomModel(randn(in, out))
  • predict(reg::RandomModel, X::AbstractMatrix) = X*reg.w
  • Base.copy(reg::RandomModel) = RandomModel(randn(size(reg.w)...))
  • train!(::MiniBatchGD, model::RandomModel, lossfunc, opt, X, Y, num_epochs) =
    nothing
```

## Models

- `AbstractModel`: This is an abstract type which is used to derive all the model types in this assignment
- `predict` : This takes a matrix of samples and returns the prediction doing the proper data transforms.
- `get_features` : This transforms the features according to the non-linear transform of the model (which is the identity for linear).
- `get_linear_model` : All models are based on a linear model with transformed features, and thus have a linear model.
- `copy` : This returns a new copy of the model.

```
Main.workspace2.AbstractModel
```

```
predict (generic function with 1 method)
```

```
update_transform! (generic function with 1 method)
```

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# Linear Model

A linear model is the linear function

$$f(x) = \mathbf{x}^\top \mathbf{w}$$

giving us a prediction  $\hat{y}$ .

Note that to query this function on more than one sample, we can use the fact that  $\mathbf{X}\mathbf{w}$  corresponds to a vector where the first element is the dot product between the first row of  $\mathbf{X}$  and  $\mathbf{w}$ , the second element is the dot product between the second row of  $\mathbf{X}$  and  $\mathbf{w}$  and so on. We exploit this in `predict`, to return predictions for the data matrix  $\mathbf{X}$  of size (samples, features).

We define `get_features`, which we will need for polynomial regression. For linear regression, the default is to return the inputs themselves. In polynomial regression, we will replace this function with one that returns polynomial features.

`get_features` (generic function with 1 method)

```
• begin
•     struct LinearModel <: AbstractModel
•         w::Matrix{Float64} # Aliased to Array{Float64, 2}
•     end
• 
•     LinearModel(in, out=1) =
•         LinearModel(zeros(in, out)) # feature size x output size
• 
•     Base.copy(lm::LinearModel) = LinearModel(copy(lm.w))
•     predict(lm::LinearModel, X::AbstractMatrix) = X * lm.w
•     get_features(m::LinearModel, x) = x
• 
```

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# (a) Mini-batch Gradient Descent

```
• begin
•     __check_MBGD = let
•
•         lm = LinearModel(3, 1)
•         opt = _LR()
•         lf = _LF()
•         X = ones(10, 3)
•         Y = collect(0.0:0.1:0.9)
•         mbgd = MiniBatchGD(5)
•         epoch!(mbgd, lm, lf, opt, X, Y)
•         all(lm.w .== -10.0)
•     end
•     str = "<h2 id=graddescent> (a) ${__check_complete(__check_MBGD)} Mini-batch Gr
•     Descent </h2>"
•     HTML(str)
```



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For this notebook we use minibatch gradient descent, and three stepsize approaches: ConstantLR, HeuristicLR, and AdaGrad. We provide a default update function below, that does gradient descent with a stepsize of 1.0. This update function will be defined for each of these three stepsize approaches later.

Notice that we use `.-=` which is the same as `lm.w = lm.w .- Δw`. The dot-minus means elementwise subtraction, for the vectors `lm.w` and `Δw`. In general, prefacing with a dot means elementwise operations: `a.*b` would mean elementwise product between vectors `a` and `b`, and `a./b` would mean elementwise division.

```
• begin
•     struct _LR <: Optimizer end
•     struct _LF <: LossFunction end
•     function gradient(lm::LinearModel, lf::_LF, X::Matrix, Y::Vector)
•         sum(X, dims=1)
•     end
•     function update!(lm::LinearModel,
•                     lf::_LF,
•                     opt::_LR,
•                     x::Matrix,
•                     y::Vector)
•
•         φ = get_features(lm, x)
•
•         Δw = gradient(lm, lf, φ, y)[1, :]
•         lm.w .-= Δw
•     end
•
•     struct MiniBatchGD
•         n::Int
```

First, you will set up the basic minibatch gradient descent code. You need to implement the function `epoch!` which goes through the data set in minibatches of size `mbgd.n`. Remember to shuffle the data for each epoch. In your code, you can call the function

