Computing Science (CMPUT) 455

Search, Knowledge, and Simulations

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455 Today - Lecture 3

Topics:

- Introduction to decision-making
- Optimal decision-making
- Some models of human decision-making

Coursework

- TA Office hours began this week
    - See website for times and meeting links
- Start coding Assignment 1
- Quiz 2, review Reinforcement Renaissance
- Read Heingartner, Maybe We Should Leave That Up to the Computer.
- Activities for Lecture 3

Lecture Topics (1)

- What is decision-making?
- Models of the world, reward, and utility
- How to evaluate alternatives in decision-making?
- Exact evaluation, expected values

Lecture Topics (2)

- How do humans make decisions?
- Heuristics, Bounded Rationality, and Satisficing
- What is the “right” decision for a program to make?
- Kahneman and Tversky experiments, criticism of utility theory

Decision Making in Humans and Machines

Image source:

`blogs-images.forbes.com/mikemyatt/files/2012/11/decision-making-processes1.jpg`

Decision making is studied in many fields

- Business
- Psychology
- Advertising
- Computing Science
- AI
- ...

Decision Making in Humans and Machines (2)

- Decision making in politics can have far-reaching consequences (war, peace, prosperity, ...)
- Decision making is big business -
  what to buy, sell, produce,...
- Decision making is studied by many people in many different ways
  - "Common sense"
  - Academic and industry research
  - Popular "how to" books
- We make decisions every day. How and why?

Decision Making in Humans and Machines (3)

Some big questions:

- Can we make better decisions?
- Can we understand and influence other people's decisions?
- Can we teach decision-making -
  to children, students, employees?
- Can we model decision-making in a computer program?

Decision Theory (Theory of Choice)

Two main strands of research:

- *Normative* decision theory
  - Analyze decision problem
  - Tell user what is best action
  - Example:
    "You should play e4, it is the best move"

Decision Theory (Theory of Choice)

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  - Analyze decision problem
  - Tell user what is best action
  - Example:
    "You should play e4, it is the best move"
- *Descriptive* decision theory
  - Analyze how real agents (people, programs?) make decisions
  - Example:
    "In our user study, 55% played move d4, 32% played e4, and 13% played some other move.
    The reasons are: ..."

Game Theory and Expected Value

- Classical game theory (e.g. von Neumann and Morgenstern 1947)
- Selfish players, try to maximize their money (*)
- Simplest case: two player zero sum games
- Zero sum - my win is your loss
- Actions can involve random outcomes, but with known probabilities
- Goal: maximize *expected value*

Expected Value

- Concept from probability theory
- Random event, with n different outcomes
- Each outcome evaluated by a number value v_i (reward, money, ...)
- Probability p_i of each outcome known
  - $\sum_{i=1}^{n} p_i = 1$
- Expected value (EV):
  - $\sum_{i=1}^{n} p_i v_i$

Expected Value Example

- Throw a six-sided fair die
- Value = the number rolled
- $n = 6$
- $p_1 = p_2 = p_3 = p_4 = p_5 = p_6 = 1/6$
- $v_1 = 1, v_2 = 2, v_3 = 3, v_4 = 4, v_5 = 5, v_6 = 6$
- Expected value
- $\sum_{i=1}^{n} p_i v_i = 1/6(1 + 2 + 3 + 4 + 5 + 6) = 21/6 = 3.5$

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- Question: what is the EV when rolling two dice?

Example for Expected Value - Fold or Bid?

- Assume you play a simple card game, and all you care about is maximizing money
- Assume you have two possible actions, fold or bid
- Fold, you lose \$1 for sure
- Bid, you either win \$5 or lose \$3

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- Question: What *additional information* do you need to answer that question?
- The *probability of winning* if you bid

Fold or Bid?

- Set p to be your probability of winning if you bid
- Fold: your value is -1.
- Bid: your value is
  - +5 with probability p
  - -3 with probability $1 - p$
- Which action is better *in expectation*?

Fold or Bid? - Analysis

- Bid: +5 with probability p
- Bid: -3 with probability $1 - p$
- Expected value after bid: $5p + (-3)(1 - p) = 8p - 3$
- When is this better, worse, or equal to -1 (Fold)?
- Depends on p, compare $8p - 3$ with -1
- When are they equal? Solve equation $8p - 3 = -1$
- Solution $p = 1/4$

Fold or Bid? - Solution

- When $p = 1/4$, you are *indifferent*
  - *Expected value (EV)* of both actions, fold and bid, is the same
  - Confirm EV for bid:

$$5 \times \frac{1}{4} + (-1) \times (1 - \frac{1}{4}) = \frac{5}{4} - \frac{9}{4} = -1$$

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- When p grows, $8p - 3$ also grows
- For $p > 1/4$, bidding is better
- For $p < 1/4$, folding is better

Fold or Bid? - Example

Example: when $p = 1/3$, you want to bid

- EV(fold) = -1
- EV(bid) = $1/3 \times 5 + 2/3 \times -3 = 5/3 - 6/3 = -1/3$, better than folding

Fold or Bid? - Scaling Up

- The analysis before was probably reasonable for most people, describes the “most rational” choice
- What happens if we scale it up?
    - Instead of -1, +5, -3 dollars,
      play with -10000, 50000, -30000

Fold or Bid? - Scaling Up

- The analysis before was probably reasonable for most people, describes the “most rational” choice
- What happens if we scale it up?
  - Instead of -1, +5, -3 dollars, play with -10000, 50000, -30000
- Question: Is optimizing expected value the “most rational” strategy *now*?

Fold or Bid? - Scaling Up

- Some people would hate to lose \$10000 without even trying
- Some people can lose \$10000 without horrible consequences, but not \$30000
- Some people would value winning \$50000 very highly
- Our **utility** of money does not always scale linearly with the amount of money
- It depends on how it affects our life

St. Petersburg Paradox (Nicolas Bernoulli, 1713)

A paradox about expected value vs. actual behavior of people

- Play a game against the bank:
- The bank puts $2 in the pot originally
- Each round you flip a coin
- If head, the bank doubles the pot
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- Q1: What is your expected value for this game?
- Q2: How much would you pay to be allowed to play this game?

Let's Play St. Petersburg Paradox...

- Start with $2
- Head - double
- Tail - game over
- See Python code `petersburg.py`, `petersburg2.py`
- Short demo now.

St. Petersburg Paradox Analysis

- probability 1/2, win $2 - tail
- probability 1/4, win $4 - head, tail
- probability 1/8, win $8 - head, head, tail
- probability 1/16, win $16 - head, head, head, tail
- ...
- Expected value of your win
  $1/2 \times 2$
  $+1/4 \times 4$
  $+...$
  $= 1 + 1 + ... = \infty$
- How much would *you* pay to play?

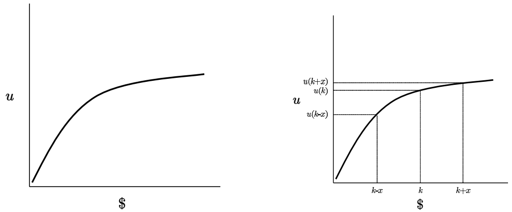
St. Petersburg Paradox vs. Reality

- The expected value is infinity but:
- It includes mostly extremely unlikely events
- Example:
  - Chance of 1/1,024 to win $1,024
  - Chance of 1/1,048,576 to win $1,048,576
  - Chance of 1/1,099,511,627,776 to win $1,099,511,627,776
- How to evaluate those in practice?

Utility

- Utility is a concept from economics
- Measures satisfaction of a consumer with an outcome (e.g., receiving a specific good)
- Utility of a good? It determines the price that a consumer is willing to pay
- But - what is the utility of money?
- Q: Is twice the money twice as desirable?
- In general, no.