```
update!(model, lossfunc, opt, X_batch, Y_batch)
```

to update your model in the epoch. Again, we will use different updates depending on the steps rules, defined in the section below on [optimizers](#).

`epoch!` (generic function with 1 method)

```
• function epoch!(mbgd::MiniBatchGD, model::LinearModel, lossfunc, opt, X, Y)
•     perm=randperm(length(Y))
•     X=X[perm,:]
•     Y=Y[perm]
•
•     batch_num=(length(Y))/Int(mbgd.n)
•     for i in 1:batch_num
•         low_range=Int((i-1)*mbgd.n +1)
•         high_range=Int(i*mbgd.n)
•         if high_range>length(Y)
•             break
•         end
•         update!(model::LinearModel, lossfunc, opt,
•             X[low_range:high_range,:],Y[low_range:high_range])
•     end
• 
```

`epoch!` (generic function with 2 methods)

```
• function epoch!(mbgd::MiniBatchGD, model::AbstractModel, lossfunc, opt, X, Y)
•     epoch!(mbgd, get_linear_model(model), lossfunc, opt, get_features(lp.model, X))
```

`train!` (generic function with 3 methods)

```
• function train!(mbgd::MiniBatchGD, model::AbstractModel, lossfunc, opt, X, Y,
•     num_epochs)
•     train!(mbgd, get_linear_model(model), lossfunc, opt, get_features(model, X),
•     num_epochs)
```

`train!` (generic function with 4 methods)

```
• function train!(mbgd::MiniBatchGD, model::LinearModel, lossfunc, opt, X, Y, num_epochs)
•     L = zeros(num_epochs + 1)
•     L[1] = loss(model, lossfunc, X, Y)
•     for i in 1:num_epochs
•         epoch!(mbgd, model, lossfunc, opt, X, Y)
•         L[i+1] = loss(model, lossfunc, X, Y)
•     end
•     L
• end
```

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## Loss Functions

For this notebook we will only be using MSE, but we still introduce the abstract type `LossFunction` for the future. Below you will need to implement the `loss`  function and the `gradient`  function for MSE.

```

• begin
•     # __check_mseGrad
•     lm1 = LinearModel(3, 1)
•     lm2 = LinearModel(3, 1)
•     lm2.w .+= 1
•     __check_mseloss = loss(lm1, MSE(), ones(4, 3), [1,2,3,4]) == 3.75 && loss(lm2,
MSE(), ones(4, 3), [1,2,3,4]) == 0.75 && loss(lm2, MSE(), ones(4, 3), [7,8,9,0])
10.75
•     __check_msegrad = all(gradient(LinearModel(3, 1), MSE(), ones(4, 3), [1,2,3,4]) ==
-2.5)
•
•     __check_MSE = __check_mseloss && __check_msegrad
•
•     md"""
• For this notebook we will only be using MSE, but we still introduce the abstract
LossFunction for the future. Below you will need to implement the 'loss'
${__check_complete(__check_mseloss)} function and the 'gradient'
${__check_complete(__check_msegrad)} function for MSE.
"""

```

## (b) Mean Squared Error

We will be implementing  $1/2$  MSE in the loss function.

$$c(\mathbf{w}) = \frac{1}{2n} \sum_i^n (f(\mathbf{x}_i) - y_i)^2$$

where  $f(\mathbf{x})$  is the prediction from the passed model.

`loss` (generic function with 1 method)

```

• function loss(lm::AbstractModel, mse::MSE, X, Y)
•     0.0
•     ##### BEGIN SOLUTION
•     sum=0
•     n=length(Y)
•     for i in 1:n
•         f_x=predict(lm,X[i,:])
•         sum+=(f_x-Y[i])^2
•     end
•     (sum/=2*n)
•
•     ##### END SOLUTION

```

## (c) ✓ Gradient of Mean Squared Error

You will implement the gradient of the MSE loss function  $c(w)$  in the gradient function with respect to  $w$ , returning a matrix of the same size of  $\text{lm.w}$ .

gradient (generic function with 1 method)