Utility Function and Risk



(c) Risk aversion (d) Risk aversion: fair lottery

Image source: (Shoham & Leyton-Brown, 2008)

- *Utility function for money* is a mapping:
  - From: Monetary outcome
  - To: utility scale reflecting personal preferences
- Linked with types of behavior:
  - Risk-averse (conservative)
    - Utility function grows slower than linear
  - Risk-neutral
  - Risk-seeking (e.g. playing lottery)
    - Utility function grows faster than linear

Marginal Utility

Image source:

`http://s3.crackedcdn.com/articleimages/dan/rags/gates3.jpg`

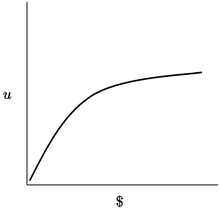
- Marginal utility: increase in consumer satisfaction from having one unit more of a good
- Example: what is the value of having $100 more?
  - Very high if you are broke
  - Very low if you are Bill Gates
  - Marginal utility of money generally decreases with wealth

Car Example

Image source: `shedsunlimited.net`

Another example:

- What is the marginal utility of owning one more car?
- High if you have no car
- Much lower if you already own 3



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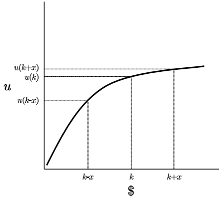

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Maximum Expected Utility (MEU)

- Principle of maximum expected utility (Ramsey; von Neumann / Morgenstern):
- Choose action which maximizes your expected utility:
- With known probabilities, we can compute expected utilities just as we computed expected value
- Just replace the values with the utilities in the computation
- Expected Utility Hypothesis: under certain conditions, people behave in that way...
- Q: Do you think people *actually* act like this?

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- Q: Do you think people *actually* act like this?
- Q: Do you think expected utility maximization is necessarily the *best* (most rational) way to act? Why or why not?

Estimating Probabilities, Risk and Insurance

- Insurance companies charge an insurance fee...
- ... in return for promising to reimburse you for a low-probability large loss
- How to come up with a “fair” insurance premium?
- Need to know all the risks - bad things that could happen - and their probabilities
- Typically, only specific types of risk are covered by a policy
  - Home insurances usually exclude war, water damage, some types of natural and human-made disasters, . . .
- Impossible to estimate singular events, “black swans”

Heuristics

Image source:

`http://archimedespalimpsest.`

`org/images/kaltoon/4.php`

- From Greek word “find” or “discover” (wikipedia)
- Any practical problem solving method
- “Mental shortcut”, rule of thumb, educated guess, common sense rule, a rough model, using a similar case for guidance, ...

Heuristic vs Exact

Image source:

`http://archimedespalimpsest.org/images/kaltoon/4.php`

- Opposites of heuristic approach: exact solution, exhaustive analysis, precise theory
- We rely on heuristics all the time
- Most of life is too complex to "solve exactly"
- Heuristic decision-making and exact methods are both used in computer programs
- Example of modern machine-learned heuristic: neural network

Heuristics in Computing Science

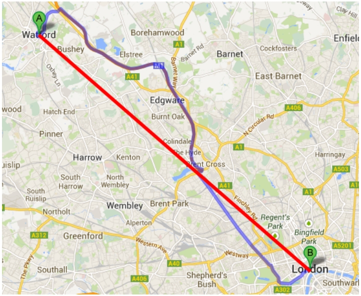

Image source:

`https://stackoverflow.com/`

`questions/17594924/`

- Typical heuristic in CS:
  solution to simplified problem
- Example problem:
  find shortest path from A to B
- Real solution:
  follow roads, avoid obstacles
- Heuristic: straight-line distance

Polya - How to Solve It

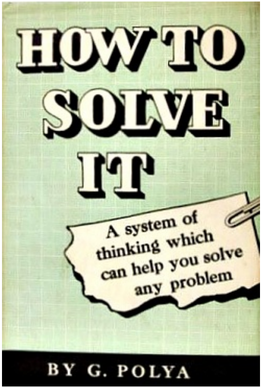

Image source:

`en.wikipedia.org/wiki/File:`

`HowToSolveIt.jpg`

- A system for human problem-solving
  - Classic book by mathematician George Polya
  - Published in 1945
  - Still popular and influential
- Four principles
- Large set of heuristics

How to Solve It - Four Principles

1. Understand the problem
2. Devise a plan
    - Find connection between data and unknown
3. Carry out the plan
4. Looking back
    - Examine the solution, review/extend

- Polya's book focuses on problem-solving mathematics
- We "translate" some ideas for CS

Principle 1: Understand the Problem

Format: *Polya's text in italic* - comments below

- *What are the data?*
  - What is given? What is the input?
- *What is the unknown?*
  - What is the output?
- *What is the condition?*
  - What are the requirements/constraints for the solution?