```
• function gradient(lm::AbstractModel, mse::MSE, X::Matrix, Y::Vector)
•     ∇w = zero(lm.w) # gradients should be the size of the weights
•
•     #### BEGIN SOLUTION
•     n=length(Y)
•     XT=X'
•     sum=zeros(lm.w)
•     for i in 1:n
•         sum.+=(X[i,:].* lm.w) .- Y[i]) .* X[i,:]
•     end
•     sum./=n
•     ∇w=sum
•     #### END SOLUTION
•     @assert size(∇w) == size(lm.w)
•     ∇w
```

## Optimizers

Below you will need to implement three optimizers

- Constant learning rate ✓
- Heuristic learning rate ✓
- AdaGrad ✓



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## (d) Constant Learning Rate

To update the weights for mini-batch gradient descent, we can use `ConstantLR` optimizer which updates the weights using a constant stepsize  $\eta$

$$w = w - \eta * g$$

where  $g$  is the gradient defined by the loss function.

Implement the `ConstantLR` optimizer.

```

• begin
•     _check_ConstantLR = let
•         lm = LinearModel(3, 1)
•         opt = ConstantLR(0.1)
•         lf = MSE()
•         X = ones(4, 3)
•         Y = [0.1, 0.2, 0.3, 0.4]
•         update!(lm, lf, opt, X, Y)
•         all(lm.w .== 0.025)
•     end
•     md"""
•     """
•     ## (d) $_check_complete(_check_ConstantLR)) Constant Learning Rate
•
•     To update the weights for mini-batch gradient descent, we can use 'ConstantLR'
•     optimizer which updates the weights using a constant stepsize 'η'
•
•     ```math
•     w = w - η*g
•     ````

•     where 'g' is the gradient defined by the loss function.
•
•     Implement the 'ConstantLR' optimizer.
•     """

```

---

```

• struct ConstantLR <: Optimizer
•     η::Float64

```

`update!` (generic function with 2 methods)

```

• function update!(lm::LinearModel,
•                 lf::LossFunction,
•                 opt::ConstantLR,
•                 x::Matrix,
•                 y::Vector)
•
•     g = gradient(lm, lf, x, y)
•
•     ##### BEGIN SOLUTION
•     lm.w.-=g.*opt.η
•     ##### END SOLUTION

```



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## (e) Heuristic Learning Rate

To update the weights for mini-batch gradient descent, we can use `HeuristicLR` optimizer which updates the weights using a stepsize  $\eta$  that is a function of the gradient. We define the stepsize at time  $t$  as:

$$\eta_t = (1 + \bar{g}_t)^{-1}$$

where  $\bar{g}_t$  is an accumulating gradient over time that uses the gradient  $g$  defined by the loss function. We use the following to compute  $\bar{g}_t$

$$\bar{g}_t = \bar{g}_{t-1} + \frac{1}{d} \sum_{j=1}^d |g_{t,j}|$$

Then, we use the update

$$w_t = w_{t-1} - \eta_t g_t$$

Implement the `HeuristicLR`.

```

• begin
•     _check_HeuristicLR = let
•         lm = LinearModel(3, 1)
•         opt = HeuristicLR()
•         lf = MSE()
•         X = ones(4, 3)
•         Y = [0.1, 0.2, 0.3, 0.4]
•         update!(lm, lf, opt, X, Y)
•         all(lm.w .≈ 0.11111111111111)
•     end
•     md"""
•     ### (e) $_check_complete(_check_HeuristicLR)) Heuristic Learning Rate
•
•     To update the weights for mini-batch gradient descent, we can use 'HeuristicLR' optimizer which updates the weights using a stepsize ' $\eta$ ' that is a function of the gradient. We define the stepsize at time $t$ as:
•
•     ```math
•     \eta_t = (1 + \bar{g}_t)^{-1}
•     ```
•     where $\bar{g}_t$ is an accumulating gradient over time that uses the gradient $g$ defined by the loss function. We use the following to compute $\bar{g}_t$:
•
•     ```math
•     \bar{g}_t = \bar{g}_{t-1} + \frac{1}{d} \sum_{j=1}^d |g_{t,j}|
•     ```
•
•     Then, we use the update
•
•     ```math
•     w_t = w_{t-1} - \eta_t g_t
•     ```
•     Implement the 'HeuristicLR'.
"""

```

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## HeuristicLR

```

• begin
•     mutable struct HeuristicLR <: Optimizer
•         g_bar::Float64
•     end
•     HeuristicLR() = HeuristicLR(1.0)

```

## update! (generic function with 3 methods)

```

• function update!(lm::LinearModel,
•                 lf::LossFunction,
•                 opt::HeuristicLR,
•                 x::Matrix,
•                 y::Vector)
•
•     g = gradient(lm, lf, x, y)
•     ##### BEGIN SOLUTION
•     d = length(g)
•     sum = 0.0
•     for i in 1:d
•         sum += abs(g[i])
•     end
•     sum /= d
•     opt.g_bar += sum
•     η = (1 + opt.g_bar)^(-1)
•     update = g .* η
•     (lm.w -= update)
•     ##### END SOLUTION

```

## (f) AdaGrad

AdaGrad is another technique for adapting the stepsize where we use a different stepsize for every element  $j$  in the weight vector.

To implement the AdaGrad optimizer, we use the following equations for each  $j$  from 1 to the length of the weight vector:

$$\bar{g}_{t,j} = \bar{g}_{t-1,j} + g_j^2$$

$$w_{t,j} = w_{t-1,j} - \frac{\eta}{\sqrt{\bar{g}_{t,j} + \epsilon}} g_j$$

where  $g$  is the gradient and  $g_j$  is the  $j$ th element of the gradient. These equations can be implemented without using a for loop, by using elementwise multiplication and division. If you are stuck on the syntax, feel free to use a for loop. Note that to get the elementwise squaring of gradient  $g$ , you would use `g.^2` and to get elementwise sqrt you would use `sqrt.(g)`.

Implement AdaGrad.

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```
• begin
•     mutable struct AdaGrad <: Optimizer
•         η::Float64 # step size
•         gbar::Matrix{Float64} # exponential decaying average
•         ε::Float64 #
•     end
•
•     AdaGrad(n) = AdaGrad(n, zeros(1, 1), 1e-5)
•     AdaGrad(n, lm::LinearModel) = AdaGrad(n, zero(lm.w), 1e-5)
•     AdaGrad(n, lm::AbstractModel) = AdaGrad(n, get_linear_model(model))
•     Base.copy(adagrad::AdaGrad) = AdaGrad(adagrad.n, zero(adagrad.gbar), adagrad.
```

update! (generic function with 4 methods)

```
•     function update!(lm::LinearModel,
•                     lf::LossFunction,
•                     opt::AdaGrad,
•                     x::Matrix,
•                     y::Vector)
•
•         g = gradient(lm, lf, x, y)
•         if size(g) !== size(opt.gbar) # need to make sure this is of the right shape.
•             opt.gbar = zero(g)
•         end
•
•         # update opt.gbar and lm.w
•         ##### BEGIN SOLUTION
•         n=length(lm.w)
•         opt.gbar.+=(g.^2)
•         lm.w.-= (opt.η./ sqrt.((opt.gbar .+ opt.ε) )) .* g
•         ##### END SOLUTION
•
•     end
```



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## (g) Polynomial Features

```

• begin
•
•     _check_Poly2 = let
•         pm = Polynomial2Model(2, 1)
•         rng = Random.MersenneTwister(1)
•         X = rand(rng, 3, 2)
•         Φ = get_features(pm, X)
•         Φ_true = [
•             1.0 0.23603334566204692 0.00790928339056074 0.05571174026441932
•             0.0018668546204633095 6.25567637522e-5;
•             1.0 0.34651701419196046 0.4886128300795012 0.12007404112451132
•             0.16931265897503248 0.2387424977182995;
•             1.0 0.3127069683360675 0.21096820215853596 0.09778564804593431
•             0.06597122691230639 0.04450758232200489]
•         check_1 = check_elements(Φ_true, Φ) #all(Φ .≈ Φ_true)
•
•         pm = Polynomial2Model(2, 1; ignore_first=true)
•         X_bias = ones(size(X, 1), size(X, 2) + 1)
•         X_bias[:, 2:end] .= X
•         Φ = get_features(pm, X_bias)
•         check_2 = check_elements(Φ_true, Φ) #all(Φ .≈ Φ_true)
•         check_1 && check_2
•     end
•
•     HTML("<h2 id=poly> (g) $_check_complete(_check_Poly2) Polynomial Features <br> <hr>")
• end