Principle 1: Understand the Problem (continued)

- *Draw a figure. Introduce suitable notation.*
  - Draw or write down the important concepts in the problem and their relations.
- *Separate the various parts of the condition*
  - Find smaller parts, functions that make up the required solution

Principle 2: Devise a Plan

Polya gives a list of general approaches to try. Examples:

- *Find connection between data and unknown*
  - How do you compute the output as a function of the input?
- *Have you seen it before? Do you know a related problem?*
  - Can you re-use the previous solution?
- *Could you restate the problem?*
  - Is there a different way to write it,
    which is more similar to things you know?

Principle 2: Devise a Plan (continued)

- *If you cannot solve the proposed problem try to solve first some related problem*
  - Solve a special case
  - Solve a concrete example
  - Drop the complicated parts for now

- *Did you use all the data?*
  - Are you using everything you know?
  - The whole specification?
  - All properties of the input?

Principle 3: Carry out the Plan

- *Carrying out your plan of the solution check each step*
  - Write functions to implement your program, test each one separately.
  - Use unit tests to help verify that each function works as expected, at least on the test cases.

- *Can you see clearly that the step is correct?*
  Can you prove that it is correct?
  - Use assertions in your code to make sure input and output are as you expect. For really tricky code, you can even try a formal proof with pre- and postconditions and loop invariants (Cmput 204 stuff).

Principle 4: Looking Back

- *Can you check the result?*
  - Examine the solution
  - Review the problem in all details, check with your solution

- *Can you use the result, or the method,*
  for some other problem?
  - Refactor code, simplify functions, clean up
  - Extend or generalize functions for other problems
  - Organize into modules

How to Solve It - Heuristics

- Dictionary of heuristics is largest part of book
- Over 60 entries
- Some are specific to mathematical problem solving
- Most are generally useful
- Next two slides show examples

Polya - Example of Heuristic

Auxiliary problem

- Find an easier problem that will help solve the original
- Example: useful helper function
- Solve problem in several small steps, each implemented in a simpler function

Polya - Example of Heuristic

Decomposing and recombining

- Break a big problem into parts
- Find out which parts are important
- Solve parts
- Put together solutions of parts
- Examples:
  - Separate UI from engine
  - Floodfill - separate scan of full board from what to do in each area
  - Separate tree search algorithm from details of what to do in each node

Herb Simon and Bounded Rationality

Image source:

http://www.cs.cmu.edu/simon/

- Herb Simon (1916 - 2001)
  - A founder of AI (and other disciplines)
  - Nobel-prize winner
  - Professor at Carnegie-Mellon
- Original background: decision-making in business, economics
- Criticized "perfect rationality" assumption of previous theorists
- Developed influential concept of "Bounded Rationality"

Activity: Watch Herb Simon Videos

See activities course page

- Video 1: The Limits or Bounds of Rationality
- Video 2: What is Intuition?
- Optional - read more about Herb Simon: `https://en.wikipedia.org/wiki/Herbert_A._Simon`

Decision-making and Optimization

- Mathematical optimization:
  find the **best possible** solution to a problem

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- Much harder example:
  what should my company produce to maximize its profit?
- Even harder:
  what should the company produce to make the most people the happiest?

Decision-making vs solving an Optimization Problem

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- Answer: sometimes...

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Decision-making vs solving an Optimization Problem

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- Answer: sometimes...
- Yes:
  - The decision concerns optimizing some quantity: money, grade average, "reward"
- No:
  - The decision involves many factors that are hard to compare
- Example:
  study more, or get more sleep?