```

  
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Now, we will implement Polynomial Model which uses the linear model on non-linear features. To do so, we apply a polynomial transformation to our data to create new polynomial features. For  $d$  inputs with a polynomial of size  $p$ , the number of features is  $m = \binom{d+p}{p}$ , giving polynomial func

$$f(\mathbf{x}) = \sum_{j=1}^m w_j \phi_j(\mathbf{x}) = \boldsymbol{\phi}(\mathbf{x})^\top \mathbf{w}$$

We simply apply this transformation to every data point  $\mathbf{x}_i$  to get the new dataset  $\{(\boldsymbol{\phi}(\mathbf{x}_i), y_i)\}$

Implement the polynomial feature transformation for  $p = 2$  degrees in the function `get_features`

```
get_linear_model (generic function with 1 method)
```

```
• begin
•     struct Polynomial2Model <: AbstractModel
•         model::LinearModel
•         ignore_first::Bool
•     end
•     Polynomial2Model(in, out=1; ignore_first=false) = if ignore_first
•         in = in - 1
•         Polynomial2Model(LinearModel(1 + in + Int(in*(in+1)/2)), out), ignore_firs
•     else
•         Polynomial2Model(LinearModel(1 + in + Int(in*(in+1)/2)), out), ignore_firs
•     end
•     Base.copy(lm::Polynomial2Model) = Polynomial2Model(copy(lm.model), lm.ignore_
•     get_linear_model(lm::Polynomial2Model) = lm.model
• 
• end
```

```
predict (generic function with 6 methods)
```



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```
get_features (generic function with 2 methods)
```

```
•   function get_features(pm::Polynomial2Model, _X::AbstractMatrix)
•
•     # If _X already has a bias remove it.
•     X = if pm.ignore_first
•           _X[:, 2:end]
•     else
•       _X
•     end
•
•     d = size(X, 2)
•     N = size(X, 1)
•     num_features = 1 + # Bias bit
•                   d + # p = 1
•                   Int(d*(d+1)/2) # combinations (i.e. x_i*x_j)
•
•     Φ = zeros(N, num_features)
•
•     # Construct Φ
•     ##### BEGIN SOLUTION
•     i1=zeros(N)
•     i2=zeros(N)
•     i3=zeros(N)
•     i4=zeros(N)
•     i5=zeros(N)
•     i6=zeros(N)
•     for i in 1:N
•       i1[i]=1
•       i2[i]=X[i,1]
•       i3[i]=X[i,2]
•       i4[i]=i2[i]*i3[i]
•       i5[i]=i2[i]^2
•       i6[i]=i3[i]^2
•     end
•     Φ[:,1]=i1
•     Φ[:,2]=i2
•     Φ[:,3]=i3
•     Φ[:,4]=i4
•     Φ[:,5]=i5
•     Φ[:,6]=i6
•
•     ##### END SOLUTION
•
•     Φ
```

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# Evaluating Models

In the following section, we provide a few helper functions and structs to make evaluating methods straightforward. The abstract type `LearningProblem` with children `GDLearningProblem` and `OLSLearningProblem` are used to construct a learning problem. You will notice these structs contain all the information needed to `train!` a model. We also provide the `run` and `run!` functions. These will update the transform according to the provided data and train the model. `run` does this with a copy of the learning problem, while `run!` does this inplace.

## Main.workspace2.GDLearningProblem

```
• """
•     GDLearningProblem
•
•     This is a struct for keeping a the necessary gradient descent learning setting
•     components together.
• """
• struct GDLearningProblem{M<:AbstractModel, O<:Optimizer, LF<:LossFunction} <:
•     LearningProblem
•     gd::MiniBatchGD
•     model)::M
•     opt::O
•     loss::LF
•
•     Base.copy(lp::GDLearningProblem) =
run! (generic function with 1 method)
•     function run!(lp::GDLearningProblem, X, Y, num_epochs)
•         update_transform!(lp.model, X, Y)
•         train!(lp.gd, lp.model, lp.loss, lp.opt, X, Y, num_epochs)
run (generic function with 1 method)
•     function run(lp::LearningProblem, args...)
•         cp_lp = copy(lp)
•         L = run!(cp_lp, args...)
•         return cp_lp, L
```



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## Run Experiment

Below are the helper functions for running an experiment.

### Main.workspace2.run\_experiment

# Experiments

In this section, we will run three experiments on the different algorithms we implemented above. We provide the data in the Data section, and then follow with the three experiments and their descriptions. You will need to analyze and understand the three experiments for the written portion of this assignment.

# Data

This section creates the datasets we will use in our comparisons. Feel free to play with them in `let` blocks.