Can We Make Perfect Decisions?

- Herb Simon:
  - Most often, no.
- Why?
- Humans (and computers) have:
  - Limited memory
  - Limited time to make a decision
  - Incomplete, or wrong, information about actions and results
  - Limited powers of logic, deduction, lookahead
  - Limited imagination to come up with new approaches
  - Limited everything ...

Bounded Rationality - Discussion

- Perfect decisions: often not possible in practice
- How can we act well,
  while acknowledging our limitations?
- How can we use what we know,
  and even what we don’t know?
- How do we deal with multiple, conflicting goals?
- When should we use heuristics,
  and when a more systematic search?
- What is the “best” thing to do, given our limitations?
  Is that even well-defined?

Exact vs Good Enough, Satisficing

- Humans use heuristics as shortcuts
- Concept of satisficing (Herb Simon)
- Trying to optimize is often too hard
- More reasonable:
  - Define criteria for "good enough"
- A satisficing solution is one that fulfills these criteria
- Example in games:
  play a "good" or "strong" move,
  even if we cannot prove it is the best

Herb Simon on Satisficing

"Decision makers can satisfice
either by finding optimum solutions
for a simplified world,
or
by finding satisfactory solutions
for a more realistic world.
Neither approach, in general, dominates the other,
and both have continued to co-exist..."

Optimum Solutions for a Simplified World

Image source: `www.simpleitsolutions.com`

- Example: what is the cost of buying a new computer?
- Simple answer: look at the price tag
- More complete answer: add tax, cost of new software, cost of time for upgrades, electricity, insurance, carrying bag, ...
- In practice, we ignore or roughly summarize many of these details and make a decision for a simplified problem
- Some costs are not known anyway, e.g. future costs of electricity, repairs, ...

Satisfactory solutions for a more realistic world

Image source:

`thehealthysubstitute.`

`wordpress.com`

- Example: what to eat for lunch?
- A myriad of choices
- Many small or large variations are possible (seasoning, extras, ...)
- In real life, we only consider a small number of choices
- We (usually) satisfice, not optimize

Comment - Model vs Direct Observation

- Remember Herb Simon's quote:

  Decision makers can satisfice either by finding optimum solutions for a simplified world, or by finding satisfactory solutions for a more realistic world.

- Two approaches to reasoning for decision-making:
- Reasoning based on a **model** of the world
- Reasoning from **direct observations** of the world
- Big topic in Reinforcement Learning
- In games, we have a *perfect* model. But that is the exception, not true for real world

Kahneman and Tversky

Image source: `www.vanityfair.com/news/2016/11/`

- Very influential psychologists
- Kahneman won Nobel prize in economics (Tversky died before)
- Humans have systematic *cognitive biases*
- Most are averse to loss and ambiguity, “losses loom larger than gains”
- Activity: Watch Daniel Kahneman Videos

Kahneman and Tversky - Anchoring

Image source: `https://www.jeremysaid.com/blog/anchoring-effect-power-conversion-optimization/`

- People tend to “anchor” on first impressions (Regular $48...)
- Later decisions made relative to this, not in absolute terms
- People focus more on *changes* in their utility than on *absolute* utilities

Anchoring - Another Car Example

Scenario A

- Monday, I offer to sell you my car for $30000.
- Tuesday, I offer it to you for $20000.

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Scenario B (same car)

- Monday, I offer to sell you my car for \$10000.
- Tuesday, I offer it to you for \$20000.

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Scenario B (same car)

- Monday, I offer to sell you my car for \$10000.
- Tuesday, I offer it to you for \$20000.

Question: In which scenario are you more likely to accept the offer on Tuesday?

Summary

- Quick tour of theories and experiments in human decision-making
- How do we make decisions?
- Limits to making “perfect” decisions
- Bounded rationality and satisficing
- Expected value, expected utility
- Cognitive biases
- Next time:
  - Formal models of decision-making in sequential games
  - Representing games