```
Main.workspace2.splitdataframe
```

```
unit_normalize_columns! (generic function with 1 method)
• function unit_normalize_columns!(df::DataFrame)
•     for name in names(df)
•         mn, mx = minimum(df[:, name]), maximum(df[:, name])
•         df[:, name] .= (df[:, name] .- mn) ./ (mx - mn)
•     end
• end
```

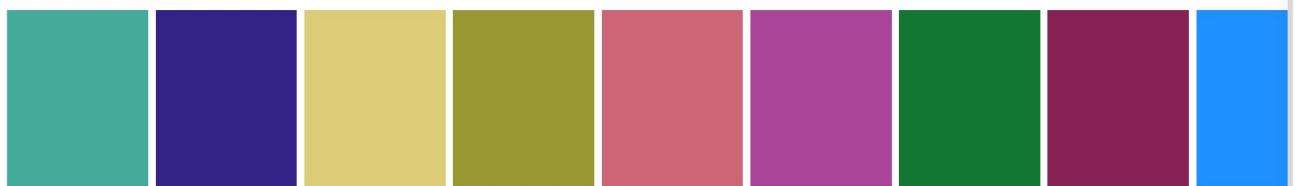
## Admissions Dataset

```
admissions_data = let
    data = CSV.read("data/admission.csv", DataFrame, delim=',', ignorerepeated=true)
    data[:, 2:end]
    data[:, 1:end-1] = unit_normalize_columns!(data[:, 1:end-1])
    data
```

## Plotting Utilities

Below we define two plotting helper functions for using PlotlyJS. You can ignore these if you want. We use them below to compare the algorithms.

```
color_scheme =
```



```
Main.workspace2.plot_results
```



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## (h) Comparing Linear Regression and Polynomial Regression

We will compare the linear regression and polynomial regression with  $p = 2$  using the a simulated data set and the admissions dataset.

To run these experiments use

This first experiment uses a simulated training set. For a given input  $\mathbf{x} \in [0.0, 1.0]^5$ , the nonlinear function that defines  $\mathbb{E}[Y|\mathbf{x}]$  is

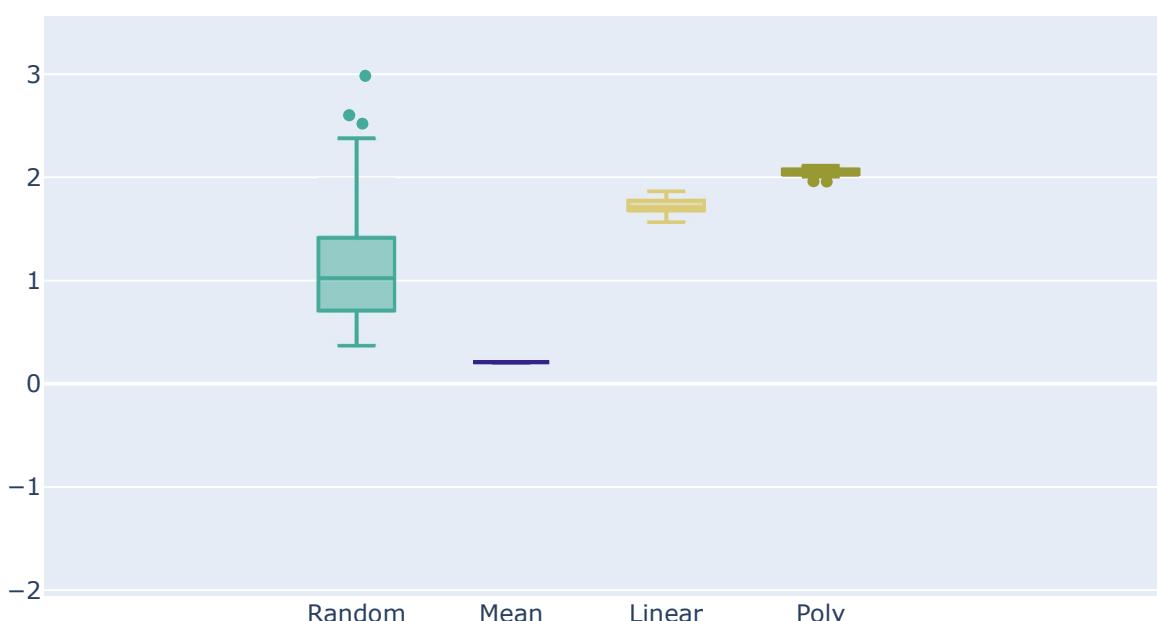
$$f(\mathbf{x}) = \sin(\pi * \mathbf{x}[1] * \mathbf{x}[2]^2) + \cos(\pi * \mathbf{x}[3]^3) + \mathbf{x}[5] * \sin(\pi * \mathbf{x}[4]^4) + 0.001 * \text{randn}()$$

To get the target, we use

$$y = f(\mathbf{x}) + 0.001 * \text{randn}()$$

namely we add a small amount of Gaussian noise. We compare a linear regression, polynomial regression and two baselines.

Synthetic Data



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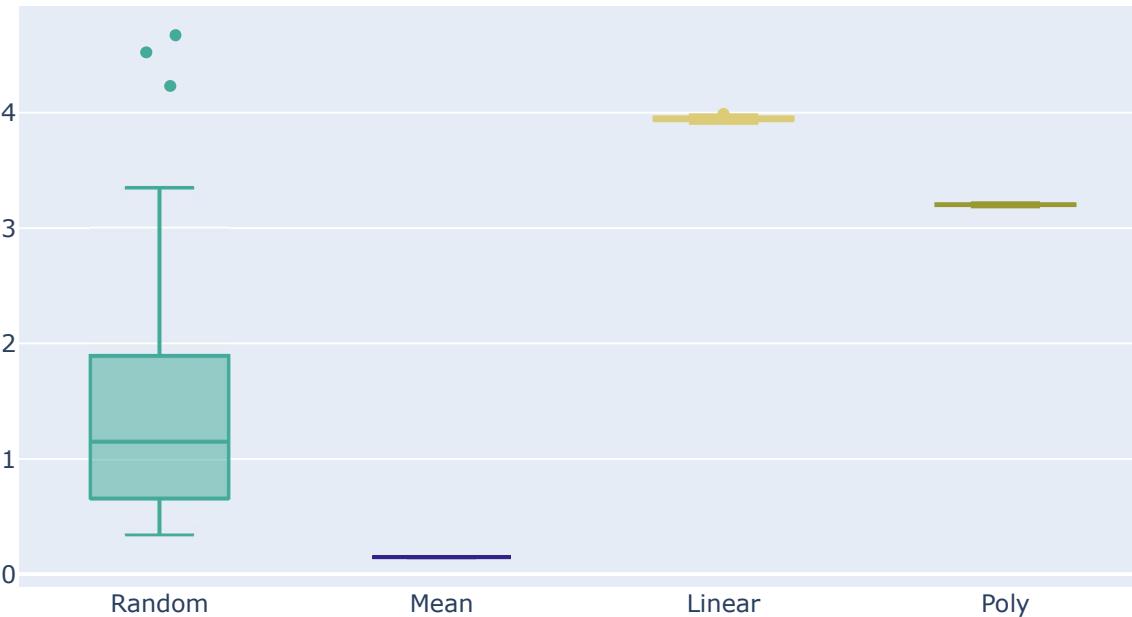
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The following experiment uses the addmisions dataset, which you should report. **You can get the average error and standard error to report from the plot or from below the plot and experiment code.**



## (i) Stepsize Adaptation

We will compare the different stepsize algorithms on a subset of the [Admissions dataset](#). From this dataset we will be predicting the likelihood of admission.

To run this experiment click

**You can get the average error and standard error to report from the plot or from the terminal where you ran this notebook or from below the plot and experiment code.**



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## Stesize Algorithm Comparisons

